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Time-Varying Feedback for Robust Regulation in Prescribed Finite Time

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14. ABSTRACT
Starting from the idea of prescribed-time stabilization, present in the heuristic Proportional Navigation feedback law, which is a PD control law with gains that grow unbounded as the missile nears the target, in this project we generalized this idea to all
linear and certain nonlinear ODE systems, extended it to PDEs, and developed disturbance-robust versions for prescribed-time stabilization under both deterministic and stochastic disturbances.

Our generalization from second-order linear systems to general ODEs has entailed systematic scaling of gains on the state by powers of the function 1/(t0 + T - t), where t is time, t0 is the initial time, and T is the prescribed terminal time. To make our feedback laws implementable without a requirement of full-state measurement, we also developed prescribed-time observers, with a similar scaling of observer gains by powers of 1/(t0 + T - t). Generalization to PDEs requires an entirely different, back stepping-based approach, with damping that grows unbounded in the target system. With this approach, in spite of infinite sums with functions that grow to infinity at terminal time, the gains converge and the states and input not only remain bounded but converge to zero, with all their
derivatives also converging to zero.

Our approach achieves a perfect disturbance rejection at the terminal time T, regardless of the size of the disturbance. This exceeds the capability of sliding mode control and related time-invariant methods, which employ an infinite gain near the origin. Such methods require a known bound on the disturbance. In addition to perfect rejection of deterministic disturbances, with a suitable combination of stochastic backstepping and time-varying gains in the stochastic target systems, we achieve a perfect mean-square
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Abstract (280 words)

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Publications

Conference:

[1] D. Steeves, M. Krstic, and R. Vazquez, “Prescribed-time H1-stabilization of reaction-diffusion equations by means of output feedback,” *European Control Conference*, 2019.

[2] P. Krishnamurthy, F. Khorrami, and M. Krstic, “Robust output-feedback prescribed-time stabilization of a class of nonlinear strict-feedback-like systems,” *European Control Conference*, 2019.

[3] D. Steeves, M. Krstic, and R. Vazquez, "Prescribed-time stabilization of reaction-diffusion equation by output feedback,” *American Control Conference*, 2019.

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[5] P. Krishnamurthy, F. Khorrami, and M. Krstic, " Adaptive output-feedback prescribed-time stabilization of uncertain nonlinear strict-feedback-like systems,” *IEEE Conference on Decision and Control*, 2019.

[6] J. I. Poveda and M. Krstic, "An extremum seeking controller with semi-global practical prescribed finite-time convergence,” *American Control Conference*, 2020.

[7] D. Steeves, L. Camacho-Solorio, M. Krstic, "Boundary prescribed-time stabilization of a pair of coupled reaction-diffusion equations,” *American Control Conference*, 2020.

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[9] W.-Q. Li, M. Krstic, “Prescribed-time mean-square nonlinear stochastic stabilization,” *IFAC World Congress*, 2020.

[10] D. Steeves, L. Camacho-Solorio, M. Benosman, and M. Krstic, "Prescribed-time tracking for triangular systems of reaction-diffusion PDEs,” *IFAC World Congress*, 2020.

[11] D. Steeves and M. Krstic, "Prescribed-time stabilization of ODEs with diffusive actuator dynamics," *24th International Symposium on Mathematical Theory of Networks and Systems*, 2021.

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[13] J. Holloway and M. Krstic, “Prescribed-time output feedback for linear systems in controllable canonical form,” *Automatica*, vol. 107, pp. 77-85, 2019.

[14] J. Holloway and M. Krstic, “Prescribed-time observers for linear systems in observer canonical form,” *IEEE Transactions on Automatic Control*, vol. 64, pp. 3905-3912, 2019.

[15] P. Krishnamurthy, F. Khorrami, and M. Krstic, "Robust adaptive prescribed-time stabilization via output feedback for uncertain nonlinear strict-feedback-like systems," *European Journal of Control*, 2020.
[16] P. Krishnamurthy, F. Khorrami, and M. Krstic, "A dynamic high-gain design for prescribed-time regulation of nonlinear systems," *Automatica*, vol. 115, paper 108860, 2020.
[17] D. Steeves, M. Krstic, and R. Vazquez, "Prescribed-time estimation and output regulation of the linearized Schrödinger equation by backstepping," *European Journal of Control*, 2020.
[18] W.-Q. Li and M. Krstic, "Stochastic nonlinear prescribed-time stabilization and inverse optimality," *IEEE Transactions on Automatic Control*, to appear, 2021.
[19] N. Espitia, D. Steeves, W. Perruquetti, and M. Krstic, "Sensor delay-compensated prescribed-time observer for LTI systems," *IEEE Transactions on Automatic Control*, under review.
[20] W.-Q. Li and M. Krstic, "Prescribed-time output-feedback control of stochastic nonlinear systems," *IEEE Transactions on Automatic Control*, under review.

Books

[21] I. Karafyllis and M. Krstic, *Input-to-State Stability for PDEs*, Birkhauser, 2017.

AWARDS

2019 *W. T. and Idalia Reid Prize,* SIAM (Society for Industrial and Applied Mathematics)
2019 *Nonlinear Control Systems Award*, International Federation for Automatic Control (IFAC)
2019 *O. Hugo Schuck Best Paper Award*, American Automatic Control Council
2018 Foreign Member, *Serbian Academy of Sciences and Arts*
2018 *Distinguished Member*, IEEE Control Systems Society

Plenary talks

2020 *IEEE Conference on Industrial Electronics and Applications*, Norway
2019 *IEEE Conference on Control Technology and Applications (CCTA)*, Hong Kong
2019 *SIAM Conference on Control and Its Applications*, Chengdu, China
2019 *IFAC Workshop on Control of Systems Governed by Partial Differential Equations*, Oaxaca, Mexico
2018 *Mathematical Theory of Systems and Control (MTNS),* Hong Kong, 2018