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## Integrating Quantum and Traditional Computing in Optimization

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<b>14. ABSTRACT</b> This proposal integrates quantum annealing with traditional computing technology to develop new optimization techniques and algorithms. Unfulfilled promises of quantum computers quickly solving complicated optimization problems have left much of the optimization community skeptical of the potential for quantum computing. There have been many times when researchers use esoteric problems to display quantum dominance, only to have the results reversed when optimization experts examine those problems. Solving challenging instances of well studied problems will help alleviate this skepticism. A unique part of the proposed approach is that we do not expect the quantum computer to directly prove optimality, something that quantum annealers are not built to do. Instead, we take advantage of quantum annealer's ability to quickly generate high-quality solutions. This ability gives optimizers the potential to, almost instantly, arrive at an inner approximation of a given model's feasible region. Such knowledge can be very useful. This proposal will demonstrate how these inner approximations can be used to verify the impact different formulations. In particular, we will use a quantum annealer to approximate the feasible regions of various formulations. Then, we can choose the formulation that is most ideal for solving the instance using traditional computing, yielding significant computational speedups.			
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Integrating Quantum and Traditional Computing in Optimization  
Jim Ostrowski  
University of Tennessee  
FA9550-19-1-0147

**Project Report:**

The goal of the second year of the project was to identify how to exploit degeneracy in the solution space with quantum computing. For this work, we took two parallel approaches, the classical optimization approach and the quantum optimization approach.

Regarding the classical approach, previous work had explored the relationship between the number of optimal/near-optimal solutions to small independent set problems and showed empirically that there is a strong positive correlation between the number of near-optimal solutions and the difficulty of the problem. We have been trying to use the idea of near-optimal solutions to help devise “optimality-preserving” constraints to help solve integer programming problems. The hope is that quantum computing can help us identify if a given problem instance has many/few optimal/near-optimal solutions, information that can then be exploited via cuts. The main application has been independent set, with some of the challenge problems being some notoriously difficult instances from the literature. Unfortunately, while we have tried a lot of ideas on these independent set problems, we haven’t yet to see any significant impact. While we are still focusing in independent set, we have also branched out to some other applications where we see the presence of multiple optimal and near-optimal solutions. We have a paper in that uses ideas of “almost symmetry” that arises when we have many near-optimal solutions that is able to help us solve some scheduling problems much faster. These results are helping us build some intuition regarding how to attack independent set (this being an important application as it is straightforward to convert to a quantum formulation). In addition, we have been looking into more general symmetry-exploiting methods. The key idea is “orbital crossover,” which can be used to arrive at a vertex to an extremely large, highly symmetric, linear program. The trick is that we can use symmetry to aggregate the LP so that we can solve it very quickly. This aggregated LP is in the center of an optimal face, so crossover is necessary. By using group theory with crossover, we can save a lot of computational time. This is a necessary tool for the following experiment (the draft of which is almost finished). When using cut generation for highly-symmetric integer programs, we would need every symmetric equivalent of a cut in order to effectively move the dual bound. However, the number of symmetric-equivalent cuts can be large. We can use orbital crossover to get around this point, quickly solving LPs that are generated by adding perhaps hundreds of millions of symmetric cuts.

Regarding the quantum approach, we simulated a large number of quantum approaches to solve combinatorial optimization problems. In previous years we looked into what graph structures seemed to indicate good/poor performance for quantum algorithms. In this past year, we investigated how to more efficiently design quantum circuits so that good solutions

can be found faster. This has immediate consequences, as error propagates through quantum circuits, meaning that deep circuits are going to be very noisy for the foreseeable future. One of the overall goals of the quantum side of the research is design quantum circuits as compactly as possible. The more logic gates that are used in a quantum circuit, the higher the probability that error will be introduced in the algorithm's execution. Error associated with gates will be something that dominates the use of quantum computing for many years, so the only hope for quantum computers to be practically useful in the mid-term is to heavily optimize the circuits, creating shallow circuits that can give meaningful results. The culmination of this effort was the paper "Multi-angle QAOA" where we were able to use classical optimization to significantly improve the efficiency of a leading quantum algorithm. In addition, papers like "Globally optimizing QAOA circuit depth" look in to how to efficiently formulate integer programs (specifically 3-SAT) in ways that make them easier to embed on quantum hardware. This work culminated in our paper "Graph decomposition techniques for solving combinatorial optimization problems with variational quantum algorithms," where we actually solved some 100 vertex MaxCut problems on real quantum hardware.

### **Publications:**

We have the following papers in print:

Alam, Rizwanul, et al. "Solving MaxCut with quantum imaginary time evolution." *Quantum Information Processing* 22.7 (2023): 281.

Lotshaw, Phillip C., et al. "Approximate Boltzmann distributions in quantum approximate optimization." *Physical Review A* 108.4 (2023): 042411.

Lotshaw, Phillip C., et al. "Scaling quantum approximate optimization on near-term hardware." *Scientific Reports* 12.1 (2022): 12388.

Herrman, Rebekah, et al. "Impact of graph structures for QAOA on MaxCut." *Quantum Information Processing* 20.9 (2021): 1-21.

Herrman, Rebekah, et al. "Multi-angle quantum approximate optimization algorithm." *Scientific Reports* 12.1 (2022): 1-10.

Herrman, Rebekah, et al. "Globally optimizing QAOA circuit depth for constrained optimization problems." *Algorithms* 14.10 (2021): 294.

Lotshaw, Phillip C., et al. "Empirical performance bounds for quantum approximate optimization." *Quantum Information Processing* 20.12 (2021): 1-32.

Herrman, Rebekah, et al. "Lower bounds on circuit depth of the quantum approximate optimization algorithm." *Quantum Information Processing* 20.2 (2021): 1-17.

Linderoth, Jeff, et al. "Orbital Conflict: Cutting Planes for Symmetric Integer Programs." *Infoms Journal on Optimization*(2021): ijoo-2019.

And in the review process:

Schrock, Jonathan, et al. "Almost Symmetry in Unit Commitment." Optimization Online

Deakins, Ethan, et al. "Orbital Crossover." In review Math Programming Computation

Ponce, Moises, et al. " Graph decomposition techniques for solving combinatorial optimization problems with variational quantum algorithms." In review Quantum Information Processing.

**Changes in research Direction:**

We had issues finding graduate students with the required skills, necessitating an extra year of no-cost-extension.

**Personnel:**

None.

**Milestones:**

Integrating Quantum and Traditional Computing in Optimization  
Jim Ostrowski  
University of Tennessee  
FA9550-19-1-0147

**Project Report:**

The goal of the this project was to identify how to exploit degeneracy in the solution space with quantum computing. For this work, we took two parallel approaches, the classical optimization approach and the quantum optimization approach.

**First Three Years of Effort:**

Regarding the classical approach, previous work had explored the relationship between the number of optimal/near-optimal solutions to small independent set problems and showed empirically that there is a strong positive correlation between the number of near-optimal solutions and the difficult of the problem. We have been trying to use the idea of near-optimal solutions to help devise “optimality-preserving” constraints to help solve integer programming problems. The hope is that quantum computing can help us identify if a given problem instance has many/few optimal/near-optimal solutions, information that can then be exploited via cuts. The main application has been independent set, with some of the challenge problems being some notoriously difficult instances from the literature. Unfortunately, while we have tried a lot of ideas on these independent set problems, we have yet to see a significant impact. While we are still focusing in independent set, we have also branched out to some other applications where we see the presence of multiple optimal and near-optimal solutions. We have a paper that uses ideas of “almost symmetry” that arises when we have many near-optimal solutions that is able to help us solve some scheduling problems much faster. These results are helping us build some intuition regarding how to attack independent set (this being an important application as it is straightforward to convert to a quantum formulation).

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**Last Year’s Effort:**

We have been looking into more general symmetry-exploiting methods. The key idea is “orbital crossover,” which can be used to arrive at a vertex to an extremely large, highly symmetric, linear program. The trick is that we can use symmetry to aggregate the LP so that we can solve it very quickly. The solution to this aggregated LP is in the center of an optimal face, so crossover is necessary. By using group theory with crossover, we can save a lot of computational time. This is a necessary tool for the following experiment (the draft of which is almost finished). When using cut generation for highly-symmetric integer programs, we would like to use every symmetric equivalent of a cut in order to effectively move the dual bound. However, the number of symmetric-equivalent cuts can be large. We can use orbital crossover to get around this problem, quickly solving the LPs that are generated by adding perhaps hundreds of millions of symmetric cuts.

Both the optimization and quantum approaches were combined in “Graph decomposition techniques for solving combinatorial optimization problems with variational quantum algorithms.” The key idea is that small local changes don’t have much of an impact on the quality of the solution, so you can use aspects of “almost-symmetry” to aggregate subgraphs in problem. As a result of this approach, we can consistently reduce MaxCut problems on 100-vertex 3-regular graphs to equivalent MaxCut problems with  $< 25$  vertices. The cost of doing so is that the resulting MaxCut problem becomes weighted and denser. However, these instances are now small enough to run on real quantum computers. We demonstrate the impact of this approach in practice. We are able to find optimal solutions to these problems using the Quantinuum quantum computer. This demonstrates one of the largest MaxCut problems actually solved on real quantum hardware. The corresponding paper is currently in review.

### **Publications:**

We have the following papers in print.

#### **New this year:**

Alam, Rizwanul, et al. "Solving MaxCut with quantum imaginary time evolution." *Quantum Information Processing* 22.7 (2023): 281.

Lotshaw, Phillip C., et al. "Approximate Boltzmann distributions in quantum approximate optimization." *Physical Review A* 108.4 (2023): 042411.

Lotshaw, Phillip C., et al. "Scaling quantum approximate optimization on near-term hardware." *Scientific Reports* 12.1 (2022): 12388.

#### **Previous Years:**

Herrman, Rebekah, et al. "Impact of graph structures for QAOA on MaxCut." *Quantum Information Processing* 20.9 (2021): 1-21.

Herrman, Rebekah, et al. "Multi-angle quantum approximate optimization algorithm." *Scientific Reports* 12.1 (2022): 1-10.

Herrman, Rebekah, et al. "Globally optimizing QAOA circuit depth for constrained optimization problems." *Algorithms* 14.10 (2021): 294.

Lotshaw, Phillip C., et al. "Empirical performance bounds for quantum approximate optimization." *Quantum Information Processing* 20.12 (2021): 1-32.

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Linderoth, Jeff, et al. "Orbital Conflict: Cutting Planes for Symmetric Integer Programs." *Informs Journal on Optimization*(2021): ijoo-2019.

#### **And in the review process:**

Schrock, Jonathan, et al. “Almost Symmetry in Unit Commitment.” *Optimization Online*

Deakins, Ethan, et al. “Orbital Crossover.” In review *Math Programming Computation*

Ponce, Moises, et al. “Graph decomposition techniques for solving combinatorial optimization problems with variational quantum algorithms.” In review *Quantum Information Processing*.

#### **Changes in research Direction:**

None

**Personnel:**

Graduate Students: Jonathan Schrock, Ethan Deakins, Moises Ponce

**Milestones:**

*Graduated Students:*

Jonathan Shrock Ph.D (Fall 2021) Currently at Lawrence Livermore National Laboratory

Ethan Deakins Ph. D (Fall 2022) Currently at Center for Naval Analysis

*Student Presentations:* (a lot of travel restricted as a result of COVID)

Jonathan Schrock: Almost Symmetries INFORMS Annual Meeting 2019

Ethan Deakins: Orbital Crossover INFORMS Annual Meeting 2019

Moises Ponce: Graph Decomposition, INFORMS Optimization Society 2022