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Direct Adaptive Control of Nonlinear Systems with Uncertain Unstable Zero Dynamics

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Abstract

This final report summarizes results obtained under AFOSR project FA9550-20-1-0028 between January 15, 2020 and January 14, 2024. This period encompasses the original 3-year plan plus a 1-year no cost extension. There was no change in scope. This report summarizes results obtained on the following topics: 1) online system identification, 2) analysis of retrospective cost adaptive control, 3) almost global convergence theory, and 4) initial undershoot due to unstable zero dynamics.

1 Accomplishments

1.1 Online System Identification

In the classical technique of recursive least squares (RLS), a forgetting factor is often used to allow the system identification to rapidly discount old data when the system undergoes a sudden change. The drawbacks of this approach are 1) a constant forgetting factor does not take into sudden changes in the system, and 2) in the absence of data persistency, the covariance matrix may diverge. We overcame this problem in prior work by developing a variable-rate forgetting version of RLS in [1]. Under this project, we obtained *weakly persistent conditions* [2], which are necessary and sufficient conditions on the regressor under which RLS is globally asymptotic convergent to the true system parameters.

1.2 Retrospective Cost Adaptive Control (RCAC)

In [3] we extended RCAC to include online identification. This development entailed several innovations motivated by the need to interconnect discrete-time linear time-varying (DTLTV) models. The key innovation was to introduce the idea of a *fixed input-argument* (FIA) filter, which provided a rigorous setting for defining the retrospective cost. We then showed that the retrospective cost performance variable can be decomposed as the sum of two terms, namely, a performance term and a model-matching term. We subsequently found that the analysis of this decomposition requires a careful treatment of realizations and interconnections of DTLTV systems. In [4], we worked out the mechanics of this approach for retrospective cost input estimation. In [5], we applied these techniques to RCAC, including a numerical demonstration and validation of the cost decomposition.

For nonlinear systems, we applied RCAC to Lur'e systems with self-excited oscillations. Self-excited systems have the property that, for almost all initial conditions, the response is bounded and nonconvergent. These systems arise in a vast range of applications, such as systems with fluid-structure interaction (for example, flutter) and combustion instabilities [6]. We developed the underlying theory of these systems in [7]. This theoretical research was motivated by experimental results obtained under a related project [6]. In [8], we used classical absolute stability criteria to show that RCAC can stabilize self-excited systems by modifying the linear dynamics of a Lur'e system in order to satisfy the classical circle criterion. This result, which is unexpected, suggests the need for additional fundamental research to understand this phenomenon.

1.3 Almost Global Convergence Theory

A dynamic system has the property of *almost global convergence* if all trajectories except those that start in a set of measure zero converge to a proper subset of the state space. Almost global convergence is similar-to-but-weaker-than global convergence. The concept of almost global convergence is introduced in [9], which presents a notion of convergence where all trajectories that start from a set of initial conditions with positive measure converge to an invariant set that is a proper subset of the state space.

In [10, 11], we developed new results on almost global convergence (AGC) for a discrete-time dynamic system $x_{k+1} = f(x_k)$. First, we developed new density-function-based sufficient conditions for demonstrating AGC. Unlike existing density-function results, these new results do not require knowledge of the inverse map f^{-1}. Additionally, these results do not require a local convergence assumption. This new density-function approach is important because f^{-1} can be difficult to express. For example, closed-loop systems that include adaptive controllers (e.g., [12,13]) often have a right-hand-side function f that is difficult to invert. Second, we developed new sufficient conditions for AGC using Lyapunov-like functions rather than density functions. Constructing density functions to prove AGC can be difficult. The new Lyapunov-like sufficient conditions can help overcome this difficulty. For example, the construction of a Lyapunov-like function is often more intuitive than a density function, because Lyapunov-like functions are often based on physical principles. Collectively, these new results can facilitate AGC analysis for systems that cannot easily be analyzed using the existing literature. For example, these results can be applied to a multi-agent system with relative-distance formation control (e.g., flocking) similar to those in [14, 15].

Our work extended past the development of new results on ABC theory. We also developed new adaptive control methods [16, 17] that do not have global convergence properties but instead have AGC properties. Finally, the work on AGC led to ideas that generated offshoot work on control barrier functions, which is in [18–22]. Notably, [18–21] present and analyze new optimal control methods that satisfy both safety constraints (i.e., state constraints) and input constraints (e.g., actuator limits).

1.4 Analysis of Initial Undershoot and Unstable Zero Dynamics in Nonlinear Discrete-Time Systems

In order to evaluate the performance of RCAC on nonlinear systems, we studied undershoot in nonlinear discrete-time systems. Within the context of linear dynamics, we laid the foundation for this work in [23]. For nonlinear systems, it turns out that the relative degree and presence of initial undershoot depend on the height of the step input.

This dependence makes the phenomenon rich and interesting. For example, we demonstrated examples for which the linearized dynamics do not exhibit initial undershoot, but the nonlinear dynamics exhibit initial undershoot locally. In these cases, the nonlinear terms dictate the initial undershoot. This situation is analogous to the situation where a nonlinear system may be controllable locally but its linearization is uncontrollable.

The paper [24], which is under review, develops the relevant theory and provides sufficient conditions under which the nonlinear dynamics exhibit initial undershoot. This work provides a fundamental contribution to this topic, which we believe warrants further investigation. For example, it is known that linear time-invariant discrete-time systems exhibit initial undershoot if and only if the system possesses an odd number of (real) positive zeros greater than 1. Although the generalization of this property for nonlinear systems is unknown, our work provides a step in this direction.

2 Impacts

We applied results from this project to predictive cost adaptive control (PCAC), which is a data-driven extension of model predictive control (MPC). In particular, we used the online identification techniques developed in [1, 2] as the basis for fast, online system identification. This technique is implementable online in real-time, and we implemented this method in laboratory experiments. Our experiments show that this method is extremely effective, and we believe it can impact real-world applications. This discovery is a success made possible by this AFOSR-funded project, and related research is continuing under ONR and NSF support with application to flow control. We expect that our research on adaptive active flow control will have applications to vehicles operating in flight regimes ranging from subsonic to hypersonic.

3 Changes

There were no changes under this project.

4 Technical Updates

There were no technical updates under this project.

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