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Hybrid Data-Driven Algorithms for Networked Multi-Agent Systems - Stability and Robustness

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14. ABSTRACT
Given the increasing complexity of networked multi-agent systems interacting with rational decision makers, including autonomous robots and human operators, as well as the increasing availability of information via sensors and communication systems, it is of great interest to design control and optimization algorithms that do not require a precise mathematical model of the system under consideration. Rather, these algorithms should be able to learn online key properties of the system via input-output measurements. These types of model-free mechanisms are usually called data-driven, and they are fundamental in many applications of interest to the Air Force. These applications include cooperative source seeking with obstacles, formation control in uncertain environments, dynamic resource allocation in multi-agent systems, and human-machine interaction in infrastructure systems. They are generally characterized by the presence of continuous-time dynamics with instantaneous state change as well, in addition to random phenomena modeling uncertain conditions and adversarial influences. Such features comprise a stochastic hybrid dynamical system, or inclusion, whose behavior can be very complex and difficult to characterize. Nevertheless, for the safe implementation of model-free and data-driven control and optimization algorithms, it is fundamental to provide useful analytical tools to assess the behavior and certify the robu

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1 Project abstract and objectives

1.1 Abstract

Given the increasing complexity of networked multi-agent systems interacting with rational decision makers, including autonomous robots and human operators, as well as the increasing availability of information via sensors and communication systems, it is of great interest to design control and optimization algorithms that do not require a precise mathematical model of the system under consideration. Rather, these algorithms should be able to learn online key properties of the system via input-output measurements. These types of model-free mechanisms are usually called *data-driven*, and they are fundamental in many applications of interest to the Air Force. These applications include cooperative source seeking with obstacles, formation control in uncertain environments, dynamic resource allocation in multi-agent systems, and human-machine interaction in infrastructure systems. They are generally characterized by the presence of continuous-time dynamics with instantaneous state change as well, in addition to random phenomena modeling uncertain conditions and adversarial influences. Such features comprise a stochastic hybrid dynamical system, or inclusion, whose behavior can be very complex and difficult to characterize. Nevertheless, for the safe implementation of model-free and data-driven control and optimization algorithms, it is fundamental to provide useful analytical tools to assess the behavior and certify the robustness of such systems. Following this reasoning, the main goal of the proposed research is to develop analytical and constructive tools for the development of model-free and data-driven stochastic hybrid dynamical systems in the context of multi-agent systems, with stability, convergence, and robustness guarantees. To achieve this objective, we will rely on and enhance the PI's recent work on a general framework for stochastic hybrid dynamical systems and model-free hybrid control systems, which allows for non-unique solutions and thus the interplay between randomness and independent decision making in multi-agent systems. Our goal is to generate novel analytical tools for the analysis of data-driven algorithms in the context of stochastic hybrid dynamical systems, as well as constructive procedures for the design of robust model-free and data-driven algorithms, with performance guarantees, for networked multi-agent dynamical systems.

1.2 Objectives

The objective of this research is to develop analysis and systematic design tools for data-driven stochastic hybrid multi-agent dynamical systems. Specific objectives include the following:

- Demonstrate how stochastic hybrid inclusions can be used to model data-driven dynamic multi-agent systems in a wide range of mission settings.
- Extend stability theory for stochastic hybrid inclusions in ways that are critical for the certification of stochastic hybrid multi-agent systems.
- Analyze the interaction of hybrid dynamics, rational decision makers, and optimization algorithms in the context of data-driven stochastic hybrid multi-agent systems.
- Analyze and design in the realm of high-fidelity models of data-driven multi-agent systems that include constraints in communication, sensing, and dynamics.
- Emphasize the deleterious effects of adversaries in data-driven stochastic hybrid multi-agent systems.
- Link the objectives of data-driven multi-agent systems to well-established stability notions for stochastic hybrid systems, and develop new notions as appropriate.
- Check progress by applying principles to specific multi-agent problems, including coordinated source seeking with obstacles, indirect coordination of autonomous agents via incentives, and stabilization of multi-agent systems on surfaces like the torus.

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4 Accomplishments

4.1 Optimization-related algorithms

Ad-hoc analyses of resetting mechanisms in optimization problems have appeared in the literature over the years. We showed how a hybrid systems framework can further motivate, extend, and certify the efficient behavior of optimization algorithms that employ resets. Our first contribution in this area [3] emphasized that no dissipation during flows is even necessary when using resets, emphasizing how using Hamiltonian flows might improve performance. This work was followed up with additional contributions to heavy-ball methods in convex optimization in [6] and [37]. In the setting of non-convex optimization, we showed how the framework of stochastic hybrid systems can inspire novel global optimization algorithms, including global optimization problems on compact manifolds. This work, which focuses on establishing almost sure global convergence to the problem's set of minimizers, appeared in [8], [12], and [18]. We also contributed new findings on the use of dynamical systems to solve optimization problems with constraints, through projections. In this work, we made several intriguing connections to our previous work on anti-windup synthesis for control systems with input constraints. This work is documented in [4], [24], and [35]. We also contributed new results to the extremum-seeking literature, showing how classical ideas can be extended to hybrid plants [17] and to systems with input dead zones [38].

4.2 Networked and distributed algorithms

Some of the optimization algorithms that we developed were considered for the case of distributed, networked systems. We especially focused on situations where the network was subject to imperfections, including adversarial attacks, sampling, and transmission delays. In this setting, we considered stability problems, convex optimization problems, and Nash equilibrium seeking. In the case of persistent adversarial attacks, we showed that, as long as the frequency and duration of attacks were not too long compared to times of attack-free behavior, desirable behavior of the distributed algorithms could be asserted. This result for convex optimization was reported in [2] while for Nash equilibrium seeking it appears in [21]. A result on stability of networked systems with transmission delays and sampling in reported in [20].

4.3 Event-triggered algorithms

Event-triggered control is a paradigm that is very useful in situations where it is important to save power when transmitting control data over a network. Event-triggered control systems are best modeled as hybrid systems, where plant variables change continuously while communication variables make jumps. We developed foundational techniques for analyzing and synthesizing event-triggered controllers in [30], developed new algorithms for event-triggered control in [28] and considered event-triggered control for stochastic systems in [39] using our novel framework for stochastic hybrid systems.

4.4 Reset controllers

Our work on optimization algorithms that employ resets inspired a new research direction on implementing reset controllers using differential inclusions, rather than hybrid systems. This modified viewpoint alleviates the need for temporal or spacial regularizations to remove the possibility of Zeno solutions in reset control systems modeled with hybrid systems. We initiated this line of research in [11] and developed the theory for very general, passivity-based control systems in [13]. We anticipate that the area of reset control systems will received renewed interest through these contributions and expect reset controllers to become more popular and effective over time.

4.5 Hybrid systems stability theory

We continued to develop tools for stability analysis for hybrid systems. For general hybrid systems with memory, including systems with delays, we develop Matrosov's theorem for asymptotic stability in [15] and [34]. We also provided new analysis techniques for singularly perturbed hybrid systems in [27]. For switched systems with distinct equilibria, we showed how to characterize a system's asymptotically stable set through a description of its Ω-limit set [31]. This analysis tool was applied in the setting of online optimization with persistent switches of certain parameters in the optimization problem in [29].

4.6 Applications to vehicles

We made significant contributions to the problem of obstacle avoidance using hybrid (hysteresis-based) control. For example, we introduced the notion of synergistic control barrier functions, as a unification of control barrier functions and synergistic Lyapunov functions (which we introduced under previous AFOSR funding) in [32]. Synergistic Lyapunov functions provide an efficient coding of effective hysteresis mechanisms that induce robustness. This unification led to algorithms for safe maneuvering, i.e., the a path-following problem where speed along the path can be specified dynamically, using synergistic control barrier functions, as documented in [16]. We also showed how to use hybrid systems concepts to develop algorithms for path following when part of the path takes a vehicle outside of a domain where it is able to obtain reliable measurements of its state [1]. Multi-agent obstacle avoidance was addressed in [25]. Related ideas were used for robust orientation control for ships in work with colleagues from Norway in [10].

4.7 Estimation, observability, and moment matching

We developed several new results on state estimation and observability for linear, time-varying and hybrid systems using hybrid estimators. Hybrid constrained estimation was developed in [7] while observability for linear hybrid systems was considered in [9], 40], and [26], the latter for the case of systems with unknown inputs. In the setting of general discrete-time nonlinear systems, we developed necessary and sufficient conditions for nonlinear detectability through its relationship to incremental input-output stability [33]. In [36] we developed a comprehensive theory of moment matching for stochastic systems, as a means toward model reduction in stochastic systems.

4.8 Output regulation and reinforcement learning

Solutions to the output regulation problem, provided long ago for linear systems with linear exosystems, have remained elusive for general nonlinear systems. Lately, a new data-driven approach to this problem has been pursued. The natural modeling framework for a data-driven approach to output regulation for continuous-time nonlinear systems is that of hybrid systems, since the data-driven component typically involves storing and processing data discretely. We have contributed new results in this area, providing an adaptive solution that employs sampled-data least-squares identifiers [5], [23]. In addition, we have provided a comprehensive solution to the linear quadratic regulator problem for linear, periodic hybrid systems in [22].

5 Impacts

Development of the principal discipline(s) of the project

The significant impact on the principal discipline of the project has been emphasized in the accomplishments section.

Development of human resource

Over the duration of this grant, two graduate students (including one female student) developed new skills through research and dissemination of their results. The female student is expecting to graduate in Fall 2022 and expects to take an industrial position with a company focused on battery development. The other graduate student will likely graduate in 2024.

6 Changes

None.