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Magnetized Electron-Ion Collision and Rydberg Atom Formation Rates in Ultracold Plasmas

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14. ABSTRACT
Ultracold neutral plasmas (UNPs) are excellent systems for studying basic plasma physics under well-characterized but also unusual plasma conditions. For example, plasmas in this work had temperatures within a few Kelvin of absolute zero. We have studied electron-ion collision rates in this project both as influenced by strong coupling and extreme magnetization. Theoretical investigations that built on prior AFOSR-funded experimental measurements showed that many-body-collisions are important for strongly-coupled plasmas, and without including those collisions theoretical predictions based only on binary collisions did not match our experimental measurements. We also report on the theoretical investigation of a heating mechanism in UNPs arising from applied DC electric fields. We have developed a new technique for measuring electron-ion collision rates through off-resonant RF heating rates that is a substantial improvement with regard to prior techniques that we used. We have used this technique to investigate electron-ion collision rates under extreme degrees of electron magnetization.

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Final Technical Report

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I. Ultracold Neutral Plasmas for Investigating Plasma Physics

The work in this project consisted of experimental and theoretical investigations of plasma physics through the study of ultracold neutral plasmas (UNPs) [1,2]. UNPs are created through photoionizing either laser-cooled atoms [1] or cold atomic [3] or molecular beams [4]. They are characterized by relatively low plasma densities and tunable electron and ion temperatures, with achievable temperatures being near absolute zero -- as low as a Kelvin or even much less [5].

UNPs are excellent systems for precise studies of basic plasma physics, especially plasma physics related to strong coupling (i.e. conditions where plasma particle correlations become significant) [6] and extreme conditions such as magnetization [7]. There are several properties of UNPs that make this so. Experiments are conducted with table-top-scale apparatuses, meaning that they are accessible with university-scale resources and can be reconfigured for new measurements and experiments in a straightforward manner. UNPs' low densities lead to low electron and ion plasma frequencies, which "slow down" plasma dynamics. That makes those dynamics accessible on timescales easily achievable with modern electronics. Precise and sensitive experimental diagnostic tools are available for UNP measurements [8,9,10]. Many UNPs are highly classical, allowing the evaluation of classical theories that form the first description of more complicated plasmas that may include quantum effects. Higher-density UNPs can be used to investigate quantum phenomena [11], too. The ionization state of UNPs is very well known and uniform. The low electron temperatures mean that collisions or other interactions with any neutral atoms are ignorable. Electrons and ions come into thermal equilibrium with themselves rapidly for many plasma conditions. The plasma spatial boundary conditions consist of open boundaries as the UNPs are finite-sized, and as a result these boundary conditions can be characterized very well. External electric and magnetic fields are easily applied. Initial UNP conditions are tunable and highly controllable.

UNP plasmas are free from turbulence unless it is deliberately introduced, allowing studies that are free from that complication (unlike many other plasmas). The number of particles in these plasmas can be very small, enabling 1:1 ratios between actual particles and the number of particles modeled in a simulation. The tunable initial conditions and typical plasma experimental parameters are very useful in studying the boundary of strong coupling physics, as characterized by the strong coupling parameter Γ [6], which is proportional to the nearest-neighbor Coulomb potential energy divided by the average thermal energy of the particles in one of the components (e.g. the electron component) of the UNP. UNPs can be controlled so that their parameters can be tuned across the strong coupling boundary, and other boundaries such as magnetization boundaries, to isolate physics of interest.

II. Areas of Physics Investigation in this Project

In this project, the electron component of UNPs was studied. Strong coupling physics in the electron component was investigated to both evaluate and eventually explain discrepancies between theory predictions and measurement [12], as well as validate recent theoretical work concerning the proper treatment of collisions in plasmas [13]. Our experiments led to successful lines of theoretical investigation of strong coupling physics that found differences between like- and unlike-sign charge collision rates that have implications for the fundamentals of plasma kinetic theory [14] and that discovered a significant role of beyond-two-body processes in transport rates in such plasmas [15].

Part of creating strong coupling in the electron component of UNPs is achieving low electron temperatures. In this project we investigated a newly discovered UNP heating mechanism [16] identified in our prior work [12]. This heating arises when a DC electric field is applied to the UNP during its formation, and since such fields are commonly used in UNP experiments for technical reasons, this is a relevant issue in many experiments. We characterized how much of a limitation it could be as a function of UNP parameters. This physics is a potential consideration for other finite-sized plasma systems as well. In addition to our investigations of the phenomenon itself, we also altered our experimental apparatus and used new techniques to radically reduce any heating from this mechanism.

We conducted measurements of electron-ion collision rates as a function of electron magnetization. As described in more detail later in this report, UNPs have highly favorable properties given their low densities and temperatures that make them relatively easy to magnetize to an extreme degree [7] with laboratory fields – especially the electron component of the plasma. This extreme degree of electron magnetization is characterized by the associated magnetic length scale being the smallest and therefore the dominant length scale in the plasma, affecting screening, transport, and even binary collision properties of the plasmas. Through scaling relations, the results we obtained can be applied to plasmas at higher densities and temperatures, with correspondingly larger magnetic fields. We were able to achieve UNP conditions in our laboratory that would correspond to plasmas with hundreds of kilotesla magnetic fields for density and temperature conditions corresponding to high-energy-density [17] or white dwarf atmospheres [18]. To our knowledge, our electron-ion collision rate measurements were conducted with the highest degree of electron magnetization in a near-neutral plasma, and with a comparable degree of magnetization as that achieved in ion storage rings [19] and non-neutral plasmas [20,21].

We have adapted theory treatments with the goal of producing predicted electron-ion collision rates for our particular UNP conditions. Additional theoretical development was necessary since no published theory treatments were directly applicable. This theory treatment is not limited to UNPs, but is generally

applicable to plasmas. We obtained electron-ion collision rate measurements for cold but not strongly coupled UNPs. These were obtained for three magnetic field values: one at the border of weakly/strongly magnetized electrons, one just below the extreme magnetization region, and one in the extreme magnetization region [7]. Initial indications are that the theory treatment we developed, which is based strongly on other theory treatments in the literature [22,23], is consistent with the lowest magnetic field data point (the one at the weakly/strongly magnetized region), but does not capture the changes in electron-ion collision rate as the magnetic field is increased to extreme magnetization. This is not necessarily surprising given likely limitations in the theory, but as our experiments represent the first measurement of electron-ion collision with this degree of magnetization and the degree of precision we were able to achieve, they are well-positioned to illuminate parameters where additional theoretical development is necessary.

These measurements were obtained through a novel experimental technique we developed for UNPs. We used off-resonant radiofrequency (RF) heating rates to obtain electron-ion collision rates. This technique was developed, tested, and calibrated as part of this project. As noted above, it has already been used to measure electron-ion collision rates in a previously experimentally unexplored region of electron temperature and magnetization, and has great promise to extend these measurements in many directions to test a range of fundamental plasma physics centered on electron-ion collisions. Additional details on these possible future directions of study will be presented briefly at the end of this report.

III. Research Products

Our work has resulted in three publications during the project period [13,15,16]. Two more manuscripts are expected to be submitted in the next three months. With regard to one of these manuscripts, the COVID-19 pandemic restricted lab access and we could not complete necessary calibrations until the system was restored to functionality after months of inability to monitor and maintain it. Required additional measurements have now been taken, but a mild amount of additional work is necessary to produce final results.

One PhD student supported by this project completed his degree during the project period (Wei Ting Chen). This student went on to a post-doc at the University of California at Berkeley in the field of quantum information. Three others were supported either fully (John Guthrie, Puchang Jiang) or through summer salary support (Bridget O'Mara). One of these students (Guthrie) is scheduled to defend his PhD before the end of the 2020 calendar year.

During the project period, the PI gave three invited talks and the group presented 11 contributed talks and posters at national conferences. No patents were produced during the project period.

IV. Transport and Collisions in Plasmas

Transport coefficients such as those associated with diffusion, viscosity, thermal conductivity, and electrical conductivity not only determine the behavior of plasmas; knowledge of these coefficients is necessary for the modeling of a wide variety of plasma systems. For sufficiently dense and cold plasmas, these transport coefficients are determined largely by collision rates in the plasma [24], through electron-electron, electron-ion, or ion-ion collisions. Through scaling relations, measurements of these properties in one plasma can be connected directly to a different plasma system despite possible wide variations in temperature and density. For instance, there are strong connections between UNPs and high-energy-density plasmas [13] through such dimensionless scaling parameters. Experimental and theoretical studies of UNPs are therefore useful for investigating and measuring quantities and physics important in other plasma systems as well.

For instance, collisions in plasmas are most often described in the so-called weakly coupled limit where temperatures are high enough and densities low enough that the spatial correlations that exist between particles in the plasma are not significant. As temperatures are decreased or densities increased, these correlations are no longer insignificant and the plasma coupling becomes stronger, with first moderate and then strong coupling. The usually weakly-coupled plasma theory breaks down under these conditions, as do many assumptions used to derive the weakly-coupled results [24,25,26,27]. Experiments allow comparisons to extensions of the weakly-coupled theories [12,28] and experiments show that careful theoretical treatments that examine and in many cases do not use typical underlying assumptions in treating collisions and transport are necessary in order to explain observed rates [13], lending validation to those theories as well as showing where they are yet incomplete. Our work touches on both of these topics – showing the need for the proper treatment of assumptions in theoretical descriptions of electron-ion collision rates, while also showing that binary collision theories do not match our observations (indicating the need to include more-than-two-body multi-body collisions).

As another example where UNP experiments can evaluate and inform theory treatments, UNP measurements can be compared to molecular dynamics (MD) simulations. In an MD simulation, the position and velocity of individual particles is traced through time as they all interact with one another via interparticle forces. MD simulations should contain all of the relevant plasma physics and as such represent a very sound theoretical model of plasmas. Comparison with experiment, however, is useful in making sure that all of the relevant parameters and physics are included. This was the case with DC electric fields in our UNPs [16], where we initially did not include them until the resulting simulation and experiment mismatch prompted us to examine where something had been left out of the MD simulation.

Even further than this, experiments can identify not just mismatched parameters or missing physics, but problems with MD codes. We encountered a situation where we had an MD code that appeared well-converged using usual energy conservation and other consistency checks. This code was designed to predict RF heating in magnetized plasmas. However, the code's predictions did not match experiments. We traced this to a failure to properly account for individual electron trajectories with sufficient consistency. We have an apparent solution for this difficulty, but at the expense of a factor of 100 to 1000 longer run times. We are presently evaluating if there are ways that can be shortened through changing integrators. Regardless, comparison to experiment was very useful in determining whether or not this code was valid.

This is true of other MD codes. By providing experimental data, the MD codes can be checked against experimental results and validated. Given the use of MD codes to determine transport coefficients and collision rates, using UNPs as a validation tool has the potential for improving the veracity of predictions for numerous plasma systems that are modeled in this way. Such data is equally applicable to evaluating the quality of predictions of analytic theories as well, and again has the potential to either verify those theories or show where they can be improved.

Before concluding this section, I will make a note concerning the fact that in many circumstances it is possible to create extreme conditions in UNPs that are not as easy to replicate in other plasmas (although this is not universally true and other plasmas can exhibit extreme conditions, too). If theories can be developed that are robust enough to account for any given extreme plasma physics, then they can be used for plasmas with less extreme conditions as well, and also used to evaluate the applicability of approximations. UNP experiments have a wider applicability than just UNPs themselves, but also wider applicability than measurements that can be directly compared to plasmas under equally extreme conditions. This is relevant for the studies of magnetized electron-ion collisions measured during this project.

V. Magnetized Electron-ion Collisions and Transport

Magnetic fields are a common feature of plasmas. In some cases, these magnetic fields are internally generated such as in plasmas described by MHD physics or in plasmas where large currents are created such as in some laser-induced high-energy-density plasmas. In other cases, magnetic fields are externally applied to confine or otherwise control laboratory plasmas of interest, including low-temperature plasmas with technological applications [29].

The strength of the magnetic field in the plasma determines how much of an impact the magnetic field has on the plasma behavior. In order to characterize this strength, it is possible to assign characteristic length

scales to different plasma properties. The smaller the length scale of a particular property, the stronger an effect the associated parameter has on the plasma. Dimensionless ratios of plasma parameters can be created, and plasmas with similar dimensionless ratios of parameters will behave similarly, even if they have radically different temperatures and densities.

In Ref. [7], the authors describe different regions of magnetization as a function of the strength of the magnetic field, the plasma density, and the degree of strong coupling in the plasma. These length scales relevant to the identified regions are a transport mean-free-path (λ_c), screening length (λ_D), binary collision characteristic length (r_L), and magnetic cyclotron radius (r_c). The usual ordering for a plasma is $\lambda_c > \lambda_D > r_L$ for those three parameters. In an unmagnetized plasma, r_c is greater than all these scales. As r_c becomes smaller than each parameter in turn as the magnetic field strength is increased, the plasma moves through weakly, strongly, and extremely magnetized regions. For matter-density plasmas of moderate electron coupling strength, the strong magnetization region occurs at 10s of kilotesla and the extreme at about an order of magnitude or two greater than that. Such magnetic fields can occur in astrophysical plasmas or laboratory plasmas with very high currents induced through high-power lasers [17,30].

Owing to their near-absolute zero temperatures, UNPs are much easier to magnetize. For instance, the ratio r_c/r_L is proportional to $T^{3/2}$, where T is the electron temperature. Extreme magnetization in UNPs can be achieved at magnetic fields as small as 10 mT or so. Easily-generated laboratory fields can thus be used to study plasma properties under extreme conditions. Moreover, experimental parameters can be controlled to allow systematic studies across magnetization ranges. These are the measurements that we performed.

Conventional descriptions of magnetized electron-ion collisions are expected to not necessarily continue to be accurate past the strong magnetization region. MD simulations can provide insight in highly magnetized plasmas, certainly at least in principle. In order to be accurate, however, MD simulations need to have short enough time steps to resolve the electron cyclotron motion in sufficiently strongly coupled cases. This places a large computational burden on these calculations at the least and may introduce difficult-to-detect errors at worst. Experiments can be used to test conventional descriptions of electron-ion collision rates and also MD predictions of those rates in this parameter region. Doing so pushes the frontier of plasma theoretical understanding, and again the further that frontier is pushed, not only is the range of validity of the theory extended, but the soundness and accuracy of the theory description at smaller degrees of magnetization is likely improved as well. This is one of the main goals of this project, and the goal of ongoing work as well.

VI. Overview of Ultracold Neutral Plasma Physics

Before reporting on the results and progress obtained in this project, a very brief overview of UNP physics is provided here. A more detailed description of the relevant UNP physics in the context of our experimental apparatus can be found in Refs [12,31].

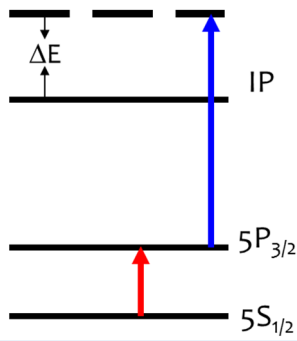


Fig. 1. Two-step photoionization of ^{85}Rb to form a UNP. The first state is an excitation from the ground state to the first excited state at 780nm. The second is a pulsed dye laser excitation to near the ionization potential at a wavelength near 479nm. By tuning the wavelength, the kinetic energy imparted during the photoionization (ΔE) can be controlled.

In our apparatus, UNPs are created through the photoionization of ultracold Rb atoms. These ultracold Rb atoms are obtained through standard laser cooling techniques. The initial starting conditions for a UNP consist of a finite-spatial-extent gas of ultracold atoms at a temperature of about 100 μK with a millimeter spatial length scale. A two-stage photoionization (see fig. 1) is used to ionize a chosen fraction of the atoms. The wavelength of the laser that excites the second stage of the ionization is tuned near the photoionization threshold. Doing so allows control over the initial electron kinetic energy and thus electron temperature of the UNP. In our experiments, typical densities are approximately $1 \times 10^7 \text{ cm}^{-3}$ and initial electron temperatures are set to 2-4K.

Shortly after the atoms are photoionized, nothing confines the electrons and some electrons escape the UNP region. The more massive ions do not move as fast, however, and so a space charge develops that eventually confines the electrons in the UNP, forming a two-component plasma of ions and electrons. The thermal pressure of the electrons ultimately causes the UNP to expand through accelerating the ions and eventually the UNP falls apart. However, the vast majority of our experiments are conducted before the expansion is significant.

There are several heating and cooling mechanisms for the electrons and ions in the UNP. As stated earlier, we measure the electron component. Electron component heating mechanisms include three-body recombination [32,33], disorder-induced heating [34,35], continuum lowering [36], and heating from an applied DC electric field [16]. Cooling mechanisms include expansion [37] and evaporative cooling [38].

Of the heating mechanisms, three-body recombination is the most significant for our conditions. Two electrons can collide near one of the ions such that one of the electrons becomes bound to the ion, forming a highly excited atom that is called a Rydberg atom. The other electron carries the binding energy into the plasma, ultimately heating the electrons. Three-body recombination limits the lowest achievable electron temperatures as has been known from the early days of UNP experiments [32]. In this project, it was discovered that three-body recombination and subsequent re-ionization of those atoms plays a role in transport physics and effective collision rates as well.

VII. Experimental Apparatus

In order to provide context for the measurements that have been made as part of this project and the new techniques that have been developed, this section briefly describes the experimental apparatus that was used. Figure 2 shows a general schematic of the experimental apparatus.

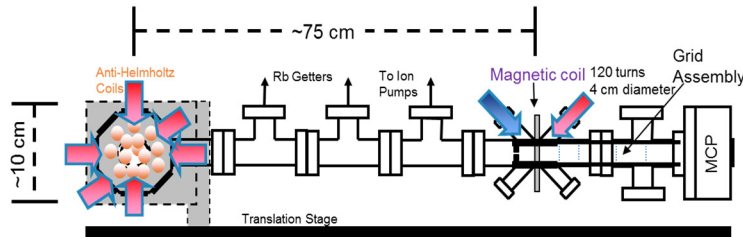


Fig. 2. Schematic of the experimental apparatus. All UNP measurements are conducted in a UNV system with the elements shown in this diagram. The laser cooling and trapping region is depicted on the left by indicating lasers that go into the chamber, and an oversized representation of the atoms being trapped, and a gray block that represents the anti-Helmholtz coils. These coils form a magnetic trap and translate to the plasma science region indicated by the two arrows that represent the two photoionization beams toward the right of the image. Between the plasma science interaction region and the MCP detector are a series of wire grids. Voltages can be applied to these grids to guide the electrons to the detector and extract electrons from the UNP. To the left of the plasma science region is depicted a conductor with a small hole in the center. Voltages can be applied to that as well and it is electrically isolated from the rest of the system. RF fields are applied to the UNP through voltages applied to this conductor.

which steady or time-varying voltages can be applied in order to apply electric fields to the UNP. Outside the chamber are two sets of wire coils. One is a 1.75" diameter 120 turn coil that is designed to produce the applied magnetic field at the UNP. This is the coil that is responsible for magnetizing the electrons in the UNP. Only 4A is required for 140G, which is enough magnetic field to magnetize the electrons to an extreme degree. The coil is operated in a pulsed configuration to avoid the need for water cooling. The second coil is a solenoid that produces a magnetic field to guide the electrons to the microchannel plate (MCP) detector, where the electron escape rate from the UNP is measured. We originally did not anticipate that this solenoid was needed, but certain electron trajectories did not connect with the MCP detector without it. The MCP signal is the primary experimental signal. Electrons that escape the plasma are accelerated by a series of wire grids with more than 92% open area toward the MCP in addition to being guided by the solenoid field.

The initial kinetic energy of the electrons can be controlled by altering the second-stage photoionization laser wavelength. We typically will run with 390 μeV to 1200 μeV of energy imparted per photoionized electron. At the lower end of this range, the resulting electron temperature is 3K. We can control the number of atoms ionized through adjusting the intensity of the first stage of the two-stage photoionization.

An experimental sequence starts with a standard Magneto-optic trap (MOT) of ^{85}Rb atoms in the laser cooling and trapping portion of the UHV chamber [31]. These atoms are then transported in a magnetic trap via physically moving anti-Helmholtz configuration wire coils to the plasma science portion of the vacuum system while the atoms remain in the magnetic trap created by a current through those coils. In the plasma science part of the chamber, there is a collection of isolated conductors and wire grids to

We performed substantial modifications to the conductors around the UNP as part of this project. The previous conductor assembly consisted of a series of ring electrodes positioned around the UNP. These allowed the application of DC and AC electric fields to the UNP, but the range of these fields was restricted because sufficiently large fields would cause the electrons being extracted from the plasma to strike the ring electrodes. Very careful arrangements of voltages were needed to both extract the electrons and prevent electron loss, and given the fact that applied magnetic field is relevant for electron trajectories we opted for a more robust system consisting of a single conductor with a hole in it on one side of the UNP and a series of open-area wire grids on the other (represented by the thin dotted lines in fig, 2). This new configuration required substantially less tuning of applied voltages to extract an electron signal, allowing for more robust operation of the system across different magnetic fields with the same voltage configuration.

There was a complication that arose with this change in electrode configuration, however. In the prior configuration, there were very steep changes in the electric field just outside the UNP spatial extent. These steep fields were ascertained from modeling combined with the measured UNP response, but even with that it was difficult to characterize them precisely. Since the physics being measured did not depend on these fields in any quantitative way, that was not a problem. When we changed the electric configuration, the electric field had a much smoother profile, but that had the unintended consequence of introducing a complication to our measurements. The prior experimental method that we used consisted of using pairs of short electric fields pulses [12] to first initiate oscillation of the electrons in the plasma (with the first pulse) and then read out the oscillation amplitude (with the second pulse) by measuring the number of electrons that escaped the UNP as a result of the second pulse. With the new electrode configuration, we found that there were electrons that left the UNP as a result of the first pulse that did not go directly to the MCP but rather remained near the UNP. These electrons complicated the interpretation of the electron escape number from the second pulse, making that the experimental technique that we used before more complicated with the new configuration (in a way that took substantial time to diagnose and understand). We could have returned to the prior electrode configuration and accepted the consequences of the more complicated operation that would have resulted. Instead, this difficulty led us to think about alternate techniques and come up with a superior one that we have then used for our measurements. We still use electron oscillations to measure electron-ion collision rates, but now use RF fields rather than pairs of short electric field pulses. Details of the new technique are included in section XII below.

The total number of electrons is proportional to the total integrated signal measured on the MCP over the full lifetime of the UNP. The density of the plasma is calibrated through applying RF fields to the UNP near its resonant frequency and measuring the plasma response to find the resonant frequency, which is a

function of the density [31]. All other experiments consisted of applying a series of fields to the UNP and measuring its response in order to measure any physics quantities of interest.

VIII. Computational Capabilities

In addition to our experimental apparatus, we have computational capabilities that are a critical part of our research efforts. There are three ways that this is true. First, we can use these capabilities to model the UNPs in our apparatus as a way of predicting the UNP behavior. We can use experimental results to validate the computational models that we have and then use those model results to investigate the physics of interest. Second, we can use our computational models to relate our results to other plasma systems. We can include theoretical treatments that have been published or have been derived from other plasma systems and use our computational capabilities to translate them into a prediction for our UNP system. For example, we can create a code that incorporates a theory of electron-ion collisions into a model of our UNP system. We can then perform calculations that in turn translate this theory to predictions for quantities that we can experimentally measure in our system, such as damping rates or heating rates. This allows us to test how well the theory predictions match the experimentally measured quantities in our plasma, which tests the quality of the theory predictions. Without our computational capabilities, this comparison of theory to experimental measurement would be at least far more difficult, if not impossible.

The third way that these computational capabilities support our research is that we need to perform calibrations of our experimental parameters for both temperature and density. The computational capabilities that we have allow us to transfer our measured experimental responses to the quantities of interest (e.g. plasma density). We can also use simulations of the UNPs to check on approximations that we have made in the course of our measurements. In many instances, there are effects that we assume to be small and can make general arguments about why they should be small. We can perform simulations to confirm that this is indeed the case for our experimental parameters.

The computations that we perform consist of MD simulations of our UNPs. We have the capability of performing simulations with a 1:1 ratio of simulated particles to particles in the experimental UNPs, meaning that we do not have to investigate number scaling relations and then do extrapolation to our system. Our MD code consists of direct computation of the Coulomb interaction between particles in the plasma in each time step to determine the net acceleration for each particle. This is a naturally N^2 problem since the Coulomb force between all pairs of particles needs to be considered. Given that we have on the order of 10^5 particles in the plasma this represented a serious computational challenge. While there are sophisticated algorithms [39,40] to reduce the order of the problem from N^2 , we instead rely on hardware solutions to perform our computations in a reasonable amount of time. We use GPUs to perform parallel computations

efficiently. The GPU code was developed in collaboration with Dominic Meiser while both his efforts at Tech-X and ours were supported by the AFOSR.

There are two types of simulations that we run. In one configuration that we call the “full MD” we include both the ions and electrons in the plasma as point particles interacting via Coulomb forces. We can apply external electric and magnetic fields to the simulation. This configuration should have all of the physics of interest, and at lower magnetic fields we have demonstrated agreement between this full MD simulation and measurements that we have made. The second type of simulation that we run was call the “Monte Carlo” or MC simulation. In these simulations, the electrons are still treated as interacting point particles. Electron-ion collisions, however, are modeled via a Monte Carlo collision operator. In addition, a uniform positive charged background is included to model the spatial confinement of the ions. The collision operator is specified by taking a theory of electron-ion collisions and translating it into an equivalent collision operator. The MC simulations thus allow the investigation of different theories and how they compare to our experimental measurements and/or MD simulations. While the MD simulations are to have all of the physics in them, it is not immediately clear what physics is important or what those simulations mean for the understanding of theory predictions. The MC simulations bridge that gap.

At the start of this project there were two challenges that we were facing with regard to our computations, one of which was resolved and one of which is still outstanding. The first problem was that while the MD simulations and experimental measurements agreed, there was a gap between the predicted UNP electron-ion collision signal that came from theory treatments derived from the MC simulation and experimental measurements/MD simulation results. This problem was resolved through finding that many-body effects were important in moderately coupled electron UNPs as mentioned earlier in this report and discussed in detail in section X below.

The other challenge is still outstanding, and was presented in section IV above. When we incorporated magnetic fields into our MD simulations, the MD simulation results and our experimental measurements did not match despite the fact that the usual checks on the convergence of the MD simulation seemed to indicate that it should have been fine.

We were able to determine that while the MD simulation was converged with respect macroscopic plasma quantities such as total energy and was also converged over time scales on the order of a few times the inverse of the electron plasma oscillation frequency. But, the MD simulation was not converged with respect to electron trajectories over longer time periods. Changing the time step would produce different trajectories. Additional numerical investigations based on single electrons colliding with single ions in strong magnetic fields showed a marked sensitivity to initial conditions [41,42] as has been known, and the

relevant time scale of the collisions in strong magnetic fields is such that the MD was not sufficiently convergent. As mentioned above, to make these collisions convergent would require decreasing the step size by over 100 according to these numerical estimates, corresponding to a commensurate increase in computation time. For our 1:1 modeling, 20 hour simulation runs were required and so this increase is a substantial increase in simulation time, and the convergence of those simulations is still an open question.

The fact that there was a disagreement between MD predictions and experiment for a situation where the MD code met typical convergence check criteria does show the utility of UNP measurements. Without the experimental check, the MD code would have looked valid while being incorrect. We conducted work looking to increase the speed of the simulations through changing aspects of the computation techniques used, but substantial increase has not yet been realized. Collaborations with the Murillo group and their AFOSR-funded work developing their SARKAS code [39] are promising in this regard and may provide the solution to this particular problem.

In the meantime, we have developed electron-ion collision rate predictions based on dielectric theory as described in section XV below. Our MC code is still valid for translating these predictions to our UNPs, and our calibration needs can still be met by the code since they do not depend on the exact (or even rough) value of the electron-ion collision rate. The computational capabilities that we have are therefore still crucial for our experimental work and sufficient for what is absolutely needed, despite there being ongoing problems associated with the physics of strong magnetic fields in plasmas.

These two computational challenges are relevant to the results described in the following sections. To start, the results obtained in small magnetic fields will be presented first, before returning to the stronger magnetic field theory and experiments.

IX. Electron-ion Collisions in Plasmas with Moderate Electron Strong Coupling

To discuss the context of work that occurred during this project, an overview of AFOSR-funded results that were published from our research group in Ref. [12] will be very briefly presented. In that work, the damping rate of electron oscillations initiated by a short electric field pulse applied to the UNP was used to measure the electron-ion collision rate. This was done under conditions where the electrons were very cold such that strong coupling physics was relevant, with a strong coupling parameter Γ of 0.35.

Most theory treatments of electron-ion collision rates are performed using assumptions that the plasma is in the weakly-coupled regime where $\Gamma \ll 1$. These theories produce a collision rate that is proportional to a term called the Coulomb logarithm [24]. This Coulomb logarithm incorporates the screening and thus collective physics of the plasma into the predicted collision rates. Theoretical results are almost invariably

published with such Coulomb logarithm terms as explicit logarithms, which is its weakly-coupled form. Multiple theories [24,25,26,27] have been published that extend collision theory to incorporate strong coupling. However, even with the extensions to strong coupling in those theories, their reliance on an explicit logarithm term (in [25,26]) results in predictions that do not match our measured electron-ion collision rates by more than a factor of 2 [13] despite including strong coupling physics in the theory.

One of the problems is the continuing use of the logarithm functional form in the Coulomb logarithm to match weakly-coupled plasma theory even once the electrons have started to become strongly coupled. Using more generalized [24] or extended [25] Coulomb logarithm terms addresses this problem at least in part and that change dramatically improves the agreement between measured (and full MD) UNP electron-ion collision rates. Our UNP experiments thus demonstrate the need for such theory and illustrate that the limitations to the standard Coulomb logarithm pointed out in these theories are relevant for plasma systems in this parameter range.

While these results and implications are present in our work in Ref. [12], the publication of the interpretation of these results and situating them in the context of relations to other plasma systems in Ref. [13] is very useful for better disseminating this physics more widely and highlighting that these considerations are necessary for accurate treatment in plasmas with moderate coupling. Not only does that publication help place this work in context, but additional work was done that specified the Coulomb logarithm form used in our MC code in an analytically-accessible way and not just through the numerics of that code. Through continuing work and increased understanding, the relevance and implications of AFOSR-funded work in our research group and elsewhere is better connected in Ref. [13] not only to UNPs but to high-energy-density plasmas with similar dimensionless plasma parameters.

X. Many-body Collisions in Moderately-coupled-electron UNPs

While using a proper treatment of the Coulomb logarithm improved the agreement between theoretical predictions of the electron-ion collision rate as expressed through our MC simulations on one hand and our experimental and MD simulation results on the other, there still was a substantial gap of over 30% between the theory predictions and our measurements/MD simulation at the coldest electron temperature that we studied (which is the highest degree of electron strong coupling, $\Gamma=0.35$). The quantity where this difference shows up is in the damping rate of electron oscillations, which for certain parameters can be made to be wholly dependent on the electron-ion collision rate to a very good approximation [43]. This gap was evidence of relevant physics not being included in the theory treatment, and so represented a chance to improve the theoretical understanding of these plasmas by determining what the source of the gap was.

One possibility is that the theories used to calculate the collision rate [25-27] were themselves incomplete in some way. There was one candidate for this incompleteness which was eventually found to be substantial for moderately- and strongly-coupled plasmas. For numerical convenience, most simulations of plasma conditions are calculated with like-signed charged particles. This avoids complications owing to metastability and the presence of bound states (unlike-signed charges will not want to remain free particles at sufficiently cold temperatures, but will combine back into atoms). In the limit of weak-coupling, the sign of the charged particles in binary scattering is irrelevant. However, that is not necessarily the case as strong coupling is increased. Our observed gap between data and predictions led to conversations with the Scott Baalrud group at the University of Iowa. They theoretically examined the difference between like- and unlike-signed charge collisions and found that indeed there is a predicted effect when strong coupling becomes relevant [14]. For some parameters, the difference between the cases is as large as a factor of 2. This work is not relevant only for UNPs, but for plasmas in general and also has implications for the understanding of the fundamentals of kinetic theory in plasma treatments. While this discussion was useful and fruitful, for our conditions this identified effect was only 6% of the approximately 30% gap.

The work that occurred in [14] was specifically designed to include binary collision effects only and not bring in complications from many-body effects, which were studiously avoided. In UNPs, though, many-body effects increase rapidly with increasing electron strong coupling. The most well-studied many-body effect is three-body recombination to Rydberg atoms (touched on briefly in section VI). While the heating and recombination rate have been the subject of study in UNPs [33], the impact of these collisions on transport properties in UNPs has not been considered until the work performed during this project which is described here.

To see how these three-body recombination collisions can affect transport, consider electrons with a non-zero center-of-mass velocity with respect to a collection of motionless ions. The electrons will collide with the ions via binary collisions, and that will deflect those electrons' velocities, ultimately causing a reduction in the center-of-mass velocity as the electrons' velocities become randomized. There is another possible process, though, for deflecting an electron's velocity. The electron could become bound to an ion in a Rydberg atom through a three-body recombination collision. Once that happens, the electron's velocity direction change as it "orbits" the ion is much faster than due to electron-ion binary collisions. In addition to three-body recombination forming Rydberg atoms, there is also a substantial probability that such loosely bound electrons can be re-ionized through a collision between the Rydberg atom and another electron [33]. If this re-ionization occurs, the net result of such a sequence is a large deflection of the electron's velocity. While Rydberg atom formation/re-ionization probability is rare compared to a binary electron-ion collision

probability, the deflection is on average much greater and so this is a process with at least the potential to have a significant impact on transport.

We used our full MD code to investigate the contribution of such many-body effects for our UNP conditions. While the full MD code should contain the relevant physics, work is necessary to identify particular effects within the calculation. We examined several signatures that are associated with the physics of interest. We could look at the distribution of the deflection of electrons over a set time period.

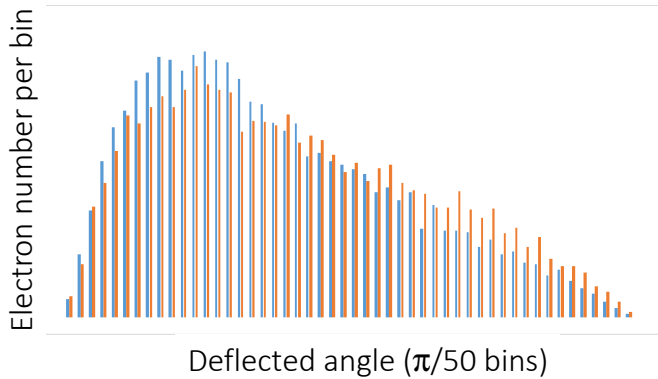


Fig. 3. Comparison of MC and MD electron angle distributions. In this data, the angle between electrons' velocity at time t_0 and the angle at time $t_0+20\text{ns}$ is computed and a histogram created with bins $\pi/50$ in width. The blue data come from the MC code, while the red from the MD. The MD code consistently has more electrons with a higher deflection angle, owing to the deflection that can occur through an electron becoming bound to an ion in a Rydberg atom.

It is possible to predict that distribution through binary collision theory, and the distribution obtained from the MD simulation could be compared to that prediction. An example of such a comparison is shown in the histogram in fig. 3. The full MD code showed electron distributions that had higher deflection angles than the binary collisions alone would predict.

The fact that we were using a simulation meant that we could do tests that were not physically possible otherwise. Three-body recombination rates scale more strongly with ion charge than binary collision rates do. By artificially lowering the ion charge while keeping the electron charge and the

average charge density the same, the ratio of three-body collisions to binary collisions could be decreased. When doing so, the discrepancy between the theory predictions from binary collisions and the results of the MD simulations decreased. We looked at the spatial localization of electrons in a fixed time period and the scaling of the discrepancy with electron strong coupling parameter and in all cases the results supported the many-body collisions as being responsible for the observed gap [15].

Our work is quantitative enough to enable the determination of whether or not such considerations are important in other plasmas when electron strong coupling is relevant. It highlights the importance of this physics and characterizes the size of its effect. It is not yet a full theory of many-body-collisions and electron transport, but does represent the first step in that direction. It also demonstrates how discrepancies between UNP theory and measurements can lead to the identification of previously unknown relevant

physics, though either simply illuminating a useful direction for study (the Baalrud group results [14]) or directly through identifying physics that needs to be considered.

One possible future direction that would be interesting to examine is to determine the impact of these effects on stopping power in strongly coupled electron plasmas. Sufficiently strong electron coupling is possible in inertial confinement fusion-type plasmas [44] for these considerations to be relevant in principle. While the ion velocities studied in [44] were fast enough that three-body recombination won't be significant, for sufficiently thick plasmas or different experimental parameters these considerations could become relevant. Transport in other strongly coupled electron plasmas would be also be impacted and so for plasmas such as atmospheric plasmas that have similarly strong coupling these considerations will be present, too – with the added complication of electron-neutral collisions being important as well.

Adaptations of our MD code would make stopping power calculations possible, albeit numerically intensive. This is actively being evaluated as a future research direction.

XI. DC Electric Field Heating of UNPs

The influence of strong coupling on the physics of the electron component of UNPs can be studied only when the electrons are sufficiently cold. While three-body recombination is predicted to limit electron temperatures such that strong coupling parameters can only be as much as about $\Gamma=0.5$ [33], achieving that degree of strong coupling can only occur if there aren't other significant heating mechanisms. In work published in Ref. [16], we describe the heating that occurs from a DC electric field applied to a UNP. Our primary tools for this work were our simulations, but it was necessary to take into account this physics to match the MD to our experimental results [12]. In addition to being a concern as a limit on achievable Γ , the presence of heating from an electric field can complicate the determination of the electron temperature in UNPs through being another contribution to the temperature that would have to be taken into account.

Electric fields are commonly applied to UNPs where the electron component is to be studied. Small fields direct the electrons out of the UNP preferentially toward the charge detector that is used to measure the UNP properties. Experiments with UNPs where Rydberg atoms are studied also use relatively large electric fields and so have the potential for electric fields to possibly be present during plasma formation.

The mechanism by which these electric fields can heat UNPs is straightforward. When atoms (or molecules) are ionized to form the UNP, the resulting electron and ion distributions overlap in space. If an electric field is present, though, the electrons and ions will be accelerated in opposite directions until a restoring force large enough to oppose that motion develops. The electrons and ions will then proceed to oscillate with respect to one another until that motion damps, turning into heat in the electron component.

This heating mechanism is complicated by the fact that as the electron motion and subsequent damping is occurring, electrons are also escaping from the plasma since there is no confinement of the electrons until a sufficiently large space charge develops. These escaping electrons will carry more-than-average energy away from the UNP, and it is not immediately clear whether or not this energy removal could offset the heating caused by the electric field acceleration. Our work measured the contributions of the heating and cooling from these electric field and electron escape mechanisms, and found that indeed heating did result for plasma conditions relevant for UNPs. We characterized the amount of heating, determined that heating scaled at least qualitatively with the applied electric field strength squared at low values of electric field magnitude (as expected), and determined when the electron escape considerations were relevant.

We were also able to verify that a very simple estimate of the amount of heating caused by a specific applied electric field was valid as an upper limit on the induced heating for the conditions that we studied. It is thus possible to quickly compute how problematic any applied electric field would be for a UNP or any other finite-sized plasmas that would exhibit the same physics.

Given the complications that could come from this heating, we designed our experiments to minimize the heating from this mechanism. The new electrode configuration radically reduced the applied fields necessary to direct electrons to the MCP detector. We also will form the UNP at near-zero external electric fields and then will slowly ramp up the strength of the applied field that we ultimately use to direct the electrons to the MCP detector. We can do this in such a way that electrons can initially escape the UNP at low values of electric field, but they can't strike a surface and be lost before the ramp field directs them to the UNP. Our work in this area was thus useful not only in quantifying the effect, but also showing us that it was something to be mitigated and minimized, and we were able to do so. Our previous studies at $\Gamma=0.35$ are consistent with higher values of strong coupling now being achievable in the absence of the applied electric field that was present in that prior work. The highest achievable Γ for electrons in a UNP remains an open question, however, that represents a possible future research direction.

XII. Off-resonant RF Heating to Measure Extremely Magnetized Electron-ion Collision Rates

As was mentioned in section VII above, the original technique of using two short electric field pulses to measure electron-ion collisions through electron-ion damping rates did not work as well once we changed our electrode configuration. We did gain advantages from the new electrode configuration that we did not wish to abandon if we did not have to, and so we developed an alternate technique for measuring electron-ion collision rates in these UNPs: off-resonant RF heating rates.

In many ways, this was a fortunate circumstance as this new technique has numerous advantages over the old technique. It has a much better signal-to-noise ratio by approximately a factor of 10. For many

experimental parameters, it is far less susceptible to the problems that can be introduced from collisionless damping contributions [43] in UNPs, meaning that it can be applied over a wider range of UNP parameters. It allows for the investigation of plasma physics beyond that originally envisioned, including investigations of the timescale of electron screening in magnetized and unmagnetized plasmas and nearly direct investigations of stopping power in magnetized plasmas by using off-resonant RF frequencies much lower than the electron plasma frequency. This technique connects through scaling relations very naturally to inverse bremsstrahlung heating in laser plasmas [45] and has some unique capabilities to isolate physics of interest in those systems, especially with respect to the nonlinearity of absorption as is briefly pointed out in section XVI below. While substantial additional development was required, in the end this is a superior technique to our prior one that will enable better and more studies of interesting plasma physics.

Before describing the technique, I will take a slight detour to note that in diagnosing possible difficulties with our magnetized plasma measurements using our old technique, we were able to determine several useful UNP properties. The UNP formation physics was observed to not depend in anything more than a minor way on whether or not a magnetic field was applied. The UNP expansion rate similarly showed little sign of substantial variation, although it is the case that quantitative differences could yet be detected. These two observations are consistent with predictions that the magnetic field should not substantially alter the initial temperature of the electrons, at least under the conditions where we operated such that three-body recombination was not expected to be a substantial heating mechanism (since we took data with electrons sufficiently weakly coupled that this should not have been the case). I will also note that we experimented with using different Rb isotopes, and while there was no difference in UNP physics (as expected), for technical reasons our original choice of ^{85}Rb was better than ^{87}Rb . The isotopic abundance of ^{85}Rb and the need to put atoms in the lower hyperfine state in the magnetic trap to protect against stray light with frequencies near the upper hyperfine state cycling transition in our system outweighed potential simplifying advantages from using ^{87}Rb trapped in its upper hyperfine state.

Returning to the off-resonant RF technique, there is a reason that off-resonant RF frequencies are used. Resonant RF frequencies have long been used in UNPs to characterize their density [37], and we use them for that purpose as well. In the resonant case, however, the amplitude of the oscillation is ultimately determined by the effective damping rate in the plasma. While in principle that could be used to determine quantities such as the electron-ion collision rate, the RF frequency being near the electron plasma frequency means that collisionless contributions could play a role and that variations in the UNP density from experimental shot to experimental shot would result in large variations in the amplitude of oscillation. In order to derive the electron-ion collision rate, the amplitude must be known, and so this presents a complication.

In the off-resonant case, the oscillation amplitude induced by the applied RF is independent of the resonant frequency of the UNP to a good approximation. If needed, corrections can be straightforwardly applied as well. This work focuses on high-frequency RF heating, but similar considerations also apply to RF heating well below the plasma frequency.

The independence of the oscillation amplitude can be derived from a simple harmonic oscillator model. At sufficiently high applied frequency, the amplitude will scale as one over the applied frequency squared. The electrons will all oscillate together in this circumstance, and so electron-electron collisions cannot change the electron energy relative to the oscillating center of mass and so cannot result in the heating of the electrons. We confirmed that this was the case through simulations. Electron-ion collisions, on the other hand, will change electron velocities with respect to the oscillating center of mass, ultimately resulting in heating on average. Thus, the heating rate of the electrons is directly proportional to the electron-ion collision rate, enabling the measurement of that rate through this technique.

In our apparatus, the RF voltages are applied to the copper disk electrode on the “MOT side” of fig. 2. Doing so produces an electric field at the UNP that oscillates in the axial direction. Applied RF and magnetic fields are thus collinear.

In order for this measurement to be useful, however, a technique is needed to measure the electron temperature in the UNP. Direct measurements of electron temperature are not readily available, and so we developed one based loosely on the ideas in Ref. [46]. A DC electric field can be slowly applied to the UNP to extract electrons from the UNP. The amount of electrons that are extracted will in part be a function of the electron temperature. It is therefore possible measure electron temperature changes through the amount of electrons extracted through the application of such a field.

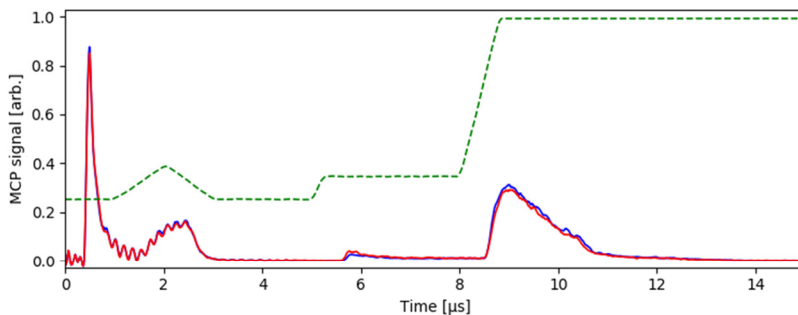


Fig. 4. Electron temperature measurement via electron extraction. The dotted green curve is shows the applied electric field, offset from zero for clarity and plotted with arbitrary units. The other two traces are MCP signals, with the red trace the signal with RF heating present and the blue trace without. The features of the MCP signal are described in the main text.

Figure 4 shows a typical experimental sequence. The UNP is formed by photoionization. The initial large peak at early times is caused by the electrons that escape the UNP shortly after formation. A slow ramp in DC electric field is applied to remove some electrons from the UNP after its formation to make

a deeper confinement for the remaining electrons once the DC field is slowly removed. The electron signal near 2 μs in fig. 4 signifies the electrons being removed in this step. Simulations indicate this process does not introduce significant heating or cooling for our conditions. After the ramp sequence, an off-resonant RF field is applied or not applied between 3 and 4 μs . If it is applied, it is turned on and off adiabatically. After time for any imparted heat to thermalize, a DC electric field is applied and the electrons that are partially removed by this field are counted, integrating the escape signal around 6 μs . A stronger applied electric field removes the rest of the electrons to determine the total number.

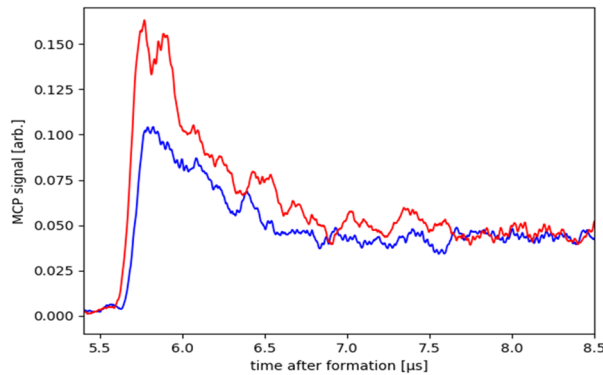


Fig. 5. The same two traces as shown in fig. 4 for the MCP signal, but focusing on the region right after the extraction pulse was applied. The difference in integrated area in these two signals is evidence of heating in the case where the RF was applied (the red trace).

As can be seen in fig. 5, the amount of extracted electrons is higher when the off-resonant RF field is applied, indicating that field does indeed heat the plasma. Through altering the RF field amplitude, the amount of imparted heating can be varied.

We conducted most of our measurements with UNPs with a density of $1.2 \times 10^7 \text{ cm}^{-3}$. The electron temperature was on average 4.5K over the heating process, starting at 3.5K and ending at 5.5K. The center-of-mass electron oscillation frequency [12] was set to be 16MHz, and

typically there were on order 10^5 electrons and ions in all measurements. Off-resonant RF fields of 60MHz were most often applied to make the planned measurements.

In order to make a useful measurement, the partially extracted electron signal has to be related to a specific amount of heating. This partially extracted signal is very sensitive to the exact details of the UNP number, spatial size, applied electric field, and electron temperature such that a quantitative determination from a theoretical or simulated description of the signal directly would have a high degree of uncertainty and would be difficult. Instead, the partial extraction signal is compared to a known amount of heating in order to calibrate it. The applied RF amplitude is then adjusted to match that heating and from the relation between the heating and the oscillation amplitude (which is ultimately related to the RF field amplitude), the electron-ion heating rate can be determined.

There are a series of calibrations that we have developed to determine an electron-ion collision rate from this experimental signal:

- The relation of the additional extracted signal to the increase in electron temperature is calibrated through changing the photoionization laser wavelength to alter the initial electron temperature by a known amount. This is done under conditions where minimal heating and cooling contributions are expected, both through estimates and through explicit simulation. The photoionization laser wavelength is known well, and changes to the wavelength are known even better (to better than 0.1K equivalent temperature change).
- Short electric field pulses heat the electrons, too. The amount of heating from a short electric field pulse is dependent only on the amplitude of the pulse for sufficiently fast pulses. So, measurements are conducted as a function of the amplitude of a short electric field pulse to find the amplitude where the heating from the short electric fields pulse matches the calibrated heating determined in the prior step. This heating will be the same for all magnetic fields. The calibrated short electric field pulse can then deliver a calibrated amount of heating to the UNP under a wide range of conditions.
- The electric field amplitude of the applied RF can be calibrated using plasma heating, too. Rather than adiabatically turning off the applied electric field, it can be turned off suddenly. If done at the right time, the electrons will have a center-of-mass velocity that will oscillate after the rapid RF field turn-off through the restoring force the ions exert. This oscillation will eventually damp and that damping will result in electron heating. The velocity of the electrons is related to the amplitude of the RF field amplitude, and so by comparing the heating from a suddenly-turned-off RF heating configuration to an adiabatically turned-off one, the calibration of the RF field amplitude can be obtained. This RF field calibration has the advantage that it occurs at the UNP's location.

These calibrations need to be performed only once, and can be performed for any workable set of UNP parameters. They all can be accomplished with sub-10% precision in our apparatus. Once the short electric field pulse heating is calibrated, the heating from that short electric field pulse can be matched to the heating from an applied off-resonant RF field by varying the amplitude of the applied RF field. Once the matching is performed, a relation is established between the applied RF field and the total amount of heating, and from that relation the electron-ion collisional heating rate can be determined.

At the time of the completion of this project and in the period shortly thereafter, all of the procedures listed above had been established, checked, and verified with one exception. The third calibration step relies on the sudden turn-off of the RF amplitude, and we realized that the finite capacitance and inductance of the electrodes in our system would be relevant for that turn-off. We have since determined that is indeed the case and are taking the relevant data to enable the proper calibration. Unfortunately, the on-campus shutdown owing the pandemic in spring 2020 both reduced needed lab access and required time to bring

the system back up to operation after a period of dormancy. We have just completed taking the necessary data but full analysis of that data is still outstanding. We expect to complete that task soon, and that is the remaining task necessary for the calibration of our data that has been collected to date.

XIII. Rapid Application of Magnetic Fields for UNP Electron-ion Collision Rate Measurements

One additional aspect of our apparatus underwent substantial development during the project period was the controls used to apply a chosen magnetic field to the plasma region of our apparatus. We cannot use a permanent or continuously-on magnetic field for technical reasons owing to transporting the atoms by use of a magnetic trap. A large magnetic field would shift the location of the atoms and would also compromise the trap if it were large enough. We thus needed a technique to turn on a magnetic field rapidly (within 1ms), but in a way that the magnetic field would be stable once it was turned on.

We were able to achieve this using high-current solid-state switches and large inductors. A circuit diagram is shown in figure 6. By flowing a current through the relatively large 10mH inductor, we store energy in the magnetic field of that inductor. All of the switches are rated with capacities up to 100A. The servo is used to regulate the current through the magnetic field coil at a low level to provide a small bias field as the atoms are being transported into the region. This small bias field of 10.7G is also used for our lowest magnetic field measurements as well. The switch configuration for this small bias field is switches #1 and #3 closed and #2 and #4 open. Several amps (depending on the desired final magnetic field for the UNP, typically up to 7A) are flowing through the inductor and the 0.5 Ω resistor in this configuration. This large current is not left on continuously, though. Switch #1 is left open until the atoms are to arrive in the plasma

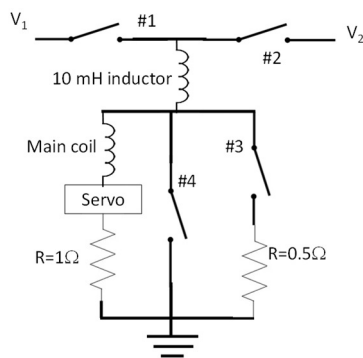


Fig. 6. Circuit diagram for rapid turn-on of magnetic field-producing coil. Numbered switches, the main coil as an inductor, sense resistors, and the large inductor used to store magnetic energy are shown. Details of circuit operation are presented in the main text.

region and is closed well before they do so.

To rapidly increase the current through the coil, the states of all of the switches are reversed. The 10mH inductor produces a large emf to rapidly increase the current in the magnetic field coil. The voltage v_2 is set to produce the current that corresponds to the desired magnetic field. The voltage v_1 is adjusted to minimize ringing when the switching occurs. Using these techniques we can turn on the magnetic field current to its desired value within 300 μs . Without the extra inductor (i.e. applying the desired voltage and waiting for the current to come to its steady-state value), the turn-on time is a factor of 100 times slower, too slow for our technical requirements.

XIV. Measurement of Magnetized Electron-ion Collision Rates

We have collected electron-ion collision rate data under three magnetic field conditions. The first was selected to be at the boundary of weakly and strongly magnetized region [7] at 10.7G. The second was selected to be close to the extreme magnetization region but still at a field classified as strongly magnetized at 65G. The third data point collected at 136G was selected to be in the extreme magnetization regions. Density and temperature parameters are given in the section above for these conditions.

The collection of each of these data points required several weeks of time each. Only four full days were required for the measurement of the RF amplitude that matched the short pulse heating to establish the heating rate itself. The rest of the required time is setting experimental parameters: getting the UNP electron and ion number set to produce the desired density, setting the guiding solenoid fields so that the electrons are successfully guided to the MCP, making sure that there is no electron loss to surfaces other than the MCP at each condition, setting the first-stage ionization laser frequency to the appropriate value given the applied magnetic field, adjusting current setting to produce the desired applied magnetic field, and measuring the value of the applied magnetic field using spectroscopic techniques. We anticipate that in the future this process will be faster as we can interpolate between known parameters, and think that ultimately it can be shortened to two weeks or less. The length of time involved at the moment, however, is the reason that we do not yet have a much larger set of data points.

In order to assign an absolute heating rate to each data point, the calibration tasks described in the prior section need to be completed. Preliminary indications are that there are no substantial (i.e. order-of-

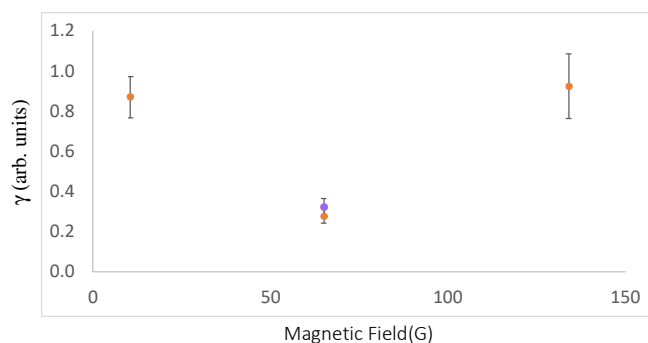


Fig. 7. Relative electron-ion collision rates as a function of applied magnetic field. The purple point was collected with the heating set to be 1/3 as much as the red points.

magnitude) differences between the measured and estimated or predicted heating rate (see the section below).

Prior to establishing the absolute calibration, we can compare the relative electron-ion collision rates for the collected data. This is done through setting the induced temperature increase (ΔT) equal to $\gamma \Delta t V_{pp}^2$ where Δt is the length of time that the RF heating pulse is applied, V_{pp} is the

amplitude of the RF signal selected on the function generator producing the signal, and γ is a constant that is proportional to the electron-ion collision rate. Figure 7 shows a comparison of the value of γ for different magnetic fields. It is clear that there is some structure as a function of magnetic field as the value of γ varies

substantially over the range of applied magnetic fields. Much more data will be needed to better characterize this structure, and that is the near-term goal for future work.

The two points at 65 G represent a check on any possible non-linear effects. The purple point is taken under conditions with only 1/3 the heating of the other points shown in the figure. Despite the reduced ΔT value, the electron-ion collision rate coefficient γ is the same to within the precision of the measurement. This indicates the absence of large non-linear effects, at least at that magnetic field.

The mild magnetic fields that were used can possibly obscure the fact that a large change in the degree of magnetization occurred over the range of magnetic fields that were applied. The upper magnetic field in this data set would correspond to hundreds of kilotesla in matter-density plasmas and represents a set of parameters where the magnetization should dominate the physics of the UNP electrons. The fact that there is hardly any change in the electron-ion collision rate is thus somewhat surprising in general terms. The fact that there is not a larger change is ultimately related to the anisotropy of magnetized collisions and this lack of substantial change is an indication of the relevant physics. In order to discuss that more fully, however, it is useful to compare the measured variation of the collision rate with magnetic field to predictions and that will be done in the following section.

The obtainable level of precision in these measurements is also quite good as compared to what can be practically obtained for many plasma measurements. We were able to achieve a precision of about 15% in the determination of the heating rates for each data point. We would thus be sensitive to even relatively small variations of collision rate with magnetic field. For this data, the level of precision that we have available means that we can clearly resolve the differences in electron-ion collision rate that are observed.

XV. Dielectric Theory Treatment of Magnetized Electron-ion Collisions

There are several possible ways that a theory prediction could be derived for UNP heating rate for our experiments. One of the most natural, though, is to look at predictions of plasma AC conductivity. The heating from the RF field can be thought of as ohmic heating through a finite resistivity of the plasma. Theories dating from decades ago [22,23] make predictions for heating rates for our plasma parameters, and we can compare these theory predictions to our measurements. Other possible theory treatments include those based on binary collisions or adaptations of stopping power theories [47,48]. These are usually derived in perturbative limits, however, and our UNPs are cold enough that perturbation theory is not applicable. Outside perturbative limits, binary collisions can be very complicated when a magnetic field is present, as collisions that exhibit chaotic behavior are common, for instance [42]. AC conductivity and other theories that rely on the dielectric response function of the plasma as opposed to binary collisions do not have the same difficulties (at least not apparently) for our UNP conditions.

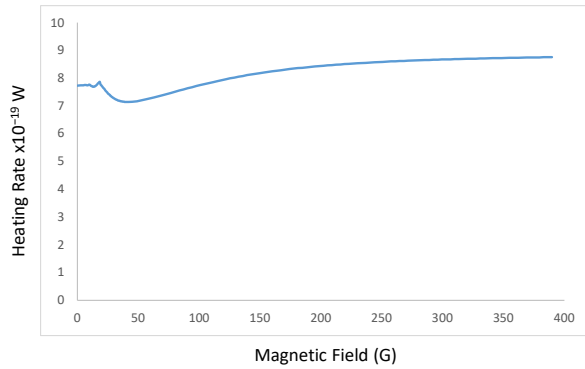


Fig. 8. Predicted per electron off-resonant heating rate as a function of magnetic field for the UNP conditions described in the main text assuming a RF amplitude of 1V/m. This prediction is derived from Ref. [23].

Figure 8 shows a prediction of heating rate as a function of magnetic field for conditions relevant for our UNPs, including the applied RF frequency that we used. This prediction is taken from the theory of Ref. [23], which is based on the theory of Ref. [22]. The plasma density and temperature parameters that are used are the ones that are specified above in section XII and correspond to the density and temperature of our UNP systems. One of the more striking features of this prediction is the lack of substantial variation

with magnetic field. Again, the degree of magnetization of the plasma is changing profoundly over this range.

By way of contrast, an example of a plasma system and collision-based property that does show variation over this range of magnetization is the energy equipartition rate in non-neutral plasmas of pure electrons. These rates have been predicted [20] and measured [21], and show a strong dependence on applied magnetic field. Over the range of magnetization that we investigated, the equipartition rate in this non-neutral system decreases by well over an order of magnitude. This non-neutral plasma system is different than our magnetized UNP in many important respects, and the collisions that are being studied are not even the same (electron-electron vs. electron-ion). But, the difference in the degree of variation of a collision rate with magnetization between the two systems is notable and indicates that magnetized collisions are not just characterized by a single rate but that the anisotropy inherent in the magnetic field is an important aspect of the physics.

The fact that the high- and low-magnetic field data points in our collected data are similar to one another lends support to the overall prediction that there is no substantial change in heating rate even in a condition of extreme magnetization. However, the data point at the intermediate field does indicate that there is a variation that is unaccounted for by the theory. Clearly, more data is needed to ascertain the variation in the electron-ion collision rate to get a sense of where the deviation from theory predictions may be occurring, and what the overall structure looks like at additional interior fields and also at higher magnetic fields as well.

While in this section the comparison has been made to AC conductivity predictions, those predictions are not strictly directly applicable to our UNPs. The RF amplitudes that are needed for adequate signal-to-noise in our system are higher than the limits of validity specified in the theories of Refs. [22,23]. We have developed an alternate theory based on the Boltzmann equation with a thermal relaxation term [49] that

allows a natural extension to larger electric field amplitudes similar to Ref. [50] in the unmagnetized case. We have shown that this theory predicts the same heating rates as Refs. [22,23] in the low-amplitude and high-frequency limit. This theory maps onto stopping power theories [51] in the low-frequency limit, too. We are still addressing some lingering convergence and numerical issues with this new theory, but do not anticipate it will predict substantially different heating rates than Refs. [22,23] for our UNP conditions. It will, however, take into account the amplitude of our applied RF fields in a consistent way. The lack of agreement between observed and predicted heating as a function of magnetic field will still be an ongoing issue in all likelihood, however. This will not preclude the potential utility of this new theory at lower magnetic fields.

We are preparing two manuscripts based on the work in this and the previous section. One details our experimental results presented in primarily in section XIV, and second details our dielectric constant theory described in the paragraph above.

XVI. Future Measurements

While there are already unanswered questions with the collection of just a few data points using our off-resonant RF heating technique for measuring electron-ion collision rates, there are many possible future research directions that this technique is well-suited to allow for further exploration:

- More data is clearly needed to investigate the unpredicted variation in RF heating rate with magnetic field that has already been observed.
- Data can be collected at even higher fields, exploring physics deeper in the extreme magnetization region.
- By going to higher temperatures, the UNP can be moved toward a regime where perturbative theories are more applicable and can thus be tested. A factor of three increase in temperature should be achievable in a straightforward way, and higher than that is possible.
- It is possible to go to lower temperatures, too, exploring the interaction between extreme magnetization and strong coupling influences. Rydberg atom formation will lead to complications in interpretation of the heating data, but there are multiple ways that this distortion can be compensated or accounted for.
- There is very little data regarding stopping power in magnetized plasmas. By performing experiments with off-resonant frequencies far below the resonant frequency, direct connections can be made to stopping power predictions and experimental tests of those predictions can be made in a systematic way. Signal-to-noise will be challenging in this region, but estimates and simulations indicate that useful measurements can be made.

- By going to higher frequencies, the time-dependence of the screening response of the UNP can be investigated in a systematic way
- By investigating the heating rate as a function of RF amplitude, predictions for the nonlinearity of the heating rate can be tested. This can be done as a function of magnetic field, and these results can be scaled to inverse bremsstrahlung heating experiments through scaling relations to help answer open questions in those systems. [52]
- With the addition of a wires running in the axial direction, perpendicular RF fields can be applied to investigate transverse heating in addition to the axial heating that is being investigated now.
- Work will continue on improving the MD simulations, and criteria necessary for agreement between the simulations and the experiments will be elucidated.

Measurements will typically be made with 20% precision or better, with several percent precision possible if that is advantageous. Off-resonant RF heating is a powerful and versatile experimental technique for investigating fundamental plasma physics, and its (somewhat fortuitous) development has enabled a platform that can be used to study many aspects of plasma physics, as indicated by the list above. I fully expect that existing plasma theories will be challenged, presenting opportunities for additional discovery.

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