AFRL-AFOSR-VA-TR-2022-0232

Dynamical Issues in Space Situational Awareness

Scheeres, Daniel
REGENTS OF THE UNIVERSITY OF COLORADO
3100 MARINE ST 572 UCB
BOULDER, CO,
US

06/10/2022
Final Technical Report

Air Force Research Laboratory
Air Force Office of Scientific Research
Arlington, Virginia 22203
Air Force Materiel Command

REPORT DOCUMENTATION PAGE

PLEASE DO NOT RETURN YOUR FORM TO THE ABOVE ORGANIZATION.

1. REPORT DATE	**2. REPORT TYPE**	**3. DATES COVERED**	
20220610	Final	**START DATE** 20180501	**END DATE** 20221231

4. TITLE AND SUBTITLE
Dynamical Issues in Space Situational Awareness

5a. CONTRACT NUMBER	**5b. GRANT NUMBER**	**5c. PROGRAM ELEMENT NUMBER**
	FA9550-18-1-0313	61102F
5d. PROJECT NUMBER	**5e. TASK NUMBER**	**5f. WORK UNIT NUMBER**

6. AUTHOR(S)
Daniel Scheeres

7. PERFORMING ORGANIZATION NAME(S) AND ADDRESS(ES)	**8. PERFORMING ORGANIZATION REPORT NUMBER**
REGENTS OF THE UNIVERSITY OF COLORADO 3100 MARINE ST 572 UCB BOULDER, CO US	

9. SPONSORING/MONITORING AGENCY NAME(S) AND ADDRESS(ES)	**10. SPONSOR/MONITOR'S ACRONYM(S)**	**11. SPONSOR/MONITOR'S REPORT NUMBER(S)**
Air Force Office of Scientific Research 875 N. Randolph St. Room 3112 Arlington, VA 22203	AFRL/AFOSR RTB1	AFRL-AFOSR-VA-TR-2022-0232

12. DISTRIBUTION/AVAILABILITY STATEMENT
A Distribution Unlimited: PB Public Release

13. SUPPLEMENTARY NOTES

14. ABSTRACT
The objective of this study was to conduct fundamental research in theeld of Space Situational Awareness related to the topics of orbital and rotational dynamics, maneuver detection and the determination of operational bounds of active satellites, and probing fundamental constraints on uncertainty mapping and orbit determination in the space environment. Our goal was to bring a rigorous approach to the study of active and non-active space object dynamics to better understand the dynamics that shape their motion and the change events that active bodies are commanded to undergo. As motivation, results from this research program can be used to develop a better understanding of the evolution of a passive or active space object's state and how models of these eects can be estimated and improved. These studies will also lead to improved tracking strategies to enable the ident cation of hypothesized physical eects, improved modeling techniques to more accuratelyt SSA observations, and the development of methods to analytically discern the actions of controlled space objects based on measurable quantities. Research methods used in this study rely on fundamental astrodynamics, optimal control and dynamical systems theory.
This was a multi-investigator and multi-institution proposal. At the University of Colorado the research is led by co-PI Prof. Daniel J. Scheeres and supports graduate students. At Texas A&M University the research is led by co-PI Prof. Kyle T. Alfriend and also supports a post-doctoral scholar and a graduate student. The current research proposal continued and extended research earlier being carried out by the same team under AFOSR grant FA9550-14-1-0348 with The University of Colorado. Research on this grant was ultimately focused on a set of core topics: Long-term rotational and orbital dynamics of defunct satellites and debris, machine learning applied to orbital dynamics and control, optimal control applied to the space domain, improved models for satellite drag and estimation of atmospheric density, maneuver detection and characterization, 1-Norm orbit determination and covariance based track association. Research in these areas were supported at least in part by the AFOSR grant, but additional sources of support were also leveraged when available, including NASA and NSF fellowships. In the following a summary of results is given.

15. SUBJECT TERMS

16. SECURITY CLASSIFICATION OF:			**17. LIMITATION OF ABSTRACT**	**18. NUMBER OF PAGES**
a. REPORT	**b. ABSTRACT**	**c. THIS PAGE**	UU	9
U	U	U		

19a. NAME OF RESPONSIBLE PERSON	**19b. PHONE NUMBER** *(Include area code)*
MICHAEL YAKES	00000000

Dynamical Issues in Space Situational Awareness
Final Report
Grant# FA9550-18-1-0313

D. J. Scheeres
Department of Aerospace Engineering Sciences
The University of Colorado
scheeres@colorado.edu

K.T. Alfriend
Department of Aerospace Engineering
Texas A&M University

May 27, 2022

Overview

The objective of this study was to conduct fundamental research in the field of Space Situational Awareness related to the topics of orbital and rotational dynamics, maneuver detection and the determination of operational bounds of active satellites, and probing fundamental constraints on uncertainty mapping and orbit determination in the space environment. Our goal was to bring a rigorous approach to the study of active and non-active space object dynamics to better understand the dynamics that shape their motion and the change events that active bodies are commanded to undergo. As motivation, results from this research program can be used to develop a better understanding of the evolution of a passive or active space object's state and how models of these effects can be estimated and improved. These studies will also lead to improved tracking strategies to enable the identification of hypothesized physical effects, improved modeling techniques to more accurately fit SSA observations, and the development of methods to analytically discern the actions of controlled space objects based on measurable quantities. Research methods used in this study rely on fundamental astrodynamics, optimal control and dynamical systems theory.

This was a multi-investigator and multi-institution proposal. At the University of Colorado the research is led by co-PI Prof. Daniel J. Scheeres and supports graduate students. At Texas A&M University the research is led by co-PI Prof. Kyle T. Alfriend and also supports a post-doctoral scholar and a graduate student. The current research proposal continued and extended research earlier being carried out by the same team under AFOSR grant FA9550-14-1-0348 with The University of Colorado.

Research on this grant was ultimately focused on a set of core topics: Long-term rotational and orbital dynamics of defunct satellites and debris, machine learning applied to orbital dynamics and control, optimal control applied to the space domain, improved models for satellite drag and estimation of atmospheric density, maneuver detection and characterization, 1-Norm orbit determination and covariance based track association. Research in these areas were supported at least in part by the AFOSR grant, but additional sources of support were also leveraged when available, including NASA and NSF fellowships. In the following a summary of results is given.

Results

Progress was made in all areas, as documented by PhD theses, peer-reviewed journal papers, and conference papers, all listed at the end of this report and summarized in brief below.

Long-term rotational and orbital dynamics of defunct satellites and debris

Long-Term Rotational Dynamics of Defunct Satellites

In this area we studied the dynamical evolution of defunct satellites in the GEO regimes. The main focus is on their rotational dynamics, as these bodies exhibit a range of spin rates and spin states that are clearly driven by the natural environment. Work was focused on the estimation of complex rotation spin states based on photometric light curve analysis, the modeling of complex rotational motion as driven by solar radiation pressure torques, and the internal energy dissipation effects for bodies driven by these external torques. Papers were presented at conferences and published in peer reviewed journals, covering a range of dynamical topics associated with this problem. Work in this area is reported in the thesis by Benson and in Journal papers [2-5] and Conference papers [21-25].

Long-Term Orbital Dynamics of Debris

In this area we are investigating the chaotic dynamics in the vicinity of the 12-hour orbits favored for GNSS systems. These chaotic regimes provide an opportunity for end of life satellite disposal, and their understanding is also crucial for mapping the likely impact of debris in this crucial regime of space. Associated with this topic, research has also been performed on the incorporation of non-linearities into standard correlation methods used in initial orbit determination. Research was also performed that evaluated the degree of targeting precision needed to have a satellite re-enter after time periods of up to 5 decades and derived a full set of doubly-averaged equations for the rapid long-term propagation of orbits in this regime. Work in this area is reported in the thesis by Pellegrino and in Journal papers [10-11] and Conference papers [34-38].

Control applied to the space domain

Machine Learning Applications to Space Trajectories and Control

Research in this area was mainly focused on the feasibility of applying new machine learning techniques to orbit design, orbit propagation and the use of control theory. Results were generally positive and captured in a few theses and papers. This area is ripe for future exploration, however the rapid advances in ML technology make it difficult to know when the optimal time for applications should be pursued. Work in this area is reported in the theses by De Smet and Parrish and in Journal papers [6-8] and Conference papers [26, 27, 33].

Optimal control applied to cooperative and non-cooperative satellite formations

In this area we are studying a range of questions that involve optimal cooperative and non-cooperative behavior in the space regime. Work in this area has included numerical approaches to efficiently solve optimal control problems ? with some initial explorations in machine learning applications. Also studied are fundamental limits on time-free optimal rendezvous and pursuit/evasion games. Work was also focused on rigorously applying differential game theory to the dynamics of cooperative and non-cooperative orbital motion. Work in this area is reported in the thesis by Venigalla and in Journal papers [19-20] and Conference papers [45-47].

Improved models for satellite drag and estimation of atmospheric density

In this area we have teamed with a local small business to develop new and improved methods for modeling the drag acting on a satellite. The motivation for this is to enable improved upper atmosphere density estimates. The improved drag models utilize harmonic expansions of body-fixed components that can be uniquely estimated, unlike most conventional drag models that lump several coefficients into a singular combination. The theory can also be extended to a harmonic expansion tied to the orbital motion of the spacecraft.

One highlight from this research is the identification of a subtle correlation between the degree and order of Earth gravity field modeled (not estimated) and the corruption of non-gravitational estimates in solar radiation pressure and drag. Our result shows how the use of a too-low degree and order model will alias unmodeled Earth gravitational field coefficients into drag and SRP models, thus making these estimates non-physical. This correlation has not been identified previously, yet will have a significant impact across many current approaches to maintenance of the Earth orbital catalog.

Research in this area has demonstrated significant improvements in the estimation accuracy of low-altitude satellites and the resulting prediction accuracy for these orbits. In addition, work has started on developing improved analytical theories for the propagation of orbits in atmosphere, which will enable improved insight into estimating the density fluctuations of the upper atmosphere, and better constraining the influence of molecular species on satellite drag. Work in this area is reported in the thesis by Ray and in Journal papers [13-18] and Conference papers [40-44].

Orbit determination and Covariance based track association

Maneuver detection and characterization

Research in this area was focused on the use of the Optimal Control based Estimator to solve for unknown maneuvers in targets that were being tracked and observed by ground stations or neighboring satellites. This capability is essential for maintaining custody of a space vehicle that has propulsion capability, and ideally would be carried out in an autonomous fashion. Our research in this area has shown that this can be achieved in the highly dynamical environment of cislunar space. Work in this area was performed by Greaves and is reported in Journal paper [9] and Conference papers [28-30].

1-Norm Orbit determination

Orbit determination is usually performed using the L2 norm with least squares of the Extended Kalman filter. Our method closely resembles the well known least squares algorithm in the sense that we compute differential corrections to an estimate of the state at epoch. Though both these algorithms share many similarities, the proposed algorithm differs in the cost function it minimizes. While the Least Squares (LS) algorithm minimizes the sum of squares of the residual error, our method minimizes the sum of absolute values of the residual error. It is thus called the Least Sum of Absolute Residuals (LSAR) orbit determination. The LSAR algorithm, as we shall see, has some valuable properties, one of them being measurement outlier rejection. This property is important, as during the process of taking observations we may inadvertently end up with bad measurements belonging to different objects in different but close-by orbits. We call these undesired measurements as outliers. We then need a method of estimating the orbit of the desired object while disregarding these outliers. At present, the outliers in the measurements are dealt with by a nonlinear procedure consisting of a preliminary screen to reject extreme observations. The surviving data is then processed by methods such as the LS algorithm to determine the orbit. The LSAR algorithm rejects the outliers in the data and estimates the orbit without any preliminary screening. Work in this area is reported in Journal paper [12] and Conference paper [39].

Covariance based track association

Past research has proposed using the Mahalanobis distance (MD) for correlating uncorrelated tracks (UCTs) of space objects obtained from radar observations [1]. For linear Hamiltonian systems the MD is constant in time. Also, the volume of the covariance, the equiprobability ellipsoid, is constant in time even though its shape changes. This means the same result is obtained regardless of the time This covariance-based track association (CBTA) has been evaluated in an operational environment using Cartesian coordinates and equinoctial elements. The neglected nonlinearities in the covariance propagation cause the CBTA to deteriorate with time. Ref. 31 showed that for equinoctial elements CBTA performed well for about 10 days in the orbit maintenance mode and about 2-3 days in the single track orbit determination mode. However, in Cartesian coordinates it was much less, about 2-3 days for orbit maintenance and about 2 hours in the single track orbit determination mode. Ref. [1] proved that CBTA is the optimum approach for UCT correlation as long as the covariance propagation is linear. Others have used information metrics for UCT correlation. The disadvantage of the information metrics as compared to the Mahalanobis distance is that they only provide the best correlation, there is no value of the metric that provides a given probability of correlation. In contrast the Mahalanobis distance is the number of standard deviations, and for 6D k=4 is about 99% correlation. Current research is focused on extending the Mahalanobis distance concept to incorporate the effect of the neglected nonlinearities in the covariance propagation that makes the uncertainty non-Gaussian. As just noted above the covariance stays linear longer on orbital element space than Cartesian coordinates. In orbital element space the nonlinearity appears in the mean anomaly equation, however the nonlinear terms cause the covariance to become non-Gaussian eventually. The new approach being investigated is to not use the semi-major axis a, but the mean motion. With this change the equations of motion for two-body motion are linear. This means the covariance stays linear and CBTA can be used for track association. A key improvement was made by using mean modified equinoctial elements to perform the correlation. In the presence of J2 perturbations it yields correlations between orbits that are good for time scales of weeks, as compared to regular equinoctial elements that have correlations over time scales of days, or Cartesian states with correlations possible only over time scales of hours. This work is continued to be pursued and a publication will be created and submitted once testing and development is complete.

Personnel Supported

- D.J. Scheeres: direct salary support.
- K.T. Alfriend: direct salary support.
- PhDs supported or contributing to research:
  - Nathan L.O. Parrish Defended May 2018.

"Low Thrust Trajectory Optimization in Cislunar and Translunar Space,"
Committee Chair
Smead Department of Aerospace Engineering Sciences, University of Colorado

  - Stijn De Smet Defended November 2018.
    "On the design of solar gravity driven planetocentric transfers using artificial neural networks,"
    Committee Chair
    Smead Department of Aerospace Engineering Sciences, University of Colorado
  - Conor Benson Defended June 2021.
    "Solar Torque and Dissipation Dynamics for Tumbling Bodies: Theory and Observations,"
    Department of Aerospace Engineering Sciences, University of Colorado
  - Marielle Pellegrino Defended July 2021.
    "Using Solar Radiation Pressure and Luni-Solar Resonances for Debris Mitigation,"
    Department of Aerospace Engineering Sciences, University of Colorado
  - Vishal Ray Defended November 2021.
    "Advances in atmospheric drag force modeling for satellite orbit prediction and density estimation,"
    Department of Aerospace Engineering Sciences, University of Colorado
  - Chandrakanth Venigalla Defended November 2021.
    "Multi-Spacecraft Cooperative and Non-Cooperative Trajectory Optimization,"
    Department of Aerospace Engineering Sciences, University of Colorado

- PhD Candidates/MS students supported or contributing to research:
  - Jesse Greaves Department of Aerospace Engineering Sciences, University of Colorado
  - Yashica Khatri Department of Aerospace Engineering Sciences, University of Colorado
  - Gavin Brown Department of Aerospace Engineering Sciences, University of Colorado

Transitions

1. Our approach to estimating satellite drag coefficients and their use in improving density estimation has been applied by Kayhan Space in a successful STTR proposal.

Achievements

1. Scheeres was elected to the International Astronautical Academy as a full member in 2021.

Publications

The following publications were supported, at least in part, by the AFOSR grant.

Peer-Reviewed Journals

1. Alfriend, K.T., "A Dynamic Algorithm for UCT Processing", Paper No. AAS 97-607, 1997 AAS/AIAA Astrodynamics Specialist Conference, Sun Valley, ID, August 1997.

2. C. Benson, D.J. Scheeres, W.H. Ryan, E.V. Ryan, N.A. Moskovitz. 2020. "GOES Spin State Diversity and the Implications for GEO Debris Mitigation," *Acta Astronautica* 167: 212-221.

3. C.J. Benson and D.J. Scheeres. 2021. "Resonance-Averaged Solar Torque Dynamics for Tumbling Satellites," *Journal of Guidance, Control and Dynamics* 44(12): 2143-2154.

4. C.J. Benson and D.J. Scheeres. 2021. "Averaged Solar Torque Rotational Dynamics for Defunct Satellites," *Journal of Guidance, Control and Dynamics* 44(4): 749-766. https://doi.org/10.2514/1.G005449

5. C.J. Benson, C.J. Naudet; D.J. Scheeres J.S. Jao, L.G. Snedeker, W.H. Ryan, E.V. Ryan, M.A. Silva, J.K. Lagrange, S.H. Bryant, P.C. Tsao, D.K. Lee, U. Yildiz, and H.D. Nguyen. 2021. "Radar and Optical Study of Defunct Geosynchronous Satellites," *Journal of the Astronautical Sciences* 68: 728-749. https://doi.org/10.1007/s40295-021-00266-z

6. S. De Smet, D.J. Scheeres, and J.S. Parker. 2019. "Representing Dynamics in the Eccentric Hill System using a Neural Network Architecture," *Journal of Astrodynamics* 3: 301–324.

7. S. De Smet, D.J. Scheeres and Jeffrey S. Parker. 2019. "Leveraging Artificial Neural Networks to Systematically Explore Solar Gravity Driven Transfers in the Martian System," *Journal of the Astronautical Sciences* 66:282.

8. S. De Smet and D.J. Scheeres. 2019. "Identifying heteroclinic connections using artificial neural networks," *Acta Astronautica* 161: 192-199.

9. J. A. Greaves and D. J. Scheeres. 2021. "Observation and Maneuver Detection for Cislunar Vehicles," *Journal of the Astronautical Sciences* 68: 826-854. http://link.springer.com/article/10.1007/s40295-021-00283-y

10. M. Pellegrino and D.J. Scheeres. 2021. "Reachability of a Passive Solar Sail in Earth Orbit," *Journal of Guidance, Control and Dynamics* 44(2): 360-369.

11. M. M. Pellegrino, D. J. Scheeres, and B. J. Streetman. 2021. "The Feasibility of Targeting Chaotic Regions in the GNSS Regime," *Journal of the Astronautical Sciences* 68: 553. https://doi.org/10.1007/s40295-021-00270-3

12. Prabhu, K., Majji, M. and Alfriend, K. T., "Least Sum of Absolute Residuals Orbit Determination," Journal of Guidance, Control and Dynamics, Jan. 2022. https://doi.org/10.2514/1.G006088

13. V. Ray and D.J. Scheeres. 2020. "Gravitational Force-Model Aliasing with Non-Gravitational Force Coefficients in Dynamic Prediction," *Journal of Guidance, Dynamics and Control* 43(11): 1984–1997. https://doi.org/10.2514/1.G005001

14. V. Ray and D.J. Scheeres. 2020. "Drag coefficient model to track variations due to attitude and orbital motion," *Journal of Guidance, Dynamics and Control* 43(10): 1915-1926.

15. V. Ray, D.J. Scheeres, S.G. Hesar and M. Duncan. 2020. "A drag coefficient modeling approach using spatial and temporal Fourier expansions for orbit determination," *Journal of the Astronautical Sciences* 67(3), 1139-1168. 10.1007/s40295-019-00200-4

16. V. Ray and D.J. Scheeres. 2021. "King-Hele orbit theory for periodic orbit and attitude variations," *Monthly Notices of the Royal Astronomical Society* 501(1): 1168-1187. https://doi.org/10.1093/mnras/staa3630

17. V. Ray, D.J. Scheeres and M. Pilinski. 2021. "Inverting gas-surface interaction parameters from Fourier drag-coefficient estimates for a given atmospheric model," *Advances in Space Research* 68: 1902-1927.

18. V. Ray, D.J. Scheeres, S. Alnaqbi, K.W. Tobiska and S.G. Hesar. 2022. "A framework to estimate local atmospheric densities with reduced drag-coefficient biases," *Space Weather* 20(3), March 2022, e2021SW002972.

19. C. Venigalla and D.J. Scheeres. 2021. "Delta-V Based Analysis of Spacecraft Pursuit-Evasion Games," *Journal of Guidance, Dynamics and Control* 44(11): 1961-1971.

20. C. Venigalla and D.J. Scheeres. 2020. "Minimum Bounds on Multi-Spacecraft ΔV Optimal Cooperative Rendezvous," *Journal of Guidance, Dynamics and Control* 43(12): 2333–2348. https://doi.org/10.2514/1.G004978

Conference Papers (note 2019 is missing)

21. C.J. Benson, D.J. Scheeres, W.H. Ryan, E.V. Ryan and N.A. Moskovitz. "GOES Tumbling Spin State Evolution and the Implications for GEO Debris Mitigation," paper presented at the 69th International Astronautical Congress, Bremen, Germany, October 2018. Paper IAC-18,A6,10-C1.7,1,x46361.

22. C.J. Benson and D.J. Scheeres. "Cyclic Complex Spin State Evolution of Defunct GEO Satellites," paper presented at the 2018 AMOS Conference, September 2018.

23. Conor Benson, Daniel Scheeres. AAS 20-470 Averaged Solar Torque Rotational Dynamics for Defunct Satellites. Paper presented at the 2020 Astrodynamics Specialist Meeting.

24. C. Benson and D.J. Scheeres. Radar and Optical Study of Defunct GEO Satellites. Paper presented at the AMOS 2020 Conference.

25. Conor Benson, D.J. Scheeres Radar-Derived Spin States of Defunct GEO Satellites and Rocket Bodies. AMOS Conference, September 15-17, 2021.

26. S. De Smet, D.J. Scheeres and J. Parker. "Systematic Exploration of Solar Gravity Driven Orbital Transfers in the Martian System using Artificial Neural Networks," paper presented at the 2018 AAS/AIAA Astrodynamics Specialist Conference, August 2018. Paper AAS 18-216.

27. S. De Smet and D.J. Scheeres. "Identifying Heteroclinic Connections using Artificial Neural Networks," paper presented at the 69th International Astronautical Congress, Bremen, Germany, October 2018. Paper IAC-18,C1,1,8,x42401.

28. J Greaves, DJ Scheeres Estimation of Stochastic Events for Vehicles in NRHOs. Paper presented at the AIAA Scitech 2020 Forum, 0227

29. J. Greaves and D.J. Scheeres. Maneuver Detection for Cislunar Vehicles using Optical Measurements Paper presented at the AMOS 2020 Conference.
Awarded "Best of Conference" paper.

30. Jesse Greaves, D.J. Scheeres Relative Estimation in the Cislunar Regime using Optical Sensors. AMOS Conference, September 15-17, 2021.

31. Hill, K., Alfriend, K.T. and Sabol, C." Covariance-based Uncorrelated Track Association", Paper No. AIAA 2008-7211, 2008 AIAA/AAS Astrodynamics Conference, Honolulu, HI, 17-21 August 2008.

32. Y. Khatri, D.J. Scheeres Nonlinear Semi-Analytical Uncertainty Propagation for Conjunction Analysis IAC Conference, October 25-29, 2021. Paper IAC-21,C1,3,12,x64896.

33. N. Parrish and D.J. Scheeres. "Optimal Low-Thrust Trajectory Correction with Neural Networks," paper presented at the 2018 AAS/AIAA Astrodynamics Specialist Conference, August 2018. Paper AAS 18-397.

34. M. Pellegrino and D.J. Scheeres. "Targeting Regions of Chaos In the GNSS Regime," paper presented at the 2018 AAS/AIAA Astrodynamics Specialist Conference, August 2018. Paper AAS 18-372.

35. M. Pellegrino and D.J. Scheeres. "Optimal Deployment of Solar Radiation Pressure Enhancement Devices for Space Debris Mitigation," paper presented at the 2018 AIAA/AAS Space Flight Mechanics Meeting, January 2018.

36. Marielle Pellegrino, Daniel Scheeres, Brett Streetman. AAS 20-476 Development and Analysis of the Doubly Averaged Model for Solar Radiation Pressure. Paper presented at the 2020 Astrodynamics Specialist Meeting.

37. Marielle Pellegrino, Daniel Scheeres, Brett Streetman. Loitering of Breakup Event Debris Near Nominal GNSS Orbits 2021 AAS/AIAA Astrodynamics Specialist Conference, Virtual August 9-11, 2021. Paper AAS 21-597

38. Marielle Pellegrino, D.J. Scheeres, B. Streetman Debris Cloud Structure in Medium Earth Orbit. AMOS Conference, September 15-17, 2021.

39. Prabhu, K., Majji, M. and Alfriend, K.T., "Least Sum of Absolute Residuals in Orbit Determination," 2nd IAA Conference on Space Situational Awareness, Washington, DC, 14-17 Jan. 2020.

40. V. Ray and D.J. Scheeres. "Drag Coefficient Modeling with Spatial and Temporal Fourier Coefficient Expansions: Theory and Application," paper presented at the 2018 AMOS Conference, September 2018.

41. Vishal Ray, Daniel Scheeres. AAS 20-542 Extension of King-Hele theory to variable drag-coefficients. Paper presented at the 2020 Astrodynamics Specialist Meeting.

42. V. Ray and D.J. Scheeres. Evaluation of Performance Metrics for Fourier Drag Models in Orbit Determination and Prediction Paper presented at the AMOS 2020 Conference.

43. Vishal Ray, Daniel Scheeres, Eric Sutton, Marcin Pilinski. Density estimation using second-order Gauss Markov processes 31ST AAS/AIAA Space Flight Mechanics Meeting, Virtual February 1-3, 2021. Paper AAS 21-340

44. Vishal Ray, D.J. Scheeres, et al. Decorrelating Density and Drag-coefficient Through Attitude Variations. AMOS Conference, September 15-17, 2021.

45. C.Venigalla and D.J. Scheeres. "Spacecraft Rendezvous and Pursuit/Evasion Analysis Using Reachable Sets," paper presented at the 2018 AIAA/AAS Space Flight Mechanics Meeting, January 2018.

46. C. Venigalla and D.J. Scheeres. "Numerical And Analytical Reachable Set Applications To Cooperative And Non-Cooperative Multi-Spacecraft Trajectory Coordination," paper presented at the 69th International Astronautical Congress, Bremen, Germany, October 2018. Paper IAC-18,C1,5,x46279.

47. CK Venigalla, Daniel Scheeres. Optimal Multi-Spacecraft Cooperative Rendezvous and Constellation Deployment Trajectories 2021 AAS/AIAA Astrodynamics Specialist Conference, Virtual August 9-11, 2021. Paper AAS 21-718