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Predictive Modeling for Complex Plasma Systems with Poorly Understood Physics

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Final Technical Report**

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14. ABSTRACT The objective of this work was to investigate a new technique for predicting the behavior of complex plasma systems of strategic importance to the Space Propulsion and Power program. Many of the most promising technologies of interest to the Air Force—even those that are considered mature—have poorly understood aspects of their operation. The lack of understanding of these processes has in many cases prevented the development of fully predictive numerical models for these technologies. Such high-fidelity models are necessary for • Predicting and optimizing performance of new and unproven technologies • Extending mature technologies beyond their operational envelopes to meet strategic goals • Predicting the response of plasma systems to external factors like environmental changes							
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1. Overview

The objective of this work was to investigate a new technique for predicting the behavior of complex plasma systems of strategic importance to the Space Propulsion and Power program. Many of the most promising technologies of interest to the Air Force—even those that are considered mature—have poorly understood aspects of their operation. The lack of understanding of these processes has in many cases prevented the development of fully predictive numerical models for these technologies. Such high-fidelity models are necessary for

- Predicting and optimizing performance of new and unproven technologies
- Extending mature technologies beyond their operational envelopes to meet strategic goals
- Predicting the response of plasma systems to external factors like environmental changes

We leveraged in this effort both physics-based and data-driven methods to inspire models for poorly understood processes in two complex plasma systems: the Hall effect thruster and rotating magnetic (RMF) thruster. The goal in analyzing the former was to demonstrate our method on a technology with extensive flight heritage but several outstanding questions related to its fundamental operation. The goal for analyzing the latter was to demonstrate the potential for our method to develop and optimize promising but unproven technologies.

We had four major elements to our investigation: (1) identification of reduced fidelity models (RFM) for critical missing pieces of physics, (2) generation of experimental datasets on representative test articles for each system, (3) model regression to determine values for model parameters in the RFMs, and (4) demonstration of a predictive numerical capability by incorporating the RFMs into numerical models. This effort spanned 3.5 years (with a 0.5 year no cost extension) and supported three doctoral dissertations. It leveraged a broad range of resources including experimental test articles, facilities, and diagnostics in house at the PI’s laboratory as well as numerical code and machine learning resources available at the University of Michigan. At the close of this work, we have established the foundation for a new, data-driven paradigm for analyzing complex plasma systems. In so doing, we have provided a unique set of tools for improving model fidelity for existing technologies and enabling model-based design strategies to develop next-generation concepts.

2. Accomplishments

We describe in the following key accomplishments from this effort. We provide a brief overview of each project and its motivation, the objectives, the key milestones, and how we disseminated our results to the broader community.

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2.1. Anomalous electron transport in hollow cathode plume

Background

The efficient operation of the Hall effect thruster depends on the ability of its electron source, the hollow cathode, to couple electrically to the thruster’s plasma discharge (Figure 1). These thermionically emitting devices enable the electrons to both ionize the thruster’s propellant and neutralize the positively charged ion exhaust after it has been accelerated out of the thruster. If changes to the cathode operation or environment impede the ability of these electrons to connect to the thruster’s main discharge, performance and stability can suffer. Indeed, it has been shown that varying key cathode parameters such as flow rate, temperature, orientation, and test environment can negatively impact the cathode coupling and by extension thruster operation.

Although these effects have been well-documented, there are aspects of the underlying physical processes governing the cathode coupling that remain poorly understood. The major challenge in building a first-principles description of the cathode coupling stems from the existence of non-classical processes in the plumes of these devices. Most notably, the electron resistivity is orders of magnitude higher than can be explained by friction from ordinary collisions. It thus historically has not been possible to model or predict the local plasma environment using a classical plasma approach. With that said, recent studies have shown that the onset of propagating electrostatic waves in the form of current-driven ion acoustic turbulence (IAT) is the dominant contributor to an effective, “anomalous” collision frequency on the electrons. Subsequent efforts have built on these findings to incorporate non-classical, wave-driven effects self-consistently into numerical results.

Despite these on-going efforts, there has yet to be a predictive engineering model for the cathode’s dependence on operating condition and facility configuration. The response to facility configuration---specifically background pressure---is a particularly critical issue as it has been suggested that measurements of the cathode performance in ground facilities may not be representative of a space like environment. This risk has given rise to a new urgency to develop models capable of extrapolating cathode behavior from ground tests to predictions on orbit.

Objective

Our goal was to develop a reduced fidelity model for the non-classical electron dynamics in the cathode plume and to leverage this model to make predictions for key aspects of cathode performance. For the latter objective, we focused primarily on the cathode coupling voltage---the electrical potential difference between the cathode and thruster plume. This is a critical factor that impacts thruster efficiency and its electrical configuration.

Milestones

- **Measured the electron dynamics directly.** We performed detailed experimental campaigns to measure the spatial and temporal dependence of the electron dynamics in a typical hollow cathode plume. This effort, which is detailed in Ref. [1], involved applying an Ohm’s law to relate local measurements of plasma density and electron current to an effective electron collision frequency. These results (Figure 2) showed that the electron resistivity in the plume,

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as represented by the electron frequency, is non-classical. These measurements similarly provided a dataset for evaluating models to explain the non-classical behavior.

- Measured properties of turbulence believed to be responsible for anomalous electron transport in the plume of a hollow cathode.** We directly measured the properties of ion acoustic turbulence [1-3] in the plumes of both a stand-alone cathode (i.e. without a thruster) and a cathode in a Hall thruster (Figure 3). We then showed through a quasi-linear formulation that the variations in the measured electron resistivity (see previous milestone) were correlated with variations in the total energy in the spectrum of the ion acoustic turbulence (Figure 4). When combined with the findings of other studies, this result showed conclusively that the turbulence was a dominant driver for the anomalous transport.
- Motivated physics-based closure model based on measurements.** We used the experimental measurements from the previous milestone to propose closure models for the electron resistivity (as represented by the electron collision frequency). We applied quasilinear theory combined with wave kinetic theory to arrive at 0D and 1D models for how the wave energy in the acoustic turbulence grows and in turn how this gives rise to an effective drag on the electrons [3,4]. We in turn showed that these closure models agreed quantitatively with experimental measurements of the electron collision frequency (Figure 5 and Figure 6). This work was recognized as a Featured article in the Physics of Plasmas. We formulated these closures in terms of local plasma properties such that they could be incorporated self-consistently into fluid-based models for the cathode plume. We leveraged this capability for the next milestone.
- Developed physics-based, reduced fidelity model for electron transport in hollow cathode plume.** We used the experimentally-validated closure models from the previous milestone to propose a new model for how cathode coupling voltage responds to background facility pressure [5]. This model, which was based on a reduced dimensional Ohm’s, had three free parameters. We showed from our physics-based analysis that these parameters should be constant for a given thruster and operating condition. This thus allowed us to leverage experimental data to determine these model parameters from regression.
- Regressed model and demonstrated ability to make predictions for cathode operation under uncertainty:** We used our reduced fidelity model to regress experimental datasets for cathode coupling voltage as a function of facility pressure from 7 different Hall thrusters and a total of 15 different operating conditions (Figure 7) [5]. We were able to show in all cases that the model was able to accurately match trends in the data—showing in some cases the ability to exhibit inflections in trends with pressure. We in turn were able to use these regressed models to make extrapolations to on-orbit conditions (zero background pressure). This was, to our knowledge, the first fully physics-based attempt to make the extrapolation for this property of the cathode. Moreover, since our approach to regression was Bayesian, we could assess confidence in our predictions. This was the first direct application of uncertainty quantification to model predictions for a Hall thruster facility effect.

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Dissemination of results (journal papers and conference proceedings)

We presented the results from this project in both peer-reviewed formats and conference proceedings.

- [1] M.P. Georgin, B.A. Jorns, and A.D. Gallimore, “Correlation of ion acoustic turbulence with self-organization in a low-temperature plasma,” *Physics of Plasmas*, Vol. 26, Issue 8, 082308, 2019.
- [2] M. Georgin, B. Jorns, and A. Gallimore, “Transient non-classical transport in the hollow cathode plume I: measurements of time-varying electron collision frequency,” *2020 Plasma Sources Sci. Technol.* 29 105010.
- [3] B. Jorns, S. Cusson, Z. Brown, and E. Dale, “Non-classical electron transport in the cathode plume of a Hall effect thruster,” *Physics of Plasmas*, 27, 022311, 2020. Awarded Featured Article.
- [4] M. Georgin and B. Jorns, “Transient non-classical transport in the hollow cathode plume II: Evaluation of models for the anomalous collision frequency,” *2020 Plasma Sources Sci. Technol.* 29 105011.
- [5] B. Jorns and M. Byrne, “Model for the dependence of cathode voltage in a Hall thruster on facility pressure,” *2021 Plasma Sources Sci. Technol.* 30 015012.
- [6] M. Georgin, “Ionization Instability of the Hollow Cathode Plume,” PhD Dissertation, University of Michigan, 2020.

2.2. Anomalous electron transport in Hall thrusters

Background

The Hall effect thruster is an attractive technology for in-space propulsion, balancing a moderate thrust density with high specific impulse (> 2000 s). These advantages have led to the widespread use of these devices for applications ranging from geocentric orbit raising to station keeping. Despite this growing maturity of this technology, there are several outstanding challenges related to their future development. These include open questions about the fidelity of ground tests for representing on-orbit performance and how to perform lifetime assessments for higher-power, longer duration operation. Many of these challenges could be addressed in part through analysis, i.e. the use of predictive numerical models.

To this end, it is common to use fast but reduced fidelity frameworks---most commonly fluid or hybrid-fluid approximations---to model Hall thruster operation. These models are not fully predictive, however. This stems from the fact that although there are well-established classical fluid equations for low temperature plasmas like that found in the Hall thruster, it has been shown experimentally that not all these classical equations apply to this device. Most notably, the cross-

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field electron current is orders of magnitude higher than can be explained by classical transport theory. In order to capture this enhanced transport, the electron diffusion in fluid-based Hall thruster codes is typically represented with an ad-hoc transport coefficient—either in the form of an anomalous collision frequency or mobility. This parameter in turn can be adjusted to allow for non-classical electron motion. While the addition of this new parameter is an expedient knob for allowing a user to better match simulations to experiment, introducing this parameter has the adverse consequence of opening the governing set of equations for the thruster. Hall thruster fluid codes with an ad-hoc coefficient thus are not self-consistent—a user needs to prescribe values for the parameter based on trying to match code outputs to experimental data. This limits the predictive capabilities of the models.

In light of this limitation, there have been several attempts to propose closure models for the anomalous transport that can be self-consistently incorporated into engineering, fluid-based codes. Many of the more recent efforts have been guided by the growing consensus that the transport is linked to the onset of current-driven electron cyclotron drift instabilities (ECDI). However, despite yielding new insights into the physics of thruster operation, these attempts at closure models to date have yet to be fully predictive. There thus remains a pressing need to identify new models for this transport coefficient.

Objective

Our goal was to discover new closure models for the anomalous electron transport in Hall effect thrusters. To this end, we leveraged extensive experimental campaigns to inform both machine learning-based and physics-based models. We in turn assessed our confidence in these closure models through an application of Bayesian inference. Finally, we had the goals of incorporating these closures self-consistently into a state-of-the-art Hall thruster fluid model (Hall2De from the Jet Propulsion Laboratory), quantifying our uncertainty in the model predictions, and evaluating the predictive capability of the code when compared to experimental data.

Milestones

- **Measured the electron transport directly in Hall thruster channel.** In order to establish datasets for validation and model discovery, it was necessary to measure the electron transport directly in the Hall thruster plasma. The challenge was that traditional, probing-based methods can be highly perturbative to the local plasma. We subsequently developed a new technique with non-invasive laser induced fluorescence to infer the electron transport (Figure 8) [1]. This method is based on using measured moments of the ion velocity distribution function coupled with the ion fluid equations and electron Ohm’s law. The application of this technique yielded the first time-resolved measurements of the electron transport fluctuations in a Hall thruster (Figure 9). We in turn showed for the first time that this transport, as characterized by an effective electron collision frequency, varies by orders of magnitude as the thruster plasma oscillates. This work was recognized with an Editor’s pick award from the Physics of Plasmas.
- **Measured properties of turbulence believed to be linked to the anomalous electron transport.** Through measurements of the spatial evolution the ECDI turbulent spectra in a Hall thruster (Figure 10), we were able to highlight the role of non-linear cross-lengthscale

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energy transfer in the saturation of this mode [2]. This work was recognized as an Editor’s pick by the Physics of Plasmas. We in turn employed quasilinear theory to determine if the measured wave properties could explain the measured electron transport (previous milestone). We ultimately found the quasilinear theory was inadequate, which we attributed to a failing of key assumptions underlying this theory [3]. In our subsequent work, we have developed new methods [4] to measure directly the growth rate, wave energy, and cross-lengthscale coupling in the plasma (Figure 11). We have shown that when these processes are directly considered in the context of quasilinear theory, the waves can explain the transport [5]. The observations of the wave properties in turn have guided new closure models for the transport.

- Motivated closure models for electron transport in Hall thruster based on measurements.** We leveraged experimental measurements from the previous milestones in two ways to propose new closure models for the electron transport. First, we applied a genetic algorithm to regress our database of measured/inferred anomalous electron transport. Through a series of evolutions and mutations, the algorithm yielded a set of symbolic expressions for how the transport relates to local plasma properties. We showed that the most promising of these purely data-driven inspired models outperformed any physics-based closure models proposed to date (Figure 12) [6]. The second method we employed to find closure models was to leverage our experimental measurements of the wave properties to propose zero-equation, one-equation, and two-equation formulations for how the wave energy and growth evolved in the plasma (Figure 13). We in turn related these through quasilinear theory to models for the anomalous electron transport coefficient [7].
- Rigorously assessed confidence in model parameters of proposed closure models.** Each of the closure models we proposed from the previous milestone had free model parameters in the form of constant coefficients. These had to be inferred by regressing the models against experimental datasets (Figure 14). To this end, we adopted a Bayesian approach to represent the values of these model parameters as probability distributions subject to inherent model-based and experimental uncertainty (Figure 15) [8,9].
- Integrated closures self-consistently into fluid-based Hall thruster model and demonstrated ability to make predictions for thruster operation under uncertainty.** We integrated our most promising closure models from the previous milestones into the Jet Propulsion Laboratory’s multi-fluid Hall thruster model, Hall2De. We then applied forward uncertainty quantification methods to sample from the distribution of model parameters (previous milestone) and run Hall2De. In this way, we were able to build a distribution of model predictions for thruster performance and the internal plasma properties (Figure 16). This was, to our knowledge, the first time uncertainty quantification had been applied to Hall thruster model predictions [8]. This work was recognized with the best paper award at the 2020 AIAA Propulsion and Energy Forum. We subsequently were able to show that we could extend our model to make predictions that were qualitatively consistent with some experimental trends from a real Hall thruster (Figure 17) [9].
- Developed algorithms to explore and rigorously quantify uncertainty in future proposed closure models.** In our exploration of different closures, we discovered a potential issue with

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our approach to model regression. We previously had used time-averaged measurements of the electron transport coefficient to regress our models. However, when we used the model parameters resulting from this regression in Hall2De, we found the predicted thruster performance could diverge substantially from experimental measurement [9,10]. This suggested that the steady-state data we were using for model regression may not be appropriate. Instead, we concluded we should use the direct output of the model (performance, acceleration profiles, etc.). The problem with this approach, however, is that generating these predictions can require several hours of model run time, which is prohibitively long for model parameter inference. As of the close of this effort, we thus have started to explore surrogate based methods to increase the speed of model discovery and regression [11].

Dissemination of results (journal papers and conference proceedings)

We presented the results from this project in peer-reviewed formats, conference proceedings, and invited talks:

- [1] E.T. Dale and B.A. Jorns, “Non-invasive time-resolved measurements of anomalous collision frequency in a Hall thruster,” *Physics of Plasmas*, Vol. 26, Issue 1, 013516, 2019. Awarded Editor’s pick.
- [2] Z. Brown and B. Jorns, “Spatial Evolution of Small Wavelength Fluctuations in a Hall Thruster,” *Physics of Plasmas*, 26, 113504, 2019. Awarded Editor’s pick.
- [3] Z. Brown and B. Jorns, “Experimental measurements of the contribution of plasma turbulence to anomalous collision frequency in a Hall thruster,” Paper presented at the 2021 AIAA Propulsion and Energy Forum, 2021, AIAA-2021-3415
- [4] Z. Brown and B. Jorns, “Growth and Saturation of the Electron Drift Instability in a Crossed-Field Plasma,” In Review (2022).
- [5] Z. Brown and B. Jorns, “Anomalous cross-field transport in a Hall effect thruster determined from experimental measurement of the electron cyclotron drift instability,” In preparation (2022).
- [6] B. Jorns, “Predictive, data-driven model for the anomalous electron collision frequency in a Hall effect thruster,” *Plasma Sources Science and Technology*, Vol. 27, No. 10, 10.1088/1361-6595, 2018.
- [7] B. Jorns, “Two Equation Closure Model for Plasma Turbulence in a Hall Effect Thruster,” Paper presented at the 36th International Electric Propulsion Conference, Vienna, Austria, IEPC-2019-129, 2019
- [8] B. Jorns, T. Marks and E. Dale, “A Predictive Hall Thruster Model Enabled by Data-Driven Closure,” Paper presented at the 2020 AIAA Propulsion and Energy Forum, 2020, AIAA-2020-3622. Awarded Best Paper.

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- [9] T. Marks, A. Lopez Ortega, I. Mikellides, and B. Jorns, “Self-consistent implementation of a zero-equation transport model into a predictive model for a Hall effect thruster,” Paper presented at the 2021 AIAA Propulsion and Energy Forum, 2021, AIAA-2021-3424.
- [10] T. Marks and B. Jorns, “Challenges with the self-consistent implementation of closure models in Hall thruster fluid simulations,” In preparation, 2022.
- [11] T. Marks and B. Jorns, “Modeling anomalous electron transport in Hall thrusters using surrogate methods,” Paper presented at the 37th International Electric Propulsion Conference, Boston, MA, IEPC-2022-344. 2022
- [12] E. Dale, “Investigation of the Hall Thruster Breathing Mode,” PhD Dissertation, University of Michigan, 2021.

2.3. Reduced fidelity model for Rotating Magnetic Field (RMF) thruster operation

Background

The Rotating Magnetic Field (RMF) thruster is an inductive pulsed plasma thruster (IPPT), which employs a rotating magnetic field to induce an azimuthal current in a plasma. This current interacts with the radial component of a steady bias magnetic field, resulting in an axial body force on the plasma, ejecting the propellant as a slug and generating impulse (Figure 18). As an IPPT, the RMF thruster shares many of its potential strengths with other in-family devices. These include high throttability while maintaining efficiency and specific impulse; high specific power; and in-situ resource utilization (ISRU). The RMF current drive scheme has the additional benefit that the induced plasma current depends on the rotational frequency of the RMF rather than its magnitude. The RMF thruster can thereby avoid the problematically large current and voltage transients required for other IPPTs.

In light of these advantages of the RMF thruster, several groups have studied this technology to date. These studies have shown, however, that the thrusters generally exhibit poor efficiency, < 10%, with specific impulse less than 400 s. These results, which are underwhelming when compared to more mature concepts like Hall and ion thrusters, are puzzling in the context of theoretical predictions for RMF thruster performance. Indeed, it has been suggested that with an optimum RMF current drive, efficiencies of up to 85% are achievable. This invites the question as to the nature of the major drivers for the efficiency loss and in turn what strategies may be employed to improve performance. To this end, there is a need for predictive models to inform our understanding of RMF thruster performance. The challenge, however, is that there are aspects of RMF operation—mostly notably the magnetic coupling between the accelerating coils and plasma—that have precluded the development of predictive models for these devices.

Objective

The goal of this project was to derive a predictive engineering model for RMF thrusters capable of providing assessments of key performance parameters like efficiency, thrust, and specific

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impulse. To this end, we developed an equivalent circuit model to approximate complex geometrical features of the thruster and its plasma. We then calibrated this model against experimental data and used the resulting model to inform new design strategies.

Milestones

- Developed equivalent circuit model for RMF thruster operation.** We derived a physics-based, equivalent circuit model [1-3] for the operation of the RMF thruster (Figure 19). This model was based on an analysis of a volume integrated electron Ohm’s law, Maxwell’s equations, and a 1D ion momentum equation. We represented the RMF coil circuits with characteristic resistance and inductance and modelled the plasma as a solid slug with fixed geometry and fixed resistance and self-inductance. We in turn represented the coupling between the RMF coils and plasma with a combination of mutual inductance and gyrator terms. As with the previous two projects, this reduced fidelity model also had a problem of closure. Specifically, we did not know a priori how the mutual inductance, i.e. the degree of magnetic coupling between coil and slug, depended on slug position. We therefore motivated a physics-based closure drawn from previous studies on pulsed plasma devices: we assumed this mutual inductance decayed exponentially with position in the device. With this closure model, the resulting equivalent circuit simulation for the RMF had several constant parameters that were not known a priori and had to be inferred from data. These included the initial mutual inductance between the driving coils and plasma, the characteristic length of magnetic coupling between the plasma and coils, the resistance of the plasma slug, the coupling of background field to the currents driven in the plasma, and the characteristic ionization time of the plasma.
- Developed RMF prototype and experimentally characterized operation.** We developed a new RMF test article [4] for this effort (Figure 20, Figure 21, and Figure 22). This was a 5-kW class device capable of operating at pulse rates less than 150 Hz and with pulse durations on the order of 150 μ s. We developed a diagnostic suite comprised of far-field probes and thrust stand to measure both the time-averaged and transient operation of this device. Key telemetry included thrust, specific impulse, efficiency, and the current/voltage characteristics of the thruster. These results were, to our knowledge, the first published direct thrust measurements of an RMF thruster. This work was recognized with a Best Paper Award from the AIAA.
- Determined model parameters from experimental measurement.** We applied a simple least-squares analysis to determine the best fit model parameters between measurements of the test article and model predictions. We exclusively employed the measured current and voltage traces to perform the model calibration (Figure 23). Once calibrated, we then were able to apply the model to make predictions for thrust, specific impulse, and efficiency. We subsequently compared parametrically the calibrated model results against experimental measurement for a range of operating conditions. As Figure 24 shows, we were able to match trends in the experimental data [2,3]. This result was, to our knowledge, the first demonstration and validation of an equivalent circuit model for RMF thrusters. The results also yielded novel physical insight into the device’s operation. In particular, we found that the plasma resistance was an order of magnitude higher than we would anticipate based on classical collisions. This suggested that energy losses could be attributed to non-collisional processes such as flux to the

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walls and enhanced radiation. We recently have been able to confirm these suppositions experimentally in follow on work.

- **Applied model to make design recommendations for future iterations.** We found from our model calibration performed for the previous milestone that the model parameters remained relatively constant over the operating conditions of the thruster. This suggested that we could use average values of these coefficients in the model and in turn leverage the model to perform a designed based optimization study [3]. Figure 25 shows an example result from this type of study where we parametrically varied the radius and length of the thruster. This result shows there are evident optima for improving performance of the thruster—a key practical result which is informing future designs.

Dissemination of results (journal papers and conference proceedings)

We presented the results from this project in peer-reviewed formats, conference proceedings, and invited talks:

- [1] J. Woods, C. Sercel, T. Gill, E. Vigas, and B. Jorns, “Data-Driven Approach to Modeling and Development of a 30 kW Field-reversed Configuration Thruster,” Paper presented at the 36th International Electric Propulsion Conference, Vienna, Austria, IEPC-2019-717, 2019
- [2] J. Woods, C. Sercel, T. Gill and B. Jorns, “Equivalent Circuit Model for a Rotating Magnetic Field Thruster,” Paper presented at the 2021 AIAA Propulsion and Energy Forum, AIAA-2021-3400, 2021.
- [3] J. Woods, “Performance of a Rotating Magnetic Field Thruster,” PhD Dissertation, University of Michigan, 2022.
- [4] C. Sercel, J. Woods, T. Gill, and B. Jorns, “Performance measurements of a 5 kW-class rotating magnetic field thruster,” Paper presented at the 2021 AIAA Propulsion and Energy, AIAA-2021-3384. 2021. Awarded Best Paper.
- [5] T. Gill, C. Sercel, J. Woods, and B. Jorns, “Experimental Characterization of Efficiency Modes in a Rotating Magnetic Field Thruster,” Paper presented at the 2022 AIAA SciTech, AIAA-2022-2191, 2022

3. Impacts

We describe in the following key impacts that have emerged from this work.

3.1. Development of the principal disciplines of the project

Our efforts led to several new advances in methods and insights that may have a broader impact on the field of electric propulsion and low temperature plasmas. We itemize these key

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developments in the following:

- **Performed first uncertainty quantification on Hall thruster model predictions.** This methodology and technique likely will be leveraged in future development programs. Indeed, this approach has formed the basis of a recently awarded NASA institute to model the impact of facility effects on high power thruster operation.
- **Performed first non-invasive measurements of electron transport directly in Hall thruster channel.** This result showed for the first time the transient nature of the anomalous transport in Hall thrusters. Our work subsequently has informed modeling and experimental efforts in the broader community.
- **Demonstrated solution to problem of anomalous transport in hollow cathodes.** Our insights have led to closure models with sufficient fidelity that we now can predict key aspects of cathode dynamics. Our work subsequently has been leveraged by NASA, peer institutions in the US, and in Europe to improve cathode model fidelity.
- **Performed first experimental assessment of growth and nonlinear saturation turbulence in Hall thrusters.** These measurements were critical to informing our closure models. More broadly, the techniques we pioneered to make these measurements can be leveraged to characterize instabilities in a wide range of plasmas.
- **Developed first first-principles model for cathode coupling voltage response to facility pressure.** Our model provides a tool for any thruster developer to map measurements from ground-based facilities to space. The methodologies we developed to do this extrapolation under uncertainty are being leveraged in a recently awarded NASA institute to model the impact of facility effects on high power thruster operation.
- **Introduced to field of electric propulsion the use of Bayesian inference for model-based uncertainty quantification.** While this is a less measurable contribution, before this effort, model-based uncertainty was largely ignored in the broader community. Simulation results were reported with no or limited error bars. We believe our work has helped highlight the importance and enabling features of performing rigorous uncertainty quantification.
- **Established new self-consistent models for transport in Hall thrusters.** While we cannot claim we have solved the problem of anomalous electron transport in Hall thrusters, our work has yielded new closure models with a higher degree of fidelity than any model previously proposed. Our work, and particularly our methodology for discovering new models for transport, has thus moved the field closer to achieve the ultimate goal of fully predictive engineering models.
- **Published first ever direct performance measurements of novel thruster technology.** Before the start of this effort, there were no published thrust-stand measurements of an RMF thruster. Our work has served to establish the baseline for this technology for field.

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- **Developed first reduced fidelity model for RMF thruster.** While the performance of the RMF thruster is underwhelming, our model has provided a template for identifying new designs. This will enable in principle the rapid optimization and demonstration of this technology. If successful, this device could offer potentially revolutionizing capabilities for next-generation propulsion.

3.2. Describe the impact in this reporting period on the development of human resources

This work supported directly the doctoral work of five students: Zachariah Brown, Thomas Marks, Dr. Marcel Georgin (graduated 2020), Dr. Ethan Dale (graduated 2021), and Dr. Josh Woods (graduated 2022). This work indirectly supported the work of current doctoral candidates Christopher Sercel and Tate Gill.

4. Changes

There were no changes to the scope of work proposed under this work. There was a no cost extension that was granted due to the pandemic. The grant was extended by an additional six months.

5. Technical updates

The following section contains the figures highlighted in the preceding discussion of accomplishments. The captions provide additional context on the information displayed.

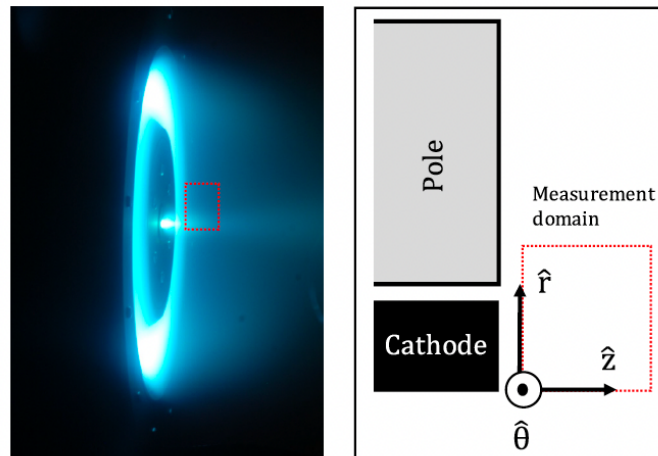


Figure 1: Hall effect thruster showing the cathode and the domain of interest for modeling the electron dynamics for this device.

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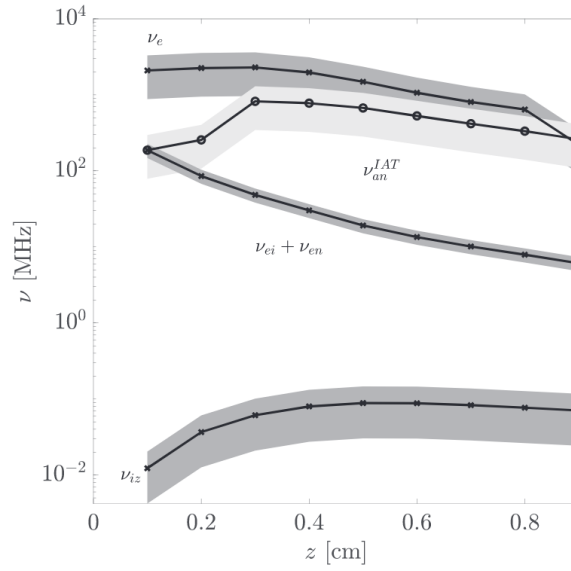


Figure 2: Measured electron collision frequency in plume of a hollow cathode along centerline as a function of distance from cathode exit. ν_e denotes the measured electron collision frequency, ν_{en} is the theoretical electron-neutral collision frequency, ν_{ei} is the theoretical electron-ion collision frequency, ν_{iz} is ionization frequency, and ν_{an}^{IAT} denotes the effective collision frequency attributed to the presence of ion acoustic turbulence. This result illustrates the non-classical nature of the electron resistivity in the cathode plume (proportional to collision frequency) and shows that the wave-driven ion acoustic effects are the dominant driver for the total collision frequency. This experiment was performed with a hollow cathode in a stand-alone, i.e. without a thruster, configuration.

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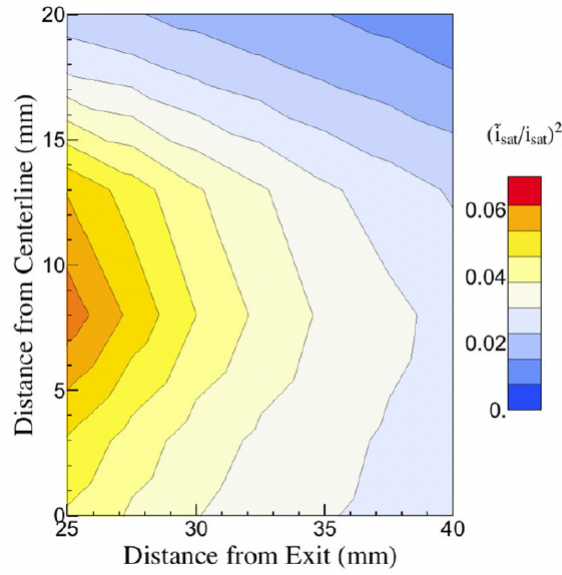


Figure 3: Measurements of turbulent energy of ion acoustic turbulence (IAT) oscillations in plume of hollow cathode as a function of distance from the cathode exit plane. This experiment was performed with a cathode operating in conjunction with a Hall thruster.

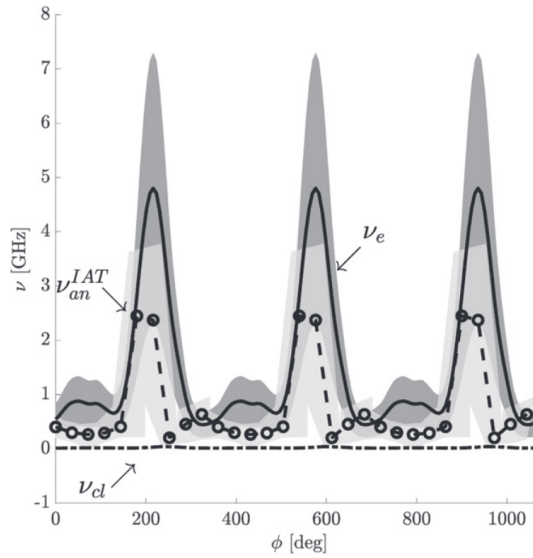


Figure 4: Time resolved measurements of the total electron collision frequency (ν_e) and IAT-induced collision frequency (ν_{an}^{IAT}) in the plume of a hollow cathode. The bottom axis is expressed in terms of phase of the plume mode oscillation. This result illustrates that even on the time scale of large amplitude transients, the effective electron resistance is driven by the presence of the IAT.

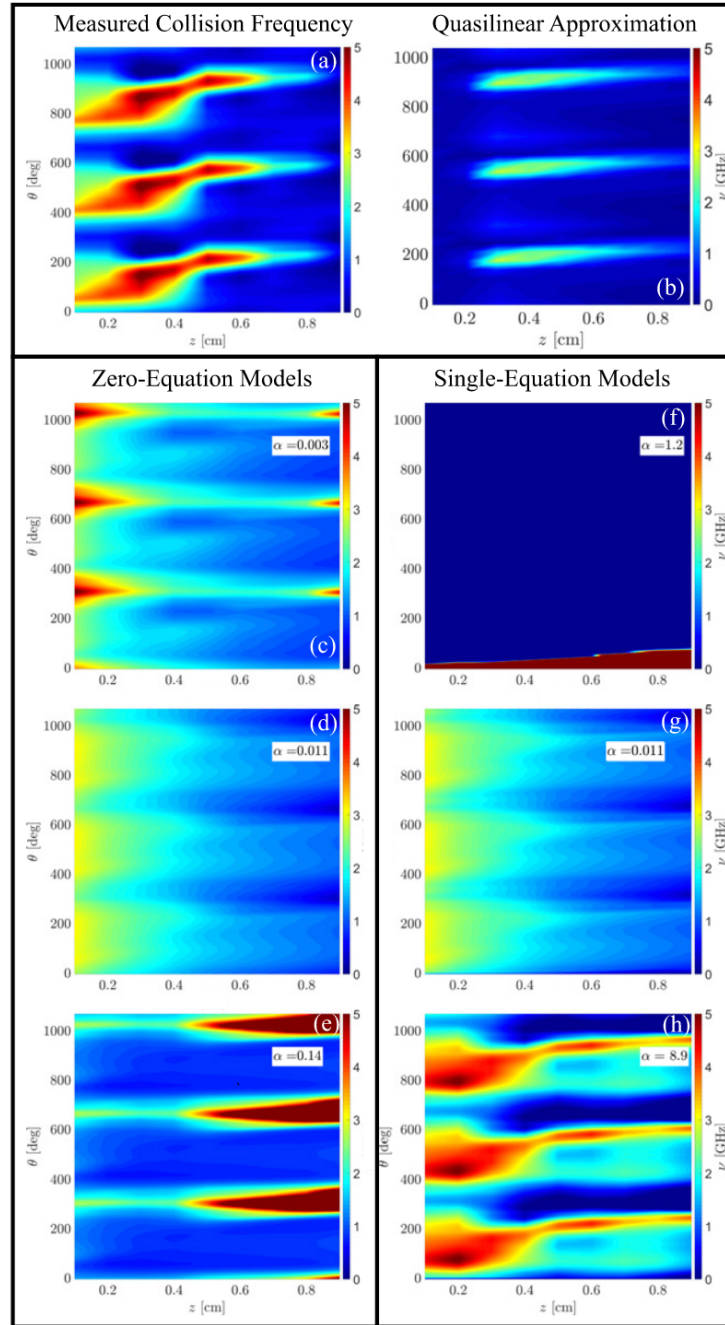


Figure 5: Top: measured electron collision frequency and collision frequency from IAT as a function of phase of plume mode oscillation and location in cathode plume. Bottom: predictions from various proposed zero and one-equation closure models. The quantitative agreement between the model in (h) and experiment indicates the quality of this closure model. These comparisons were performed for a cathode operating in a stand-alone configuration.

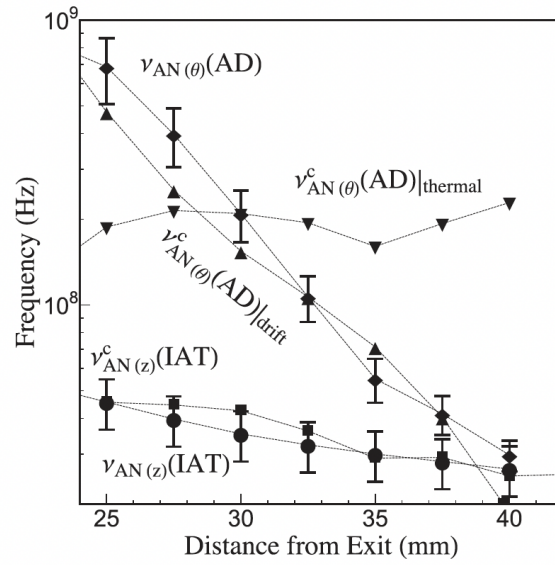


Figure 6: Comparison of measured and predicted electron collision frequency along centerline of a hollow cathode operating with a Hall thruster discharge. $\nu_{AN(\theta)}(AD)$ is a measured cross-field effective collision frequency due to anti-drive waves, $\nu_{AN(z)}(IAT)$ is an effective collision frequency along the applied magnetic field due to ion acoustic turbulence. Quantities with a “c” superscript denote proposed closure models based on assumptions about the underlying plasma turbulence. As can be seen, two of the closure models exhibit excellent agreement with measurement, providing validation for these approaches to simulating the non-classical electron dynamics.

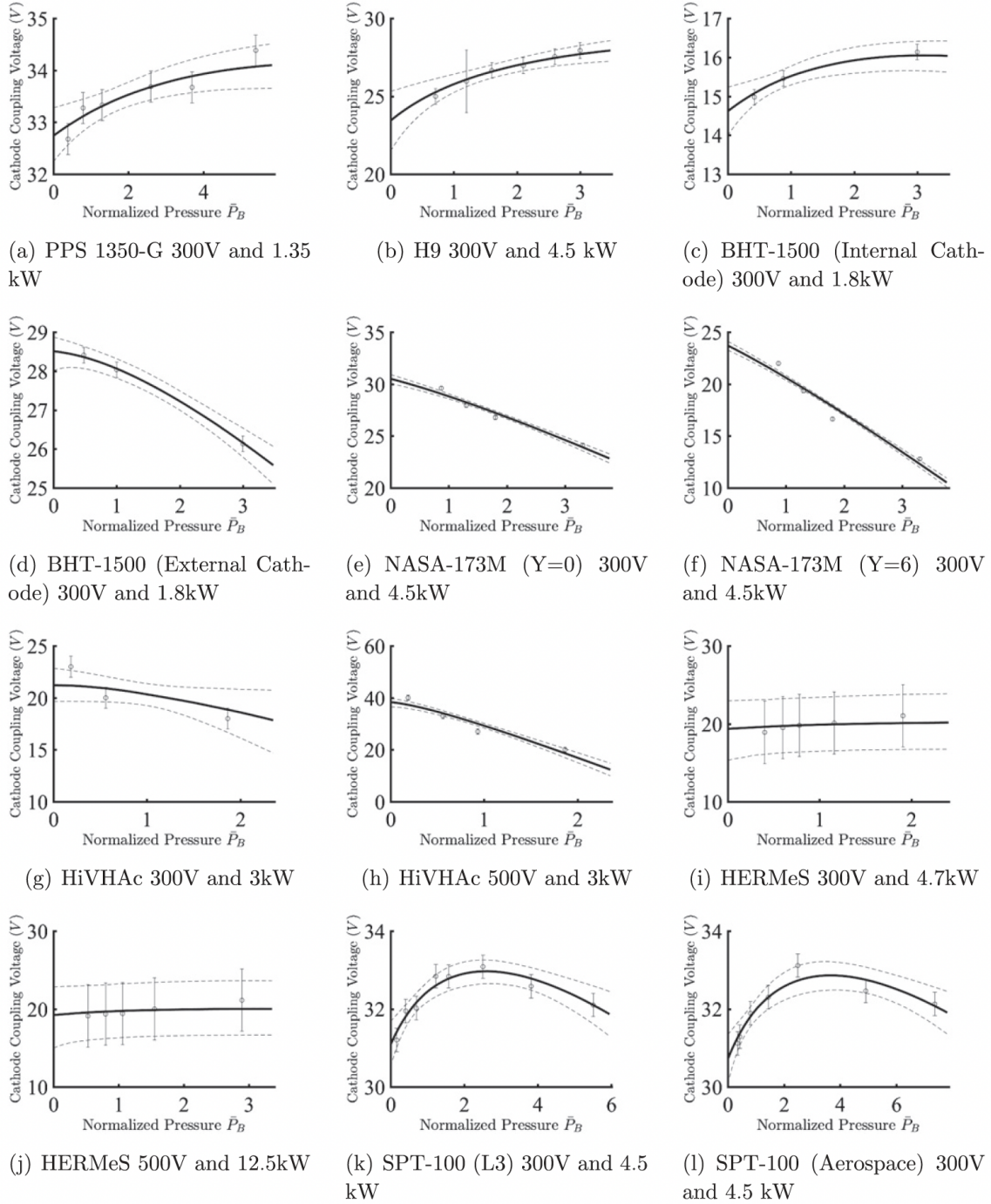


Figure 7: Experimental measurements of cathode coupling voltage as a function of facility pressure for 15 Hall thruster configurations. Solid lines denote median predictions from our model while dashed lines indicate 5 and 95% credible intervals for these predictions. The model is extrapolated to zero pressure thus providing a prediction for on-orbit behavior. This model for facility effects was enabled by the closures we developed and validated in previous work.

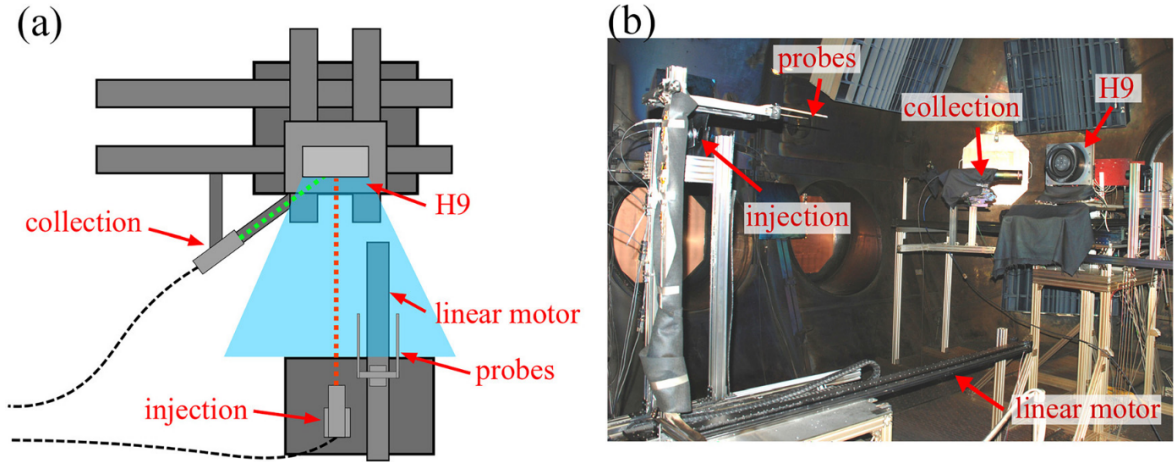


Figure 8: a) Top-down schematic of the experimental configuration we employed to measure plasma properties along the channel centerline of a 9-kW class Hall thruster. b) Image of the experimental layout in the Large Vacuum Test Facility at the University of Michigan.

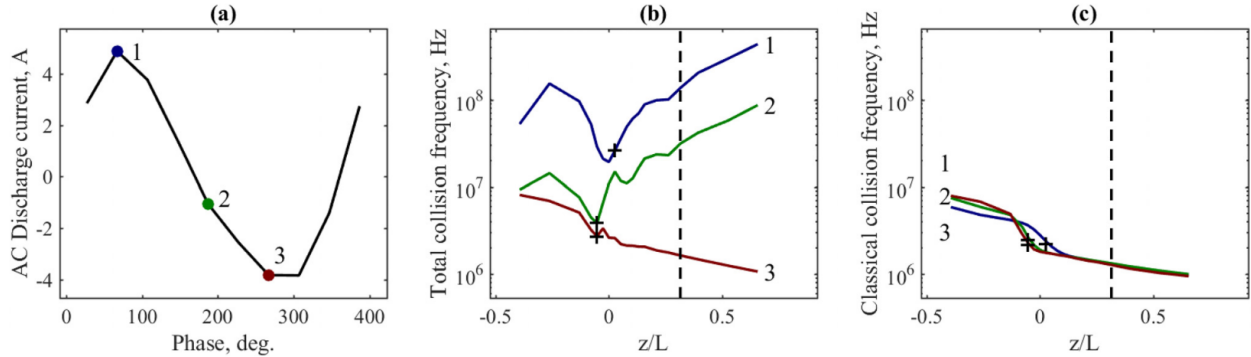


Figure 9: Measurements along channel centerline as a function of distance from the thruster exit plane of the (a) total discharge, (b) total electron collision frequency, and (c) classical collision frequency. The spatial measurements are time-indexed according to the phase of the large amplitude breathing mode oscillation illustrated in (a). This result illustrates our ability to measure the electron collision frequency non-invasively and with time resolution. The physical significance of this result is that it showed experimentally that the anomalous collision frequency in Hall thrusters can vary by orders of magnitude on the time scale of the breathing mode oscillation. This observation was critical for helping inform closure models for this effect.

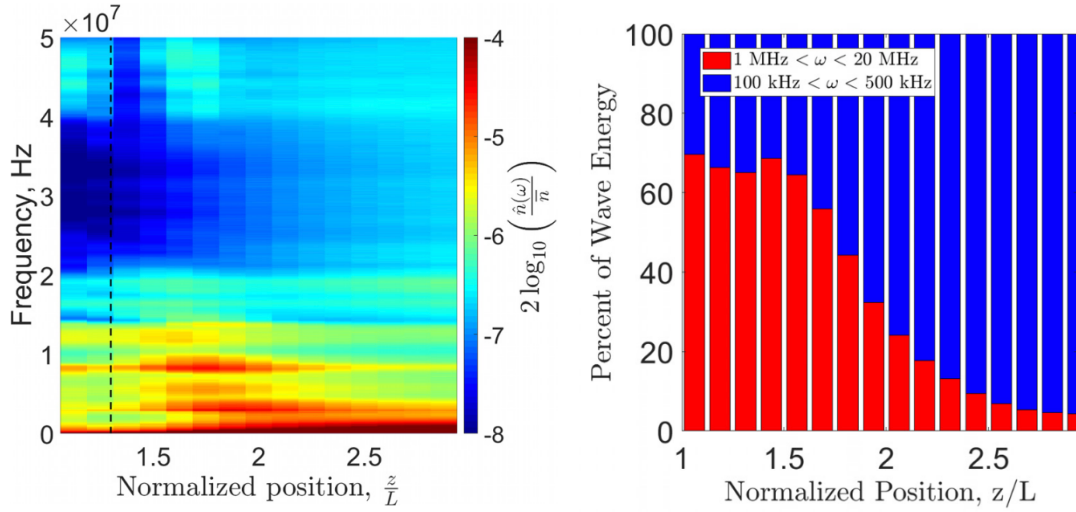


Figure 10: Left: experimental measurement of the power spectrum in the electron cyclotron drift instability along the channel centerline of a Hall thruster as a function of distance from the thruster exit plane. This result shows correlational evidence of an inverse energy cascade from higher frequency (shorter wavelength) to smaller frequency (longer wavelengths). Right: The fraction of total energy in the ECDI power spectrum in long (blue) versus short (red) wavelength modes as a function of position along channel centerline. This result also suggests the presence of an inverse energy cascade. The significance of this result in the context of this effort is that it helped inform models for the mechanisms that saturate the growth and amplitude of the waves responsible for cross-field transport. We leveraged these results to invoke several assumptions that led to new closure models for the electron dynamics.

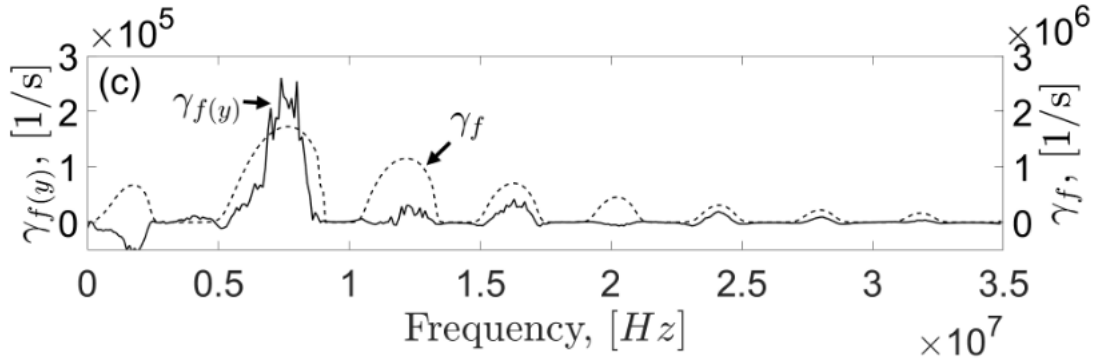


Figure 11: Comparison of the measured ($\gamma_{f(y)}$) and theoretical (γ_f) growth rates for the electron cyclotron drift instability in the channel of a Hall effect thruster. These results show that the measured growth is an order of magnitude lower than the prediction from linear theory. This novel experimental insight has been a major factor in informing our most sophisticated closure models for the electron transport.

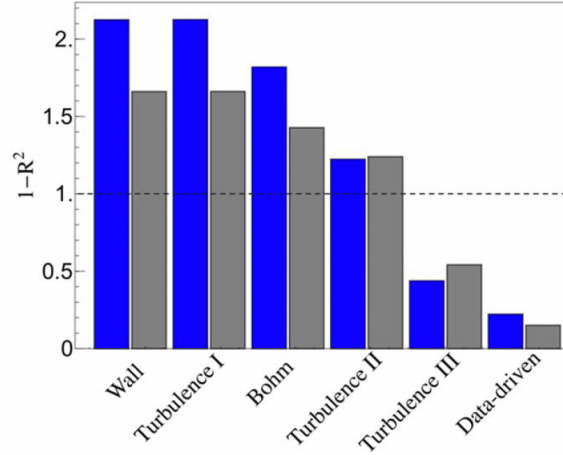


Figure 12: Sum of residuals resulting from comparing closure model predictions and time-averaged measurements of the anomalous electron collision frequency in the channel of a Hall thruster. We found the data-driven model from an application of machine learning to a training data set. The other models are physics-based closures previously proposed by other authors. Our result shows that the data-driven method outperforms all other closure models in terms of accuracy of prediction for steady-state data.

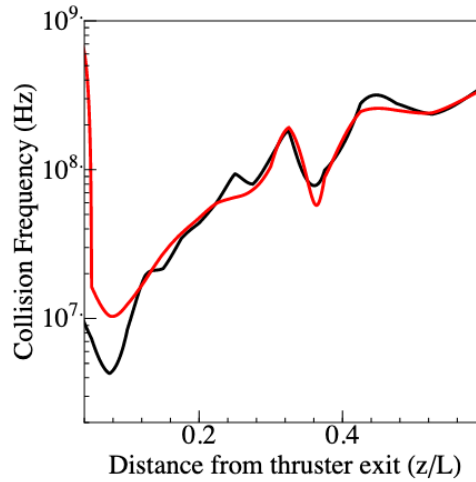


Figure 13: Comparison of experimental measurement (black) and closure model predictions (red) for the anomalous electron collision frequency along channel centerline in a Hall thruster. This closure is a two-equation model inspired by our experimental measurements of the plasma turbulence. The quantitative agreement suggests this model may be a plausible candidate to recreate the electron dynamics in the channel.

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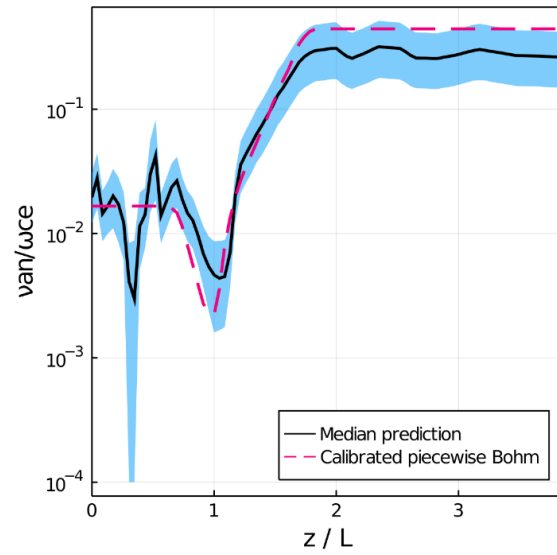


Figure 14: Comparison of electron collision frequency along channel centerline in a Hall thruster inferred from experimental data (red) and the predictions from a data-driven closure model (black). The model predictions are subject to uncertainty stemming from uncertainty about the values of the model parameters. The solid line denotes the median prediction while the blue shading denotes the 5 and 95% credible intervals. This result is significant in the context of the project as it was one of the first attempts, to our knowledge, to incorporate model and physics-based uncertainty into Hall thruster model predictions.

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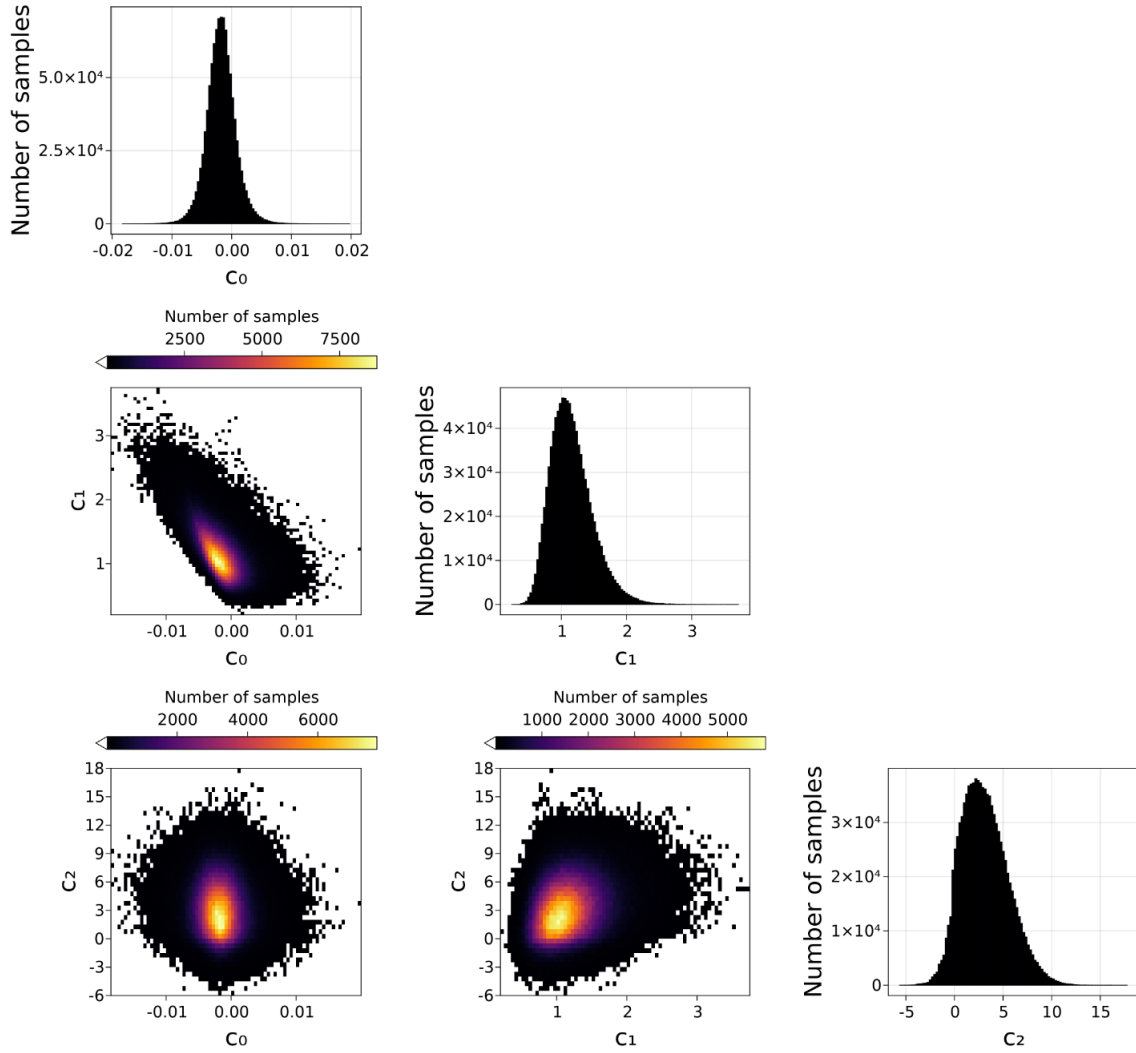


Figure 15: Examples of joint and marginalized posterior distributions for three model parameters in a data-driven closure model for electron transport in a Hall effect thruster. These results reflect the confidence in the model parameters considering the experimental data we employed to regress the data. We in turn were able to sample from these distributions when implementing this closure model self-consistently in a Hall thruster code, Hall2De, to generate predictions under uncertainty for thruster operation.

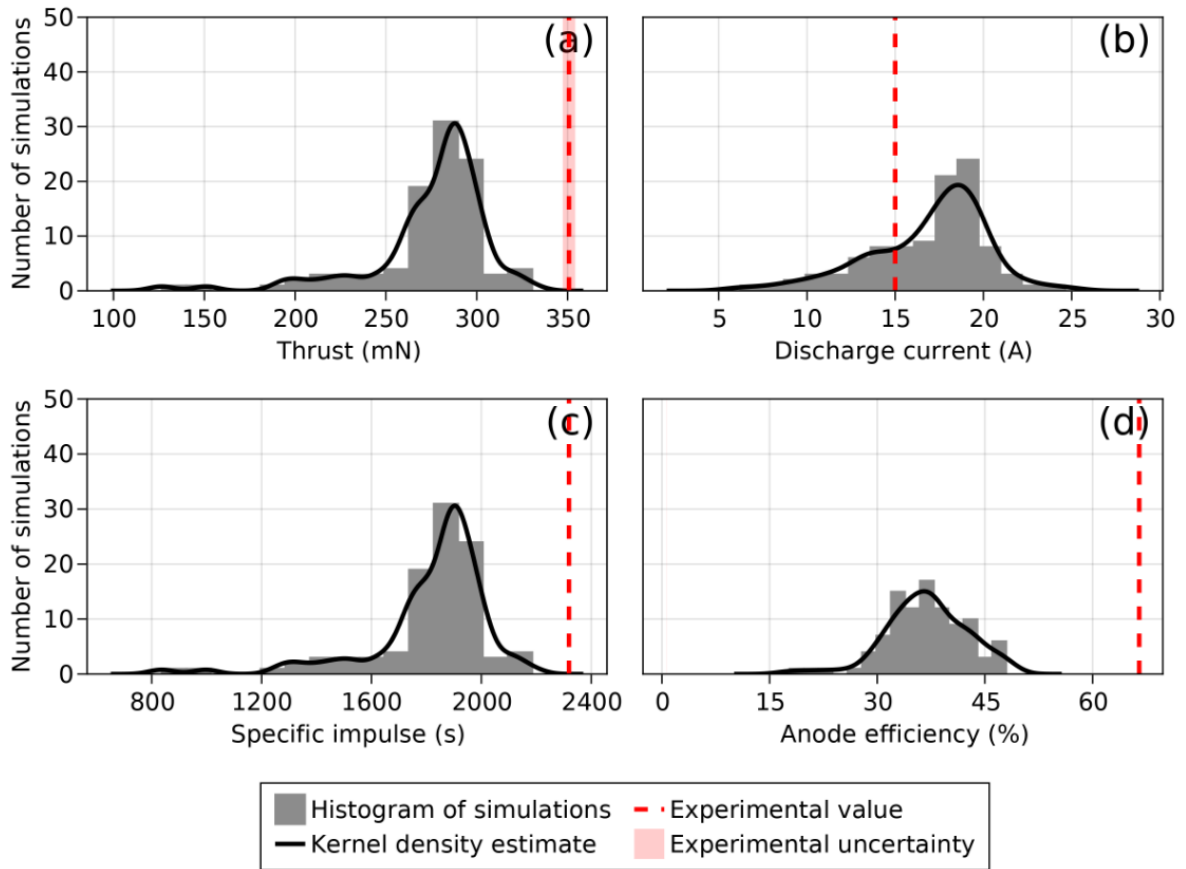


Figure 16: Probabilistic predictions from self-consistent Hall thruster model using one of our data-driven closure models for electron transport. The predictions are presented as probability distributions which reflects uncertainty in the values of the model parameters. The experimental measurements for this operating condition are shown as red vertical lines for comparison. The self-consistent medians do not match the experimental measurements, suggesting this closure model’s predictive capability is limited. With that said, the significance of this result is that it represents one of the first attempts to generate model predictions under uncertainty for Hall thruster performance.

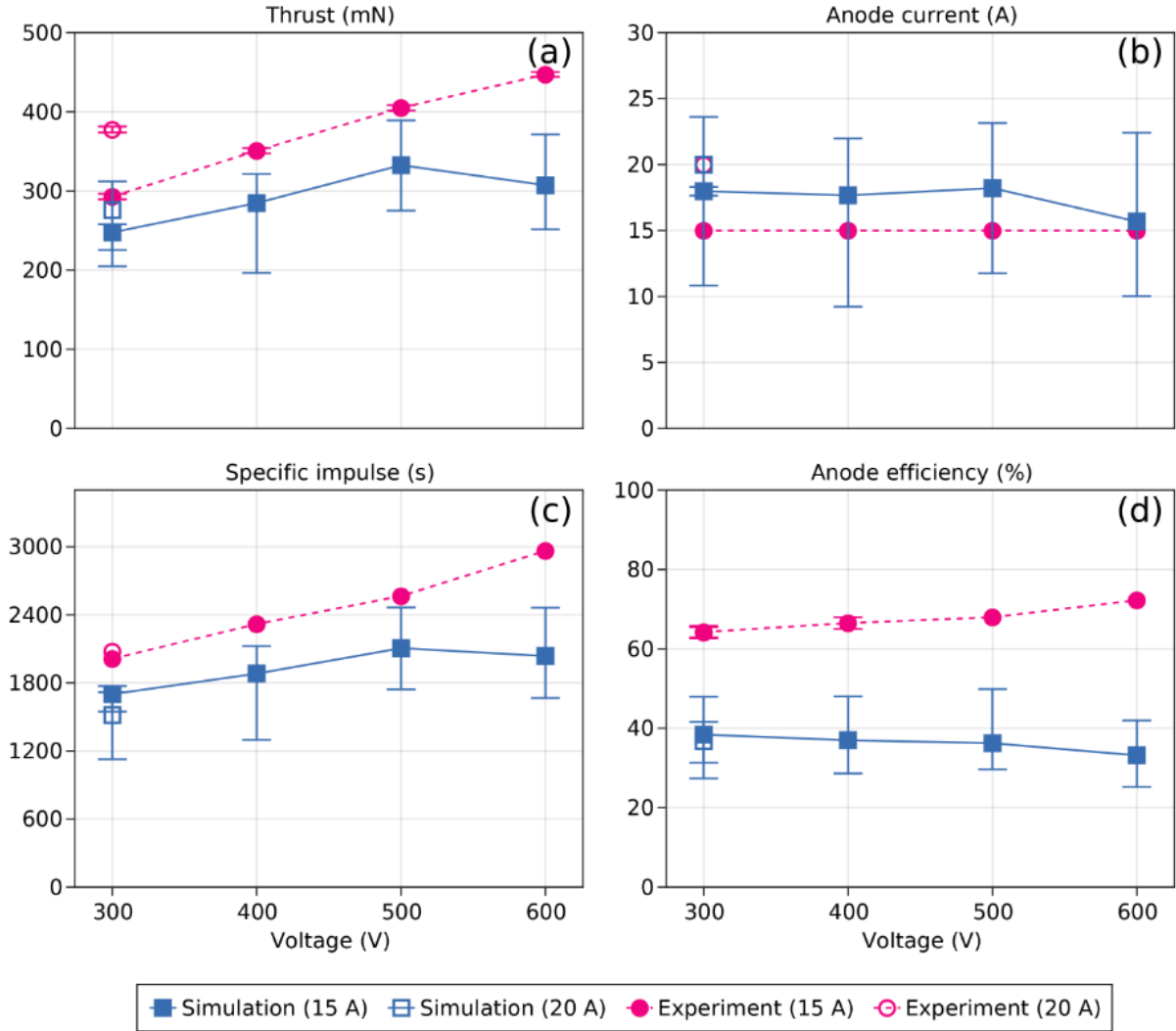


Figure 17: Comparison of model predictions (blue) and experimental measurements (red) of performance of a Hall effect thruster as a function of operating condition. The error bars denote 5 and 95% credible intervals for the predictions. The Hall thruster model, Hall2De, employed in this case had a data-driven closure for the electron collision frequency. The credible intervals stem from uncertainty in the model parameters. While the median model predictions do not match experiment, qualitatively the simulations follow within experimental error the trends in three of the performance metrics. This result is significant as represents a critical target application for our approach, i.e. the ability to leverage models to predict within experimental uncertainty the performance of thrusters when extended to new operating regimes.

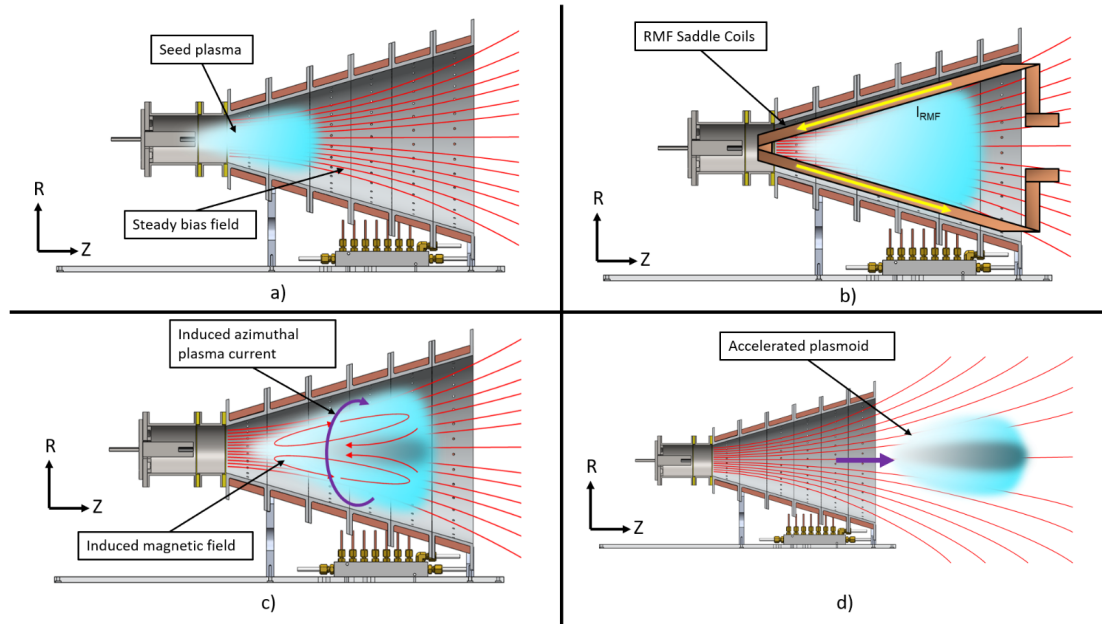


Figure 18: Side view cross-section in the r - z plane of an RMF thruster illustrating the key steps in its operation: a) ionized gas injected into the discharge chamber with a steady bias magnetic field with radial gradient; b) RMF coils are discharged; c) An azimuthal current is generated in the plasma, which also induces an opposing magnetic field; d) The plasma is accelerated out of the thruster.

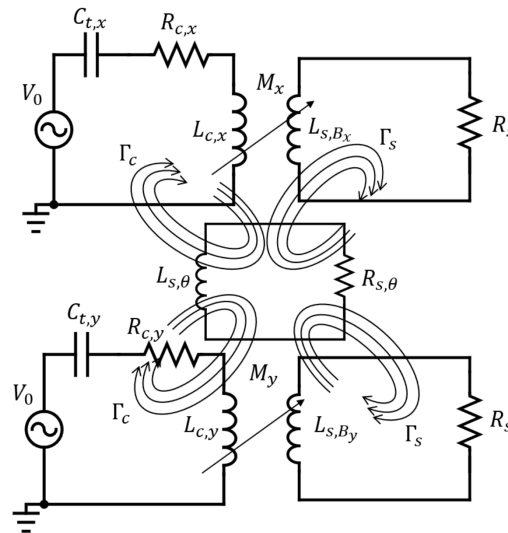


Figure 19: Equivalent circuit model for RMF thruster. The antennae current loops are the two circuit elements on the left. The current loops representing axial currents driven in the plasma are on the right. The center loop represents the azimuthal current driven in the plasma. “x” and “y”

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indices denote the directions of the antenna loop, “s” denotes plasma slug, and θ denotes component in the azimuthal direction. The lump circuit elements have their standard definitions while Γ denotes a gyrator terms. The arrows denote magnetic coupling between different elements of the circuit. We used this equivalent circuit representation combined with Kirchhoff’s laws to derive a set of ODEs for the current and voltage in the thruster and for key performance metrics including thrust, efficiency, and specific impulse.

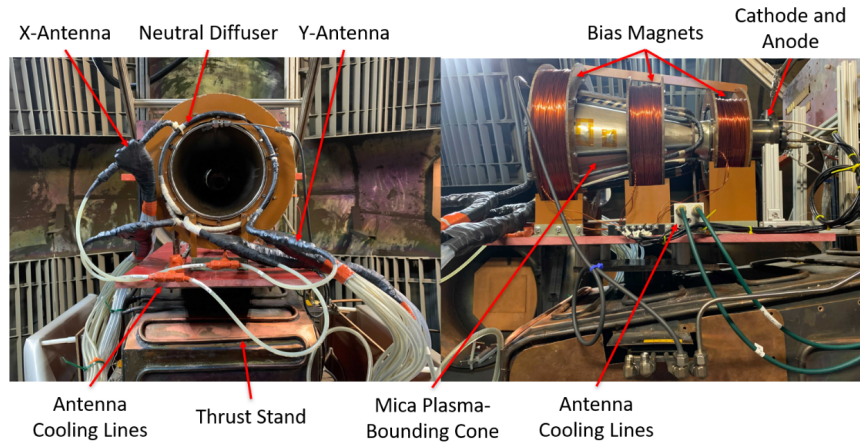


Figure 20: Image of the RMF test article developed at the University of Michigan for this test effort. This device is capable of 5 kW steady-state operation. With this system and the supporting diagnostic infrastructure, we were able to publish for the first time direct (i.e. with thrust stand) performance measurements of this device.



Figure 21: 10 s exposure image of the RMF thruster operating at the Large Vacuum Test Facility at the University of Michigan.

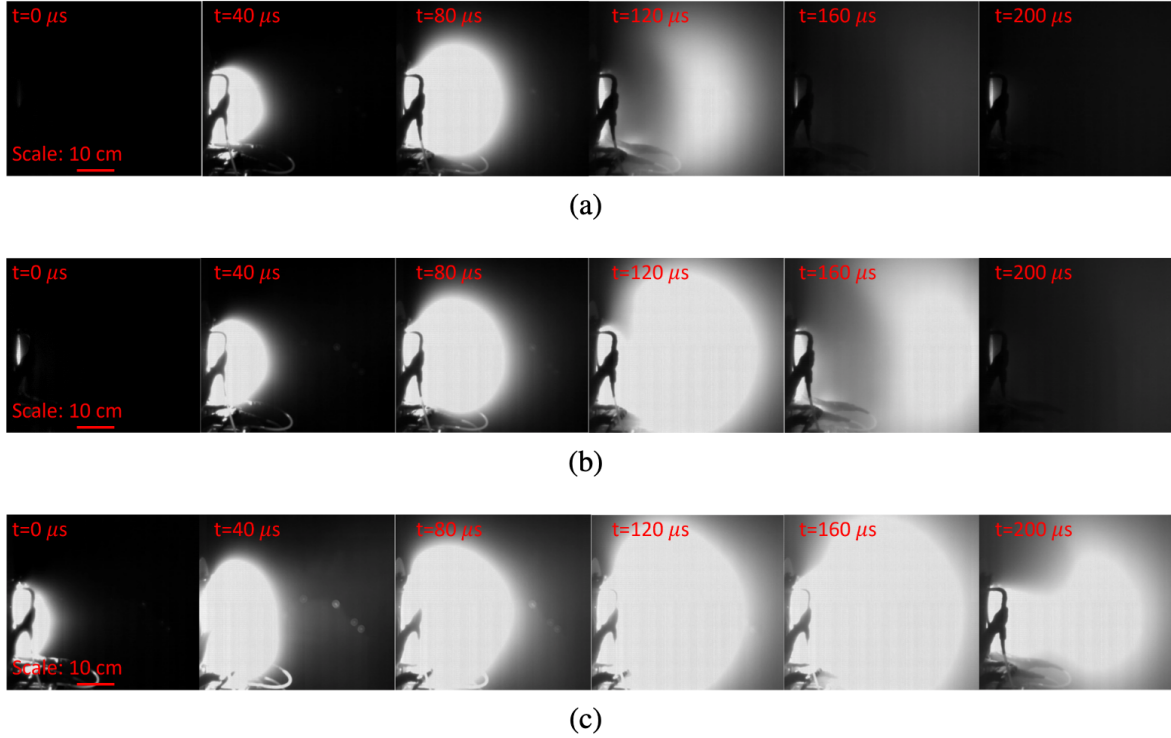


Figure 22: High speed images of an accelerated plasmoid from the RMF thruster. This illustrates the formation of the plasmoid and its ejection from the thruster geometry.

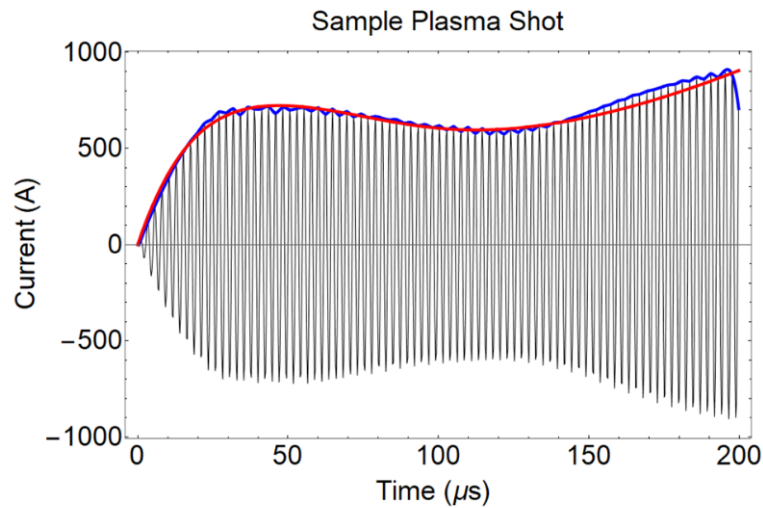


Figure 23 Comparison of measurement of current oscillations (blue) and equivalent circuit model fit (red) for a current trace during an RMF pulse. The ability of the model to match the simulation serves as validation of this approach and suggests the model, despite its simplicity, is able to capture key aspects of the acceleration process.

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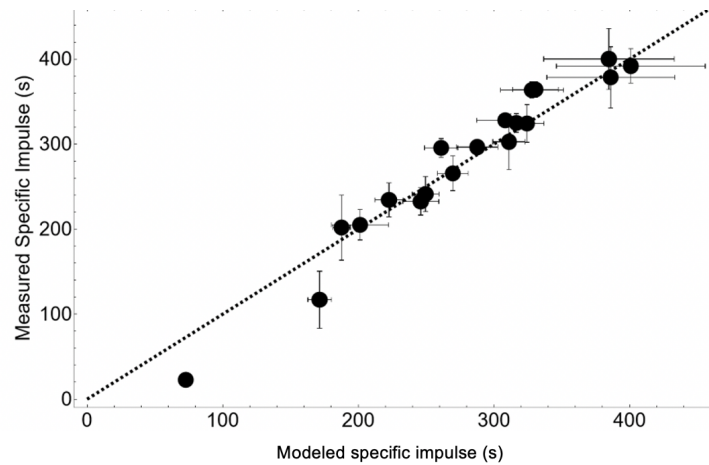


Figure 24: Comparison of the measured specific impulse and modeled specific impulse for the RMF thruster. This result is significant as it shows that the equivalent circuit model, which was trained exclusively on current traces, can accurately capture key aspects of thruster performance. This is a critical capability for leveraging the model to inform design decisions to optimize performance.

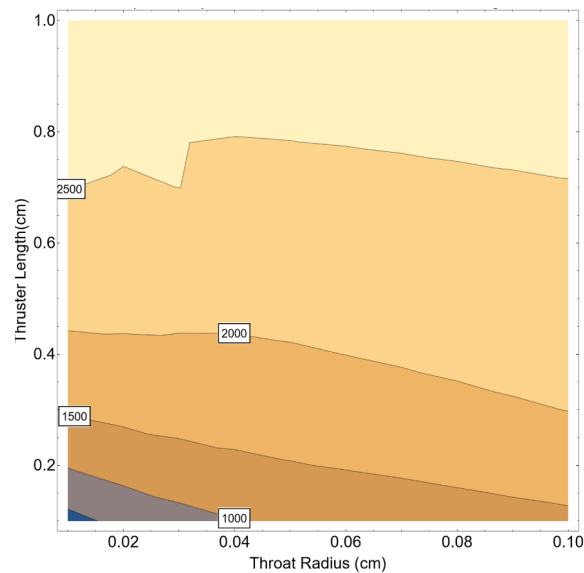


Figure 25: Model-based parametric study of the RMF’s predicted specific impulse as a function of radius and thruster length. This result is based on using the mean best fit model parameters inferred from experimental campaigns and extrapolating the model predictions to new design configurations. This result illustrates one of the key goals of this effort: the ability to use reduced fidelity models combined with simplified closure models to inform design decisions to improve performance. These results are being leveraged to inform another iteration for the thruster that will be tested in a campaign that is scheduled to occur after the conclusion of this effort.