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# Optimal Sensor Location for Distributed Parameter Systems

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The long term goal of this research is a unified approach to estimation and sensor design for distributed parameter systems. Estimation is of interest to numerous communities outside of control, for example, in the study of fluid dynamics and improvements in building comfort and efficiency. Proper sensor placement is part of effective estimation. This research program includes a number of important applications; reducing bridge vibrations, thermostat placement to improve building efficiency and sensor placement to study temperature and currents in large lakes. Progress towards the long-term goal of better estimation in distributed parameter systems requires fundamental theoretical contributions, in particular to estimation of nonlinear systems.

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# OPTIMAL SENSOR LOCATION FOR DISTRIBUTED PARAMETER SYSTEMS

## AFOSR GRANT FA9550-16-1-0061
## Program Officer: Fred Leve

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Final Report

## 1 Summary

The long term goal of this research is a unified approach to estimation and sensor design for distributed parameter systems. Estimation is of interest to numerous communities outside of control, for example, in the study of fluid dynamics and improvements in building comfort and efficiency. Proper sensor placement is part of effective estimation. This research program includes a number of important applications; reducing bridge vibrations, thermostat placement to improve building efficiency and sensor placement to study temperature and currents in large lakes. Progress towards the long-term goal of better estimation in distributed parameter systems requires fundamental theoretical contributions, in particular to estimation of nonlinear systems.

# 2 Results

For control systems that are distributed in space, such as structural vibrations, and fluid dynamics, the locations of the control hardware, the actuators and sensors, are important design variables. Performance of controlled systems was shown in earlier work of the Principal Investigator (PI) to be affected by actuator location. Computation with examples motivated by applications showed estimator accuracy to be affected by sensor location [8,11,12]. The optimal locations of actuators and sensors are often different from locations chosen based on physical intuition. This shows that analysis of control hardware placement is crucial, as it is often to difficult move hardware once it is placed.

A mechatronic approach, where the controller design is integrated with design, and estimation with sensor design, has been followed by the PI. Previously funded research by the PI and collaborators provided a theoretical framework for the well-posedness of various actuator location problems and also conditions under which approximations provide correct results. Theory for sensor design that minimizes the error variance for estimation of a linear system was completed [6,8]. These same publications also describe a computational framework.

Optimality equations for optimal actuator/controller design in semi-linear systems have been obtained. It was demonstrated that the results apply to a number of practical problems [3,7]. Results on calculation of optimal shapes has been obtained [2,9,13]. It is expected that collaborative research with Dante Kalise (Imperial College, U.K.) on optimal shape calculation for nonlinear dynamics will continue.

Many systems, for example non-divergent fluid flow, lithium-ion electrochemistry and many chemical reactions are modelled by coupled differential and algebraic equations. Such system present both theoretical and computational challenges. A comparison of several estimation algorithms, Extended Kalman Filter (EKF) and Unscented Kalman Filter (UKF) based on conversion to standard differential form versus the original formulation was done [4]. Results indicate that for comparable computation time, implementation of an estimator with the original form provides more accurate estimation.

There is little theory for estimation of nonlinear systems, even for ordinary differential equation models. Theoretical results for extended Kalman fil-

tering have been obtained [1,5]. These results are new even for ordinary differential equations in that it is not assumed that the system is locally observable. Furthermore, the results apply to semi-linear PDEs.

Research with Marius Tucsnak (Université of Bordeaux, France) on extending theory/numerics to thermostat placement as part of energy efficient buildings is in progress. The mathematical problems are challenging since the models are coupled nonlinear PDEs and both control and sensing occur on the boundary. This means that the control and observation operators are unbounded on the energy space. Establishing well-posedness and continuity as well as a computation method is non-trivial but it is anticipated that the collaboration with Tucsnak well as interaction with colleagues who have expertise in fluid dynamics will be fruitful. We have already completed the theory for linear systems. Computational work has started, as well as an extension of the theory to nonlinear models.

The results in optimal actuator and sensor location are the foundation of a strategy for joint actuator/sensor location as part of a mechatronic approach to output feedback controller design. This is currently being investigated by a Master's student in the context of reduction of bridge vibrations. This is in collaboration with a civil engineering colleague Sriram Narasimhan (UCLA).

Many of the results obtained during this grant and the previous AFOSR grant held by the PI are explained in the recently published book

> K. A. Morris, *Controller Design for Distributed Parameter Systems*, Springer, 2020.

# 3 Refereed Publications

1. S. Afshar, F. Germ and K. A. Morris, "Extended Kalman filter based observer design for semilinear infinite-dimensional systems", *IEEE Trans. on Automatic Control,* to be submitted.

2. M. S. Edalatzadeh, D. Kalise, K. A. Morris and K. Sturm, "Shape optimization of actuators over Banach spaces for nonlinear systems", *IEEE Confence on Decision and Control,* submitted.

3. M. S. Edalatzadeh and K. A. Morris, "Optimal controller and actuator design for nonlinear parabolic systems", *IEEE Trans. on Automatic*

*Control,* submitted.

4. B. Sherbak and K. A. Morris, "Implementation of Kalman filtering for differential algebraic equations", *Proc. of American Control Conference*, 2021.

5. S. Afshar, F. Germ and K. A. Morris, "Well-posedness of Extended Kalman Filter equations for semilinear infinite-dimensional systems", *Proc. of IEEE Conference on Decision and Control,* 2020.

6. K. A. Morris,"Optimal Output Estimation for Infinite-dimensional Systems with Disturbances", *Systems and Control Letters*, vol. 146, 2020.

7. M. S. Edalatzadeh and K. A. Morris, "Optimal Actuator Design for Semi-linear Systems", *SIAM Jour. of Control and Optimization*, 2019.

8. Minxin Zhang and Kirsten Morris, "Sensor Choice for Minimum Error Variance Estimation", *IEEE Trans. on Automatic Control*, vol. 63-2, pg. 315-330, 2018.

9. Shu-xia Tang and Kirsten Morris, "Optimal Sensor Design for Infinite-Time Kalman Filters", *Proc. of IEEE Conference on Decision and Control*, 2017.

10. K.A. Morris, "The Role of Sensor and Actuator Models in Control of Distributed Parameter Systems", *Emerging Applications in Control and Systems Theory*, ed. P. Misra and S. Yurkovich, Springer, 2017.

11. M. Zhang and K. A. Morris, "Effect of Sensor Noise on Estimation of Diffusion", *Proc. of IFAC Conf. on Control of Partial Differential Equations,* Italy, 2016.

12. Weiwei Hu, Kirsten Morris and Yangwen Zhang, "Sensor Location in a Controlled Thermal Fluid", *Proc. of IEEE Conference on Decision & Control,* USA, 2016.

13. Kirsten Morris and Ambroise Vest, "Design of damping for optimal energy dissipation of vibrations", *Proc. of 2016 IEEE Conference on Decision & Control,* USA, 2016.

# 4 Invited presentations

The following invited presentations disseminated research funded by this grant. Travel expenses for presentations indicated by (*) were funded by the inviting organization.

1. *Optimal Controller and Actuator Design for Partial Differential Equations,* plenary lecture, Canadian Mathematical Society Winter Meeting, Toronto, Canada, Dec. 6, 2019. (*)

2. *short course on Controller Design for Partial Differential Equations,* Canadian Mathematical Society Winter Meeting, Toronto, Canada, Dec. 6, 2019. (*)

3. *Concurrent optimal actuator and controller design for partial differential equations,* Third AFOSR Monterey Workshop on Computational Issues in Nonlinear Control, Monterey, USA, Oct. 7 , 2019. (*)

4. *Design of Estimators for Partial Differential Equations with Disturbances*, SIAM Conference on Control and its Applications, China, June 20, 2019.

5. *Optimal sensor/estimator design for distributed parameter systems,* 3rd IFAC Workshop on Control of Partial Differential Equations, Oaxaca, Mexico, May 24, 2019. (*)

6. *Concurrent optimal controller and actuator design for partial differential equations,* Séminaire de Physique Mathé matique, Université de Bordeaux, January 15, 2019.

7. *Issues in estimator and sensor design for PDEs,* Workshop on Analysis, Control and Inverse Problems for PDEs, Naples, Italy, Nov. 28, 2018. (*)

8. *Estimation for infinite-dimensional systems*, Applied functional analysis seminar, University of Wuppertal, Wuppertal, Germany, Oct. 18, 2018. (*)

9. *Optimal sensor location,* 2nd Workshop on Stability and Control of Infinite-Dimensional Systems (SCINDIS-2018), Wurzburg, Germany, Oct. 11, 2018. (*)

10. *Estimation of infinite-dimensional systems with unknown disturbances*, 23rd International Symposium on Mathematical Theory of Networks and Systems, Hong Kong, July 17, 2018.

11. *Concurrent Optimal Control and Actuator Design for Semi-linear Systems*, 23rd International Symposium on Mathematical Theory of Networks and Systems, Hong Kong, July 17, 2018. (talk given by M. S. Edalatzadeh)

12. *Optimal sensor design for partial differential equations,* Mathematical Congress of the Americas, July 27, 2017.

13. *Distributed Parameter Systems: from theory to applications*, tutorial workshop speaker, IEEE Conference on Decision and Control, Melbourne, Australia, Dec. 10, 2017.

14. *Actuators and sensors in control of distributed parameter systems*, Institute for Systems Research, Distinguished Lecturer Series, University of Maryland, USA, October 19, 2017.(*)

15. *Optimal actuator/sensor location in distributed parameter systems*, keynote talk, Workshop on Emerging Applications in Control and Systems Theory, University of Texas (Dallas), September 28, 2017.(*)

16. *Optimal sensor design*, Conference on distributed parameter systems, keynote talk, Bordeaux, France, July 5, 2017. (*)

17. *Optimal Sensor Design,* Mathematical Congress of the Americas, Montréal, July 27 2017.

18. *The role of sensors and actuators in control of infinite-dimensional systems*, KAUST, Saudi Arabia, April 19, 2017.(*)

19. *The role of actuators and sensors in control of distributed parameter systems*, Institute for Mathematics and Applications, Minneapolis, USA, March 17, 2016.

20. *Using approximations in controller synthesis for systems modelled by partial differential equations*, Institute for Mathematics and Applications, Minneapolis, USA, Feb. 4, 2016.

21. *The role of actuators and sensors in control of distributed parameter systems*, University of Groningen, Netherlands, May 17, 2016. (*)

22. *Optimal actuator design with a linear-quadratic performance measure*, 22nd International Symposium on Mathematical Theory of Networks and Systems, Minneapolis, USA, July 14, 2016.

# 5 Honors & awards

- IEEE Fellow
- SIAM Fellow
- IEEE CSS Distinguished Member Award 2020
- Initiative d'Excellence, visiting professor, Université de Bordeaux, 2018-2019

# 6 Supported Personnel

| | |
|---|---|
| Adrian Gammon | undergraduate research assistant |
| Fabian Germ | Master's student |
| Fei Liu | Master's student |
| Xuanrui Lui | Master's student |
| Bogdan Sherbak | Master's student |
| Purushottam Sinha | Master's student |
| Shuxia Tang | PDF |
| Ambroise Vest | PDF |
| Minxin Zhang | Master's student |
| Stan Zonov | Master's student |

# 7 Conference & Workshop Organization

- Program co-chair, *3rd IFAC Workshop on Control of Partial Differential Equations*, Oaxaca, May 2019
- Co-organizer, *Sensor Location in Distributed Parameter Systems*, Institute for Mathematics & Applications, September 2017 (funded by IMA and AFOSR)
- Co-organizer, *Women in Control: New Trends in Infinite Dimensions*, Banff, July 2017 (funded by PIMS)
- Co-organizer, *Control of Distributed Parameter Systems,* Institute for Mathematics & Applications, March 2016 (funded by IMA)
- Steering Committee, *International Symposium on Mathematical Theory of Networks and Systems*, 2016-
- Chair of Steering Committee, *Conference on Distributed Parameter Systems,* 2020-

## AFOSR Point of Contact

Fred Leve Program Manager, Control & Dynamics AFOSR/RSL.

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