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Prescribed-Time Stabilization: Noise-Robustness, Generalizations, Applications

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14. ABSTRACT
The following activities were conducted on this one-year project: • PT non-overshooting control of multi-input systems with one or more high relative degree outputs and control barrier functions. Models of aerospace vehicles with actuator dynamics give rise to this type of PT control problems. • PT and non-overshooting control for stochastic systems. Under an appropriate assumption on the decay of the covariance of the Brownian motion driving the system, we have pursued PT trajectory tracking with no overshoot in the mean. • PT extremum seeking for fully actuated and nonholonomic vehicles, such as seeking sources in GPS-denied environments, have been pursued. Additionally, output maximization was tackled for maps with large delays on the input or sensing, as well as maps cascaded with parabolic PDEs, which occur in additive manufacturing problems (under the thermal dynamics of melting and solidification).

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PI: Miroslav Krstic

Final report to AFOSR on grant FA9550-22-1-0265

Program officer: Frederick Leve

ACCOMPLISHMENTS

In the following list we review the results that we have obtained in the course of this one-year grant on *prescribed-time stabilization*.

1. **Prescribed-time stochastic control.** In the years leading to this grant, we have been developing results on prescribed-time control of uncertain deterministic systems. The most recent in the progression of these results is our development of prescribed-time safety filters for a chain of integrators (conference publication), generalized to nonlinear strict-feedback systems (under review for TAC). Over the past year we have advanced from deterministic uncertain systems to stochastic nonlinear systems, with both vanishing and non-vanishing stochastic disturbances, and developed prescribed-time stabilizing output-feedback stochastic controllers. Additionally, we have developed safe stabilizing controllers for stochastic systems with prescribed-time guarantees of both regulation to a setpoint and safety in the mean (the so-called "mean-nonovershooting controllers").
2. **Fixed-time homogeneous controllers.** Prescribed-time controllers employ time-varying feedback, with gains that increase towards the terminal (prescribed) time. It is well known that finite-time regulation is also achievable by time-invariant feedback, provided the feedback is non-smooth. Sliding-mode control is a particular ("primitive") version of this principle. We have developed such a time-invariant non-smooth counterpart of fixed-time stabilizing controllers, where regulation is completed no later than a fixed user-assignable terminal time.
3. **Prescribed-time extremum seeking.** Our work a quarter of a century ago established convergence guarantees for extremum seeking. This convergence is, conventionally, exponential and at a rate that is not known a priori because the map is unknown. Over the past year we have set out to develop extremum seeking algorithms with which convergence to the optimum is completed by a user-assigned time, regardless of the unknown map. Our prescribed-time ESC developments have gone in three directions. After developing prescribed-time convergent ESC algorithms for basic static maps, we extended these algorithms to cascades of PDEs into static maps. We then extended

prescribed-time extremum seeking to prescribed-time source seeking, for nonholonomic vehicle models.

4. **Robotic experiments with prescribed-time control.** We have had access to a humanoid 7DOF robotic arm manipulator "Baxter" at San Diego State University and have performed the algorithm designs and conducted experimental testing for prescribed-time tracking with the manipulator and prescribed-time safety. Baxter has succeeded at pick-and-place tasks, within a prescribed time of task completion, and in the face of large obstacles, with our algorithms, outperforming the alternative exponential safety filters, which are either overly conservative or overly aggressive, since they use gains that are constant both near and far from the target and near and far from an obstacle.

Recognitions received by the PI during the one-year grant

- **Adaptive and Learning Systems Award** (triennial), International Federation for Automatic Control (IFAC), 2023
- **Ruth Curtain Distributed Parameter Systems Award** (triennial, inaugural recipient), International Federation for Automatic Control (IFAC), 2022
- **A. V. Balakrishnan Award for Scientific Excellence in Research on Mathematics of Systems** (biennial, inaugural recipient), University of Southern California, 2022

IMPACTS

Development of the principal discipline

Pioneering advances on prescribed-time stabilization and prescribed-time safety filters.

Impact on other disciplines

Prescribed-time stabilization and safe control demonstrated on robotic platform Baxter.

Impact on the development of human resources and opportunities for training

The PI advises more than half a dozen doctoral students, gives workshops that introduces students to advanced research, and regularly publishes books that synthesize a decade of advances on a research subject for an easier entry for junior researchers into advanced fields.

He mentors students on selecting research topics of high relevance and impact, guides them in the publication process and in conference presentations, and supports them in their career launches and much beyond their graduation.

Impact on teaching and educational experiences

Teaching graduate course nonlinear systems, always with 100% student approval, supplemented with examples from latest research, including from the current grant on prescribed-time stabilization.

We list here the papers that we have published under the effort on this grant. Some are not picked up by Cross Reference, which is fine.

Journal Papers

[J1] W.-Q. Li and M. Krstic, "Stochastic nonlinear prescribed-time stabilization and inverse optimality," *IEEE Transactions on Automatic Control*, pp. 1179-1193, vol. 67, 2022.

[J2] N. Espitia, D. Steeves, W. Perruquetti, and M. Krstic, "Sensor delay-compensated prescribed-time observer for LTI systems," *Automatica*, paper 110005, vol. 135, 2022.

[J3] W.-Q. Li and M. Krstic, "Prescribed-time output-feedback control of stochastic nonlinear systems," *IEEE Transactions on Automatic Control*, vol. 68, pp. 1431-1446, 2023.

[J4] A. Bertino, P. Naseradinmousavi, and M. Krstic, "Design and experiment of a prescribed-time trajectory tracking controller for a 7-DOF robot manipulator," *ASME Journal of Dynamic Systems, Measurement, and Control*, vol. 144, paper 101005-1, 2022.

[J5] W.-Q. Li and M. Krstic, "Prescribed-time mean-nonovershooting control under finite-time vanishing noise," *SIAM Journal of Control and Optimization*, to appear.

[J6] A. Polyakov and M. Krstic, "Finite- and fixed-time nonovershooting stabilizers and safety filters by homogeneous feedback," *IEEE Transactions on Automatic Control*, to appear.

[J7] V. Todorovski and M. Krstic, "Practical prescribed-time seeking of a repulsive source by unicycle angular velocity tuning," *Automatica*, to appear.

[J8] W.-Q. Li and M. Krstic, "Prescribed-time control of nonlinear systems with linearly vanishing multiplicative measurement noise," *IEEE Transactions on Automatic Control*, to appear.

Refereed Conference Papers

[C1] A. Bertino, P. Naseradinmousavi, M. Krstic, "Experimental and analytical prescribed-time trajectory tracking control of a 7-DOF robot manipulator," *American Control Conference*, 2022.

[C2] I. Abel, D. Steeves, M. Krstic, "Prescribed-time safety design for a chain of integrators," *American Control Conference*, 2022.

[C3] D. Steeves, M. Krstic, "Prescribed-time stabilization robust to measurement disturbances," *American Control Conference*, 2022.

[C4] V. Todorovski, M. Krstic, "Prescribed-time seeking of a repulsive source by angular velocity tuning," *American Control Conference*, 2022.

[C5] C. T. Yilmaz, M. Krstic, "Prescribed-time extremum seeking with chirpy probing for PDEs—Part I: Delay," *American Control Conference*, 2022.

[C6] C. T. Yilmaz, M. Krstic, "Prescribed-time extremum seeking with chirpy probing for PDEs—Part II: Heat PDE," *American Control Conference*, 2022.

[C7] A. Polyakov and M. Krstic, "Homogeneous nonovershooting stabilizers and safety filters rejecting matched disturbances," *IEEE Conference on Decision and Control*, 2022.

[C8] W.-Q. Li and M. Krstic, "Prescribed-time nonlinear control with multiplicative noise," *IEEE Conference on Decision and Control,* 2023.