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QIS 3.0: Highly Entangled States of Light

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Final Technical Report**

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Grant FA9550-22-1-0062
Highly Entangled States of Light

Final Report

Awarded to:
Ki3 Photonics Technologies Inc.
Quantum Bridge Technologies Inc (QBT).

Recipient Principal Investigator:
Dr. Yoann Jestin

1. Executive summary

Problem statement:

- Graph quantum states are an important resource for fundamental research and technological applications
- Designing quantum circuits that output specific and desired graph states is challenging
- Real quantum circuits are subject to noise and loss, which are necessary to include for an accurate and realistic model

Project scope:

- Construct a theoretical and numerical framework for the study of graph states and of circuits that generate them
- Develop schemes for the construction of quantum circuits that output desired graph states
- Create models for the noise and loss effects that are present in real quantum setups

General outcome:

- To date, we have defined a framework that allows us to find and to simulate quantum circuits that output low-complexity graph states
- Our schemes successfully reproduce results from literature and are also capable of finding alternative setups to choose between
- We have developed noise and loss models that are representative of effects found in common photonics platforms
- Our framework is a flexible and extensible tool that is soon to be open-sourced and which can be easily adapted to simulate various physical platforms

Future Plans:

- Maintain and develop the framework alongside the open-source quantum community to find solutions for higher complexity graph states
- Develop fusion-gate based protocols for producing large graph states
- Implement encoding strategies for qubit protection
- Model quantum-dot-based sources and interface with current framework
- Develop models for integrated-photonics-based components as an experimental basis for quantum circuits
- Develop more sophisticated noise and distribution channel models
- Extend our ability to inject experimental details into our framework, further connecting results with real setups

2. Introduction

Graph states are a family of quantum states with high levels of entanglement that are of great fundamental interest [1], as well as being a key resource for applications in entanglement distribution and quantum networking [2-4], quantum error correction [5,6], and quantum metrology [7]. Among various physical platforms, photonics shows much promise for graph state realizations [8,9] as photons interact very weakly with their environment, enabling stable qubits. Additionally, photons are compatible with mature integrated and fiber technologies, granting additional support in the generation and distribution of graph states.

As well, for the generation of photons, various sources are available within photonics-based platforms, including solid-state emitters such as quantum dots that can emit single photons on demand [10], or nonlinear elements generating pairs of entangled photons [11,12].

Quantum circuits describe the sequence of quantum operations necessary to generate such graph states. However, connecting a desired graph state with the circuit that can output it is a non-trivial design task that only becomes more difficult with increasing state size and complexity. Recent work [13] has led to the development of an impressive deterministic method for finding quantum circuits that produce targeted graph states. However, such methods do not guarantee well-performing quantum circuits, allow little flexibility, and cannot consider impacts associated with noise or photonic loss, which is present in all real setups.

Towards addressing this research gap, the goal of our work was to develop a flexible theoretical and computational framework for the study of realistic graph states and of the quantum circuits that generate them. To accomplish this, we must be able to represent both the quantum states and the quantum circuits that produce them. Both representations must also be flexible enough to support noise and photonic loss modeling. The performance of different circuits must also be quantifiable, in order to drive schemes that search for best performing setups. All of these concepts must also be realized in a flexible and modular way that allows for co-development of solutions with the broader research community, driving collaborative and fundamental work into the understanding of entangled states as a resource.

Over the past year of our project's progress, we have successfully identified and realized a framework comprising the following core concepts: *Quantum state representations*, *quantum circuit representations*, *realistic component models*, *quantum circuit performance metrics*, and *quantum circuit optimization schemes*. Figure 1 shows these key concepts and their mutual interaction within our framework.

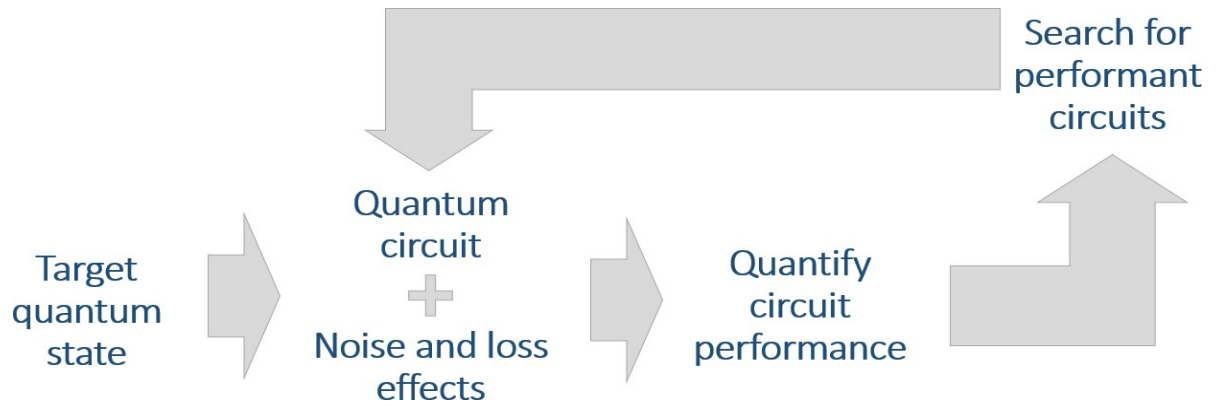


Figure 1: Key concepts within our framework. The performance metric is calculated with respect to the target graph state, guiding the search for performant circuits.

3. Accomplishments

In what follows, we explain each of the core concepts of our framework alongside our respective research efforts in developing them.

3.1. Research objectives

3.3.1. Representing quantum states

The objects of interest, graph states, are quantum systems of photonic and emitter qubits that can be mathematically represented using various formalisms. Our computational framework presently considers two mathematical representations, the *density matrix* and the *stabilizer*, with the modularity to easily include further representations in the future.

3.1.2. Representing quantum circuits

Quantum circuits represent the sequence of quantum gates and operations applied on solid state emitter qubits and photonic qubits. To computationally read, save, and manipulate these quantum circuits, we represent them as Directed Acyclic Graphs (DAGs) [19], which find common use in the field of quantum information [20]. Using the DAG representation, our computational framework can easily modify and evolve the quantum circuit topology to optimize the generated quantum state towards a target graph state. For the visualization of quantum circuits, our DAG representation was also made compatible with the openQASM format, a standard within the field [21].

3.1.3. Realistic component models

Towards increasing the realism of our framework, we support diverse models for noise and photonic loss. For example, our framework presently includes noise models for depolarizing noise, amplitude damping, and Pauli noise.

Additionally, we have developed a new representation to model mixed states using stabilizers. This new representation keeps track of all potential branchings of a mixed state, along with the probability of such a branch being measured. This novel approach allows us to consider probabilistic noise, more akin to real experimental systems.

Motivated by the physics of today's solid-state emitter and photonic qubit systems, we have furthermore developed a time-dependent noise model within the stabilizer formalism. In experiments, the number of errors that occur is strongly correlated with the time spent during and between each quantum gate. In our time-dependent model, we make use of an effective noise (simulated only at certain gates), maintaining model accuracy while reducing the computational load of the model.

3.1.4. Assessing circuit performance

When optimizing a quantum circuit for the generation of a target graph state, it is important to properly quantify the performance of said circuit using quantitative metrics. In fact, having proper metrics can heavily influence the performance of a circuit search algorithm. To date, we have investigated the following metrics as potential candidates [14], [22], [23]:

Fidelity	Relative entropy
Entanglement witness	Squashed entanglement
Graph edit distance	Adjacency matrix fidelity
Concurrence	Geometric measure
Negativity	Fisher information
Trace distance	Mermin inequality
Entanglement distillation	Stabilizer tableau fidelity

3.1.5. Searching for performant quantum circuits

We have investigated the following schemes for converging on suitable quantum circuits that output a desired graph state:

- **Random:** This scheme initializes a list of random circuits subject to physical

constraints, which we call a population. At each iteration and for each circuit within the population, the algorithm applies a random operation, which includes addition, removal, or replacement of a quantum gate. Each circuit in the population is modified independently from the other circuits. The scheme then keeps track of a certain number of top performing circuits.

- **Evolutionary:** Similarly to the random search approach, this scheme also applies random operations in each iteration. However, in contrast, after each iteration it constructs a new population by putting circuits into a N-wise competition and selecting the best performers.
- **Deterministic:** This scheme is our implementation of Ref. [13]’s approach to deterministically find a quantum circuit for a target graph state.
- **Hybrid:** In this scheme, we use the deterministic method from Ref. [13] to produce an initial solution. We then perturb this solution and use the random/evolutionary schemes to search for quantum circuit alternatives.
- **Gradient-based:** This is a density matrix specific scheme where we consider tunable/parametrized gates that are compatible with automatic differentiation and gradient descent optimization techniques.

3.2 Details of accomplishments

3.2.1. Representing quantum states

The *density matrix* representation can simulate a state’s full Hilbert space and can inherently represent mixed states, such as those resulting from the effects of noise and photonic loss [14]. However, this representation does not scale well with the number of qubits, making it impractical to investigate large graph states, and thus motivating the need for complementary representations. We have used this representation: 1) To study small graph states with the inclusion of noise effects; 2) As a benchmark for other representations; and 3) For the study of noise, photonic loss, mixed states, and gate parametrization.

The *Stabilizer* representation [15], [16], on the other hand, scales well with the number of qubits, making it more efficient for the study of large graph states (relative to the density matrix formalism). However, only a subset of quantum operations can be efficiently simulated within the stabilizer representation, limiting the scope for circuit optimization, and making this representation unsuited for the simulation of noise without judicious changes to the formalism. Presently, the study of representing mixed states within the stabilizer representation is very limited in the literature. To be able to consider large graph states that are mixed due to noise or loss, we have developed a probability tree approach as a way to represent such states. This point of research is presented further below in Sec. 3.3.

We have also developed relevant concepts from *graph* representations, which are also well-suited for many qubits, and contrary to the stabilizer representation, there is more progress made on describing mixed states [17,18]. In particular, we have worked on leveraging graph state equivalency through Clifford gates. As discussed in Sec. 3.4, the graph representation also offers unique metrics for characterizing the performance of quantum circuits.

Each of the quantum state representations has advantages and disadvantages, making it useful to be able to move between them. Most of these representation transformations can be found within literature [3], [14-18], except for obtaining a graph from a density matrix. We have resolved this issue by designing a novel protocol, as discussed in Sec. 3.5.

3.2.2. Representing quantum circuits

Although we have focused on solid state emitter qubits and photonic qubits, we have designed our framework with extensibility in mind. The types of operations, and therefore the

physical circuits we can represent and simulate, are flexible, meaning we can adapt our framework towards different physical platforms with relative ease.

Lastly, our quantum circuit representation also includes parameterized quantum gates and automatic differentiation (see Sec. 3.5 for further discussion).

3.2.3. Realistic component models

The Figure 2 shows an example of the model in practice, deriving the fidelity of an output graph state as a function of qubit error rate from various sources of noise.

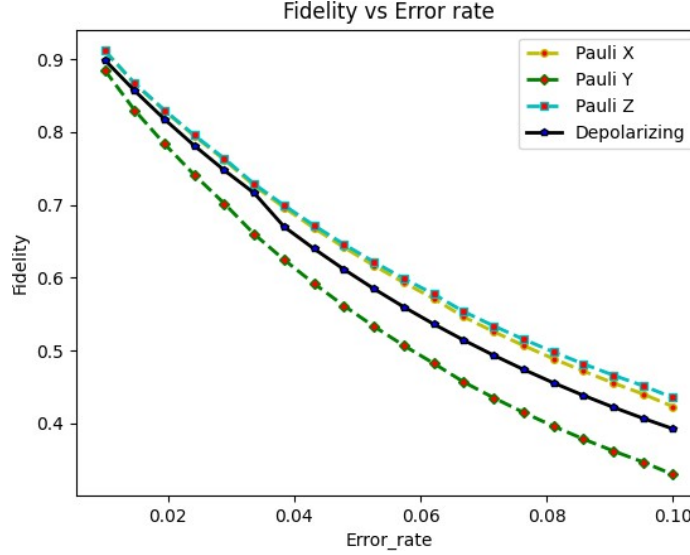


Figure 2: Fidelity versus error rate for the generation of a 4-photon linear cluster state assuming X -gate errors (yellow), Y -gate errors (green), Z -gate errors (blue) and a depolarizing channel (black). We find that the generation of linear cluster states is equally sensitive to bit-flip (X -gate) and phase-flip (Z -gate) errors, and more sensitive when both errors occur (Y -gate).

As photonic losses have non-trivial impacts on quantum states, generally producing mixed states, we have explored various approaches to simulate their effects. One approach is to extend the standard qubit space to a qutrit space so that, by accounting for a vacuum term, the state can still be represented as pure. Another approach is to keep track of the survival probability of a state after potential photonic loss events. Interestingly, we observe that the state fidelity calculation only depends on branches that see no loss, weighted by their respective probabilities. By tracking transmission probabilities, we gain information useful for approximating state generation rates, which are a very important experimental insight. We also calculate the survival probability for each photon, which indicates which portions of the graph state are more prone to loss. This information is essential for efficiently implementing encoding for error correction.

3.2.4. Assessing circuit performance

Fidelity has been our main metric thus far, but it is not ideal for our task as it can change abruptly, which is partially due to the discrete nature of adding or removing quantum operations within the quantum circuit. Entanglement witnesses have potential [24]–[26], but our investigations of GHZ and linear cluster states suggest they may be too robust and not sufficiently monotonic in the neighborhood of optimums. The graph edit metric, which is a metric based on the graph representation of a quantum state, holds potential as it is fundamentally different from other metrics. There are however challenges as it is not obvious

how fundamental graph operations, such as local complementations, would be represented in this metric. Most metrics investigated thus proved to not be useful towards search/optimization schemes or well suited for multipartite states.

3.2.5. Searching for performant quantum circuits

Using the random and evolutionary schemes within the density matrix representation, we were able to:

- Reproduce results from [13] for 4-photon linear cluster states.
- Find alternative, but equivalent circuits. Given some circuits may be more sensitive to noise compared to others or that some gates may be more difficult to implement than others, this gives options to experimentalists in implementing whichever protocol is more practical.
- Find cases where the circuit depth is reduced relative to circuits produced by Ref. [13]'s approach. An example of this depth decrease is shown in Fig. 3. Decreases in circuit depth imply a simplified protocol as there are fewer gates to be implemented or more gates can be implemented in parallel.
- In terms of convergence, we found that the random and evolutionary schemes performed similarly. Figure 4 shows the fraction of search runs that converge on a quantum circuit producing the target graph state.

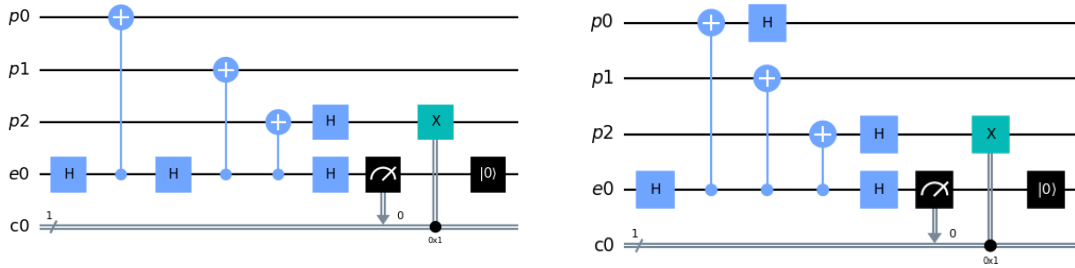


Figure 3: Quantum circuits for the generation of a 3-photon linear cluster state as determined by (left) the deterministic methods of [13] and (right) our framework. Our solution presents a reduced circuit depth relative to the deterministic method.

Using the stabilizer representation, we are able to efficiently simulate large graph states, up to hundreds of qubits, but it remains difficult to converge to an optimal circuit due to the vast optimization landscape. For this reason, we have also investigated the hybrid approach, which leverages the deterministic scheme of Ref. [13] to produce a starting point for random/evolutionary-based search approaches. Using this method, we have been able to find alternative quantum circuits to those proposed by the deterministic approach, as well as access larger graph states.

Additionally, the deterministic solutions do not include the effects of noise, which degrade the quality of the output graph state for a given circuit. Using our hybrid scheme, we have begun investigations into re-optimizing the deterministic solution after the introduction of noise, to regain state fidelity. This investigation is expected to greatly benefit from experimental feedback to more accurately represent noise found within real photonic platforms.



	Linear Cluster (3 qubits)	GHZ (3 qubits)	Linear Cluster (4 qubits)	GHZ (4 qubits)
Random scheme	92%	60%	24%	24%
Evolutionary scheme	96%	16%	24%	28%

Figure 4: Fraction of simulations that converge on a solution for a quantum circuit using a density matrix representation. Target graph states for each case are indicated above columns. Both the random and evolutionary schemes have difficulty converging with an increasing number of qubits. This is due to the vastness of the optimization landscape, indicating the need for better metrics and/or optimization schemes.

Within the density matrix representation, we have also implemented parameterized quantum gates and the automatic differentiation of quantum circuits. This enables the gradient-based optimization of circuit evaluation metrics (using, e.g., gradient-descent optimizers). This aspect of our framework enables quantum circuits to be optimized in the presence of noise, and enables gradient descent to be combined with circuit topology optimization schemes. An example of this scheme in action is shown in Fig. 5, where we perform quantum gate optimization to recover fidelity lost due to noise. Although the noise model requires refinement to accurately represent experimentally-found errors, these are promising results for managing noise-related deterioration of graph states.

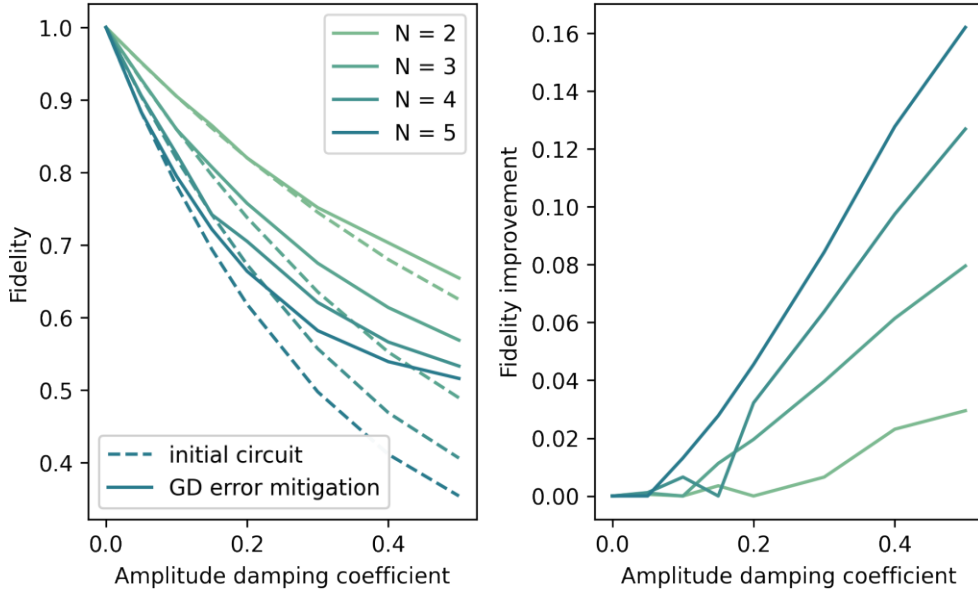


Figure 5: Regaining lost fidelity by parametrized gates optimization for linear cluster states consisting of $N=1-5$ photons as function of noise amplitude, where an amplitude damping model was uniformly applied on all qubits. Left panel shows the fidelity of the initial circuit after introducing noise (dashed) and the fidelity after gate optimization (solid). Right panel shows the relative difference between the solid and dashed lines. We find that gate optimization led to absolute fidelity recoveries of up to 15%.

3.2.5. Summary of accomplishments

During year 1 of this project, we have accomplished the following basic research milestones:

- Constructed a theoretical and numerical framework for the study of graph states and of the quantum circuits that generate them within photonics-based platforms.
- Reproduced known quantum circuits for the generation of graph states found within recent literature. In some cases, such as a 3-photon GHZ state, we have found alternative, yet equivalent quantum circuits, providing experimentalists with setup variety.
- For a 3-photon linear cluster state, we have found an improved quantum circuit with a shorter depth relative to published setups.
- Our framework supports diverse noise models, including depolarizing noise, amplitude damping, and Pauli errors.
- We have developed an experimentally-motivated, time-dependent noise model leveraging the stabilizer representation. This model reduces the number of potential outcomes within mixed states and expresses the noise in terms of circuit time-scales.
- Demonstrated that the use of parameterized gates in conjunction with gradient-based optimization can remediate the impacts of noise-driven fidelity degradation, with an absolute fidelity recovery of up to 15% for 5-photon linear cluster states.
- Developed a novel way of representing mixed states using stabilizers, allowing the simulation of more realistic noise effects and circuit dynamics.
- Developed a novel method for generating the graph representation of a quantum state from a density matrix.
- We have developed random and evolutionary schemes that search for well-performing quantum circuits given a target output state.
- We have developed a hybrid quantum circuit search scheme by combining deterministic and stochastic methods.
- Our framework is modular, and can easily be extended to other physical platforms and noise models.

3.2.6. Dissemination to the communities of interest.

Nothing to report. Nevertheless, manuscripts are in preparation and will be shared with the program officer.

4. Impacts

4.1. Development of the principal discipline of the project

We have developed a framework for the study and optimization of graph state generation in the presence of realistic effects such as noise and photonic loss, making it the first of its kind. Such a tool is a driver for further basic research into entangled resource preparation and distribution for various applications.

The framework is implemented in Python and was designed to be extensible, with community co-development in mind. To support the community and future partners in using these tools, we have prepared thorough documentation, code examples, and interactive tutorials.

Our long-term plans are discussed in detail in Sec. 6, but our near-term plan for disseminating our work to the community includes the following:

- Make our framework available to the community by open-sourcing our codebase

- alongside documentation, use examples, and tutorials
- Use the open-sourced code to enhance and establish research partnerships
- Prepare abstracts for conferences such as the International Conference on Quantum Communication, Measurement and Computing (QCMC), Q2B22 Silicon Valley, Photonics West, and CLEO
- Prepare a manuscript for peer-review and publication

4.2. Impact on other disciplines

In addition to the dissemination of our framework, we also intend to collaborate directly with colleagues from the community. This includes collaborating with the National Research Council of Canada (NRC), specialized in the fabrication of quantum dots that can be used as emitter qubits, l’Institut National de la Recherche Scientifique (INRS), giving us access to knowledge on integrated photonics, as well as Caltech and Fermilab, who can provide realistic entanglement distribution channel metrics from their respective testbeds. Additionally, Quantum Bridge Technologies, Inc. (QBT) is a University of Toronto spin-off, with many local research collaboration opportunities. Our collaborations will allow us to test and validate our framework, as well as improve its level of realism.

5. Future goals

As part of future work, our team intends to maintain and develop our framework alongside the open-source quantum community to find solutions for higher complexity graph states. In parallel, our work internally will focus on extending the framework with more numerous and realistic components as well as qubit protection strategies. To perform this work, we hope for continued support of the project in the years to come and provide below a Gantt chart presenting our future project goals for the following year. Indicated within each task are assigned lead researchers, with a team overview following the chart.

Task description	Months											
	M1-M4				M4-M8				M8-M12			
Modeling fusion gate components												
Using fusion gates [27], we can form large graph states from smaller graph states. We will implement such gates within our framework, giving an alternative route to realizing graph states. This task will be led by PDF1 and supported by PDF2 and PhD1.												
Literature review and implementing fusion gates within framework												
Investigate fusion gate-based quantum circuits												
Encoding for error correction and fusion gate protection												
We will explore encoding methods for protecting qubits against random errors (e.g., bitflip) and failed fusion gate attempts, which lead to the loss of a qubit. This task will be led by PDF1, and supported by PDF2 and PhD1.												
Investigate encoding strategies in literature												
Implement encoding for qubit protection												

Open innovation and community impact As discussed in Sec. 5, we aim to disseminate our work to support the quantum research community. Open-sourcing will be co-led by I1 and I2. Reaching out to community members will be co-led by PM1, PM2, PM3 and PM4. The writing of a manuscript will be co-led by PDF2 and PDF3 with support from the full team.															
Publish codebase and finalize public documentation and tutorials															
Prepare manuscript and submit to journal															
Project management and administrative support Project management will continue to be led by PM1, with the support of PM2, PM3 and PM4.															
Ongoing management and project reporting															

6. Team overview:

PM1: Piotr Roztocki

CTO, Ki3 Photonics Technologies Inc. Principal project manager responsible for day-to-day activities. Honors B.Sc. in Integrated Science (Physics), McMaster University 2014; Ph.D. in Energy and Materials, INRS-EMT 2022.

PM2: Hoi-Kwong Lo

CTO, Quantum Bridge Technologies, Inc. (QBT). Plays a supervisory role in the project. Professor of Physics and ECE at the University of Toronto, BA in Mathematics, Cambridge University, 1989, Ph.D. in Physics, Caltech, 1994. Co-inventor of measurement-device-independent quantum key distribution (QKD) and all photonics quantum repeaters. Fellow of the American Physical Society (APS) and Optica Fellow.

PM3: Mattia Montagna

CEO, Quantum Bridge Technologies, Inc. (QBT), PhD in Applied Math. Founder of the ECB (European Central Bank) Bank Interconnected Coordination Group (BICG). Has been leading QBT since its founding in 2019.

PM4: Yoann Jestin

CEO, Ki3 Photonics Technologies Inc. Principal Investigator of the project. B.Sc in Chemistry, University of Rennes, 1997; M.Sc. in Material Science, University of Rennes, 1999; Ph.D in Material Science, University of Le Mans, 2002.

PDF1: Jie Lin

QBT quantum research scientist with specializations in quantum optics and security analysis of quantum key distribution. Will lead development of both fusion gates and qubit encoding. B. A. Mathematics, Carleton College; M. A. S. Theoretical Physics, University of Cambridge; M. Sc. Physics and Ph.D. Physics (quantum information), University of Waterloo.

PDF2: Luc Robichaud

QBT quantum research scientist who specializes in quantum dot modeling and their application to entanglement generation. Will lead modeling of various sources and incorporating said models into the framework. Will also co-lead with PDF3 the writing of papers for peer-reviewed journals. B.Sc. Physics and M.Sc. Physics, Université de Moncton;

Ph.D. Physics, University of Ottawa.

PDF3: Stefania Sciara

Postdoc at l'Institut National de la Recherche Scientifique (INRS) and Ki3 researcher with specializations in both experimental and theoretical quantum optics, and entanglement generation. B.Sc. Physics and M.Sc. Physics, University of Palermo; Ph.D. Quantum optics, INRS.

PhD1: Sobhan Ghanbari

QBT intern. Ph.D. candidate at the University of Toronto with specialization in quantum information. Sobhan has led and will continue to lead development of various noise models within the framework. B.Sc. Physics/ B.Sc. Mechanical Engineering (dual- degree), Sharif University of Technology.

PhD2: Benjamin MacLellan

Ph.D. candidate at the University of Waterloo with specialization in quantum information and inverse design of photonic systems. Will lead the development of a more comprehensive physical layer, including various integrated-photonics-based components. B.Sc. Physics, University of Waterloo; M.Sc., Energy & Materials, INRS.

M1: Kai Sum Chan

QBT quantum research scientist. M.Phil. candidate at the University of Hong Kong, and QBT experimental and theoretical quantum researcher with specialization in quantum information, adaptive optics and quantum dots. Will play a supporting role in calibrating models according to experiments.

I1: Khuong Tran

B.Sc. candidate in computer sciences at the University of Ontario Institute of Technology and intern at Ki3. Will co-lead with I2 preparation of open-sourcing, framework documentation and tutorials.

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Key: PM: Project manager, PDF: Postdoctoral fellow, PhD: PhD student, M: Master's student, I: Intern.

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Highly Entangled States of Light

Final Report

Awarded to:
Ki3 Photonics Technologies Inc.
Quantum Bridge Technologies Inc (QBT).

Recipient Principal Investigator:
Dr. Yoann Jestin

1. Executive summary

Problem statement:

- Graph quantum states are an important resource for fundamental research and technological applications
- Designing quantum circuits that output specific and desired graph states is challenging
- Real quantum circuits are subject to noise and loss, which are necessary to include for an accurate and realistic model

Project scope:

- Construct a theoretical and numerical framework for the study of graph states and of circuits that generate them
- Develop schemes for the construction of quantum circuits that output desired graph states
- Create models for the noise and loss effects that are present in real quantum setups

General outcome:

- To date, we have defined a framework that allows us to find and to simulate quantum circuits that output low-complexity graph states
- Our schemes successfully reproduce results from literature and are also capable of finding alternative setups to choose between
- We have developed noise and loss models that are representative of effects found in common photonics platforms
- Our framework is a flexible and extensible tool that is soon to be open-sourced and which can be easily adapted to simulate various physical platforms

Future Plans:

- Maintain and develop the framework alongside the open-source quantum community to find solutions for higher complexity graph states
- Develop fusion-gate based protocols for producing large graph states
- Implement encoding strategies for qubit protection
- Model quantum-dot-based sources and interface with current framework
- Develop models for integrated-photonics-based components as an experimental basis for quantum circuits
- Develop more sophisticated noise and distribution channel models
- Extend our ability to inject experimental details into our framework, further connecting results with real setups

2. Introduction

Graph states are a family of quantum states with high levels of entanglement that are of great fundamental interest [1], as well as being a key resource for applications in entanglement distribution and quantum networking [2-4], quantum error correction [5,6], and quantum metrology [7]. Among various physical platforms, photonics shows much promise for graph state realizations [8,9] as photons interact very weakly with their environment, enabling stable qubits. Additionally, photons are compatible with mature integrated and fiber technologies, granting additional support in the generation and distribution of graph states.

As well, for the generation of photons, various sources are available within photonics-based platforms, including solid-state emitters such as quantum dots that can emit single photons on demand [10], or nonlinear elements generating pairs of entangled photons [11,12].

Quantum circuits describe the sequence of quantum operations necessary to generate such graph states. However, connecting a desired graph state with the circuit that can output it is a non-trivial design task that only becomes more difficult with increasing state size and complexity. Recent work [13] has led to the development of an impressive deterministic method for finding quantum circuits that produce targeted graph states. However, such methods do not guarantee well-performing quantum circuits, allow little flexibility, and cannot consider impacts associated with noise or photonic loss, which is present in all real setups.

Towards addressing this research gap, the goal of our work was to develop a flexible theoretical and computational framework for the study of realistic graph states and of the quantum circuits that generate them. To accomplish this, we must be able to represent both the quantum states and the quantum circuits that produce them. Both representations must also be flexible enough to support noise and photonic loss modeling. The performance of different circuits must also be quantifiable, in order to drive schemes that search for best performing setups. All of these concepts must also be realized in a flexible and modular way that allows for co-development of solutions with the broader research community, driving collaborative and fundamental work into the understanding of entangled states as a resource.

Over the past year of our project's progress, we have successfully identified and realized a framework comprising the following core concepts: *Quantum state representations*, *quantum circuit representations*, *realistic component models*, *quantum circuit performance metrics*, and *quantum circuit optimization schemes*. Figure 1 shows these key concepts and their mutual interaction within our framework.

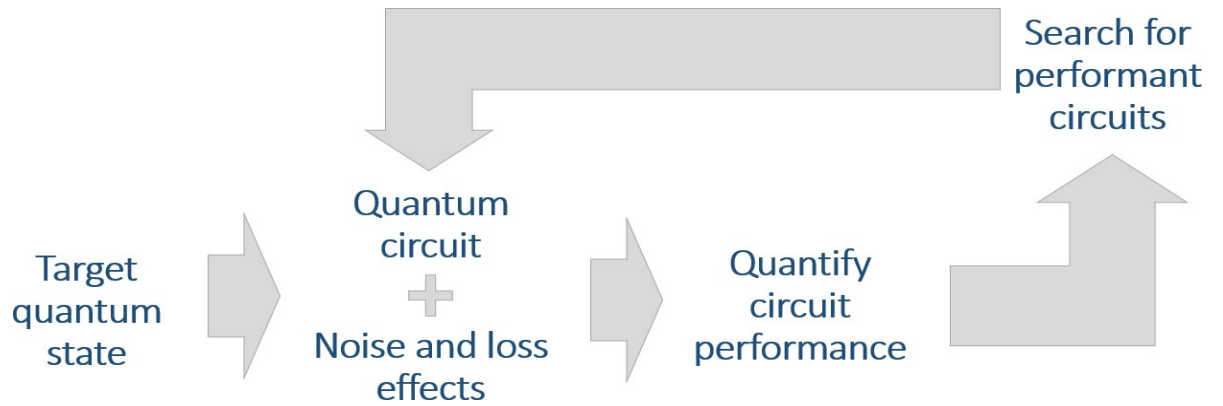


Figure 1: Key concepts within our framework. The performance metric is calculated with respect to the target graph state, guiding the search for performant circuits.

3. Accomplishments

In what follows, we explain each of the core concepts of our framework alongside our respective research efforts in developing them.

3.1. Research objectives

3.3.1. Representing quantum states

The objects of interest, graph states, are quantum systems of photonic and emitter qubits that can be mathematically represented using various formalisms. Our computational framework presently considers two mathematical representations, the *density matrix* and the *stabilizer*, with the modularity to easily include further representations in the future.

3.1.2. Representing quantum circuits

Quantum circuits represent the sequence of quantum gates and operations applied on solid state emitter qubits and photonic qubits. To computationally read, save, and manipulate these quantum circuits, we represent them as Directed Acyclic Graphs (DAGs) [19], which find common use in the field of quantum information [20]. Using the DAG representation, our computational framework can easily modify and evolve the quantum circuit topology to optimize the generated quantum state towards a target graph state. For the visualization of quantum circuits, our DAG representation was also made compatible with the openQASM format, a standard within the field [21].

3.1.3. Realistic component models

Towards increasing the realism of our framework, we support diverse models for noise and photonic loss. For example, our framework presently includes noise models for depolarizing noise, amplitude damping, and Pauli noise.

Additionally, we have developed a new representation to model mixed states using stabilizers. This new representation keeps track of all potential branchings of a mixed state, along with the probability of such a branch being measured. This novel approach allows us to consider probabilistic noise, more akin to real experimental systems.

Motivated by the physics of today's solid-state emitter and photonic qubit systems, we have furthermore developed a time-dependent noise model within the stabilizer formalism. In experiments, the number of errors that occur is strongly correlated with the time spent during and between each quantum gate. In our time-dependent model, we make use of an effective noise (simulated only at certain gates), maintaining model accuracy while reducing the computational load of the model.

3.1.4. Assessing circuit performance

When optimizing a quantum circuit for the generation of a target graph state, it is important to properly quantify the performance of said circuit using quantitative metrics. In fact, having proper metrics can heavily influence the performance of a circuit search algorithm. To date, we have investigated the following metrics as potential candidates [14], [22], [23]:

Fidelity	Relative entropy
Entanglement witness	Squashed entanglement
Graph edit distance	Adjacency matrix fidelity
Concurrence	Geometric measure
Negativity	Fisher information
Trace distance	Mermin inequality
Entanglement distillation	Stabilizer tableau fidelity

3.1.5. Searching for performant quantum circuits

We have investigated the following schemes for converging on suitable quantum circuits that output a desired graph state:

- **Random:** This scheme initializes a list of random circuits subject to physical

constraints, which we call a population. At each iteration and for each circuit within the population, the algorithm applies a random operation, which includes addition, removal, or replacement of a quantum gate. Each circuit in the population is modified independently from the other circuits. The scheme then keeps track of a certain number of top performing circuits.

- **Evolutionary:** Similarly to the random search approach, this scheme also applies random operations in each iteration. However, in contrast, after each iteration it constructs a new population by putting circuits into a N-wise competition and selecting the best performers.
- **Deterministic:** This scheme is our implementation of Ref. [13]’s approach to deterministically find a quantum circuit for a target graph state.
- **Hybrid:** In this scheme, we use the deterministic method from Ref. [13] to produce an initial solution. We then perturb this solution and use the random/evolutionary schemes to search for quantum circuit alternatives.
- **Gradient-based:** This is a density matrix specific scheme where we consider tunable/parametrized gates that are compatible with automatic differentiation and gradient descent optimization techniques.

3.2 Details of accomplishments

3.2.1. Representing quantum states

The *density matrix* representation can simulate a state’s full Hilbert space and can inherently represent mixed states, such as those resulting from the effects of noise and photonic loss [14]. However, this representation does not scale well with the number of qubits, making it impractical to investigate large graph states, and thus motivating the need for complementary representations. We have used this representation: 1) To study small graph states with the inclusion of noise effects; 2) As a benchmark for other representations; and 3) For the study of noise, photonic loss, mixed states, and gate parametrization.

The *Stabilizer* representation [15], [16], on the other hand, scales well with the number of qubits, making it more efficient for the study of large graph states (relative to the density matrix formalism). However, only a subset of quantum operations can be efficiently simulated within the stabilizer representation, limiting the scope for circuit optimization, and making this representation unsuited for the simulation of noise without judicious changes to the formalism. Presently, the study of representing mixed states within the stabilizer representation is very limited in the literature. To be able to consider large graph states that are mixed due to noise or loss, we have developed a probability tree approach as a way to represent such states. This point of research is presented further below in Sec. 3.3.

We have also developed relevant concepts from *graph* representations, which are also well-suited for many qubits, and contrary to the stabilizer representation, there is more progress made on describing mixed states [17,18]. In particular, we have worked on leveraging graph state equivalency through Clifford gates. As discussed in Sec. 3.4, the graph representation also offers unique metrics for characterizing the performance of quantum circuits.

Each of the quantum state representations has advantages and disadvantages, making it useful to be able to move between them. Most of these representation transformations can be found within literature [3], [14-18], except for obtaining a graph from a density matrix. We have resolved this issue by designing a novel protocol, as discussed in Sec. 3.5.

3.2.2. Representing quantum circuits

Although we have focused on solid state emitter qubits and photonic qubits, we have designed our framework with extensibility in mind. The types of operations, and therefore the

physical circuits we can represent and simulate, are flexible, meaning we can adapt our framework towards different physical platforms with relative ease.

Lastly, our quantum circuit representation also includes parameterized quantum gates and automatic differentiation (see Sec. 3.5 for further discussion).

3.2.3. Realistic component models

The Figure 2 shows an example of the model in practice, deriving the fidelity of an output graph state as a function of qubit error rate from various sources of noise.

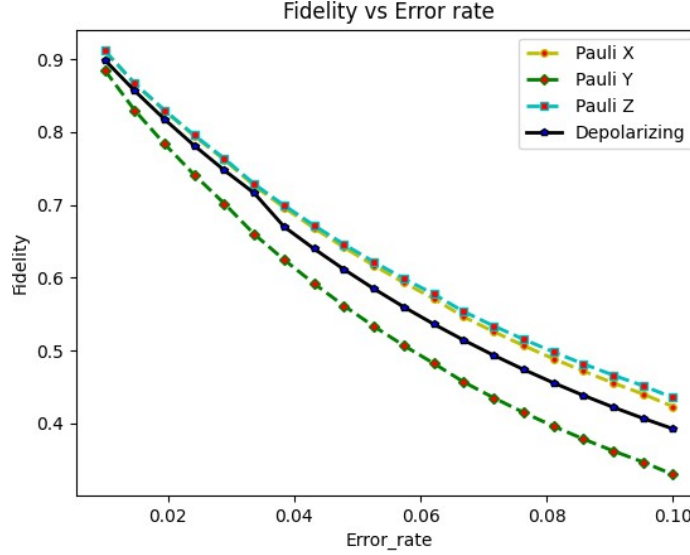


Figure 2: Fidelity versus error rate for the generation of a 4-photon linear cluster state assuming X -gate errors (yellow), Y -gate errors (green), Z -gate errors (blue) and a depolarizing channel (black). We find that the generation of linear cluster states is equally sensitive to bit-flip (X -gate) and phase-flip (Z -gate) errors, and more sensitive when both errors occur (Y -gate).

As photonic losses have non-trivial impacts on quantum states, generally producing mixed states, we have explored various approaches to simulate their effects. One approach is to extend the standard qubit space to a qutrit space so that, by accounting for a vacuum term, the state can still be represented as pure. Another approach is to keep track of the survival probability of a state after potential photonic loss events. Interestingly, we observe that the state fidelity calculation only depends on branches that see no loss, weighted by their respective probabilities. By tracking transmission probabilities, we gain information useful for approximating state generation rates, which are a very important experimental insight. We also calculate the survival probability for each photon, which indicates which portions of the graph state are more prone to loss. This information is essential for efficiently implementing encoding for error correction.

3.2.4. Assessing circuit performance

Fidelity has been our main metric thus far, but it is not ideal for our task as it can change abruptly, which is partially due to the discrete nature of adding or removing quantum operations within the quantum circuit. Entanglement witnesses have potential [24]–[26], but our investigations of GHZ and linear cluster states suggest they may be too robust and not sufficiently monotonic in the neighborhood of optimums. The graph edit metric, which is a metric based on the graph representation of a quantum state, holds potential as it is fundamentally different from other metrics. There are however challenges as it is not obvious

how fundamental graph operations, such as local complementations, would be represented in this metric. Most metrics investigated thus proved to not be useful towards search/optimization schemes or well suited for multipartite states.

3.2.5. Searching for performant quantum circuits

Using the random and evolutionary schemes within the density matrix representation, we were able to:

- Reproduce results from [13] for 4-photon linear cluster states.
- Find alternative, but equivalent circuits. Given some circuits may be more sensitive to noise compared to others or that some gates may be more difficult to implement than others, this gives options to experimentalists in implementing whichever protocol is more practical.
- Find cases where the circuit depth is reduced relative to circuits produced by Ref. [13]'s approach. An example of this depth decrease is shown in Fig. 3. Decreases in circuit depth imply a simplified protocol as there are fewer gates to be implemented or more gates can be implemented in parallel.
- In terms of convergence, we found that the random and evolutionary schemes performed similarly. Figure 4 shows the fraction of search runs that converge on a quantum circuit producing the target graph state.

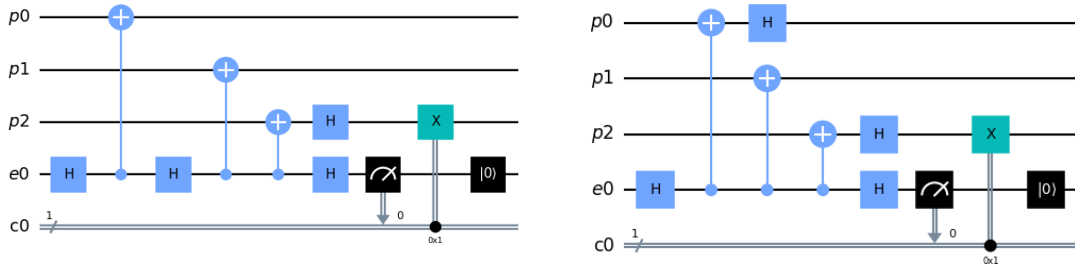


Figure 3: Quantum circuits for the generation of a 3-photon linear cluster state as determined by (left) the deterministic methods of [13] and (right) our framework. Our solution presents a reduced circuit depth relative to the deterministic method.

Using the stabilizer representation, we are able to efficiently simulate large graph states, up to hundreds of qubits, but it remains difficult to converge to an optimal circuit due to the vast optimization landscape. For this reason, we have also investigated the hybrid approach, which leverages the deterministic scheme of Ref. [13] to produce a starting point for random/evolutionary-based search approaches. Using this method, we have been able to find alternative quantum circuits to those proposed by the deterministic approach, as well as access larger graph states.

Additionally, the deterministic solutions do not include the effects of noise, which degrade the quality of the output graph state for a given circuit. Using our hybrid scheme, we have begun investigations into re-optimizing the deterministic solution after the introduction of noise, to regain state fidelity. This investigation is expected to greatly benefit from experimental feedback to more accurately represent noise found within real photonic platforms.



	Linear Cluster (3 qubits)	GHZ (3 qubits)	Linear Cluster (4 qubits)	GHZ (4 qubits)
Random scheme	92%	60%	24%	24%
Evolutionary scheme	96%	16%	24%	28%

Figure 4: Fraction of simulations that converge on a solution for a quantum circuit using a density matrix representation. Target graph states for each case are indicated above columns. Both the random and evolutionary schemes have difficulty converging with an increasing number of qubits. This is due to the vastness of the optimization landscape, indicating the need for better metrics and/or optimization schemes.

Within the density matrix representation, we have also implemented parameterized quantum gates and the automatic differentiation of quantum circuits. This enables the gradient-based optimization of circuit evaluation metrics (using, e.g., gradient-descent optimizers). This aspect of our framework enables quantum circuits to be optimized in the presence of noise, and enables gradient descent to be combined with circuit topology optimization schemes. An example of this scheme in action is shown in Fig. 5, where we perform quantum gate optimization to recover fidelity lost due to noise. Although the noise model requires refinement to accurately represent experimentally-found errors, these are promising results for managing noise-related deterioration of graph states.

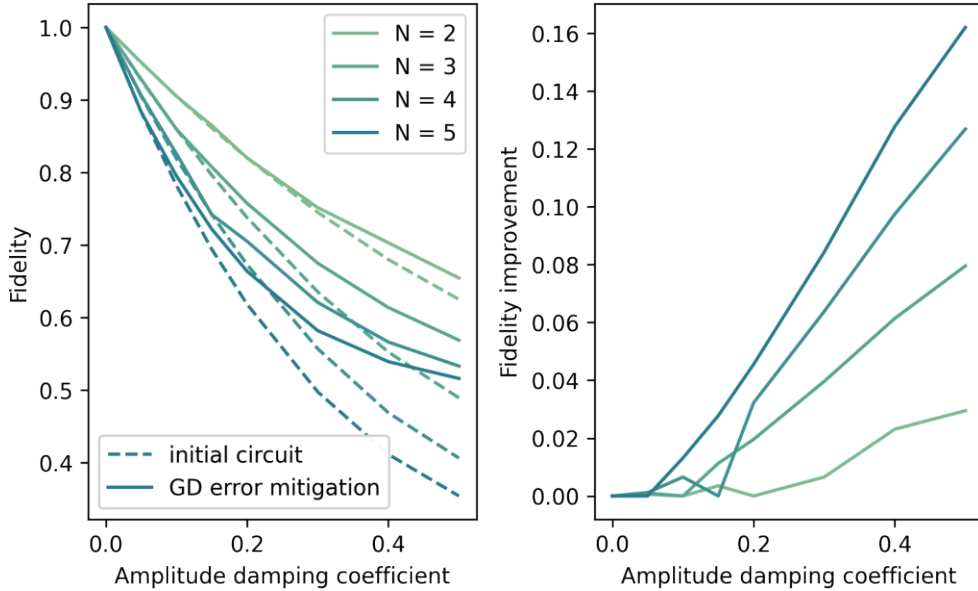


Figure 5: Regaining lost fidelity by parametrized gates optimization for linear cluster states consisting of $N=1-5$ photons as function of noise amplitude, where an amplitude damping model was uniformly applied on all qubits. Left panel shows the fidelity of the initial circuit after introducing noise (dashed) and the fidelity after gate optimization (solid). Right panel shows the relative difference between the solid and dashed lines. We find that gate optimization led to absolute fidelity recoveries of up to 15%.

3.2.5. Summary of accomplishments

During year 1 of this project, we have accomplished the following basic research milestones:

- Constructed a theoretical and numerical framework for the study of graph states and of the quantum circuits that generate them within photonics-based platforms.
- Reproduced known quantum circuits for the generation of graph states found within recent literature. In some cases, such as a 3-photon GHZ state, we have found alternative, yet equivalent quantum circuits, providing experimentalists with setup variety.
- For a 3-photon linear cluster state, we have found an improved quantum circuit with a shorter depth relative to published setups.
- Our framework supports diverse noise models, including depolarizing noise, amplitude damping, and Pauli errors.
- We have developed an experimentally-motivated, time-dependent noise model leveraging the stabilizer representation. This model reduces the number of potential outcomes within mixed states and expresses the noise in terms of circuit time-scales.
- Demonstrated that the use of parameterized gates in conjunction with gradient-based optimization can remediate the impacts of noise-driven fidelity degradation, with an absolute fidelity recovery of up to 15% for 5-photon linear cluster states.
- Developed a novel way of representing mixed states using stabilizers, allowing the simulation of more realistic noise effects and circuit dynamics.
- Developed a novel method for generating the graph representation of a quantum state from a density matrix.
- We have developed random and evolutionary schemes that search for well-performing quantum circuits given a target output state.
- We have developed a hybrid quantum circuit search scheme by combining deterministic and stochastic methods.
- Our framework is modular, and can easily be extended to other physical platforms and noise models.

3.2.6. Dissemination to the communities of interest.

Nothing to report. Nevertheless, manuscripts are in preparation and will be shared with the program officer.

4. Impacts

4.1. Development of the principal discipline of the project

We have developed a framework for the study and optimization of graph state generation in the presence of realistic effects such as noise and photonic loss, making it the first of its kind. Such a tool is a driver for further basic research into entangled resource preparation and distribution for various applications.

The framework is implemented in Python and was designed to be extensible, with community co-development in mind. To support the community and future partners in using these tools, we have prepared thorough documentation, code examples, and interactive tutorials.

Our long-term plans are discussed in detail in Sec. 6, but our near-term plan for disseminating our work to the community includes the following:

- Make our framework available to the community by open-sourcing our codebase

- alongside documentation, use examples, and tutorials
- Use the open-sourced code to enhance and establish research partnerships
- Prepare abstracts for conferences such as the International Conference on Quantum Communication, Measurement and Computing (QCMC), Q2B22 Silicon Valley, Photonics West, and CLEO
- Prepare a manuscript for peer-review and publication

4.2. Impact on other disciplines

In addition to the dissemination of our framework, we also intend to collaborate directly with colleagues from the community. This includes collaborating with the National Research Council of Canada (NRC), specialized in the fabrication of quantum dots that can be used as emitter qubits, l’Institut National de la Recherche Scientifique (INRS), giving us access to knowledge on integrated photonics, as well as Caltech and Fermilab, who can provide realistic entanglement distribution channel metrics from their respective testbeds. Additionally, Quantum Bridge Technologies, Inc. (QBT) is a University of Toronto spin-off, with many local research collaboration opportunities. Our collaborations will allow us to test and validate our framework, as well as improve its level of realism.

5. Future goals

As part of future work, our team intends to maintain and develop our framework alongside the open-source quantum community to find solutions for higher complexity graph states. In parallel, our work internally will focus on extending the framework with more numerous and realistic components as well as qubit protection strategies. To perform this work, we hope for continued support of the project in the years to come and provide below a Gantt chart presenting our future project goals for the following year. Indicated within each task are assigned lead researchers, with a team overview following the chart.

Task description	Months											
	M1-M4				M4-M8				M8-M12			
Modeling fusion gate components Using fusion gates [27], we can form large graph states from smaller graph states. We will implement such gates within our framework, giving an alternative route to realizing graph states. This task will be led by PDF1 and supported by PDF2 and PhD1.												
Literature review and implementing fusion gates within framework												
Investigate fusion gate-based quantum circuits												
Encoding for error correction and fusion gate protection We will explore encoding methods for protecting qubits against random errors (e.g., bitflip) and failed fusion gate attempts, which lead to the loss of a qubit. This task will be led by PDF1, and supported by PDF2 and PhD1.												
Investigate encoding strategies in literature												
Implement encoding for qubit protection												

Open innovation and community impact As discussed in Sec. 5, we aim to disseminate our work to support the quantum research community. Open-sourcing will be co-led by I1 and I2. Reaching out to community members will be co-led by PM1, PM2, PM3 and PM4. The writing of a manuscript will be co-led by PDF2 and PDF3 with support from the full team.												
Publish codebase and finalize public documentation and tutorials												
Prepare manuscript and submit to journal												
Project management and administrative support Project management will continue to be led by PM1, with the support of PM2, PM3 and PM4.												
Ongoing management and project reporting												

6. Team overview:

PM1: Piotr Roztocki

CTO, Ki3 Photonics Technologies Inc. Principal project manager responsible for day-to-day activities. Honors B.Sc. in Integrated Science (Physics), McMaster University 2014; Ph.D. in Energy and Materials, INRS-EMT 2022.

PM2: Hoi-Kwong Lo

CTO, Quantum Bridge Technologies, Inc. (QBT). Plays a supervisory role in the project. Professor of Physics and ECE at the University of Toronto, BA in Mathematics, Cambridge University, 1989, Ph.D. in Physics, Caltech, 1994. Co-inventor of measurement-device-independent quantum key distribution (QKD) and all photonics quantum repeaters. Fellow of the American Physical Society (APS) and Optica Fellow.

PM3: Mattia Montagna

CEO, Quantum Bridge Technologies, Inc. (QBT), PhD in Applied Math. Founder of the ECB (European Central Bank) Bank Interconnected Coordination Group (BICG). Has been leading QBT since its founding in 2019.

PM4: Yoann Jestin

CEO, Ki3 Photonics Technologies Inc. Principal Investigator of the project. B.Sc in Chemistry, University of Rennes, 1997; M.Sc. in Material Science, University of Rennes, 1999; Ph.D in Material Science, University of Le Mans, 2002.

PDF1: Jie Lin

QBT quantum research scientist with specializations in quantum optics and security analysis of quantum key distribution. Will lead development of both fusion gates and qubit encoding. B. A. Mathematics, Carleton College; M. A. S. Theoretical Physics, University of Cambridge; M. Sc. Physics and Ph.D. Physics (quantum information), University of Waterloo.

PDF2: Luc Robichaud

QBT quantum research scientist who specializes in quantum dot modeling and their application to entanglement generation. Will lead modeling of various sources and incorporating said models into the framework. Will also co-lead with PDF3 the writing of papers for peer-reviewed journals. B.Sc. Physics and M.Sc. Physics, Université de Moncton;

Ph.D. Physics, University of Ottawa.

PDF3: Stefania Sciara

Postdoc at l'Institut National de la Recherche Scientifique (INRS) and Ki3 researcher with specializations in both experimental and theoretical quantum optics, and entanglement generation. B.Sc. Physics and M.Sc. Physics, University of Palermo; Ph.D. Quantum optics, INRS.

PhD1: Sobhan Ghanbari

QBT intern. Ph.D. candidate at the University of Toronto with specialization in quantum information. Sobhan has led and will continue to lead development of various noise models within the framework. B.Sc. Physics/ B.Sc. Mechanical Engineering (dual- degree), Sharif University of Technology.

PhD2: Benjamin MacLellan

Ph.D. candidate at the University of Waterloo with specialization in quantum information and inverse design of photonic systems. Will lead the development of a more comprehensive physical layer, including various integrated-photonics-based components. B.Sc. Physics, University of Waterloo; M.Sc., Energy & Materials, INRS.

M1: Kai Sum Chan

QBT quantum research scientist. M.Phil. candidate at the University of Hong Kong, and QBT experimental and theoretical quantum researcher with specialization in quantum information, adaptive optics and quantum dots. Will play a supporting role in calibrating models according to experiments.

I1: Khuong Tran

B.Sc. candidate in computer sciences at the University of Ontario Institute of Technology and intern at Ki3. Will co-lead with I2 preparation of open-sourcing, framework documentation and tutorials.

I2: Aayush Soni

B.A.Sc. candidate in mechatronics engineering at the University of Waterloo and intern at Ki3. Will co-lead with I1 the preparation of open-sourcing, framework documentation and tutorials.

Key: PM: Project manager, PDF: Postdoctoral fellow, PhD: PhD student, M: Master's student, I: Intern.

7. References

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Grant FA9550-22-1-0062
Highly Entangled States of Light

Final Report

Awarded to:
Ki3 Photonics Technologies Inc.
Quantum Bridge Technologies Inc (QBT).

Recipient Principal Investigator:
Dr. Yoann Jestin

1. Executive summary

Problem statement:

- Graph quantum states are an important resource for fundamental research and technological applications
- Designing quantum circuits that output specific and desired graph states is challenging
- Real quantum circuits are subject to noise and loss, which are necessary to include for an accurate and realistic model

Project scope:

- Construct a theoretical and numerical framework for the study of graph states and of circuits that generate them
- Develop schemes for the construction of quantum circuits that output desired graph states
- Create models for the noise and loss effects that are present in real quantum setups

General outcome:

- To date, we have defined a framework that allows us to find and to simulate quantum circuits that output low-complexity graph states
- Our schemes successfully reproduce results from literature and are also capable of finding alternative setups to choose between
- We have developed noise and loss models that are representative of effects found in common photonics platforms
- Our framework is a flexible and extensible tool that is soon to be open-sourced and which can be easily adapted to simulate various physical platforms

Future Plans:

- Maintain and develop the framework alongside the open-source quantum community to find solutions for higher complexity graph states
- Develop fusion-gate based protocols for producing large graph states
- Implement encoding strategies for qubit protection
- Model quantum-dot-based sources and interface with current framework
- Develop models for integrated-photonics-based components as an experimental basis for quantum circuits
- Develop more sophisticated noise and distribution channel models
- Extend our ability to inject experimental details into our framework, further connecting results with real setups

2. Introduction

Graph states are a family of quantum states with high levels of entanglement that are of great fundamental interest [1], as well as being a key resource for applications in entanglement distribution and quantum networking [2-4], quantum error correction [5,6], and quantum metrology [7]. Among various physical platforms, photonics shows much promise for graph state realizations [8,9] as photons interact very weakly with their environment, enabling stable qubits. Additionally, photons are compatible with mature integrated and fiber technologies, granting additional support in the generation and distribution of graph states.

As well, for the generation of photons, various sources are available within photonics-based platforms, including solid-state emitters such as quantum dots that can emit single photons on demand [10], or nonlinear elements generating pairs of entangled photons [11,12].

Quantum circuits describe the sequence of quantum operations necessary to generate such graph states. However, connecting a desired graph state with the circuit that can output it is a non-trivial design task that only becomes more difficult with increasing state size and complexity. Recent work [13] has led to the development of an impressive deterministic method for finding quantum circuits that produce targeted graph states. However, such methods do not guarantee well-performing quantum circuits, allow little flexibility, and cannot consider impacts associated with noise or photonic loss, which is present in all real setups.

Towards addressing this research gap, the goal of our work was to develop a flexible theoretical and computational framework for the study of realistic graph states and of the quantum circuits that generate them. To accomplish this, we must be able to represent both the quantum states and the quantum circuits that produce them. Both representations must also be flexible enough to support noise and photonic loss modeling. The performance of different circuits must also be quantifiable, in order to drive schemes that search for best performing setups. All of these concepts must also be realized in a flexible and modular way that allows for co-development of solutions with the broader research community, driving collaborative and fundamental work into the understanding of entangled states as a resource.

Over the past year of our project's progress, we have successfully identified and realized a framework comprising the following core concepts: *Quantum state representations*, *quantum circuit representations*, *realistic component models*, *quantum circuit performance metrics*, and *quantum circuit optimization schemes*. Figure 1 shows these key concepts and their mutual interaction within our framework.

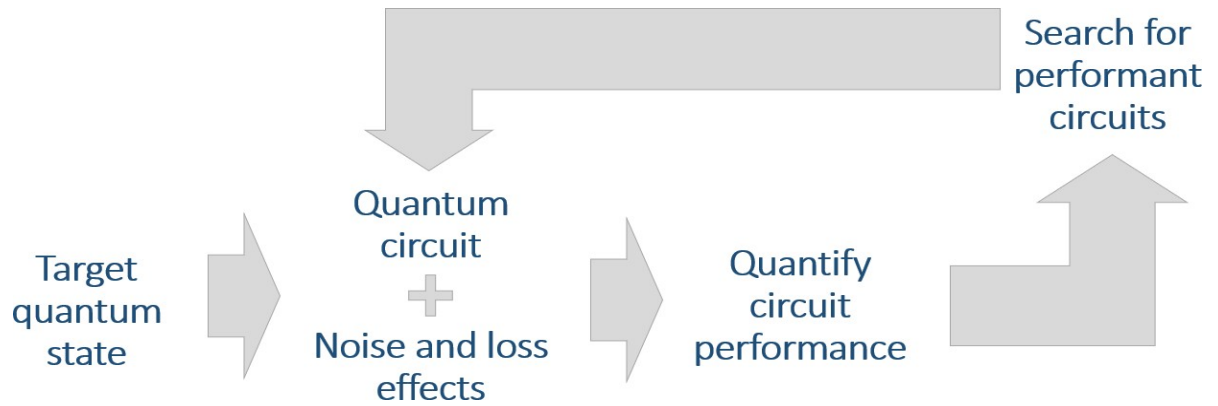


Figure 1: Key concepts within our framework. The performance metric is calculated with respect to the target graph state, guiding the search for performant circuits.

3. Accomplishments

In what follows, we explain each of the core concepts of our framework alongside our respective research efforts in developing them.

3.1. Research objectives

3.3.1. Representing quantum states

The objects of interest, graph states, are quantum systems of photonic and emitter qubits that can be mathematically represented using various formalisms. Our computational framework presently considers two mathematical representations, the *density matrix* and the *stabilizer*, with the modularity to easily include further representations in the future.

3.1.2. Representing quantum circuits

Quantum circuits represent the sequence of quantum gates and operations applied on solid state emitter qubits and photonic qubits. To computationally read, save, and manipulate these quantum circuits, we represent them as Directed Acyclic Graphs (DAGs) [19], which find common use in the field of quantum information [20]. Using the DAG representation, our computational framework can easily modify and evolve the quantum circuit topology to optimize the generated quantum state towards a target graph state. For the visualization of quantum circuits, our DAG representation was also made compatible with the openQASM format, a standard within the field [21].

3.1.3. Realistic component models

Towards increasing the realism of our framework, we support diverse models for noise and photonic loss. For example, our framework presently includes noise models for depolarizing noise, amplitude damping, and Pauli noise.

Additionally, we have developed a new representation to model mixed states using stabilizers. This new representation keeps track of all potential branchings of a mixed state, along with the probability of such a branch being measured. This novel approach allows us to consider probabilistic noise, more akin to real experimental systems.

Motivated by the physics of today's solid-state emitter and photonic qubit systems, we have furthermore developed a time-dependent noise model within the stabilizer formalism. In experiments, the number of errors that occur is strongly correlated with the time spent during and between each quantum gate. In our time-dependent model, we make use of an effective noise (simulated only at certain gates), maintaining model accuracy while reducing the computational load of the model.

3.1.4. Assessing circuit performance

When optimizing a quantum circuit for the generation of a target graph state, it is important to properly quantify the performance of said circuit using quantitative metrics. In fact, having proper metrics can heavily influence the performance of a circuit search algorithm. To date, we have investigated the following metrics as potential candidates [14], [22], [23]:

Fidelity	Relative entropy
Entanglement witness	Squashed entanglement
Graph edit distance	Adjacency matrix fidelity
Concurrence	Geometric measure
Negativity	Fisher information
Trace distance	Mermin inequality
Entanglement distillation	Stabilizer tableau fidelity

3.1.5. Searching for performant quantum circuits

We have investigated the following schemes for converging on suitable quantum circuits that output a desired graph state:

- **Random:** This scheme initializes a list of random circuits subject to physical

constraints, which we call a population. At each iteration and for each circuit within the population, the algorithm applies a random operation, which includes addition, removal, or replacement of a quantum gate. Each circuit in the population is modified independently from the other circuits. The scheme then keeps track of a certain number of top performing circuits.

- **Evolutionary:** Similarly to the random search approach, this scheme also applies random operations in each iteration. However, in contrast, after each iteration it constructs a new population by putting circuits into a N-wise competition and selecting the best performers.
- **Deterministic:** This scheme is our implementation of Ref. [13]’s approach to deterministically find a quantum circuit for a target graph state.
- **Hybrid:** In this scheme, we use the deterministic method from Ref. [13] to produce an initial solution. We then perturb this solution and use the random/evolutionary schemes to search for quantum circuit alternatives.
- **Gradient-based:** This is a density matrix specific scheme where we consider tunable/parametrized gates that are compatible with automatic differentiation and gradient descent optimization techniques.

3.2 Details of accomplishments

3.2.1. Representing quantum states

The *density matrix* representation can simulate a state’s full Hilbert space and can inherently represent mixed states, such as those resulting from the effects of noise and photonic loss [14]. However, this representation does not scale well with the number of qubits, making it impractical to investigate large graph states, and thus motivating the need for complementary representations. We have used this representation: 1) To study small graph states with the inclusion of noise effects; 2) As a benchmark for other representations; and 3) For the study of noise, photonic loss, mixed states, and gate parametrization.

The *Stabilizer* representation [15], [16], on the other hand, scales well with the number of qubits, making it more efficient for the study of large graph states (relative to the density matrix formalism). However, only a subset of quantum operations can be efficiently simulated within the stabilizer representation, limiting the scope for circuit optimization, and making this representation unsuited for the simulation of noise without judicious changes to the formalism. Presently, the study of representing mixed states within the stabilizer representation is very limited in the literature. To be able to consider large graph states that are mixed due to noise or loss, we have developed a probability tree approach as a way to represent such states. This point of research is presented further below in Sec. 3.3.

We have also developed relevant concepts from *graph* representations, which are also well-suited for many qubits, and contrary to the stabilizer representation, there is more progress made on describing mixed states [17,18]. In particular, we have worked on leveraging graph state equivalency through Clifford gates. As discussed in Sec. 3.4, the graph representation also offers unique metrics for characterizing the performance of quantum circuits.

Each of the quantum state representations has advantages and disadvantages, making it useful to be able to move between them. Most of these representation transformations can be found within literature [3], [14-18], except for obtaining a graph from a density matrix. We have resolved this issue by designing a novel protocol, as discussed in Sec. 3.5.

3.2.2. Representing quantum circuits

Although we have focused on solid state emitter qubits and photonic qubits, we have designed our framework with extensibility in mind. The types of operations, and therefore the

physical circuits we can represent and simulate, are flexible, meaning we can adapt our framework towards different physical platforms with relative ease.

Lastly, our quantum circuit representation also includes parameterized quantum gates and automatic differentiation (see Sec. 3.5 for further discussion).

3.2.3. Realistic component models

The Figure 2 shows an example of the model in practice, deriving the fidelity of an output graph state as a function of qubit error rate from various sources of noise.

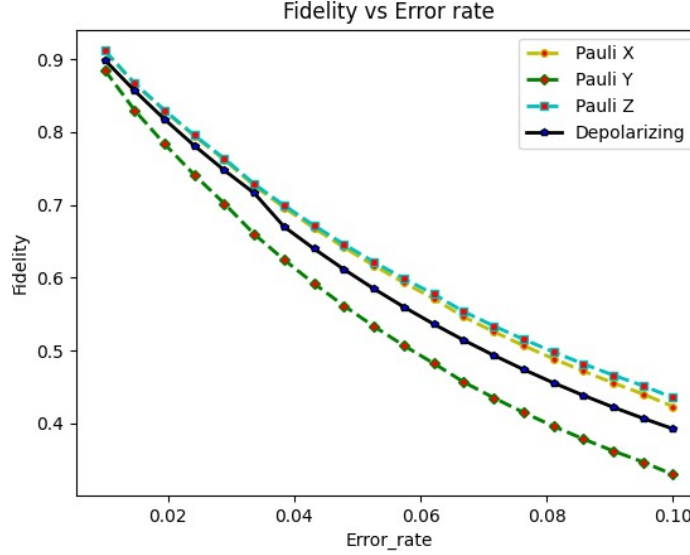


Figure 2: Fidelity versus error rate for the generation of a 4-photon linear cluster state assuming X -gate errors (yellow), Y -gate errors (green), Z -gate errors (blue) and a depolarizing channel (black). We find that the generation of linear cluster states is equally sensitive to bit-flip (X -gate) and phase-flip (Z -gate) errors, and more sensitive when both errors occur (Y -gate).

As photonic losses have non-trivial impacts on quantum states, generally producing mixed states, we have explored various approaches to simulate their effects. One approach is to extend the standard qubit space to a qutrit space so that, by accounting for a vacuum term, the state can still be represented as pure. Another approach is to keep track of the survival probability of a state after potential photonic loss events. Interestingly, we observe that the state fidelity calculation only depends on branches that see no loss, weighted by their respective probabilities. By tracking transmission probabilities, we gain information useful for approximating state generation rates, which are a very important experimental insight. We also calculate the survival probability for each photon, which indicates which portions of the graph state are more prone to loss. This information is essential for efficiently implementing encoding for error correction.

3.2.4. Assessing circuit performance

Fidelity has been our main metric thus far, but it is not ideal for our task as it can change abruptly, which is partially due to the discrete nature of adding or removing quantum operations within the quantum circuit. Entanglement witnesses have potential [24]–[26], but our investigations of GHZ and linear cluster states suggest they may be too robust and not sufficiently monotonic in the neighborhood of optimums. The graph edit metric, which is a metric based on the graph representation of a quantum state, holds potential as it is fundamentally different from other metrics. There are however challenges as it is not obvious

how fundamental graph operations, such as local complementations, would be represented in this metric. Most metrics investigated thus proved to not be useful towards search/optimization schemes or well suited for multipartite states.

3.2.5. Searching for performant quantum circuits

Using the random and evolutionary schemes within the density matrix representation, we were able to:

- Reproduce results from [13] for 4-photon linear cluster states.
- Find alternative, but equivalent circuits. Given some circuits may be more sensitive to noise compared to others or that some gates may be more difficult to implement than others, this gives options to experimentalists in implementing whichever protocol is more practical.
- Find cases where the circuit depth is reduced relative to circuits produced by Ref. [13]'s approach. An example of this depth decrease is shown in Fig. 3. Decreases in circuit depth imply a simplified protocol as there are fewer gates to be implemented or more gates can be implemented in parallel.
- In terms of convergence, we found that the random and evolutionary schemes performed similarly. Figure 4 shows the fraction of search runs that converge on a quantum circuit producing the target graph state.

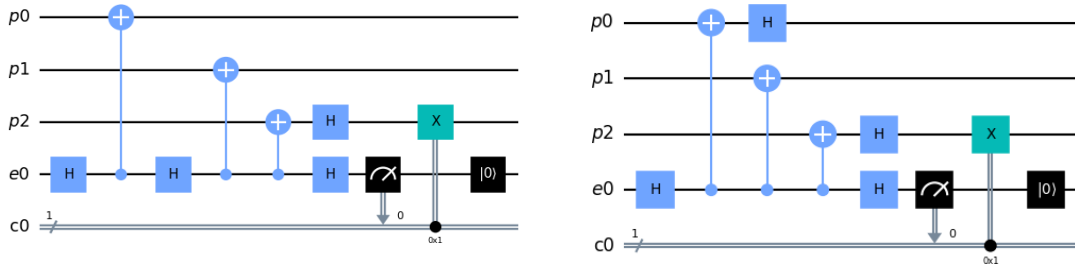


Figure 3: Quantum circuits for the generation of a 3-photon linear cluster state as determined by (left) the deterministic methods of [13] and (right) our framework. Our solution presents a reduced circuit depth relative to the deterministic method.

Using the stabilizer representation, we are able to efficiently simulate large graph states, up to hundreds of qubits, but it remains difficult to converge to an optimal circuit due to the vast optimization landscape. For this reason, we have also investigated the hybrid approach, which leverages the deterministic scheme of Ref. [13] to produce a starting point for random/evolutionary-based search approaches. Using this method, we have been able to find alternative quantum circuits to those proposed by the deterministic approach, as well as access larger graph states.

Additionally, the deterministic solutions do not include the effects of noise, which degrade the quality of the output graph state for a given circuit. Using our hybrid scheme, we have begun investigations into re-optimizing the deterministic solution after the introduction of noise, to regain state fidelity. This investigation is expected to greatly benefit from experimental feedback to more accurately represent noise found within real photonic platforms.



	Linear Cluster (3 qubits)	GHZ (3 qubits)	Linear Cluster (4 qubits)	GHZ (4 qubits)
Random scheme	92%	60%	24%	24%
Evolutionary scheme	96%	16%	24%	28%

Figure 4: Fraction of simulations that converge on a solution for a quantum circuit using a density matrix representation. Target graph states for each case are indicated above columns. Both the random and evolutionary schemes have difficulty converging with an increasing number of qubits. This is due to the vastness of the optimization landscape, indicating the need for better metrics and/or optimization schemes.

Within the density matrix representation, we have also implemented parameterized quantum gates and the automatic differentiation of quantum circuits. This enables the gradient-based optimization of circuit evaluation metrics (using, e.g., gradient-descent optimizers). This aspect of our framework enables quantum circuits to be optimized in the presence of noise, and enables gradient descent to be combined with circuit topology optimization schemes. An example of this scheme in action is shown in Fig. 5, where we perform quantum gate optimization to recover fidelity lost due to noise. Although the noise model requires refinement to accurately represent experimentally-found errors, these are promising results for managing noise-related deterioration of graph states.

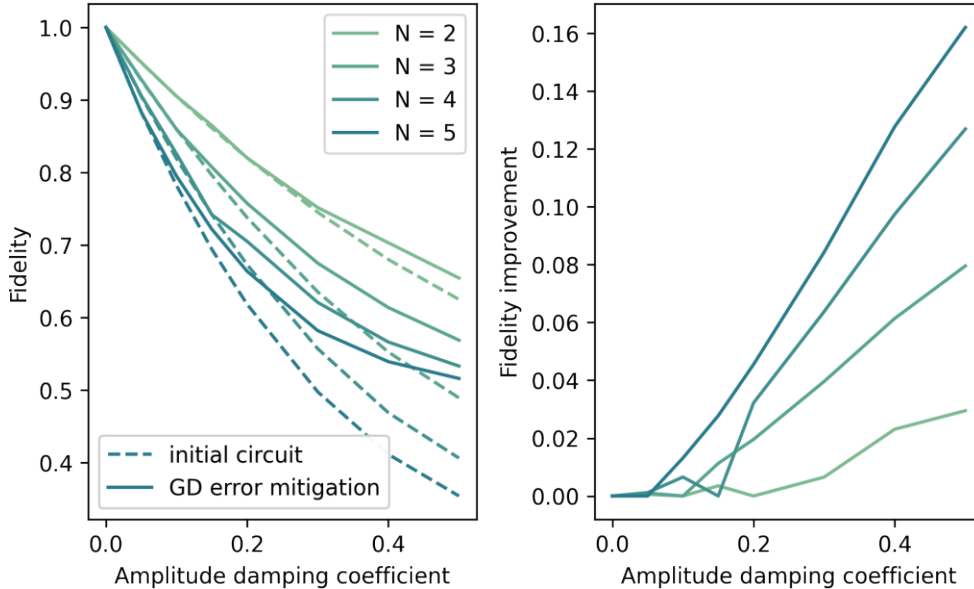


Figure 5: Regaining lost fidelity by parametrized gates optimization for linear cluster states consisting of $N=1-5$ photons as function of noise amplitude, where an amplitude damping model was uniformly applied on all qubits. Left panel shows the fidelity of the initial circuit after introducing noise (dashed) and the fidelity after gate optimization (solid). Right panel shows the relative difference between the solid and dashed lines. We find that gate optimization led to absolute fidelity recoveries of up to 15%.

3.2.5. Summary of accomplishments

During year 1 of this project, we have accomplished the following basic research milestones:

- Constructed a theoretical and numerical framework for the study of graph states and of the quantum circuits that generate them within photonics-based platforms.
- Reproduced known quantum circuits for the generation of graph states found within recent literature. In some cases, such as a 3-photon GHZ state, we have found alternative, yet equivalent quantum circuits, providing experimentalists with setup variety.
- For a 3-photon linear cluster state, we have found an improved quantum circuit with a shorter depth relative to published setups.
- Our framework supports diverse noise models, including depolarizing noise, amplitude damping, and Pauli errors.
- We have developed an experimentally-motivated, time-dependent noise model leveraging the stabilizer representation. This model reduces the number of potential outcomes within mixed states and expresses the noise in terms of circuit time-scales.
- Demonstrated that the use of parameterized gates in conjunction with gradient-based optimization can remediate the impacts of noise-driven fidelity degradation, with an absolute fidelity recovery of up to 15% for 5-photon linear cluster states.
- Developed a novel way of representing mixed states using stabilizers, allowing the simulation of more realistic noise effects and circuit dynamics.
- Developed a novel method for generating the graph representation of a quantum state from a density matrix.
- We have developed random and evolutionary schemes that search for well-performing quantum circuits given a target output state.
- We have developed a hybrid quantum circuit search scheme by combining deterministic and stochastic methods.
- Our framework is modular, and can easily be extended to other physical platforms and noise models.

3.2.6. Dissemination to the communities of interest.

Nothing to report. Nevertheless, manuscripts are in preparation and will be shared with the program officer.

4. Impacts

4.1. Development of the principal discipline of the project

We have developed a framework for the study and optimization of graph state generation in the presence of realistic effects such as noise and photonic loss, making it the first of its kind. Such a tool is a driver for further basic research into entangled resource preparation and distribution for various applications.

The framework is implemented in Python and was designed to be extensible, with community co-development in mind. To support the community and future partners in using these tools, we have prepared thorough documentation, code examples, and interactive tutorials.

Our long-term plans are discussed in detail in Sec. 6, but our near-term plan for disseminating our work to the community includes the following:

- Make our framework available to the community by open-sourcing our codebase

- alongside documentation, use examples, and tutorials
- Use the open-sourced code to enhance and establish research partnerships
- Prepare abstracts for conferences such as the International Conference on Quantum Communication, Measurement and Computing (QCMC), Q2B22 Silicon Valley, Photonics West, and CLEO
- Prepare a manuscript for peer-review and publication

4.2. Impact on other disciplines

In addition to the dissemination of our framework, we also intend to collaborate directly with colleagues from the community. This includes collaborating with the National Research Council of Canada (NRC), specialized in the fabrication of quantum dots that can be used as emitter qubits, l’Institut National de la Recherche Scientifique (INRS), giving us access to knowledge on integrated photonics, as well as Caltech and Fermilab, who can provide realistic entanglement distribution channel metrics from their respective testbeds. Additionally, Quantum Bridge Technologies, Inc. (QBT) is a University of Toronto spin-off, with many local research collaboration opportunities. Our collaborations will allow us to test and validate our framework, as well as improve its level of realism.

5. Future goals

As part of future work, our team intends to maintain and develop our framework alongside the open-source quantum community to find solutions for higher complexity graph states. In parallel, our work internally will focus on extending the framework with more numerous and realistic components as well as qubit protection strategies. To perform this work, we hope for continued support of the project in the years to come and provide below a Gantt chart presenting our future project goals for the following year. Indicated within each task are assigned lead researchers, with a team overview following the chart.

[illegible]

Open innovation and community impact As discussed in Sec. 5, we aim to disseminate our work to support the quantum research community. Open-sourcing will be co-led by I1 and I2. Reaching out to community members will be co-led by PM1, PM2, PM3 and PM4. The writing of a manuscript will be co-led by PDF2 and PDF3 with support from the full team.															
Publish codebase and finalize public documentation and tutorials															
Prepare manuscript and submit to journal															
Project management and administrative support Project management will continue to be led by PM1, with the support of PM2, PM3 and PM4.															
Ongoing management and project reporting															

6. Team overview:

PM1: Piotr Roztocki

CTO, Ki3 Photonics Technologies Inc. Principal project manager responsible for day-to-day activities. Honors B.Sc. in Integrated Science (Physics), McMaster University 2014; Ph.D. in Energy and Materials, INRS-EMT 2022.

PM2: Hoi-Kwong Lo

CTO, Quantum Bridge Technologies, Inc. (QBT). Plays a supervisory role in the project. Professor of Physics and ECE at the University of Toronto, BA in Mathematics, Cambridge University, 1989, Ph.D. in Physics, Caltech, 1994. Co-inventor of measurement-device-independent quantum key distribution (QKD) and all photonics quantum repeaters. Fellow of the American Physical Society (APS) and Optica Fellow.

PM3: Mattia Montagna

CEO, Quantum Bridge Technologies, Inc. (QBT), PhD in Applied Math. Founder of the ECB (European Central Bank) Bank Interconnected Coordination Group (BICG). Has been leading QBT since its founding in 2019.

PM4: Yoann Jestin

CEO, Ki3 Photonics Technologies Inc. Principal Investigator of the project. B.Sc in Chemistry, University of Rennes, 1997; M.Sc. in Material Science, University of Rennes, 1999; Ph.D in Material Science, University of Le Mans, 2002.

PDF1: Jie Lin

QBT quantum research scientist with specializations in quantum optics and security analysis of quantum key distribution. Will lead development of both fusion gates and qubit encoding. B. A. Mathematics, Carleton College; M. A. S. Theoretical Physics, University of Cambridge; M. Sc. Physics and Ph.D. Physics (quantum information), University of Waterloo.

PDF2: Luc Robichaud

QBT quantum research scientist who specializes in quantum dot modeling and their application to entanglement generation. Will lead modeling of various sources and incorporating said models into the framework. Will also co-lead with PDF3 the writing of papers for peer-reviewed journals. B.Sc. Physics and M.Sc. Physics, Université de Moncton;

Ph.D. Physics, University of Ottawa.

PDF3: Stefania Sciara

Postdoc at l'Institut National de la Recherche Scientifique (INRS) and Ki3 researcher with specializations in both experimental and theoretical quantum optics, and entanglement generation. B.Sc. Physics and M.Sc. Physics, University of Palermo; Ph.D. Quantum optics, INRS.

PhD1: Sobhan Ghanbari

QBT intern. Ph.D. candidate at the University of Toronto with specialization in quantum information. Sobhan has led and will continue to lead development of various noise models within the framework. B.Sc. Physics/ B.Sc. Mechanical Engineering (dual- degree), Sharif University of Technology.

PhD2: Benjamin MacLellan

Ph.D. candidate at the University of Waterloo with specialization in quantum information and inverse design of photonic systems. Will lead the development of a more comprehensive physical layer, including various integrated-photonics-based components. B.Sc. Physics, University of Waterloo; M.Sc., Energy & Materials, INRS.

M1: Kai Sum Chan

QBT quantum research scientist. M.Phil. candidate at the University of Hong Kong, and QBT experimental and theoretical quantum researcher with specialization in quantum information, adaptive optics and quantum dots. Will play a supporting role in calibrating models according to experiments.

I1: Khuong Tran

B.Sc. candidate in computer sciences at the University of Ontario Institute of Technology and intern at Ki3. Will co-lead with I2 preparation of open-sourcing, framework documentation and tutorials.

I2: Aayush Soni

B.A.Sc. candidate in mechatronics engineering at the University of Waterloo and intern at Ki3. Will co-lead with I1 the preparation of open-sourcing, framework documentation and tutorials.

Key: PM: Project manager, PDF: Postdoctoral fellow, PhD: PhD student, M: Master's student, I: Intern.

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Highly Entangled States of Light

Final Report

Awarded to:
Ki3 Photonics Technologies Inc.
Quantum Bridge Technologies Inc (QBT).

Recipient Principal Investigator:
Dr. Yoann Jestin

1. Executive summary

Problem statement:

- Graph quantum states are an important resource for fundamental research and technological applications
- Designing quantum circuits that output specific and desired graph states is challenging
- Real quantum circuits are subject to noise and loss, which are necessary to include for an accurate and realistic model

Project scope:

- Construct a theoretical and numerical framework for the study of graph states and of circuits that generate them
- Develop schemes for the construction of quantum circuits that output desired graph states
- Create models for the noise and loss effects that are present in real quantum setups

General outcome:

- To date, we have defined a framework that allows us to find and to simulate quantum circuits that output low-complexity graph states
- Our schemes successfully reproduce results from literature and are also capable of finding alternative setups to choose between
- We have developed noise and loss models that are representative of effects found in common photonics platforms
- Our framework is a flexible and extensible tool that is soon to be open-sourced and which can be easily adapted to simulate various physical platforms

Future Plans:

- Maintain and develop the framework alongside the open-source quantum community to find solutions for higher complexity graph states
- Develop fusion-gate based protocols for producing large graph states
- Implement encoding strategies for qubit protection
- Model quantum-dot-based sources and interface with current framework
- Develop models for integrated-photonics-based components as an experimental basis for quantum circuits
- Develop more sophisticated noise and distribution channel models
- Extend our ability to inject experimental details into our framework, further connecting results with real setups

2. Introduction

Graph states are a family of quantum states with high levels of entanglement that are of great fundamental interest [1], as well as being a key resource for applications in entanglement distribution and quantum networking [2-4], quantum error correction [5,6], and quantum metrology [7]. Among various physical platforms, photonics shows much promise for graph state realizations [8,9] as photons interact very weakly with their environment, enabling stable qubits. Additionally, photons are compatible with mature integrated and fiber technologies, granting additional support in the generation and distribution of graph states.

As well, for the generation of photons, various sources are available within photonics-based platforms, including solid-state emitters such as quantum dots that can emit single photons on demand [10], or nonlinear elements generating pairs of entangled photons [11,12].

Quantum circuits describe the sequence of quantum operations necessary to generate such graph states. However, connecting a desired graph state with the circuit that can output it is a non-trivial design task that only becomes more difficult with increasing state size and complexity. Recent work [13] has led to the development of an impressive deterministic method for finding quantum circuits that produce targeted graph states. However, such methods do not guarantee well-performing quantum circuits, allow little flexibility, and cannot consider impacts associated with noise or photonic loss, which is present in all real setups.

Towards addressing this research gap, the goal of our work was to develop a flexible theoretical and computational framework for the study of realistic graph states and of the quantum circuits that generate them. To accomplish this, we must be able to represent both the quantum states and the quantum circuits that produce them. Both representations must also be flexible enough to support noise and photonic loss modeling. The performance of different circuits must also be quantifiable, in order to drive schemes that search for best performing setups. All of these concepts must also be realized in a flexible and modular way that allows for co-development of solutions with the broader research community, driving collaborative and fundamental work into the understanding of entangled states as a resource.

Over the past year of our project's progress, we have successfully identified and realized a framework comprising the following core concepts: *Quantum state representations*, *quantum circuit representations*, *realistic component models*, *quantum circuit performance metrics*, and *quantum circuit optimization schemes*. Figure 1 shows these key concepts and their mutual interaction within our framework.

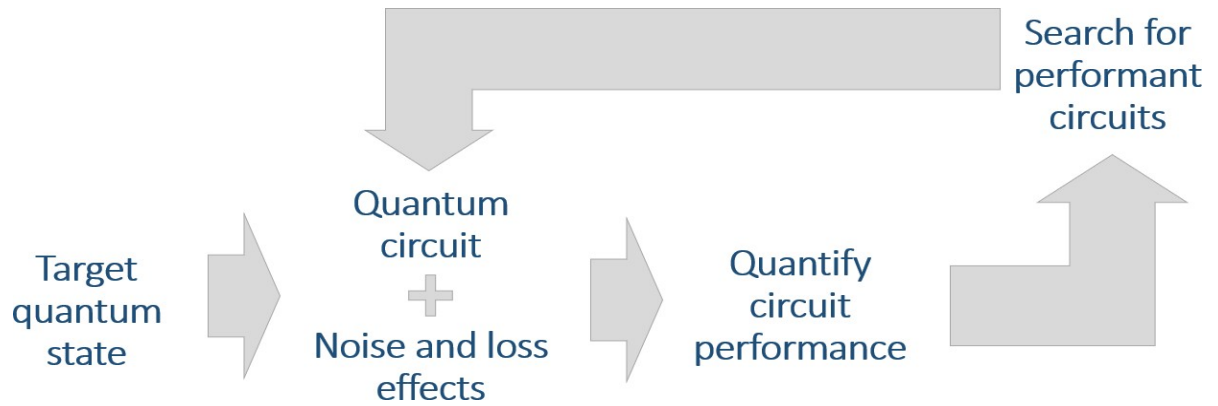


Figure 1: Key concepts within our framework. The performance metric is calculated with respect to the target graph state, guiding the search for performant circuits.

3. Accomplishments

In what follows, we explain each of the core concepts of our framework alongside our respective research efforts in developing them.

3.1. Research objectives

3.3.1. Representing quantum states

The objects of interest, graph states, are quantum systems of photonic and emitter qubits that can be mathematically represented using various formalisms. Our computational framework presently considers two mathematical representations, the *density matrix* and the *stabilizer*, with the modularity to easily include further representations in the future.

3.1.2. Representing quantum circuits

Quantum circuits represent the sequence of quantum gates and operations applied on solid state emitter qubits and photonic qubits. To computationally read, save, and manipulate these quantum circuits, we represent them as Directed Acyclic Graphs (DAGs) [19], which find common use in the field of quantum information [20]. Using the DAG representation, our computational framework can easily modify and evolve the quantum circuit topology to optimize the generated quantum state towards a target graph state. For the visualization of quantum circuits, our DAG representation was also made compatible with the openQASM format, a standard within the field [21].

3.1.3. Realistic component models

Towards increasing the realism of our framework, we support diverse models for noise and photonic loss. For example, our framework presently includes noise models for depolarizing noise, amplitude damping, and Pauli noise.

Additionally, we have developed a new representation to model mixed states using stabilizers. This new representation keeps track of all potential branchings of a mixed state, along with the probability of such a branch being measured. This novel approach allows us to consider probabilistic noise, more akin to real experimental systems.

Motivated by the physics of today's solid-state emitter and photonic qubit systems, we have furthermore developed a time-dependent noise model within the stabilizer formalism. In experiments, the number of errors that occur is strongly correlated with the time spent during and between each quantum gate. In our time-dependent model, we make use of an effective noise (simulated only at certain gates), maintaining model accuracy while reducing the computational load of the model.

3.1.4. Assessing circuit performance

When optimizing a quantum circuit for the generation of a target graph state, it is important to properly quantify the performance of said circuit using quantitative metrics. In fact, having proper metrics can heavily influence the performance of a circuit search algorithm. To date, we have investigated the following metrics as potential candidates [14], [22], [23]:

Fidelity	Relative entropy
Entanglement witness	Squashed entanglement
Graph edit distance	Adjacency matrix fidelity
Concurrence	Geometric measure
Negativity	Fisher information
Trace distance	Mermin inequality
Entanglement distillation	Stabilizer tableau fidelity

3.1.5. Searching for performant quantum circuits

We have investigated the following schemes for converging on suitable quantum circuits that output a desired graph state:

- **Random:** This scheme initializes a list of random circuits subject to physical

constraints, which we call a population. At each iteration and for each circuit within the population, the algorithm applies a random operation, which includes addition, removal, or replacement of a quantum gate. Each circuit in the population is modified independently from the other circuits. The scheme then keeps track of a certain number of top performing circuits.

- **Evolutionary:** Similarly to the random search approach, this scheme also applies random operations in each iteration. However, in contrast, after each iteration it constructs a new population by putting circuits into a N-wise competition and selecting the best performers.
- **Deterministic:** This scheme is our implementation of Ref. [13]’s approach to deterministically find a quantum circuit for a target graph state.
- **Hybrid:** In this scheme, we use the deterministic method from Ref. [13] to produce an initial solution. We then perturb this solution and use the random/evolutionary schemes to search for quantum circuit alternatives.
- **Gradient-based:** This is a density matrix specific scheme where we consider tunable/parametrized gates that are compatible with automatic differentiation and gradient descent optimization techniques.

3.2 Details of accomplishments

3.2.1. Representing quantum states

The *density matrix* representation can simulate a state’s full Hilbert space and can inherently represent mixed states, such as those resulting from the effects of noise and photonic loss [14]. However, this representation does not scale well with the number of qubits, making it impractical to investigate large graph states, and thus motivating the need for complementary representations. We have used this representation: 1) To study small graph states with the inclusion of noise effects; 2) As a benchmark for other representations; and 3) For the study of noise, photonic loss, mixed states, and gate parametrization.

The *Stabilizer* representation [15], [16], on the other hand, scales well with the number of qubits, making it more efficient for the study of large graph states (relative to the density matrix formalism). However, only a subset of quantum operations can be efficiently simulated within the stabilizer representation, limiting the scope for circuit optimization, and making this representation unsuited for the simulation of noise without judicious changes to the formalism. Presently, the study of representing mixed states within the stabilizer representation is very limited in the literature. To be able to consider large graph states that are mixed due to noise or loss, we have developed a probability tree approach as a way to represent such states. This point of research is presented further below in Sec. 3.3.

We have also developed relevant concepts from *graph* representations, which are also well-suited for many qubits, and contrary to the stabilizer representation, there is more progress made on describing mixed states [17,18]. In particular, we have worked on leveraging graph state equivalency through Clifford gates. As discussed in Sec. 3.4, the graph representation also offers unique metrics for characterizing the performance of quantum circuits.

Each of the quantum state representations has advantages and disadvantages, making it useful to be able to move between them. Most of these representation transformations can be found within literature [3], [14-18], except for obtaining a graph from a density matrix. We have resolved this issue by designing a novel protocol, as discussed in Sec. 3.5.

3.2.2. Representing quantum circuits

Although we have focused on solid state emitter qubits and photonic qubits, we have designed our framework with extensibility in mind. The types of operations, and therefore the

physical circuits we can represent and simulate, are flexible, meaning we can adapt our framework towards different physical platforms with relative ease.

Lastly, our quantum circuit representation also includes parameterized quantum gates and automatic differentiation (see Sec. 3.5 for further discussion).

3.2.3. Realistic component models

The Figure 2 shows an example of the model in practice, deriving the fidelity of an output graph state as a function of qubit error rate from various sources of noise.

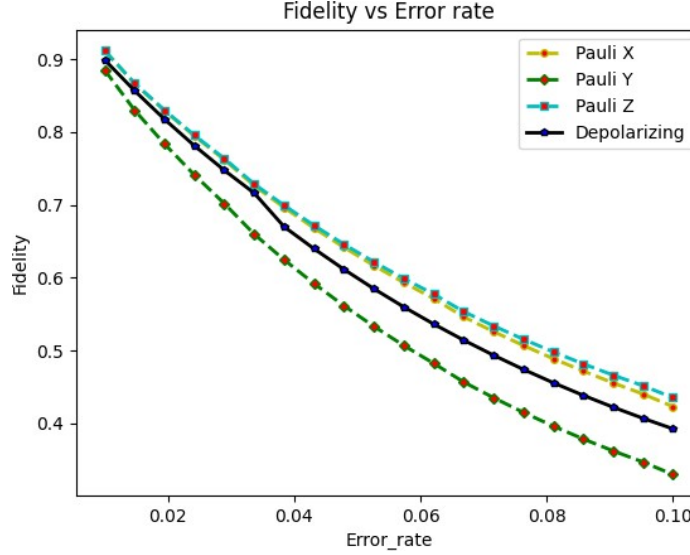


Figure 2: Fidelity versus error rate for the generation of a 4-photon linear cluster state assuming X -gate errors (yellow), Y -gate errors (green), Z -gate errors (blue) and a depolarizing channel (black). We find that the generation of linear cluster states is equally sensitive to bit-flip (X -gate) and phase-flip (Z -gate) errors, and more sensitive when both errors occur (Y -gate).

As photonic losses have non-trivial impacts on quantum states, generally producing mixed states, we have explored various approaches to simulate their effects. One approach is to extend the standard qubit space to a qutrit space so that, by accounting for a vacuum term, the state can still be represented as pure. Another approach is to keep track of the survival probability of a state after potential photonic loss events. Interestingly, we observe that the state fidelity calculation only depends on branches that see no loss, weighted by their respective probabilities. By tracking transmission probabilities, we gain information useful for approximating state generation rates, which are a very important experimental insight. We also calculate the survival probability for each photon, which indicates which portions of the graph state are more prone to loss. This information is essential for efficiently implementing encoding for error correction.

3.2.4. Assessing circuit performance

Fidelity has been our main metric thus far, but it is not ideal for our task as it can change abruptly, which is partially due to the discrete nature of adding or removing quantum operations within the quantum circuit. Entanglement witnesses have potential [24]–[26], but our investigations of GHZ and linear cluster states suggest they may be too robust and not sufficiently monotonic in the neighborhood of optimums. The graph edit metric, which is a metric based on the graph representation of a quantum state, holds potential as it is fundamentally different from other metrics. There are however challenges as it is not obvious

how fundamental graph operations, such as local complementations, would be represented in this metric. Most metrics investigated thus proved to not be useful towards search/optimization schemes or well suited for multipartite states.

3.2.5. Searching for performant quantum circuits

Using the random and evolutionary schemes within the density matrix representation, we were able to:

- Reproduce results from [13] for 4-photon linear cluster states.
- Find alternative, but equivalent circuits. Given some circuits may be more sensitive to noise compared to others or that some gates may be more difficult to implement than others, this gives options to experimentalists in implementing whichever protocol is more practical.
- Find cases where the circuit depth is reduced relative to circuits produced by Ref. [13]'s approach. An example of this depth decrease is shown in Fig. 3. Decreases in circuit depth imply a simplified protocol as there are fewer gates to be implemented or more gates can be implemented in parallel.
- In terms of convergence, we found that the random and evolutionary schemes performed similarly. Figure 4 shows the fraction of search runs that converge on a quantum circuit producing the target graph state.

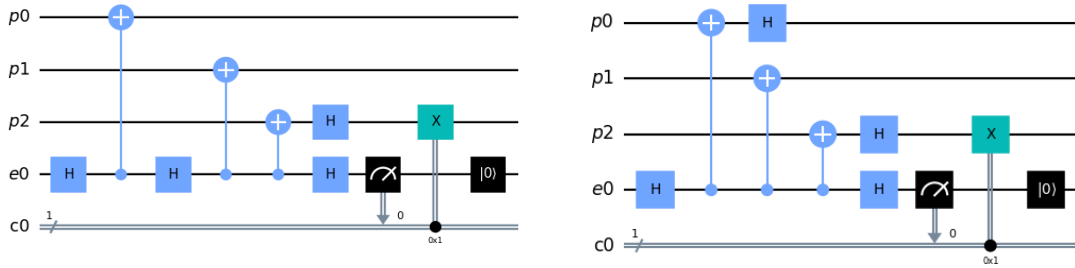


Figure 3: Quantum circuits for the generation of a 3-photon linear cluster state as determined by (left) the deterministic methods of [13] and (right) our framework. Our solution presents a reduced circuit depth relative to the deterministic method.

Using the stabilizer representation, we are able to efficiently simulate large graph states, up to hundreds of qubits, but it remains difficult to converge to an optimal circuit due to the vast optimization landscape. For this reason, we have also investigated the hybrid approach, which leverages the deterministic scheme of Ref. [13] to produce a starting point for random/evolutionary-based search approaches. Using this method, we have been able to find alternative quantum circuits to those proposed by the deterministic approach, as well as access larger graph states.

Additionally, the deterministic solutions do not include the effects of noise, which degrade the quality of the output graph state for a given circuit. Using our hybrid scheme, we have begun investigations into re-optimizing the deterministic solution after the introduction of noise, to regain state fidelity. This investigation is expected to greatly benefit from experimental feedback to more accurately represent noise found within real photonic platforms.



	Linear Cluster (3 qubits)	GHZ (3 qubits)	Linear Cluster (4 qubits)	GHZ (4 qubits)
Random scheme	92%	60%	24%	24%
Evolutionary scheme	96%	16%	24%	28%

Figure 4: Fraction of simulations that converge on a solution for a quantum circuit using a density matrix representation. Target graph states for each case are indicated above columns. Both the random and evolutionary schemes have difficulty converging with an increasing number of qubits. This is due to the vastness of the optimization landscape, indicating the need for better metrics and/or optimization schemes.

Within the density matrix representation, we have also implemented parameterized quantum gates and the automatic differentiation of quantum circuits. This enables the gradient-based optimization of circuit evaluation metrics (using, e.g., gradient-descent optimizers). This aspect of our framework enables quantum circuits to be optimized in the presence of noise, and enables gradient descent to be combined with circuit topology optimization schemes. An example of this scheme in action is shown in Fig. 5, where we perform quantum gate optimization to recover fidelity lost due to noise. Although the noise model requires refinement to accurately represent experimentally-found errors, these are promising results for managing noise-related deterioration of graph states.

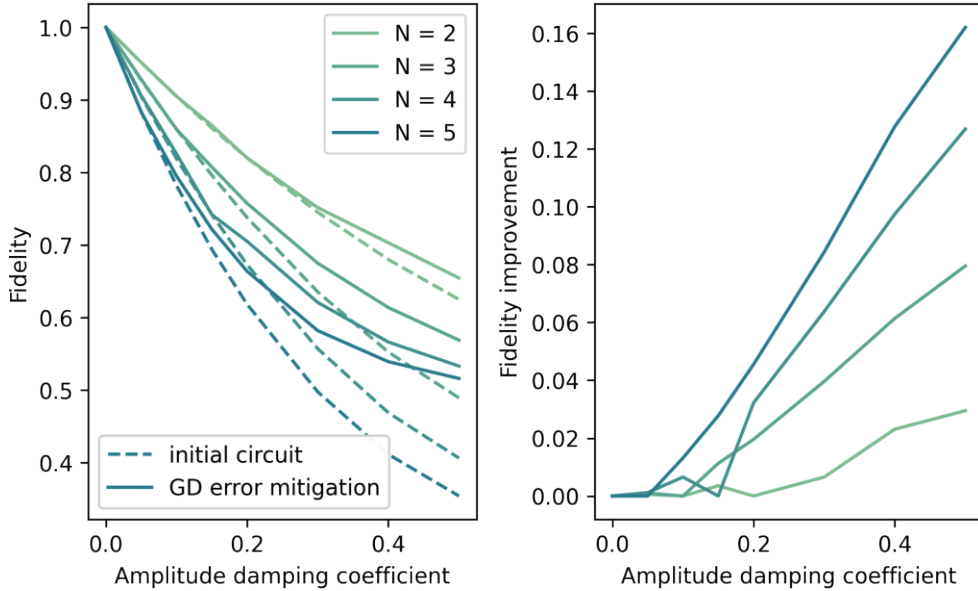


Figure 5: Regaining lost fidelity by parametrized gates optimization for linear cluster states consisting of $N=1-5$ photons as function of noise amplitude, where an amplitude damping model was uniformly applied on all qubits. Left panel shows the fidelity of the initial circuit after introducing noise (dashed) and the fidelity after gate optimization (solid). Right panel shows the relative difference between the solid and dashed lines. We find that gate optimization led to absolute fidelity recoveries of up to 15%.

3.2.5. Summary of accomplishments

During year 1 of this project, we have accomplished the following basic research milestones:

- Constructed a theoretical and numerical framework for the study of graph states and of the quantum circuits that generate them within photonics-based platforms.
- Reproduced known quantum circuits for the generation of graph states found within recent literature. In some cases, such as a 3-photon GHZ state, we have found alternative, yet equivalent quantum circuits, providing experimentalists with setup variety.
- For a 3-photon linear cluster state, we have found an improved quantum circuit with a shorter depth relative to published setups.
- Our framework supports diverse noise models, including depolarizing noise, amplitude damping, and Pauli errors.
- We have developed an experimentally-motivated, time-dependent noise model leveraging the stabilizer representation. This model reduces the number of potential outcomes within mixed states and expresses the noise in terms of circuit time-scales.
- Demonstrated that the use of parameterized gates in conjunction with gradient-based optimization can remediate the impacts of noise-driven fidelity degradation, with an absolute fidelity recovery of up to 15% for 5-photon linear cluster states.
- Developed a novel way of representing mixed states using stabilizers, allowing the simulation of more realistic noise effects and circuit dynamics.
- Developed a novel method for generating the graph representation of a quantum state from a density matrix.
- We have developed random and evolutionary schemes that search for well-performing quantum circuits given a target output state.
- We have developed a hybrid quantum circuit search scheme by combining deterministic and stochastic methods.
- Our framework is modular, and can easily be extended to other physical platforms and noise models.

3.2.6. Dissemination to the communities of interest.

Nothing to report. Nevertheless, manuscripts are in preparation and will be shared with the program officer.

4. Impacts

4.1. Development of the principal discipline of the project

We have developed a framework for the study and optimization of graph state generation in the presence of realistic effects such as noise and photonic loss, making it the first of its kind. Such a tool is a driver for further basic research into entangled resource preparation and distribution for various applications.

The framework is implemented in Python and was designed to be extensible, with community co-development in mind. To support the community and future partners in using these tools, we have prepared thorough documentation, code examples, and interactive tutorials.

Our long-term plans are discussed in detail in Sec. 6, but our near-term plan for disseminating our work to the community includes the following:

- Make our framework available to the community by open-sourcing our codebase

- alongside documentation, use examples, and tutorials
- Use the open-sourced code to enhance and establish research partnerships
- Prepare abstracts for conferences such as the International Conference on Quantum Communication, Measurement and Computing (QCMC), Q2B22 Silicon Valley, Photonics West, and CLEO
- Prepare a manuscript for peer-review and publication

4.2. Impact on other disciplines

In addition to the dissemination of our framework, we also intend to collaborate directly with colleagues from the community. This includes collaborating with the National Research Council of Canada (NRC), specialized in the fabrication of quantum dots that can be used as emitter qubits, l’Institut National de la Recherche Scientifique (INRS), giving us access to knowledge on integrated photonics, as well as Caltech and Fermilab, who can provide realistic entanglement distribution channel metrics from their respective testbeds. Additionally, Quantum Bridge Technologies, Inc. (QBT) is a University of Toronto spin-off, with many local research collaboration opportunities. Our collaborations will allow us to test and validate our framework, as well as improve its level of realism.

5. Future goals

As part of future work, our team intends to maintain and develop our framework alongside the open-source quantum community to find solutions for higher complexity graph states. In parallel, our work internally will focus on extending the framework with more numerous and realistic components as well as qubit protection strategies. To perform this work, we hope for continued support of the project in the years to come and provide below a Gantt chart presenting our future project goals for the following year. Indicated within each task are assigned lead researchers, with a team overview following the chart.

Task description	Months											
	M1-M4				M4-M8				M8-M12			
Modeling fusion gate components Using fusion gates [27], we can form large graph states from smaller graph states. We will implement such gates within our framework, giving an alternative route to realizing graph states. This task will be led by PDF1 and supported by PDF2 and PhD1.												
Literature review and implementing fusion gates within framework												
Investigate fusion gate-based quantum circuits												
Encoding for error correction and fusion gate protection We will explore encoding methods for protecting qubits against random errors (e.g., bitflip) and failed fusion gate attempts, which lead to the loss of a qubit. This task will be led by PDF1, and supported by PDF2 and PhD1.												
Investigate encoding strategies in literature												
Implement encoding for qubit protection												

Open innovation and community impact As discussed in Sec. 5, we aim to disseminate our work to support the quantum research community. Open-sourcing will be co-led by I1 and I2. Reaching out to community members will be co-led by PM1, PM2, PM3 and PM4. The writing of a manuscript will be co-led by PDF2 and PDF3 with support from the full team.												
Publish codebase and finalize public documentation and tutorials												
Prepare manuscript and submit to journal												
Project management and administrative support Project management will continue to be led by PM1, with the support of PM2, PM3 and PM4.												
Ongoing management and project reporting												

6. Team overview:

PM1: Piotr Roztocki

CTO, Ki3 Photonics Technologies Inc. Principal project manager responsible for day-to-day activities. Honors B.Sc. in Integrated Science (Physics), McMaster University 2014; Ph.D. in Energy and Materials, INRS-EMT 2022.

PM2: Hoi-Kwong Lo

CTO, Quantum Bridge Technologies, Inc. (QBT). Plays a supervisory role in the project. Professor of Physics and ECE at the University of Toronto, BA in Mathematics, Cambridge University, 1989, Ph.D. in Physics, Caltech, 1994. Co-inventor of measurement-device-independent quantum key distribution (QKD) and all photonics quantum repeaters. Fellow of the American Physical Society (APS) and Optica Fellow.

PM3: Mattia Montagna

CEO, Quantum Bridge Technologies, Inc. (QBT), PhD in Applied Math. Founder of the ECB (European Central Bank) Bank Interconnected Coordination Group (BICG). Has been leading QBT since its founding in 2019.

PM4: Yoann Jestin

CEO, Ki3 Photonics Technologies Inc. Principal Investigator of the project. B.Sc in Chemistry, University of Rennes, 1997; M.Sc. in Material Science, University of Rennes, 1999; Ph.D in Material Science, University of Le Mans, 2002.

PDF1: Jie Lin

QBT quantum research scientist with specializations in quantum optics and security analysis of quantum key distribution. Will lead development of both fusion gates and qubit encoding. B. A. Mathematics, Carleton College; M. A. S. Theoretical Physics, University of Cambridge; M. Sc. Physics and Ph.D. Physics (quantum information), University of Waterloo.

PDF2: Luc Robichaud

QBT quantum research scientist who specializes in quantum dot modeling and their application to entanglement generation. Will lead modeling of various sources and incorporating said models into the framework. Will also co-lead with PDF3 the writing of papers for peer-reviewed journals. B.Sc. Physics and M.Sc. Physics, Université de Moncton;

Ph.D. Physics, University of Ottawa.

PDF3: Stefania Sciara

Postdoc at l'Institut National de la Recherche Scientifique (INRS) and Ki3 researcher with specializations in both experimental and theoretical quantum optics, and entanglement generation. B.Sc. Physics and M.Sc. Physics, University of Palermo; Ph.D. Quantum optics, INRS.

PhD1: Sobhan Ghanbari

QBT intern. Ph.D. candidate at the University of Toronto with specialization in quantum information. Sobhan has led and will continue to lead development of various noise models within the framework. B.Sc. Physics/ B.Sc. Mechanical Engineering (dual- degree), Sharif University of Technology.

PhD2: Benjamin MacLellan

Ph.D. candidate at the University of Waterloo with specialization in quantum information and inverse design of photonic systems. Will lead the development of a more comprehensive physical layer, including various integrated-photonics-based components. B.Sc. Physics, University of Waterloo; M.Sc., Energy & Materials, INRS.

M1: Kai Sum Chan

QBT quantum research scientist. M.Phil. candidate at the University of Hong Kong, and QBT experimental and theoretical quantum researcher with specialization in quantum information, adaptive optics and quantum dots. Will play a supporting role in calibrating models according to experiments.

I1: Khuong Tran

B.Sc. candidate in computer sciences at the University of Ontario Institute of Technology and intern at Ki3. Will co-lead with I2 preparation of open-sourcing, framework documentation and tutorials.

I2: Aayush Soni

B.A.Sc. candidate in mechatronics engineering at the University of Waterloo and intern at Ki3. Will co-lead with I1 the preparation of open-sourcing, framework documentation and tutorials.

Key: PM: Project manager, PDF: Postdoctoral fellow, PhD: PhD student, M: Master's student, I: Intern.

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Highly Entangled States of Light

Final Report

Awarded to:
Ki3 Photonics Technologies Inc.
Quantum Bridge Technologies Inc (QBT).

Recipient Principal Investigator:
Dr. Yoann Jestin

1. Executive summary

Problem statement:

- Graph quantum states are an important resource for fundamental research and technological applications
- Designing quantum circuits that output specific and desired graph states is challenging
- Real quantum circuits are subject to noise and loss, which are necessary to include for an accurate and realistic model

Project scope:

- Construct a theoretical and numerical framework for the study of graph states and of circuits that generate them
- Develop schemes for the construction of quantum circuits that output desired graph states
- Create models for the noise and loss effects that are present in real quantum setups

General outcome:

- To date, we have defined a framework that allows us to find and to simulate quantum circuits that output low-complexity graph states
- Our schemes successfully reproduce results from literature and are also capable of finding alternative setups to choose between
- We have developed noise and loss models that are representative of effects found in common photonics platforms
- Our framework is a flexible and extensible tool that is soon to be open-sourced and which can be easily adapted to simulate various physical platforms

Future Plans:

- Maintain and develop the framework alongside the open-source quantum community to find solutions for higher complexity graph states
- Develop fusion-gate based protocols for producing large graph states
- Implement encoding strategies for qubit protection
- Model quantum-dot-based sources and interface with current framework
- Develop models for integrated-photonics-based components as an experimental basis for quantum circuits
- Develop more sophisticated noise and distribution channel models
- Extend our ability to inject experimental details into our framework, further connecting results with real setups

2. Introduction

Graph states are a family of quantum states with high levels of entanglement that are of great fundamental interest [1], as well as being a key resource for applications in entanglement distribution and quantum networking [2-4], quantum error correction [5,6], and quantum metrology [7]. Among various physical platforms, photonics shows much promise for graph state realizations [8,9] as photons interact very weakly with their environment, enabling stable qubits. Additionally, photons are compatible with mature integrated and fiber technologies, granting additional support in the generation and distribution of graph states.

As well, for the generation of photons, various sources are available within photonics-based platforms, including solid-state emitters such as quantum dots that can emit single photons on demand [10], or nonlinear elements generating pairs of entangled photons [11,12].

Quantum circuits describe the sequence of quantum operations necessary to generate such graph states. However, connecting a desired graph state with the circuit that can output it is a non-trivial design task that only becomes more difficult with increasing state size and complexity. Recent work [13] has led to the development of an impressive deterministic method for finding quantum circuits that produce targeted graph states. However, such methods do not guarantee well-performing quantum circuits, allow little flexibility, and cannot consider impacts associated with noise or photonic loss, which is present in all real setups.

Towards addressing this research gap, the goal of our work was to develop a flexible theoretical and computational framework for the study of realistic graph states and of the quantum circuits that generate them. To accomplish this, we must be able to represent both the quantum states and the quantum circuits that produce them. Both representations must also be flexible enough to support noise and photonic loss modeling. The performance of different circuits must also be quantifiable, in order to drive schemes that search for best performing setups. All of these concepts must also be realized in a flexible and modular way that allows for co-development of solutions with the broader research community, driving collaborative and fundamental work into the understanding of entangled states as a resource.

Over the past year of our project's progress, we have successfully identified and realized a framework comprising the following core concepts: *Quantum state representations*, *quantum circuit representations*, *realistic component models*, *quantum circuit performance metrics*, and *quantum circuit optimization schemes*. Figure 1 shows these key concepts and their mutual interaction within our framework.

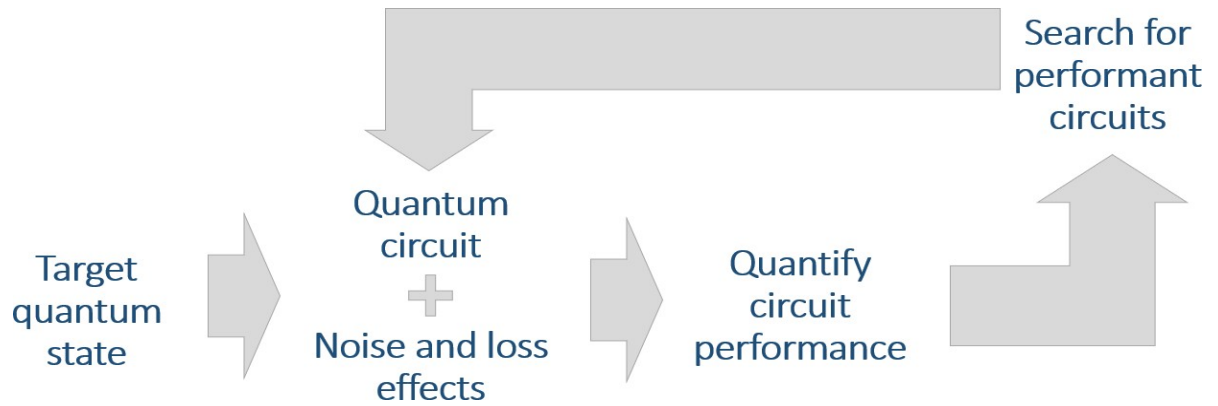


Figure 1: Key concepts within our framework. The performance metric is calculated with respect to the target graph state, guiding the search for performant circuits.

3. Accomplishments

In what follows, we explain each of the core concepts of our framework alongside our respective research efforts in developing them.

3.1. Research objectives

3.3.1. Representing quantum states

The objects of interest, graph states, are quantum systems of photonic and emitter qubits that can be mathematically represented using various formalisms. Our computational framework presently considers two mathematical representations, the *density matrix* and the *stabilizer*, with the modularity to easily include further representations in the future.

3.1.2. Representing quantum circuits

Quantum circuits represent the sequence of quantum gates and operations applied on solid state emitter qubits and photonic qubits. To computationally read, save, and manipulate these quantum circuits, we represent them as Directed Acyclic Graphs (DAGs) [19], which find common use in the field of quantum information [20]. Using the DAG representation, our computational framework can easily modify and evolve the quantum circuit topology to optimize the generated quantum state towards a target graph state. For the visualization of quantum circuits, our DAG representation was also made compatible with the openQASM format, a standard within the field [21].

3.1.3. Realistic component models

Towards increasing the realism of our framework, we support diverse models for noise and photonic loss. For example, our framework presently includes noise models for depolarizing noise, amplitude damping, and Pauli noise.

Additionally, we have developed a new representation to model mixed states using stabilizers. This new representation keeps track of all potential branchings of a mixed state, along with the probability of such a branch being measured. This novel approach allows us to consider probabilistic noise, more akin to real experimental systems.

Motivated by the physics of today's solid-state emitter and photonic qubit systems, we have furthermore developed a time-dependent noise model within the stabilizer formalism. In experiments, the number of errors that occur is strongly correlated with the time spent during and between each quantum gate. In our time-dependent model, we make use of an effective noise (simulated only at certain gates), maintaining model accuracy while reducing the computational load of the model.

3.1.4. Assessing circuit performance

When optimizing a quantum circuit for the generation of a target graph state, it is important to properly quantify the performance of said circuit using quantitative metrics. In fact, having proper metrics can heavily influence the performance of a circuit search algorithm. To date, we have investigated the following metrics as potential candidates [14], [22], [23]:

Fidelity	Relative entropy
Entanglement witness	Squashed entanglement
Graph edit distance	Adjacency matrix fidelity
Concurrence	Geometric measure
Negativity	Fisher information
Trace distance	Mermin inequality
Entanglement distillation	Stabilizer tableau fidelity

3.1.5. Searching for performant quantum circuits

We have investigated the following schemes for converging on suitable quantum circuits that output a desired graph state:

- **Random:** This scheme initializes a list of random circuits subject to physical

constraints, which we call a population. At each iteration and for each circuit within the population, the algorithm applies a random operation, which includes addition, removal, or replacement of a quantum gate. Each circuit in the population is modified independently from the other circuits. The scheme then keeps track of a certain number of top performing circuits.

- **Evolutionary:** Similarly to the random search approach, this scheme also applies random operations in each iteration. However, in contrast, after each iteration it constructs a new population by putting circuits into a N-wise competition and selecting the best performers.
- **Deterministic:** This scheme is our implementation of Ref. [13]’s approach to deterministically find a quantum circuit for a target graph state.
- **Hybrid:** In this scheme, we use the deterministic method from Ref. [13] to produce an initial solution. We then perturb this solution and use the random/evolutionary schemes to search for quantum circuit alternatives.
- **Gradient-based:** This is a density matrix specific scheme where we consider tunable/parametrized gates that are compatible with automatic differentiation and gradient descent optimization techniques.

3.2 Details of accomplishments

3.2.1. Representing quantum states

The *density matrix* representation can simulate a state’s full Hilbert space and can inherently represent mixed states, such as those resulting from the effects of noise and photonic loss [14]. However, this representation does not scale well with the number of qubits, making it impractical to investigate large graph states, and thus motivating the need for complementary representations. We have used this representation: 1) To study small graph states with the inclusion of noise effects; 2) As a benchmark for other representations; and 3) For the study of noise, photonic loss, mixed states, and gate parametrization.

The *Stabilizer* representation [15], [16], on the other hand, scales well with the number of qubits, making it more efficient for the study of large graph states (relative to the density matrix formalism). However, only a subset of quantum operations can be efficiently simulated within the stabilizer representation, limiting the scope for circuit optimization, and making this representation unsuited for the simulation of noise without judicious changes to the formalism. Presently, the study of representing mixed states within the stabilizer representation is very limited in the literature. To be able to consider large graph states that are mixed due to noise or loss, we have developed a probability tree approach as a way to represent such states. This point of research is presented further below in Sec. 3.3.

We have also developed relevant concepts from *graph* representations, which are also well-suited for many qubits, and contrary to the stabilizer representation, there is more progress made on describing mixed states [17,18]. In particular, we have worked on leveraging graph state equivalency through Clifford gates. As discussed in Sec. 3.4, the graph representation also offers unique metrics for characterizing the performance of quantum circuits.

Each of the quantum state representations has advantages and disadvantages, making it useful to be able to move between them. Most of these representation transformations can be found within literature [3], [14-18], except for obtaining a graph from a density matrix. We have resolved this issue by designing a novel protocol, as discussed in Sec. 3.5.

3.2.2. Representing quantum circuits

Although we have focused on solid state emitter qubits and photonic qubits, we have designed our framework with extensibility in mind. The types of operations, and therefore the

physical circuits we can represent and simulate, are flexible, meaning we can adapt our framework towards different physical platforms with relative ease.

Lastly, our quantum circuit representation also includes parameterized quantum gates and automatic differentiation (see Sec. 3.5 for further discussion).

3.2.3. Realistic component models

The Figure 2 shows an example of the model in practice, deriving the fidelity of an output graph state as a function of qubit error rate from various sources of noise.

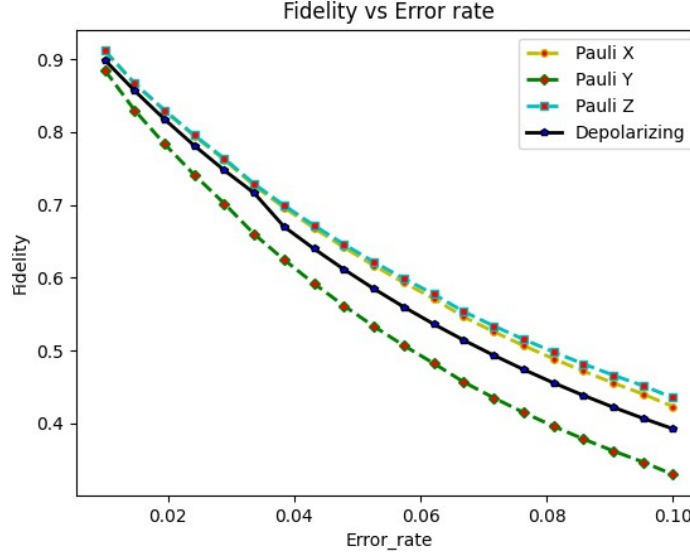


Figure 2: Fidelity versus error rate for the generation of a 4-photon linear cluster state assuming X -gate errors (yellow), Y -gate errors (green), Z -gate errors (blue) and a depolarizing channel (black). We find that the generation of linear cluster states is equally sensitive to bit-flip (X -gate) and phase-flip (Z -gate) errors, and more sensitive when both errors occur (Y -gate).

As photonic losses have non-trivial impacts on quantum states, generally producing mixed states, we have explored various approaches to simulate their effects. One approach is to extend the standard qubit space to a qutrit space so that, by accounting for a vacuum term, the state can still be represented as pure. Another approach is to keep track of the survival probability of a state after potential photonic loss events. Interestingly, we observe that the state fidelity calculation only depends on branches that see no loss, weighted by their respective probabilities. By tracking transmission probabilities, we gain information useful for approximating state generation rates, which are a very important experimental insight. We also calculate the survival probability for each photon, which indicates which portions of the graph state are more prone to loss. This information is essential for efficiently implementing encoding for error correction.

3.2.4. Assessing circuit performance

Fidelity has been our main metric thus far, but it is not ideal for our task as it can change abruptly, which is partially due to the discrete nature of adding or removing quantum operations within the quantum circuit. Entanglement witnesses have potential [24]–[26], but our investigations of GHZ and linear cluster states suggest they may be too robust and not sufficiently monotonic in the neighborhood of optimums. The graph edit metric, which is a metric based on the graph representation of a quantum state, holds potential as it is fundamentally different from other metrics. There are however challenges as it is not obvious

how fundamental graph operations, such as local complementations, would be represented in this metric. Most metrics investigated thus proved to not be useful towards search/optimization schemes or well suited for multipartite states.

3.2.5. Searching for performant quantum circuits

Using the random and evolutionary schemes within the density matrix representation, we were able to:

- Reproduce results from [13] for 4-photon linear cluster states.
- Find alternative, but equivalent circuits. Given some circuits may be more sensitive to noise compared to others or that some gates may be more difficult to implement than others, this gives options to experimentalists in implementing whichever protocol is more practical.
- Find cases where the circuit depth is reduced relative to circuits produced by Ref. [13]'s approach. An example of this depth decrease is shown in Fig. 3. Decreases in circuit depth imply a simplified protocol as there are fewer gates to be implemented or more gates can be implemented in parallel.
- In terms of convergence, we found that the random and evolutionary schemes performed similarly. Figure 4 shows the fraction of search runs that converge on a quantum circuit producing the target graph state.

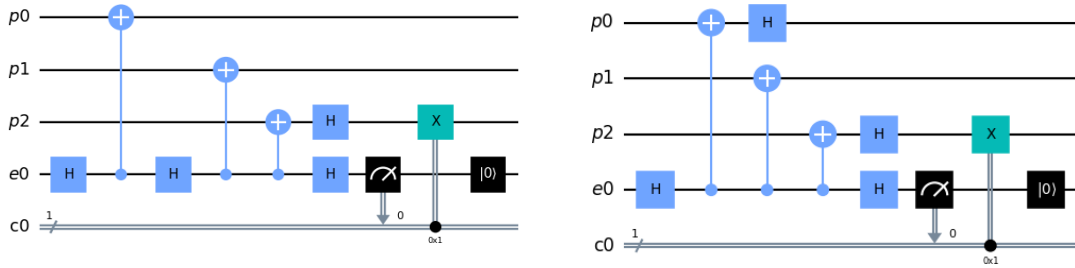


Figure 3: Quantum circuits for the generation of a 3-photon linear cluster state as determined by (left) the deterministic methods of [13] and (right) our framework. Our solution presents a reduced circuit depth relative to the deterministic method.

Using the stabilizer representation, we are able to efficiently simulate large graph states, up to hundreds of qubits, but it remains difficult to converge to an optimal circuit due to the vast optimization landscape. For this reason, we have also investigated the hybrid approach, which leverages the deterministic scheme of Ref. [13] to produce a starting point for random/evolutionary-based search approaches. Using this method, we have been able to find alternative quantum circuits to those proposed by the deterministic approach, as well as access larger graph states.

Additionally, the deterministic solutions do not include the effects of noise, which degrade the quality of the output graph state for a given circuit. Using our hybrid scheme, we have begun investigations into re-optimizing the deterministic solution after the introduction of noise, to regain state fidelity. This investigation is expected to greatly benefit from experimental feedback to more accurately represent noise found within real photonic platforms.



	Linear Cluster (3 qubits)	GHZ (3 qubits)	Linear Cluster (4 qubits)	GHZ (4 qubits)
Random scheme	92%	60%	24%	24%
Evolutionary scheme	96%	16%	24%	28%

Figure 4: Fraction of simulations that converge on a solution for a quantum circuit using a density matrix representation. Target graph states for each case are indicated above columns. Both the random and evolutionary schemes have difficulty converging with an increasing number of qubits. This is due to the vastness of the optimization landscape, indicating the need for better metrics and/or optimization schemes.

Within the density matrix representation, we have also implemented parameterized quantum gates and the automatic differentiation of quantum circuits. This enables the gradient-based optimization of circuit evaluation metrics (using, e.g., gradient-descent optimizers). This aspect of our framework enables quantum circuits to be optimized in the presence of noise, and enables gradient descent to be combined with circuit topology optimization schemes. An example of this scheme in action is shown in Fig. 5, where we perform quantum gate optimization to recover fidelity lost due to noise. Although the noise model requires refinement to accurately represent experimentally-found errors, these are promising results for managing noise-related deterioration of graph states.

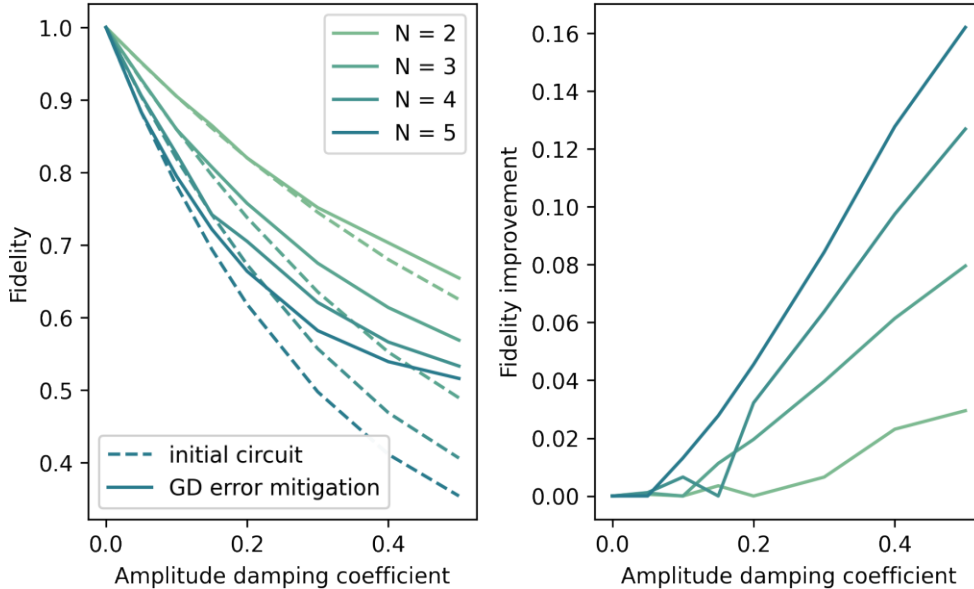


Figure 5: Regaining lost fidelity by parametrized gates optimization for linear cluster states consisting of $N=1-5$ photons as function of noise amplitude, where an amplitude damping model was uniformly applied on all qubits. Left panel shows the fidelity of the initial circuit after introducing noise (dashed) and the fidelity after gate optimization (solid). Right panel shows the relative difference between the solid and dashed lines. We find that gate optimization led to absolute fidelity recoveries of up to 15%.

3.2.5. Summary of accomplishments

During year 1 of this project, we have accomplished the following basic research milestones:

- Constructed a theoretical and numerical framework for the study of graph states and of the quantum circuits that generate them within photonics-based platforms.
- Reproduced known quantum circuits for the generation of graph states found within recent literature. In some cases, such as a 3-photon GHZ state, we have found alternative, yet equivalent quantum circuits, providing experimentalists with setup variety.
- For a 3-photon linear cluster state, we have found an improved quantum circuit with a shorter depth relative to published setups.
- Our framework supports diverse noise models, including depolarizing noise, amplitude damping, and Pauli errors.
- We have developed an experimentally-motivated, time-dependent noise model leveraging the stabilizer representation. This model reduces the number of potential outcomes within mixed states and expresses the noise in terms of circuit time-scales.
- Demonstrated that the use of parameterized gates in conjunction with gradient-based optimization can remediate the impacts of noise-driven fidelity degradation, with an absolute fidelity recovery of up to 15% for 5-photon linear cluster states.
- Developed a novel way of representing mixed states using stabilizers, allowing the simulation of more realistic noise effects and circuit dynamics.
- Developed a novel method for generating the graph representation of a quantum state from a density matrix.
- We have developed random and evolutionary schemes that search for well-performing quantum circuits given a target output state.
- We have developed a hybrid quantum circuit search scheme by combining deterministic and stochastic methods.
- Our framework is modular, and can easily be extended to other physical platforms and noise models.

3.2.6. Dissemination to the communities of interest.

Nothing to report. Nevertheless, manuscripts are in preparation and will be shared with the program officer.

4. Impacts

4.1. Development of the principal discipline of the project

We have developed a framework for the study and optimization of graph state generation in the presence of realistic effects such as noise and photonic loss, making it the first of its kind. Such a tool is a driver for further basic research into entangled resource preparation and distribution for various applications.

The framework is implemented in Python and was designed to be extensible, with community co-development in mind. To support the community and future partners in using these tools, we have prepared thorough documentation, code examples, and interactive tutorials.

Our long-term plans are discussed in detail in Sec. 6, but our near-term plan for disseminating our work to the community includes the following:

- Make our framework available to the community by open-sourcing our codebase

- alongside documentation, use examples, and tutorials
- Use the open-sourced code to enhance and establish research partnerships
- Prepare abstracts for conferences such as the International Conference on Quantum Communication, Measurement and Computing (QCMC), Q2B22 Silicon Valley, Photonics West, and CLEO
- Prepare a manuscript for peer-review and publication

4.2. Impact on other disciplines

In addition to the dissemination of our framework, we also intend to collaborate directly with colleagues from the community. This includes collaborating with the National Research Council of Canada (NRC), specialized in the fabrication of quantum dots that can be used as emitter qubits, l’Institut National de la Recherche Scientifique (INRS), giving us access to knowledge on integrated photonics, as well as Caltech and Fermilab, who can provide realistic entanglement distribution channel metrics from their respective testbeds. Additionally, Quantum Bridge Technologies, Inc. (QBT) is a University of Toronto spin-off, with many local research collaboration opportunities. Our collaborations will allow us to test and validate our framework, as well as improve its level of realism.

5. Future goals

As part of future work, our team intends to maintain and develop our framework alongside the open-source quantum community to find solutions for higher complexity graph states. In parallel, our work internally will focus on extending the framework with more numerous and realistic components as well as qubit protection strategies. To perform this work, we hope for continued support of the project in the years to come and provide below a Gantt chart presenting our future project goals for the following year. Indicated within each task are assigned lead researchers, with a team overview following the chart.

[illegible]

Open innovation and community impact As discussed in Sec. 5, we aim to disseminate our work to support the quantum research community. Open-sourcing will be co-led by I1 and I2. Reaching out to community members will be co-led by PM1, PM2, PM3 and PM4. The writing of a manuscript will be co-led by PDF2 and PDF3 with support from the full team.															
Publish codebase and finalize public documentation and tutorials															
Prepare manuscript and submit to journal															
Project management and administrative support Project management will continue to be led by PM1, with the support of PM2, PM3 and PM4.															
Ongoing management and project reporting															

6. Team overview:

PM1: Piotr Roztocki

CTO, Ki3 Photonics Technologies Inc. Principal project manager responsible for day-to-day activities. Honors B.Sc. in Integrated Science (Physics), McMaster University 2014; Ph.D. in Energy and Materials, INRS-EMT 2022.

PM2: Hoi-Kwong Lo

CTO, Quantum Bridge Technologies, Inc. (QBT). Plays a supervisory role in the project. Professor of Physics and ECE at the University of Toronto, BA in Mathematics, Cambridge University, 1989, Ph.D. in Physics, Caltech, 1994. Co-inventor of measurement-device-independent quantum key distribution (QKD) and all photonics quantum repeaters. Fellow of the American Physical Society (APS) and Optica Fellow.

PM3: Mattia Montagna

CEO, Quantum Bridge Technologies, Inc. (QBT), PhD in Applied Math. Founder of the ECB (European Central Bank) Bank Interconnected Coordination Group (BICG). Has been leading QBT since its founding in 2019.

PM4: Yoann Jestin

CEO, Ki3 Photonics Technologies Inc. Principal Investigator of the project. B.Sc in Chemistry, University of Rennes, 1997; M.Sc. in Material Science, University of Rennes, 1999; Ph.D in Material Science, University of Le Mans, 2002.

PDF1: Jie Lin

QBT quantum research scientist with specializations in quantum optics and security analysis of quantum key distribution. Will lead development of both fusion gates and qubit encoding. B. A. Mathematics, Carleton College; M. A. S. Theoretical Physics, University of Cambridge; M. Sc. Physics and Ph.D. Physics (quantum information), University of Waterloo.

PDF2: Luc Robichaud

QBT quantum research scientist who specializes in quantum dot modeling and their application to entanglement generation. Will lead modeling of various sources and incorporating said models into the framework. Will also co-lead with PDF3 the writing of papers for peer-reviewed journals. B.Sc. Physics and M.Sc. Physics, Université de Moncton;

Ph.D. Physics, University of Ottawa.

PDF3: Stefania Sciara

Postdoc at l'Institut National de la Recherche Scientifique (INRS) and Ki3 researcher with specializations in both experimental and theoretical quantum optics, and entanglement generation. B.Sc. Physics and M.Sc. Physics, University of Palermo; Ph.D. Quantum optics, INRS.

PhD1: Sobhan Ghanbari

QBT intern. Ph.D. candidate at the University of Toronto with specialization in quantum information. Sobhan has led and will continue to lead development of various noise models within the framework. B.Sc. Physics/ B.Sc. Mechanical Engineering (dual- degree), Sharif University of Technology.

PhD2: Benjamin MacLellan

Ph.D. candidate at the University of Waterloo with specialization in quantum information and inverse design of photonic systems. Will lead the development of a more comprehensive physical layer, including various integrated-photonics-based components. B.Sc. Physics, University of Waterloo; M.Sc., Energy & Materials, INRS.

M1: Kai Sum Chan

QBT quantum research scientist. M.Phil. candidate at the University of Hong Kong, and QBT experimental and theoretical quantum researcher with specialization in quantum information, adaptive optics and quantum dots. Will play a supporting role in calibrating models according to experiments.

I1: Khuong Tran

B.Sc. candidate in computer sciences at the University of Ontario Institute of Technology and intern at Ki3. Will co-lead with I2 preparation of open-sourcing, framework documentation and tutorials.

I2: Aayush Soni

B.A.Sc. candidate in mechatronics engineering at the University of Waterloo and intern at Ki3. Will co-lead with I1 the preparation of open-sourcing, framework documentation and tutorials.

Key: PM: Project manager, PDF: Postdoctoral fellow, PhD: PhD student, M: Master's student, I: Intern.

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Highly Entangled States of Light

Final Report

Awarded to:
Ki3 Photonics Technologies Inc.
Quantum Bridge Technologies Inc (QBT).

Recipient Principal Investigator:
Dr. Yoann Jestin

1. Executive summary

Problem statement:

- Graph quantum states are an important resource for fundamental research and technological applications
- Designing quantum circuits that output specific and desired graph states is challenging
- Real quantum circuits are subject to noise and loss, which are necessary to include for an accurate and realistic model

Project scope:

- Construct a theoretical and numerical framework for the study of graph states and of circuits that generate them
- Develop schemes for the construction of quantum circuits that output desired graph states
- Create models for the noise and loss effects that are present in real quantum setups

General outcome:

- To date, we have defined a framework that allows us to find and to simulate quantum circuits that output low-complexity graph states
- Our schemes successfully reproduce results from literature and are also capable of finding alternative setups to choose between
- We have developed noise and loss models that are representative of effects found in common photonics platforms
- Our framework is a flexible and extensible tool that is soon to be open-sourced and which can be easily adapted to simulate various physical platforms

Future Plans:

- Maintain and develop the framework alongside the open-source quantum community to find solutions for higher complexity graph states
- Develop fusion-gate based protocols for producing large graph states
- Implement encoding strategies for qubit protection
- Model quantum-dot-based sources and interface with current framework
- Develop models for integrated-photonics-based components as an experimental basis for quantum circuits
- Develop more sophisticated noise and distribution channel models
- Extend our ability to inject experimental details into our framework, further connecting results with real setups

2. Introduction

Graph states are a family of quantum states with high levels of entanglement that are of great fundamental interest [1], as well as being a key resource for applications in entanglement distribution and quantum networking [2-4], quantum error correction [5,6], and quantum metrology [7]. Among various physical platforms, photonics shows much promise for graph state realizations [8,9] as photons interact very weakly with their environment, enabling stable qubits. Additionally, photons are compatible with mature integrated and fiber technologies, granting additional support in the generation and distribution of graph states.

As well, for the generation of photons, various sources are available within photonics-based platforms, including solid-state emitters such as quantum dots that can emit single photons on demand [10], or nonlinear elements generating pairs of entangled photons [11,12].

Quantum circuits describe the sequence of quantum operations necessary to generate such graph states. However, connecting a desired graph state with the circuit that can output it is a non-trivial design task that only becomes more difficult with increasing state size and complexity. Recent work [13] has led to the development of an impressive deterministic method for finding quantum circuits that produce targeted graph states. However, such methods do not guarantee well-performing quantum circuits, allow little flexibility, and cannot consider impacts associated with noise or photonic loss, which is present in all real setups.

Towards addressing this research gap, the goal of our work was to develop a flexible theoretical and computational framework for the study of realistic graph states and of the quantum circuits that generate them. To accomplish this, we must be able to represent both the quantum states and the quantum circuits that produce them. Both representations must also be flexible enough to support noise and photonic loss modeling. The performance of different circuits must also be quantifiable, in order to drive schemes that search for best performing setups. All of these concepts must also be realized in a flexible and modular way that allows for co-development of solutions with the broader research community, driving collaborative and fundamental work into the understanding of entangled states as a resource.

Over the past year of our project's progress, we have successfully identified and realized a framework comprising the following core concepts: *Quantum state representations*, *quantum circuit representations*, *realistic component models*, *quantum circuit performance metrics*, and *quantum circuit optimization schemes*. Figure 1 shows these key concepts and their mutual interaction within our framework.

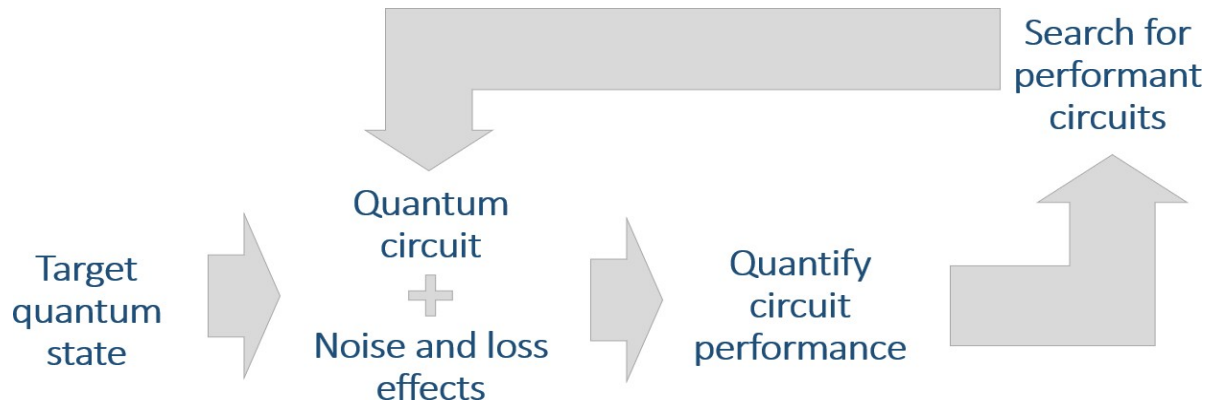


Figure 1: Key concepts within our framework. The performance metric is calculated with respect to the target graph state, guiding the search for performant circuits.

3. Accomplishments

In what follows, we explain each of the core concepts of our framework alongside our respective research efforts in developing them.

3.1. Research objectives

3.3.1. Representing quantum states

The objects of interest, graph states, are quantum systems of photonic and emitter qubits that can be mathematically represented using various formalisms. Our computational framework presently considers two mathematical representations, the *density matrix* and the *stabilizer*, with the modularity to easily include further representations in the future.

3.1.2. Representing quantum circuits

Quantum circuits represent the sequence of quantum gates and operations applied on solid state emitter qubits and photonic qubits. To computationally read, save, and manipulate these quantum circuits, we represent them as Directed Acyclic Graphs (DAGs) [19], which find common use in the field of quantum information [20]. Using the DAG representation, our computational framework can easily modify and evolve the quantum circuit topology to optimize the generated quantum state towards a target graph state. For the visualization of quantum circuits, our DAG representation was also made compatible with the openQASM format, a standard within the field [21].

3.1.3. Realistic component models

Towards increasing the realism of our framework, we support diverse models for noise and photonic loss. For example, our framework presently includes noise models for depolarizing noise, amplitude damping, and Pauli noise.

Additionally, we have developed a new representation to model mixed states using stabilizers. This new representation keeps track of all potential branchings of a mixed state, along with the probability of such a branch being measured. This novel approach allows us to consider probabilistic noise, more akin to real experimental systems.

Motivated by the physics of today's solid-state emitter and photonic qubit systems, we have furthermore developed a time-dependent noise model within the stabilizer formalism. In experiments, the number of errors that occur is strongly correlated with the time spent during and between each quantum gate. In our time-dependent model, we make use of an effective noise (simulated only at certain gates), maintaining model accuracy while reducing the computational load of the model.

3.1.4. Assessing circuit performance

When optimizing a quantum circuit for the generation of a target graph state, it is important to properly quantify the performance of said circuit using quantitative metrics. In fact, having proper metrics can heavily influence the performance of a circuit search algorithm. To date, we have investigated the following metrics as potential candidates [14], [22], [23]:

Fidelity	Relative entropy
Entanglement witness	Squashed entanglement
Graph edit distance	Adjacency matrix fidelity
Concurrence	Geometric measure
Negativity	Fisher information
Trace distance	Mermin inequality
Entanglement distillation	Stabilizer tableau fidelity

3.1.5. Searching for performant quantum circuits

We have investigated the following schemes for converging on suitable quantum circuits that output a desired graph state:

- **Random:** This scheme initializes a list of random circuits subject to physical

constraints, which we call a population. At each iteration and for each circuit within the population, the algorithm applies a random operation, which includes addition, removal, or replacement of a quantum gate. Each circuit in the population is modified independently from the other circuits. The scheme then keeps track of a certain number of top performing circuits.

- **Evolutionary:** Similarly to the random search approach, this scheme also applies random operations in each iteration. However, in contrast, after each iteration it constructs a new population by putting circuits into a N-wise competition and selecting the best performers.
- **Deterministic:** This scheme is our implementation of Ref. [13]’s approach to deterministically find a quantum circuit for a target graph state.
- **Hybrid:** In this scheme, we use the deterministic method from Ref. [13] to produce an initial solution. We then perturb this solution and use the random/evolutionary schemes to search for quantum circuit alternatives.
- **Gradient-based:** This is a density matrix specific scheme where we consider tunable/parametrized gates that are compatible with automatic differentiation and gradient descent optimization techniques.

3.2 Details of accomplishments

3.2.1. Representing quantum states

The *density matrix* representation can simulate a state’s full Hilbert space and can inherently represent mixed states, such as those resulting from the effects of noise and photonic loss [14]. However, this representation does not scale well with the number of qubits, making it impractical to investigate large graph states, and thus motivating the need for complementary representations. We have used this representation: 1) To study small graph states with the inclusion of noise effects; 2) As a benchmark for other representations; and 3) For the study of noise, photonic loss, mixed states, and gate parametrization.

The *Stabilizer* representation [15], [16], on the other hand, scales well with the number of qubits, making it more efficient for the study of large graph states (relative to the density matrix formalism). However, only a subset of quantum operations can be efficiently simulated within the stabilizer representation, limiting the scope for circuit optimization, and making this representation unsuited for the simulation of noise without judicious changes to the formalism. Presently, the study of representing mixed states within the stabilizer representation is very limited in the literature. To be able to consider large graph states that are mixed due to noise or loss, we have developed a probability tree approach as a way to represent such states. This point of research is presented further below in Sec. 3.3.

We have also developed relevant concepts from *graph* representations, which are also well-suited for many qubits, and contrary to the stabilizer representation, there is more progress made on describing mixed states [17,18]. In particular, we have worked on leveraging graph state equivalency through Clifford gates. As discussed in Sec. 3.4, the graph representation also offers unique metrics for characterizing the performance of quantum circuits.

Each of the quantum state representations has advantages and disadvantages, making it useful to be able to move between them. Most of these representation transformations can be found within literature [3], [14-18], except for obtaining a graph from a density matrix. We have resolved this issue by designing a novel protocol, as discussed in Sec. 3.5.

3.2.2. Representing quantum circuits

Although we have focused on solid state emitter qubits and photonic qubits, we have designed our framework with extensibility in mind. The types of operations, and therefore the

physical circuits we can represent and simulate, are flexible, meaning we can adapt our framework towards different physical platforms with relative ease.

Lastly, our quantum circuit representation also includes parameterized quantum gates and automatic differentiation (see Sec. 3.5 for further discussion).

3.2.3. Realistic component models

The Figure 2 shows an example of the model in practice, deriving the fidelity of an output graph state as a function of qubit error rate from various sources of noise.

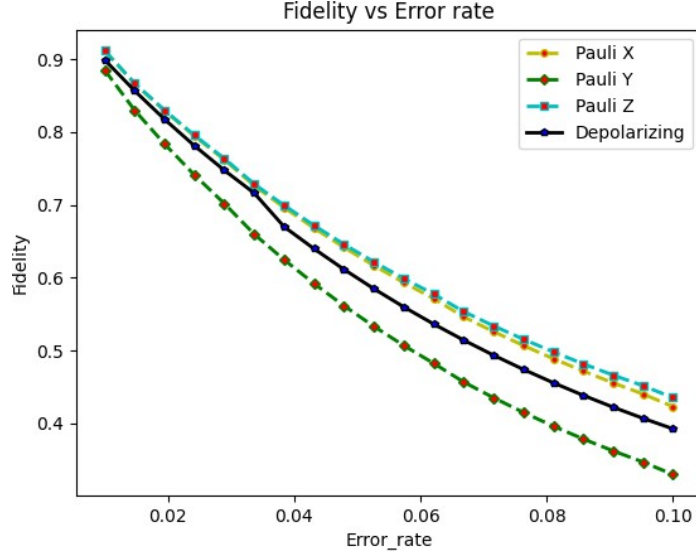


Figure 2: Fidelity versus error rate for the generation of a 4-photon linear cluster state assuming X -gate errors (yellow), Y -gate errors (green), Z -gate errors (blue) and a depolarizing channel (black). We find that the generation of linear cluster states is equally sensitive to bit-flip (X -gate) and phase-flip (Z -gate) errors, and more sensitive when both errors occur (Y -gate).

As photonic losses have non-trivial impacts on quantum states, generally producing mixed states, we have explored various approaches to simulate their effects. One approach is to extend the standard qubit space to a qutrit space so that, by accounting for a vacuum term, the state can still be represented as pure. Another approach is to keep track of the survival probability of a state after potential photonic loss events. Interestingly, we observe that the state fidelity calculation only depends on branches that see no loss, weighted by their respective probabilities. By tracking transmission probabilities, we gain information useful for approximating state generation rates, which are a very important experimental insight. We also calculate the survival probability for each photon, which indicates which portions of the graph state are more prone to loss. This information is essential for efficiently implementing encoding for error correction.

3.2.4. Assessing circuit performance

Fidelity has been our main metric thus far, but it is not ideal for our task as it can change abruptly, which is partially due to the discrete nature of adding or removing quantum operations within the quantum circuit. Entanglement witnesses have potential [24]–[26], but our investigations of GHZ and linear cluster states suggest they may be too robust and not sufficiently monotonic in the neighborhood of optimums. The graph edit metric, which is a metric based on the graph representation of a quantum state, holds potential as it is fundamentally different from other metrics. There are however challenges as it is not obvious

how fundamental graph operations, such local complementations, would be represented in this metric. Most metrics investigated thus proved to not be useful towards search/optimization schemes or well suited for multipartite states.

3.2.5. Searching for performant quantum circuits

Using the random and evolutionary schemes within the density matrix representation, we were able to:

- Reproduce results from [13] for 4-photon linear cluster states.
- Find alternative, but equivalent circuits. Given some circuits may be more sensitive to noise compared to others or that some gates may be more difficult to implement than others, this gives options to experimentalists in implementing whichever protocol is more practical.
- Find cases where the circuit depth is reduced relative to circuits produced by Ref. [13]'s approach. An example of this depth decrease is shown in Fig. 3. Decreases in circuit depth imply a simplified protocol as there are fewer gates to be implemented or more gates can be implemented in parallel.
- In terms of convergence, we found that the random and evolutionary schemes performed similarly. Figure 4 shows the fraction of search runs that converge on a quantum circuit producing the target graph state.

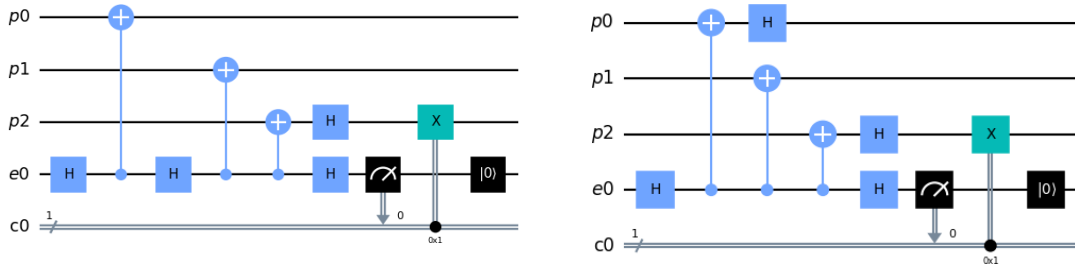


Figure 3: Quantum circuits for the generation of a 3-photon linear cluster state as determined by (left) the deterministic methods of [13] and (right) our framework. Our solution presents a reduced circuit depth relative to the deterministic method.

Using the stabilizer representation, we are able to efficiently simulate large graph states, up to hundreds of qubits, but it remains difficult to converge to an optimal circuit due to the vast optimization landscape. For this reason, we have also investigated the hybrid approach, which leverages the deterministic scheme of Ref. [13] to produce a starting point for random/evolutionary-based search approaches. Using this method, we have been able to find alternative quantum circuits to those proposed by the deterministic approach, as well as access larger graph states.

Additionally, the deterministic solutions do not include the effects of noise, which degrade the quality of the output graph state for a given circuit. Using our hybrid scheme, we have begun investigations into re-optimizing the deterministic solution after the introduction of noise, to regain state fidelity. This investigation is expected to greatly benefit from experimental feedback to more accurately represent noise found within real photonic platforms.



	Linear Cluster (3 qubits)	GHZ (3 qubits)	Linear Cluster (4 qubits)	GHZ (4 qubits)
Random scheme	92%	60%	24%	24%
Evolutionary scheme	96%	16%	24%	28%

Figure 4: Fraction of simulations that converge on a solution for a quantum circuit using a density matrix representation. Target graph states for each case are indicated above columns. Both the random and evolutionary schemes have difficulty converging with an increasing number of qubits. This is due to the vastness of the optimization landscape, indicating the need for better metrics and/or optimization schemes.

Within the density matrix representation, we have also implemented parameterized quantum gates and the automatic differentiation of quantum circuits. This enables the gradient-based optimization of circuit evaluation metrics (using, e.g., gradient-descent optimizers). This aspect of our framework enables quantum circuits to be optimized in the presence of noise, and enables gradient descent to be combined with circuit topology optimization schemes. An example of this scheme in action is shown in Fig. 5, where we perform quantum gate optimization to recover fidelity lost due to noise. Although the noise model requires refinement to accurately represent experimentally-found errors, these are promising results for managing noise-related deterioration of graph states.

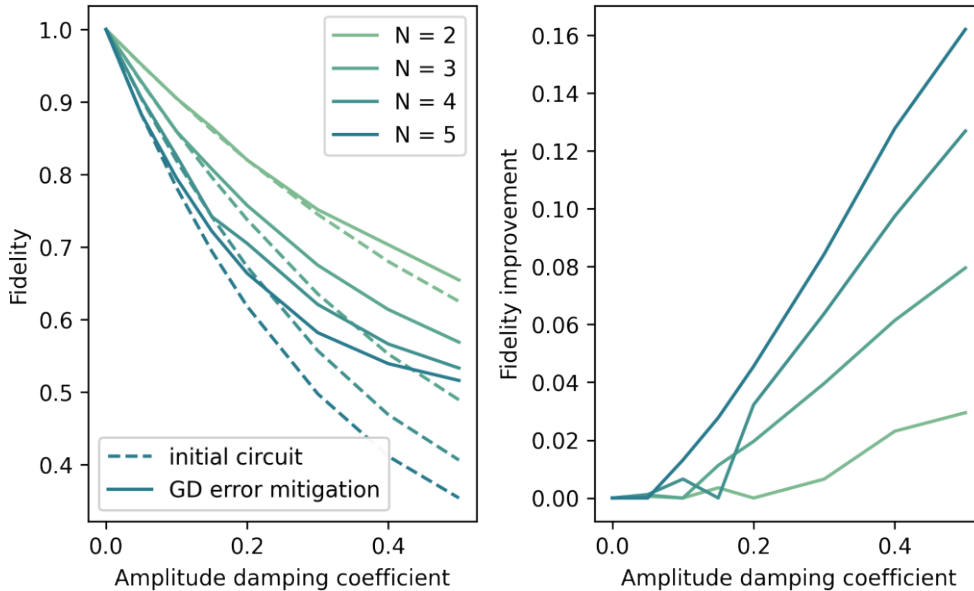


Figure 5: Regaining lost fidelity by parametrized gates optimization for linear cluster states consisting of $N=1-5$ photons as function of noise amplitude, where an amplitude damping model was uniformly applied on all qubits. Left panel shows the fidelity of the initial circuit after introducing noise (dashed) and the fidelity after gate optimization (solid). Right panel shows the relative difference between the solid and dashed lines. We find that gate optimization led to absolute fidelity recoveries of up to 15%.

3.2.5. Summary of accomplishments

During year 1 of this project, we have accomplished the following basic research milestones:

- Constructed a theoretical and numerical framework for the study of graph states and of the quantum circuits that generate them within photonics-based platforms.
- Reproduced known quantum circuits for the generation of graph states found within recent literature. In some cases, such as a 3-photon GHZ state, we have found alternative, yet equivalent quantum circuits, providing experimentalists with setup variety.
- For a 3-photon linear cluster state, we have found an improved quantum circuit with a shorter depth relative to published setups.
- Our framework supports diverse noise models, including depolarizing noise, amplitude damping, and Pauli errors.
- We have developed an experimentally-motivated, time-dependent noise model leveraging the stabilizer representation. This model reduces the number of potential outcomes within mixed states and expresses the noise in terms of circuit time-scales.
- Demonstrated that the use of parameterized gates in conjunction with gradient-based optimization can remediate the impacts of noise-driven fidelity degradation, with an absolute fidelity recovery of up to 15% for 5-photon linear cluster states.
- Developed a novel way of representing mixed states using stabilizers, allowing the simulation of more realistic noise effects and circuit dynamics.
- Developed a novel method for generating the graph representation of a quantum state from a density matrix.
- We have developed random and evolutionary schemes that search for well-performing quantum circuits given a target output state.
- We have developed a hybrid quantum circuit search scheme by combining deterministic and stochastic methods.
- Our framework is modular, and can easily be extended to other physical platforms and noise models.

3.2.6. Dissemination to the communities of interest.

Nothing to report. Nevertheless, manuscripts are in preparation and will be shared with the program officer.

4. Impacts

4.1. Development of the principal discipline of the project

We have developed a framework for the study and optimization of graph state generation in the presence of realistic effects such as noise and photonic loss, making it the first of its kind. Such a tool is a driver for further basic research into entangled resource preparation and distribution for various applications.

The framework is implemented in Python and was designed to be extensible, with community co-development in mind. To support the community and future partners in using these tools, we have prepared thorough documentation, code examples, and interactive tutorials.

Our long-term plans are discussed in detail in Sec. 6, but our near-term plan for disseminating our work to the community includes the following:

- Make our framework available to the community by open-sourcing our codebase

- alongside documentation, use examples, and tutorials
- Use the open-sourced code to enhance and establish research partnerships
- Prepare abstracts for conferences such as the International Conference on Quantum Communication, Measurement and Computing (QCMC), Q2B22 Silicon Valley, Photonics West, and CLEO
- Prepare a manuscript for peer-review and publication

4.2. Impact on other disciplines

In addition to the dissemination of our framework, we also intend to collaborate directly with colleagues from the community. This includes collaborating with the National Research Council of Canada (NRC), specialized in the fabrication of quantum dots that can be used as emitter qubits, l’Institut National de la Recherche Scientifique (INRS), giving us access to knowledge on integrated photonics, as well as Caltech and Fermilab, who can provide realistic entanglement distribution channel metrics from their respective testbeds. Additionally, Quantum Bridge Technologies, Inc. (QBT) is a University of Toronto spin-off, with many local research collaboration opportunities. Our collaborations will allow us to test and validate our framework, as well as improve its level of realism.

5. Future goals

As part of future work, our team intends to maintain and develop our framework alongside the open-source quantum community to find solutions for higher complexity graph states. In parallel, our work internally will focus on extending the framework with more numerous and realistic components as well as qubit protection strategies. To perform this work, we hope for continued support of the project in the years to come and provide below a Gantt chart presenting our future project goals for the following year. Indicated within each task are assigned lead researchers, with a team overview following the chart.

Task description	Months											
	M1-M4				M4-M8				M8-M12			
Modeling fusion gate components Using fusion gates [27], we can form large graph states from smaller graph states. We will implement such gates within our framework, giving an alternative route to realizing graph states. This task will be led by PDF1 and supported by PDF2 and PhD1.												
Literature review and implementing fusion gates within framework												
Investigate fusion gate-based quantum circuits												
Encoding for error correction and fusion gate protection We will explore encoding methods for protecting qubits against random errors (e.g., bitflip) and failed fusion gate attempts, which lead to the loss of a qubit. This task will be led by PDF1, and supported by PDF2 and PhD1.												
Investigate encoding strategies in literature												
Implement encoding for qubit protection												

Open innovation and community impact As discussed in Sec. 5, we aim to disseminate our work to support the quantum research community. Open-sourcing will be co-led by I1 and I2. Reaching out to community members will be co-led by PM1, PM2, PM3 and PM4. The writing of a manuscript will be co-led by PDF2 and PDF3 with support from the full team.															
Publish codebase and finalize public documentation and tutorials															
Prepare manuscript and submit to journal															
Project management and administrative support Project management will continue to be led by PM1, with the support of PM2, PM3 and PM4.															
Ongoing management and project reporting															

6. Team overview:

PM1: Piotr Roztocki

CTO, Ki3 Photonics Technologies Inc. Principal project manager responsible for day-to-day activities. Honors B.Sc. in Integrated Science (Physics), McMaster University 2014; Ph.D. in Energy and Materials, INRS-EMT 2022.

PM2: Hoi-Kwong Lo

CTO, Quantum Bridge Technologies, Inc. (QBT). Plays a supervisory role in the project. Professor of Physics and ECE at the University of Toronto, BA in Mathematics, Cambridge University, 1989, Ph.D. in Physics, Caltech, 1994. Co-inventor of measurement-device-independent quantum key distribution (QKD) and all photonics quantum repeaters. Fellow of the American Physical Society (APS) and Optica Fellow.

PM3: Mattia Montagna

CEO, Quantum Bridge Technologies, Inc. (QBT), PhD in Applied Math. Founder of the ECB (European Central Bank) Bank Interconnected Coordination Group (BICG). Has been leading QBT since its founding in 2019.

PM4: Yoann Jestin

CEO, Ki3 Photonics Technologies Inc. Principal Investigator of the project. B.Sc in Chemistry, University of Rennes, 1997; M.Sc. in Material Science, University of Rennes, 1999; Ph.D in Material Science, University of Le Mans, 2002.

PDF1: Jie Lin

QBT quantum research scientist with specializations in quantum optics and security analysis of quantum key distribution. Will lead development of both fusion gates and qubit encoding. B. A. Mathematics, Carleton College; M. A. S. Theoretical Physics, University of Cambridge; M. Sc. Physics and Ph.D. Physics (quantum information), University of Waterloo.

PDF2: Luc Robichaud

QBT quantum research scientist who specializes in quantum dot modeling and their application to entanglement generation. Will lead modeling of various sources and incorporating said models into the framework. Will also co-lead with PDF3 the writing of papers for peer-reviewed journals. B.Sc. Physics and M.Sc. Physics, Université de Moncton;

Ph.D. Physics, University of Ottawa.

PDF3: Stefania Sciara

Postdoc at l'Institut National de la Recherche Scientifique (INRS) and Ki3 researcher with specializations in both experimental and theoretical quantum optics, and entanglement generation. B.Sc. Physics and M.Sc. Physics, University of Palermo; Ph.D. Quantum optics, INRS.

PhD1: Sobhan Ghanbari

QBT intern. Ph.D. candidate at the University of Toronto with specialization in quantum information. Sobhan has led and will continue to lead development of various noise models within the framework. B.Sc. Physics/ B.Sc. Mechanical Engineering (dual- degree), Sharif University of Technology.

PhD2: Benjamin MacLellan

Ph.D. candidate at the University of Waterloo with specialization in quantum information and inverse design of photonic systems. Will lead the development of a more comprehensive physical layer, including various integrated-photonics-based components. B.Sc. Physics, University of Waterloo; M.Sc., Energy & Materials, INRS.

M1: Kai Sum Chan

QBT quantum research scientist. M.Phil. candidate at the University of Hong Kong, and QBT experimental and theoretical quantum researcher with specialization in quantum information, adaptive optics and quantum dots. Will play a supporting role in calibrating models according to experiments.

I1: Khuong Tran

B.Sc. candidate in computer sciences at the University of Ontario Institute of Technology and intern at Ki3. Will co-lead with I2 preparation of open-sourcing, framework documentation and tutorials.

I2: Aayush Soni

B.A.Sc. candidate in mechatronics engineering at the University of Waterloo and intern at Ki3. Will co-lead with I1 the preparation of open-sourcing, framework documentation and tutorials.

Key: PM: Project manager, PDF: Postdoctoral fellow, PhD: PhD student, M: Master's student, I: Intern.

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Highly Entangled States of Light

Final Report

Awarded to:
Ki3 Photonics Technologies Inc.
Quantum Bridge Technologies Inc (QBT).

Recipient Principal Investigator:
Dr. Yoann Jestin

1. Executive summary

Problem statement:

- Graph quantum states are an important resource for fundamental research and technological applications
- Designing quantum circuits that output specific and desired graph states is challenging
- Real quantum circuits are subject to noise and loss, which are necessary to include for an accurate and realistic model

Project scope:

- Construct a theoretical and numerical framework for the study of graph states and of circuits that generate them
- Develop schemes for the construction of quantum circuits that output desired graph states
- Create models for the noise and loss effects that are present in real quantum setups

General outcome:

- To date, we have defined a framework that allows us to find and to simulate quantum circuits that output low-complexity graph states
- Our schemes successfully reproduce results from literature and are also capable of finding alternative setups to choose between
- We have developed noise and loss models that are representative of effects found in common photonics platforms
- Our framework is a flexible and extensible tool that is soon to be open-sourced and which can be easily adapted to simulate various physical platforms

Future Plans:

- Maintain and develop the framework alongside the open-source quantum community to find solutions for higher complexity graph states
- Develop fusion-gate based protocols for producing large graph states
- Implement encoding strategies for qubit protection
- Model quantum-dot-based sources and interface with current framework
- Develop models for integrated-photonics-based components as an experimental basis for quantum circuits
- Develop more sophisticated noise and distribution channel models
- Extend our ability to inject experimental details into our framework, further connecting results with real setups

2. Introduction

Graph states are a family of quantum states with high levels of entanglement that are of great fundamental interest [1], as well as being a key resource for applications in entanglement distribution and quantum networking [2-4], quantum error correction [5,6], and quantum metrology [7]. Among various physical platforms, photonics shows much promise for graph state realizations [8,9] as photons interact very weakly with their environment, enabling stable qubits. Additionally, photons are compatible with mature integrated and fiber technologies, granting additional support in the generation and distribution of graph states.

As well, for the generation of photons, various sources are available within photonics-based platforms, including solid-state emitters such as quantum dots that can emit single photons on demand [10], or nonlinear elements generating pairs of entangled photons [11,12].

Quantum circuits describe the sequence of quantum operations necessary to generate such graph states. However, connecting a desired graph state with the circuit that can output it is a non-trivial design task that only becomes more difficult with increasing state size and complexity. Recent work [13] has led to the development of an impressive deterministic method for finding quantum circuits that produce targeted graph states. However, such methods do not guarantee well-performing quantum circuits, allow little flexibility, and cannot consider impacts associated with noise or photonic loss, which is present in all real setups.

Towards addressing this research gap, the goal of our work was to develop a flexible theoretical and computational framework for the study of realistic graph states and of the quantum circuits that generate them. To accomplish this, we must be able to represent both the quantum states and the quantum circuits that produce them. Both representations must also be flexible enough to support noise and photonic loss modeling. The performance of different circuits must also be quantifiable, in order to drive schemes that search for best performing setups. All of these concepts must also be realized in a flexible and modular way that allows for co-development of solutions with the broader research community, driving collaborative and fundamental work into the understanding of entangled states as a resource.

Over the past year of our project's progress, we have successfully identified and realized a framework comprising the following core concepts: *Quantum state representations*, *quantum circuit representations*, *realistic component models*, *quantum circuit performance metrics*, and *quantum circuit optimization schemes*. Figure 1 shows these key concepts and their mutual interaction within our framework.

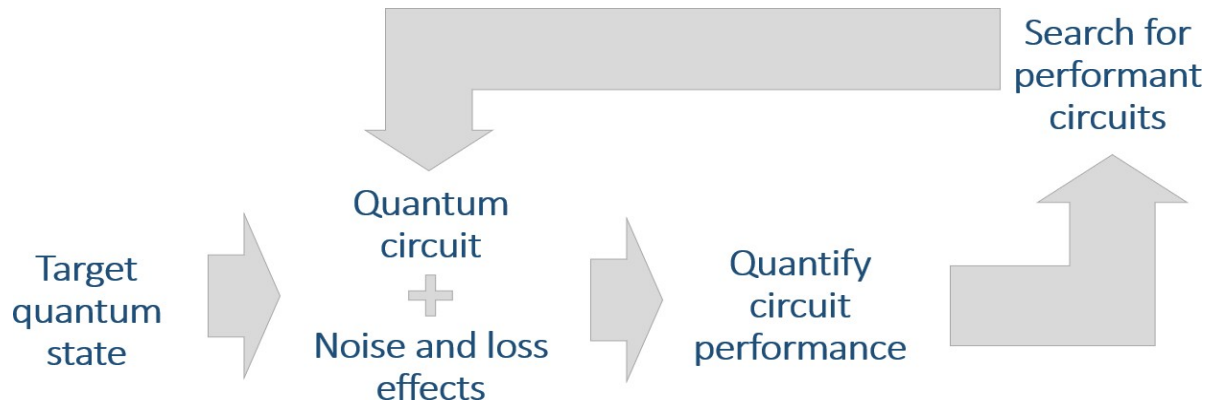


Figure 1: Key concepts within our framework. The performance metric is calculated with respect to the target graph state, guiding the search for performant circuits.

3. Accomplishments

In what follows, we explain each of the core concepts of our framework alongside our respective research efforts in developing them.

3.1. Research objectives

3.3.1. Representing quantum states

The objects of interest, graph states, are quantum systems of photonic and emitter qubits that can be mathematically represented using various formalisms. Our computational framework presently considers two mathematical representations, the *density matrix* and the *stabilizer*, with the modularity to easily include further representations in the future.

3.1.2. Representing quantum circuits

Quantum circuits represent the sequence of quantum gates and operations applied on solid state emitter qubits and photonic qubits. To computationally read, save, and manipulate these quantum circuits, we represent them as Directed Acyclic Graphs (DAGs) [19], which find common use in the field of quantum information [20]. Using the DAG representation, our computational framework can easily modify and evolve the quantum circuit topology to optimize the generated quantum state towards a target graph state. For the visualization of quantum circuits, our DAG representation was also made compatible with the openQASM format, a standard within the field [21].

3.1.3. Realistic component models

Towards increasing the realism of our framework, we support diverse models for noise and photonic loss. For example, our framework presently includes noise models for depolarizing noise, amplitude damping, and Pauli noise.

Additionally, we have developed a new representation to model mixed states using stabilizers. This new representation keeps track of all potential branchings of a mixed state, along with the probability of such a branch being measured. This novel approach allows us to consider probabilistic noise, more akin to real experimental systems.

Motivated by the physics of today's solid-state emitter and photonic qubit systems, we have furthermore developed a time-dependent noise model within the stabilizer formalism. In experiments, the number of errors that occur is strongly correlated with the time spent during and between each quantum gate. In our time-dependent model, we make use of an effective noise (simulated only at certain gates), maintaining model accuracy while reducing the computational load of the model.

3.1.4. Assessing circuit performance

When optimizing a quantum circuit for the generation of a target graph state, it is important to properly quantify the performance of said circuit using quantitative metrics. In fact, having proper metrics can heavily influence the performance of a circuit search algorithm. To date, we have investigated the following metrics as potential candidates [14], [22], [23]:

Fidelity	Relative entropy
Entanglement witness	Squashed entanglement
Graph edit distance	Adjacency matrix fidelity
Concurrence	Geometric measure
Negativity	Fisher information
Trace distance	Mermin inequality
Entanglement distillation	Stabilizer tableau fidelity

3.1.5. Searching for performant quantum circuits

We have investigated the following schemes for converging on suitable quantum circuits that output a desired graph state:

- **Random:** This scheme initializes a list of random circuits subject to physical

constraints, which we call a population. At each iteration and for each circuit within the population, the algorithm applies a random operation, which includes addition, removal, or replacement of a quantum gate. Each circuit in the population is modified independently from the other circuits. The scheme then keeps track of a certain number of top performing circuits.

- **Evolutionary:** Similarly to the random search approach, this scheme also applies random operations in each iteration. However, in contrast, after each iteration it constructs a new population by putting circuits into a N-wise competition and selecting the best performers.
- **Deterministic:** This scheme is our implementation of Ref. [13]’s approach to deterministically find a quantum circuit for a target graph state.
- **Hybrid:** In this scheme, we use the deterministic method from Ref. [13] to produce an initial solution. We then perturb this solution and use the random/evolutionary schemes to search for quantum circuit alternatives.
- **Gradient-based:** This is a density matrix specific scheme where we consider tunable/parametrized gates that are compatible with automatic differentiation and gradient descent optimization techniques.

3.2 Details of accomplishments

3.2.1. Representing quantum states

The *density matrix* representation can simulate a state’s full Hilbert space and can inherently represent mixed states, such as those resulting from the effects of noise and photonic loss [14]. However, this representation does not scale well with the number of qubits, making it impractical to investigate large graph states, and thus motivating the need for complementary representations. We have used this representation: 1) To study small graph states with the inclusion of noise effects; 2) As a benchmark for other representations; and 3) For the study of noise, photonic loss, mixed states, and gate parametrization.

The *Stabilizer* representation [15], [16], on the other hand, scales well with the number of qubits, making it more efficient for the study of large graph states (relative to the density matrix formalism). However, only a subset of quantum operations can be efficiently simulated within the stabilizer representation, limiting the scope for circuit optimization, and making this representation unsuited for the simulation of noise without judicious changes to the formalism. Presently, the study of representing mixed states within the stabilizer representation is very limited in the literature. To be able to consider large graph states that are mixed due to noise or loss, we have developed a probability tree approach as a way to represent such states. This point of research is presented further below in Sec. 3.3.

We have also developed relevant concepts from *graph* representations, which are also well-suited for many qubits, and contrary to the stabilizer representation, there is more progress made on describing mixed states [17,18]. In particular, we have worked on leveraging graph state equivalency through Clifford gates. As discussed in Sec. 3.4, the graph representation also offers unique metrics for characterizing the performance of quantum circuits.

Each of the quantum state representations has advantages and disadvantages, making it useful to be able to move between them. Most of these representation transformations can be found within literature [3], [14-18], except for obtaining a graph from a density matrix. We have resolved this issue by designing a novel protocol, as discussed in Sec. 3.5.

3.2.2. Representing quantum circuits

Although we have focused on solid state emitter qubits and photonic qubits, we have designed our framework with extensibility in mind. The types of operations, and therefore the

physical circuits we can represent and simulate, are flexible, meaning we can adapt our framework towards different physical platforms with relative ease.

Lastly, our quantum circuit representation also includes parameterized quantum gates and automatic differentiation (see Sec. 3.5 for further discussion).

3.2.3. Realistic component models

The Figure 2 shows an example of the model in practice, deriving the fidelity of an output graph state as a function of qubit error rate from various sources of noise.

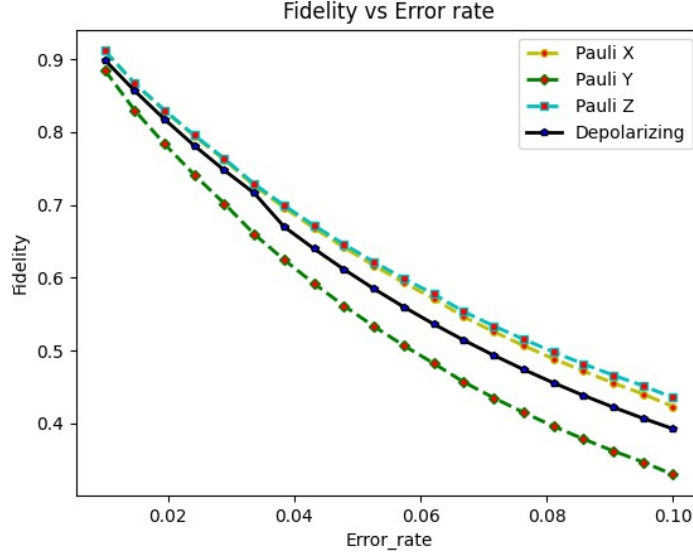


Figure 2: Fidelity versus error rate for the generation of a 4-photon linear cluster state assuming X -gate errors (yellow), Y -gate errors (green), Z -gate errors (blue) and a depolarizing channel (black). We find that the generation of linear cluster states is equally sensitive to bit-flip (X -gate) and phase-flip (Z -gate) errors, and more sensitive when both errors occur (Y -gate).

As photonic losses have non-trivial impacts on quantum states, generally producing mixed states, we have explored various approaches to simulate their effects. One approach is to extend the standard qubit space to a qutrit space so that, by accounting for a vacuum term, the state can still be represented as pure. Another approach is to keep track of the survival probability of a state after potential photonic loss events. Interestingly, we observe that the state fidelity calculation only depends on branches that see no loss, weighted by their respective probabilities. By tracking transmission probabilities, we gain information useful for approximating state generation rates, which are a very important experimental insight. We also calculate the survival probability for each photon, which indicates which portions of the graph state are more prone to loss. This information is essential for efficiently implementing encoding for error correction.

3.2.4. Assessing circuit performance

Fidelity has been our main metric thus far, but it is not ideal for our task as it can change abruptly, which is partially due to the discrete nature of adding or removing quantum operations within the quantum circuit. Entanglement witnesses have potential [24]–[26], but our investigations of GHZ and linear cluster states suggest they may be too robust and not sufficiently monotonic in the neighborhood of optimums. The graph edit metric, which is a metric based on the graph representation of a quantum state, holds potential as it is fundamentally different from other metrics. There are however challenges as it is not obvious

how fundamental graph operations, such as local complementations, would be represented in this metric. Most metrics investigated thus proved to not be useful towards search/optimization schemes or well suited for multipartite states.

3.2.5. Searching for performant quantum circuits

Using the random and evolutionary schemes within the density matrix representation, we were able to:

- Reproduce results from [13] for 4-photon linear cluster states.
- Find alternative, but equivalent circuits. Given some circuits may be more sensitive to noise compared to others or that some gates may be more difficult to implement than others, this gives options to experimentalists in implementing whichever protocol is more practical.
- Find cases where the circuit depth is reduced relative to circuits produced by Ref. [13]'s approach. An example of this depth decrease is shown in Fig. 3. Decreases in circuit depth imply a simplified protocol as there are fewer gates to be implemented or more gates can be implemented in parallel.
- In terms of convergence, we found that the random and evolutionary schemes performed similarly. Figure 4 shows the fraction of search runs that converge on a quantum circuit producing the target graph state.

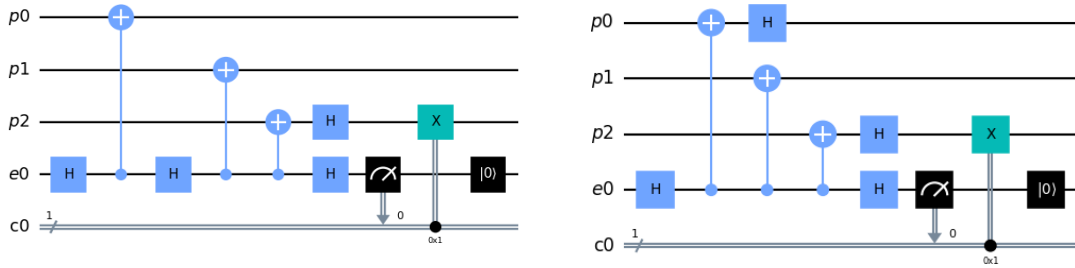


Figure 3: Quantum circuits for the generation of a 3-photon linear cluster state as determined by (left) the deterministic methods of [13] and (right) our framework. Our solution presents a reduced circuit depth relative to the deterministic method.

Using the stabilizer representation, we are able to efficiently simulate large graph states, up to hundreds of qubits, but it remains difficult to converge to an optimal circuit due to the vast optimization landscape. For this reason, we have also investigated the hybrid approach, which leverages the deterministic scheme of Ref. [13] to produce a starting point for random/evolutionary-based search approaches. Using this method, we have been able to find alternative quantum circuits to those proposed by the deterministic approach, as well as access larger graph states.

Additionally, the deterministic solutions do not include the effects of noise, which degrade the quality of the output graph state for a given circuit. Using our hybrid scheme, we have begun investigations into re-optimizing the deterministic solution after the introduction of noise, to regain state fidelity. This investigation is expected to greatly benefit from experimental feedback to more accurately represent noise found within real photonic platforms.



	Linear Cluster (3 qubits)	GHZ (3 qubits)	Linear Cluster (4 qubits)	GHZ (4 qubits)
Random scheme	92%	60%	24%	24%
Evolutionary scheme	96%	16%	24%	28%

Figure 4: Fraction of simulations that converge on a solution for a quantum circuit using a density matrix representation. Target graph states for each case are indicated above columns. Both the random and evolutionary schemes have difficulty converging with an increasing number of qubits. This is due to the vastness of the optimization landscape, indicating the need for better metrics and/or optimization schemes.

Within the density matrix representation, we have also implemented parameterized quantum gates and the automatic differentiation of quantum circuits. This enables the gradient-based optimization of circuit evaluation metrics (using, e.g., gradient-descent optimizers). This aspect of our framework enables quantum circuits to be optimized in the presence of noise, and enables gradient descent to be combined with circuit topology optimization schemes. An example of this scheme in action is shown in Fig. 5, where we perform quantum gate optimization to recover fidelity lost due to noise. Although the noise model requires refinement to accurately represent experimentally-found errors, these are promising results for managing noise-related deterioration of graph states.

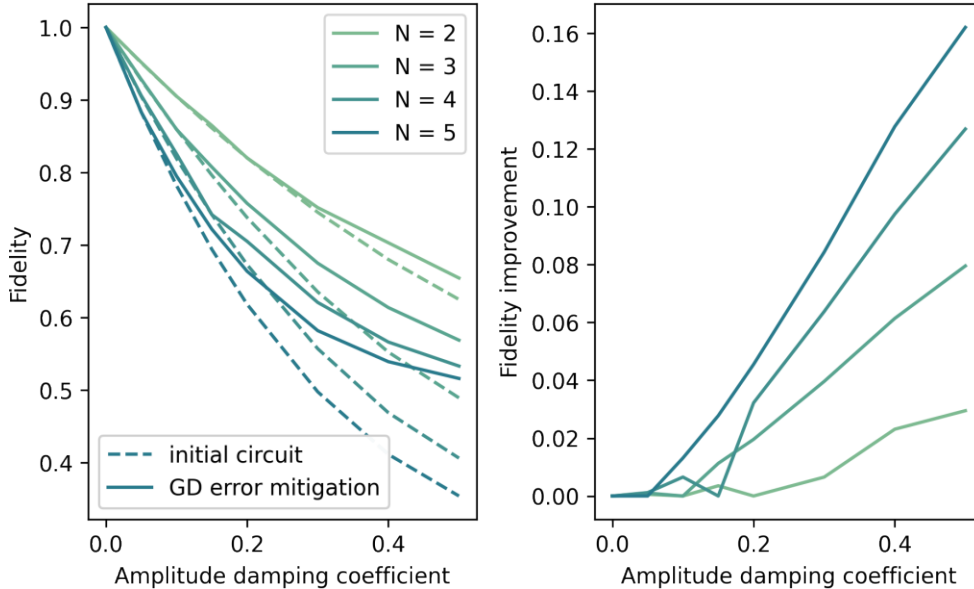


Figure 5: Regaining lost fidelity by parametrized gates optimization for linear cluster states consisting of $N=1-5$ photons as function of noise amplitude, where an amplitude damping model was uniformly applied on all qubits. Left panel shows the fidelity of the initial circuit after introducing noise (dashed) and the fidelity after gate optimization (solid). Right panel shows the relative difference between the solid and dashed lines. We find that gate optimization led to absolute fidelity recoveries of up to 15%.

3.2.5. Summary of accomplishments

During year 1 of this project, we have accomplished the following basic research milestones:

- Constructed a theoretical and numerical framework for the study of graph states and of the quantum circuits that generate them within photonics-based platforms.
- Reproduced known quantum circuits for the generation of graph states found within recent literature. In some cases, such as a 3-photon GHZ state, we have found alternative, yet equivalent quantum circuits, providing experimentalists with setup variety.
- For a 3-photon linear cluster state, we have found an improved quantum circuit with a shorter depth relative to published setups.
- Our framework supports diverse noise models, including depolarizing noise, amplitude damping, and Pauli errors.
- We have developed an experimentally-motivated, time-dependent noise model leveraging the stabilizer representation. This model reduces the number of potential outcomes within mixed states and expresses the noise in terms of circuit time-scales.
- Demonstrated that the use of parameterized gates in conjunction with gradient-based optimization can remediate the impacts of noise-driven fidelity degradation, with an absolute fidelity recovery of up to 15% for 5-photon linear cluster states.
- Developed a novel way of representing mixed states using stabilizers, allowing the simulation of more realistic noise effects and circuit dynamics.
- Developed a novel method for generating the graph representation of a quantum state from a density matrix.
- We have developed random and evolutionary schemes that search for well-performing quantum circuits given a target output state.
- We have developed a hybrid quantum circuit search scheme by combining deterministic and stochastic methods.
- Our framework is modular, and can easily be extended to other physical platforms and noise models.

3.2.6. Dissemination to the communities of interest.

Nothing to report. Nevertheless, manuscripts are in preparation and will be shared with the program officer.

4. Impacts

4.1. Development of the principal discipline of the project

We have developed a framework for the study and optimization of graph state generation in the presence of realistic effects such as noise and photonic loss, making it the first of its kind. Such a tool is a driver for further basic research into entangled resource preparation and distribution for various applications.

The framework is implemented in Python and was designed to be extensible, with community co-development in mind. To support the community and future partners in using these tools, we have prepared thorough documentation, code examples, and interactive tutorials.

Our long-term plans are discussed in detail in Sec. 6, but our near-term plan for disseminating our work to the community includes the following:

- Make our framework available to the community by open-sourcing our codebase

- alongside documentation, use examples, and tutorials
- Use the open-sourced code to enhance and establish research partnerships
- Prepare abstracts for conferences such as the International Conference on Quantum Communication, Measurement and Computing (QCMC), Q2B22 Silicon Valley, Photonics West, and CLEO
- Prepare a manuscript for peer-review and publication

4.2. Impact on other disciplines

In addition to the dissemination of our framework, we also intend to collaborate directly with colleagues from the community. This includes collaborating with the National Research Council of Canada (NRC), specialized in the fabrication of quantum dots that can be used as emitter qubits, l’Institut National de la Recherche Scientifique (INRS), giving us access to knowledge on integrated photonics, as well as Caltech and Fermilab, who can provide realistic entanglement distribution channel metrics from their respective testbeds. Additionally, Quantum Bridge Technologies, Inc. (QBT) is a University of Toronto spin-off, with many local research collaboration opportunities. Our collaborations will allow us to test and validate our framework, as well as improve its level of realism.

5. Future goals

As part of future work, our team intends to maintain and develop our framework alongside the open-source quantum community to find solutions for higher complexity graph states. In parallel, our work internally will focus on extending the framework with more numerous and realistic components as well as qubit protection strategies. To perform this work, we hope for continued support of the project in the years to come and provide below a Gantt chart presenting our future project goals for the following year. Indicated within each task are assigned lead researchers, with a team overview following the chart.

Task description	Months											
	M1-M4				M4-M8				M8-M12			
Modeling fusion gate components												
Using fusion gates [27], we can form large graph states from smaller graph states. We will implement such gates within our framework, giving an alternative route to realizing graph states. This task will be led by PDF1 and supported by PDF2 and PhD1.												
Literature review and implementing fusion gates within framework												
Investigate fusion gate-based quantum circuits												
Encoding for error correction and fusion gate protection												
We will explore encoding methods for protecting qubits against random errors (e.g., bitflip) and failed fusion gate attempts, which lead to the loss of a qubit. This task will be led by PDF1, and supported by PDF2 and PhD1.												
Investigate encoding strategies in literature												
Implement encoding for qubit protection												

Open innovation and community impact As discussed in Sec. 5, we aim to disseminate our work to support the quantum research community. Open-sourcing will be co-led by I1 and I2. Reaching out to community members will be co-led by PM1, PM2, PM3 and PM4. The writing of a manuscript will be co-led by PDF2 and PDF3 with support from the full team.															
Publish codebase and finalize public documentation and tutorials															
Prepare manuscript and submit to journal															
Project management and administrative support Project management will continue to be led by PM1, with the support of PM2, PM3 and PM4.															
Ongoing management and project reporting															

6. Team overview:

PM1: Piotr Roztocki

CTO, Ki3 Photonics Technologies Inc. Principal project manager responsible for day-to-day activities. Honors B.Sc. in Integrated Science (Physics), McMaster University 2014; Ph.D. in Energy and Materials, INRS-EMT 2022.

PM2: Hoi-Kwong Lo

CTO, Quantum Bridge Technologies, Inc. (QBT). Plays a supervisory role in the project. Professor of Physics and ECE at the University of Toronto, BA in Mathematics, Cambridge University, 1989, Ph.D. in Physics, Caltech, 1994. Co-inventor of measurement-device-independent quantum key distribution (QKD) and all photonics quantum repeaters. Fellow of the American Physical Society (APS) and Optica Fellow.

PM3: Mattia Montagna

CEO, Quantum Bridge Technologies, Inc. (QBT), PhD in Applied Math. Founder of the ECB (European Central Bank) Bank Interconnected Coordination Group (BICG). Has been leading QBT since its founding in 2019.

PM4: Yoann Jestin

CEO, Ki3 Photonics Technologies Inc. Principal Investigator of the project. B.Sc in Chemistry, University of Rennes, 1997; M.Sc. in Material Science, University of Rennes, 1999; Ph.D in Material Science, University of Le Mans, 2002.

PDF1: Jie Lin

QBT quantum research scientist with specializations in quantum optics and security analysis of quantum key distribution. Will lead development of both fusion gates and qubit encoding. B. A. Mathematics, Carleton College; M. A. S. Theoretical Physics, University of Cambridge; M. Sc. Physics and Ph.D. Physics (quantum information), University of Waterloo.

PDF2: Luc Robichaud

QBT quantum research scientist who specializes in quantum dot modeling and their application to entanglement generation. Will lead modeling of various sources and incorporating said models into the framework. Will also co-lead with PDF3 the writing of papers for peer-reviewed journals. B.Sc. Physics and M.Sc. Physics, Université de Moncton;

Ph.D. Physics, University of Ottawa.

PDF3: Stefania Sciara

Postdoc at l'Institut National de la Recherche Scientifique (INRS) and Ki3 researcher with specializations in both experimental and theoretical quantum optics, and entanglement generation. B.Sc. Physics and M.Sc. Physics, University of Palermo; Ph.D. Quantum optics, INRS.

PhD1: Sobhan Ghanbari

QBT intern. Ph.D. candidate at the University of Toronto with specialization in quantum information. Sobhan has led and will continue to lead development of various noise models within the framework. B.Sc. Physics/ B.Sc. Mechanical Engineering (dual- degree), Sharif University of Technology.

PhD2: Benjamin MacLellan

Ph.D. candidate at the University of Waterloo with specialization in quantum information and inverse design of photonic systems. Will lead the development of a more comprehensive physical layer, including various integrated-photonics-based components. B.Sc. Physics, University of Waterloo; M.Sc., Energy & Materials, INRS.

M1: Kai Sum Chan

QBT quantum research scientist. M.Phil. candidate at the University of Hong Kong, and QBT experimental and theoretical quantum researcher with specialization in quantum information, adaptive optics and quantum dots. Will play a supporting role in calibrating models according to experiments.

I1: Khuong Tran

B.Sc. candidate in computer sciences at the University of Ontario Institute of Technology and intern at Ki3. Will co-lead with I2 preparation of open-sourcing, framework documentation and tutorials.

I2: Aayush Soni

B.A.Sc. candidate in mechatronics engineering at the University of Waterloo and intern at Ki3. Will co-lead with I1 the preparation of open-sourcing, framework documentation and tutorials.

Key: PM: Project manager, PDF: Postdoctoral fellow, PhD: PhD student, M: Master's student, I: Intern.

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Highly Entangled States of Light

Final Report

Awarded to:
Ki3 Photonics Technologies Inc.
Quantum Bridge Technologies Inc (QBT).

Recipient Principal Investigator:
Dr. Yoann Jestin

1. Executive summary

Problem statement:

- Graph quantum states are an important resource for fundamental research and technological applications
- Designing quantum circuits that output specific and desired graph states is challenging
- Real quantum circuits are subject to noise and loss, which are necessary to include for an accurate and realistic model

Project scope:

- Construct a theoretical and numerical framework for the study of graph states and of circuits that generate them
- Develop schemes for the construction of quantum circuits that output desired graph states
- Create models for the noise and loss effects that are present in real quantum setups

General outcome:

- To date, we have defined a framework that allows us to find and to simulate quantum circuits that output low-complexity graph states
- Our schemes successfully reproduce results from literature and are also capable of finding alternative setups to choose between
- We have developed noise and loss models that are representative of effects found in common photonics platforms
- Our framework is a flexible and extensible tool that is soon to be open-sourced and which can be easily adapted to simulate various physical platforms

Future Plans:

- Maintain and develop the framework alongside the open-source quantum community to find solutions for higher complexity graph states
- Develop fusion-gate based protocols for producing large graph states
- Implement encoding strategies for qubit protection
- Model quantum-dot-based sources and interface with current framework
- Develop models for integrated-photonics-based components as an experimental basis for quantum circuits
- Develop more sophisticated noise and distribution channel models
- Extend our ability to inject experimental details into our framework, further connecting results with real setups

2. Introduction

Graph states are a family of quantum states with high levels of entanglement that are of great fundamental interest [1], as well as being a key resource for applications in entanglement distribution and quantum networking [2-4], quantum error correction [5,6], and quantum metrology [7]. Among various physical platforms, photonics shows much promise for graph state realizations [8,9] as photons interact very weakly with their environment, enabling stable qubits. Additionally, photons are compatible with mature integrated and fiber technologies, granting additional support in the generation and distribution of graph states.

As well, for the generation of photons, various sources are available within photonics-based platforms, including solid-state emitters such as quantum dots that can emit single photons on demand [10], or nonlinear elements generating pairs of entangled photons [11,12].

Quantum circuits describe the sequence of quantum operations necessary to generate such graph states. However, connecting a desired graph state with the circuit that can output it is a non-trivial design task that only becomes more difficult with increasing state size and complexity. Recent work [13] has led to the development of an impressive deterministic method for finding quantum circuits that produce targeted graph states. However, such methods do not guarantee well-performing quantum circuits, allow little flexibility, and cannot consider impacts associated with noise or photonic loss, which is present in all real setups.

Towards addressing this research gap, the goal of our work was to develop a flexible theoretical and computational framework for the study of realistic graph states and of the quantum circuits that generate them. To accomplish this, we must be able to represent both the quantum states and the quantum circuits that produce them. Both representations must also be flexible enough to support noise and photonic loss modeling. The performance of different circuits must also be quantifiable, in order to drive schemes that search for best performing setups. All of these concepts must also be realized in a flexible and modular way that allows for co-development of solutions with the broader research community, driving collaborative and fundamental work into the understanding of entangled states as a resource.

Over the past year of our project's progress, we have successfully identified and realized a framework comprising the following core concepts: *Quantum state representations*, *quantum circuit representations*, *realistic component models*, *quantum circuit performance metrics*, and *quantum circuit optimization schemes*. Figure 1 shows these key concepts and their mutual interaction within our framework.

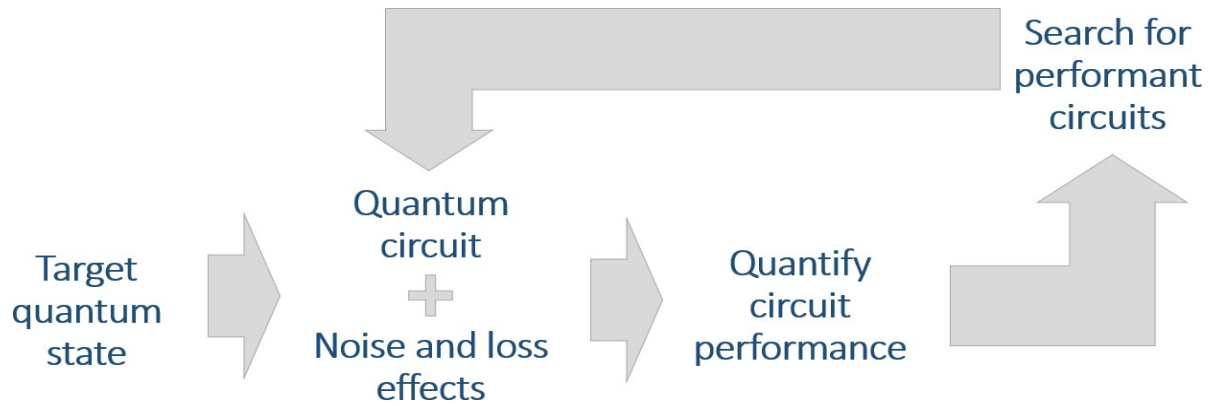


Figure 1: Key concepts within our framework. The performance metric is calculated with respect to the target graph state, guiding the search for performant circuits.

3. Accomplishments

In what follows, we explain each of the core concepts of our framework alongside our respective research efforts in developing them.

3.1. Research objectives

3.3.1. Representing quantum states

The objects of interest, graph states, are quantum systems of photonic and emitter qubits that can be mathematically represented using various formalisms. Our computational framework presently considers two mathematical representations, the *density matrix* and the *stabilizer*, with the modularity to easily include further representations in the future.

3.1.2. Representing quantum circuits

Quantum circuits represent the sequence of quantum gates and operations applied on solid state emitter qubits and photonic qubits. To computationally read, save, and manipulate these quantum circuits, we represent them as Directed Acyclic Graphs (DAGs) [19], which find common use in the field of quantum information [20]. Using the DAG representation, our computational framework can easily modify and evolve the quantum circuit topology to optimize the generated quantum state towards a target graph state. For the visualization of quantum circuits, our DAG representation was also made compatible with the openQASM format, a standard within the field [21].

3.1.3. Realistic component models

Towards increasing the realism of our framework, we support diverse models for noise and photonic loss. For example, our framework presently includes noise models for depolarizing noise, amplitude damping, and Pauli noise.

Additionally, we have developed a new representation to model mixed states using stabilizers. This new representation keeps track of all potential branchings of a mixed state, along with the probability of such a branch being measured. This novel approach allows us to consider probabilistic noise, more akin to real experimental systems.

Motivated by the physics of today's solid-state emitter and photonic qubit systems, we have furthermore developed a time-dependent noise model within the stabilizer formalism. In experiments, the number of errors that occur is strongly correlated with the time spent during and between each quantum gate. In our time-dependent model, we make use of an effective noise (simulated only at certain gates), maintaining model accuracy while reducing the computational load of the model.

3.1.4. Assessing circuit performance

When optimizing a quantum circuit for the generation of a target graph state, it is important to properly quantify the performance of said circuit using quantitative metrics. In fact, having proper metrics can heavily influence the performance of a circuit search algorithm. To date, we have investigated the following metrics as potential candidates [14], [22], [23]:

Fidelity	Relative entropy
Entanglement witness	Squashed entanglement
Graph edit distance	Adjacency matrix fidelity
Concurrence	Geometric measure
Negativity	Fisher information
Trace distance	Mermin inequality
Entanglement distillation	Stabilizer tableau fidelity

3.1.5. Searching for performant quantum circuits

We have investigated the following schemes for converging on suitable quantum circuits that output a desired graph state:

- **Random:** This scheme initializes a list of random circuits subject to physical

constraints, which we call a population. At each iteration and for each circuit within the population, the algorithm applies a random operation, which includes addition, removal, or replacement of a quantum gate. Each circuit in the population is modified independently from the other circuits. The scheme then keeps track of a certain number of top performing circuits.

- **Evolutionary:** Similarly to the random search approach, this scheme also applies random operations in each iteration. However, in contrast, after each iteration it constructs a new population by putting circuits into a N-wise competition and selecting the best performers.
- **Deterministic:** This scheme is our implementation of Ref. [13]’s approach to deterministically find a quantum circuit for a target graph state.
- **Hybrid:** In this scheme, we use the deterministic method from Ref. [13] to produce an initial solution. We then perturb this solution and use the random/evolutionary schemes to search for quantum circuit alternatives.
- **Gradient-based:** This is a density matrix specific scheme where we consider tunable/parametrized gates that are compatible with automatic differentiation and gradient descent optimization techniques.

3.2 Details of accomplishments

3.2.1. Representing quantum states

The *density matrix* representation can simulate a state’s full Hilbert space and can inherently represent mixed states, such as those resulting from the effects of noise and photonic loss [14]. However, this representation does not scale well with the number of qubits, making it impractical to investigate large graph states, and thus motivating the need for complementary representations. We have used this representation: 1) To study small graph states with the inclusion of noise effects; 2) As a benchmark for other representations; and 3) For the study of noise, photonic loss, mixed states, and gate parametrization.

The *Stabilizer* representation [15], [16], on the other hand, scales well with the number of qubits, making it more efficient for the study of large graph states (relative to the density matrix formalism). However, only a subset of quantum operations can be efficiently simulated within the stabilizer representation, limiting the scope for circuit optimization, and making this representation unsuited for the simulation of noise without judicious changes to the formalism. Presently, the study of representing mixed states within the stabilizer representation is very limited in the literature. To be able to consider large graph states that are mixed due to noise or loss, we have developed a probability tree approach as a way to represent such states. This point of research is presented further below in Sec. 3.3.

We have also developed relevant concepts from *graph* representations, which are also well-suited for many qubits, and contrary to the stabilizer representation, there is more progress made on describing mixed states [17,18]. In particular, we have worked on leveraging graph state equivalency through Clifford gates. As discussed in Sec. 3.4, the graph representation also offers unique metrics for characterizing the performance of quantum circuits.

Each of the quantum state representations has advantages and disadvantages, making it useful to be able to move between them. Most of these representation transformations can be found within literature [3], [14-18], except for obtaining a graph from a density matrix. We have resolved this issue by designing a novel protocol, as discussed in Sec. 3.5.

3.2.2. Representing quantum circuits

Although we have focused on solid state emitter qubits and photonic qubits, we have designed our framework with extensibility in mind. The types of operations, and therefore the

physical circuits we can represent and simulate, are flexible, meaning we can adapt our framework towards different physical platforms with relative ease.

Lastly, our quantum circuit representation also includes parameterized quantum gates and automatic differentiation (see Sec. 3.5 for further discussion).

3.2.3. Realistic component models

The Figure 2 shows an example of the model in practice, deriving the fidelity of an output graph state as a function of qubit error rate from various sources of noise.

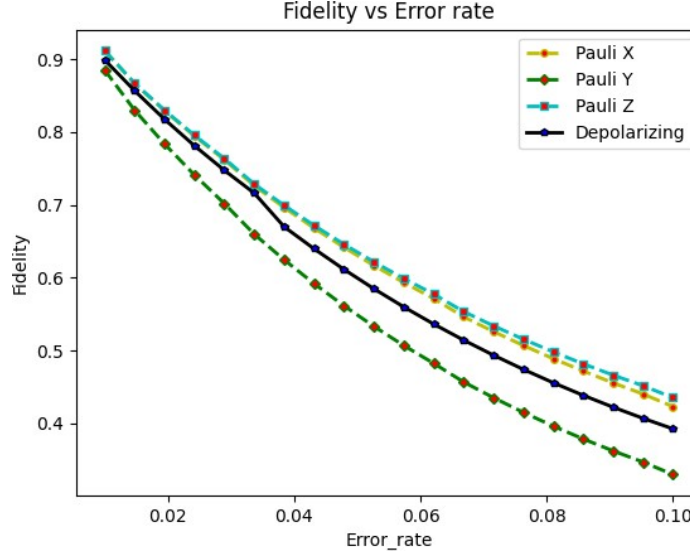


Figure 2: Fidelity versus error rate for the generation of a 4-photon linear cluster state assuming X -gate errors (yellow), Y -gate errors (green), Z -gate errors (blue) and a depolarizing channel (black). We find that the generation of linear cluster states is equally sensitive to bit-flip (X -gate) and phase-flip (Z -gate) errors, and more sensitive when both errors occur (Y -gate).

As photonic losses have non-trivial impacts on quantum states, generally producing mixed states, we have explored various approaches to simulate their effects. One approach is to extend the standard qubit space to a qutrit space so that, by accounting for a vacuum term, the state can still be represented as pure. Another approach is to keep track of the survival probability of a state after potential photonic loss events. Interestingly, we observe that the state fidelity calculation only depends on branches that see no loss, weighted by their respective probabilities. By tracking transmission probabilities, we gain information useful for approximating state generation rates, which are a very important experimental insight. We also calculate the survival probability for each photon, which indicates which portions of the graph state are more prone to loss. This information is essential for efficiently implementing encoding for error correction.

3.2.4. Assessing circuit performance

Fidelity has been our main metric thus far, but it is not ideal for our task as it can change abruptly, which is partially due to the discrete nature of adding or removing quantum operations within the quantum circuit. Entanglement witnesses have potential [24]–[26], but our investigations of GHZ and linear cluster states suggest they may be too robust and not sufficiently monotonic in the neighborhood of optimums. The graph edit metric, which is a metric based on the graph representation of a quantum state, holds potential as it is fundamentally different from other metrics. There are however challenges as it is not obvious

how fundamental graph operations, such as local complementations, would be represented in this metric. Most metrics investigated thus proved to not be useful towards search/optimization schemes or well suited for multipartite states.

3.2.5. Searching for performant quantum circuits

Using the random and evolutionary schemes within the density matrix representation, we were able to:

- Reproduce results from [13] for 4-photon linear cluster states.
- Find alternative, but equivalent circuits. Given some circuits may be more sensitive to noise compared to others or that some gates may be more difficult to implement than others, this gives options to experimentalists in implementing whichever protocol is more practical.
- Find cases where the circuit depth is reduced relative to circuits produced by Ref. [13]'s approach. An example of this depth decrease is shown in Fig. 3. Decreases in circuit depth imply a simplified protocol as there are fewer gates to be implemented or more gates can be implemented in parallel.
- In terms of convergence, we found that the random and evolutionary schemes performed similarly. Figure 4 shows the fraction of search runs that converge on a quantum circuit producing the target graph state.

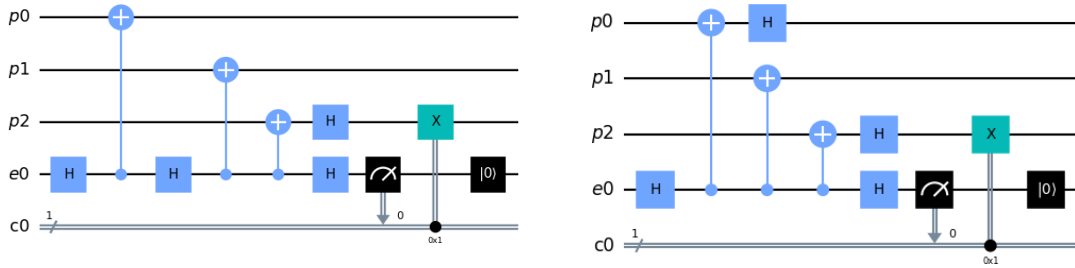


Figure 3: Quantum circuits for the generation of a 3-photon linear cluster state as determined by (left) the deterministic methods of [13] and (right) our framework. Our solution presents a reduced circuit depth relative to the deterministic method.

Using the stabilizer representation, we are able to efficiently simulate large graph states, up to hundreds of qubits, but it remains difficult to converge to an optimal circuit due to the vast optimization landscape. For this reason, we have also investigated the hybrid approach, which leverages the deterministic scheme of Ref. [13] to produce a starting point for random/evolutionary-based search approaches. Using this method, we have been able to find alternative quantum circuits to those proposed by the deterministic approach, as well as access larger graph states.

Additionally, the deterministic solutions do not include the effects of noise, which degrade the quality of the output graph state for a given circuit. Using our hybrid scheme, we have begun investigations into re-optimizing the deterministic solution after the introduction of noise, to regain state fidelity. This investigation is expected to greatly benefit from experimental feedback to more accurately represent noise found within real photonic platforms.



	Linear Cluster (3 qubits)	GHZ (3 qubits)	Linear Cluster (4 qubits)	GHZ (4 qubits)
Random scheme	92%	60%	24%	24%
Evolutionary scheme	96%	16%	24%	28%

Figure 4: Fraction of simulations that converge on a solution for a quantum circuit using a density matrix representation. Target graph states for each case are indicated above columns. Both the random and evolutionary schemes have difficulty converging with an increasing number of qubits. This is due to the vastness of the optimization landscape, indicating the need for better metrics and/or optimization schemes.

Within the density matrix representation, we have also implemented parameterized quantum gates and the automatic differentiation of quantum circuits. This enables the gradient-based optimization of circuit evaluation metrics (using, e.g., gradient-descent optimizers). This aspect of our framework enables quantum circuits to be optimized in the presence of noise, and enables gradient descent to be combined with circuit topology optimization schemes. An example of this scheme in action is shown in Fig. 5, where we perform quantum gate optimization to recover fidelity lost due to noise. Although the noise model requires refinement to accurately represent experimentally-found errors, these are promising results for managing noise-related deterioration of graph states.

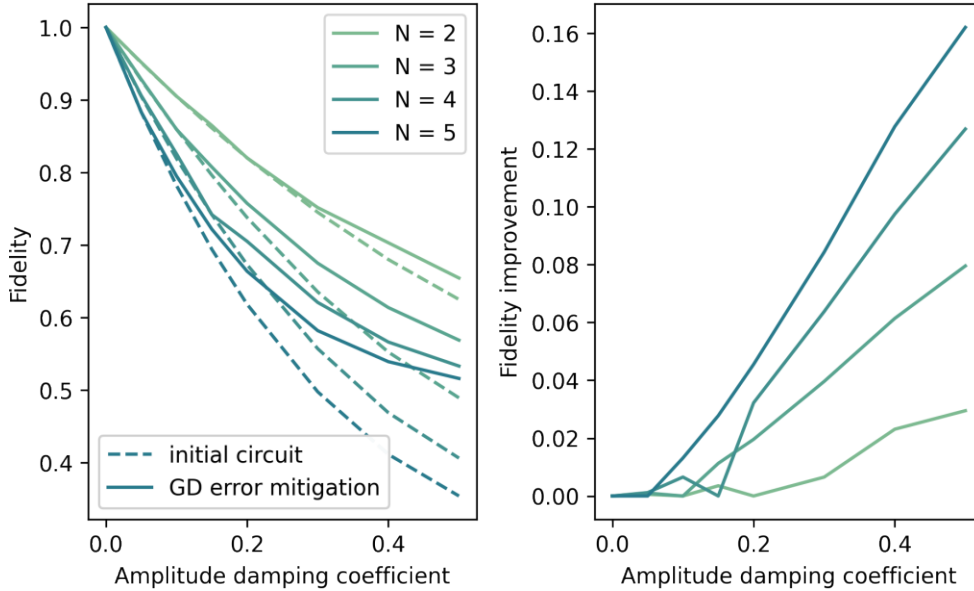


Figure 5: Regaining lost fidelity by parametrized gates optimization for linear cluster states consisting of $N=1-5$ photons as function of noise amplitude, where an amplitude damping model was uniformly applied on all qubits. Left panel shows the fidelity of the initial circuit after introducing noise (dashed) and the fidelity after gate optimization (solid). Right panel shows the relative difference between the solid and dashed lines. We find that gate optimization led to absolute fidelity recoveries of up to 15%.

3.2.5. Summary of accomplishments

During year 1 of this project, we have accomplished the following basic research milestones:

- Constructed a theoretical and numerical framework for the study of graph states and of the quantum circuits that generate them within photonics-based platforms.
- Reproduced known quantum circuits for the generation of graph states found within recent literature. In some cases, such as a 3-photon GHZ state, we have found alternative, yet equivalent quantum circuits, providing experimentalists with setup variety.
- For a 3-photon linear cluster state, we have found an improved quantum circuit with a shorter depth relative to published setups.
- Our framework supports diverse noise models, including depolarizing noise, amplitude damping, and Pauli errors.
- We have developed an experimentally-motivated, time-dependent noise model leveraging the stabilizer representation. This model reduces the number of potential outcomes within mixed states and expresses the noise in terms of circuit time-scales.
- Demonstrated that the use of parameterized gates in conjunction with gradient-based optimization can remediate the impacts of noise-driven fidelity degradation, with an absolute fidelity recovery of up to 15% for 5-photon linear cluster states.
- Developed a novel way of representing mixed states using stabilizers, allowing the simulation of more realistic noise effects and circuit dynamics.
- Developed a novel method for generating the graph representation of a quantum state from a density matrix.
- We have developed random and evolutionary schemes that search for well-performing quantum circuits given a target output state.
- We have developed a hybrid quantum circuit search scheme by combining deterministic and stochastic methods.
- Our framework is modular, and can easily be extended to other physical platforms and noise models.

3.2.6. Dissemination to the communities of interest.

Nothing to report. Nevertheless, manuscripts are in preparation and will be shared with the program officer.

4. Impacts

4.1. Development of the principal discipline of the project

We have developed a framework for the study and optimization of graph state generation in the presence of realistic effects such as noise and photonic loss, making it the first of its kind. Such a tool is a driver for further basic research into entangled resource preparation and distribution for various applications.

The framework is implemented in Python and was designed to be extensible, with community co-development in mind. To support the community and future partners in using these tools, we have prepared thorough documentation, code examples, and interactive tutorials.

Our long-term plans are discussed in detail in Sec. 6, but our near-term plan for disseminating our work to the community includes the following:

- Make our framework available to the community by open-sourcing our codebase

- alongside documentation, use examples, and tutorials
- Use the open-sourced code to enhance and establish research partnerships
- Prepare abstracts for conferences such as the International Conference on Quantum Communication, Measurement and Computing (QCMC), Q2B22 Silicon Valley, Photonics West, and CLEO
- Prepare a manuscript for peer-review and publication

4.2. Impact on other disciplines

In addition to the dissemination of our framework, we also intend to collaborate directly with colleagues from the community. This includes collaborating with the National Research Council of Canada (NRC), specialized in the fabrication of quantum dots that can be used as emitter qubits, l’Institut National de la Recherche Scientifique (INRS), giving us access to knowledge on integrated photonics, as well as Caltech and Fermilab, who can provide realistic entanglement distribution channel metrics from their respective testbeds. Additionally, Quantum Bridge Technologies, Inc. (QBT) is a University of Toronto spin-off, with many local research collaboration opportunities. Our collaborations will allow us to test and validate our framework, as well as improve its level of realism.

5. Future goals

As part of future work, our team intends to maintain and develop our framework alongside the open-source quantum community to find solutions for higher complexity graph states. In parallel, our work internally will focus on extending the framework with more numerous and realistic components as well as qubit protection strategies. To perform this work, we hope for continued support of the project in the years to come and provide below a Gantt chart presenting our future project goals for the following year. Indicated within each task are assigned lead researchers, with a team overview following the chart.

Task description	Months											
	M1-M4				M4-M8				M8-M12			
Modeling fusion gate components												
Using fusion gates [27], we can form large graph states from smaller graph states. We will implement such gates within our framework, giving an alternative route to realizing graph states. This task will be led by PDF1 and supported by PDF2 and PhD1.												
Literature review and implementing fusion gates within framework												
Investigate fusion gate-based quantum circuits												
Encoding for error correction and fusion gate protection												
We will explore encoding methods for protecting qubits against random errors (e.g., bitflip) and failed fusion gate attempts, which lead to the loss of a qubit. This task will be led by PDF1, and supported by PDF2 and PhD1.												
Investigate encoding strategies in literature												
Implement encoding for qubit protection												

Open innovation and community impact As discussed in Sec. 5, we aim to disseminate our work to support the quantum research community. Open-sourcing will be co-led by I1 and I2. Reaching out to community members will be co-led by PM1, PM2, PM3 and PM4. The writing of a manuscript will be co-led by PDF2 and PDF3 with support from the full team.															
Publish codebase and finalize public documentation and tutorials															
Prepare manuscript and submit to journal															
Project management and administrative support Project management will continue to be led by PM1, with the support of PM2, PM3 and PM4.															
Ongoing management and project reporting															

6. Team overview:

PM1: Piotr Roztocki

CTO, Ki3 Photonics Technologies Inc. Principal project manager responsible for day-to-day activities. Honors B.Sc. in Integrated Science (Physics), McMaster University 2014; Ph.D. in Energy and Materials, INRS-EMT 2022.

PM2: Hoi-Kwong Lo

CTO, Quantum Bridge Technologies, Inc. (QBT). Plays a supervisory role in the project. Professor of Physics and ECE at the University of Toronto, BA in Mathematics, Cambridge University, 1989, Ph.D. in Physics, Caltech, 1994. Co-inventor of measurement-device-independent quantum key distribution (QKD) and all photonics quantum repeaters. Fellow of the American Physical Society (APS) and Optica Fellow.

PM3: Mattia Montagna

CEO, Quantum Bridge Technologies, Inc. (QBT), PhD in Applied Math. Founder of the ECB (European Central Bank) Bank Interconnected Coordination Group (BICG). Has been leading QBT since its founding in 2019.

PM4: Yoann Jestin

CEO, Ki3 Photonics Technologies Inc. Principal Investigator of the project. B.Sc in Chemistry, University of Rennes, 1997; M.Sc. in Material Science, University of Rennes, 1999; Ph.D in Material Science, University of Le Mans, 2002.

PDF1: Jie Lin

QBT quantum research scientist with specializations in quantum optics and security analysis of quantum key distribution. Will lead development of both fusion gates and qubit encoding. B. A. Mathematics, Carleton College; M. A. S. Theoretical Physics, University of Cambridge; M. Sc. Physics and Ph.D. Physics (quantum information), University of Waterloo.

PDF2: Luc Robichaud

QBT quantum research scientist who specializes in quantum dot modeling and their application to entanglement generation. Will lead modeling of various sources and incorporating said models into the framework. Will also co-lead with PDF3 the writing of papers for peer-reviewed journals. B.Sc. Physics and M.Sc. Physics, Université de Moncton;

Ph.D. Physics, University of Ottawa.

PDF3: Stefania Sciara

Postdoc at l'Institut National de la Recherche Scientifique (INRS) and Ki3 researcher with specializations in both experimental and theoretical quantum optics, and entanglement generation. B.Sc. Physics and M.Sc. Physics, University of Palermo; Ph.D. Quantum optics, INRS.

PhD1: Sobhan Ghanbari

QBT intern. Ph.D. candidate at the University of Toronto with specialization in quantum information. Sobhan has led and will continue to lead development of various noise models within the framework. B.Sc. Physics/ B.Sc. Mechanical Engineering (dual- degree), Sharif University of Technology.

PhD2: Benjamin MacLellan

Ph.D. candidate at the University of Waterloo with specialization in quantum information and inverse design of photonic systems. Will lead the development of a more comprehensive physical layer, including various integrated-photonics-based components. B.Sc. Physics, University of Waterloo; M.Sc., Energy & Materials, INRS.

M1: Kai Sum Chan

QBT quantum research scientist. M.Phil. candidate at the University of Hong Kong, and QBT experimental and theoretical quantum researcher with specialization in quantum information, adaptive optics and quantum dots. Will play a supporting role in calibrating models according to experiments.

I1: Khuong Tran

B.Sc. candidate in computer sciences at the University of Ontario Institute of Technology and intern at Ki3. Will co-lead with I2 preparation of open-sourcing, framework documentation and tutorials.

I2: Aayush Soni

B.A.Sc. candidate in mechatronics engineering at the University of Waterloo and intern at Ki3. Will co-lead with I1 the preparation of open-sourcing, framework documentation and tutorials.

Key: PM: Project manager, PDF: Postdoctoral fellow, PhD: PhD student, M: Master's student, I: Intern.

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Highly Entangled States of Light

Final Report

Awarded to:
Ki3 Photonics Technologies Inc.
Quantum Bridge Technologies Inc (QBT).

Recipient Principal Investigator:
Dr. Yoann Jestin

1. Executive summary

Problem statement:

- Graph quantum states are an important resource for fundamental research and technological applications
- Designing quantum circuits that output specific and desired graph states is challenging
- Real quantum circuits are subject to noise and loss, which are necessary to include for an accurate and realistic model

Project scope:

- Construct a theoretical and numerical framework for the study of graph states and of circuits that generate them
- Develop schemes for the construction of quantum circuits that output desired graph states
- Create models for the noise and loss effects that are present in real quantum setups

General outcome:

- To date, we have defined a framework that allows us to find and to simulate quantum circuits that output low-complexity graph states
- Our schemes successfully reproduce results from literature and are also capable of finding alternative setups to choose between
- We have developed noise and loss models that are representative of effects found in common photonics platforms
- Our framework is a flexible and extensible tool that is soon to be open-sourced and which can be easily adapted to simulate various physical platforms

Future Plans:

- Maintain and develop the framework alongside the open-source quantum community to find solutions for higher complexity graph states
- Develop fusion-gate based protocols for producing large graph states
- Implement encoding strategies for qubit protection
- Model quantum-dot-based sources and interface with current framework
- Develop models for integrated-photonics-based components as an experimental basis for quantum circuits
- Develop more sophisticated noise and distribution channel models
- Extend our ability to inject experimental details into our framework, further connecting results with real setups

2. Introduction

Graph states are a family of quantum states with high levels of entanglement that are of great fundamental interest [1], as well as being a key resource for applications in entanglement distribution and quantum networking [2-4], quantum error correction [5,6], and quantum metrology [7]. Among various physical platforms, photonics shows much promise for graph state realizations [8,9] as photons interact very weakly with their environment, enabling stable qubits. Additionally, photons are compatible with mature integrated and fiber technologies, granting additional support in the generation and distribution of graph states.

As well, for the generation of photons, various sources are available within photonics-based platforms, including solid-state emitters such as quantum dots that can emit single photons on demand [10], or nonlinear elements generating pairs of entangled photons [11,12].

Quantum circuits describe the sequence of quantum operations necessary to generate such graph states. However, connecting a desired graph state with the circuit that can output it is a non-trivial design task that only becomes more difficult with increasing state size and complexity. Recent work [13] has led to the development of an impressive deterministic method for finding quantum circuits that produce targeted graph states. However, such methods do not guarantee well-performing quantum circuits, allow little flexibility, and cannot consider impacts associated with noise or photonic loss, which is present in all real setups.

Towards addressing this research gap, the goal of our work was to develop a flexible theoretical and computational framework for the study of realistic graph states and of the quantum circuits that generate them. To accomplish this, we must be able to represent both the quantum states and the quantum circuits that produce them. Both representations must also be flexible enough to support noise and photonic loss modeling. The performance of different circuits must also be quantifiable, in order to drive schemes that search for best performing setups. All of these concepts must also be realized in a flexible and modular way that allows for co-development of solutions with the broader research community, driving collaborative and fundamental work into the understanding of entangled states as a resource.

Over the past year of our project's progress, we have successfully identified and realized a framework comprising the following core concepts: *Quantum state representations*, *quantum circuit representations*, *realistic component models*, *quantum circuit performance metrics*, and *quantum circuit optimization schemes*. Figure 1 shows these key concepts and their mutual interaction within our framework.

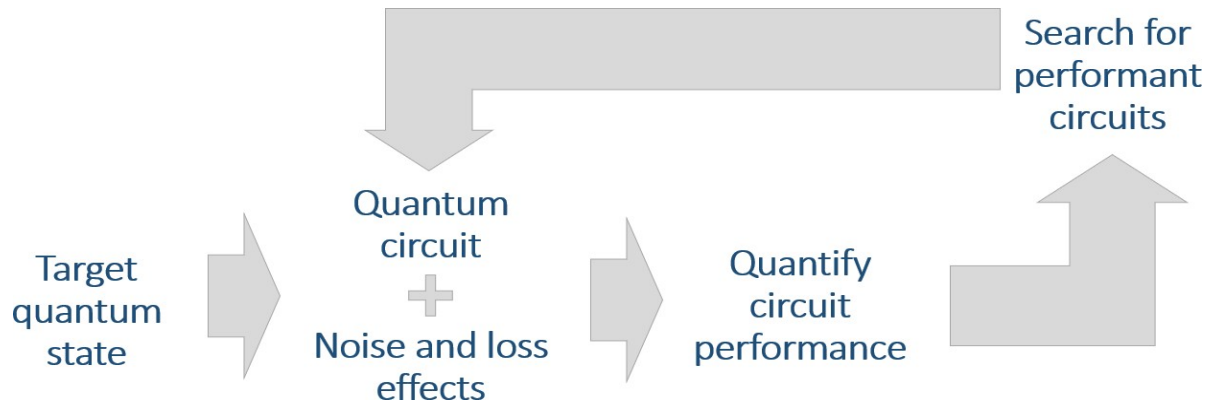


Figure 1: Key concepts within our framework. The performance metric is calculated with respect to the target graph state, guiding the search for performant circuits.

3. Accomplishments

In what follows, we explain each of the core concepts of our framework alongside our respective research efforts in developing them.

3.1. Research objectives

3.3.1. Representing quantum states

The objects of interest, graph states, are quantum systems of photonic and emitter qubits that can be mathematically represented using various formalisms. Our computational framework presently considers two mathematical representations, the *density matrix* and the *stabilizer*, with the modularity to easily include further representations in the future.

3.1.2. Representing quantum circuits

Quantum circuits represent the sequence of quantum gates and operations applied on solid state emitter qubits and photonic qubits. To computationally read, save, and manipulate these quantum circuits, we represent them as Directed Acyclic Graphs (DAGs) [19], which find common use in the field of quantum information [20]. Using the DAG representation, our computational framework can easily modify and evolve the quantum circuit topology to optimize the generated quantum state towards a target graph state. For the visualization of quantum circuits, our DAG representation was also made compatible with the openQASM format, a standard within the field [21].

3.1.3. Realistic component models

Towards increasing the realism of our framework, we support diverse models for noise and photonic loss. For example, our framework presently includes noise models for depolarizing noise, amplitude damping, and Pauli noise.

Additionally, we have developed a new representation to model mixed states using stabilizers. This new representation keeps track of all potential branchings of a mixed state, along with the probability of such a branch being measured. This novel approach allows us to consider probabilistic noise, more akin to real experimental systems.

Motivated by the physics of today's solid-state emitter and photonic qubit systems, we have furthermore developed a time-dependent noise model within the stabilizer formalism. In experiments, the number of errors that occur is strongly correlated with the time spent during and between each quantum gate. In our time-dependent model, we make use of an effective noise (simulated only at certain gates), maintaining model accuracy while reducing the computational load of the model.

3.1.4. Assessing circuit performance

When optimizing a quantum circuit for the generation of a target graph state, it is important to properly quantify the performance of said circuit using quantitative metrics. In fact, having proper metrics can heavily influence the performance of a circuit search algorithm. To date, we have investigated the following metrics as potential candidates [14], [22], [23]:

Fidelity	Relative entropy
Entanglement witness	Squashed entanglement
Graph edit distance	Adjacency matrix fidelity
Concurrence	Geometric measure
Negativity	Fisher information
Trace distance	Mermin inequality
Entanglement distillation	Stabilizer tableau fidelity

3.1.5. Searching for performant quantum circuits

We have investigated the following schemes for converging on suitable quantum circuits that output a desired graph state:

- **Random:** This scheme initializes a list of random circuits subject to physical

constraints, which we call a population. At each iteration and for each circuit within the population, the algorithm applies a random operation, which includes addition, removal, or replacement of a quantum gate. Each circuit in the population is modified independently from the other circuits. The scheme then keeps track of a certain number of top performing circuits.

- **Evolutionary:** Similarly to the random search approach, this scheme also applies random operations in each iteration. However, in contrast, after each iteration it constructs a new population by putting circuits into a N-wise competition and selecting the best performers.
- **Deterministic:** This scheme is our implementation of Ref. [13]’s approach to deterministically find a quantum circuit for a target graph state.
- **Hybrid:** In this scheme, we use the deterministic method from Ref. [13] to produce an initial solution. We then perturb this solution and use the random/evolutionary schemes to search for quantum circuit alternatives.
- **Gradient-based:** This is a density matrix specific scheme where we consider tunable/parametrized gates that are compatible with automatic differentiation and gradient descent optimization techniques.

3.2 Details of accomplishments

3.2.1. Representing quantum states

The *density matrix* representation can simulate a state’s full Hilbert space and can inherently represent mixed states, such as those resulting from the effects of noise and photonic loss [14]. However, this representation does not scale well with the number of qubits, making it impractical to investigate large graph states, and thus motivating the need for complementary representations. We have used this representation: 1) To study small graph states with the inclusion of noise effects; 2) As a benchmark for other representations; and 3) For the study of noise, photonic loss, mixed states, and gate parametrization.

The *Stabilizer* representation [15], [16], on the other hand, scales well with the number of qubits, making it more efficient for the study of large graph states (relative to the density matrix formalism). However, only a subset of quantum operations can be efficiently simulated within the stabilizer representation, limiting the scope for circuit optimization, and making this representation unsuited for the simulation of noise without judicious changes to the formalism. Presently, the study of representing mixed states within the stabilizer representation is very limited in the literature. To be able to consider large graph states that are mixed due to noise or loss, we have developed a probability tree approach as a way to represent such states. This point of research is presented further below in Sec. 3.3.

We have also developed relevant concepts from *graph* representations, which are also well-suited for many qubits, and contrary to the stabilizer representation, there is more progress made on describing mixed states [17,18]. In particular, we have worked on leveraging graph state equivalency through Clifford gates. As discussed in Sec. 3.4, the graph representation also offers unique metrics for characterizing the performance of quantum circuits.

Each of the quantum state representations has advantages and disadvantages, making it useful to be able to move between them. Most of these representation transformations can be found within literature [3], [14-18], except for obtaining a graph from a density matrix. We have resolved this issue by designing a novel protocol, as discussed in Sec. 3.5.

3.2.2. Representing quantum circuits

Although we have focused on solid state emitter qubits and photonic qubits, we have designed our framework with extensibility in mind. The types of operations, and therefore the

physical circuits we can represent and simulate, are flexible, meaning we can adapt our framework towards different physical platforms with relative ease. Lastly, our quantum circuit representation also includes parameterized quantum gates and automatic differentiation (see Sec. 3.5 for further discussion).

3.2.3. Realistic component models

The Figure 2 shows an example of the model in practice, deriving the fidelity of an output graph state as a function of qubit error rate from various sources of noise.

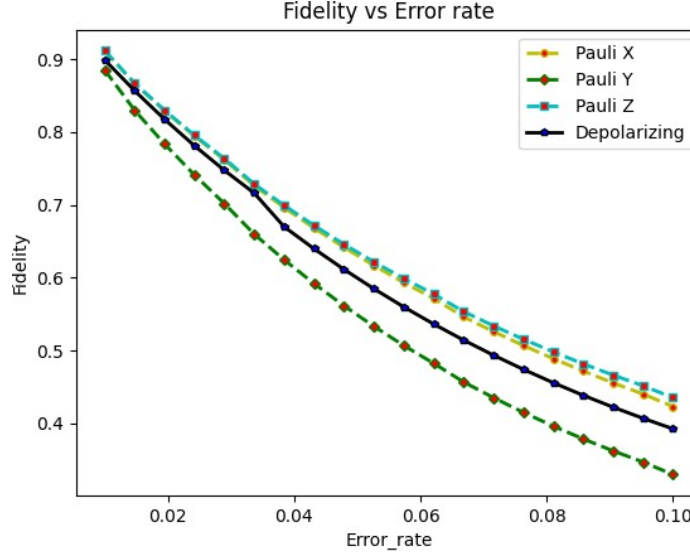


Figure 2: Fidelity versus error rate for the generation of a 4-photon linear cluster state assuming X -gate errors (yellow), Y -gate errors (green), Z -gate errors (blue) and a depolarizing channel (black). We find that the generation of linear cluster states is equally sensitive to bit-flip (X -gate) and phase-flip (Z -gate) errors, and more sensitive when both errors occur (Y -gate).

As photonic losses have non-trivial impacts on quantum states, generally producing mixed states, we have explored various approaches to simulate their effects. One approach is to extend the standard qubit space to a qutrit space so that, by accounting for a vacuum term, the state can still be represented as pure. Another approach is to keep track of the survival probability of a state after potential photonic loss events. Interestingly, we observe that the state fidelity calculation only depends on branches that see no loss, weighted by their respective probabilities. By tracking transmission probabilities, we gain information useful for approximating state generation rates, which are a very important experimental insight. We also calculate the survival probability for each photon, which indicates which portions of the graph state are more prone to loss. This information is essential for efficiently implementing encoding for error correction.

3.2.4. Assessing circuit performance

Fidelity has been our main metric thus far, but it is not ideal for our task as it can change abruptly, which is partially due to the discrete nature of adding or removing quantum operations within the quantum circuit. Entanglement witnesses have potential [24]–[26], but our investigations of GHZ and linear cluster states suggest they may be too robust and not sufficiently monotonic in the neighborhood of optimums. The graph edit metric, which is a metric based on the graph representation of a quantum state, holds potential as it is fundamentally different from other metrics. There are however challenges as it is not obvious



	Linear Cluster (3 qubits)	GHZ (3 qubits)	Linear Cluster (4 qubits)	GHZ (4 qubits)
Random scheme	92%	60%	24%	24%
Evolutionary scheme	96%	16%	24%	28%

Figure 4: Fraction of simulations that converge on a solution for a quantum circuit using a density matrix representation. Target graph states for each case are indicated above columns. Both the random and evolutionary schemes have difficulty converging with an increasing number of qubits. This is due to the vastness of the optimization landscape, indicating the need for better metrics and/or optimization schemes.

Within the density matrix representation, we have also implemented parameterized quantum gates and the automatic differentiation of quantum circuits. This enables the gradient-based optimization of circuit evaluation metrics (using, e.g., gradient-descent optimizers). This aspect of our framework enables quantum circuits to be optimized in the presence of noise, and enables gradient descent to be combined with circuit topology optimization schemes. An example of this scheme in action is shown in Fig. 5, where we perform quantum gate optimization to recover fidelity lost due to noise. Although the noise model requires refinement to accurately represent experimentally-found errors, these are promising results for managing noise-related deterioration of graph states.

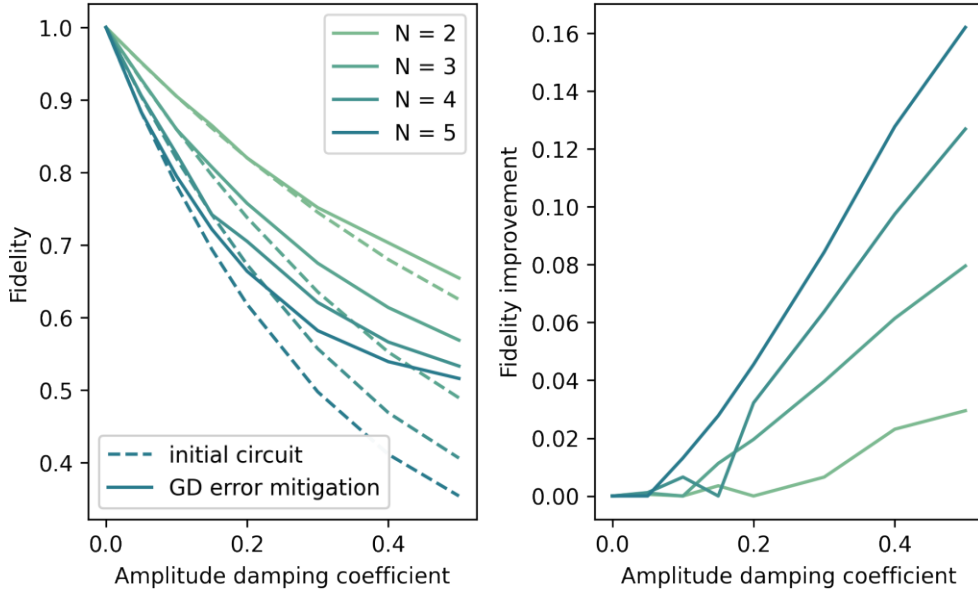


Figure 5: Regaining lost fidelity by parametrized gates optimization for linear cluster states consisting of $N=1-5$ photons as function of noise amplitude, where an amplitude damping model was uniformly applied on all qubits. Left panel shows the fidelity of the initial circuit after introducing noise (dashed) and the fidelity after gate optimization (solid). Right panel shows the relative difference between the solid and dashed lines. We find that gate optimization led to absolute fidelity recoveries of up to 15%.

3.2.5. Summary of accomplishments

During year 1 of this project, we have accomplished the following basic research milestones:

- Constructed a theoretical and numerical framework for the study of graph states and of the quantum circuits that generate them within photonics-based platforms.
- Reproduced known quantum circuits for the generation of graph states found within recent literature. In some cases, such as a 3-photon GHZ state, we have found alternative, yet equivalent quantum circuits, providing experimentalists with setup variety.
- For a 3-photon linear cluster state, we have found an improved quantum circuit with a shorter depth relative to published setups.
- Our framework supports diverse noise models, including depolarizing noise, amplitude damping, and Pauli errors.
- We have developed an experimentally-motivated, time-dependent noise model leveraging the stabilizer representation. This model reduces the number of potential outcomes within mixed states and expresses the noise in terms of circuit time-scales.
- Demonstrated that the use of parameterized gates in conjunction with gradient-based optimization can remediate the impacts of noise-driven fidelity degradation, with an absolute fidelity recovery of up to 15% for 5-photon linear cluster states.
- Developed a novel way of representing mixed states using stabilizers, allowing the simulation of more realistic noise effects and circuit dynamics.
- Developed a novel method for generating the graph representation of a quantum state from a density matrix.
- We have developed random and evolutionary schemes that search for well-performing quantum circuits given a target output state.
- We have developed a hybrid quantum circuit search scheme by combining deterministic and stochastic methods.
- Our framework is modular, and can easily be extended to other physical platforms and noise models.

3.2.6. Dissemination to the communities of interest.

Nothing to report. Nevertheless, manuscripts are in preparation and will be shared with the program officer.

4. Impacts

4.1. Development of the principal discipline of the project

We have developed a framework for the study and optimization of graph state generation in the presence of realistic effects such as noise and photonic loss, making it the first of its kind. Such a tool is a driver for further basic research into entangled resource preparation and distribution for various applications.

The framework is implemented in Python and was designed to be extensible, with community co-development in mind. To support the community and future partners in using these tools, we have prepared thorough documentation, code examples, and interactive tutorials.

Our long-term plans are discussed in detail in Sec. 6, but our near-term plan for disseminating our work to the community includes the following:

- Make our framework available to the community by open-sourcing our codebase

- alongside documentation, use examples, and tutorials
- Use the open-sourced code to enhance and establish research partnerships
- Prepare abstracts for conferences such as the International Conference on Quantum Communication, Measurement and Computing (QCMC), Q2B22 Silicon Valley, Photonics West, and CLEO
- Prepare a manuscript for peer-review and publication

4.2. Impact on other disciplines

In addition to the dissemination of our framework, we also intend to collaborate directly with colleagues from the community. This includes collaborating with the National Research Council of Canada (NRC), specialized in the fabrication of quantum dots that can be used as emitter qubits, l’Institut National de la Recherche Scientifique (INRS), giving us access to knowledge on integrated photonics, as well as Caltech and Fermilab, who can provide realistic entanglement distribution channel metrics from their respective testbeds. Additionally, Quantum Bridge Technologies, Inc. (QBT) is a University of Toronto spin-off, with many local research collaboration opportunities. Our collaborations will allow us to test and validate our framework, as well as improve its level of realism.

5. Future goals

As part of future work, our team intends to maintain and develop our framework alongside the open-source quantum community to find solutions for higher complexity graph states. In parallel, our work internally will focus on extending the framework with more numerous and realistic components as well as qubit protection strategies. To perform this work, we hope for continued support of the project in the years to come and provide below a Gantt chart presenting our future project goals for the following year. Indicated within each task are assigned lead researchers, with a team overview following the chart.

Task description	Months								
	M1-M4	M4-M8	M8-M12						
Modeling fusion gate components									
Using fusion gates [27], we can form large graph states from smaller graph states. We will implement such gates within our framework, giving an alternative route to realizing graph states. This task will be led by PDF1 and supported by PDF2 and PhD1.									
Literature review and implementing fusion gates within framework									
Investigate fusion gate-based quantum circuits									
Encoding for error correction and fusion gate protection									
We will explore encoding methods for protecting qubits against random errors (e.g., bitflip) and failed fusion gate attempts, which lead to the loss of a qubit. This task will be led by PDF1, and supported by PDF2 and PhD1.									
Investigate encoding strategies in literature									
Implement encoding for qubit protection									

Open innovation and community impact As discussed in Sec. 5, we aim to disseminate our work to support the quantum research community. Open-sourcing will be co-led by I1 and I2. Reaching out to community members will be co-led by PM1, PM2, PM3 and PM4. The writing of a manuscript will be co-led by PDF2 and PDF3 with support from the full team.															
Publish codebase and finalize public documentation and tutorials															
Prepare manuscript and submit to journal															
Project management and administrative support Project management will continue to be led by PM1, with the support of PM2, PM3 and PM4.															
Ongoing management and project reporting															

6. Team overview:

PM1: Piotr Roztocki

CTO, Ki3 Photonics Technologies Inc. Principal project manager responsible for day-to-day activities. Honors B.Sc. in Integrated Science (Physics), McMaster University 2014; Ph.D. in Energy and Materials, INRS-EMT 2022.

PM2: Hoi-Kwong Lo

CTO, Quantum Bridge Technologies, Inc. (QBT). Plays a supervisory role in the project. Professor of Physics and ECE at the University of Toronto, BA in Mathematics, Cambridge University, 1989, Ph.D. in Physics, Caltech, 1994. Co-inventor of measurement-device-independent quantum key distribution (QKD) and all photonics quantum repeaters. Fellow of the American Physical Society (APS) and Optica Fellow.

PM3: Mattia Montagna

CEO, Quantum Bridge Technologies, Inc. (QBT), PhD in Applied Math. Founder of the ECB (European Central Bank) Bank Interconnected Coordination Group (BICG). Has been leading QBT since its founding in 2019.

PM4: Yoann Jestin

CEO, Ki3 Photonics Technologies Inc. Principal Investigator of the project. B.Sc in Chemistry, University of Rennes, 1997; M.Sc. in Material Science, University of Rennes, 1999; Ph.D in Material Science, University of Le Mans, 2002.

PDF1: Jie Lin

QBT quantum research scientist with specializations in quantum optics and security analysis of quantum key distribution. Will lead development of both fusion gates and qubit encoding. B. A. Mathematics, Carleton College; M. A. S. Theoretical Physics, University of Cambridge; M. Sc. Physics and Ph.D. Physics (quantum information), University of Waterloo.

PDF2: Luc Robichaud

QBT quantum research scientist who specializes in quantum dot modeling and their application to entanglement generation. Will lead modeling of various sources and incorporating said models into the framework. Will also co-lead with PDF3 the writing of papers for peer-reviewed journals. B.Sc. Physics and M.Sc. Physics, Université de Moncton;

Ph.D. Physics, University of Ottawa.

PDF3: Stefania Sciara

Postdoc at l'Institut National de la Recherche Scientifique (INRS) and Ki3 researcher with specializations in both experimental and theoretical quantum optics, and entanglement generation. B.Sc. Physics and M.Sc. Physics, University of Palermo; Ph.D. Quantum optics, INRS.

PhD1: Sobhan Ghanbari

QBT intern. Ph.D. candidate at the University of Toronto with specialization in quantum information. Sobhan has led and will continue to lead development of various noise models within the framework. B.Sc. Physics/ B.Sc. Mechanical Engineering (dual- degree), Sharif University of Technology.

PhD2: Benjamin MacLellan

Ph.D. candidate at the University of Waterloo with specialization in quantum information and inverse design of photonic systems. Will lead the development of a more comprehensive physical layer, including various integrated-photonics-based components. B.Sc. Physics, University of Waterloo; M.Sc., Energy & Materials, INRS.

M1: Kai Sum Chan

QBT quantum research scientist. M.Phil. candidate at the University of Hong Kong, and QBT experimental and theoretical quantum researcher with specialization in quantum information, adaptive optics and quantum dots. Will play a supporting role in calibrating models according to experiments.

I1: Khuong Tran

B.Sc. candidate in computer sciences at the University of Ontario Institute of Technology and intern at Ki3. Will co-lead with I2 preparation of open-sourcing, framework documentation and tutorials.

I2: Aayush Soni

B.A.Sc. candidate in mechatronics engineering at the University of Waterloo and intern at Ki3. Will co-lead with I1 the preparation of open-sourcing, framework documentation and tutorials.

Key: PM: Project manager, PDF: Postdoctoral fellow, PhD: PhD student, M: Master's student, I: Intern.

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Highly Entangled States of Light

Final Report

Awarded to:
Ki3 Photonics Technologies Inc.
Quantum Bridge Technologies Inc (QBT).

Recipient Principal Investigator:
Dr. Yoann Jestin

1. Executive summary

Problem statement:

- Graph quantum states are an important resource for fundamental research and technological applications
- Designing quantum circuits that output specific and desired graph states is challenging
- Real quantum circuits are subject to noise and loss, which are necessary to include for an accurate and realistic model

Project scope:

- Construct a theoretical and numerical framework for the study of graph states and of circuits that generate them
- Develop schemes for the construction of quantum circuits that output desired graph states
- Create models for the noise and loss effects that are present in real quantum setups

General outcome:

- To date, we have defined a framework that allows us to find and to simulate quantum circuits that output low-complexity graph states
- Our schemes successfully reproduce results from literature and are also capable of finding alternative setups to choose between
- We have developed noise and loss models that are representative of effects found in common photonics platforms
- Our framework is a flexible and extensible tool that is soon to be open-sourced and which can be easily adapted to simulate various physical platforms

Future Plans:

- Maintain and develop the framework alongside the open-source quantum community to find solutions for higher complexity graph states
- Develop fusion-gate based protocols for producing large graph states
- Implement encoding strategies for qubit protection
- Model quantum-dot-based sources and interface with current framework
- Develop models for integrated-photonics-based components as an experimental basis for quantum circuits
- Develop more sophisticated noise and distribution channel models
- Extend our ability to inject experimental details into our framework, further connecting results with real setups

2. Introduction

Graph states are a family of quantum states with high levels of entanglement that are of great fundamental interest [1], as well as being a key resource for applications in entanglement distribution and quantum networking [2-4], quantum error correction [5,6], and quantum metrology [7]. Among various physical platforms, photonics shows much promise for graph state realizations [8,9] as photons interact very weakly with their environment, enabling stable qubits. Additionally, photons are compatible with mature integrated and fiber technologies, granting additional support in the generation and distribution of graph states.

As well, for the generation of photons, various sources are available within photonics-based platforms, including solid-state emitters such as quantum dots that can emit single photons on demand [10], or nonlinear elements generating pairs of entangled photons [11,12].

Quantum circuits describe the sequence of quantum operations necessary to generate such graph states. However, connecting a desired graph state with the circuit that can output it is a non-trivial design task that only becomes more difficult with increasing state size and complexity. Recent work [13] has led to the development of an impressive deterministic method for finding quantum circuits that produce targeted graph states. However, such methods do not guarantee well-performing quantum circuits, allow little flexibility, and cannot consider impacts associated with noise or photonic loss, which is present in all real setups.

Towards addressing this research gap, the goal of our work was to develop a flexible theoretical and computational framework for the study of realistic graph states and of the quantum circuits that generate them. To accomplish this, we must be able to represent both the quantum states and the quantum circuits that produce them. Both representations must also be flexible enough to support noise and photonic loss modeling. The performance of different circuits must also be quantifiable, in order to drive schemes that search for best performing setups. All of these concepts must also be realized in a flexible and modular way that allows for co-development of solutions with the broader research community, driving collaborative and fundamental work into the understanding of entangled states as a resource.

Over the past year of our project's progress, we have successfully identified and realized a framework comprising the following core concepts: *Quantum state representations*, *quantum circuit representations*, *realistic component models*, *quantum circuit performance metrics*, and *quantum circuit optimization schemes*. Figure 1 shows these key concepts and their mutual interaction within our framework.

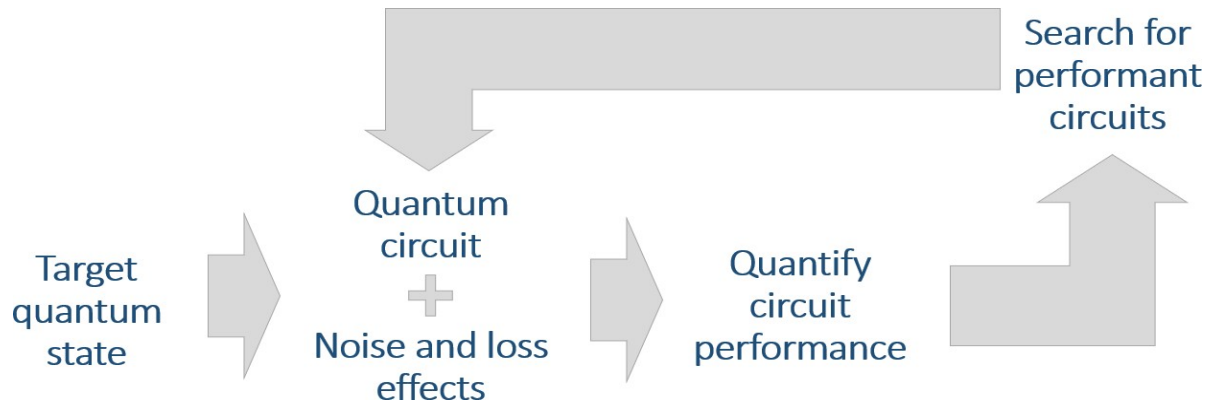


Figure 1: Key concepts within our framework. The performance metric is calculated with respect to the target graph state, guiding the search for performant circuits.

3. Accomplishments

In what follows, we explain each of the core concepts of our framework alongside our respective research efforts in developing them.

3.1. Research objectives

3.3.1. Representing quantum states

The objects of interest, graph states, are quantum systems of photonic and emitter qubits that can be mathematically represented using various formalisms. Our computational framework presently considers two mathematical representations, the *density matrix* and the *stabilizer*, with the modularity to easily include further representations in the future.

3.1.2. Representing quantum circuits

Quantum circuits represent the sequence of quantum gates and operations applied on solid state emitter qubits and photonic qubits. To computationally read, save, and manipulate these quantum circuits, we represent them as Directed Acyclic Graphs (DAGs) [19], which find common use in the field of quantum information [20]. Using the DAG representation, our computational framework can easily modify and evolve the quantum circuit topology to optimize the generated quantum state towards a target graph state. For the visualization of quantum circuits, our DAG representation was also made compatible with the openQASM format, a standard within the field [21].

3.1.3. Realistic component models

Towards increasing the realism of our framework, we support diverse models for noise and photonic loss. For example, our framework presently includes noise models for depolarizing noise, amplitude damping, and Pauli noise.

Additionally, we have developed a new representation to model mixed states using stabilizers. This new representation keeps track of all potential branchings of a mixed state, along with the probability of such a branch being measured. This novel approach allows us to consider probabilistic noise, more akin to real experimental systems.

Motivated by the physics of today's solid-state emitter and photonic qubit systems, we have furthermore developed a time-dependent noise model within the stabilizer formalism. In experiments, the number of errors that occur is strongly correlated with the time spent during and between each quantum gate. In our time-dependent model, we make use of an effective noise (simulated only at certain gates), maintaining model accuracy while reducing the computational load of the model.

3.1.4. Assessing circuit performance

When optimizing a quantum circuit for the generation of a target graph state, it is important to properly quantify the performance of said circuit using quantitative metrics. In fact, having proper metrics can heavily influence the performance of a circuit search algorithm. To date, we have investigated the following metrics as potential candidates [14], [22], [23]:

Fidelity	Relative entropy
Entanglement witness	Squashed entanglement
Graph edit distance	Adjacency matrix fidelity
Concurrence	Geometric measure
Negativity	Fisher information
Trace distance	Mermin inequality
Entanglement distillation	Stabilizer tableau fidelity

3.1.5. Searching for performant quantum circuits

We have investigated the following schemes for converging on suitable quantum circuits that output a desired graph state:

- **Random:** This scheme initializes a list of random circuits subject to physical

constraints, which we call a population. At each iteration and for each circuit within the population, the algorithm applies a random operation, which includes addition, removal, or replacement of a quantum gate. Each circuit in the population is modified independently from the other circuits. The scheme then keeps track of a certain number of top performing circuits.

- **Evolutionary:** Similarly to the random search approach, this scheme also applies random operations in each iteration. However, in contrast, after each iteration it constructs a new population by putting circuits into a N-wise competition and selecting the best performers.
- **Deterministic:** This scheme is our implementation of Ref. [13]’s approach to deterministically find a quantum circuit for a target graph state.
- **Hybrid:** In this scheme, we use the deterministic method from Ref. [13] to produce an initial solution. We then perturb this solution and use the random/evolutionary schemes to search for quantum circuit alternatives.
- **Gradient-based:** This is a density matrix specific scheme where we consider tunable/parametrized gates that are compatible with automatic differentiation and gradient descent optimization techniques.

3.2 Details of accomplishments

3.2.1. Representing quantum states

The *density matrix* representation can simulate a state’s full Hilbert space and can inherently represent mixed states, such as those resulting from the effects of noise and photonic loss [14]. However, this representation does not scale well with the number of qubits, making it impractical to investigate large graph states, and thus motivating the need for complementary representations. We have used this representation: 1) To study small graph states with the inclusion of noise effects; 2) As a benchmark for other representations; and 3) For the study of noise, photonic loss, mixed states, and gate parametrization.

The *Stabilizer* representation [15], [16], on the other hand, scales well with the number of qubits, making it more efficient for the study of large graph states (relative to the density matrix formalism). However, only a subset of quantum operations can be efficiently simulated within the stabilizer representation, limiting the scope for circuit optimization, and making this representation unsuited for the simulation of noise without judicious changes to the formalism. Presently, the study of representing mixed states within the stabilizer representation is very limited in the literature. To be able to consider large graph states that are mixed due to noise or loss, we have developed a probability tree approach as a way to represent such states. This point of research is presented further below in Sec. 3.3.

We have also developed relevant concepts from *graph* representations, which are also well-suited for many qubits, and contrary to the stabilizer representation, there is more progress made on describing mixed states [17,18]. In particular, we have worked on leveraging graph state equivalency through Clifford gates. As discussed in Sec. 3.4, the graph representation also offers unique metrics for characterizing the performance of quantum circuits.

Each of the quantum state representations has advantages and disadvantages, making it useful to be able to move between them. Most of these representation transformations can be found within literature [3], [14-18], except for obtaining a graph from a density matrix. We have resolved this issue by designing a novel protocol, as discussed in Sec. 3.5.

3.2.2. Representing quantum circuits

Although we have focused on solid state emitter qubits and photonic qubits, we have designed our framework with extensibility in mind. The types of operations, and therefore the

physical circuits we can represent and simulate, are flexible, meaning we can adapt our framework towards different physical platforms with relative ease.

Lastly, our quantum circuit representation also includes parameterized quantum gates and automatic differentiation (see Sec. 3.5 for further discussion).

3.2.3. Realistic component models

The Figure 2 shows an example of the model in practice, deriving the fidelity of an output graph state as a function of qubit error rate from various sources of noise.

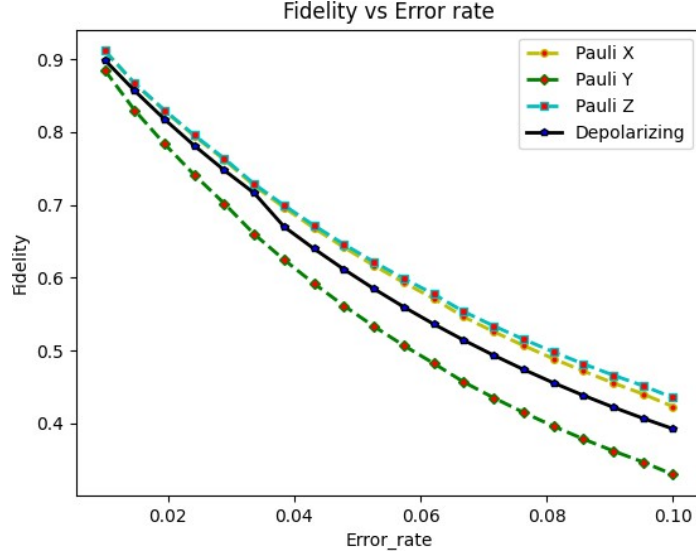


Figure 2: Fidelity versus error rate for the generation of a 4-photon linear cluster state assuming X -gate errors (yellow), Y -gate errors (green), Z -gate errors (blue) and a depolarizing channel (black). We find that the generation of linear cluster states is equally sensitive to bit-flip (X -gate) and phase-flip (Z -gate) errors, and more sensitive when both errors occur (Y -gate).

As photonic losses have non-trivial impacts on quantum states, generally producing mixed states, we have explored various approaches to simulate their effects. One approach is to extend the standard qubit space to a qutrit space so that, by accounting for a vacuum term, the state can still be represented as pure. Another approach is to keep track of the survival probability of a state after potential photonic loss events. Interestingly, we observe that the state fidelity calculation only depends on branches that see no loss, weighted by their respective probabilities. By tracking transmission probabilities, we gain information useful for approximating state generation rates, which are a very important experimental insight. We also calculate the survival probability for each photon, which indicates which portions of the graph state are more prone to loss. This information is essential for efficiently implementing encoding for error correction.

3.2.4. Assessing circuit performance

Fidelity has been our main metric thus far, but it is not ideal for our task as it can change abruptly, which is partially due to the discrete nature of adding or removing quantum operations within the quantum circuit. Entanglement witnesses have potential [24]–[26], but our investigations of GHZ and linear cluster states suggest they may be too robust and not sufficiently monotonic in the neighborhood of optimums. The graph edit metric, which is a metric based on the graph representation of a quantum state, holds potential as it is fundamentally different from other metrics. There are however challenges as it is not obvious

how fundamental graph operations, such as local complementations, would be represented in this metric. Most metrics investigated thus proved to not be useful towards search/optimization schemes or well suited for multipartite states.

3.2.5. Searching for performant quantum circuits

Using the random and evolutionary schemes within the density matrix representation, we were able to:

- Reproduce results from [13] for 4-photon linear cluster states.
- Find alternative, but equivalent circuits. Given some circuits may be more sensitive to noise compared to others or that some gates may be more difficult to implement than others, this gives options to experimentalists in implementing whichever protocol is more practical.
- Find cases where the circuit depth is reduced relative to circuits produced by Ref. [13]'s approach. An example of this depth decrease is shown in Fig. 3. Decreases in circuit depth imply a simplified protocol as there are fewer gates to be implemented or more gates can be implemented in parallel.
- In terms of convergence, we found that the random and evolutionary schemes performed similarly. Figure 4 shows the fraction of search runs that converge on a quantum circuit producing the target graph state.

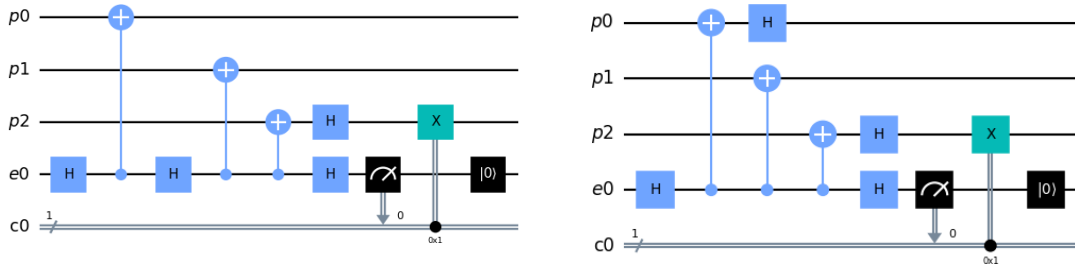


Figure 3: Quantum circuits for the generation of a 3-photon linear cluster state as determined by (left) the deterministic methods of [13] and (right) our framework. Our solution presents a reduced circuit depth relative to the deterministic method.

Using the stabilizer representation, we are able to efficiently simulate large graph states, up to hundreds of qubits, but it remains difficult to converge to an optimal circuit due to the vast optimization landscape. For this reason, we have also investigated the hybrid approach, which leverages the deterministic scheme of Ref. [13] to produce a starting point for random/evolutionary-based search approaches. Using this method, we have been able to find alternative quantum circuits to those proposed by the deterministic approach, as well as access larger graph states.

Additionally, the deterministic solutions do not include the effects of noise, which degrade the quality of the output graph state for a given circuit. Using our hybrid scheme, we have begun investigations into re-optimizing the deterministic solution after the introduction of noise, to regain state fidelity. This investigation is expected to greatly benefit from experimental feedback to more accurately represent noise found within real photonic platforms.



	Linear Cluster (3 qubits)	GHZ (3 qubits)	Linear Cluster (4 qubits)	GHZ (4 qubits)
Random scheme	92%	60%	24%	24%
Evolutionary scheme	96%	16%	24%	28%

Figure 4: Fraction of simulations that converge on a solution for a quantum circuit using a density matrix representation. Target graph states for each case are indicated above columns. Both the random and evolutionary schemes have difficulty converging with an increasing number of qubits. This is due to the vastness of the optimization landscape, indicating the need for better metrics and/or optimization schemes.

Within the density matrix representation, we have also implemented parameterized quantum gates and the automatic differentiation of quantum circuits. This enables the gradient-based optimization of circuit evaluation metrics (using, e.g., gradient-descent optimizers). This aspect of our framework enables quantum circuits to be optimized in the presence of noise, and enables gradient descent to be combined with circuit topology optimization schemes. An example of this scheme in action is shown in Fig. 5, where we perform quantum gate optimization to recover fidelity lost due to noise. Although the noise model requires refinement to accurately represent experimentally-found errors, these are promising results for managing noise-related deterioration of graph states.

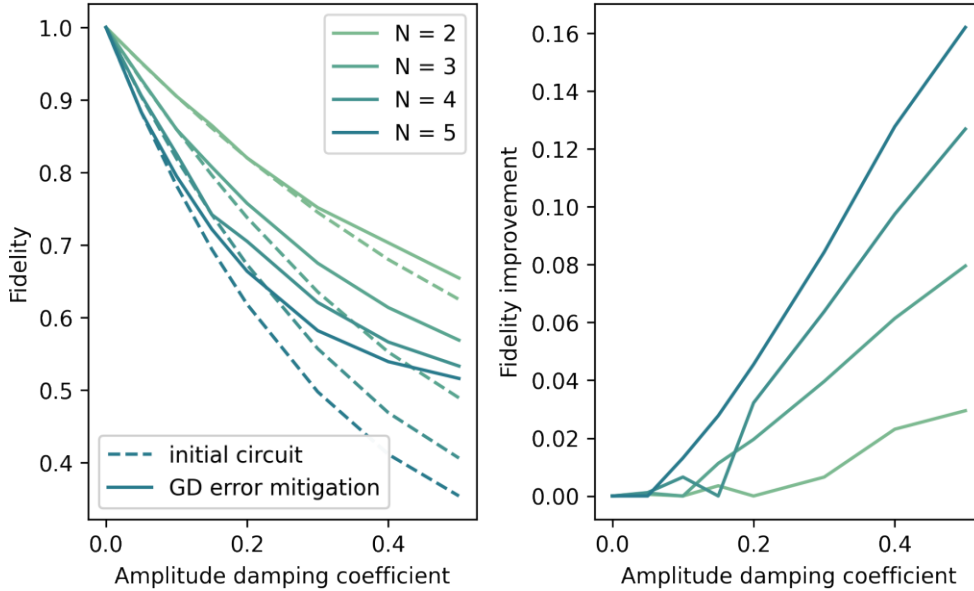


Figure 5: Regaining lost fidelity by parametrized gates optimization for linear cluster states consisting of $N=1-5$ photons as function of noise amplitude, where an amplitude damping model was uniformly applied on all qubits. Left panel shows the fidelity of the initial circuit after introducing noise (dashed) and the fidelity after gate optimization (solid). Right panel shows the relative difference between the solid and dashed lines. We find that gate optimization led to absolute fidelity recoveries of up to 15%.

3.2.5. Summary of accomplishments

During year 1 of this project, we have accomplished the following basic research milestones:

- Constructed a theoretical and numerical framework for the study of graph states and of the quantum circuits that generate them within photonics-based platforms.
- Reproduced known quantum circuits for the generation of graph states found within recent literature. In some cases, such as a 3-photon GHZ state, we have found alternative, yet equivalent quantum circuits, providing experimentalists with setup variety.
- For a 3-photon linear cluster state, we have found an improved quantum circuit with a shorter depth relative to published setups.
- Our framework supports diverse noise models, including depolarizing noise, amplitude damping, and Pauli errors.
- We have developed an experimentally-motivated, time-dependent noise model leveraging the stabilizer representation. This model reduces the number of potential outcomes within mixed states and expresses the noise in terms of circuit time-scales.
- Demonstrated that the use of parameterized gates in conjunction with gradient-based optimization can remediate the impacts of noise-driven fidelity degradation, with an absolute fidelity recovery of up to 15% for 5-photon linear cluster states.
- Developed a novel way of representing mixed states using stabilizers, allowing the simulation of more realistic noise effects and circuit dynamics.
- Developed a novel method for generating the graph representation of a quantum state from a density matrix.
- We have developed random and evolutionary schemes that search for well-performing quantum circuits given a target output state.
- We have developed a hybrid quantum circuit search scheme by combining deterministic and stochastic methods.
- Our framework is modular, and can easily be extended to other physical platforms and noise models.

3.2.6. Dissemination to the communities of interest.

Nothing to report. Nevertheless, manuscripts are in preparation and will be shared with the program officer.

4. Impacts

4.1. Development of the principal discipline of the project

We have developed a framework for the study and optimization of graph state generation in the presence of realistic effects such as noise and photonic loss, making it the first of its kind. Such a tool is a driver for further basic research into entangled resource preparation and distribution for various applications.

The framework is implemented in Python and was designed to be extensible, with community co-development in mind. To support the community and future partners in using these tools, we have prepared thorough documentation, code examples, and interactive tutorials.

Our long-term plans are discussed in detail in Sec. 6, but our near-term plan for disseminating our work to the community includes the following:

- Make our framework available to the community by open-sourcing our codebase

alongside documentation, use examples, and tutorials

- Use the open-sourced code to enhance and establish research partnerships
- Prepare abstracts for conferences such as the International Conference on Quantum Communication, Measurement and Computing (QCMC), Q2B22 Silicon Valley, Photonics West, and CLEO
- Prepare a manuscript for peer-review and publication

4.2. Impact on other disciplines

In addition to the dissemination of our framework, we also intend to collaborate directly with colleagues from the community. This includes collaborating with the National Research Council of Canada (NRC), specialized in the fabrication of quantum dots that can be used as emitter qubits, l'Institut National de la Recherche Scientifique (INRS), giving us access to knowledge on integrated photonics, as well as Caltech and Fermilab, who can provide realistic entanglement distribution channel metrics from their respective testbeds. Additionally, Quantum Bridge Technologies, Inc. (QBT) is a University of Toronto spin-off, with many local research collaboration opportunities. Our collaborations will allow us to test and validate our framework, as well as improve its level of realism.

5. Future goals

As part of future work, our team intends to maintain and develop our framework alongside the open-source quantum community to find solutions for higher complexity graph states. In parallel, our work internally will focus on extending the framework with more numerous and realistic components as well as qubit protection strategies. To perform this work, we hope for continued support of the project in the years to come and provide below a Gantt chart presenting our future project goals for the following year. Indicated within each task are assigned lead researchers, with a team overview following the chart.

Task description	Months											
	M1-M4				M4-M8				M8-M12			
Modeling fusion gate components Using fusion gates [27], we can form large graph states from smaller graph states. We will implement such gates within our framework, giving an alternative route to realizing graph states. This task will be led by PDF1 and supported by PDF2 and PhD1.												
Literature review and implementing fusion gates within framework												
Investigate fusion gate-based quantum circuits												
Encoding for error correction and fusion gate protection We will explore encoding methods for protecting qubits against random errors (e.g., bitflip) and failed fusion gate attempts, which lead to the loss of a qubit. This task will be led by PDF1, and supported by PDF2 and PhD1.												
Investigate encoding strategies in literature												
Implement encoding for qubit protection												

Open innovation and community impact As discussed in Sec. 5, we aim to disseminate our work to support the quantum research community. Open-sourcing will be co-led by I1 and I2. Reaching out to community members will be co-led by PM1, PM2, PM3 and PM4. The writing of a manuscript will be co-led by PDF2 and PDF3 with support from the full team.															
Publish codebase and finalize public documentation and tutorials															
Prepare manuscript and submit to journal															
Project management and administrative support Project management will continue to be led by PM1, with the support of PM2, PM3 and PM4.															
Ongoing management and project reporting															

6. Team overview:

PM1: Piotr Roztocki

CTO, Ki3 Photonics Technologies Inc. Principal project manager responsible for day-to-day activities. Honors B.Sc. in Integrated Science (Physics), McMaster University 2014; Ph.D. in Energy and Materials, INRS-EMT 2022.

PM2: Hoi-Kwong Lo

CTO, Quantum Bridge Technologies, Inc. (QBT). Plays a supervisory role in the project. Professor of Physics and ECE at the University of Toronto, BA in Mathematics, Cambridge University, 1989, Ph.D. in Physics, Caltech, 1994. Co-inventor of measurement-device-independent quantum key distribution (QKD) and all photonics quantum repeaters. Fellow of the American Physical Society (APS) and Optica Fellow.

PM3: Mattia Montagna

CEO, Quantum Bridge Technologies, Inc. (QBT), PhD in Applied Math. Founder of the ECB (European Central Bank) Bank Interconnected Coordination Group (BICG). Has been leading QBT since its founding in 2019.

PM4: Yoann Jestin

CEO, Ki3 Photonics Technologies Inc. Principal Investigator of the project. B.Sc in Chemistry, University of Rennes, 1997; M.Sc. in Material Science, University of Rennes, 1999; Ph.D in Material Science, University of Le Mans, 2002.

PDF1: Jie Lin

QBT quantum research scientist with specializations in quantum optics and security analysis of quantum key distribution. Will lead development of both fusion gates and qubit encoding. B. A. Mathematics, Carleton College; M. A. S. Theoretical Physics, University of Cambridge; M. Sc. Physics and Ph.D. Physics (quantum information), University of Waterloo.

PDF2: Luc Robichaud

QBT quantum research scientist who specializes in quantum dot modeling and their application to entanglement generation. Will lead modeling of various sources and incorporating said models into the framework. Will also co-lead with PDF3 the writing of papers for peer-reviewed journals. B.Sc. Physics and M.Sc. Physics, Université de Moncton;

Ph.D. Physics, University of Ottawa.

PDF3: Stefania Sciara

Postdoc at l'Institut National de la Recherche Scientifique (INRS) and Ki3 researcher with specializations in both experimental and theoretical quantum optics, and entanglement generation. B.Sc. Physics and M.Sc. Physics, University of Palermo; Ph.D. Quantum optics, INRS.

PhD1: Sobhan Ghanbari

QBT intern. Ph.D. candidate at the University of Toronto with specialization in quantum information. Sobhan has led and will continue to lead development of various noise models within the framework. B.Sc. Physics/ B.Sc. Mechanical Engineering (dual- degree), Sharif University of Technology.

PhD2: Benjamin MacLellan

Ph.D. candidate at the University of Waterloo with specialization in quantum information and inverse design of photonic systems. Will lead the development of a more comprehensive physical layer, including various integrated-photonics-based components. B.Sc. Physics, University of Waterloo; M.Sc., Energy & Materials, INRS.

M1: Kai Sum Chan

QBT quantum research scientist. M.Phil. candidate at the University of Hong Kong, and QBT experimental and theoretical quantum researcher with specialization in quantum information, adaptive optics and quantum dots. Will play a supporting role in calibrating models according to experiments.

I1: Khuong Tran

B.Sc. candidate in computer sciences at the University of Ontario Institute of Technology and intern at Ki3. Will co-lead with I2 preparation of open-sourcing, framework documentation and tutorials.

I2: Aayush Soni

B.A.Sc. candidate in mechatronics engineering at the University of Waterloo and intern at Ki3. Will co-lead with I1 the preparation of open-sourcing, framework documentation and tutorials.

Key: PM: Project manager, PDF: Postdoctoral fellow, PhD: PhD student, M: Master's student, I: Intern.

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Highly Entangled States of Light

Final Report

Awarded to:
Ki3 Photonics Technologies Inc.
Quantum Bridge Technologies Inc (QBT).

Recipient Principal Investigator:
Dr. Yoann Jestin

1. Executive summary

Problem statement:

- Graph quantum states are an important resource for fundamental research and technological applications
- Designing quantum circuits that output specific and desired graph states is challenging
- Real quantum circuits are subject to noise and loss, which are necessary to include for an accurate and realistic model

Project scope:

- Construct a theoretical and numerical framework for the study of graph states and of circuits that generate them
- Develop schemes for the construction of quantum circuits that output desired graph states
- Create models for the noise and loss effects that are present in real quantum setups

General outcome:

- To date, we have defined a framework that allows us to find and to simulate quantum circuits that output low-complexity graph states
- Our schemes successfully reproduce results from literature and are also capable of finding alternative setups to choose between
- We have developed noise and loss models that are representative of effects found in common photonics platforms
- Our framework is a flexible and extensible tool that is soon to be open-sourced and which can be easily adapted to simulate various physical platforms

Future Plans:

- Maintain and develop the framework alongside the open-source quantum community to find solutions for higher complexity graph states
- Develop fusion-gate based protocols for producing large graph states
- Implement encoding strategies for qubit protection
- Model quantum-dot-based sources and interface with current framework
- Develop models for integrated-photonics-based components as an experimental basis for quantum circuits
- Develop more sophisticated noise and distribution channel models
- Extend our ability to inject experimental details into our framework, further connecting results with real setups

2. Introduction

Graph states are a family of quantum states with high levels of entanglement that are of great fundamental interest [1], as well as being a key resource for applications in entanglement distribution and quantum networking [2-4], quantum error correction [5,6], and quantum metrology [7]. Among various physical platforms, photonics shows much promise for graph state realizations [8,9] as photons interact very weakly with their environment, enabling stable qubits. Additionally, photons are compatible with mature integrated and fiber technologies, granting additional support in the generation and distribution of graph states.

As well, for the generation of photons, various sources are available within photonics-based platforms, including solid-state emitters such as quantum dots that can emit single photons on demand [10], or nonlinear elements generating pairs of entangled photons [11,12].

Quantum circuits describe the sequence of quantum operations necessary to generate such graph states. However, connecting a desired graph state with the circuit that can output it is a non-trivial design task that only becomes more difficult with increasing state size and complexity. Recent work [13] has led to the development of an impressive deterministic method for finding quantum circuits that produce targeted graph states. However, such methods do not guarantee well-performing quantum circuits, allow little flexibility, and cannot consider impacts associated with noise or photonic loss, which is present in all real setups.

Towards addressing this research gap, the goal of our work was to develop a flexible theoretical and computational framework for the study of realistic graph states and of the quantum circuits that generate them. To accomplish this, we must be able to represent both the quantum states and the quantum circuits that produce them. Both representations must also be flexible enough to support noise and photonic loss modeling. The performance of different circuits must also be quantifiable, in order to drive schemes that search for best performing setups. All of these concepts must also be realized in a flexible and modular way that allows for co-development of solutions with the broader research community, driving collaborative and fundamental work into the understanding of entangled states as a resource.

Over the past year of our project's progress, we have successfully identified and realized a framework comprising the following core concepts: *Quantum state representations*, *quantum circuit representations*, *realistic component models*, *quantum circuit performance metrics*, and *quantum circuit optimization schemes*. Figure 1 shows these key concepts and their mutual interaction within our framework.

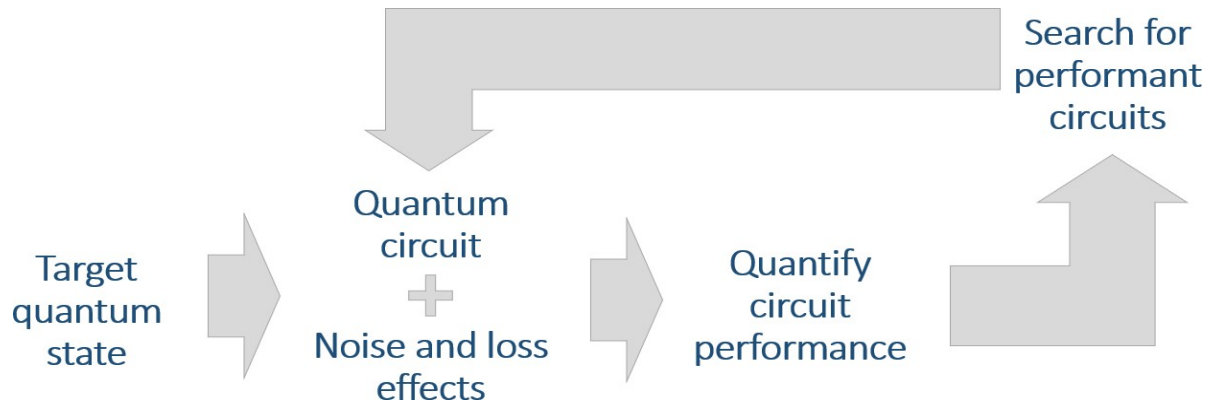


Figure 1: Key concepts within our framework. The performance metric is calculated with respect to the target graph state, guiding the search for performant circuits.

3. Accomplishments

In what follows, we explain each of the core concepts of our framework alongside our respective research efforts in developing them.

3.1. Research objectives

3.3.1. Representing quantum states

The objects of interest, graph states, are quantum systems of photonic and emitter qubits that can be mathematically represented using various formalisms. Our computational framework presently considers two mathematical representations, the *density matrix* and the *stabilizer*, with the modularity to easily include further representations in the future.

3.1.2. Representing quantum circuits

Quantum circuits represent the sequence of quantum gates and operations applied on solid state emitter qubits and photonic qubits. To computationally read, save, and manipulate these quantum circuits, we represent them as Directed Acyclic Graphs (DAGs) [19], which find common use in the field of quantum information [20]. Using the DAG representation, our computational framework can easily modify and evolve the quantum circuit topology to optimize the generated quantum state towards a target graph state. For the visualization of quantum circuits, our DAG representation was also made compatible with the openQASM format, a standard within the field [21].

3.1.3. Realistic component models

Towards increasing the realism of our framework, we support diverse models for noise and photonic loss. For example, our framework presently includes noise models for depolarizing noise, amplitude damping, and Pauli noise.

Additionally, we have developed a new representation to model mixed states using stabilizers. This new representation keeps track of all potential branchings of a mixed state, along with the probability of such a branch being measured. This novel approach allows us to consider probabilistic noise, more akin to real experimental systems.

Motivated by the physics of today's solid-state emitter and photonic qubit systems, we have furthermore developed a time-dependent noise model within the stabilizer formalism. In experiments, the number of errors that occur is strongly correlated with the time spent during and between each quantum gate. In our time-dependent model, we make use of an effective noise (simulated only at certain gates), maintaining model accuracy while reducing the computational load of the model.

3.1.4. Assessing circuit performance

When optimizing a quantum circuit for the generation of a target graph state, it is important to properly quantify the performance of said circuit using quantitative metrics. In fact, having proper metrics can heavily influence the performance of a circuit search algorithm. To date, we have investigated the following metrics as potential candidates [14], [22], [23]:

Fidelity	Relative entropy
Entanglement witness	Squashed entanglement
Graph edit distance	Adjacency matrix fidelity
Concurrence	Geometric measure
Negativity	Fisher information
Trace distance	Mermin inequality
Entanglement distillation	Stabilizer tableau fidelity

3.1.5. Searching for performant quantum circuits

We have investigated the following schemes for converging on suitable quantum circuits that output a desired graph state:

- **Random:** This scheme initializes a list of random circuits subject to physical

constraints, which we call a population. At each iteration and for each circuit within the population, the algorithm applies a random operation, which includes addition, removal, or replacement of a quantum gate. Each circuit in the population is modified independently from the other circuits. The scheme then keeps track of a certain number of top performing circuits.

- **Evolutionary:** Similarly to the random search approach, this scheme also applies random operations in each iteration. However, in contrast, after each iteration it constructs a new population by putting circuits into a N-wise competition and selecting the best performers.
- **Deterministic:** This scheme is our implementation of Ref. [13]’s approach to deterministically find a quantum circuit for a target graph state.
- **Hybrid:** In this scheme, we use the deterministic method from Ref. [13] to produce an initial solution. We then perturb this solution and use the random/evolutionary schemes to search for quantum circuit alternatives.
- **Gradient-based:** This is a density matrix specific scheme where we consider tunable/parametrized gates that are compatible with automatic differentiation and gradient descent optimization techniques.

3.2 Details of accomplishments

3.2.1. Representing quantum states

The *density matrix* representation can simulate a state’s full Hilbert space and can inherently represent mixed states, such as those resulting from the effects of noise and photonic loss [14]. However, this representation does not scale well with the number of qubits, making it impractical to investigate large graph states, and thus motivating the need for complementary representations. We have used this representation: 1) To study small graph states with the inclusion of noise effects; 2) As a benchmark for other representations; and 3) For the study of noise, photonic loss, mixed states, and gate parametrization.

The *Stabilizer* representation [15], [16], on the other hand, scales well with the number of qubits, making it more efficient for the study of large graph states (relative to the density matrix formalism). However, only a subset of quantum operations can be efficiently simulated within the stabilizer representation, limiting the scope for circuit optimization, and making this representation unsuited for the simulation of noise without judicious changes to the formalism. Presently, the study of representing mixed states within the stabilizer representation is very limited in the literature. To be able to consider large graph states that are mixed due to noise or loss, we have developed a probability tree approach as a way to represent such states. This point of research is presented further below in Sec. 3.3.

We have also developed relevant concepts from *graph* representations, which are also well-suited for many qubits, and contrary to the stabilizer representation, there is more progress made on describing mixed states [17,18]. In particular, we have worked on leveraging graph state equivalency through Clifford gates. As discussed in Sec. 3.4, the graph representation also offers unique metrics for characterizing the performance of quantum circuits.

Each of the quantum state representations has advantages and disadvantages, making it useful to be able to move between them. Most of these representation transformations can be found within literature [3], [14-18], except for obtaining a graph from a density matrix. We have resolved this issue by designing a novel protocol, as discussed in Sec. 3.5.

3.2.2. Representing quantum circuits

Although we have focused on solid state emitter qubits and photonic qubits, we have designed our framework with extensibility in mind. The types of operations, and therefore the

physical circuits we can represent and simulate, are flexible, meaning we can adapt our framework towards different physical platforms with relative ease. Lastly, our quantum circuit representation also includes parameterized quantum gates and automatic differentiation (see Sec. 3.5 for further discussion).

3.2.3. Realistic component models

The Figure 2 shows an example of the model in practice, deriving the fidelity of an output graph state as a function of qubit error rate from various sources of noise.

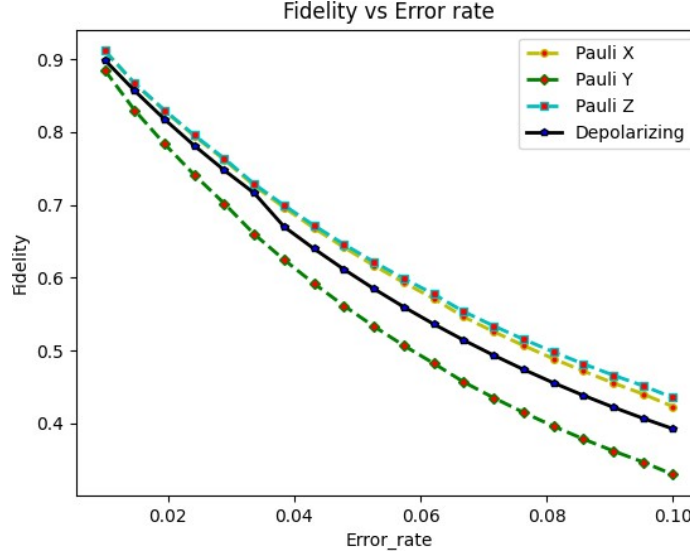


Figure 2: Fidelity versus error rate for the generation of a 4-photon linear cluster state assuming X -gate errors (yellow), Y -gate errors (green), Z -gate errors (blue) and a depolarizing channel (black). We find that the generation of linear cluster states is equally sensitive to bit-flip (X -gate) and phase-flip (Z -gate) errors, and more sensitive when both errors occur (Y -gate).

As photonic losses have non-trivial impacts on quantum states, generally producing mixed states, we have explored various approaches to simulate their effects. One approach is to extend the standard qubit space to a qutrit space so that, by accounting for a vacuum term, the state can still be represented as pure. Another approach is to keep track of the survival probability of a state after potential photonic loss events. Interestingly, we observe that the state fidelity calculation only depends on branches that see no loss, weighted by their respective probabilities. By tracking transmission probabilities, we gain information useful for approximating state generation rates, which are a very important experimental insight. We also calculate the survival probability for each photon, which indicates which portions of the graph state are more prone to loss. This information is essential for efficiently implementing encoding for error correction.

3.2.4. Assessing circuit performance

Fidelity has been our main metric thus far, but it is not ideal for our task as it can change abruptly, which is partially due to the discrete nature of adding or removing quantum operations within the quantum circuit. Entanglement witnesses have potential [24]–[26], but our investigations of GHZ and linear cluster states suggest they may be too robust and not sufficiently monotonic in the neighborhood of optimums. The graph edit metric, which is a metric based on the graph representation of a quantum state, holds potential as it is fundamentally different from other metrics. There are however challenges as it is not obvious

how fundamental graph operations, such as local complementations, would be represented in this metric. Most metrics investigated thus proved to not be useful towards search/optimization schemes or well suited for multipartite states.

3.2.5. Searching for performant quantum circuits

Using the random and evolutionary schemes within the density matrix representation, we were able to:

- Reproduce results from [13] for 4-photon linear cluster states.
- Find alternative, but equivalent circuits. Given some circuits may be more sensitive to noise compared to others or that some gates may be more difficult to implement than others, this gives options to experimentalists in implementing whichever protocol is more practical.
- Find cases where the circuit depth is reduced relative to circuits produced by Ref. [13]'s approach. An example of this depth decrease is shown in Fig. 3. Decreases in circuit depth imply a simplified protocol as there are fewer gates to be implemented or more gates can be implemented in parallel.
- In terms of convergence, we found that the random and evolutionary schemes performed similarly. Figure 4 shows the fraction of search runs that converge on a quantum circuit producing the target graph state.

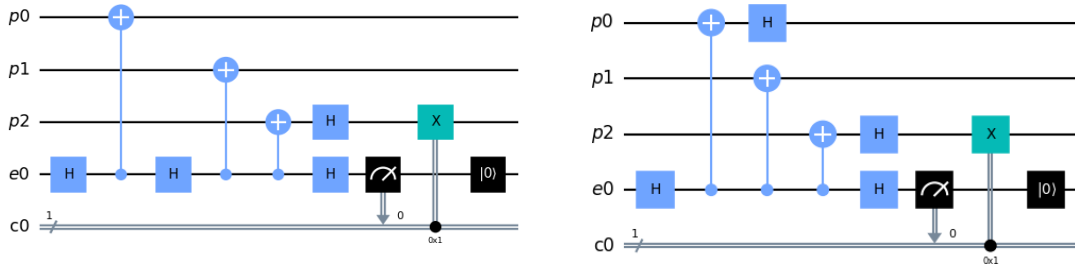


Figure 3: Quantum circuits for the generation of a 3-photon linear cluster state as determined by (left) the deterministic methods of [13] and (right) our framework. Our solution presents a reduced circuit depth relative to the deterministic method.

Using the stabilizer representation, we are able to efficiently simulate large graph states, up to hundreds of qubits, but it remains difficult to converge to an optimal circuit due to the vast optimization landscape. For this reason, we have also investigated the hybrid approach, which leverages the deterministic scheme of Ref. [13] to produce a starting point for random/evolutionary-based search approaches. Using this method, we have been able to find alternative quantum circuits to those proposed by the deterministic approach, as well as access larger graph states.

Additionally, the deterministic solutions do not include the effects of noise, which degrade the quality of the output graph state for a given circuit. Using our hybrid scheme, we have begun investigations into re-optimizing the deterministic solution after the introduction of noise, to regain state fidelity. This investigation is expected to greatly benefit from experimental feedback to more accurately represent noise found within real photonic platforms.



	Linear Cluster (3 qubits)	GHZ (3 qubits)	Linear Cluster (4 qubits)	GHZ (4 qubits)
Random scheme	92%	60%	24%	24%
Evolutionary scheme	96%	16%	24%	28%

Figure 4: Fraction of simulations that converge on a solution for a quantum circuit using a density matrix representation. Target graph states for each case are indicated above columns. Both the random and evolutionary schemes have difficulty converging with an increasing number of qubits. This is due to the vastness of the optimization landscape, indicating the need for better metrics and/or optimization schemes.

Within the density matrix representation, we have also implemented parameterized quantum gates and the automatic differentiation of quantum circuits. This enables the gradient-based optimization of circuit evaluation metrics (using, e.g., gradient-descent optimizers). This aspect of our framework enables quantum circuits to be optimized in the presence of noise, and enables gradient descent to be combined with circuit topology optimization schemes. An example of this scheme in action is shown in Fig. 5, where we perform quantum gate optimization to recover fidelity lost due to noise. Although the noise model requires refinement to accurately represent experimentally-found errors, these are promising results for managing noise-related deterioration of graph states.

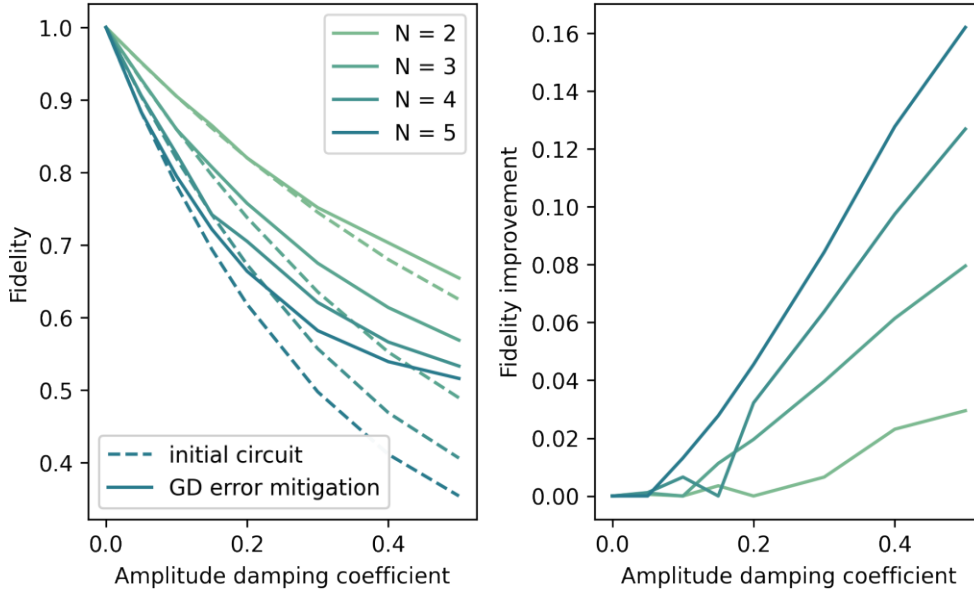


Figure 5: Regaining lost fidelity by parametrized gates optimization for linear cluster states consisting of $N=1-5$ photons as function of noise amplitude, where an amplitude damping model was uniformly applied on all qubits. Left panel shows the fidelity of the initial circuit after introducing noise (dashed) and the fidelity after gate optimization (solid). Right panel shows the relative difference between the solid and dashed lines. We find that gate optimization led to absolute fidelity recoveries of up to 15%.

3.2.5. Summary of accomplishments

During year 1 of this project, we have accomplished the following basic research milestones:

- Constructed a theoretical and numerical framework for the study of graph states and of the quantum circuits that generate them within photonics-based platforms.
- Reproduced known quantum circuits for the generation of graph states found within recent literature. In some cases, such as a 3-photon GHZ state, we have found alternative, yet equivalent quantum circuits, providing experimentalists with setup variety.
- For a 3-photon linear cluster state, we have found an improved quantum circuit with a shorter depth relative to published setups.
- Our framework supports diverse noise models, including depolarizing noise, amplitude damping, and Pauli errors.
- We have developed an experimentally-motivated, time-dependent noise model leveraging the stabilizer representation. This model reduces the number of potential outcomes within mixed states and expresses the noise in terms of circuit time-scales.
- Demonstrated that the use of parameterized gates in conjunction with gradient-based optimization can remediate the impacts of noise-driven fidelity degradation, with an absolute fidelity recovery of up to 15% for 5-photon linear cluster states.
- Developed a novel way of representing mixed states using stabilizers, allowing the simulation of more realistic noise effects and circuit dynamics.
- Developed a novel method for generating the graph representation of a quantum state from a density matrix.
- We have developed random and evolutionary schemes that search for well-performing quantum circuits given a target output state.
- We have developed a hybrid quantum circuit search scheme by combining deterministic and stochastic methods.
- Our framework is modular, and can easily be extended to other physical platforms and noise models.

3.2.6. Dissemination to the communities of interest.

Nothing to report. Nevertheless, manuscripts are in preparation and will be shared with the program officer.

4. Impacts

4.1. Development of the principal discipline of the project

We have developed a framework for the study and optimization of graph state generation in the presence of realistic effects such as noise and photonic loss, making it the first of its kind. Such a tool is a driver for further basic research into entangled resource preparation and distribution for various applications.

The framework is implemented in Python and was designed to be extensible, with community co-development in mind. To support the community and future partners in using these tools, we have prepared thorough documentation, code examples, and interactive tutorials.

Our long-term plans are discussed in detail in Sec. 6, but our near-term plan for disseminating our work to the community includes the following:

- Make our framework available to the community by open-sourcing our codebase

alongside documentation, use examples, and tutorials

- Use the open-sourced code to enhance and establish research partnerships
- Prepare abstracts for conferences such as the International Conference on Quantum Communication, Measurement and Computing (QCMC), Q2B22 Silicon Valley, Photonics West, and CLEO
- Prepare a manuscript for peer-review and publication

4.2. Impact on other disciplines

In addition to the dissemination of our framework, we also intend to collaborate directly with colleagues from the community. This includes collaborating with the National Research Council of Canada (NRC), specialized in the fabrication of quantum dots that can be used as emitter qubits, l'Institut National de la Recherche Scientifique (INRS), giving us access to knowledge on integrated photonics, as well as Caltech and Fermilab, who can provide realistic entanglement distribution channel metrics from their respective testbeds. Additionally, Quantum Bridge Technologies, Inc. (QBT) is a University of Toronto spin-off, with many local research collaboration opportunities. Our collaborations will allow us to test and validate our framework, as well as improve its level of realism.

5. Future goals

As part of future work, our team intends to maintain and develop our framework alongside the open-source quantum community to find solutions for higher complexity graph states. In parallel, our work internally will focus on extending the framework with more numerous and realistic components as well as qubit protection strategies. To perform this work, we hope for continued support of the project in the years to come and provide below a Gantt chart presenting our future project goals for the following year. Indicated within each task are assigned lead researchers, with a team overview following the chart.

Task description	Months											
	M1-M4				M4-M8				M8-M12			
Modeling fusion gate components Using fusion gates [27], we can form large graph states from smaller graph states. We will implement such gates within our framework, giving an alternative route to realizing graph states. This task will be led by PDF1 and supported by PDF2 and PhD1.												
Literature review and implementing fusion gates within framework												
Investigate fusion gate-based quantum circuits												
Encoding for error correction and fusion gate protection We will explore encoding methods for protecting qubits against random errors (e.g., bitflip) and failed fusion gate attempts, which lead to the loss of a qubit. This task will be led by PDF1, and supported by PDF2 and PhD1.												
Investigate encoding strategies in literature												
Implement encoding for qubit protection												

Open innovation and community impact As discussed in Sec. 5, we aim to disseminate our work to support the quantum research community. Open-sourcing will be co-led by I1 and I2. Reaching out to community members will be co-led by PM1, PM2, PM3 and PM4. The writing of a manuscript will be co-led by PDF2 and PDF3 with support from the full team.															
Publish codebase and finalize public documentation and tutorials															
Prepare manuscript and submit to journal															
Project management and administrative support Project management will continue to be led by PM1, with the support of PM2, PM3 and PM4.															
Ongoing management and project reporting															

6. Team overview:

PM1: Piotr Roztocki

CTO, Ki3 Photonics Technologies Inc. Principal project manager responsible for day-to-day activities. Honors B.Sc. in Integrated Science (Physics), McMaster University 2014; Ph.D. in Energy and Materials, INRS-EMT 2022.

PM2: Hoi-Kwong Lo

CTO, Quantum Bridge Technologies, Inc. (QBT). Plays a supervisory role in the project. Professor of Physics and ECE at the University of Toronto, BA in Mathematics, Cambridge University, 1989, Ph.D. in Physics, Caltech, 1994. Co-inventor of measurement-device-independent quantum key distribution (QKD) and all photonics quantum repeaters. Fellow of the American Physical Society (APS) and Optica Fellow.

PM3: Mattia Montagna

CEO, Quantum Bridge Technologies, Inc. (QBT), PhD in Applied Math. Founder of the ECB (European Central Bank) Bank Interconnected Coordination Group (BICG). Has been leading QBT since its founding in 2019.

PM4: Yoann Jestin

CEO, Ki3 Photonics Technologies Inc. Principal Investigator of the project. B.Sc in Chemistry, University of Rennes, 1997; M.Sc. in Material Science, University of Rennes, 1999; Ph.D in Material Science, University of Le Mans, 2002.

PDF1: Jie Lin

QBT quantum research scientist with specializations in quantum optics and security analysis of quantum key distribution. Will lead development of both fusion gates and qubit encoding. B. A. Mathematics, Carleton College; M. A. S. Theoretical Physics, University of Cambridge; M. Sc. Physics and Ph.D. Physics (quantum information), University of Waterloo.

PDF2: Luc Robichaud

QBT quantum research scientist who specializes in quantum dot modeling and their application to entanglement generation. Will lead modeling of various sources and incorporating said models into the framework. Will also co-lead with PDF3 the writing of papers for peer-reviewed journals. B.Sc. Physics and M.Sc. Physics, Université de Moncton;

Ph.D. Physics, University of Ottawa.

PDF3: Stefania Sciara

Postdoc at l'Institut National de la Recherche Scientifique (INRS) and Ki3 researcher with specializations in both experimental and theoretical quantum optics, and entanglement generation. B.Sc. Physics and M.Sc. Physics, University of Palermo; Ph.D. Quantum optics, INRS.

PhD1: Sobhan Ghanbari

QBT intern. Ph.D. candidate at the University of Toronto with specialization in quantum information. Sobhan has led and will continue to lead development of various noise models within the framework. B.Sc. Physics/ B.Sc. Mechanical Engineering (dual- degree), Sharif University of Technology.

PhD2: Benjamin MacLellan

Ph.D. candidate at the University of Waterloo with specialization in quantum information and inverse design of photonic systems. Will lead the development of a more comprehensive physical layer, including various integrated-photonics-based components. B.Sc. Physics, University of Waterloo; M.Sc., Energy & Materials, INRS.

M1: Kai Sum Chan

QBT quantum research scientist. M.Phil. candidate at the University of Hong Kong, and QBT experimental and theoretical quantum researcher with specialization in quantum information, adaptive optics and quantum dots. Will play a supporting role in calibrating models according to experiments.

I1: Khuong Tran

B.Sc. candidate in computer sciences at the University of Ontario Institute of Technology and intern at Ki3. Will co-lead with I2 preparation of open-sourcing, framework documentation and tutorials.

I2: Aayush Soni

B.A.Sc. candidate in mechatronics engineering at the University of Waterloo and intern at Ki3. Will co-lead with I1 the preparation of open-sourcing, framework documentation and tutorials.

Key: PM: Project manager, PDF: Postdoctoral fellow, PhD: PhD student, M: Master's student, I: Intern.

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Highly Entangled States of Light

Final Report

Awarded to:
Ki3 Photonics Technologies Inc.
Quantum Bridge Technologies Inc (QBT).

Recipient Principal Investigator:
Dr. Yoann Jestin

1. Executive summary

Problem statement:

- Graph quantum states are an important resource for fundamental research and technological applications
- Designing quantum circuits that output specific and desired graph states is challenging
- Real quantum circuits are subject to noise and loss, which are necessary to include for an accurate and realistic model

Project scope:

- Construct a theoretical and numerical framework for the study of graph states and of circuits that generate them
- Develop schemes for the construction of quantum circuits that output desired graph states
- Create models for the noise and loss effects that are present in real quantum setups

General outcome:

- To date, we have defined a framework that allows us to find and to simulate quantum circuits that output low-complexity graph states
- Our schemes successfully reproduce results from literature and are also capable of finding alternative setups to choose between
- We have developed noise and loss models that are representative of effects found in common photonics platforms
- Our framework is a flexible and extensible tool that is soon to be open-sourced and which can be easily adapted to simulate various physical platforms

Future Plans:

- Maintain and develop the framework alongside the open-source quantum community to find solutions for higher complexity graph states
- Develop fusion-gate based protocols for producing large graph states
- Implement encoding strategies for qubit protection
- Model quantum-dot-based sources and interface with current framework
- Develop models for integrated-photonics-based components as an experimental basis for quantum circuits
- Develop more sophisticated noise and distribution channel models
- Extend our ability to inject experimental details into our framework, further connecting results with real setups

2. Introduction

Graph states are a family of quantum states with high levels of entanglement that are of great fundamental interest [1], as well as being a key resource for applications in entanglement distribution and quantum networking [2-4], quantum error correction [5,6], and quantum metrology [7]. Among various physical platforms, photonics shows much promise for graph state realizations [8,9] as photons interact very weakly with their environment, enabling stable qubits. Additionally, photons are compatible with mature integrated and fiber technologies, granting additional support in the generation and distribution of graph states.

As well, for the generation of photons, various sources are available within photonics-based platforms, including solid-state emitters such as quantum dots that can emit single photons on demand [10], or nonlinear elements generating pairs of entangled photons [11,12].

Quantum circuits describe the sequence of quantum operations necessary to generate such graph states. However, connecting a desired graph state with the circuit that can output it is a non-trivial design task that only becomes more difficult with increasing state size and complexity. Recent work [13] has led to the development of an impressive deterministic method for finding quantum circuits that produce targeted graph states. However, such methods do not guarantee well-performing quantum circuits, allow little flexibility, and cannot consider impacts associated with noise or photonic loss, which is present in all real setups.

Towards addressing this research gap, the goal of our work was to develop a flexible theoretical and computational framework for the study of realistic graph states and of the quantum circuits that generate them. To accomplish this, we must be able to represent both the quantum states and the quantum circuits that produce them. Both representations must also be flexible enough to support noise and photonic loss modeling. The performance of different circuits must also be quantifiable, in order to drive schemes that search for best performing setups. All of these concepts must also be realized in a flexible and modular way that allows for co-development of solutions with the broader research community, driving collaborative and fundamental work into the understanding of entangled states as a resource.

Over the past year of our project's progress, we have successfully identified and realized a framework comprising the following core concepts: *Quantum state representations*, *quantum circuit representations*, *realistic component models*, *quantum circuit performance metrics*, and *quantum circuit optimization schemes*. Figure 1 shows these key concepts and their mutual interaction within our framework.

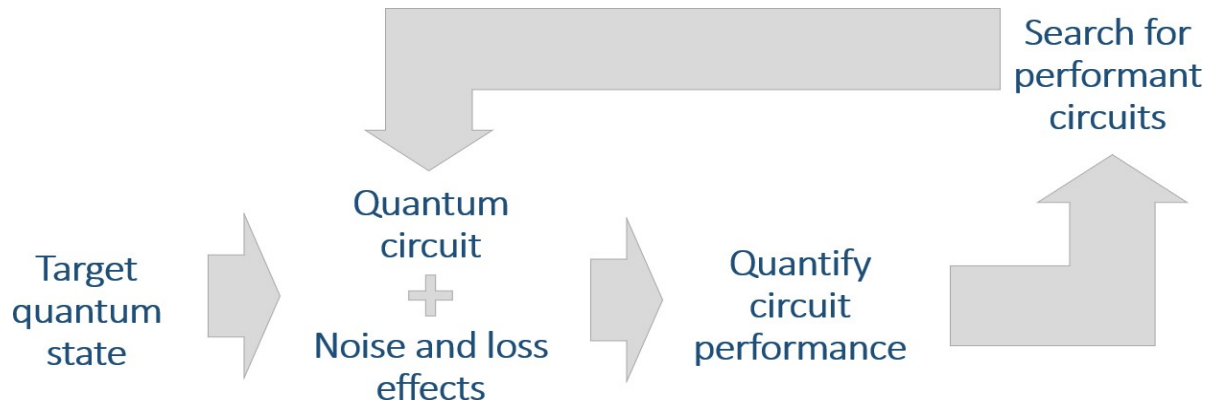


Figure 1: Key concepts within our framework. The performance metric is calculated with respect to the target graph state, guiding the search for performant circuits.

3. Accomplishments

In what follows, we explain each of the core concepts of our framework alongside our respective research efforts in developing them.

3.1. Research objectives

3.3.1. Representing quantum states

The objects of interest, graph states, are quantum systems of photonic and emitter qubits that can be mathematically represented using various formalisms. Our computational framework presently considers two mathematical representations, the *density matrix* and the *stabilizer*, with the modularity to easily include further representations in the future.

3.1.2. Representing quantum circuits

Quantum circuits represent the sequence of quantum gates and operations applied on solid state emitter qubits and photonic qubits. To computationally read, save, and manipulate these quantum circuits, we represent them as Directed Acyclic Graphs (DAGs) [19], which find common use in the field of quantum information [20]. Using the DAG representation, our computational framework can easily modify and evolve the quantum circuit topology to optimize the generated quantum state towards a target graph state. For the visualization of quantum circuits, our DAG representation was also made compatible with the openQASM format, a standard within the field [21].

3.1.3. Realistic component models

Towards increasing the realism of our framework, we support diverse models for noise and photonic loss. For example, our framework presently includes noise models for depolarizing noise, amplitude damping, and Pauli noise.

Additionally, we have developed a new representation to model mixed states using stabilizers. This new representation keeps track of all potential branchings of a mixed state, along with the probability of such a branch being measured. This novel approach allows us to consider probabilistic noise, more akin to real experimental systems.

Motivated by the physics of today's solid-state emitter and photonic qubit systems, we have furthermore developed a time-dependent noise model within the stabilizer formalism. In experiments, the number of errors that occur is strongly correlated with the time spent during and between each quantum gate. In our time-dependent model, we make use of an effective noise (simulated only at certain gates), maintaining model accuracy while reducing the computational load of the model.

3.1.4. Assessing circuit performance

When optimizing a quantum circuit for the generation of a target graph state, it is important to properly quantify the performance of said circuit using quantitative metrics. In fact, having proper metrics can heavily influence the performance of a circuit search algorithm. To date, we have investigated the following metrics as potential candidates [14], [22], [23]:

Fidelity	Relative entropy
Entanglement witness	Squashed entanglement
Graph edit distance	Adjacency matrix fidelity
Concurrence	Geometric measure
Negativity	Fisher information
Trace distance	Mermin inequality
Entanglement distillation	Stabilizer tableau fidelity

3.1.5. Searching for performant quantum circuits

We have investigated the following schemes for converging on suitable quantum circuits that output a desired graph state:

- **Random:** This scheme initializes a list of random circuits subject to physical

constraints, which we call a population. At each iteration and for each circuit within the population, the algorithm applies a random operation, which includes addition, removal, or replacement of a quantum gate. Each circuit in the population is modified independently from the other circuits. The scheme then keeps track of a certain number of top performing circuits.

- **Evolutionary:** Similarly to the random search approach, this scheme also applies random operations in each iteration. However, in contrast, after each iteration it constructs a new population by putting circuits into a N-wise competition and selecting the best performers.
- **Deterministic:** This scheme is our implementation of Ref. [13]’s approach to deterministically find a quantum circuit for a target graph state.
- **Hybrid:** In this scheme, we use the deterministic method from Ref. [13] to produce an initial solution. We then perturb this solution and use the random/evolutionary schemes to search for quantum circuit alternatives.
- **Gradient-based:** This is a density matrix specific scheme where we consider tunable/parametrized gates that are compatible with automatic differentiation and gradient descent optimization techniques.

3.2 Details of accomplishments

3.2.1. Representing quantum states

The *density matrix* representation can simulate a state’s full Hilbert space and can inherently represent mixed states, such as those resulting from the effects of noise and photonic loss [14]. However, this representation does not scale well with the number of qubits, making it impractical to investigate large graph states, and thus motivating the need for complementary representations. We have used this representation: 1) To study small graph states with the inclusion of noise effects; 2) As a benchmark for other representations; and 3) For the study of noise, photonic loss, mixed states, and gate parametrization.

The *Stabilizer* representation [15], [16], on the other hand, scales well with the number of qubits, making it more efficient for the study of large graph states (relative to the density matrix formalism). However, only a subset of quantum operations can be efficiently simulated within the stabilizer representation, limiting the scope for circuit optimization, and making this representation unsuited for the simulation of noise without judicious changes to the formalism. Presently, the study of representing mixed states within the stabilizer representation is very limited in the literature. To be able to consider large graph states that are mixed due to noise or loss, we have developed a probability tree approach as a way to represent such states. This point of research is presented further below in Sec. 3.3.

We have also developed relevant concepts from *graph* representations, which are also well-suited for many qubits, and contrary to the stabilizer representation, there is more progress made on describing mixed states [17,18]. In particular, we have worked on leveraging graph state equivalency through Clifford gates. As discussed in Sec. 3.4, the graph representation also offers unique metrics for characterizing the performance of quantum circuits.

Each of the quantum state representations has advantages and disadvantages, making it useful to be able to move between them. Most of these representation transformations can be found within literature [3], [14-18], except for obtaining a graph from a density matrix. We have resolved this issue by designing a novel protocol, as discussed in Sec. 3.5.

3.2.2. Representing quantum circuits

Although we have focused on solid state emitter qubits and photonic qubits, we have designed our framework with extensibility in mind. The types of operations, and therefore the

physical circuits we can represent and simulate, are flexible, meaning we can adapt our framework towards different physical platforms with relative ease.

Lastly, our quantum circuit representation also includes parameterized quantum gates and automatic differentiation (see Sec. 3.5 for further discussion).

3.2.3. Realistic component models

The Figure 2 shows an example of the model in practice, deriving the fidelity of an output graph state as a function of qubit error rate from various sources of noise.

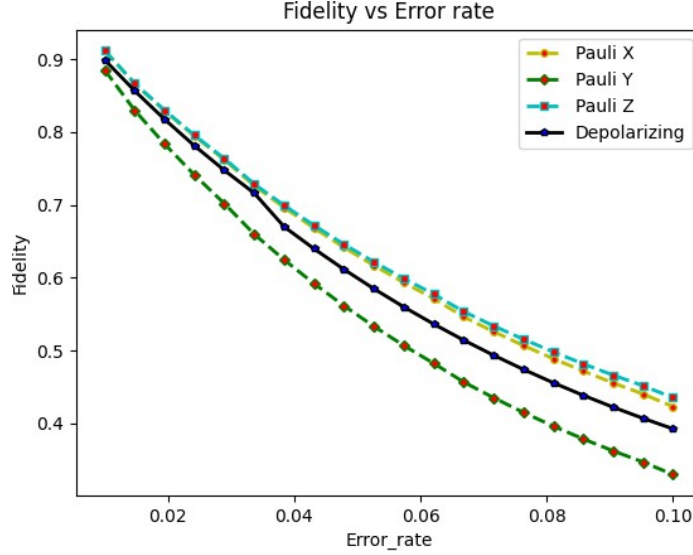


Figure 2: Fidelity versus error rate for the generation of a 4-photon linear cluster state assuming X -gate errors (yellow), Y -gate errors (green), Z -gate errors (blue) and a depolarizing channel (black). We find that the generation of linear cluster states is equally sensitive to bit-flip (X -gate) and phase-flip (Z -gate) errors, and more sensitive when both errors occur (Y -gate).

As photonic losses have non-trivial impacts on quantum states, generally producing mixed states, we have explored various approaches to simulate their effects. One approach is to extend the standard qubit space to a qutrit space so that, by accounting for a vacuum term, the state can still be represented as pure. Another approach is to keep track of the survival probability of a state after potential photonic loss events. Interestingly, we observe that the state fidelity calculation only depends on branches that see no loss, weighted by their respective probabilities. By tracking transmission probabilities, we gain information useful for approximating state generation rates, which are a very important experimental insight. We also calculate the survival probability for each photon, which indicates which portions of the graph state are more prone to loss. This information is essential for efficiently implementing encoding for error correction.

3.2.4. Assessing circuit performance

Fidelity has been our main metric thus far, but it is not ideal for our task as it can change abruptly, which is partially due to the discrete nature of adding or removing quantum operations within the quantum circuit. Entanglement witnesses have potential [24]–[26], but our investigations of GHZ and linear cluster states suggest they may be too robust and not sufficiently monotonic in the neighborhood of optimums. The graph edit metric, which is a metric based on the graph representation of a quantum state, holds potential as it is fundamentally different from other metrics. There are however challenges as it is not obvious

how fundamental graph operations, such as local complementations, would be represented in this metric. Most metrics investigated thus proved to not be useful towards search/optimization schemes or well suited for multipartite states.

3.2.5. Searching for performant quantum circuits

Using the random and evolutionary schemes within the density matrix representation, we were able to:

- Reproduce results from [13] for 4-photon linear cluster states.
- Find alternative, but equivalent circuits. Given some circuits may be more sensitive to noise compared to others or that some gates may be more difficult to implement than others, this gives options to experimentalists in implementing whichever protocol is more practical.
- Find cases where the circuit depth is reduced relative to circuits produced by Ref. [13]'s approach. An example of this depth decrease is shown in Fig. 3. Decreases in circuit depth imply a simplified protocol as there are fewer gates to be implemented or more gates can be implemented in parallel.
- In terms of convergence, we found that the random and evolutionary schemes performed similarly. Figure 4 shows the fraction of search runs that converge on a quantum circuit producing the target graph state.

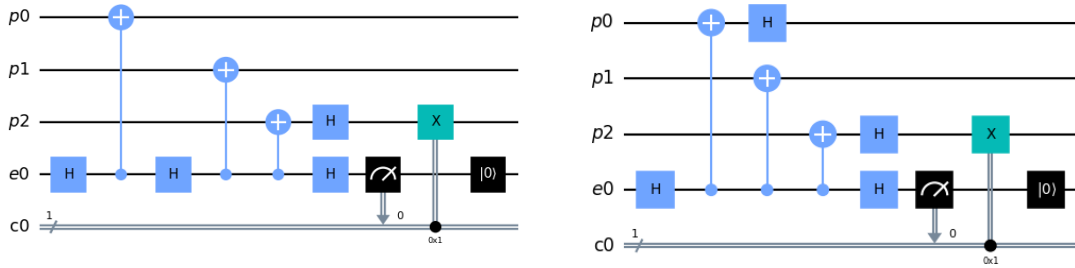


Figure 3: Quantum circuits for the generation of a 3-photon linear cluster state as determined by (left) the deterministic methods of [13] and (right) our framework. Our solution presents a reduced circuit depth relative to the deterministic method.

Using the stabilizer representation, we are able to efficiently simulate large graph states, up to hundreds of qubits, but it remains difficult to converge to an optimal circuit due to the vast optimization landscape. For this reason, we have also investigated the hybrid approach, which leverages the deterministic scheme of Ref. [13] to produce a starting point for random/evolutionary-based search approaches. Using this method, we have been able to find alternative quantum circuits to those proposed by the deterministic approach, as well as access larger graph states.

Additionally, the deterministic solutions do not include the effects of noise, which degrade the quality of the output graph state for a given circuit. Using our hybrid scheme, we have begun investigations into re-optimizing the deterministic solution after the introduction of noise, to regain state fidelity. This investigation is expected to greatly benefit from experimental feedback to more accurately represent noise found within real photonic platforms.



	Linear Cluster (3 qubits)	GHZ (3 qubits)	Linear Cluster (4 qubits)	GHZ (4 qubits)
Random scheme	92%	60%	24%	24%
Evolutionary scheme	96%	16%	24%	28%

Figure 4: Fraction of simulations that converge on a solution for a quantum circuit using a density matrix representation. Target graph states for each case are indicated above columns. Both the random and evolutionary schemes have difficulty converging with an increasing number of qubits. This is due to the vastness of the optimization landscape, indicating the need for better metrics and/or optimization schemes.

Within the density matrix representation, we have also implemented parameterized quantum gates and the automatic differentiation of quantum circuits. This enables the gradient-based optimization of circuit evaluation metrics (using, e.g., gradient-descent optimizers). This aspect of our framework enables quantum circuits to be optimized in the presence of noise, and enables gradient descent to be combined with circuit topology optimization schemes. An example of this scheme in action is shown in Fig. 5, where we perform quantum gate optimization to recover fidelity lost due to noise. Although the noise model requires refinement to accurately represent experimentally-found errors, these are promising results for managing noise-related deterioration of graph states.

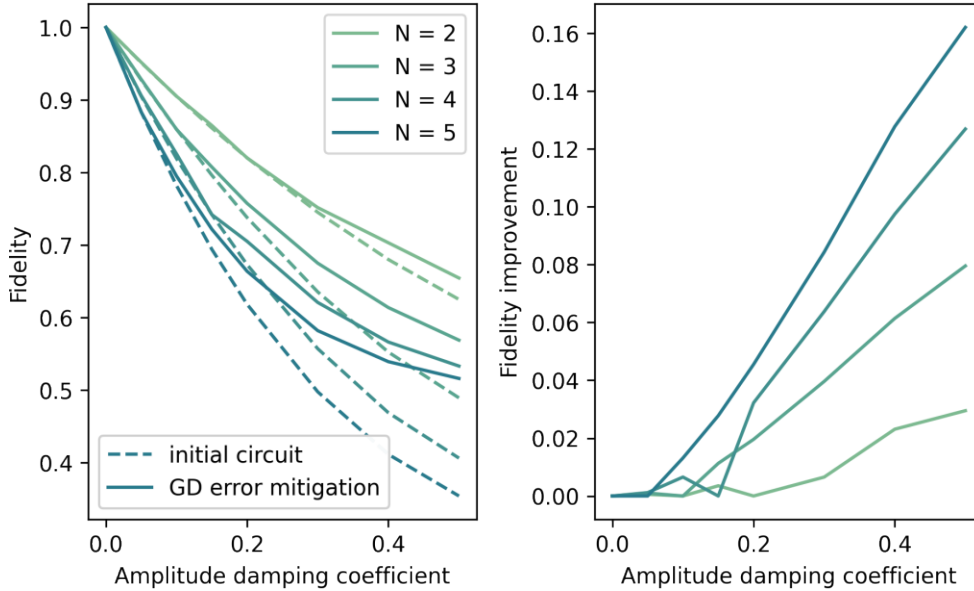


Figure 5: Regaining lost fidelity by parametrized gates optimization for linear cluster states consisting of $N=1-5$ photons as function of noise amplitude, where an amplitude damping model was uniformly applied on all qubits. Left panel shows the fidelity of the initial circuit after introducing noise (dashed) and the fidelity after gate optimization (solid). Right panel shows the relative difference between the solid and dashed lines. We find that gate optimization led to absolute fidelity recoveries of up to 15%.

3.2.5. Summary of accomplishments

During year 1 of this project, we have accomplished the following basic research milestones:

- Constructed a theoretical and numerical framework for the study of graph states and of the quantum circuits that generate them within photonics-based platforms.
- Reproduced known quantum circuits for the generation of graph states found within recent literature. In some cases, such as a 3-photon GHZ state, we have found alternative, yet equivalent quantum circuits, providing experimentalists with setup variety.
- For a 3-photon linear cluster state, we have found an improved quantum circuit with a shorter depth relative to published setups.
- Our framework supports diverse noise models, including depolarizing noise, amplitude damping, and Pauli errors.
- We have developed an experimentally-motivated, time-dependent noise model leveraging the stabilizer representation. This model reduces the number of potential outcomes within mixed states and expresses the noise in terms of circuit time-scales.
- Demonstrated that the use of parameterized gates in conjunction with gradient-based optimization can remediate the impacts of noise-driven fidelity degradation, with an absolute fidelity recovery of up to 15% for 5-photon linear cluster states.
- Developed a novel way of representing mixed states using stabilizers, allowing the simulation of more realistic noise effects and circuit dynamics.
- Developed a novel method for generating the graph representation of a quantum state from a density matrix.
- We have developed random and evolutionary schemes that search for well-performing quantum circuits given a target output state.
- We have developed a hybrid quantum circuit search scheme by combining deterministic and stochastic methods.
- Our framework is modular, and can easily be extended to other physical platforms and noise models.

3.2.6. Dissemination to the communities of interest.

Nothing to report. Nevertheless, manuscripts are in preparation and will be shared with the program officer.

4. Impacts

4.1. Development of the principal discipline of the project

We have developed a framework for the study and optimization of graph state generation in the presence of realistic effects such as noise and photonic loss, making it the first of its kind. Such a tool is a driver for further basic research into entangled resource preparation and distribution for various applications.

The framework is implemented in Python and was designed to be extensible, with community co-development in mind. To support the community and future partners in using these tools, we have prepared thorough documentation, code examples, and interactive tutorials.

Our long-term plans are discussed in detail in Sec. 6, but our near-term plan for disseminating our work to the community includes the following:

- Make our framework available to the community by open-sourcing our codebase

alongside documentation, use examples, and tutorials

- Use the open-sourced code to enhance and establish research partnerships
- Prepare abstracts for conferences such as the International Conference on Quantum Communication, Measurement and Computing (QCMC), Q2B22 Silicon Valley, Photonics West, and CLEO
- Prepare a manuscript for peer-review and publication

4.2. Impact on other disciplines

In addition to the dissemination of our framework, we also intend to collaborate directly with colleagues from the community. This includes collaborating with the National Research Council of Canada (NRC), specialized in the fabrication of quantum dots that can be used as emitter qubits, l'Institut National de la Recherche Scientifique (INRS), giving us access to knowledge on integrated photonics, as well as Caltech and Fermilab, who can provide realistic entanglement distribution channel metrics from their respective testbeds. Additionally, Quantum Bridge Technologies, Inc. (QBT) is a University of Toronto spin-off, with many local research collaboration opportunities. Our collaborations will allow us to test and validate our framework, as well as improve its level of realism.

5. Future goals

As part of future work, our team intends to maintain and develop our framework alongside the open-source quantum community to find solutions for higher complexity graph states. In parallel, our work internally will focus on extending the framework with more numerous and realistic components as well as qubit protection strategies. To perform this work, we hope for continued support of the project in the years to come and provide below a Gantt chart presenting our future project goals for the following year. Indicated within each task are assigned lead researchers, with a team overview following the chart.

Task description	Months											
	M1-M4				M4-M8				M8-M12			
Modeling fusion gate components Using fusion gates [27], we can form large graph states from smaller graph states. We will implement such gates within our framework, giving an alternative route to realizing graph states. This task will be led by PDF1 and supported by PDF2 and PhD1.												
Literature review and implementing fusion gates within framework												
Investigate fusion gate-based quantum circuits												
Encoding for error correction and fusion gate protection We will explore encoding methods for protecting qubits against random errors (e.g., bitflip) and failed fusion gate attempts, which lead to the loss of a qubit. This task will be led by PDF1, and supported by PDF2 and PhD1.												
Investigate encoding strategies in literature												
Implement encoding for qubit protection												

Open innovation and community impact As discussed in Sec. 5, we aim to disseminate our work to support the quantum research community. Open-sourcing will be co-led by I1 and I2. Reaching out to community members will be co-led by PM1, PM2, PM3 and PM4. The writing of a manuscript will be co-led by PDF2 and PDF3 with support from the full team.												
Publish codebase and finalize public documentation and tutorials												
Prepare manuscript and submit to journal												
Project management and administrative support Project management will continue to be led by PM1, with the support of PM2, PM3 and PM4.												
Ongoing management and project reporting												

6. Team overview:

PM1: Piotr Roztocki

CTO, Ki3 Photonics Technologies Inc. Principal project manager responsible for day-to-day activities. Honors B.Sc. in Integrated Science (Physics), McMaster University 2014; Ph.D. in Energy and Materials, INRS-EMT 2022.

PM2: Hoi-Kwong Lo

CTO, Quantum Bridge Technologies, Inc. (QBT). Plays a supervisory role in the project. Professor of Physics and ECE at the University of Toronto, BA in Mathematics, Cambridge University, 1989, Ph.D. in Physics, Caltech, 1994. Co-inventor of measurement-device-independent quantum key distribution (QKD) and all photonics quantum repeaters. Fellow of the American Physical Society (APS) and Optica Fellow.

PM3: Mattia Montagna

CEO, Quantum Bridge Technologies, Inc. (QBT), PhD in Applied Math. Founder of the ECB (European Central Bank) Bank Interconnected Coordination Group (BICG). Has been leading QBT since its founding in 2019.

PM4: Yoann Jestin

CEO, Ki3 Photonics Technologies Inc. Principal Investigator of the project. B.Sc in Chemistry, University of Rennes, 1997; M.Sc. in Material Science, University of Rennes, 1999; Ph.D in Material Science, University of Le Mans, 2002.

PDF1: Jie Lin

QBT quantum research scientist with specializations in quantum optics and security analysis of quantum key distribution. Will lead development of both fusion gates and qubit encoding. B. A. Mathematics, Carleton College; M. A. S. Theoretical Physics, University of Cambridge; M. Sc. Physics and Ph.D. Physics (quantum information), University of Waterloo.

PDF2: Luc Robichaud

QBT quantum research scientist who specializes in quantum dot modeling and their application to entanglement generation. Will lead modeling of various sources and incorporating said models into the framework. Will also co-lead with PDF3 the writing of papers for peer-reviewed journals. B.Sc. Physics and M.Sc. Physics, Université de Moncton;

Ph.D. Physics, University of Ottawa.

PDF3: Stefania Sciara

Postdoc at l'Institut National de la Recherche Scientifique (INRS) and Ki3 researcher with specializations in both experimental and theoretical quantum optics, and entanglement generation. B.Sc. Physics and M.Sc. Physics, University of Palermo; Ph.D. Quantum optics, INRS.

PhD1: Sobhan Ghanbari

QBT intern. Ph.D. candidate at the University of Toronto with specialization in quantum information. Sobhan has led and will continue to lead development of various noise models within the framework. B.Sc. Physics/ B.Sc. Mechanical Engineering (dual- degree), Sharif University of Technology.

PhD2: Benjamin MacLellan

Ph.D. candidate at the University of Waterloo with specialization in quantum information and inverse design of photonic systems. Will lead the development of a more comprehensive physical layer, including various integrated-photonics-based components. B.Sc. Physics, University of Waterloo; M.Sc., Energy & Materials, INRS.

M1: Kai Sum Chan

QBT quantum research scientist. M.Phil. candidate at the University of Hong Kong, and QBT experimental and theoretical quantum researcher with specialization in quantum information, adaptive optics and quantum dots. Will play a supporting role in calibrating models according to experiments.

I1: Khuong Tran

B.Sc. candidate in computer sciences at the University of Ontario Institute of Technology and intern at Ki3. Will co-lead with I2 preparation of open-sourcing, framework documentation and tutorials.

I2: Aayush Soni

B.A.Sc. candidate in mechatronics engineering at the University of Waterloo and intern at Ki3. Will co-lead with I1 the preparation of open-sourcing, framework documentation and tutorials.

Key: PM: Project manager, PDF: Postdoctoral fellow, PhD: PhD student, M: Master's student, I: Intern.

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Highly Entangled States of Light

Final Report

Awarded to:
Ki3 Photonics Technologies Inc.
Quantum Bridge Technologies Inc (QBT).

Recipient Principal Investigator:
Dr. Yoann Jestin

1. Executive summary

Problem statement:

- Graph quantum states are an important resource for fundamental research and technological applications
- Designing quantum circuits that output specific and desired graph states is challenging
- Real quantum circuits are subject to noise and loss, which are necessary to include for an accurate and realistic model

Project scope:

- Construct a theoretical and numerical framework for the study of graph states and of circuits that generate them
- Develop schemes for the construction of quantum circuits that output desired graph states
- Create models for the noise and loss effects that are present in real quantum setups

General outcome:

- To date, we have defined a framework that allows us to find and to simulate quantum circuits that output low-complexity graph states
- Our schemes successfully reproduce results from literature and are also capable of finding alternative setups to choose between
- We have developed noise and loss models that are representative of effects found in common photonics platforms
- Our framework is a flexible and extensible tool that is soon to be open-sourced and which can be easily adapted to simulate various physical platforms

Future Plans:

- Maintain and develop the framework alongside the open-source quantum community to find solutions for higher complexity graph states
- Develop fusion-gate based protocols for producing large graph states
- Implement encoding strategies for qubit protection
- Model quantum-dot-based sources and interface with current framework
- Develop models for integrated-photonics-based components as an experimental basis for quantum circuits
- Develop more sophisticated noise and distribution channel models
- Extend our ability to inject experimental details into our framework, further connecting results with real setups

2. Introduction

Graph states are a family of quantum states with high levels of entanglement that are of great fundamental interest [1], as well as being a key resource for applications in entanglement distribution and quantum networking [2-4], quantum error correction [5,6], and quantum metrology [7]. Among various physical platforms, photonics shows much promise for graph state realizations [8,9] as photons interact very weakly with their environment, enabling stable qubits. Additionally, photons are compatible with mature integrated and fiber technologies, granting additional support in the generation and distribution of graph states.

As well, for the generation of photons, various sources are available within photonics-based platforms, including solid-state emitters such as quantum dots that can emit single photons on demand [10], or nonlinear elements generating pairs of entangled photons [11,12].

Quantum circuits describe the sequence of quantum operations necessary to generate such graph states. However, connecting a desired graph state with the circuit that can output it is a non-trivial design task that only becomes more difficult with increasing state size and complexity. Recent work [13] has led to the development of an impressive deterministic method for finding quantum circuits that produce targeted graph states. However, such methods do not guarantee well-performing quantum circuits, allow little flexibility, and cannot consider impacts associated with noise or photonic loss, which is present in all real setups.

Towards addressing this research gap, the goal of our work was to develop a flexible theoretical and computational framework for the study of realistic graph states and of the quantum circuits that generate them. To accomplish this, we must be able to represent both the quantum states and the quantum circuits that produce them. Both representations must also be flexible enough to support noise and photonic loss modeling. The performance of different circuits must also be quantifiable, in order to drive schemes that search for best performing setups. All of these concepts must also be realized in a flexible and modular way that allows for co-development of solutions with the broader research community, driving collaborative and fundamental work into the understanding of entangled states as a resource.

Over the past year of our project's progress, we have successfully identified and realized a framework comprising the following core concepts: *Quantum state representations*, *quantum circuit representations*, *realistic component models*, *quantum circuit performance metrics*, and *quantum circuit optimization schemes*. Figure 1 shows these key concepts and their mutual interaction within our framework.

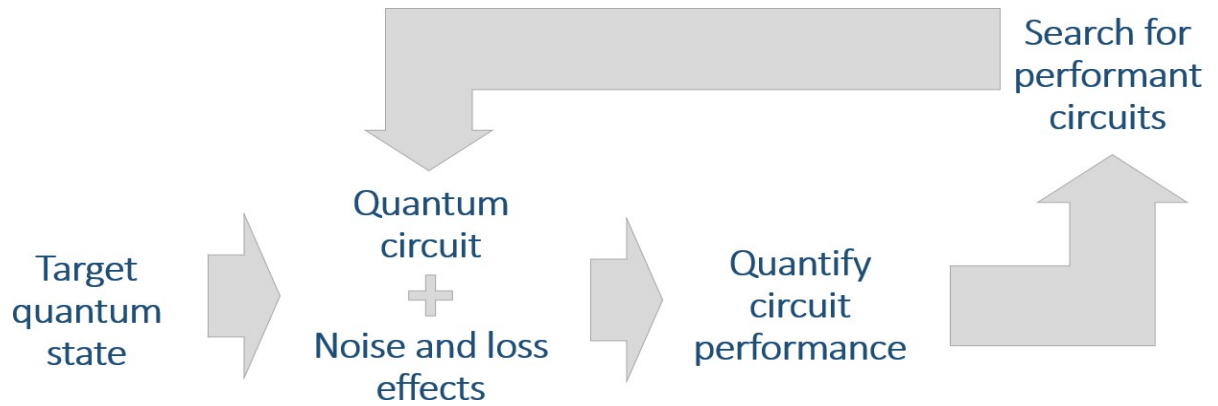


Figure 1: Key concepts within our framework. The performance metric is calculated with respect to the target graph state, guiding the search for performant circuits.

3. Accomplishments

In what follows, we explain each of the core concepts of our framework alongside our respective research efforts in developing them.

3.1. Research objectives

3.3.1. Representing quantum states

The objects of interest, graph states, are quantum systems of photonic and emitter qubits that can be mathematically represented using various formalisms. Our computational framework presently considers two mathematical representations, the *density matrix* and the *stabilizer*, with the modularity to easily include further representations in the future.

3.1.2. Representing quantum circuits

Quantum circuits represent the sequence of quantum gates and operations applied on solid state emitter qubits and photonic qubits. To computationally read, save, and manipulate these quantum circuits, we represent them as Directed Acyclic Graphs (DAGs) [19], which find common use in the field of quantum information [20]. Using the DAG representation, our computational framework can easily modify and evolve the quantum circuit topology to optimize the generated quantum state towards a target graph state. For the visualization of quantum circuits, our DAG representation was also made compatible with the openQASM format, a standard within the field [21].

3.1.3. Realistic component models

Towards increasing the realism of our framework, we support diverse models for noise and photonic loss. For example, our framework presently includes noise models for depolarizing noise, amplitude damping, and Pauli noise.

Additionally, we have developed a new representation to model mixed states using stabilizers. This new representation keeps track of all potential branchings of a mixed state, along with the probability of such a branch being measured. This novel approach allows us to consider probabilistic noise, more akin to real experimental systems.

Motivated by the physics of today's solid-state emitter and photonic qubit systems, we have furthermore developed a time-dependent noise model within the stabilizer formalism. In experiments, the number of errors that occur is strongly correlated with the time spent during and between each quantum gate. In our time-dependent model, we make use of an effective noise (simulated only at certain gates), maintaining model accuracy while reducing the computational load of the model.

3.1.4. Assessing circuit performance

When optimizing a quantum circuit for the generation of a target graph state, it is important to properly quantify the performance of said circuit using quantitative metrics. In fact, having proper metrics can heavily influence the performance of a circuit search algorithm. To date, we have investigated the following metrics as potential candidates [14], [22], [23]:

Fidelity	Relative entropy
Entanglement witness	Squashed entanglement
Graph edit distance	Adjacency matrix fidelity
Concurrence	Geometric measure
Negativity	Fisher information
Trace distance	Mermin inequality
Entanglement distillation	Stabilizer tableau fidelity

3.1.5. Searching for performant quantum circuits

We have investigated the following schemes for converging on suitable quantum circuits that output a desired graph state:

- **Random:** This scheme initializes a list of random circuits subject to physical

constraints, which we call a population. At each iteration and for each circuit within the population, the algorithm applies a random operation, which includes addition, removal, or replacement of a quantum gate. Each circuit in the population is modified independently from the other circuits. The scheme then keeps track of a certain number of top performing circuits.

- **Evolutionary:** Similarly to the random search approach, this scheme also applies random operations in each iteration. However, in contrast, after each iteration it constructs a new population by putting circuits into a N-wise competition and selecting the best performers.
- **Deterministic:** This scheme is our implementation of Ref. [13]’s approach to deterministically find a quantum circuit for a target graph state.
- **Hybrid:** In this scheme, we use the deterministic method from Ref. [13] to produce an initial solution. We then perturb this solution and use the random/evolutionary schemes to search for quantum circuit alternatives.
- **Gradient-based:** This is a density matrix specific scheme where we consider tunable/parametrized gates that are compatible with automatic differentiation and gradient descent optimization techniques.

3.2 Details of accomplishments

3.2.1. Representing quantum states

The *density matrix* representation can simulate a state’s full Hilbert space and can inherently represent mixed states, such as those resulting from the effects of noise and photonic loss [14]. However, this representation does not scale well with the number of qubits, making it impractical to investigate large graph states, and thus motivating the need for complementary representations. We have used this representation: 1) To study small graph states with the inclusion of noise effects; 2) As a benchmark for other representations; and 3) For the study of noise, photonic loss, mixed states, and gate parametrization.

The *Stabilizer* representation [15], [16], on the other hand, scales well with the number of qubits, making it more efficient for the study of large graph states (relative to the density matrix formalism). However, only a subset of quantum operations can be efficiently simulated within the stabilizer representation, limiting the scope for circuit optimization, and making this representation unsuited for the simulation of noise without judicious changes to the formalism. Presently, the study of representing mixed states within the stabilizer representation is very limited in the literature. To be able to consider large graph states that are mixed due to noise or loss, we have developed a probability tree approach as a way to represent such states. This point of research is presented further below in Sec. 3.3.

We have also developed relevant concepts from *graph* representations, which are also well-suited for many qubits, and contrary to the stabilizer representation, there is more progress made on describing mixed states [17,18]. In particular, we have worked on leveraging graph state equivalency through Clifford gates. As discussed in Sec. 3.4, the graph representation also offers unique metrics for characterizing the performance of quantum circuits.

Each of the quantum state representations has advantages and disadvantages, making it useful to be able to move between them. Most of these representation transformations can be found within literature [3], [14-18], except for obtaining a graph from a density matrix. We have resolved this issue by designing a novel protocol, as discussed in Sec. 3.5.

3.2.2. Representing quantum circuits

Although we have focused on solid state emitter qubits and photonic qubits, we have designed our framework with extensibility in mind. The types of operations, and therefore the

physical circuits we can represent and simulate, are flexible, meaning we can adapt our framework towards different physical platforms with relative ease.

Lastly, our quantum circuit representation also includes parameterized quantum gates and automatic differentiation (see Sec. 3.5 for further discussion).

3.2.3. Realistic component models

The Figure 2 shows an example of the model in practice, deriving the fidelity of an output graph state as a function of qubit error rate from various sources of noise.

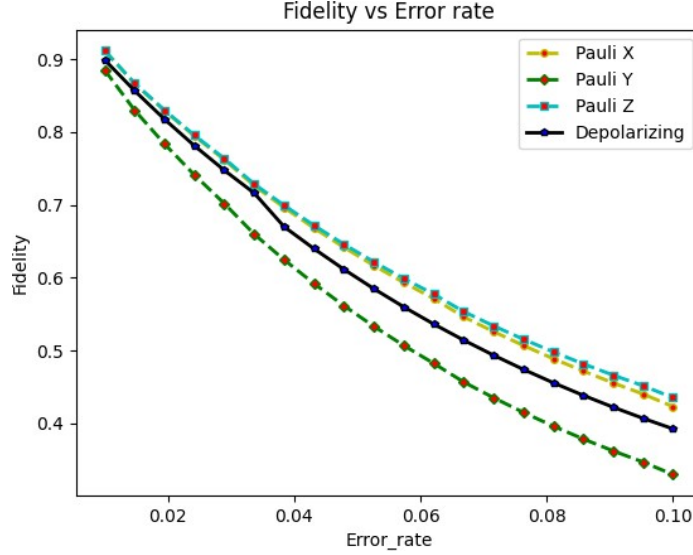


Figure 2: Fidelity versus error rate for the generation of a 4-photon linear cluster state assuming X -gate errors (yellow), Y -gate errors (green), Z -gate errors (blue) and a depolarizing channel (black). We find that the generation of linear cluster states is equally sensitive to bit-flip (X -gate) and phase-flip (Z -gate) errors, and more sensitive when both errors occur (Y -gate).

As photonic losses have non-trivial impacts on quantum states, generally producing mixed states, we have explored various approaches to simulate their effects. One approach is to extend the standard qubit space to a qutrit space so that, by accounting for a vacuum term, the state can still be represented as pure. Another approach is to keep track of the survival probability of a state after potential photonic loss events. Interestingly, we observe that the state fidelity calculation only depends on branches that see no loss, weighted by their respective probabilities. By tracking transmission probabilities, we gain information useful for approximating state generation rates, which are a very important experimental insight. We also calculate the survival probability for each photon, which indicates which portions of the graph state are more prone to loss. This information is essential for efficiently implementing encoding for error correction.

3.2.4. Assessing circuit performance

Fidelity has been our main metric thus far, but it is not ideal for our task as it can change abruptly, which is partially due to the discrete nature of adding or removing quantum operations within the quantum circuit. Entanglement witnesses have potential [24]–[26], but our investigations of GHZ and linear cluster states suggest they may be too robust and not sufficiently monotonic in the neighborhood of optimums. The graph edit metric, which is a metric based on the graph representation of a quantum state, holds potential as it is fundamentally different from other metrics. There are however challenges as it is not obvious

how fundamental graph operations, such as local complementations, would be represented in this metric. Most metrics investigated thus proved to not be useful towards search/optimization schemes or well suited for multipartite states.

3.2.5. Searching for performant quantum circuits

Using the random and evolutionary schemes within the density matrix representation, we were able to:

- Reproduce results from [13] for 4-photon linear cluster states.
- Find alternative, but equivalent circuits. Given some circuits may be more sensitive to noise compared to others or that some gates may be more difficult to implement than others, this gives options to experimentalists in implementing whichever protocol is more practical.
- Find cases where the circuit depth is reduced relative to circuits produced by Ref. [13]'s approach. An example of this depth decrease is shown in Fig. 3. Decreases in circuit depth imply a simplified protocol as there are fewer gates to be implemented or more gates can be implemented in parallel.
- In terms of convergence, we found that the random and evolutionary schemes performed similarly. Figure 4 shows the fraction of search runs that converge on a quantum circuit producing the target graph state.

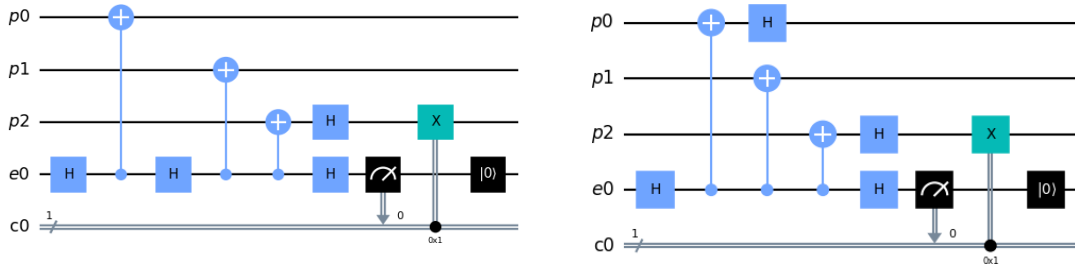


Figure 3: Quantum circuits for the generation of a 3-photon linear cluster state as determined by (left) the deterministic methods of [13] and (right) our framework. Our solution presents a reduced circuit depth relative to the deterministic method.

Using the stabilizer representation, we are able to efficiently simulate large graph states, up to hundreds of qubits, but it remains difficult to converge to an optimal circuit due to the vast optimization landscape. For this reason, we have also investigated the hybrid approach, which leverages the deterministic scheme of Ref. [13] to produce a starting point for random/evolutionary-based search approaches. Using this method, we have been able to find alternative quantum circuits to those proposed by the deterministic approach, as well as access larger graph states.

Additionally, the deterministic solutions do not include the effects of noise, which degrade the quality of the output graph state for a given circuit. Using our hybrid scheme, we have begun investigations into re-optimizing the deterministic solution after the introduction of noise, to regain state fidelity. This investigation is expected to greatly benefit from experimental feedback to more accurately represent noise found within real photonic platforms.



	Linear Cluster (3 qubits)	GHZ (3 qubits)	Linear Cluster (4 qubits)	GHZ (4 qubits)
Random scheme	92%	60%	24%	24%
Evolutionary scheme	96%	16%	24%	28%

Figure 4: Fraction of simulations that converge on a solution for a quantum circuit using a density matrix representation. Target graph states for each case are indicated above columns. Both the random and evolutionary schemes have difficulty converging with an increasing number of qubits. This is due to the vastness of the optimization landscape, indicating the need for better metrics and/or optimization schemes.

Within the density matrix representation, we have also implemented parameterized quantum gates and the automatic differentiation of quantum circuits. This enables the gradient-based optimization of circuit evaluation metrics (using, e.g., gradient-descent optimizers). This aspect of our framework enables quantum circuits to be optimized in the presence of noise, and enables gradient descent to be combined with circuit topology optimization schemes. An example of this scheme in action is shown in Fig. 5, where we perform quantum gate optimization to recover fidelity lost due to noise. Although the noise model requires refinement to accurately represent experimentally-found errors, these are promising results for managing noise-related deterioration of graph states.

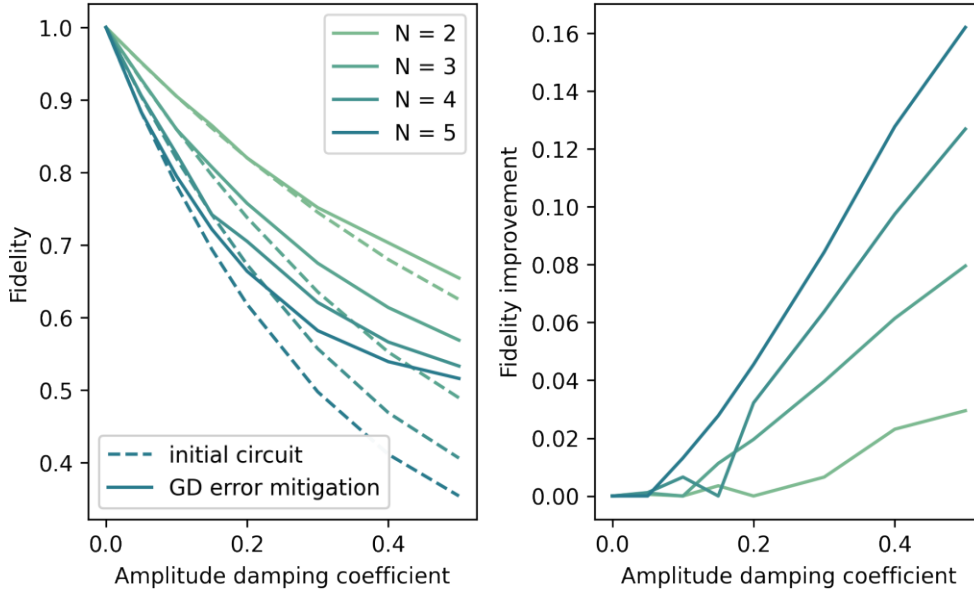


Figure 5: Regaining lost fidelity by parametrized gates optimization for linear cluster states consisting of $N=1-5$ photons as function of noise amplitude, where an amplitude damping model was uniformly applied on all qubits. Left panel shows the fidelity of the initial circuit after introducing noise (dashed) and the fidelity after gate optimization (solid). Right panel shows the relative difference between the solid and dashed lines. We find that gate optimization led to absolute fidelity recoveries of up to 15%.

3.2.5. Summary of accomplishments

During year 1 of this project, we have accomplished the following basic research milestones:

- Constructed a theoretical and numerical framework for the study of graph states and of the quantum circuits that generate them within photonics-based platforms.
- Reproduced known quantum circuits for the generation of graph states found within recent literature. In some cases, such as a 3-photon GHZ state, we have found alternative, yet equivalent quantum circuits, providing experimentalists with setup variety.
- For a 3-photon linear cluster state, we have found an improved quantum circuit with a shorter depth relative to published setups.
- Our framework supports diverse noise models, including depolarizing noise, amplitude damping, and Pauli errors.
- We have developed an experimentally-motivated, time-dependent noise model leveraging the stabilizer representation. This model reduces the number of potential outcomes within mixed states and expresses the noise in terms of circuit time-scales.
- Demonstrated that the use of parameterized gates in conjunction with gradient-based optimization can remediate the impacts of noise-driven fidelity degradation, with an absolute fidelity recovery of up to 15% for 5-photon linear cluster states.
- Developed a novel way of representing mixed states using stabilizers, allowing the simulation of more realistic noise effects and circuit dynamics.
- Developed a novel method for generating the graph representation of a quantum state from a density matrix.
- We have developed random and evolutionary schemes that search for well-performing quantum circuits given a target output state.
- We have developed a hybrid quantum circuit search scheme by combining deterministic and stochastic methods.
- Our framework is modular, and can easily be extended to other physical platforms and noise models.

3.2.6. Dissemination to the communities of interest.

Nothing to report. Nevertheless, manuscripts are in preparation and will be shared with the program officer.

4. Impacts

4.1. Development of the principal discipline of the project

We have developed a framework for the study and optimization of graph state generation in the presence of realistic effects such as noise and photonic loss, making it the first of its kind. Such a tool is a driver for further basic research into entangled resource preparation and distribution for various applications.

The framework is implemented in Python and was designed to be extensible, with community co-development in mind. To support the community and future partners in using these tools, we have prepared thorough documentation, code examples, and interactive tutorials.

Our long-term plans are discussed in detail in Sec. 6, but our near-term plan for disseminating our work to the community includes the following:

- Make our framework available to the community by open-sourcing our codebase

alongside documentation, use examples, and tutorials

- Use the open-sourced code to enhance and establish research partnerships
- Prepare abstracts for conferences such as the International Conference on Quantum Communication, Measurement and Computing (QCMC), Q2B22 Silicon Valley, Photonics West, and CLEO
- Prepare a manuscript for peer-review and publication

4.2. Impact on other disciplines

In addition to the dissemination of our framework, we also intend to collaborate directly with colleagues from the community. This includes collaborating with the National Research Council of Canada (NRC), specialized in the fabrication of quantum dots that can be used as emitter qubits, l'Institut National de la Recherche Scientifique (INRS), giving us access to knowledge on integrated photonics, as well as Caltech and Fermilab, who can provide realistic entanglement distribution channel metrics from their respective testbeds. Additionally, Quantum Bridge Technologies, Inc. (QBT) is a University of Toronto spin-off, with many local research collaboration opportunities. Our collaborations will allow us to test and validate our framework, as well as improve its level of realism.

5. Future goals

As part of future work, our team intends to maintain and develop our framework alongside the open-source quantum community to find solutions for higher complexity graph states. In parallel, our work internally will focus on extending the framework with more numerous and realistic components as well as qubit protection strategies. To perform this work, we hope for continued support of the project in the years to come and provide below a Gantt chart presenting our future project goals for the following year. Indicated within each task are assigned lead researchers, with a team overview following the chart.

Task description	Months											
	M1-M4				M4-M8				M8-M12			
Modeling fusion gate components Using fusion gates [27], we can form large graph states from smaller graph states. We will implement such gates within our framework, giving an alternative route to realizing graph states. This task will be led by PDF1 and supported by PDF2 and PhD1.												
Literature review and implementing fusion gates within framework												
Investigate fusion gate-based quantum circuits												
Encoding for error correction and fusion gate protection We will explore encoding methods for protecting qubits against random errors (e.g., bitflip) and failed fusion gate attempts, which lead to the loss of a qubit. This task will be led by PDF1, and supported by PDF2 and PhD1.												
Investigate encoding strategies in literature												
Implement encoding for qubit protection												

Open innovation and community impact As discussed in Sec. 5, we aim to disseminate our work to support the quantum research community. Open-sourcing will be co-led by I1 and I2. Reaching out to community members will be co-led by PM1, PM2, PM3 and PM4. The writing of a manuscript will be co-led by PDF2 and PDF3 with support from the full team.															
Publish codebase and finalize public documentation and tutorials															
Prepare manuscript and submit to journal															
Project management and administrative support Project management will continue to be led by PM1, with the support of PM2, PM3 and PM4.															
Ongoing management and project reporting															

6. Team overview:

PM1: Piotr Roztocki

CTO, Ki3 Photonics Technologies Inc. Principal project manager responsible for day-to-day activities. Honors B.Sc. in Integrated Science (Physics), McMaster University 2014; Ph.D. in Energy and Materials, INRS-EMT 2022.

PM2: Hoi-Kwong Lo

CTO, Quantum Bridge Technologies, Inc. (QBT). Plays a supervisory role in the project. Professor of Physics and ECE at the University of Toronto, BA in Mathematics, Cambridge University, 1989, Ph.D. in Physics, Caltech, 1994. Co-inventor of measurement-device-independent quantum key distribution (QKD) and all photonics quantum repeaters. Fellow of the American Physical Society (APS) and Optica Fellow.

PM3: Mattia Montagna

CEO, Quantum Bridge Technologies, Inc. (QBT), PhD in Applied Math. Founder of the ECB (European Central Bank) Bank Interconnected Coordination Group (BICG). Has been leading QBT since its founding in 2019.

PM4: Yoann Jestin

CEO, Ki3 Photonics Technologies Inc. Principal Investigator of the project. B.Sc in Chemistry, University of Rennes, 1997; M.Sc. in Material Science, University of Rennes, 1999; Ph.D in Material Science, University of Le Mans, 2002.

PDF1: Jie Lin

QBT quantum research scientist with specializations in quantum optics and security analysis of quantum key distribution. Will lead development of both fusion gates and qubit encoding. B. A. Mathematics, Carleton College; M. A. S. Theoretical Physics, University of Cambridge; M. Sc. Physics and Ph.D. Physics (quantum information), University of Waterloo.

PDF2: Luc Robichaud

QBT quantum research scientist who specializes in quantum dot modeling and their application to entanglement generation. Will lead modeling of various sources and incorporating said models into the framework. Will also co-lead with PDF3 the writing of papers for peer-reviewed journals. B.Sc. Physics and M.Sc. Physics, Université de Moncton;

Ph.D. Physics, University of Ottawa.

PDF3: Stefania Sciara

Postdoc at l'Institut National de la Recherche Scientifique (INRS) and Ki3 researcher with specializations in both experimental and theoretical quantum optics, and entanglement generation. B.Sc. Physics and M.Sc. Physics, University of Palermo; Ph.D. Quantum optics, INRS.

PhD1: Sobhan Ghanbari

QBT intern. Ph.D. candidate at the University of Toronto with specialization in quantum information. Sobhan has led and will continue to lead development of various noise models within the framework. B.Sc. Physics/ B.Sc. Mechanical Engineering (dual- degree), Sharif University of Technology.

PhD2: Benjamin MacLellan

Ph.D. candidate at the University of Waterloo with specialization in quantum information and inverse design of photonic systems. Will lead the development of a more comprehensive physical layer, including various integrated-photonics-based components. B.Sc. Physics, University of Waterloo; M.Sc., Energy & Materials, INRS.

M1: Kai Sum Chan

QBT quantum research scientist. M.Phil. candidate at the University of Hong Kong, and QBT experimental and theoretical quantum researcher with specialization in quantum information, adaptive optics and quantum dots. Will play a supporting role in calibrating models according to experiments.

I1: Khuong Tran

B.Sc. candidate in computer sciences at the University of Ontario Institute of Technology and intern at Ki3. Will co-lead with I2 preparation of open-sourcing, framework documentation and tutorials.

I2: Aayush Soni

B.A.Sc. candidate in mechatronics engineering at the University of Waterloo and intern at Ki3. Will co-lead with I1 the preparation of open-sourcing, framework documentation and tutorials.

Key: PM: Project manager, PDF: Postdoctoral fellow, PhD: PhD student, M: Master's student, I: Intern.

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Grant FA9550-22-1-0062
Highly Entangled States of Light

Final Report

Awarded to:
Ki3 Photonics Technologies Inc.
Quantum Bridge Technologies Inc (QBT).

Recipient Principal Investigator:
Dr. Yoann Jestin

1. Executive summary

Problem statement:

- Graph quantum states are an important resource for fundamental research and technological applications
- Designing quantum circuits that output specific and desired graph states is challenging
- Real quantum circuits are subject to noise and loss, which are necessary to include for an accurate and realistic model

Project scope:

- Construct a theoretical and numerical framework for the study of graph states and of circuits that generate them
- Develop schemes for the construction of quantum circuits that output desired graph states
- Create models for the noise and loss effects that are present in real quantum setups

General outcome:

- To date, we have defined a framework that allows us to find and to simulate quantum circuits that output low-complexity graph states
- Our schemes successfully reproduce results from literature and are also capable of finding alternative setups to choose between
- We have developed noise and loss models that are representative of effects found in common photonics platforms
- Our framework is a flexible and extensible tool that is soon to be open-sourced and which can be easily adapted to simulate various physical platforms

Future Plans:

- Maintain and develop the framework alongside the open-source quantum community to find solutions for higher complexity graph states
- Develop fusion-gate based protocols for producing large graph states
- Implement encoding strategies for qubit protection
- Model quantum-dot-based sources and interface with current framework
- Develop models for integrated-photonics-based components as an experimental basis for quantum circuits
- Develop more sophisticated noise and distribution channel models
- Extend our ability to inject experimental details into our framework, further connecting results with real setups

2. Introduction

Graph states are a family of quantum states with high levels of entanglement that are of great fundamental interest [1], as well as being a key resource for applications in entanglement distribution and quantum networking [2-4], quantum error correction [5,6], and quantum metrology [7]. Among various physical platforms, photonics shows much promise for graph state realizations [8,9] as photons interact very weakly with their environment, enabling stable qubits. Additionally, photons are compatible with mature integrated and fiber technologies, granting additional support in the generation and distribution of graph states.

As well, for the generation of photons, various sources are available within photonics-based platforms, including solid-state emitters such as quantum dots that can emit single photons on demand [10], or nonlinear elements generating pairs of entangled photons [11,12].

Quantum circuits describe the sequence of quantum operations necessary to generate such graph states. However, connecting a desired graph state with the circuit that can output it is a non-trivial design task that only becomes more difficult with increasing state size and complexity. Recent work [13] has led to the development of an impressive deterministic method for finding quantum circuits that produce targeted graph states. However, such methods do not guarantee well-performing quantum circuits, allow little flexibility, and cannot consider impacts associated with noise or photonic loss, which is present in all real setups.

Towards addressing this research gap, the goal of our work was to develop a flexible theoretical and computational framework for the study of realistic graph states and of the quantum circuits that generate them. To accomplish this, we must be able to represent both the quantum states and the quantum circuits that produce them. Both representations must also be flexible enough to support noise and photonic loss modeling. The performance of different circuits must also be quantifiable, in order to drive schemes that search for best performing setups. All of these concepts must also be realized in a flexible and modular way that allows for co-development of solutions with the broader research community, driving collaborative and fundamental work into the understanding of entangled states as a resource.

Over the past year of our project's progress, we have successfully identified and realized a framework comprising the following core concepts: *Quantum state representations*, *quantum circuit representations*, *realistic component models*, *quantum circuit performance metrics*, and *quantum circuit optimization schemes*. Figure 1 shows these key concepts and their mutual interaction within our framework.

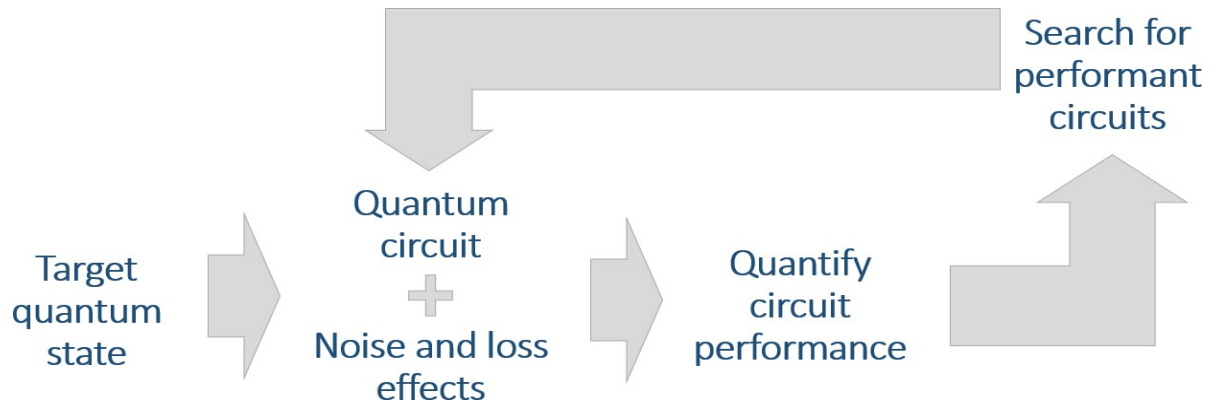


Figure 1: Key concepts within our framework. The performance metric is calculated with respect to the target graph state, guiding the search for performant circuits.

3. Accomplishments

In what follows, we explain each of the core concepts of our framework alongside our respective research efforts in developing them.

3.1. Research objectives

3.3.1. Representing quantum states

The objects of interest, graph states, are quantum systems of photonic and emitter qubits that can be mathematically represented using various formalisms. Our computational framework presently considers two mathematical representations, the *density matrix* and the *stabilizer*, with the modularity to easily include further representations in the future.

3.1.2. Representing quantum circuits

Quantum circuits represent the sequence of quantum gates and operations applied on solid state emitter qubits and photonic qubits. To computationally read, save, and manipulate these quantum circuits, we represent them as Directed Acyclic Graphs (DAGs) [19], which find common use in the field of quantum information [20]. Using the DAG representation, our computational framework can easily modify and evolve the quantum circuit topology to optimize the generated quantum state towards a target graph state. For the visualization of quantum circuits, our DAG representation was also made compatible with the openQASM format, a standard within the field [21].

3.1.3. Realistic component models

Towards increasing the realism of our framework, we support diverse models for noise and photonic loss. For example, our framework presently includes noise models for depolarizing noise, amplitude damping, and Pauli noise.

Additionally, we have developed a new representation to model mixed states using stabilizers. This new representation keeps track of all potential branchings of a mixed state, along with the probability of such a branch being measured. This novel approach allows us to consider probabilistic noise, more akin to real experimental systems.

Motivated by the physics of today's solid-state emitter and photonic qubit systems, we have furthermore developed a time-dependent noise model within the stabilizer formalism. In experiments, the number of errors that occur is strongly correlated with the time spent during and between each quantum gate. In our time-dependent model, we make use of an effective noise (simulated only at certain gates), maintaining model accuracy while reducing the computational load of the model.

3.1.4. Assessing circuit performance

When optimizing a quantum circuit for the generation of a target graph state, it is important to properly quantify the performance of said circuit using quantitative metrics. In fact, having proper metrics can heavily influence the performance of a circuit search algorithm. To date, we have investigated the following metrics as potential candidates [14], [22], [23]:

Fidelity	Relative entropy
Entanglement witness	Squashed entanglement
Graph edit distance	Adjacency matrix fidelity
Concurrence	Geometric measure
Negativity	Fisher information
Trace distance	Mermin inequality
Entanglement distillation	Stabilizer tableau fidelity

3.1.5. Searching for performant quantum circuits

We have investigated the following schemes for converging on suitable quantum circuits that output a desired graph state:

- **Random:** This scheme initializes a list of random circuits subject to physical

constraints, which we call a population. At each iteration and for each circuit within the population, the algorithm applies a random operation, which includes addition, removal, or replacement of a quantum gate. Each circuit in the population is modified independently from the other circuits. The scheme then keeps track of a certain number of top performing circuits.

- **Evolutionary:** Similarly to the random search approach, this scheme also applies random operations in each iteration. However, in contrast, after each iteration it constructs a new population by putting circuits into a N-wise competition and selecting the best performers.
- **Deterministic:** This scheme is our implementation of Ref. [13]’s approach to deterministically find a quantum circuit for a target graph state.
- **Hybrid:** In this scheme, we use the deterministic method from Ref. [13] to produce an initial solution. We then perturb this solution and use the random/evolutionary schemes to search for quantum circuit alternatives.
- **Gradient-based:** This is a density matrix specific scheme where we consider tunable/parametrized gates that are compatible with automatic differentiation and gradient descent optimization techniques.

3.2 Details of accomplishments

3.2.1. Representing quantum states

The *density matrix* representation can simulate a state’s full Hilbert space and can inherently represent mixed states, such as those resulting from the effects of noise and photonic loss [14]. However, this representation does not scale well with the number of qubits, making it impractical to investigate large graph states, and thus motivating the need for complementary representations. We have used this representation: 1) To study small graph states with the inclusion of noise effects; 2) As a benchmark for other representations; and 3) For the study of noise, photonic loss, mixed states, and gate parametrization.

The *Stabilizer* representation [15], [16], on the other hand, scales well with the number of qubits, making it more efficient for the study of large graph states (relative to the density matrix formalism). However, only a subset of quantum operations can be efficiently simulated within the stabilizer representation, limiting the scope for circuit optimization, and making this representation unsuited for the simulation of noise without judicious changes to the formalism. Presently, the study of representing mixed states within the stabilizer representation is very limited in the literature. To be able to consider large graph states that are mixed due to noise or loss, we have developed a probability tree approach as a way to represent such states. This point of research is presented further below in Sec. 3.3.

We have also developed relevant concepts from *graph* representations, which are also well-suited for many qubits, and contrary to the stabilizer representation, there is more progress made on describing mixed states [17,18]. In particular, we have worked on leveraging graph state equivalency through Clifford gates. As discussed in Sec. 3.4, the graph representation also offers unique metrics for characterizing the performance of quantum circuits.

Each of the quantum state representations has advantages and disadvantages, making it useful to be able to move between them. Most of these representation transformations can be found within literature [3], [14-18], except for obtaining a graph from a density matrix. We have resolved this issue by designing a novel protocol, as discussed in Sec. 3.5.

3.2.2. Representing quantum circuits

Although we have focused on solid state emitter qubits and photonic qubits, we have designed our framework with extensibility in mind. The types of operations, and therefore the

physical circuits we can represent and simulate, are flexible, meaning we can adapt our framework towards different physical platforms with relative ease.

Lastly, our quantum circuit representation also includes parameterized quantum gates and automatic differentiation (see Sec. 3.5 for further discussion).

3.2.3. Realistic component models

The Figure 2 shows an example of the model in practice, deriving the fidelity of an output graph state as a function of qubit error rate from various sources of noise.

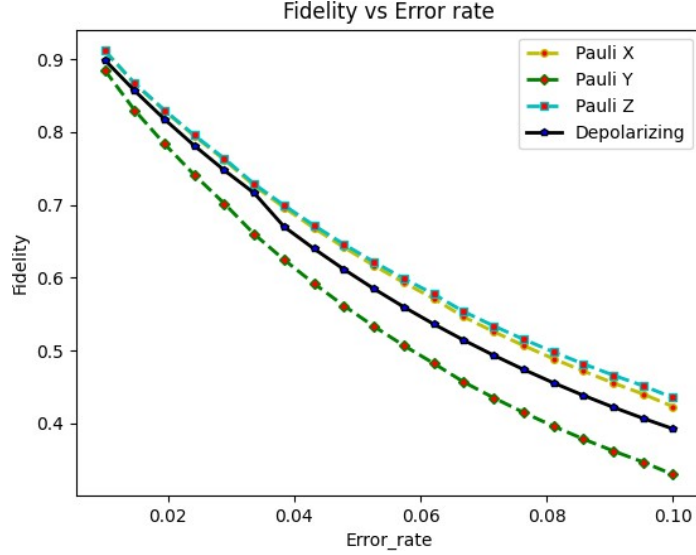


Figure 2: Fidelity versus error rate for the generation of a 4-photon linear cluster state assuming X -gate errors (yellow), Y -gate errors (green), Z -gate errors (blue) and a depolarizing channel (black). We find that the generation of linear cluster states is equally sensitive to bit-flip (X -gate) and phase-flip (Z -gate) errors, and more sensitive when both errors occur (Y -gate).

As photonic losses have non-trivial impacts on quantum states, generally producing mixed states, we have explored various approaches to simulate their effects. One approach is to extend the standard qubit space to a qutrit space so that, by accounting for a vacuum term, the state can still be represented as pure. Another approach is to keep track of the survival probability of a state after potential photonic loss events. Interestingly, we observe that the state fidelity calculation only depends on branches that see no loss, weighted by their respective probabilities. By tracking transmission probabilities, we gain information useful for approximating state generation rates, which are a very important experimental insight. We also calculate the survival probability for each photon, which indicates which portions of the graph state are more prone to loss. This information is essential for efficiently implementing encoding for error correction.

3.2.4. Assessing circuit performance

Fidelity has been our main metric thus far, but it is not ideal for our task as it can change abruptly, which is partially due to the discrete nature of adding or removing quantum operations within the quantum circuit. Entanglement witnesses have potential [24]–[26], but our investigations of GHZ and linear cluster states suggest they may be too robust and not sufficiently monotonic in the neighborhood of optimums. The graph edit metric, which is a metric based on the graph representation of a quantum state, holds potential as it is fundamentally different from other metrics. There are however challenges as it is not obvious

how fundamental graph operations, such as local complementations, would be represented in this metric. Most metrics investigated thus proved to not be useful towards search/optimization schemes or well suited for multipartite states.

3.2.5. Searching for performant quantum circuits

Using the random and evolutionary schemes within the density matrix representation, we were able to:

- Reproduce results from [13] for 4-photon linear cluster states.
- Find alternative, but equivalent circuits. Given some circuits may be more sensitive to noise compared to others or that some gates may be more difficult to implement than others, this gives options to experimentalists in implementing whichever protocol is more practical.
- Find cases where the circuit depth is reduced relative to circuits produced by Ref. [13]'s approach. An example of this depth decrease is shown in Fig. 3. Decreases in circuit depth imply a simplified protocol as there are fewer gates to be implemented or more gates can be implemented in parallel.
- In terms of convergence, we found that the random and evolutionary schemes performed similarly. Figure 4 shows the fraction of search runs that converge on a quantum circuit producing the target graph state.

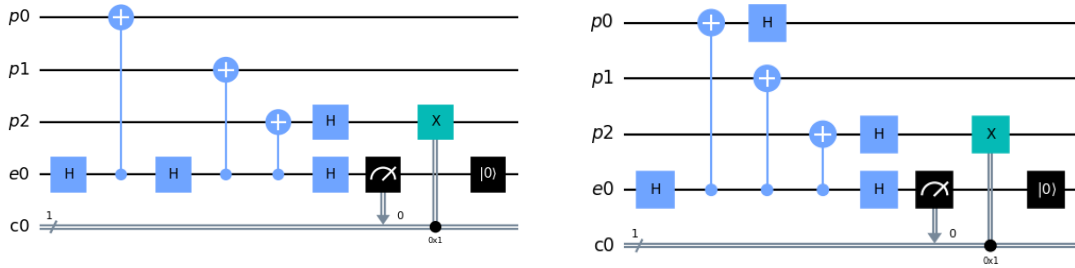


Figure 3: Quantum circuits for the generation of a 3-photon linear cluster state as determined by (left) the deterministic methods of [13] and (right) our framework. Our solution presents a reduced circuit depth relative to the deterministic method.

Using the stabilizer representation, we are able to efficiently simulate large graph states, up to hundreds of qubits, but it remains difficult to converge to an optimal circuit due to the vast optimization landscape. For this reason, we have also investigated the hybrid approach, which leverages the deterministic scheme of Ref. [13] to produce a starting point for random/evolutionary-based search approaches. Using this method, we have been able to find alternative quantum circuits to those proposed by the deterministic approach, as well as access larger graph states.

Additionally, the deterministic solutions do not include the effects of noise, which degrade the quality of the output graph state for a given circuit. Using our hybrid scheme, we have begun investigations into re-optimizing the deterministic solution after the introduction of noise, to regain state fidelity. This investigation is expected to greatly benefit from experimental feedback to more accurately represent noise found within real photonic platforms.



	Linear Cluster (3 qubits)	GHZ (3 qubits)	Linear Cluster (4 qubits)	GHZ (4 qubits)
Random scheme	92%	60%	24%	24%
Evolutionary scheme	96%	16%	24%	28%

Figure 4: Fraction of simulations that converge on a solution for a quantum circuit using a density matrix representation. Target graph states for each case are indicated above columns. Both the random and evolutionary schemes have difficulty converging with an increasing number of qubits. This is due to the vastness of the optimization landscape, indicating the need for better metrics and/or optimization schemes.

Within the density matrix representation, we have also implemented parameterized quantum gates and the automatic differentiation of quantum circuits. This enables the gradient-based optimization of circuit evaluation metrics (using, e.g., gradient-descent optimizers). This aspect of our framework enables quantum circuits to be optimized in the presence of noise, and enables gradient descent to be combined with circuit topology optimization schemes. An example of this scheme in action is shown in Fig. 5, where we perform quantum gate optimization to recover fidelity lost due to noise. Although the noise model requires refinement to accurately represent experimentally-found errors, these are promising results for managing noise-related deterioration of graph states.

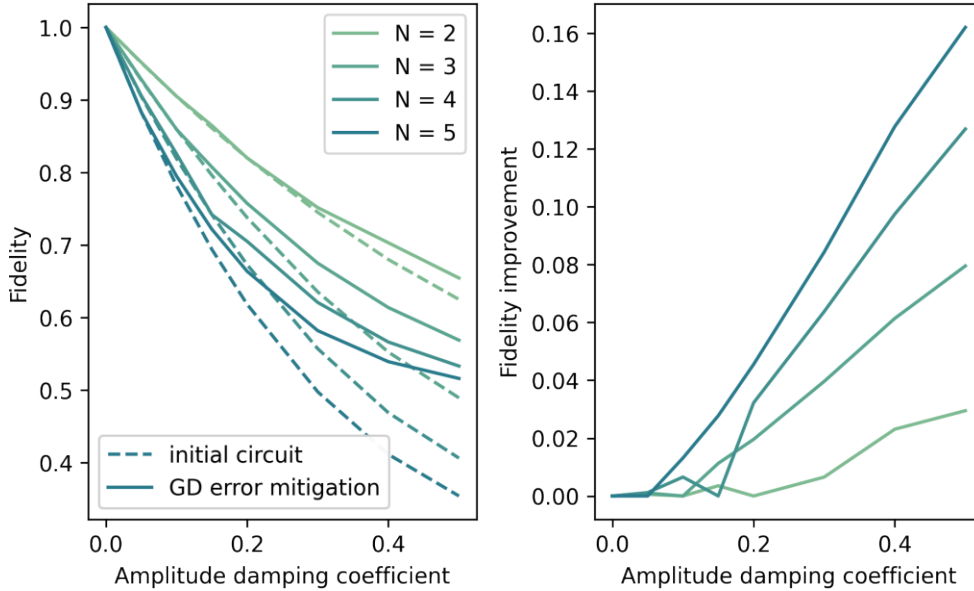


Figure 5: Regaining lost fidelity by parametrized gates optimization for linear cluster states consisting of $N=1-5$ photons as function of noise amplitude, where an amplitude damping model was uniformly applied on all qubits. Left panel shows the fidelity of the initial circuit after introducing noise (dashed) and the fidelity after gate optimization (solid). Right panel shows the relative difference between the solid and dashed lines. We find that gate optimization led to absolute fidelity recoveries of up to 15%.

3.2.5. Summary of accomplishments

During year 1 of this project, we have accomplished the following basic research milestones:

- Constructed a theoretical and numerical framework for the study of graph states and of the quantum circuits that generate them within photonics-based platforms.
- Reproduced known quantum circuits for the generation of graph states found within recent literature. In some cases, such as a 3-photon GHZ state, we have found alternative, yet equivalent quantum circuits, providing experimentalists with setup variety.
- For a 3-photon linear cluster state, we have found an improved quantum circuit with a shorter depth relative to published setups.
- Our framework supports diverse noise models, including depolarizing noise, amplitude damping, and Pauli errors.
- We have developed an experimentally-motivated, time-dependent noise model leveraging the stabilizer representation. This model reduces the number of potential outcomes within mixed states and expresses the noise in terms of circuit time-scales.
- Demonstrated that the use of parameterized gates in conjunction with gradient-based optimization can remediate the impacts of noise-driven fidelity degradation, with an absolute fidelity recovery of up to 15% for 5-photon linear cluster states.
- Developed a novel way of representing mixed states using stabilizers, allowing the simulation of more realistic noise effects and circuit dynamics.
- Developed a novel method for generating the graph representation of a quantum state from a density matrix.
- We have developed random and evolutionary schemes that search for well-performing quantum circuits given a target output state.
- We have developed a hybrid quantum circuit search scheme by combining deterministic and stochastic methods.
- Our framework is modular, and can easily be extended to other physical platforms and noise models.

3.2.6. Dissemination to the communities of interest.

Nothing to report. Nevertheless, manuscripts are in preparation and will be shared with the program officer.

4. Impacts

4.1. Development of the principal discipline of the project

We have developed a framework for the study and optimization of graph state generation in the presence of realistic effects such as noise and photonic loss, making it the first of its kind. Such a tool is a driver for further basic research into entangled resource preparation and distribution for various applications.

The framework is implemented in Python and was designed to be extensible, with community co-development in mind. To support the community and future partners in using these tools, we have prepared thorough documentation, code examples, and interactive tutorials.

Our long-term plans are discussed in detail in Sec. 6, but our near-term plan for disseminating our work to the community includes the following:

- Make our framework available to the community by open-sourcing our codebase

alongside documentation, use examples, and tutorials

- Use the open-sourced code to enhance and establish research partnerships
- Prepare abstracts for conferences such as the International Conference on Quantum Communication, Measurement and Computing (QCMC), Q2B22 Silicon Valley, Photonics West, and CLEO
- Prepare a manuscript for peer-review and publication

4.2. Impact on other disciplines

In addition to the dissemination of our framework, we also intend to collaborate directly with colleagues from the community. This includes collaborating with the National Research Council of Canada (NRC), specialized in the fabrication of quantum dots that can be used as emitter qubits, l'Institut National de la Recherche Scientifique (INRS), giving us access to knowledge on integrated photonics, as well as Caltech and Fermilab, who can provide realistic entanglement distribution channel metrics from their respective testbeds. Additionally, Quantum Bridge Technologies, Inc. (QBT) is a University of Toronto spin-off, with many local research collaboration opportunities. Our collaborations will allow us to test and validate our framework, as well as improve its level of realism.

5. Future goals

As part of future work, our team intends to maintain and develop our framework alongside the open-source quantum community to find solutions for higher complexity graph states. In parallel, our work internally will focus on extending the framework with more numerous and realistic components as well as qubit protection strategies. To perform this work, we hope for continued support of the project in the years to come and provide below a Gantt chart presenting our future project goals for the following year. Indicated within each task are assigned lead researchers, with a team overview following the chart.

Task description	Months											
	M1-M4				M4-M8				M8-M12			
Modeling fusion gate components Using fusion gates [27], we can form large graph states from smaller graph states. We will implement such gates within our framework, giving an alternative route to realizing graph states. This task will be led by PDF1 and supported by PDF2 and PhD1.												
Literature review and implementing fusion gates within framework												
Investigate fusion gate-based quantum circuits												
Encoding for error correction and fusion gate protection We will explore encoding methods for protecting qubits against random errors (e.g., bitflip) and failed fusion gate attempts, which lead to the loss of a qubit. This task will be led by PDF1, and supported by PDF2 and PhD1.												
Investigate encoding strategies in literature												
Implement encoding for qubit protection												

Open innovation and community impact As discussed in Sec. 5, we aim to disseminate our work to support the quantum research community. Open-sourcing will be co-led by I1 and I2. Reaching out to community members will be co-led by PM1, PM2, PM3 and PM4. The writing of a manuscript will be co-led by PDF2 and PDF3 with support from the full team.												
Publish codebase and finalize public documentation and tutorials												
Prepare manuscript and submit to journal												
Project management and administrative support Project management will continue to be led by PM1, with the support of PM2, PM3 and PM4.												
Ongoing management and project reporting												

6. Team overview:

PM1: Piotr Roztocki

CTO, Ki3 Photonics Technologies Inc. Principal project manager responsible for day-to-day activities. Honors B.Sc. in Integrated Science (Physics), McMaster University 2014; Ph.D. in Energy and Materials, INRS-EMT 2022.

PM2: Hoi-Kwong Lo

CTO, Quantum Bridge Technologies, Inc. (QBT). Plays a supervisory role in the project. Professor of Physics and ECE at the University of Toronto, BA in Mathematics, Cambridge University, 1989, Ph.D. in Physics, Caltech, 1994. Co-inventor of measurement-device-independent quantum key distribution (QKD) and all photonics quantum repeaters. Fellow of the American Physical Society (APS) and Optica Fellow.

PM3: Mattia Montagna

CEO, Quantum Bridge Technologies, Inc. (QBT), PhD in Applied Math. Founder of the ECB (European Central Bank) Bank Interconnected Coordination Group (BICG). Has been leading QBT since its founding in 2019.

PM4: Yoann Jestin

CEO, Ki3 Photonics Technologies Inc. Principal Investigator of the project. B.Sc in Chemistry, University of Rennes, 1997; M.Sc. in Material Science, University of Rennes, 1999; Ph.D in Material Science, University of Le Mans, 2002.

PDF1: Jie Lin

QBT quantum research scientist with specializations in quantum optics and security analysis of quantum key distribution. Will lead development of both fusion gates and qubit encoding. B. A. Mathematics, Carleton College; M. A. S. Theoretical Physics, University of Cambridge; M. Sc. Physics and Ph.D. Physics (quantum information), University of Waterloo.

PDF2: Luc Robichaud

QBT quantum research scientist who specializes in quantum dot modeling and their application to entanglement generation. Will lead modeling of various sources and incorporating said models into the framework. Will also co-lead with PDF3 the writing of papers for peer-reviewed journals. B.Sc. Physics and M.Sc. Physics, Université de Moncton;

Ph.D. Physics, University of Ottawa.

PDF3: Stefania Sciara

Postdoc at l'Institut National de la Recherche Scientifique (INRS) and Ki3 researcher with specializations in both experimental and theoretical quantum optics, and entanglement generation. B.Sc. Physics and M.Sc. Physics, University of Palermo; Ph.D. Quantum optics, INRS.

PhD1: Sobhan Ghanbari

QBT intern. Ph.D. candidate at the University of Toronto with specialization in quantum information. Sobhan has led and will continue to lead development of various noise models within the framework. B.Sc. Physics/ B.Sc. Mechanical Engineering (dual- degree), Sharif University of Technology.

PhD2: Benjamin MacLellan

Ph.D. candidate at the University of Waterloo with specialization in quantum information and inverse design of photonic systems. Will lead the development of a more comprehensive physical layer, including various integrated-photonics-based components. B.Sc. Physics, University of Waterloo; M.Sc., Energy & Materials, INRS.

M1: Kai Sum Chan

QBT quantum research scientist. M.Phil. candidate at the University of Hong Kong, and QBT experimental and theoretical quantum researcher with specialization in quantum information, adaptive optics and quantum dots. Will play a supporting role in calibrating models according to experiments.

I1: Khuong Tran

B.Sc. candidate in computer sciences at the University of Ontario Institute of Technology and intern at Ki3. Will co-lead with I2 preparation of open-sourcing, framework documentation and tutorials.

I2: Aayush Soni

B.A.Sc. candidate in mechatronics engineering at the University of Waterloo and intern at Ki3. Will co-lead with I1 the preparation of open-sourcing, framework documentation and tutorials.

Key: PM: Project manager, PDF: Postdoctoral fellow, PhD: PhD student, M: Master's student, I: Intern.

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