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**Intelligent Distributed Systems**

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<b>14. ABSTRACT</b> The aim of this project has been to develop new tools and techniques for controlling distributed systems. The main objectives pursued were to devise distributed state observers and distributed feedback controller techniques. Breakthrough advances were made in both areas.					
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# ACCOMPLISHMENTS

- **Research Objectives:** The central objective of this project has been to develop new tools for analyzing distributed dynamical systems and for controlling them. Specific objectives included 1. developing techniques for estimating the state of a distributed linear system with a time-varying communication graph, 2. stabilizing a linear system with distributed feedback, and 3. studying the consensus problem with the added constraint of limited information transfer across the associated communication network.
- **Detailed Accomplishments:** During this project three research papers have been published as full-length journal articles, namely [1, 2, 3], and two have been accepted for publication, namely [4, 5]. Among these, the most significant are the results reported in [2] and [4]. In [2] we developed what we believe to be the world’s first provably correct distributed state estimator capable of estimating exponentially fast at a pre-assigned rate, the state of a  $m > 0$  channel,  $n$ -dimensional, continuous-time, jointly controllable, jointly observable, linear system with a time varying neighbor graph which is jointly connected at each time instant. The same idea applies also to distributed discrete-time models.  
In [4] it is shown that any jointly controllable, jointly observable, multi-channel, discrete or continuous time linear system with a strongly connected communication graph can be exponentially stabilized with any pre-specified convergence rate using a time-invariant distributed linear control. This is proved constructively using a distributed observer-based certainty equivalence control and the distributed observer described in [6].
- **Dissemination:** All of the aforementioned research was or will be presented at conferences [7, 8, 9, 10, 11] and was or will be published in leading journals [1, 2, 3, 4, 5]. Some of the work was also presented as invited lectures at universities or at special workshops.

# IMPACTS

- **Development of the principle disciplines of the project:** It is fair to say that [1] has expanded our understanding of applied game theory in a social setting while [3] pushes forward the state of the art in our understanding of the generic structure of linear systems. Both of these papers have appeared in refereed journals.

References [2] and [4] strongly impact on mainstream multi-agent control system theory in fundamental ways. In particular, as of this writing it appears that [2] is the *only published approach* to distributed observer design about which all of the claims mentioned above can be made. Meanwhile [4], is clearly *a breakthrough contribution*: as of this date there does not exist any other provably correct certainty-equivalence based solution to the distributed stabilization problem, except under restrictive conditions.

Perhaps the most widely studied problem in the area of intelligent distributed systems has been the “distributed consensus problem.” In its simplest form, the *consensus problem* is to devise a distributed algorithm which will enable a network  $\mathbb{N}$  of  $m > 1$  autonomous agents {e.g., drones, robots, processors, etc.} possessing consensus variables  $x_1, x_2, \dots, x_n$  to iteratively adjust their variables to a single common value by sharing information across a network. Consensus has been widely studied within computer science since the early 1980s because of its close relation to distributed computation. Around twenty years ago it was recognized that consensus had a much broader role to play in areas of interest to the dynamics

and control community such as swarm modeling, distributed optimization and distributed state estimation. One especially important consensus problem which only now is beginning to receive in depth attention deals with how to reach a consensus in the face of limited information sharing. One version of the problem is formulated and solved in [5]. While the results therein are preliminary, the basic problem is practical, intriguing and likely to inspire many to consider it further.

## CHANGES

Nothing to report.

## TECHNICAL UPDATES

Nothing to report.

## References

- [1] Y. Li and A. S. Morse. Games on signed graphs. *Automatica*, 140 C, June 2022.
- [2] Lili Wang, A. Stephen Morse, and Ji Liu. A hybrid observer for estimating the state of a distributed linear system. *Automatica*, 146:1–13, 2022.
- [3] F. Liu and A. S. Morse. Structural completeness of a multichannel linear system with dependent parameters. *IEEE Transactions on Automatic Control*, 67(1):267–278, 2022.
- [4] F. Liu, L. Wang, D. Fullmer, and A. S. Morse. Distributed feedback control of multi-channel linear systems. *IEEE Transactions on Automatic Control*, 2023. to appear.
- [5] J. Zhu, Y. Lin, J. Liu, and A. S. Morse. Reaching a consensus with limited information. *Systems and Control Letters*, 2023. to appear.
- [6] L. Wang and A. S. Morse. A distributed observer for an time-invariant linear system. *IEEE Transactions on Automatic Control*, 63(7):2123–2130, 2018.
- [7] L. Wang, D. Fullmer, F. Liu, and A. S. Morse. Distributed control of linear multi-channel systems: Summary of results. In *Proceedings of the 2020 American Automatic Control Conference*, pages 4576–4581, 2020.
- [8] F. Liu and A. S. Morse. Overcoming the obstacle of fixed eigenvalues in decentralized control. In *Proceedings of the 2020 American Automatic Control Conference*, page 13, 2020. poster.
- [9] F. Liu and A. S. Morse. What does it take to control a multi-channel linear system with distributed feedback? In *Proceedings of the 2020 IEEE Conference on Decision and Control*, pages 5188–5193, 2020.
- [10] J. Zhu, Y. Lin, J. Liu, and A. S. Morse. Reaching a consensus with limited information. In *Proceedings of the 2022 IEEE Conference on Decision and Control*, pages 4581–4584, 2022.
- [11] K. Frieswijk, L. Zino, A. S. Morse, and M. Cao. Modeling the co-evolution of climate impact and population behavior: A mean-field analysis. In *Proceedings of the 2023 Congress of the International Federation of Automatic Control*, 2023. to appear.