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A high damping cellular material with integrated arrays of nanocomposite web-like vibration absorbers

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This report presents a comprehensive overview of the research carried out during the performance period spanning from June 1, 2020, to May 31, 2023. The project represents a multidimensional exploration, focusing on the conceptualization, creation, and experimental assessment of an innovative cellular material hosting nanocomposite membrane resonators. The overarching objective was to generate wide, nonlinear bandgaps, transcending the limitations of linear, amplitude-invariant bandgaps, thereby achieving superior control over wave propagation dynamics. Drawing inspiration from the intrinsic periodicity of honeycombs and the ultralight, resilient, nonlinear characteristics of spider webs, our resonators were designed to exhibit multiple nonlinear local resonances, encompassing both axi-symmetric and skew symmetric modes.

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Executive Summary

This report presents a comprehensive overview of the research carried out during the performance period spanning from June 1, 2020, to May 31, 2023. The project represents a multidimensional exploration, focusing on the conceptualization, creation, and experimental assessment of an innovative cellular material hosting nanocomposite membrane resonators. The overarching objective was to generate wide, nonlinear bandgaps, transcending the limitations of linear, amplitude-invariant bandgaps, thereby achieving superior control over wave propagation dynamics. Drawing inspiration from the intrinsic periodicity of honeycombs and the ultralight, resilient, nonlinear characteristics of spider webs, our resonators were designed to exhibit multiple nonlinear local resonances, encompassing both axi-symmetric and skew-symmetric modes.

Collaboration between two groups defined our project efforts – the Sapienza group, concentrating on the concept design, modeling, optimization, and dynamic testing of the metamaterial, and RomeTre, dedicated to the design, fabrication, and testing of the metamaterial samples.

The project extensively delved into a range of fabrication strategies, aiming to strike a balance between precision and cost-effectiveness in constructing the honeycomb structure with embedded spider web-inspired membranes. Addressing critical challenges such as creating a free standing membrane anchored on the hexagonal cell and supporting a central mass, web uniformity, web-to-structure connection, and web rigidity, diverse process design solutions were explored. These solutions considered the web as either randomly distributed electrospun nano/microfibers or fibers unconventionally drawn in predefined patterns. While the latter approaches showed innovation, they lacked maturity for large-scale, cost-effective production. Consequently, the project narrowed its focus to successful process design strategies with the potential for economical mass production and effective bandgap behavior.

The traditional electrospinning process generates non-woven webs with randomly distributed fibers deposited on a collecting substrate. The project dedicated substantial effort to adapting far-field electrospinning to a 3D-printed, non-continuous, and electrified PLA-based cellular substrate. Although shaping the PLA structure was straightforward, challenges arose in locally electrifying these structures, controlling the nano/microfiber patterns for uniform distribution from cell to cell, ensuring enhanced adhesion between the electrospun web edges and the hexagonal structure, and supporting the centered mass in its free-standing midspan. Additionally, the process design aimed to ensure web rigidity for proper load transfer to the hexagonal structure, achieved by manipulating process parameters and inducing controlled shrinking during polymerization. This implemented process design emerged as a pivotal contribution, offering insights into achieving uniform webs across cells and proposing a potential approach for scalable manufacturing in a cost-effective manner.

Alternative approaches for electrification and direct mass integration into each cell were explored, progressing from simplified methods to sophisticated solutions. However, these approaches were not further pursued due to perceived scalability challenges for large-scale applications.

The fabricated resonators, incorporating a PVDF web attached to the hexagonal boundary cell, were subject to extensive LSV testing which unveiled the activation of local membrane resonances within clusters of resonators, accompanied by the emergence of acoustic and optical waves. Our experimental analyses aimed at acquiring Operational Deflection Shapes (ODS) derived from velocity fields, FRFs under chirp excitations focused on transitions across bandgaps, delineating acoustic modes preceding and optical modes following each bandgap. Although responses within the range indicated characteristics akin to bandgaps, the absence of perfect

membrane and stretching uniformity likely impeded the formation of ideal acoustic/optical modes typically observed in perfectly periodic lattices where the emergence of these modes entails a collective in-phase and out-of-phase motion of the resonators, involving the first or higher membrane vibration modes, with respect to the honeycomb cells. To achieve completely realized multiple bandgaps, 3D-printed samples were created with a spider web design, ensuring improved repeatability and optimal mass distribution. These samples, consisting of a larger central hexagonal mass and lateral masses interconnected via wires, underwent LSV analysis, capturing intriguing wave propagation properties, including the formation of three large bandgaps – an exceptional experimental result.

The experimental findings were complemented by extensive modeling efforts and analytical studies. Metamaterial modeling reduced the honeycomb to an equivalent orthotropic plate model and the resonators to a set of equivalent nonlinear modal mass-spring systems. The application of the Floquet-Bloch theory facilitated the pursuit of quasi-periodic solutions, providing insights into waves traveling within the metamaterial plane with motion prevalently in the out-of-plane direction. Validation through experiments, exploring specific resonator arrays, demonstrated excellent agreement between experimentally obtained and theoretically predicted bandgap central frequencies and sizes.

Given our focus on achieving multiple bandgaps and considering variations in spider web geometric parameters, up to four complete bandgaps and several incomplete ones were obtained in the low-frequency spectrum. Delving into nonlinear resonances, we explored both softening and hardening behaviors within the PVDF resonators. The introduction of modal springs with cubic terms, representative of different spider web designs, allowed the derivation of asymptotic solutions, demonstrating the profound impact of nonlinear resonances on bandgap width. Optimization studies resulted in a promising streamlined process for the optimal design of nonlinear metamaterials, summarized by a design chart illustrating regions of resonator modal mass and stiffness ensuring optimal gains in nonlinear bandgaps provided that suitable resonator nonlinear features are engineered.

Frequency response analysis, employing harmonic excitation signals and various levels of amplitudes, provided deeper insights into resonator behavior. Utilizing base excitation via a shaker, we identified nonlinear resonances and demonstrated the hardening behavior of the resonator, establishing the attainability of nonlinear optimality. The challenge of optimal damping in metamaterial design became another critical focus. By incorporating damping into our model, we determined optimal damping ratios through a fixed-point approach. Remarkably, optimal damping minimized wave amplitudes across bandgaps while preserving small resonant wave amplitudes along the resonance branches for the acoustic and optical propagation modes.

Expanding our investigation beyond bandgaps, we delved into edge waves within a non-axi-symmetric sample. This exploration unveiled localized phenomena along the boundary, demonstrating alignment between experimental observations and numerical simulations. In a spin-off study conducted in the project's final months, we focused on harnessing the piezoelectric effect exhibited by the PVDF membrane stretching in the metamaterial resonators. The objective was to explore the untapped potential of on-demand frequency shifting, consequently modulating bandgaps. Through the manipulation of applied voltage and pre-stress in the membranes, our metamaterial seamlessly transitioned into a semi-adaptive state. Numerical sensitivity optimization studies showcased the power of suitably architected resonators in quenching incoming waves within a tunable frequency range, achieving real-time control over wave propagation.

This final report, organized into five chapters, provides a comprehensive overview of our project efforts. Commencing with a list of publications, invited talks, and dedicated students involved, the first chapter, 'The concept and key findings,' offers a non-technical overview of the project achievements. Subsequent chapters delve into the intricacies of metamaterial modeling, computation, experimental validation, design optimization tackling resonator damping and nonlinearity, and metamaterial tunability. The report concludes with final considerations and future outlooks in Chapter 5.

Collaboration between two groups defined our project efforts – the Sapienza group, concentrating on the concept design, modeling, optimization, and dynamic testing of the metamaterial, and RomeTre, dedicated to the design, fabrication, and testing of the metamaterial samples.

The project extensively addressed a variety of fabrication strategies to achieve accuracy and cost-effectiveness in the construction of the honeycomb structure with the embedded spider web-inspired membranes. Critical challenges related to web uniformity, web-to-structure connection, and web rigidity were addressed through diverse process design solutions. These solutions considered the web as either randomly distributed electrospun nano/microfibers or fibers unconventionally drawn in pre-defined patterns. While innovative, the latter approaches were not mature enough for large-scale, cost-effective production. The focus narrowed down to successful fabrication strategies with potential for mass production and effective bandgap behavior. The traditional electrospinning process, creating non-woven webs from an electrified polymeric solution, typically involves random fiber collection. However, the project devoted considerable effort to adapt far-field electrospinning to a 3D-printed, non-continuous, and electrified PLA-based cellular substrate. Shaping the PLA structure was straightforward, but challenges arose in electrification, ensuring adhesion with a stretched web, and integrating cells with a centered mass. This integration process emerged as a key contribution, providing insights into achieving uniform webs across cells. Alternative approaches for electrification and direct mass integration into each cell were explored, evolving from simplified methods to sophisticated solutions, aligning with the vision of large-scale and cost-effective advanced metamaterial manufacturing.

The fabricated resonators, incorporating a PVDF web attached to the hexagonal boundary cell, were subject to extensive LSV testing which unveiled the activation of local membrane resonances within clusters of resonators, accompanied by the emergence of acoustic and optical waves. Our experimental analyses aimed at acquiring Operational Deflection Shapes (ODS) derived from velocity fields, FRFs under chirp excitations focused on transitions across bandgaps, delineating acoustic modes preceding and optical modes following each bandgap. Although responses within the range indicated characteristics akin to bandgaps, the absence of perfect membrane and stretching uniformity likely impeded the formation of ideal acoustic/optical modes typically observed in perfectly periodic lattices where the emergence of these modes entails a collective in-phase and out-of-phase motion of the resonators, involving the first or higher membrane vibration modes, with respect to the honeycomb cells.

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Expanding our investigation beyond bandgaps, we delved into edge waves within a non-axi-symmetric sample featuring 9x12 hexagonal cells. This exploration unveiled localized phenomena along the boundary, demonstrating alignment between experimental observations and numerical simulations. In a spin-off study conducted in the project's final months, we focused on harnessing the piezoelectric effect exhibited by the PVDF membrane stretching in the metamaterial resonators. The objective was to explore the untapped potential of on-demand frequency shifting, consequently modulating bandgaps. Through the manipulation of applied voltage and pre-stress in the membranes, our metamaterial seamlessly transitioned into a semi-adaptive state. Numerical sensitivity optimization studies showcased the power of suitably architected resonators in quenching incoming waves within a tunable frequency range, achieving real-time control over wave propagation.

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Papers

1. Y. Shen and W. Lacarbonara (2022) "Dispersion properties of nonlinear metamaterial beams hosting nonlinear resonators," Paper No. IDETC2022-97757, *ASME 2022 International Design Engineering Technical Conferences & Computers and Information in Engineering Conference (IDETC/CIE2022), St. Louis (MI), August 14 –17*, 2022.

2. M. Murer, W. Lacarbonara, and G. Formica (2022) "Multi-stop band wave honeycomb metamaterial with embedded resonators," Paper No. IDETC2022-91070, *ASME 2022 International Design Engineering Technical Conferences & Computers and Information in Engineering Conference (IDETC/CIE2022), St. Louis (MI), August 14 –17*, 2022.

3. Y. Shen, W. Lacarbonara (2023) Nonlinear dispersion properties of metamaterial beams hosting nonlinear resonators and stopband optimization, *Mechanical Systems and Signal Processing*, **187**, 109920, doi:10.1016/j.ymssp.2022.109920

4. M. Murer, S. K. Guruva, G. Formica, W. Lacarbonara (2023) A multi-band gap metamaterial with multi-frequency resonators, *Journal of Composite Materials* **57**(4):783-804, doi:10.1177/00219983231151578.

5. Y. Shen, W. Lacarbonara (2023) Optimal resonator damping for wave propagation control in mechanical metamaterials, *Mechanics Research Communications* **130**, 104124, doi: 10.1016/j.mechrescom.2023.104124.

6. W. Lacarbonara, S. K. Guruva, B. Carboni, B. Krause, A. Janke, G. Formica, G. Lanzara (2023) Unusual nonlinear switching in branched carbon nanotube nanocomposites, *Scientific Reports* **13**, 5185. https://doi.org/10.1038/s41598-023-32331-y.

7. Y. Shen, W. Lacarbonara (2023) Nonlinearity enhanced wave bandgaps in metamaterial honeycombs embedding spider web-like resonators, *Journal of Sound and Vibration* **562**, 117821, doi:10.1016/j.jsv.2023.117821.

8. A. Fortunati, A. Arena, A. Bacigalupo, M. Lepidi, W. Lacarbonara (2023) Free propagation of resonant waves in nonlinear dissipative metamaterials, *Proceedings of the Royal Society A*, submitted on Oct. 20, 2023.

9. Y. Shen, W. Lacarbonara (2023) Wave propagation and multi-stopband behavior of metamaterial lattices with nonlinear locally resonant membranes, *International Journal of Non-Linear Mechanics*, submitted on Sept 5, 2023.

Invited talks

1. Multi-band gap nonlinear metamaterials, *2nd ICMSD 2023,* September 1- 5, 2023, Beijing, China.

2. 11th DINAME 2023, February 26- March 3, 2023, Pirenopolis, Brazil.

3. 2nd International Conference on Computations for Science and Engineering, 30 August - 2 September 2022, Rimini Riviera, Italy.

4. International Conference on Mathematical Analysis and Applications in Science and Engineering ICMA2SC'22, June 27-29, 2022 Porto, Portugal.
5. 10th ENOC, 17-22 July 2022, Lyon, France.
6. Symposium on Aeroelasticity, Fluid-Structure Interaction and Vibrations, Virtual, October 14-15, 2021.

Students and postdocs

1. Mauro Murer, Postdoc, Sapienza University of Rome
2. Krishna Chytanya Chinnam, Postdoc, University of Rome, RomaTre.
3. Luigi di Rosa (2022), MS thesis 'Stopband analysis of a metamaterial honeycomb hosting nonlinear resonators,' MS in Aeronautical Engineering, Sapienza University of Rome.
4. Enrico Corradini (2023), BS thesis 'Design, numerical, and experimental investigation of an origami based metamaterial,' BS in Aerospace Engineering, Sapienza University of Rome.
5. Domenico Pascale (2023), MS thesis 'Metamaterial isolation system exploiting nonlinearity,' MS in Aeronautical Engineering, Sapienza University of Rome.
6. Sawan Kumar Guruva, PhD student, Sapienza University of Rome.
7. Pranath Kumar Gourishetty, PhD student, Sapienza University of Rome.

Contents

The concept and key findings

This project centers on the design, fabrication, and testing of a cellular material incorporating nanocomposite membrane resonators. Drawing inspiration from the periodicity of honeycombs and the ultralight, resilient, nonlinear nature of spider webs (refer to Fig. 1.1), our focus is on creating resonators that exhibit multiple nonlinear local resonances, encompassing axi-symmetric and skew-symmetric modes (see Fig. 1.1(c)). The primary objective is to generate multiple nonlinear bandgaps that are significantly larger than the linear bandgaps. This optimization is crucial for achieving enhanced control over wave propagation in the resulting cellular material. This initial chapter serves as an overview of the project's accomplishments, with specific technical details and challenges deferred to subsequent chapters.

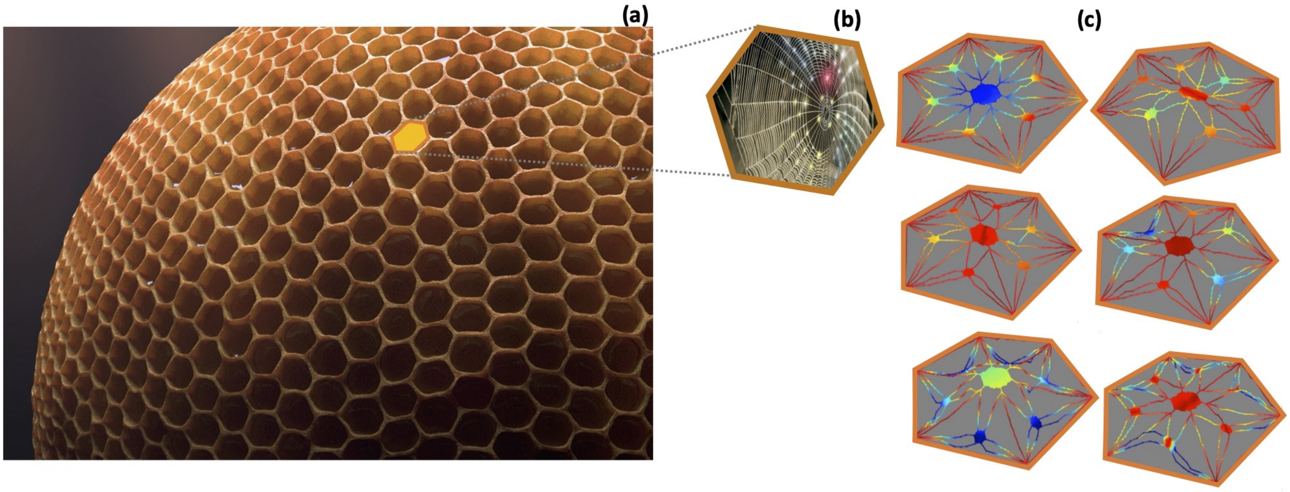

Figure 1.1: (a) The cellular material concept drawing inspiration from honeycombs which host (b) spider web-like resonators that exhibit a variety of (c) local resonances resulting into multiple bandgaps. The honeycomb and the spider web pictures are, respectively, a courtesy of www.turbosquid.com/3d-models/honey-honeycombs-3d-max/986045 and Kwang Lee et al., Adv. Mat. 96 (2020).

The fabrication process involves a unique combination of 3D printing for the honeycomb and electrospinning for the spider web-inspired membranes forming the resonators (see Fig. 1.2). These resonators consist of a central larger mass and an axi-symmetric arrangement of smaller masses. The archetypal resonator comprises a PVDF web electrospun onto the hexagonal boundary cell, periodically replicated to obtain a honeycomb cellular structure. The nanocomposite web functions as a nonlinear, dissipative spring for the central mass, facilitating oscillation in various modes orthogonal to the web plane. Comprehensive insights into the fabrication process will be detailed in a forthcoming chapter.

The modeling of the metamaterial consists of two main steps (see Fig. 1.7): 1) the honeycomb is reduced to an equivalent orthotropic plate, 2) the infinite-dimensional resonators are

Conclusions and future outlooks

This project revolved round a multidimensional, multidisciplinary exploration of an innovative cellular material housing nanocomposite membrane resonators. The overarching objective aimed at creating wide nonlinear bandgaps, transcending the limitations of linear bandgaps to achieve superior control over wave propagation dynamics. Drawing inspiration from the striking periodicity of honeycombs and the resilient, ultralight, nonlinear nature of spider webs, the resonators were designed to undergo multiple nonlinear local resonances, involving axi-symmetric and skew-symmetric modes. The goal was to harness the wide nonlinear local resonances - arising from the nonlinear response of spider web membranes - in the formation of bandgaps between an acoustic mode and an optical mode characterized by a synchronized, collective in-phase and out-of-phase motion of the resonators relative to the hosting cells. Operational Deflection Shapes (ODS) derived from velocity fields and average FRFs provided tangible indicators of acoustic and optical waves and demonstrated the existence of wide multiple bandgaps. Moreover, metamaterial modeling and analysis, employing the Floquet-Bloch theory, was validated by excellent agreement with experimental results.

The project extensively delved into a range of process design strategies, aiming to strike a balance between precision and cost-effectiveness in constructing the honeycomb structure with embedded spider web-inspired membranes. Addressing critical challenges such as creating a free standing membrane anchored on the hexagonal cell and supporting a central mass, web uniformity, web-to-structure connection, and web rigidity, a range of process design solutions was explored. These solutions considered the web as either randomly distributed electrospun nano/microfibers or fibers unconventionally drawn in predefined patterns. While the latter approaches showed innovation, they lacked maturity for large-scale, cost-effective production. Consequently, the project narrowed its focus to successful process design strategies with the potential for cost-effective mass production and robust bandgap behavior.

The project dedicated a substantial effort to design a process which could allow the realization of a free-standing membrane that hosts a mass in its mid-span and is constrained to the honeycomb structure. A number of challenges were solved by exploiting a hybrid solution that takes advantage of 3D printing and non-standard electrospinning that allowed to form webs with a pre-defined orientation and stretching. This implemented process design emerged as a pivotal contribution, offering insights into achieving uniform webs across cells and proposing a potential approach for scalable manufacturing in a cost-effective manner.

Despite the technical demands of achieving complete uniformity in membrane assembly and prestress levels, special (in-phase) acoustic or (out-of-phase) optical modes were detected. These modes selectively involved groups of resonators, deviating from the ideal infinitely extended, perfectly periodic lattice in which all resonators participate collectively in these global-local resonances. Between these acoustic/optical modes, responses indicative of bandgaps were observed. A plausible explanation for the presence of partially accomplished acoustic/optical waves with the interposed bandgaps is probably related to the nonideal uniformity in the membranes, particularly in the membrane stretching, leading to some frequency detuning between the resonators. Enhancing uniformity and control of the fibers pre-stretching is one of the goals of future endeavors. The implemented strategy of focusing the membrane into a localized cell by canceling out charges from neighboring cells appears to have a potential for large scale manufacturing especially if coupled with a moving and indeed hidden electroded cell which can be stretched to the desired extent prior being fully constrained to the honoeycomb structure.

In the pursuit of complete multiple bandgaps and in-depth exploration of the bandgap formation mechanism, our persistent efforts yielded fully 3D-printed samples with an effective discrete spider web design, demonstrating improved repeatability and optimal mass distribution. These samples featured a larger central hexagonal mass and a constellation of lateral masses interconnected by wires, fostering modal mass participation in skew-symmetric modes. LSV analysis unveiled three large bandgaps, marking an exceptional experimental outcome. Focused on achieving multiple bandgaps, the project explored variations in spider web geometric parameters, obtaining up to four complete bandgaps and several incomplete ones. Delving into nonlinear resonances, softening and hardening behaviors within the PVDF resonators were investigated. The introduction of modal springs with cubic terms representative of different spider web designs led to asymptotic solutions, highlighting the significant impact of nonlinear resonances on bandgap width. Our model and theoretical study demonstrated the achievement of optimal nonlinear design, summarized by a design chart providing regions of resonators' modal mass and stiffness with the associated nonlinearity type for optimal gains in nonlinear bandgaps regardless of the incoming wave amplitudes within a reasonable nonlinear range. Frequency response analysis, employing harmonic excitation signals via a shaker and varied amplitudes, provided deeper insights into resonator behavior. Nonlinear resonances and hardening behavior of the resonators were experimentally confirmed, demonstrating the attainment of nonlinear optimality. A notable accomplishment was the definition of optimal damping in metamaterial design, minimizing wave amplitudes across bandgaps while preserving reduced resonant wave amplitudes along resonance branches for acoustic and optical propagation modes. The ongoing challenge involves developing a theory accounting for damping in nonlinear resonators, with a clear roadmap based on our pioneering work.

In the project's final year, a spin-off study focused on leveraging the piezoelectric effect exhibited by the PVDF membrane, enabling on-demand frequency shifting and modulation of bandgaps. Manipulating applied voltage and pre-stress facilitated the seamless transition of the metamaterial into a semi-adaptive state. Numerical sensitivity and optimization studies highlighted the power of suitably architected resonators in quenching incoming waves within a tunable frequency range, achieving real-time control over wave propagation—an encouraging direction for further exploration.

Beyond bandgaps, the project delved into edge waves in a non-axi-symmetric sample, revealing localized phenomena along the boundary with experimental and numerical agreement. These phenomena call for deeper investigations into their onset, management, and optimization mechanisms.

The project was a fully rewarding and thought-provoking experience, and the PI and co-PI express their deep gratitude to the EOARD/AFOSR international program.

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