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**Acceleration Techniques for Vehicle Routing Heuristics**

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# FINAL PERFORMANCE REPORT

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Acceleration Techniques for Vehicle Routing Heuristics

Research team:

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## Table of Contents

In this report, we present the research we performed during the activity period of Grant Number FA9550-17-1-0234 and the achieved results. Due to COVID-19 Pandemy several activities planned for this year and, in particular, those related to the dissemination of the achieved results had to be cancelled and postponed. For this reason the research team has requested a no-cost extension of the grant that has been accepted.

The report is organized as follows. In Section 1, we give a summary of the main contributions. Section 2 describes the studied problems, while Section 3 reports the proposed solving methods. The results we obtained are reported in Section 4. Finally, we conclude the report in Section 5 and give a summary of the produced publications in Section 6.

## 1 Summary

During the project, we worked on the following problems:

- (Very Large Scale) Capacitated Vehicle Routing Problem
- Hybridization of Machine Learning and Optimization for the solution of Vehicle Routing Problems.
- A variant of VRP with profits and service level requirements.

All such problems fall into the category of Vehicle Routing Problems (VRPs). In particular, the first problem is one the most studied combinatorial optimization problem proposed in the Operations Research area. It is the natural generalization of the well-known Traveling Salesman Problem which is known to be NP-Hard, hence computationally difficult. For this problem, we studied various acceleration techniques, mainly based on Static Move Descriptors data structure which greatly enhance the computational effectiveness of local search techniques. We also proposed a fast heuristic algorithm that uses the combination of two acceleration techniques, namely the Static Move Descriptors data structure and the Granular Local Search within an Iterated Local Search framework. The combination of these two techniques allow us to obtain good

solutions within a very limited amount of time. Several new algorithmic components such as localized and history-based shaking have been developed and analyzed. The resulting algorithm shows a linear growth of the computing time with respect to problem instances and proved able to improve most of the existing large-scale instances with up to 30000 vertices to be served.

A second line of research, initiated during the previous year and continued in the last one, is devoted to the enhancement of the performance of VRP heuristics through the use of guidance mechanisms based on machine learning methodologies. The hybridization of machine learning and optimization algorithms is an extremely hot research area and we have obtained very interesting results in this field. The research has been slowed down due to COVID pandemy and waiting to finalize the algorithm for CVRP previously described.

The last problem is instead inspired by a problem arising in cash logistics. The problem is called the Capacitated Routing Problem with Profits and Service Level Requirements (CRPPSLR), and extends the class of Routing Problems with Profits by considering customers requesting deliveries to their (possibly multiple) service points. Moreover, each customer imposes a service level requirement specifying a minimum-acceptable bound on the fraction of its service points being delivered. A customer-specific financial penalty is incurred by the logistics service provider when this requirement is not met.

## **2 Problem description**

The vehicle routing problem (VRP) is one of the most studied combinatorial optimization problems which plays a central role in logistics and transportation. Since its definition, occurred more than fifty years ago, several exact methods and hundreds of heuristic optimization algorithms were proposed to tackle different problem variants. Even though real-world applications, such as waste collection and parcel distribution, can require up to tens of thousands of customer visits, almost all research so far has been targeted at solving problems with no more than a few hundreds of customers. Only recently, some researchers started working on the so called very-large-scale instances developing the first interesting methods to effectively tackle them. Due to the instance size (involving thousands or tens of thousands of customers), exact methods become computationally prohibitive and even standard heuristic methods cannot be used to effectively tackle those instances, despite the increasing processing capabilities of modern processors and parallel work decomposition. In fact, complete exploration of even simple neighborhoods with a polynomial cardinality cannot be computationally afforded.

Despite its importance in the vehicle routing area, most of the algorithms available in the literature still focus on relatively small CVRP instances. When scaling to very large-scale instances the design of each algorithm components becomes of crucial importance. Moreover, the employment of acceleration techniques is mandatory to design scalable algorithms that still retain their effectiveness.

The CRPPSLR consists in finding vehicle routes maximizing the difference between the collected revenues and the incurred transportation and penalty costs in such a way that vehicle capacity and route duration constraints are met. A fleet of homogeneous vehicles is available for serving the customers.

### 3 Methods, Assumptions and Procedures

For each of the described problems, we propose either a fast heuristic algorithm or an exact method which can be used to assess the performance of future heuristics.

In particular, for CVRP during the last period we finalized the study of several acceleration techniques started during the previous year, which are summarized in the following.

- We introduced a more fine-grained and generalized way of handling move generators in dynamic granular neighborhoods. This approach, called *vertex-wise management* of move generators, allows to control move generators involving each particular vertex thus enabling the designer to specifically intensify the optimization process on very focused and confined areas of the solution (e.g., those parts of the solution that were not improved after several attempts).
- We introduced a *selective caching* of vertices to identify solution's parts that are more likely to be relevant for forthcoming decisions because changed more recently.

We refined the integration of Granular Neighborhoods and Static Move Descriptors in the light of the vertex-wise management of move generators and the selective caching of vertices, making the exploration of a local search neighborhood extremely fast.

Those techniques are embedded into a heuristic based on the Iterated Local Search, called FILO, making use of an innovative shaking strategy that iteratively tailors its strength according to the instance and solution under examination.

The resulting paper:

- L. Accorsi and D. Vigo “A Fast and Scalable Heuristic for the Solution of Large-Scale Capacitated Vehicle Routing Problems” Research Report DEI-OR-2020-2

is currently under proof reading and will be soon submitted to Transportation Science.

As to the research line which studies how heuristic algorithms can be guided by exploiting past knowledge (typically, encoded into trained machine learning models). In this area, we are following two distinct sub-topics. The first one studies whether being able to identify the next most promising local search operator in a Variable Neighborhood Descent context can boost the effectiveness and efficiency of local search-based methods. The second one deals with the estimation of the optimal solution value of an instance that can be employed to prematurely terminate an algorithm execution when satisfying enough solutions have been found.

Both topics require the analysis of several instances and solutions to extract features and characteristics to be used during the training and inference of the learning models. Given the successful results of the above described and finalized algorithm, it can be used to generate a high-quality dataset of solutions.

The main challenge in those approaches consists in being able to effectively use those machine learning models during a more classical algorithm execution. In fact, despite several attempts have

been made in the literature to hybridize heuristic approaches with learning techniques. Most of them fail to be competitive with real state-of-the-art (standard) algorithms. In fact, those models need to analyze the current solution by extracting some features to be used during the inference pass. However, this process, if executed within the core of an iterative algorithm, may come with a significant computational overhead that may render useless an accurate prediction. We are thus studying whether a simple image-like representation describing a solution that can be computed in linear time can represent with a highly enough expressive power the original solution.

For the CRPPSLR we designed a branch-and-cut algorithm and evaluate the usefulness of valid inequalities that have been effectively used for the capacitated vehicle routing problem and, more recently, for other routing problems with profits.

## 4 Results

The proposed models and methods have been tested on benchmark instances from the literature or on real-world scenarios. In this section, we summarize the main achievements we obtained.

For the CVRP the algorithm, called FILO, has been used to solve literature instances with up to 30000 customers and contrarily to several existing competitors, it exhibits a computing time which is linear in the number of customers. In fact, it finds solutions better than those found by existing state-of-the-art algorithms in a few minutes of computing times compared to the several hundred minutes typically required by competing methods.

As to the CRPPSLR, a real-life case study taken from the cash supply chain in the Netherlands highlights the relevance of the problem under consideration. Computational results illustrate the performance of the proposed solution approach under different input parameter settings for the synthetic instances. For instances of real-life problems, we distinguish between coin and banknote distribution, as vehicle capacities only matter when considering the former. Finally, we report on the effectiveness of the valid inequalities in closing the optimality gap at the root node for both the synthetic and the real-life instances and conclude with a sensitivity analysis on the most significant input parameters of our model.

The work performed during the project led to the preparation of the following publications:

- L. Accorsi and D. Vigo. A Fast and Scalable Heuristic for the Solution of Large-Scale Capacitated Vehicle Routing Problem. Research Report DEI-OR-2020-2, to be submitted for publication to *Transportation Science*, 2020.
- C. Orlics, D. Laganá, W. Dullaert, D. Vigo. Distribution with Quality of Service Considerations: The Capacitated Routing Problem with Profits and Service Level Requirements. *Omega*, volume 93, 2020.

In addition, previously submitted papers (reported in our previous report) have been now published:

- S. Dabia, D. Lai, D. Vigo. An Exact Algorithm for a Rich Vehicle Routing Problem with Private Fleet and Common Carrier. *Transportation Science*, volume 53, issue 4, pages 986-1000, 2019.
- A. Baller, S. Dabia, W. Dullaert, D. Vigo. The Vehicle Routing Problem with Partial Outsourcing. *Transportation Science*, volume 54, issue 4, pages 1034-1052, 2020.

## **5 Conclusions and further work**

The proposed methods show their effectiveness and novelty, and will be presented in international conferences and submitted to reputed scientific journals. We will continue our investigation on hybridization techniques to improve the developed algorithms and will consolidate in publications the results achieved so far. As to the CVRP, given the effectiveness and scalability of the method we proposed, possible future research directions might consider tackling even larger instances in a distributed environment by applying the above described concepts on top of an instance decomposition.

## 6 List of publications produced during the project

1. O. Beek, B. Raa, W. Dullaert, D. Vigo, An Efficient Implementation of a Static Move Descriptor-based Local Search Heuristic. *Computers & Operations Research*, volume 94, pages 1-10, 2018.
2. S. Dabia, D. Lai, D. Vigo. An Exact Algorithm for a Rich Vehicle Routing Problem with Private Fleet and Common Carrier. *Transportation Science*, volume 53, issue 4, pages 986-1000, 2019.
3. A. Bettinelli, V. Cacchiani, T. G. Crainic, D. Vigo, A Branch-and-Cut-and-Price algorithm for the Multi-trip Separate Pickup and Delivery Problem with Time Windows at Customers and Facilities, *European Journal of Operational Research*, volume 279, issue 3, pages 824-839, 2019.
4. A. Baller, S. Dabia, W. Dullaert, D. Vigo. The Vehicle Routing Problem with Partial Outsourcing. *Transportation Science*, volume 54, issue 4, pages 1034-1052, 2020.
5. C. Orlis, D. Laganá, W. Dullaert, D. Vigo. Distribution with Quality of Service Considerations: The Capacitated Routing Problem with Profits and Service Level Requirements. *Omega*, volume 93, 2020.
6. L. Accorsi, D. Vigo, A Fast Metaheuristic Algorithm for the General Single Truck and Trailer Routing Problem, *Transportation Science*, Published Online:18 May 2020. <https://doi.org/10.1287/trsc.2019.0943>.
7. L. Accorsi and D. Vigo. A Fast and Scalable Heuristic for the Solution of Large-Scale Capacitated Vehicle Routing Problem. Research Report DEI-OR-2020-2, to be submitted for publication to *Transportation Science*, 2020.