AFRL-AFOSR-VA-TR-2022-0227

Adaptive Horizon Model Predictive Control and Regulation, Short Horizon Estimation

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06/10/2022
Final Technical Report

Air Force Research Laboratory
Air Force Office of Scientific Research
Arlington, Virginia 22203
Air Force Materiel Command

REPORT DOCUMENTATION PAGE

PLEASE DO NOT RETURN YOUR FORM TO THE ABOVE ORGANIZATION.

1. REPORT DATE	**2. REPORT TYPE**	**3. DATES COVERED**	
20220610	Final	**START DATE** 20170701	**END DATE** 20210630

4. TITLE AND SUBTITLE
Adaptive Horizon Model Predictive Control and Regulation, Short Horizon Estimation

5a. CONTRACT NUMBER	**5b. GRANT NUMBER**	**5c. PROGRAM ELEMENT NUMBER**
	FA9550-17-1-0219	61102F
5d. PROJECT NUMBER	**5e. TASK NUMBER**	**5f. WORK UNIT NUMBER**

6. AUTHOR(S)
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8. PERFORMING ORGANIZATION REPORT NUMBER

9. SPONSORING/MONITORING AGENCY NAME(S) AND ADDRESS(ES)	**10. SPONSOR/MONITOR'S ACRONYM(S)**	**11. SPONSOR/MONITOR'S REPORT NUMBER(S)**
Air Force Office of Scientific Research 875 N. Randolph St. Room 3112 Arlington, VA 22203	AFRL/AFOSR RTA2	AFRL-AFOSR-VA-TR-2022-0227

12. DISTRIBUTION/AVAILABILITY STATEMENT
A Distribution Unlimited: PB Public Release

13. SUPPLEMENTARY NOTES

14. ABSTRACT
In the past year, we have accomplished two of the goals of this project. The first goal was to develop and refine the Adaptive Horizon Model Predictive Control (AHMPC) methodology so that it can be used to stabilize fast processes. Recall AHMPC is a way to verify in real time that the Model Predictive Control (MPC) methodology is actually stabilizing the plant. It does this by adapting the MPC horizon length in real time keeping it as short as possible consistent with stabilization and feasibility. AHMPC does a simple check at each iteration to see if the current horizon is long enough. If the check is true, then the horizon is kept constant or shortened. If the check is false, then the horizon is lengthened.

15. SUBJECT TERMS

16. SECURITY CLASSIFICATION OF:			**17. LIMITATION OF ABSTRACT**	**18. NUMBER OF PAGES**
a. REPORT	**b. ABSTRACT**	**c. THIS PAGE**	UU	3
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19a. NAME OF RESPONSIBLE PERSON	**19b. PHONE NUMBER** *(Include area code)*
FREDERICK LEVE	696-9730

Final Report, AFOSR FA9550-17-1-0219

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In the past year, we have accomplished two of the goals of this project. The first goal was to develop and refine the Adaptive Horizon Model Predictive Control (AHMPC) methodology so that it can be used to stabilize fast processes. Recall AHMPC is a way to verify in real time that the Model Predictive Control (MPC) methodology is actually stabilizing the plant. It does this by adapting the MPC horizon length in real time keeping it as short as possible consistent with stabilization and feasibility. AHMPC does a simple check at each iteration to see if the current horizon is long enough. If the check is true, then the horizon is kept constant or shortened. If the check is false, then the horizon is lengthened.

AHMPC needs a positive definite function that is a control Lyapunov function in some neighborhood of the operating point and a feedback control law such the control Lyapunov function is a valid Lyapunov function under the closed loop dynamics. One way of getting such a pair is take the Jacobian linearization of the dynamics around the operating point, choose a quadratic Lagrangian and solve the resulting Linear Quadratic Regulator (LQR) problem. The positive definite quadratic form that is the optimal cost of the LQR is the needed control Lyapunov function and optimal linear feedback is the desired control law. But the neighborhood on which the LQR closed loop nonlinear dynamics is asymptotically stable can be very small necessitating a long horizon.

One can find a higher degree Taylor approximations to the nonlinear optimal cost and optimal feedback by the discrete time extension of Al'brekht's method that was developed by Aguilar and Krener supported by an early AFOSR grant. But a Taylor approximation to the nonlinear optimal cost of degree higher than two need not be positive definite. Therefore, it cannot be used as the terminal cost in an MPC scheme. To remedy this deficiency, we developed a technique which we call Completing the Squares (CSQ). CSQ takes a polynomial of degrees 2 through d+1 whose quadratic part is positive definite and adds terms of degrees d+2 through 2d. The expanded polynomial is a sum of squares and hence at least nonnegative definite. This expanded polynomial can be used as the terminal cost in an MPC scheme.

We showed that the degree five Taylor polynomial feedback can more smoothly stabilize a double pendula with shorter horizons than the LQR feedback in a noise free example. In a noisy example the LQR feedback failed to stabilize while the degree five feedback was able to do so.

AHMPC is a method based on three techniques that we developed with AFOSR. The first technique is a way to verify in real time that stabilization is occurring. The second technique is to extend Al'brekht's method to discrete time problems. The third technique is completing the square.

AHMPC is a way to stabilize a nonlinear plant to an operating point. Using the nonlinear regulation methodology developed by Francis, Byrnes and Isidori it can be extended to Adaptive Horizon Model Predictive Regulation (AHMPR), a methodology that can be used to track reference signals or reject disturbances.

The dual of MPC is Moving Horizon Estimation (MHE) which is a way of estimating the state of a plant using measurements of the control input and observed output over a backward horizon. Slack variables are added to the dynamics and the measurements over this backward horizon and the estimate of the state at the start of the backward horizon. Then one solves and optimization problem of minimizing a quadratic form in the slack variables consistent with the control inputs and observed outputs over the backward horizon. The end point at the current time of the minimizing state trajectory is the MHE estimate. So at each time instant one has to solve a nonlinear program over the backward horizon. The shorter the horizon the easier the nonlinear program is to solve.

This is actually a very old idea that goes back to the least squares estimate of the trajectory of planetoid Ceres by Gauss. Mortenson rediscovered it and introduced a dynamic programming approach which if it could be solved would reduce the horizon to just one time step. But Mortenson's approach requires solving in real time a dynamic programming equation driven by the control inputs and observed outputs, a daunting task. The other goal of this project was to develop a methodology that simplifies solving the dynamic programming equation. Again we did it by finding equations for the Taylor polynomial of the solution of the dynamic programming equation. At the lowest degree one, this reduces to an Extended Kalman Filter (EKF) in information form. We compared this EKF with a higher degree three approach on a difficult problem, estimating the state of the three dimensional Lorenz Attractor from a one dimensional measurement. The average error of the degree three filter was 40% less than the average error of the degree one filter (EKF).

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