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Deep Electromagnetic Probing and Imaging Through Complex Media

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14. ABSTRACT
In the project we have made important progress on the problem of understanding deep probing through atmospheric turbulence. A main goal is to understand how the wave field statistics is affected by the turbulence. As the wave propagates the wave energy is transferred into an incoherent part. Understanding this process is important in particular for imaging and communication applications using electromagnetic waves. On the one hand, in the context of classic techniques exploiting the coherent or partly-coherent wave field one would like to know how the performance is affected by the clutter and to be able to optimize the scheme depending on the character of the turbulence. On the other hand the statistics of the medium clutter or turbulence will be imprinted in the statistics of the incoherent part of the wave and this can then also be used for imaging. In the project we have worked on various aspect of such challenges, both from the point of view of fundamental theory as well as from the point
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In the project we have made important progress on the problem of understanding deep probing through atmospheric turbulence. A main goal is to understand how the wave field statistics is affected by the turbulence. As the wave propagates the wave energy is transferred into an incoherent part. Understanding this process is important in particular for imaging and communication applications using electromagnetic waves. On the one hand, in the context of classic techniques exploiting the coherent or partly-coherent wave field one would like to know how the performance is affected by the clutter and to be able to optimize the scheme depending on the character of the turbulence. On the other hand the statistics of the medium clutter or turbulence will be imprinted in the statistics of the incoherent part of the wave and this can then also be used for imaging. In the project we have worked on various aspect of such challenges, both from the point of view of fundamental theory as well as from the point of view of applications. We describe below in more detail the various contributions from our work in the project period 2-1-2018 till 1-31-2022.

Contents

1 Wave Scintillation Theory; Section 2 in Proposal

In the paper "Scintillation of Partially Coherent Light in Time Varying Complex Media" [17], joint with Josselin Garnier Ecole Polytechnique France, we develop a general theory for the scintillation of a partly coherent wave field propagating through the time varying turbulent atmosphere. Indeed, a fundamental challenge associated with laser propagation in the atmosphere is to understand the scintillation. The scintillation describes the relative fluctuations in the intensity of the wave field. The intensity, also referred to as the irradiance, is the square magnitude of the wave field. In applications to imaging and communication understanding the signal to noise ratio and how it can be controlled or maximized, corresponding to minimizing the scintillation, is a fundamental challenge [14, 15, 17]. Note also that the case when the laser beam source is itself modeled as a random field is important to understand. In fact such sources, often referred to as partly coherent sources, has been used in particular to mitigate the effect of atmospheric turbulence. Understanding the scintillation is then a challenging multiscale problem. Some of the important scales are the spatial correlation lengths of the source and of the medium fluctuations, moreover, the beam waist or width. We here also consider the situation when the medium, the atmospheric turbulence, changes in time. Modeling such changes is important since the atmospheric turbulence is time dependent, moreover, since a physical detector has a finite response time and the atmospheric temporal changes affect the measurements. In the paper we deduce a general theory for how the scintillation can be described depending on relative magnitudes of the scales in the problem. We find that the central parameter is the spatial scale of the variations in the medium relative to the source beam width. Depending on the relative size of this ratio we identify three canonical regimes where we can obtain explicit expressions for the scintillation. These expressions can then be used to understand the performance of communication and imaging algorithms and how to choose configuration parameters for optimal performance. In the paper we also compare the theoretical prediction of the scintillation measurements with actual scintillation measurements for over the water propagation. The scaling regime of the experiment corresponds to relatively slow variations in the atmosphere and we obtain excellent agreement in between theory and observations, see Figure 1. In [17] we also give a detailed account for the second moment of the wave field and how this evolves. This description is useful in particular if one have access to the field itself and can use this for instance for correlation based imaging schemes.

2 Partly Coherent Polarized Beams; Section 5 in Proposal

The question of a rigorous understanding of the joint statistics of polarization modes in the context of propagation of electromagnetic beams has been an open question. In the paper "Partially Coherent Electromagnetic Beam Propagation in Random Media" [19] we present a theory for this based on a multiscale analysis for electromagnetic beams. The analysis

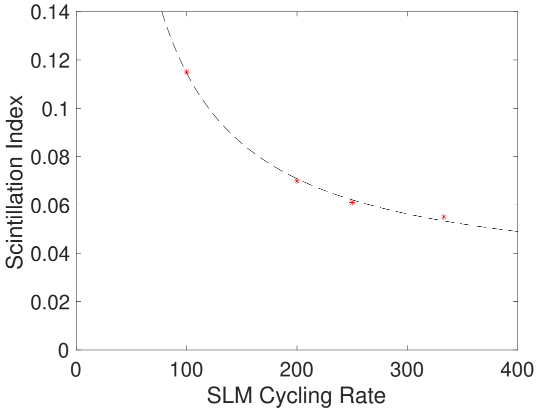

Figure 1: Data (red stars) taken from the paper [23] by Avramov-Zamurovic et al. The observations conform well with the theoretical predictions (dashed line) assuming a frozen medium. The experiment in [23] involves an over-the-water laser beam link at the United States Naval Academy. The source is partially coherent (Multi-Gaussian Schell Model) and realized via a Spatial-Light-Modulator(SLM). Shown is the measured scintillation index as function of the SLM cycling rate.

builds on the results in [9] and [10] which derives generalized Itô-Schrödinger equations for symmetric hyperbolic systems which are equations that describe the evolution of the probabilistic distribution of the modes. The motivating setting is propagation of laser beams through the turbulent atmosphere. We consider the so called saturated regime when the scattering is strong and the wave beam strongly effected by the turbulence. We moreover consider propagation from partly coherent sources. As mentioned partly coherent sources have been promoted as being resilient to turbulence and therefore being promoted in communication and imaging applications. However, the work dealing with this challenge is mostly based on numerical simulations and there is a need for an analytic frameworks that can be used for general analysis of such problems. In [19] we present such a framework. The central aspect here is that we can characterize the fourth moment without resorting to formal/perturbative approximations like the Huygens-Fresnel principle whose range of validity is unknown. The fourth moment is the central quantity needed to understand performance aspects, like the signal to noise ratios, in the mentioned applications. Our analysis moreover provides a rigorous justification of the often used Gaussian approximation and a characterization of the scaling regime when it is valid. The representation we arrive at can seen as a generalization of the context of the Michelson stellar interferometer presented in [6] to the case with propagation from partly coherent sources, moreover, to the case when the medium is turbulent. Our results also give a framework for interferometry based on intensities rather than on the field itself. We remark that the so called Hopkin's formula presented in [6] is a transmission function modification which aims to incorporate the effects

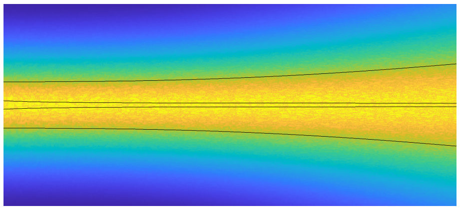

Figure 2: The figure illustrates the fluctuations in the harmonic beam field and how the character of the fluctuations and also the beam width changes with propagation distance when the beam propagates through turbulence.

of turbulence. In our framework we show that this approach is not valid in general since it does not properly take into account lateral coherence properties in the transmitted wave field. In Figure 2 we illustrate by showing the fluctuations in the laser beam and how it evolves as the beam propagates through the turbulence. The source in the partly coherent case it random with certain statistics and these statistics evolve as the beam propagates through the turbulence. A deep understanding of this evolution of the wave field statistics is important in applications. Note that in Figure 2 we show only one component of the electromagnetic field while in [19] we present the general theory which characterizes the joint statistics of all the polarization modes.

3 Enhanced Backscattering and Fabric Imaging; Section 4 in Proposal

An important challenge in wave propagation is to separate and identify loss and scattering parameters as these can be markers for anomalous sections of the medium. The so called enhanced backscattering cone refers to how wave energy backscattered from the medium is distributed over direction. This directional energy distribution is used in particular in the biomedical imaging modality referred to as Low-coherence Enhanced Backscattering (LEBS) [22]. In [16] we present the first rigorous theory that links in a quantitative way the wave energy angle distribution to the medium parameters, see Figure 3. Such insight form the basis for using the backscattering cone for imaging purposes. Our medium modeling in is motivated by deep probing through anisotropic and turbulent media, which is appropriate for atmospheric turbulence which indeed is often anisotropic due to a vertical temperature gradient. We have developed the theory which describes the statistics of the wavefield in this case. We use this theory to analyze in particular the enhanced backscattering cone and how this can be used for estimation of medium fabric. By looking at the statistics of the backscattered signal we can estimate the parameters of the turbulence and also the presence of say an anomalous layer which could be a height section with very intense

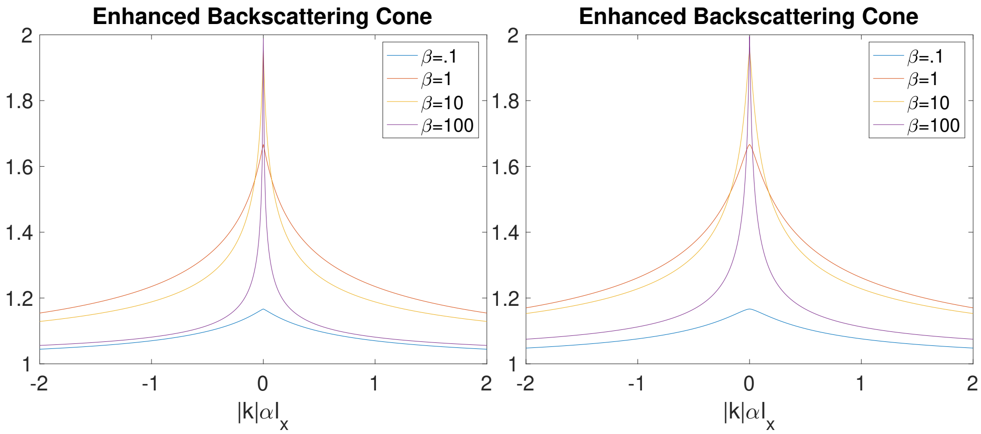

Figure 3: The figure shows the enhanced backscattering cone from two media models with the left plot corresponding to a rougher medium. The abscissa corresponds to the relative backscattering direction. The parameters α, β derives from the scattering and attenuation properties of the medium with α being strength of diffraction relative to attenuation and β being strength of random scattering relative to attenuation. In the paper we show explicitly how the shape of the cone can be related to the medium structure or fabric.

turbulence. A rigorous mathematical analysis of the fundamental phenomenon of enhanced backscattering associated with medium clutter has been a long standing open question. This problem has received a lot of attention in the physics community, both from the experimental and formal asymptotics viewpoints, and we complement this in [16] with a rigorous scaling limit analysis in the so called scintillation regime.

4 A Quantitative Stability Theory for Time Reversal of Waves; Section 3 in Proposal

We develop in [18] the theory that allows us to analyze broad band wave signals, in particular from the point of view of signal to noise ratio. The paper generalizes the results of [11] to the multifrequency case which is indeed needed to analyze broad and narrow band signals and the multifrequency speckle memory effect.

A motivating model problem is the time harmonic refocusing experiment. Here we record the time harmonic wave field emanating from a point source after it has propagated through a complex and turbulent medium. We then observe in particular a wave phase that has been distorted due to the random medium. If we change the frequency a bit how much is this phase distortion modified, or more generally how does the wave field corruption due to clutter change when we change the frequency a relatively small amount? The time harmonic refocusing experiment is important to understand deep focusing of

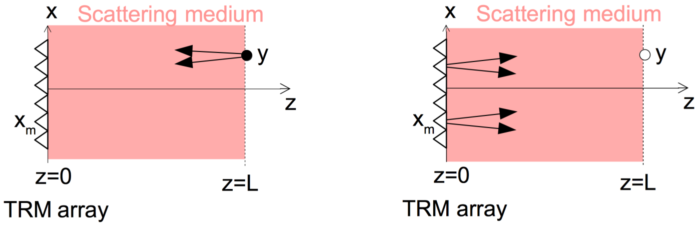

Figure 4: Phase conjugation experiment. Left: first step of the experiment (a point source transmits from $(\boldsymbol{y}, L)$ and the mirror in the plane $z = 0$ is used as an array of receivers). Right: second step of the experiment (the mirror is used as an array of sources emitting the phase conjugated wave field).

monochromatic light through a scattering medium. Indeed, this can be achieved by using a "guide-star" or point source on the other side of complex section and then record the field and next re-emit it with the opposite phase. In this case the re-emitted wave field will refocus at the original source location. The configuration is described in Figure 4. A question is then if we re-emit at a slightly different frequency how will the performance degrade. Note that to describe the first moment of the refocussed field we need a second moment of the propagated field since the wave propagates both forward and backward and we can use reciprocity. Then to describe the signal to noise ratio we need the fourth moment of the wave field, with the wave field components evaluated at different frequencies. Important parameters in this context are

- Ω the frequency offset.
- c_0 wave speed.
- D the effective strength of the random medium.

In the refocusing we then observe an *exponential* decay in the amplitude as:

$$\exp\Big(-\frac{L}{\ell_\Omega}\Big), \tag{1}$$

Similarly, in fact it follows from our analysis that the signal to noise ratio is damped by the same factor as in Eq. (1). Thus, refocusing can be achieved if

$$L \ll \ell_\Omega.$$

Here, L is the total travel distance and $\ell_\Omega = \sqrt{c_0/D\Omega}$ is the travel distance at which the frequency offset starts to randomly modulate the wave due to the turbulence. We have therefore identified rigorously the fundamental differential frequency scattering mean free path which determines how the wave field decorrelates in frequency.

We remark that the same question is relevant when we work with a guide star in the context of adaptive optics. Here we use the phase associated with the guide star to clean up the distorted image from "nearby" sources by compensating with respect to the phase distortion associated with the guide star. The question is how does this phase compensation of adaptive optics degrades with respect to a frequency offset, the image to be cleaned may be at a small offset frequency or relatively broad band. Our theory characterizes precisely this degradation which can be seen as a fundamental theoretical question associated with adaptive optics.

In the paper we also look at another central question associated with the modern theory of wave propagation, that of statistical stability in the time reversal experiment. The time reversal experiment corresponds to the situation in Figure 4 only that we use broad band or time domain signals so that the time reversal step to the left side of the complex section corresponds to true time reversal, rather than only phase conjugation. The time reversed signal will refocus at the original source point, the central question then is what is the resolution and signal to noise ratio in the refocussed signal. Before one only knew that the refocussed signal was stable in some scaling limit, while we here, with the multifrequency moment theory, for the first time can give a quantitative description of the stability that is the signal to noise ratio. A main aspect of the statistical stability result is that for deep probing the signal to noise ratio is

$$\left(\frac{L}{\ell_\Omega}\right)^2 N$$

where here the frequency offset is the bandwidth: $\Omega = B$ and N is the number of elements in the mirror. It is seen that in the broad band case we have polynomial, rather than exponential, decay in the signal to noise ratio. Thus, the differential frequency scattering mean free path is the central parameter in explaining the performance.

5 Communication through turbulence; Section 2 and 3 in Proposal

In [4] we analyze beam propagation and how to encode information for communication through weak turbulence. This paper is in part motivated by the many recent papers on so-called Orbital Angular Momentum type sources, a particular beam source. We show here analytically how the optimal source can be understood from the point of view of the Singular Value Decomposition and compare the performance of this optimal scheme with classic schemes. Again here there have been many experimental papers and there was a need for a rigorous theory that can characterize the relative performance of schemes in specific scaling regimes without having to resort to Monte Carlo simulations. We show That our theory conforms well with results from numerical simulations based on the so called phase screen approach.

6 Transport Equations for Wave Mode Coupling; Section 2 in Proposal

In papers [5, 8] we deal with the challenging problem of developing a theory for waves in random media in the situation when interfaces or boundaries are present. A number of recent fascinating applications have accentuated the need for such a theory. One such application comes from geophysics when one uses waves generated by earthquakes to say something about the earth's interior. The interface at the earth's surface then plays an important role for the propagation phenomenon. We have developed a theory that describes how such a propagation phenomenon can be described in terms of a system of transport equations. The important challenge here is to characterize the propagation and coupling of wave modes and how this depends on the properties of the random medium and the presence of the medium interface. Our analysis of the transport equations shows how one may first observe a rapid evolution toward a quasi-equilibrium for a set of surface wave modes followed by a slow relaxation toward a global equilibrium. Such behavior had been observed empirically before, but not explained theoretically. We illustrate such a behavior in Figure 5. In the papers we consider both a propagation domain of finite width as well as a random half space. The analytic framework that we use derives from that we have developed for random waveguides. An important application of the theory is to analyze and understand how one can use so called coda waves for imaging. The code waves are the incoherent part of the wave field following in the wake of the main arrival. The statistics of these coda waves carry information about the medium and the challenge is to process the wave appropriately to identify this information in a robust way. We are currently working on this challenge. A similar challenge can be found for atmospheric propagation in the context of over the horizon propagation in the ground wave system modality. Diffraction at the surface creates surface waves that can be used for long-range propagation and imaging. Although the mathematical formulation bears similarities to the one of the geophysical problem the application is very different, corresponding to electromagnetic propagation above the earth's surface rather than elastic wave propagation below the earth's surface. Note that another context of imaging in the presence of interfaces is presented in [7] where we discuss how passive imaging techniques can be used to image medium parameters in stratified earth like model.

7 Propagation Through Time Dynamic Media; Section 2 in proposal

In [2] we consider waves probing through a wave-guide or pipe with a time dependent medium. We show how the flow in the medium actually affect the coupling of propagating wave guide modes in the pipe. This coupling, and in particular how it it is affected by turbulent sections in the pipe, can be used for imaging and we present an analysis of this

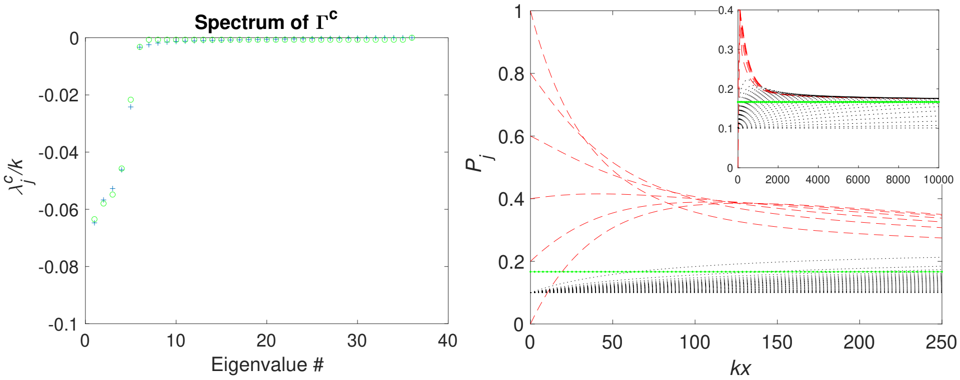

Figure 5: *Left Plot:* The eigenvalues of the matrix Γ^c which is the matrix that captures the specific statistics of the random medium that is important for wave mode coupling. Actual values are shown with blue crosses and predicted estimates are shown with the green circles. The spectrum corresponds to 6 surface wave modes. Here the random medium is modeled with a Gaussian correlation function. *Right Plot:* Evolution of the mode powers with increasing relative propagation distance. The surface mode powers are shown with red dashed lines, the body mode powers are shown with black dotted lines and the green line is the average mode power. As predicted by the theory, we see a rapid relaxation of the surface wave powers that lasts up to the scaled range $kx \approx 50$ (for k wave number and x propagation distance), after which there is a slow transition towards equipartition. The body mode powers evolve very slowly, as predicted by the analysis. The inset corresponds to a longer propagation distance and displays the evolution toward equipartition, albeit very slowly for the body modes.

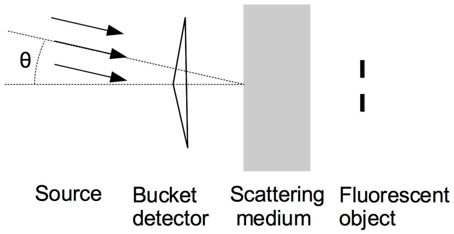

Figure 6: An example of a speckle imaging configuration. The source emits a time-harmonic wave for various incoming angles θ. The object is fluorescent and located behind a strongly scattering medium. For each incident angle the total backscattered fluorescent light is measured and this can be used to image the hidden object.

challenge. In [20] we present the theory for propagation in complex wave-guides that are static, however with long range correlations so that the random fluctuations are persistent and with slow decay of correlations.

In the case of atmospheric turbulence the medium may also be time dependent due to strong wind for instance. The rigorous analysis of wave propagation in time dependent media has been lacking. In the earlier paper [1] we carried out such an analysis for one dimensional or layered media while in [3] we push trough an analysis of wave propagation in time dependent general 3D media. We here also show how imaging algorithms can be constructed in such media and how in fact the time dependence can be used in certain situations to improve the imaging.

8 Analysis of Speckle Based Imaging; Section 3 in Proposal

When waves propagate through a strongly scattering medium the coherent wave component is lost and the wave intensity forms a random speckle pattern seemingly without much useful information. However, a number of recent physical experiments show that one can extract useful information from the statistics of this speckle pattern [21, 24]. During the first part of the grant period we completed work on the analysis of this problem. In [12, 13] we analyze how information in the speckle statistics can be used for imaging. Using our theory for the 4th moment of beam waves we show mathematically how speckle statistics can be used to image an object hidden in clutter for various imaging configurations, see Figure 6 for an example configuration. Indeed, this was conjectured by physicists and we now have a theoretical explanation that shows explicitly when the procedure will work, moreover, a resolution and stability theory.

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