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Imaging theory and mitigation in extreme turbulence-induced anisoplanatism

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14. ABSTRACT
The objective of this project was to improve our overall understanding of imaging science in scenarios where the isoplanatic angle is on the order of the diffraction-limit of the imaging system. We also aimed to understand if Light-Field (LF) or plenoptic imaging techniques may be useful in scene-recovery when turbulence induced distortions are dominated by anisoplanatic effects. We found that for scenes where the ratio of the aperture size to Fried parameter are less than the anisoplanatic error often saturates to a value less than 1 radian squared allowing for scene recovery using traditional post-processing techniques. Also, that in the scenarios LF techniques allow for single-shot (one frame) recovery via post-processing. This final finding hints at the possibility of real-time scene recovery without block-processing and likely allows for simultaneous volume phase recovery.

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Final Report: AFOSR Grant FA9550-17-1-0201 Imaging theory and mitigation in extreme, turbulence-induced anisoplanatism.

PI: Jeremy Bos, Michigan Technological University
AFOSR Program Officer: Dr. Michael Yakes

Abstract: The objective of this project was to improve our overall understanding of imaging science in scenarios where the isoplanatic angle is on the order of the diffraction-limit of the imaging system. We also aimed to understand if Light-Field (LF) or plenoptic imaging techniques may be useful in scene-recovery when turbulence induced distortions are dominated by anisoplanatic effects. We found that for scenes where the ratio of the aperture size to Fried parameter are less than the anisoplanatic error often saturates to a value less than 1 radian squared allowing for scene recovery using traditional post-processing techniques. Also, that in the scenarios LF techniques allow for single-shot (one frame) recovery via post-processing. This final finding hints at the possibility of real-time scene recovery without block-processing and likely allows for simultaneous volume phase recovery.

Accomplishments:

Research Objectives: *Improve our understanding of imaging under conditions where the isoplanatic angle is on the order of, or smaller than, the sample rate of the imaging system. Also, evaluate a technique combining concepts from plenoptic imaging and Speckle Imaging (SI) for simultaneous wave front sensing and scene recovery in extreme anisoplanatism .*

Research aims toward objectives:

1. **The piston-removed anisoplanatic error saturates to value less than 1 rad^2 as D/r_0 approaches 1 (completed, confirmed) (90% complete work awaiting final dissemination via journal paper)**

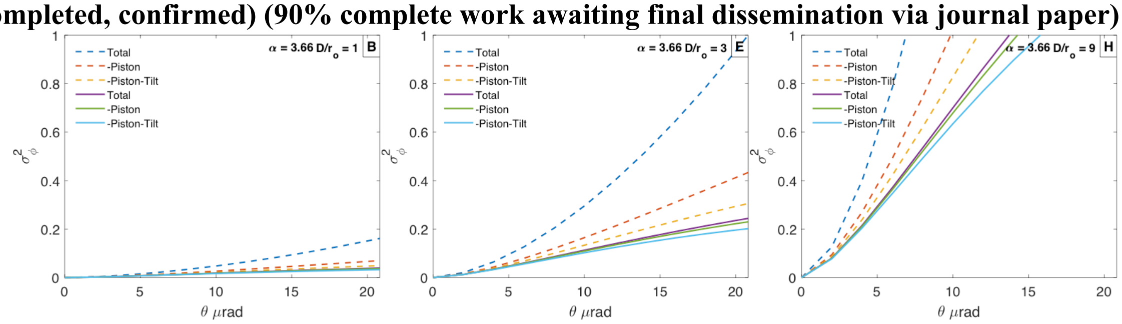

Figure 1. Anisoplanatic error as a funciton of angle for D/r0 =1 (left), 3 (center), and 9 (right) with D/L0 =1. Dashed lines are the L0 = Inf. case included for comparison. Lines labelled as Piston and Piston-Tilt indicate those terms have been subtracted from the anisoplanatic error. Takeaway: both the outer scale and piston contribute significantly to anisoplanatic error.

Associated Research Tasks:

- Evaluate the piston-removed anisoplanatic error as a function of angular separation in extremely anisoplanatic scenarios
- In we [1] evaluated Stone's expression of piston and tilt removed anisoplanatic error for turbulence conditions where the aperture size, D, is on the order of the atmospheric coherence cell size, r_0. Stone's results are for upward-looking cases with relatively large aperture sizes and assume the Hufnagel-Valley model for C_n^2 as a function of altitude. To achieve our aim we first adapted Stone's model for horizontal imaging conditions (constant C_n^2) and for small apertures. Jeff's thesis verified Stone's work and extended the work to horizontal imaging scenarios where C_n^2 is constant but results in the same D/ r_0.

- This work was published in JOSA A [2] . A key result of this work is an analytical expression for the anisoplanatic error in non-Kolmogorov turbulence:

$$\sigma^2_{\phi,T}(\theta,\alpha) = 4B(\alpha)c_1(\alpha)\int_0^L \mathrm{d}z \frac{\hat{r}_0^{2-\alpha}}{L}\int_0^\infty \mathrm{d}\kappa\kappa f(\kappa,\alpha)\times\left(1-J_0(\kappa\theta L)\right)\left[\frac{4J_2(\kappa D/2)}{\kappa D/2}\right]^2$$

and for anisoplanatic error in non-Kolmogorov turbulence with a finite outer scale

$$\sigma^2_\phi(\theta,\alpha) = 4B(\alpha)c_1(\alpha)\int_0^L \mathrm{d}z \frac{\hat{r}_0^{2-\alpha}}{L} \times \left(\frac{\kappa_0^{2-\alpha}}{-2+\alpha} - \frac{2^{1-\frac{\alpha}{2}}\left(\frac{\kappa_0}{\theta L}\right)^{1-\frac{\alpha}{2}} \mathrm{K}_{1-\frac{\alpha}{2}}[\theta L\kappa_0]}{\Gamma\left[\frac{\alpha}{2}\right]}\right)$$

- This work was extended to spherical wave sources and slant paths and can be found in [1] and is in preparation for submission to a journal.

2. **Multiple contributions from extended scenes combine such that their effect can be modeled as a zero-mean Gaussian random process under the central limit theorem**.
   Associated Research Tasks (dropped due to insufficient staffing, work confirmed by other researchers):
   - Develop analytical models for the anisoplanatic cross-spectrum, and bispectrum for two and three point sources.
   - Evaluation of the anisoplanatic cross-spectrum and bispectrum transfer functions for 2, 3, and N point sources via Monte Carlo simulation.
   - Empirical evaluation of the cross-spectrum and bispectrum SNR from field data, and comparison to results obtained via analysis and simulation.

   Expected Outcomes:
   - A model for the extended scene anisoplanatic speckle transfer function.
   - A field-validated model for the amplitude, phase, and SNR of the anisoplanatic cross-spectrum and bispectrum for extended scenes.
   - Guidelines on the range of spatial offset frequencies, aperture sizes, and engagement scenarios where anisoplanatic SI is most effective.

Note: this research task was covered in part by Holmes and Gudimetla [3]. The task itself was always supposed to be taken on by a second graduate student to continue Beck's work. Due to the pandemic and other external factors that student was never hired, and the work never started in earnest. The expected outcomes, however, are addressed in Beck's thesis [1] where a D/r0 of greater than 9 resulted in significant anisoplanatic error. The implication here being that integrated turbulence strength, rather than anisoplanatism drives the limits on scene recovery. This is true even when the isoplanatic angle is small.

3. **A plenoptic imaging model can be used for simultaneous wave front sensing and scene recovery in extreme anisoplanatism using SI (70% complete work in preparation for dissemination).**
   Associated Research Tasks:
   - Modify an existing horizontal simulation model to support plenoptic imaging (*complete*).
   - Explore the use of SI techniques for scene recovery from plenoptic focal stacks featuring angular diversity (*complete*).

- Investigate atmospheric tomographic wave front reconstruction on extended scenes using the proposed model (*complete*).

Outcomes:

- In [4] I described a method of establishing equivalency between a camera array and a plenoptic sensor. This work has been expanded to a full journal article in [5].
- The equivalent model allows for the generation of simulated Light Field (LF) imagery by translating the turbulence volume in front of a static imaging system over the bounds of the array. Example results can be found in [6].
- Simulation model uses the technique described in [7]. We are in the process of expanding this work to a full journal article have been waiting for publication of [5]
- Confirming the results in [8] angular viewpoints are not perfectly correlated this work was reported in [9] [10].
- Scene recovery methods that use multiple frames can complete recovery using a single LF image using both MFBD and the bispectrum-based SI. The former was reported in [9] the latter was presented in [11].
- The key results below clearly indicate that Phase Diversity can be used for tomographic scene recovery. The process for volumetric phase recovery from PSF estimates has been reported previously and elsewhere by others.

Key results:

Figure 2. Simulated LF imagery featuring extreme anisoplanatism. Each image represents a different angular viewpoint captured in the LF within the modeled aperture. The left and right images differ by the relative turbulence strength in the volume with the right being the more severe case.

- In [12] we demonstrated that we could generate synthetically refocused images from turbulence-corrupted light fields (See Figure 2).
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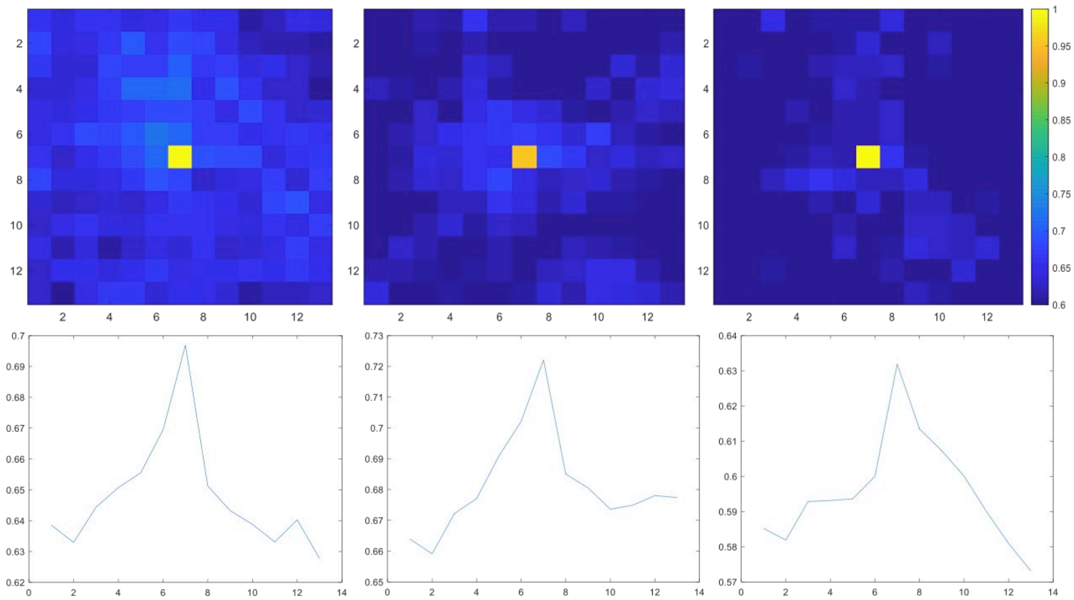

Figure 3. Cross-Correlation as viewpoint image index is varied compared to the center viewpoint image. Correlation decreases as viewpoint differs away from center. The lack of correlation allows for snapshot scene recovery as reported in [10] [9]

Figure 4. Diffraction-limited quality scene recovery from turbulence LF .

- In [9] we showed that viewpoint images in LFs show an off-axis de-correlation behavior (Figure 2) and that this can be exploited for single-shot scene recovery (Figure 3)

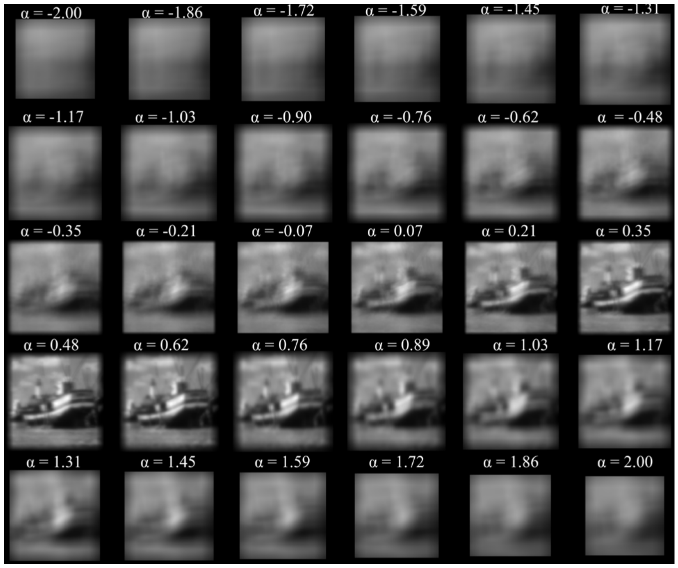

Figure 5. Synthetically refocused images from a single volume turbulence light field.

- Using the varied focal depths from and generating the corresponding defocus aberration we can use Phase Diversity (PD) to recover both a pristine image and the associated point-spread function. Figure 3 demonstrates that nearly-diffraction imagery can be recovered from a single LF much like with MFBD and the bispectrum using PD. The benefit of the PD approach is an estimate of the phase at each focal depth is available potentially allowing for volumetric phase reconstruction or simultaneous scene recovery and wave-front estimation.

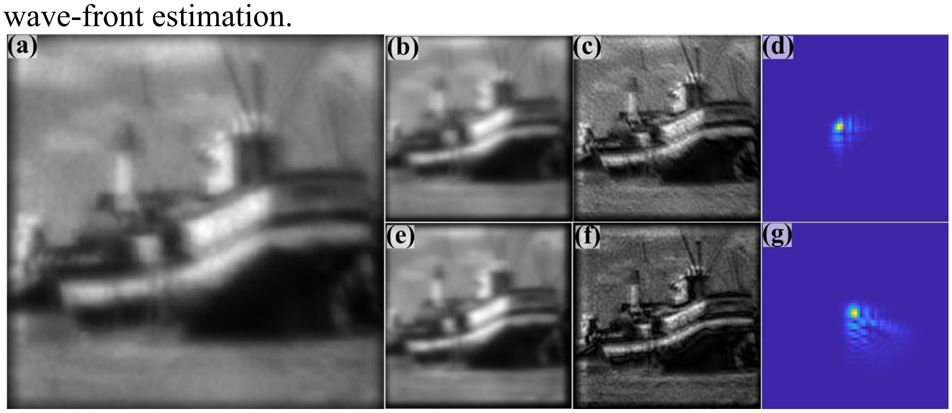

Figure 6. Phase Diversity from Light-Fields for scene recovery and turbulence phase estimation. A LF viewpoint image (a), diversity images (b,e), corresponding PD recoveries (c,f) and PSF estimates (d,g) for two different refocusing depths.

Associated Works:

- [13] was expanded upon by PI-Bos and submitted to SPIE Optical Engineering in May of this year.
- Single-shot scene recovery from LFs was demonstrated in [11]
- Phase diversity and volumetric phase recovery was demonstrated in [12]
- A paper started by undergraduate student Stephen Grulke (currently at NAVSEA Crane) was published in SPIE Optical Engineering [14]
- Planned works:
  - A letter outlining a method for generating turbulence LF and demonstrating decorrelation behavior with angular sample (in preparation)

- A journal article outlining phase diversity for turbulence LF refocusing and scene recovery (in preparation).
- A journal article demonstrating the efficacy of single-shot bispectrum scene recovery from turbulence LFs (in preparation).
- A journal article demonstrating the efficacy of single-shot MFBD scene recovery from turbulence LFs (in preparation).
- These last three items will make up PhD student Rahman's dissertation and will be completed with department support by spring 2023.

Details of accomplishments during this reporting period:

1. Major activities: graduation of PhD student Beck and promotion of student Rahman to PhD candidate. Publication of two journal articles and Beck's dissertation. Completed and published related paper started by one of the undergraduate students funded by this project.
2. Specific objectives: complete student's Beck PhD process (complete), provide student Rahman with a solid footing to complete his PhD process (in progress).
3. Significant results or key outcomes:
    a. Beck thesis indicates that effective isoplanatic patch size is very large out to D/r_0 of 9 or above.
    b. Submitted paper to Optical Engineering on wave optics simulation indicates a linear relationship between medium power-law and scintillation.
    c. Demonstrated that a single LF can be used for snapshot scene recovery using bispectrum or phase diversity. The latter allows for volume phase retrieval.
    d. Our aim to explore multiple contributions in the bispectrum while still mathematically interesting and challenging to prove has been published by other authors as a prima facie argument.
4. During the reporting period PI-Bos was asked to join as program chair for the Optica Propagation and Characterization of Atmospheric and Oceanic Phenomenon and the Applied Imaging Congress. He was also asked to join the program committee for the Laser Sensing & Communication Conference and promoted to Senior Member of Optica.

How were the results disseminated to communities of interest: a complete list of all conference talks and associated papers and journal articles are found in the "References" section of this document.

In addition, PI-Bos gave invited talks at SPIE Defense and Security in 2018, the Fraunhofer ISOB Workshop on Non-Kolmogorov Turbulence in 2019, and Optica pcAOP in 2020, and as a guest lecturer at Tufts University in 2022. Bos also delivered a keynote address for the Workshop on Information Optics at Laval in University in 2018.

Outreach: in 2017, during the first year of this award Bos organized a community solar eclipse viewing event on Michigan Tech's campus. The event was attended by hundreds of students and community members. Bos later donated a telescope purchased using his start-up funds to Michigan Tech's AMJOCH observatory and continues to be actively involved in it operation and development.

Impacts

- *Development of the principal discipline(s) of the project*: this work deals primarily with imaging and beam propagation through atmospheric turbulence. As a result of this work, we understand that the effects of anisoplanatism are severely restricted with the ratio of the aperture size to the Fried coherence diameter is less than 10. In this regime anisoplanatic error is limited to less than 1 rad^2 excluding the piston phase and global tilt. Also, that using light-field (LF) techniques for image acquisition allows single-shot scene recovery. Also, that LF techniques likely allow for recovery of the volume turbulence phase as a wavefront sensor. These techniques may result in faster or better turbulence compensation systems operating in deep turbulence with extended, non-cooperative targets.

- *Other disciplines*: this work may find use in other areas of imaging such as microscopy and other application areas such as Free Space Optical (FSO) communications.
- *Describe the impact in this reporting period on the development of human resources*: this project supported four undergraduate students, one of whom completed his PhD and is now working as a research engineer in this field. A second is currently a government employee/scientist at NAVSEA Crane. A third went on to win the DJ Lovell Optics and Photonics Scholarship from SPIE as a MS graduate student and University of Central Florida (CREOL). He is currently a PhD student and the University of Arizona scheduled to complete his PhD this spring.
- *Impact on society beyond science and technology:* PI-Bos maintains an active social media presence where he further disseminates the results of this research with good engagement from the community.

Changes:

- *Changes in approach:* the most substantial change in approach over the project are associated with Aim #2. As a result of changes in staffing, it took longer than expected to complete Aim #1. In fact, some of that work remains unpublished (in peer-reviewed form). By the time work began on Aim #2 the work in [3] had been published and we determined we would not like make progress beyond their work in the time left available in the project.
- *Problems or delays:* early in this project a student was dismissed for research misconduct. Other issues relate to student graduation timelines or choosing to purse graduate education at other institutions. Attracting and retaining high quality students remains a problem.
- *Expenditure Impacts:* at the start of the global pandemic we aimed to hire one graduate and one undergraduate student. Those hires did not happen resulting in a rather an impact to expenditures. A late no-cost extension alleviated the problem somewhat.

Technical Updates:

- As this is a final report all technical updates are provided in the "Accomplishments" section of this document.

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