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Robust Characterization of Nonlinear Structural Dynamics in Extreme Environments

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14. ABSTRACT The development of air-breathing hypersonic vehicles represents a 'perfect storm' of challenges for the aerospace structures community. This is because they must be lightweight and slender, but able to sustain extreme thermal, aerodynamic, and mechanical loads. To analyze, design, and maintain a fleet of hypersonic vehicles there is therefore a need to achieve the following overarching goals: (1) advance the understanding of the dynamics of slender aerostructures, and (2) build accurate and fast predictive tools for design engineers. A specific challenge problem of interest is the interaction of buckling due to constrained thermal expansion of skin panels at the high temperatures that occur during high-speed flight, the associated effects on aerodynamic behavior, and the potential for violent snap-through response which may cause sudden damage or accelerated fatigue. This scenario provides a useful research test bed that pushes current prediction tools beyond their limits. This report describes three projects directed at achieving the overarching goals. The first project focuses on predicting internal stresses in skin panels during extreme nonlinear response. This is an important step predicting useful service life. The second project addresses issues in the development of NonLinear Reduced Order Models (NLRMs), which have been an essential tool in unlocking the ability to simulate nonlinear structural dynamics but require some special considerations. Finally, the third project focuses on extending NLRMs to better capture local defects or features. This is an important step as NLRMs are not naturally suited to capturing fine characteristics, though these may be important in controlling damage and life-cycle.			
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1. Motivation

The development of air-breathing hypersonic vehicles represents a ‘perfect storm’ of challenges for the aerospace structures community. This is because they must be lightweight, but able to sustain extreme thermal, aerodynamic, and mechanical loads. To analyze, design, and maintain a fleet of hypersonic vehicles there is therefore a need to achieve the following goals: (1) advance the understanding of the dynamics of slender aerostructures, and (2) build efficient and accurate predictive tools for design engineers.

A specific challenge problem of interest is the interaction of buckling due to constrained (or mismatched) thermal expansion of skin panels at high temperatures, the associated effects on aerodynamic behavior, and the potential for violent snap-through response which may cause sudden damage or accelerated fatigue. This scenario provides a useful research test bed that pushes current prediction tools beyond their limits. Furthermore, it is also of practical relevance as it may represent a controlling limit state in real-world design. A possible real-world example is the permanently warped acreage of the SR-71 as shown in Fig. 1, a characteristic also seen on other high-speed aircraft. Though this may have other causes, such as extreme maneuver loading, a likely culprit is accumulated plastic deformation from mismatched thermal expansion and buckling.



Figure 1: Permanently deformed panels on SR-71 at Museum of Flight in Seattle, WA.

2. Research Framework

The physics domains and the coupling interactions between them for aerostructures in extreme environments are illustrated in Fig. 2. Due to the complex geometrically- and materially-nonlinear behavior of aerostructures in extreme environments, closed-form ‘general solutions’ of this problem that predict deformation, stress, and failure will be intractable. This work instead seeks to advance the fundamental understanding and simulation tools by exploring a suite of problems of increasing complexity, herein termed the Challenge Problem Framework. This

framework is not new for hypersonic structures research, as it has grown naturally out of prior efforts by the Air Force Research Laboratory (AFRL) and multiple university research teams. A central feature of these efforts has been combined experimental-numerical projects, in which full-field experimental data of temperatures, pressures, and deformations has been collected by AFRL to provide validation data for developed simulation tools. Several challenge problems have and continue to be explored, ranging from nonlinear structural dynamics of post-buckled beams with known forcing functions (simplest, though not simple) to fully coupled aero-elastic panel deformation in high-speed wind tunnels (complex). These have allowed for advancement within the individual domains and their coupling around the perimeter of Fig. 2. One of these advancements has been in the refinement of NonLinear Reduced Order Models (NLRMs), which may be used to quickly predict extreme structural dynamics over long time spans not tractable with high-fidelity Finite Element Models (FEMs).

The challenge problem of interest for this work, shown in Fig. 3, additionally incorporates complex local features and the potential for damage progression. The refined zone (shaded region) in the center represents a local area where high-fidelity information is needed to accurately predict stresses and potentially propagation of damage (in this case a delamination). A cohesive model is shown here as a potential tool for damage propagation prediction, but other methods may also be implemented depending on the type of damage being explored. The less-refined zone, which extends beyond the picture to the supports, represents most of the structural domain and can be modeled by low-cost models such as Equivalent Single Layer (ESL) theory or by fast NLRMs. This problem is motivated by the potential for delamination defects in laminated composite beams and panels. However, beyond this specific material system and damage mechanism, it is intended to challenge NLRMs to better handle local features. Such problems have large local gradients which cannot be accurately captured by NLRMs.

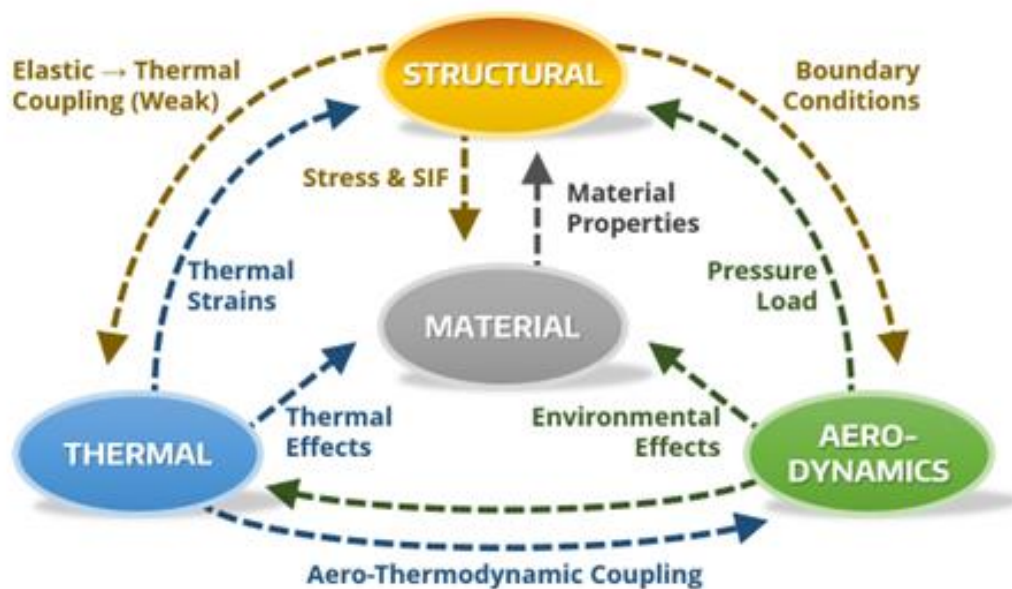


Figure 2: Schematic of the physics domains and interactions for coupled aero-thermo-structural-material mechanics.

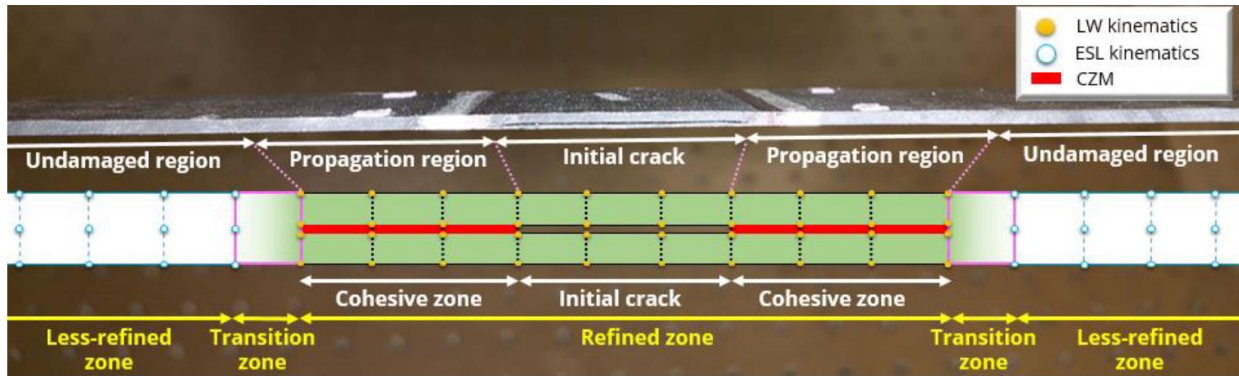


Figure 3: Challenge problem for aero-thermo-structural-material dynamics with local features: Delaminated slender composite beam. The shaded area represents where a high-fidelity model such as LayerWise (LW) theory or 2D/3D solids are used, while the remainder of the domain uses simpler Equivalent Single Layer (ESL) models or NLROMs.

3. Objectives

The challenge problem was selected to mimic the complexities of nonlinear structural dynamics of thermally post-buckled panels. The focus on laminated composites was motivated by the fact that while still highly complex, the knowledge of likely crack planes induced by delamination is very helpful in the future development of damage models. Through this challenge problem, this research pursues the two overarching goals and five objectives listed below.

Goal 1: Advance the understanding of the dynamics of slender aerostructures through the following objectives:

- i. characterize large-deformation dynamics buckled panels under extreme transverse loads representing aerodynamic pressures;
- ii. advance beyond deformations to assess internal stress time histories within pristine panels; and
- iii. assess local stress concentrations, stress intensities, or other metrics needed to predict degradation.

Goal 2: Build fast and accurate predictive tools for design engineers through the following objectives:

- iv. (fast) develop fast substructuring approaches that handle both large features using NLROMs and fine local features with high-fidelity models; and
- v. (accurate) create a suite of validation data from experiments and high-fidelity simulation of special cases.

The two overarching goals are roughly divided between “fundamental understanding” and “tools”, however, these two necessarily inform one another. High-fidelity (and computationally expensive) simulations, which are used as “truth models” to augment experimental results, are also built into several objectives as research tools, but are not envisioned as final products.

As noted earlier, the first objective has already been the subject of study from a material-agnostic perspective by the PI, AFRL, and others. Additionally, the experimental validation data research efforts in objective (v) have been largely carried out by AFRL, as part of a large suite of challenge problems. These are included here to provide context for objectives (ii) to (iv), which are the main pursuits of this research.

The listed objectives do not fully achieve the overarching goals. Changing the material system, damage mechanisms, or other features may require additional work or new challenge problems. Objective (iii), in particular, is highly dependent on the material system and damage mechanism of interest. Likewise, the high-fidelity modeling listed in Objective (iv) will be dictated by the type of local features (e.g., delamination, plasticity, notches, etc.) being investigated, and a vast array of failure theories with requisite high-fidelity models exists in the literature. Thus, this research converged toward a focus on ensuring that the developed NLROMs could be robustly “plugged into” a broad range of high-fidelity models without inducing spurious effects. It is hoped that this will broaden the impact of this work to handle new challenge problems with different materials and failure mechanisms.

4. Accomplishments

This section summarizes the three primary projects that were carried out throughout this research effort and the accomplishments within each.

4.1 Stress and Fatigue Prediction in Composite Plates

As noted earlier, significant efforts have been placed on predicting the nonlinear structural dynamics of post-buckled structures. However, there has been less work on assessing the consequences of these large deformations on the life cycle of panels. This work sought to address this by studying the stresses and their possible impact on fatigue of initially pristine panels (objective (ii)). The case study structure, a mechanically buckled (mimicking constrained thermal expansion) laminated panel, is shown in Fig. 4. This panel was also the subject of a prior study focused on objective (i), in which an FEM was validated with experimental data. The experimentally observed buckled rise is shown in part (c), while the results of a calibrated FEM are shown in part (d). Additional details are in [1] and [6] listed in the dissemination section.

The calibrated FEM was used in this study to determine stress time histories. The effects of thermal buckling, and consequent nonlinear dynamic response with snap-through on stresses are shown in Fig. 5. This first part of this figure shows the midspan deformation time history of the buckled plate (from Fig. 4), and the same plate after the axial end restraints are released to allow it to return to an un-buckled configuration (all results from simulation). The forcing function is

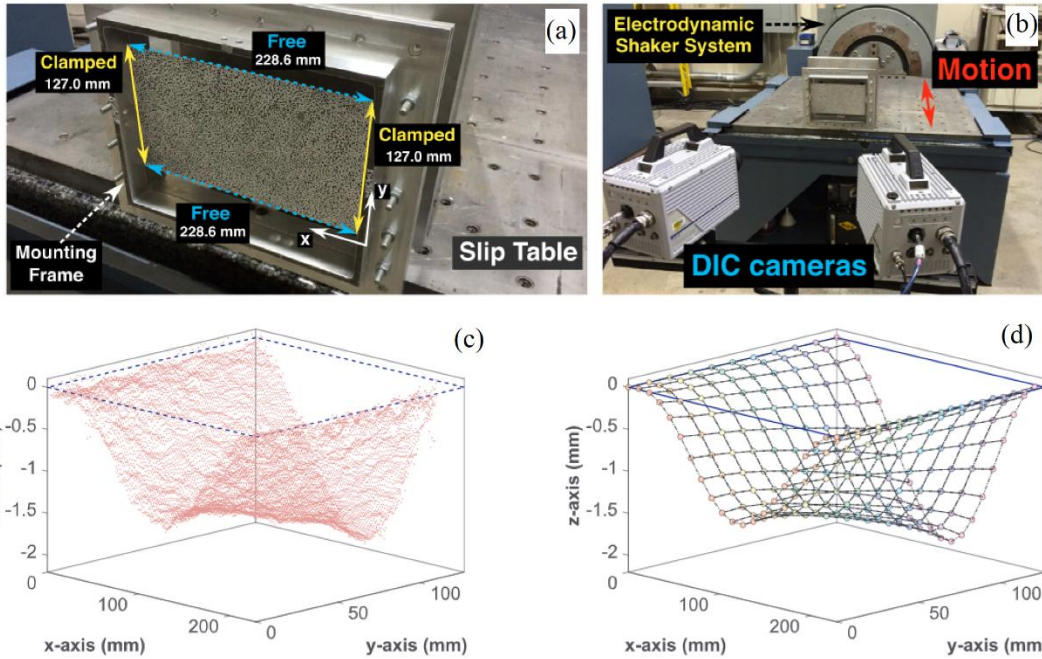


Figure 4: Case study post-buckled laminated PMC panel. (a) Panel geometry, (b) electrodynamic shaker configuration used to apply dynamic loads, (c) experimentally-measured buckled configuration after mechanical buckling, and (d) calibrated FEM buckled shape.

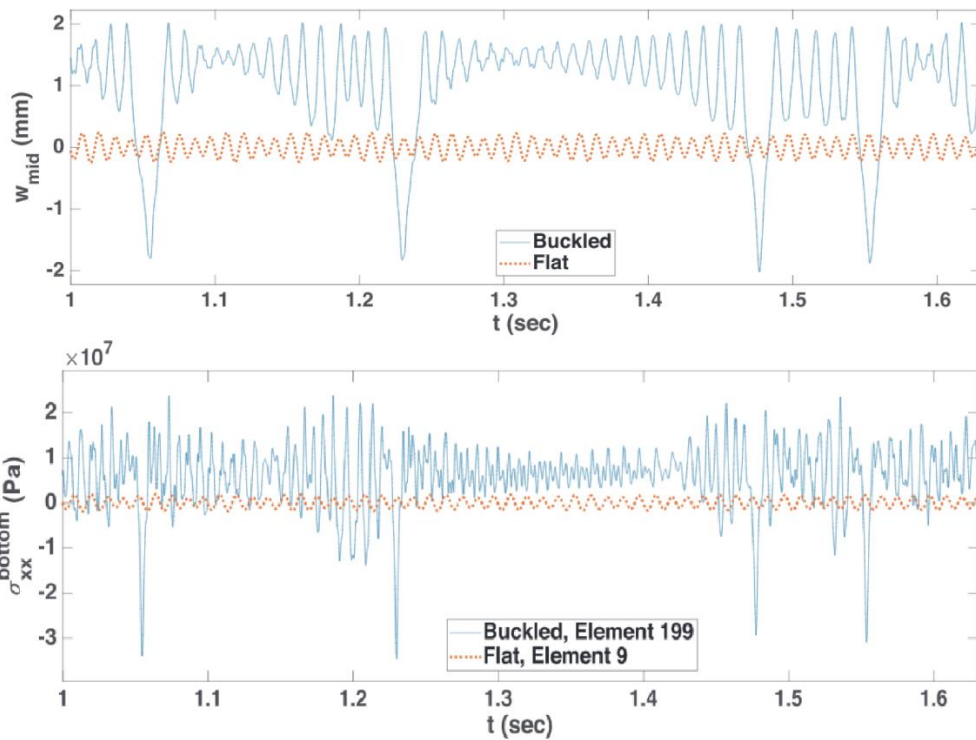


Figure 5: Time history samples of the (top) midspan displacement and (bottom) bending stress near the clamped end under harmonic distributed loading.

identical for both the flat and buckled plate. For the post-buckled scenario, the response is centered about a non-zero buckled ‘up’ value (approximately 1.4mm), but also demonstrates occasional large amplitude oscillations where it snaps through to the remote buckled ‘down’ configuration. The second part of the figure shows consequences on the bending stresses near the extreme bottom fibers of the panel near the support. It shows how both the mean and alternating components of stress are amplified by buckling when the load environment is otherwise unchanged. This may have a significant effect on the fatigue life of panels, though these results should be interpreted qualitatively, as a quantitative study would depend on specific conditions of a structure of interest.

This is just one sample of the response for a single harmonic forcing scenario, but it is representative of the amplification caused by post-buckling and snap-through that occurs over a broad range of forcing parameters. Additional details can be found in [2], [4-6], and [8-9] listed in the dissemination section.

4.2 Guided NLROM and Robust Identification

The second project focused on developing accurate NLROMs suitable for the prediction of stresses as an initial pursuit of Objectives (iii) and (iv).

A reduction in the number of DOFs of a nonlinear structural model can significantly decrease the computational time of dynamic analyses, especially in problems where long analyses are required. Perhaps the most common and widespread framework for developing low-order models of continuous systems is the classical Ritz method in conjunction with a smooth modal basis. The Ritz method requires access to the closed-form governing differential equations, and it is difficult to implement for problems with complex boundary conditions. In recent decades, significant research efforts have focused on advancing NLROMs, which can be identified using FEMs and do not require direct access to closed-form models. For this reason, NLROMs are often considered “non-intrusive” methods. They can be used to handle complex boundary conditions so long as the FEM can do the same. However, NLROMs also have some limitations and potential problems. One important limitation is that they start from an assumed functional form of the structural restoring force. The standard functional form is not guaranteed to be the optimal choice for all cases. Another concern is that restoring force identification for beams and panels with strong initial curvature has frequently been accompanied by numerical stability issues. This project focused on addressing these two concerns.

The governing equations assumed for NLROMs typically use a general cubic restoring force as follows:

$$M_{ij}\ddot{d}_j + C_{ij}\dot{d}_j + K_{ij}^L d_j + K_{ijk}^q d_i d_j + K_{ijkl}^c d_i d_j d_k = F_i(t),$$

for the i^{th} governing equation where, \mathbf{M} is the mass matrix, \mathbf{C} is the damping matrix, and \mathbf{K}^L , \mathbf{K}^q , and \mathbf{K}^c are the linear, quadratic, and cubic stiffness tensors, respectively. The key aspect of NLROMs is that these stiffness terms are identified from FEMs using test cases, as opposed to being derived from the governing equations. The cubic tensor largely controls accuracy of the

NLRROM. It also controls the cost of the identification process as the number of components to be determined, and therefore the number of test cases needed (each of which involves an FEM simulation) to identify the model, increases with the fourth power of the number of modes used in the model.

A general cubic is used in NLRROMs as it is especially versatile. It is a truncated Taylor expansion and is truly representative of intermediate geometric nonlinearities in solid mechanics. However, it does not always provide the most efficient template as a functional form. In this project, the von Kármán equations, which are appropriate for slender plates under intermediate deformations, are used to obtain a definition of the quadratic and cubic tensors which are significantly simpler. The primary outcome is that the quadratic and cubic tensors can be shown to functions of a simpler square matrix. Thus, the number of test cases needed increases only with only the second power of the number of modes used, a significant reduction from the fourth power dependence for the general cubic. It also results in more stable and accurate predictions than the general form. Additional details can be found in [3] listed in the dissemination section.

The von Kármán functional form, termed the Guided ROM (GROM), was also used to investigate and improve a long-standing stability issue in which the restoring force diverges during snap-through of curved panels. This is a relevant concern as structural skin panels are frequently curved. The GROM naturally reduced this issue, but it was also observed that both GROMs and general cubic NLRROMs can be improved by careful selection of the suite of test cases used to identify the model. Additional details on this work can be found in the PhD Dissertation [7] listed in the dissemination section and will also be the subject of a forthcoming journal article.

4.3 Coupling NLRROMs with High-Fidelity Models

This project focused on developing methods to couple NLRROMs with high-fidelity models. The work achieved objective (iv), but for beam geometry as a simplification of shells. It also provides a tool which can be applied to pursue objective (iii), though only limited features were studied within the scope of this project (clamped supports, and graded beams). The developed methods are well-suited to extension to shells, and more complex features (e.g., delaminations) which will be a topic of future research.

The approach is illustrated in Fig. 6, which shows a segment of a beam at the interface between a fast NLRROM (left) and a high-fidelity model (right). The two regions are divided by a transition zone in the center. The high-fidelity model used in this study was a 2D FEM. This would be suitable for handling irregular features such as holes, concentrated point loads, notches, or fasteners. Other high-fidelity approaches, such as Generalized FEM (GEM) could also be used. Additionally, any damage model of interest could be used within the high-fidelity region. Note that while ‘elements’ are shown in the NLRROM region, they are used only for post-processing calculations. The nodes of these elements do not provide any DOFs to the governing equations as the displacement field in this region is described by a modal decomposition.

The primary advancements in this research were the inclusion of additional DOFs within the NLROM framework, and the approach used to construct the displacement field in the transition region. The additional DOFs are through thickness deformation mode shapes shown in Fig. 7 which are applied at the FEM interface in Fig. 6. The uniform mode is already frequently used in NLROMs to represent membrane stretching. The linear and cubic modes, however, are new additions that allow the NLROM to better represent the through-thickness deformation field within the NLROM region. Without these, the NLROM-FEM interface imparts inaccurate deformations in the FEM. This results in spurious stresses within the FEM near the interface. This can be alleviated to some extent by using a larger FEM that reaches farther from the local feature of interest, however, that comes with additional cost. Using accurate through-thickness modes in the NLROM allows the FEM region size to be minimized. The transition region further improves performance. In this region, some components of deformation are dictated by the NLROM DOFs and others by the FEM.

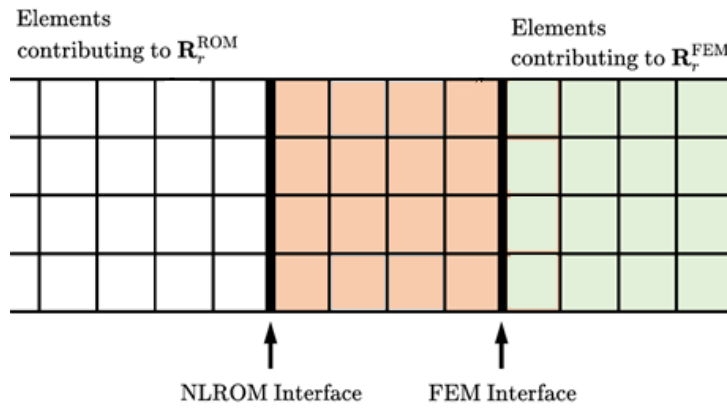


Figure 6: Coupled NLROM-FEM model with transition region. The left region represents the less-refined domain, the right region an FEM of a local area of interest with special features, and the center is a transition region with mixed definitions of the deformations.

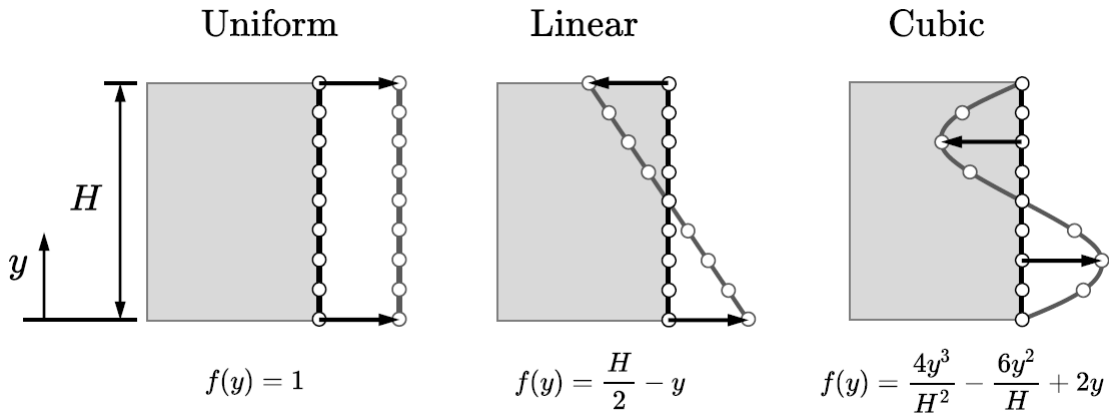


Figure 7: Through-thickness NLROM modes.

The improvements are illustrated through the simple example shown in Fig. 8, a multi-layered beam representing a laminated composite under a transverse distributed load. Representative results for the shear stress are shown in Fig. 9. The FEM region is right of the vertical black line, the fully-NLROM region is to the left of the two markers, while the transition region is in between. In this example, the local feature of interest is simply the clamped support on the right (which can be compared to the clamped support on the left which is left within the NLROM region), however, as noted earlier the FEM region could be used to assess a broad range of defects and damage mechanisms. Note that the entire NLROM region includes only 7 DOFs, while the FEM requires 2 DOFs per node, or hundreds of DOFs overall. Thus, shrinking the FEM domain results in significant computational cost reduction. This problem was chosen for illustrative purposes, if computational cost savings were needed, the transition could be placed much closer to the feature of interest (the clamped end on the right).

These plots show the error in shear stress for three different through-thickness modal selections, with an otherwise equal model. The top plot is for the case where both the uniform and linear shapes are used, which represents the “Timoshenko” assumption. The maximum errors observed (approximately 2.1 MPa) within both the transition and FEM regions are roughly equal to the absolute value of the shear stress at this same location, i.e., this has a 100% error. The middle plot shows the results when the cubic shape is added to the model. This cuts the magnitude of the error roughly in half (note that the legend scale changes), and significantly reduces the spatial extent of the large errors. The bottom plot shows the results when the cubic function is replaced with a new shape identified from the FEM of the beam. The shape has a nominally similar character to the cubic, but better captures the through-thickness behavior of this graded beam. This requires only a single test case to identify and does not need to be modified for other loading conditions, i.e., it is a property of the beam’s stiffness grading just like a cubic shape is appropriate for homogeneous beams. For this approach, the shear stress errors at the interface drop to 10% of the absolute stress within the transition region and less than 5% within the FEM region. The latter is more important, as the purpose of the NLROM coupling is to reduce the number of DOFs required, without imparting spurious stresses in the FEM region wherein the features of interest and potential stress concentrations are located. Further error reductions could be obtained by adding more through-thickness mode or by using a wider transition region. However, this is likely unnecessary, as this shows that within just one element of the interface, the errors in the FEM are virtually zero. This will allow for the interface to be placed very close to the feature of interest, thereby significantly reducing the number of DOFs.

The errors on the left end of the plot (fully within the NLROM region) demonstrate one of the many reasons why NLROMs (and traditional single layer theories such as ESL) are less suitable in the vicinity of local features due to their in-built kinematic assumptions. Other more extreme features such as sharp cracks, if left within an NLROM region would have much larger errors. Note that while Fig. 9 appears to show that the errors at the left support increase with additional modes, they are in fact unchanged (legend scale change) as this region is too far to be affected by changes at the interface.

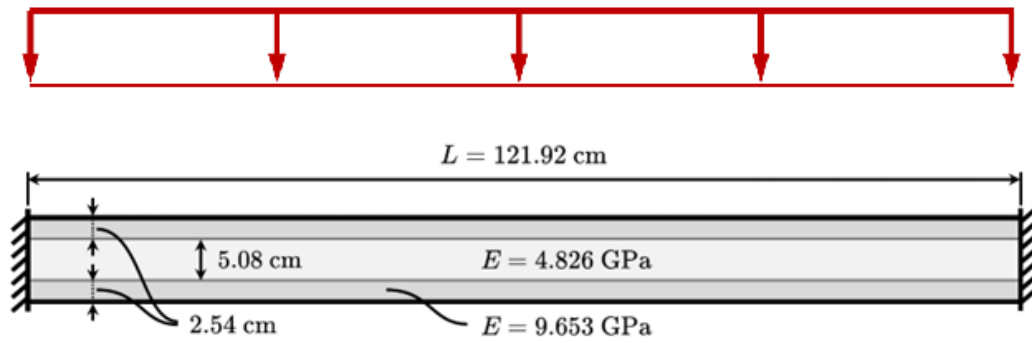


Figure 8: Test case, a multi-layered beam.

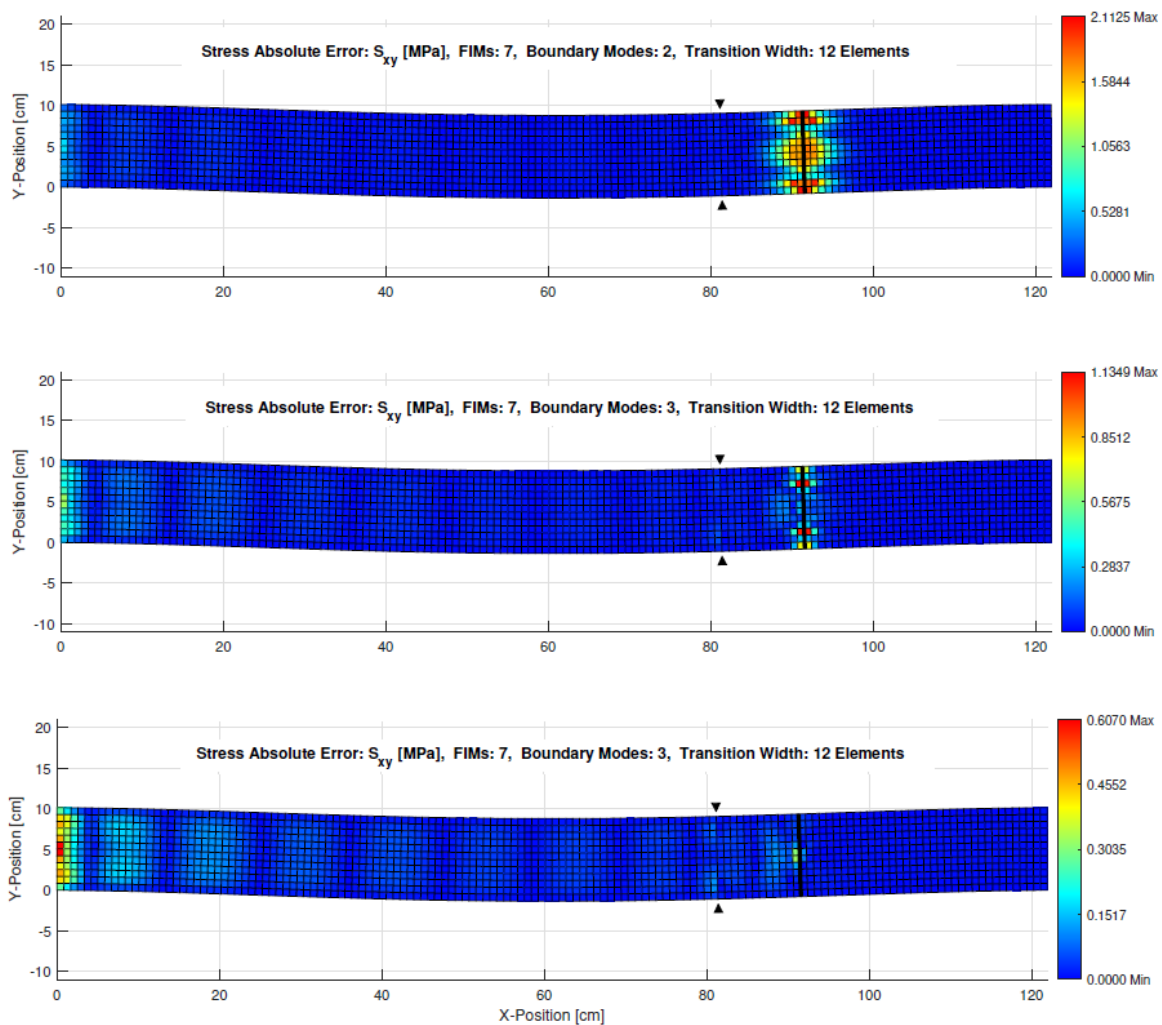


Figure 9: Shear stress error when compared with a full FEM of the entire domain. Note that the true shear stress at the interface (vertical solid line) is approximately 2 MPa for the given scenario.

The errors over the majority of the NLROM domain are very small. The small spatially oscillating errors in the bottom plot are also present in the top two figures but cannot be seen due to the scale differences. These oscillations are a result of the modal basis used in NLROM region (again, only 7 modes in total are used in this region), which are constructed from vibration modes. This error could be reduced if desired by adding more modes to the NLROM, but it is likely unnecessary as the errors are small, and more importantly, do not pollute the FEM stress predictions.

Ultimately, the NLROM domains are envisioned as “*super elements*” that can be used between features of interest in beam and plate models. These special features could be supports, e.g., for the example shown, both the left and right sides could be modeled with FEMs only a few elements wide to nearly eliminate errors. But they could also be other locations where loads are applied on the top or bottom surfaces, or the plethora of defects or local features that have already been discussed. The NLROM super elements can be used between special features to replace huge numbers of FEM DOFs with only 3-10 transverse deformation modes, and 6 through-thickness modes (3 on each end of the super element). Crucially, the through-thickness modes ensure that the NLROM is minimally invasive in its effect on the FEM region in which accurate stress predictions are required. Further details about this work can be found in the PhD dissertation [7] and will also be published in a forthcoming article.

5. Dissemination

This section lists the dissemination of the research. The listed articles provide additional details to the summary provided in this report. Additionally, they provide detailed background information and literature surveys into the topic areas studied.

Archival Journal Articles

1. H.-G. Kim and R. Wiebe, “Experimental and numerical investigation of nonlinear dynamics and snap-through boundaries of post-buckled laminated composite plates”, *Journal of Sound and Vibration*, Vol. 439, pp. 362-387, 2019.
2. H.-G. Kim and R. Wiebe, “Numerical investigation of stress states in buckled laminated composite plates under dynamic loading”, *Composite Structures*, Vol. 235, pp. 111743, 2020.
3. J.M. Seawright, R. Wiebe, and R.A. Perez, “Guided identification of nonlinear reduced-order models via the incorporation of von Kármán beam theory”, *International Journal of Non-Linear Mechanics*, Vol. 150, pp. 104348, 2023.

Conference Presentations

4. H.-G. Kim, R. Wiebe, “Stress Analysis in Post-buckled Thin Panels During Snap-through”, Presented at the 38th SEM International Modal Analysis Conference, Houston, TX, USA, February, 2020.
5. K. Wielgus, R. Wiebe, “Nonlinear structural dynamics and stress analysis of composite aerostructural skin panels”, Presented at virtual ASME International Design Engineering Technical Conferences, August, 2020.

PhD Dissertations

6. Han-Gyu Kim, “Experimental and Numerical Nonlinear Dynamics and Stress Field Analysis of Post-Buckled Composite Plates,” PhD Dissertation, University of Washington, 2019.
- Jordan M. Seawright, “Non-Intrusive Nonlinear Reduced-Order Model Identification and Substructuring of Geometrically Nonlinear Structures”, April, 2023.

MS Theses

8. Anne Magnus, “Experimental & Numerical Study of Mode II Delamination in Laminated Composite Beams”, MS Thesis, University of Washington, 2019.
9. Kayla Wielgus, “Finite Element Analysis of Post-buckled Composite Structures”, MS Thesis, University of Washington, 2020.

Additional Collaborative Projects

10. P.S. Harvey, R. Wiebe, T.M.N. Cain, “Inextensibility and its effect on the number of equilibria of shallow buckled beams”, ASME Journal of Applied Mechanics, Vol. 87, No. 12, pp. 121007, 2020.
11. K.R. Brouwer, R.A. Perez, T.J. Beberniss, S.M. Spottswood, D.A. Ehrhardt, R. Wiebe, “Investigation of aeroelastic instabilities for a thin panel in turbulent flow”, Nonlinear Dynamics, Vol. 104, pp. 3323-2246, 2021.

6. Impacts

Important impacts of this research are:

Principal Topic Area

- Advancement from material-agnostic NLROM deformation simulation (the perimeter of Fig. 2) toward prediction of stress, fatigue, and the potential to analyze degradation real material systems. Some work has been done in this topic area before, but much more is needed.
- Addressing issues in the NLROM functional form, and stability issues in the identification of NLROMs for curved beams.
- Extending NLROM formulations to accurately predict shear stresses. This is an important component in some failure progression mechanisms which is not naturally handled well by some traditional kinematic assumptions.

Other Disciplines

- The study of NLRoms has largely been limited to beam and shell problems where transverse deformations and mid-line/plane stretching are the dominant deformation mechanisms. The extension to include other deformation types (through-thickness deformations), is just one possible extension of NLRoms. This work is a first step toward generalizing NLRoms for use in more complex 2D and 3D problems to allow use in a much broader range of applications.
- Similarly, the super-element approach may allow NLRoms to be used in complex systems where 2D and 3D features are handled by FEMs or other high-fidelity models.

Human Resources and Education

- This research project has resulted in 2 PhD dissertations, and 2 MS theses.
- One of the PhD graduates, Dr. Han-Gyu Kim, is currently an assistant professor continuing research aerospace structures.