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Improving robustness, efficiency and accuracy of synthetic aperture radar (SAR) imaging techniques using multi-measurement vectors

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14. ABSTRACT
The proposed research will develop numerical algorithms that effectively use multi-measurement data collections to extract actionable information from acquired sensing data. Much research has recently been devoted to sparse signal and image recovery from multiple measurement vectors (MMV). Sometimes, as in synthetic aperture radar (SAR) over a small aperture, the collected data may not vary much. In other cases, such as MIMO SAR, the data can vary significantly. The PI will focus on these applications as prototypical sensing models. The assumption in all cases is that the underlying signals or images are jointly sparse, meaning they have some features in common with sparse representations that can be recovered from the measurement vectors. Standard sparse recovery techniques can be used separately to recover each signal or image. Joint sparsity (JS) algorithms use additional constraints to exploit this measurement coupling. The L2,1 minimization, a natural analogue of the popular L1 minimization used for single measurement sparse recovery, is commonly used.

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Abstract

Synthetic aperture radar (SAR) is a day or night any-weather complex imaging modality that is an important tool in remote sensing. Most existing SAR image formation methods result in a maximum a posteriori (MAP) image which approximates the reflectivity of an unknown ground scene. Due to the sparsity of the magnitude of the underlying SAR signal, compressive sensing (CS) algorithms are also commonly employed. Although typically typically derived from different vantage points, the single image recovered using either a MAP estimate or a CS algorithm typically differ only because of how the corresponding parameters are chosen. Indeed, it is possible to to obtain exactly the same result for a given choice of parameters. What is important to note that regardless of the approach, this single image provides no quantification of the certainty with which the features in the estimate should be trusted, *especially* because the estimate relies so heavily on how the parameters are chosen, which is difficult without prior knowledge about noise in the data.

The research conducted for this project aims to tackle this issue, as well as provide additional information about the solution, using both *deterministic* and *probabilistic* approaches, as well as combining these ideas. In addition, using the probabilistic framework, investigations done by the PI and collaborators produce a sampling framework to SAR image formation.

Summary of Research Activities

Listed below are some techniques that the PI and collaborators have developed over the duration of this award:

Joint Image Formation and Phase Error Correction (Autofocusing) Methods, [5]:

Errors in estimating round trip times in SAR imaging models typically manifest as *phase* errors in the data and can lead to defocused imagery as well as complications in information extraction. While most current methods for correcting such errors work on range compressed data, recent work by the PI and collaborators [5] has shown the benefits of applying autofocusing methods to raw phase history data. The proposed additional task will include (i) incorporation of novel *phase synchronization* methods (see for example, [9], [4]) in joint image reconstruction and phase error correction algorithms, (ii) adapting such phase synchronization methods to speckle noise models, and (iii) formulating the joint image reconstruction and autofocusing methods in an Empirical Bayesian Inference framework.

Variance based joint sparsity, [1, 3, 6, 7]:

Much research has recently been devoted to sparse signal and image recovery from multiple measurement vectors. Thus far, the PI has mainly focused on developing new numerical techniques to improve synthetic aperture radar (SAR) image recovery when multiple observations of phase history data within a small aperture are available. Since the underlying scene presumably has some features with sparse representation, *compressive sensing* (CS) techniques, which use ℓ_1 regularization to exploit such sparsity, are commonly used. Since the data collected in nearby aperture windows for the same underlying scene does not vary significantly, one can view the problem as an image recovery problem from multiple (indirect) measurements. Moreover, the underlying images are assumed to be *jointly sparse*, meaning they have some features in common with sparse representations that can be recovered from the measurement vectors. Finally, we note that while the measurements are redundant, the data collected at each azimuth angle may still be under-sampled, which is also why the CS framework is suitable.

Joint sparsity (JS) algorithms use additional constraints to exploit this measurement coupling. The $\ell_{2,1}$ minimization, a natural analogue of the popular ℓ_1 minimization used for single measurement sparse recovery, is commonly used. Two main drawbacks of existing JS algorithms are (1) they are inherently coupled and therefore difficult to parallelize; and (2) the $\ell_{2,1}$-norm provides a uniform measure across all vectors, making it more susceptible to false injections, noise, or other types of data misrepresentation.

Because of these difficulties, the PI and collaborators were motivated to develop a new approach that would be particularly suitable for SAR image recovery. In this regard, they observed that joint sparsity also suggests that the pixel-wise variance across measurement vectors should convey information about their shared support. They used this observation to develop the *variance based* joint sparsity (VBJS) method, which replaces the $\ell_{2,1}$ norm by a weighted ℓ_1 norm, where the weights scale as the reciprocals of the pixel-wise variance. As a consequence, the regularization term is more heavily penalized in regions unlikely to be in the joint support, that is, in places where there is true sparse representation. Their investigations have shown the VBJS approach to be accurate, efficient, and robust for sparse signal and image recovery, [1], especially in cases of false injections, [3]. It also helps to reduce speckle [6]. One advantage of the method is that it is able to use the phase history data *directly*, which prevents information loss in the recovery, as well as improves computational efficiency and robustness, [7].

We note that VBJS has been used to analyze several Air Force datasets. As demonstrated in [6], VBJS can be used to reduce speckle in synthetic aperture radar (SAR) imagery while maintaining target fidelity. Due to its success in clutter suppression, the VBJS algorithm was then used to curate a database of several million images for training machine learning neural networks to recognize military targets in SAR imagery. The results provided in [8] show that for the classification task, although only minimal improvement in accuracy is obtained, VBJS ensures that the network is learning based only on target signatures rather than the noise, thereby improving the explainability of the network and preventing overfitting.

Empirical Bayesian Inference Approach, [12]:

Although it is possible to improve the robustness and efficacy of CS algorithms in both the single and multiple measurement cases, the recovery is inherently limited to a point estimate solution, and it is therefore not possible to quantify the uncertainty. This disadvantage is non-trivial because in practice it is often crucial to know how reliable the recovery is, especially in the case of noisy or limited data availability, as the result of natural or intentional obstruction. To address this issue, the PI and colleagues developed a new empirical Bayesian inference algorithm for solving a linear inverse problem given multiple measurements of noisy observable data in [12]. As was done in the VBJS approach, the variance in the spatial domain across multiple measurements was used to determine the support in the sparse domain of the underlying signal or image. From this, a new *support informed* sparsity promoting prior was constructed, which was in turn used to recover the posterior distribution of the unknown. Numerical experiments demonstrated that using this new prior not only improved accuracy of the recovery (given by the mean of the sampled posterior), but also reduced the uncertainty in the posterior when compared to standard sparsity producing priors, such as the Laplace prior.

SAR image formation with uncertainty quantification, [2]:

In [2] we specifically addressed the issues of speckle reduction and uncertainty quantification in SAR image reconstruction, while maintaining enough efficiency to enable working with image sizes typical in real-world applications. Significantly, the SAR image may be recovered **directly** from phase history data, as opposed to relying on altering images that have already been reconstructed or otherwise processed. This is accomplished by taking a more robust approach to estimation and then sampling an entire posterior density estimate rather than just computing a point estimate. This new approach allows us to compute estimates and uncertainty quantification information such as standard deviation and confidence intervals for all unknown parameters in the model. The approach uses the hierarchical Bayesian prior structure from [10] and directly incorporates coherent imaging and speckle into the prior density. The prior density is also formed to encourage sparsity in order to reduce speckle and increase contrast. Conjugate priors are used so that the resulting posterior can be efficiently sampled by using a Gibbs sampler and a NUFFT. It is important to note that all parameters in the model are prescribed, requiring **no user input**. In particular, the sparse prior model is used in a sampling-based framework for spotlight mode airborne SAR image reconstruction directly from phase history data.

Sequential image recovery from under-sampled Fourier data, [11]:

The PI and collaborators developed a new algorithm to jointly recover a temporal sequence of images from noisy and under-sampled Fourier data. Specifically considered was the case where each data set is missing vital information that prevents its (individual) accurate recovery. Our new method was designed to restore the missing information in each individual image by "borrowing" it from the other images in the sequence. As a result, *all* of the individual reconstructions yield improved accuracy. The use of high resolution Fourier edge detection methods is essential to our algorithm. In particular, edge information is obtained directly from the Fourier data which leads to an accurate coupling term between data sets. Moreover, data loss is largely avoided as coarse reconstructions are not required to process inter- and intra-image information. Numerical tests in [11] include SAR images.

AFRL Collaborations

The PI and students supported by this award collaborated with research scientists in the Sensors Directorate at AFRL Wright Patterson. Two Dartmouth undergraduate students participated in the ATRC Summer Internship Program, and both students completed undergraduate honors theses at Dartmouth based on work begun during the program. Dr. Scarnati (formerly at AFRL and now at Qualis Corporation) traveled to Dartmouth (2019) to participate in several collaborative research meetings. The PI and he PhD student Victor Churchill met with Dr. Scarnati and Ed Zelnio during the annual AFOSR program meeting in January 2020. Dartmouth Ph.D. student Dylan Green work with Jan Rainer (JR) Jamora during the 2022 ATRC Summer Internship Program, and Mr. Jamora visited Dartmouth in January 2023 to continue collaborative efforts. A paper regarding 3D SAR is forthcoming.

Publications

Below is a list of the relevant peer-reviewed journal and conference papers that were accepted or published during this reporting cycle.

1. V. Churchill and A. Gelb, *Sub-aperture SAR imaging with uncertainty quantification*, Inverse Problems, **39**:5 (2023).
2. V. Churchill and A. Gelb, *Estimation and Uncertainty Quantification for Piecewise Smooth Signal Recovery*, Journal of Computational Mathematics, **41**:2 (2023).
3. J. Glaubitz, A. Gelb and G. Song, *Generalized sparse Bayesian learning and application to image reconstruction*, SIAM/ASA Journal on Uncertainty Quantification, **11**:1 pp. 262-284 (2023).
4. Y. Xiao, J. Glaubitz, A. Gelb and G. Song, *Sequential image recovery from noisy and under-sampled Fourier data*, J. Sci. Comput., **91**:3 (2022).
5. V. Churchill and A. Gelb, *Sampling-based spotlight SAR image reconstruction from phase history data for speckle reduction and uncertainty quantification*, SIAM/ASA Journal on Uncertainty Quantification, **10**:3 pp. 1225-1249 (2022).
6. J. Zhang*, A. Gelb, and T. Scarnati, *Empirical Bayesian Inference Using a Support Informed Prior*, SIAM/ASA Journal on Uncertainty Quantification, **10**:2 745-774 (2022) doi: 10.1137/21M140794X.
7. D. Green, A. Gelb and G. Luke, *Sparsity-Based Recovery of Three-Dimensional Photoacoustic Images from Compressed Single-Shot Optical Detection*, Journal of Imaging, **7**(10), 201 (2021). doi: 10.3390/jimaging7100201.
8. J. Glaubitz and A. Gelb, *Stabilizing Radial Basis Function Methods for Conservation Laws Using Weakly Enforced Boundary Conditions*, J. Sci. Comput., **87** 40 (2021). doi:10.1007/s10915-021-01453-8.
9. T. Scarnati and A. Gelb, *Accurate and Efficient Image Reconstruction from Multiple Measurements of Fourier Samples*, Journal of Computational Mathematics, **38** 798-828 (2020). DOI: 10.4208/jcm.2002-m2019-0192.
10. J. Glaubitz and A. Gelb, High order Edge Sensors with ℓ^1 Regularization for Enhanced Discontinuous Galerkin Mehtods, SIAM Journal on Scientific Computing, **41:2** A1304–A1330, (2019). (DOI:10:1137/18M1195280).
11. V. Churchill, R. Archibald, and A. Gelb, *Edge-adaptive ℓ_2 regularization image reconstruction from non-uniform Fourier data*, Inverse Problems & Imaging, **13** (2019). doi: 10.3934/ipi.2019042

12. V. Churchill and A. Gelb, *Detecting Edges from Non-uniform Fourier Data via Sparse Bayesian Learning*, J. Sci. Comput., **80:2** 762–783 (2019) DOI: 10.1007/s10915-019-00955-w

13. B. Adcock, A. Gelb, G. Song, and Y. Sui, *Joint sparse recovery based on variances*, SIAM Journal on Scientific Computing, **41:1** A246-A268 (2019) DOI: 10.1137/17M1155983.

14. A. Gelb, X. Hou, and Q. Li, *Numerical Analysis for Conservation Laws Using ℓ_1 Minimization*, J. Sci. Comput., **81** 1240-1265 (2019). DOI: 10.1007/s10915-019-00982-7.

15. A. Gelb and T. Scarnati, *Reducing Effects of Bad Data Using Variance Based Joint Sparsity Recovery*, J. Sci. Comput., **78:1** 94-120 (2019). DOI: 10.1007/s10915-018-0754-2

16. R. Shang, R. Archibald, A. Gelb and G. Luke, *Sparsity-based photoacoustic image reconstruction with a linear array transducer and direct measurement of the forward model*, Journal of Biomedical Optics, **24**(3) (2019). DOI: 10.1117/1.JBO.24.8.089801.

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