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Efficient Solution of Large-Scale Vehicle Routing Problems

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14. ABSTRACT
This project is a follow-up of project “Acceleration Techniques for Vehicle Routing Heuristics” (Grant FA9550-17-1-0234). The previous project was aimed at developing efficient, effective, and easy-to-implement acceleration techniques for the VRP which have a huge impact on the state-ofthe- art for such problem family and for the real-world applications related to it. The scientific achievements of the project were perfectly in line with the goals and resulted in at least seven publications in top quality scientific journals (see [1,2,3,4,5,6,7]). In particular, the main achievement of the project was the development of FILO (see Accorsi and Vigo [2]) a fast and scalable heuristic for the Capacitated VRP (CVRP), the basic variant of the problem in which just vehicle capacity constraints are considered, which incorporates efficient implementations of combined granular neighborhoods (see Toth and Vigo [9]) and static move descriptors (see Zachariadis and Kiranoudis [11]) The project should have ended in Q2/2020 but has been extended to Q2/2021 due to the COVID pandemic which made impossible to complete the dissemination phase.

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Efficient Solution of Large-Scale Vehicle Routing Problems

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Table of Contents

In this report, we present the research we performed during the first year of activity for Grant Number FA8655-21-1-7046 and the achieved results.

The report is organized as follows. In Section 1, we give a summary of the main contributions. Section 2 describes the studied problems, while Section 3 reports the proposed solving methods. The results we obtained are reported in Section 4. Finally, we conclude the report in Section 5 and give a summary of the produced publications in Section 6.

1 Summary

This project is a follow-up of project "Acceleration Techniques for Vehicle Routing Heuristics" (Grant FA9550-17-1-0234). The previous project was aimed at developing efficient, effective, and easy-to-implement acceleration techniques for the VRP which have a huge impact on the state-of-the-art for such problem family and for the real-world applications related to it. The scientific achievements of the project were perfectly in line with the goals and resulted in at least seven publications in top quality scientific journals (see [1,2,3,4,5,6,7]). In particular, the main achievement of the project was the development of FILO (see Accorsi and Vigo [2]) a fast and scalable heuristic for the Capacitated VRP (CVRP), the basic variant of the problem in which just vehicle capacity constraints are considered, which incorporates efficient implementations of combined granular neighborhoods (see Toth and Vigo [9]) and static move descriptors (see Zachariadis and Kiranoudis [11]) The project should have ended in Q2/2020 but has been extended to Q2/2021 due to the COVID pandemic which made impossible to complete the dissemination phase.

This project is aimed at consolidating and expanding the achievements of Grant FA9550-17-1-0234 by considering relevant variants of the VRP which are closer to the practical logistics

requirements, and by further enhancing the efficiency of the FILO approach through the hybridization of machine learning technologies and distributed computing. In particular, the main goals of our proposal are:

- Extend the acceleration techniques already successfully applied to the CVRP to other more constrained variants that have higher practical relevance, e.g. the VRP with Time Windows (VRPTW), the VRP with Pickups and Deliveries (VRPPD) and the VRP with heterogeneous fleet (HVRP).
- Explore the application of machine learning techniques to further enhance the performances of the current state-of-the-art algorithms.
- Develop new effective parallel schemes to tackle very large scale instances.

During the project we mainly concentrated on the first topic by enhancing the FILO framework and extending to relevant variants such as the VRPPD. In addition, new research lines were faced aiming at studying relevant variants of routing problems involving drones or other practically motivated characteristics. Several preliminary research activities on machine learning integration and parallel algorithms were initiated during this project and may be completed in the future.

2 Problems description

The vehicle routing problem (VRP) is one of the most studied combinatorial optimization problems which plays a central role in logistics and transportation. Since its definition, occurred more than fifty years ago, several exact methods and hundreds of heuristic optimization algorithms were proposed to tackle different problem variants. Even though real-world applications, such as waste collection and parcel distribution, can require up to tens of thousands of customer visits, almost all research so far has been targeted at solving problems with no more than a few hundreds of customers. Only recently, some researchers started working on the so called very-large-scale instances developing the first interesting methods to effectively tackle them. Due to the instance size (involving thousands or tens of thousands of customers), exact methods become computationally prohibitive and even standard heuristic methods cannot be used to effectively tackle those instances, despite the increasing processing capabilities of modern processors and parallel work decomposition. In fact, complete exploration of even simple neighborhoods with a polynomial cardinality cannot be computationally afforded.
Despite its importance in the vehicle routing area, most of the algorithms available in the literature still focus on relatively small CVRP instances. When scaling to very large-scale instances the design of each algorithm components becomes of crucial importance. Moreover, the employment of acceleration techniques is mandatory to design scalable algorithms that still retain their effectiveness.

The VRP with Pickups and Deliveries (VRPPD) consists in finding vehicle routes with minimum total cost which serve customers requiring either the delivery or the collection (or both) of goods served from a central depot. Different sub-variants falls within such a framework such as the VRP with Simultaneous PD in which customers may have a positive or negative demand which is the net difference between delivery and pickup demand. Another well studied variant is the so-called

VRP with Backhauls (VRPB) in which customers are partitioned in two groups, the first includes Linehaul customers requiring deliveries and the other is that of Backhauls customers requiring pickpus. The routes may serve both types of customers but no Backhaul customer may be served before all Linehaul customers in the route have been visited.

Drone routing problems studied in this project involve the scheduling of operations of a drone and a mobile rover station which provide the drone a mobile infrastructure for battery exchange so that its limited autonomy is improved and large range operations in surveillance and rescuing can be performed.

The latency location routing problem (LLRP), a combination of the facility location problem and the cumulative capacitated vehicle routing problem, is a recently proposed variant of location routing problems. It corresponds to a customer-centric problem, in which the aim is to minimize the sum of the arrival times at the customers.

3 Methods, Assumptions and Procedures

Acceleration techniques, such as *Granular Neighborhoods* (GN, see Toth and Vigo, [9]) and Static Move Descriptors (SMD, see Zachariadis and Kiranoudis [11]), have already been studied and applied successfully to the CVRP, leading to the development of a state-of-the-art Iterated Local Search heuristic called FILO (see Accorsi and Vigo, [2]) capable of effectively tackle instances of very large size with up to 30,000 vertices. Given the promising results obtained on the CVRP, a natural prosecution of this research is the application of a similar paradigm to other important variants of the VRP family, such as the VRPPD that, although more difficult, is also of greater practical relevance in real-world applications. The extension of SMDs to efficiently perform local search procedures on such variants is an extremely interesting yet challenging (and to the best of our knowledge, still unexplored) research direction potentially yielding to significant improvements of the existing algorithms, which because of the complexity of the problem tend to use simple local search operators or none at all. Furthermore, in order to create a framework capable of easily accommodating the extension to other relevant variants a substantial redesign of the whole FILO code is necessary to segregate the problem-dependent features in easily manageable formats. To this end a substantial effort was devoted to incorporating the Resource Extension idea for feasibility evaluation which makes very general the functions which evaluate the feasibility of a solution.

Several novel features were also introduced in FILO such as a final refinement step based on Set partitioning which builds on a pool of high quality solutions produced during the search and extracts a new, possibly better, solution from them.

Furthermore, we designed and developed a new and simple state-of-the art Set-Partitioning (SP) heuristic with the objective to coupling it with VRP heuristics.
The SP problem is one of the most known NP-hard problems in Combinatorial Optimization. Given a finite set of elements U and a finite set of subsets of U each one associated with a given cost, the SP problem asks to find a partition of U of minimum cost, i.e., to find a set of subsets of

U of minimum cost such that their union is equal to U and the pair-wise intersection of any two of the selected subsets is the empty-set.
Although the definition is very general, when we apply this problem to the vehicle routing context (using the so-called SP formulation of VRP), the set U simply refers to the set of the customers to visit in the graph, while the subsets of U are the routes (depot excluded), with their cost which is equal to the length of the route. In this sense, solving the SP means finding the set of routes that visit all the customers once while minimizing the total length, which is the generic VRP definition. Notice also that, since the routes are a variable of the problem, all the usual constraints (e.g., capacity, pickup&delivery, time windows, etc.) are implicitly mapped within the set of all the possible feasible routes, so the SP formulation for the VRP can represent many different VRP variants. Therefore, a heuristic that tackles this problem can be adopted to solve many different VRP variants without any needed customization.

The new version of FILO, integrated with the SP approach, was extensively tested and submitted to the prestigious DIMACS challenge on Vehicle Routing for the CVRP category. During the challenge FILO confirmed it is by far the best existing code for the large-scale instances and ranked fourth on the small scale ones. However, it is worth noting that the ideas of FILO were hybridized with other existing codes for small-scale instances resulting in the code by Huawei team which ranked first.

For the general VRP a new version of FILO called FILO2 was developed with the aim of solving huge scale instances with up to one million customers thus being two orders of magnitude larger than those than can be solved by existing algorithms. The method incorporates many technical improvements of the basic FILO approach designed to control memory use and speedup computation. The paper is under review for Computers and Operations Research.

For the VRPPD we completed the development and testing of a version of FILO which incorporates all the novel features and implements the resource extension function. The preliminary testing conducted so far shows that FILO is superior to all existing algorithms from the literature on VRPSPD and other similar variants. The resulting paper is under revision for Transportation Research E.

A further line of research is that of testing decomposition techniques to be coupled with heuristics for routing. We tested several approaches and compared them with state-of-the-art methods without decomposition. This research is also instrumental in the development of parallel algorithms for routing problems.

For drone-rover routing problems we developed an innovative and general approach based on Very Large Neighborhood Search paradigm capable of solving a large class of these problems and can also be used in practical applications in surveillance and rescuing.

As to the Latency Location-Routing we developed and tested several metaheuristics which outperformed existing approaches from the literature.

4 Results

The proposed models and methods have been tested on benchmark instances from the literature or on new scenarios. In this section, we summarize the main achievements we obtained.

For the VRPSPD and VRPB the algorithm has been used to solve literature instances with up to 10000 customers and contrarily to several existing competitors, it exhibits a computing time which is linear in the number of customers. In fact, it finds solutions better that those found by existing state-of-the-art algorithms in a few minutes of computing times compared to the several hundred minutes typically required by competing methods.

The current work is leading to the preparation of a manuscript to submitted for publication to Transportation Research E and currently under reviewco-authored by L. Accorsi, F. Cavaliere, D. Laganà, D. Musmanno and D. Vigo.

On the CVRP the new version of FILO, integrated with the SP approach, was extensively tested and submitted to the prestigious DIMACS challenge on Vehicle Routing for the CVRP category. During the challenge FILO confirmed it is by far the best existing code for the large-scale instances and ranked fourth on the small scale ones. However, it is worth noting that the ideas of FILO were hybridized with other existing codes for small-scale instances resulting in the code by Huawei team which ranked first.

- Cavaliere, F., Bendotti, E. & Fischetti, M. An integrated local-search/set-partitioning refinement heuristic for the Capacitated Vehicle Routing Problem. Math. Prog. Comp. (2022). https://doi.org/10.1007/s12532-022-00224-2

In this work, we took Keld Helsgaun's LKH-3, a generic and state-of-the-art heuristic capable of solving many VRP variants. We improved it for the CVRP and VRPTW case and combined it with a very simple SP phase using a restricted SP formulation (solved with CPLEX ILP solver) considering a part of the routes found by the LKH-3. The resulting technique has been able to repeatedly refine the best-known solution for many instances found in the literature of the CVRP.
From this first iteration, we expanded further the work on the SP side designing a propre SP heuristic (SPH), which we have adopted in conjunction with the LKH-3 and FILO during DIMACS 12th implementation challenge. Starting from our implementation of the Caparara, Fischetti and Toth state-of-the-art heuristic for the Set-Covering problem (a similar problem to the SP, where overlaps between subsets are allowed), we simplified and specialized the algorithm, integrating it with CPLEX ILP Solver used heuristically to solve multiple very small sub instances of the given SP instance.
The work accomplished during the challenge has been presented at multiple international conferences:

- DIMACS 12th implementation Challenge with our FSP4D (FILO + SPH) and LKHSP (LKH-3 + SPH)
- Route 2022, n Copenhagen from 22nd to 25th of May 2022.
- VeRoLog, in Hamburg from 12th to 15th of June 2022.

Our technique has been also briefly described in an extended abstract presented to the competition:

- Cavaliere, F., Fischetti, M., & Helsgaun, K. (2022). A Vehicle Routing heuristic based on accelerated LKH-3 coupled with Set Partitioning. Paper presented at 12th DIMACS Implementation Challenge, Piscataway, NJ, United States. https://drive.google.com/file/d/1nFm6OTCnrzsM5mo2mEIggWlAFi2Q-6Et/view
- Accorsi, L., Cavaliere, F., Vigo, D., (2022). Iterative Fast Optimization for the Capacitated Vehicle Routing Problem presented at 12th DIMACS Implementation Challenge, Piscataway, NJ, United States.

The decomposition techniques for routing problems are described in a paper co-authored by A. Santini, M. Schneider, T. Vidal and D. Vigo which has been published in the prestigious INFORMS Journal of Computing.

- Santini A., Schneider M., Vidal T., Vigo D. "Decomposition strategies for vehicle routing heuristics". INFORMS Journal of Computing 35(3), 543-559, 2023.

Preliminary results on the application of decomposition-based parallel approaches for routing are will be presented at the forthcoming Odysseus 2024 conference.

The extension of FILO to huge scale routing problems is described in the following paper:

- Accorsi L., Vigo D. "Routing one Million Customers in a Handful of Minutes", under review for Computers and Operations Research.

The computational testing of machine learning algorithms on routing problems has been thoroughly analyzed in

- Accorsi L., Lodi A., Vigo D. "Guidelines for the computational testing of machine learning approaches to vehicle routing problems", Operations Research Letters, 50(2), 229-234, 2022.

The drone-rover routing approach is described in two papers one published and the other currently under revision for the European Journal of Operational Research.

- Kloster K., Moeini M., Vigo D., Wendt O. "The multiple traveling salesman problem in presence of drone- and robot-supported packet stations", EJOR Volume 305, Issue 2, 2023, 630-643.
- Lorenz C., Mimmo N., Otto A., Vigo D. "Very large-scale neighborhood search for drone routing with energy replenishment", under review for EJOR. arXiv:2207.13940

The latency location routing approaches have been described in the following paper:

- Osorio-Mora A., Rey C., Toth P., Vigo D. "Effective metaheuristics for the latency location routing problem". International Transactions in Operations Research 30(6), 3801-3832, 2023.

Finally, two further research lines were developed during the project. The first one aims at exploring interesting areas in the application of routing to waste collection building on the long lasting experience of the group in this domain. To this end, a fruitful collaboration with the research group of Prof. Doerner at UniWien has been established and an extensive survey on routing problems arising in waste collection applications has been published in the paper:

- Hess, C., Dragomir, A.G., Doerner, K.F., Vigo D.. "Waste collection routing: a survey on problems and methods". Central European Journal of Operations Research H (2023). https://doi.org/10.1007/s10100-023-00892-y

The second line was one of the objectives of the present project and is devoted to the integration of machine learning and optimization techniques in the routing area. A preliminary scan of the current literature on the subject revealed that there are numerous inconsistencies in the computational testing techniques between the OR and the ML communities. Therefore, useful guidelines for the computational experimentation of routing algorithms has been published in

- Accorsi L., Lodi A., Vigo D.. "Guidelines for the computational testing of machine learning approaches to vehicle routing problems", Operations Research Letters, 50(2), 229-234, 2022.

5 List of publications produced so far during the project

1. Cavaliere F., Bendotti E., Fischetti M. An integrated local-search/set-partitioning refinement heuristic for the Capacitated Vehicle Routing Problem. Math. Prog. Comp. 2022. https://doi.org/10.1007/s12532-022-00224-2.
2. Accorsi L., Lodi A., Vigo D.. "Guidelines for the computational testing of machine learning approaches to vehicle routing problems", Operations Research Letters, 50(2), 229-234, 2022.
3. Lorenz C., Mimmo N., Otto A., Vigo D. "Very large-scale neighborhood search for drone routing with energy replenishment", under review for EJOR. arXiv:2207.13940, 2022.
4. Accorsi, L., Cavaliere, F., Vigo, D. Iterative Fast Optimization for the Capacitated Vehicle Routing Problem presented at 12th DIMACS Implementation Challenge, Piscataway, NJ, United States, 2022.
5. Santini A., Schneider M., Vidal T., Vigo D. "Decomposition strategies for vehicle routing heuristics". INFORMS Journal of Computing 35(3), 543-559, 2023.
6. Kloster K., Moeini M., Vigo D., Wendt O. "The multiple traveling salesman problem in presence of drone- and robot-supported packet stations", EJOR Volume 305, Issue 2, 630-643, 2023.
7. Accorsi L., Vigo D. "Routing one Million Customers in a Handful of Minutes", under review for Computers and Operations Research, 2023.
8. Hess, C., Dragomir, A.G., Doerner, K.F., Vigo D.. "Waste collection routing: a survey on problems and methods". Central European Journal of Operations Research H (2023). https://doi.org/10.1007/s10100-023-00892-y
9. Osorio-Mora A., Rey C., Toth P., Vigo D. "Effective metaheuristics for the latency location routing problem". International Transactions in Operations Research 30(6), 3801-3832, 2023.

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[7] C. Orlis, D. Laganá, W. Dullaert, D. Vigo. Distribution with Quality of Service Considerations: The Capacitated Routing Problem with Profits and Service Level Requirements. Omega, volume 93, 2020.
[8] P. Toth and D. Vigo (editors). The Vehicle Routing Problem. Monographs on Discrete Mathematics and Applications. S.I.A.M., Philadelpia, PA, 2002.
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