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Gratings with preselected Plane-Wave Propagation Directions

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Abstract

A generalized Poisson summation formula produces exact expressions for the locations and strengths of line currents that radiate preselected plane waves. The line currents can be in free space or placed above a perfectly conducting ground plane. Closed-form expressions are obtained for the line current locations and amplitudes when the plane-wave parameters have periodic amplitudes and propagation vectors. A corresponding wire-impedance solution employs thin-wire approximations and takes into account all multiple interactions between wires. For general non-periodic plane-wave parameters, the line-source locations and amplitudes can be determined from an FFT approach involving a complex contour integral and the corresponding wire impedances can be computed numerically. These results generalize to 2D arrays of electric and magnetic dipoles characterized by polarizability dyads. Specifically, the plane-wave parameters for the 2D array are selected in accordance with two independent periodic constructs, and the dipole strengths and polarizability dyads are expressed as the product of two functions, each of which has the form of the line-source solutions. With this technique, the number of degrees of freedom for planar gratings has been increased significantly to allow for the creation of a wide range of plane-wave fields.

Accomplishments

As indicated by the title, the research objective is to determine a grating geometry that radiates *preselected* plane waves. This approach is different from previous work where the starting point is a preselected grating geometry that contains a number of free parameters that are adjusted to obtain a desired field. To start with the preselected field requires a convenient procedure for specifying a set of plane waves that is compatible with the type of grating under consideration. This step is nontrivial since, for example, one must specify infinitely many plane waves in order to achieve a discrete grating. Indeed, if only a finite number of plane waves are specified, the resulting scattering structure must represent a continuous source. Therefore, previous publications started with the grating geometry instead of the desired field. During this project we developed a general theory for (i) specifying plane waves consistent with a discrete source and (ii) showed how the required

grating geometry can be determined directly from preselected plane-wave parameters.

The cornerstone of this work is the conversion of Bochner's generalized Poisson summation formula into a representation that can be used to explicitly determine grating geometries from preselected plane-wave parameters. Specifically, this Poisson summation formula allows an array of line currents [1] or a 2D array of dipoles [2] to be determined explicitly from preselected plane-wave parameters. This result holds generally and even works when the preselected plane waves require a continuous source region rather than a discrete one. Moreover, this theory predicts which set of plane-wave parameters require a discrete source and which require a continuous source. Some sets of plane-wave parameters result in periodic sources whereas others result in non-periodic sources.

For use in most practical applications, we developed a periodic plane-wave construct that allows the specification of a set of infinitely many plane waves that are guaranteed to produce a discrete array with equal spacing between elements [1], [2]. Here the plane-wave parameters are specified for mode numbers up to a certain cutoff and then repeated outside this range. With this infinite set of plane waves, a discrete array of either line sources [1] or dipoles [2] emerges whose locations depend only on the period of the plane-wave propagation directions. These locations are independent of the number of plane waves and their parameters within each period. We demonstrated how the periodic plane-wave construct allows the design of arrays that radiate identical propagating waves but different evanescent waves [3]. This feature of the theory makes it possible to design arrays that radiate very little imaginary (reactive) power without affecting the real power.

Using the line-source solutions from [1], a wire-impedance solution was derived that employs thin-wire approximations and takes into account all multiple interactions between wires [4], [5]. Remarkably, this solution holds even when the required impedance values are non-periodic. The plane-wave parameters determine the period of the impedance values and thus determine how many different types of loaded wires are required. For general non-periodic plane-wave parameters, the line-source locations and amplitudes can be determined from an FFT approach involving a complex contour integral and the corresponding wire impedances can be computed numerically. With these impedance solutions, the number of degrees of freedom for the planar grating has been increased significantly to allow for the creation of a wide range of plane-wave fields. In [2], we developed a corresponding 2D electric and magnetic dipole solution represented by polarizability dyads. Specifically, the required polarizability dyads and dipole locations are determined explicitly from the preselected plane-wave parameters.

In addition, three papers [6]–[8] were published during the period 15-Jan-2019 to 14-Jan-2022 that are outside the scope of the present contract but related to previous AFOSR contracts; ([6] and [7] were submitted well before 15-Jan-2019). In [6] we showed that the condition number of the normal matrix for a least-squares solution used in near-field scanning can be dramatically improved by restricting the number of far-field pattern values so that it corresponds to the actual number of degrees of freedom of the antenna under test. The paper [7] formulated an inverse source problem in terms of the singular value decomposition (SVD) for scalar sources that radiate the time-domain wavefields of a complex point source. Finally, [8] compares three translation operators applied to a near-field scanning problem. This paper demonstrates the superiority of the Gaussian translation operator developed under previous AFOSR contracts.

Impacts

The theory developed in this project will likely impact the area of grating design, which applies to the manufacturing of lenses, mirrors, and beam splitters that are building blocks of many electromagnetic and optical systems. In addition, Bochner’s generalized Poisson summation formula was examined and re-written in the project to make it eminently usable in other areas of electromagnetics and acoustics. In particular, it can possibly be incorporated into integral-equation methods that involve the Ewald transform to take into account the mutual coupling between non-uniformly spaced scattering elements. Finally, the papers produced through this project contain a number of canonical solutions that are generalizations of expressions that can be found currently in many text books. For example, the expression for the wire impedances in terms of preselected plane-wave parameters, which is a generalization of a classical formula, could become standard issue in advanced text books on electromagnetics.

Changes

The project proceeded as expected and the original plan was not modified.

Technical Updates

It was very recently discovered by the PI that the theory developed during the project can be augmented to cover the situation where a layered dielectric slab occupies the space between the loaded wires and the ground plane; see Figure 1. This augmentation results in a modification of

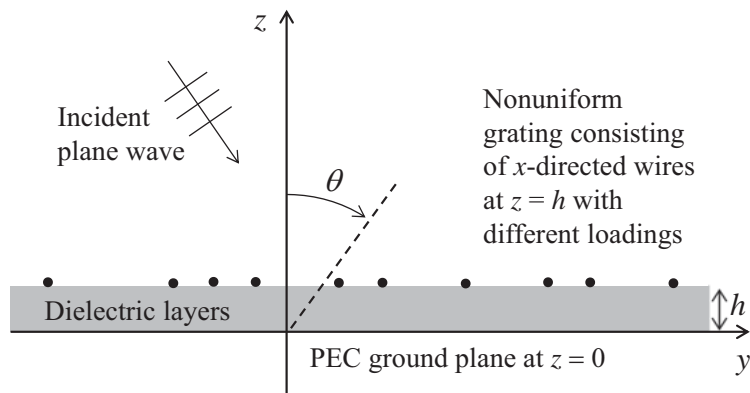


Figure 1: A grating consisting of x -directed loaded wires that lie in the plane $z = h$ with $h > 0$ above a layered dielectric structure backed by a perfectly conducting ground plane at $z = 0$. In the dielectric layer region, the boundaries between regions with constant propagation constants are $z = z_j$ where $0 \leq z_j < h$.

the expression for the wire impedances, so that it depends on both the preselected plane-wave parameters and a reflection coefficient for the dielectric layers. A similar augmentation works for the 2D array of electric and magnetic dipoles. Two benefits are achieved with these dielectric layers: (i) an increase in the degrees of freedom for the grating structure, and (ii) a functional support structure for the loaded wires that is different from the free-space dielectric assumed in the project. The additional degrees of freedom may make it possible to create a much larger class of locally lossless gratings than was previously possible.

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