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DIRECTIONAL DOPPLER DETECTION FOR IF-CORRELATOR FM RANGING SYSTEMS USING GENERAL FM MODULATIONS

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A new IF processing technique for obtaining directional doppler ranging is presented. This technique does not require a specific type of frequency modulation waveform (such as a sawtooth) but may be used with any practical FM waveform. Examples of measured directional doppler range responses are shown for both a summing circuit and a product circuit for obtaining the directional range law from two quadrature doppler channels.

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1. INTRODUCTION

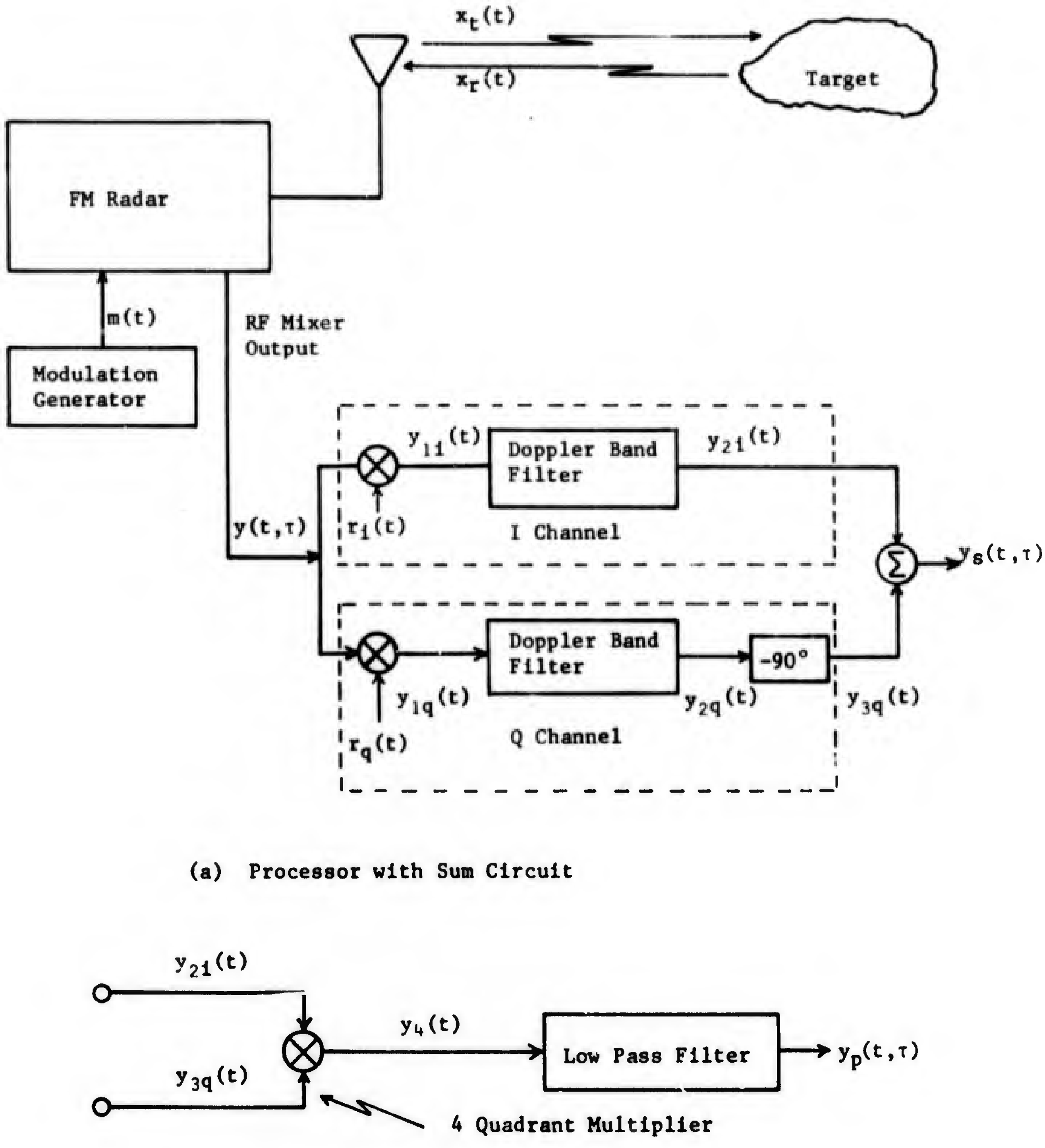
Directional doppler detection is a radar technique wherein a signal resulting from a moving target is processed in such a manner as to yield the sense of the range rate of the target. Directional doppler detection was first described by Kalmus [1] for an unmodulated CW oscillator. That paper described a method of detecting the quadrature doppler signals necessary for directional doppler detection by coherently detecting the returned signal with quadrature reference signals derived from the transmitter. The quadrature relationship between the two references was implemented with a short delay line to achieve a 90° phase shift between the reference signals. Kalmus' system requires a receiving antenna by which the target signal can be separated from the transmitted signal. In systems employing a single loop oscillator which serves simultaneously as transmitter and receiver, this separation is not possible.

Small loop oscillators, where the loop serves as both transmit and receiver antenna, have been developed which are capable of wide band FM with low incidental AM [2]. Detection of the envelope of the RF loop voltage with a target present yields an IF signal which can be processed to give range responses which peak at a desired height without requiring a delay line and which have range resolution related to the transmitted RF bandwidth. Various aspects of these systems (IF correlator-FM ranging system) have been investigated since they provide economical low height radar systems[3-9].

Tozzi has shown that directional doppler can be detected using the IF signal (post-mixing) for periodic sawtooth frequency modulation [3]. For this case, the IF signal appears as a single-sideband signal about the modulation harmonics. Its quadrature components can be detected by coherently detecting the IF signal with harmonic references in time quadrature.

It will be shown herein that directional doppler detection can be achieved for periodic or non-periodic waveforms for which the IF signal does not have the single-sideband characteristic provided the proper quadrature IF references are used. These quadrature references are described as $\cos[\theta(t,\tau_0)]$ and $\sin[\theta(t,\tau_0)]$ where $\theta(t,\tau_0)$ is an estimate of the phase of the IF signal due to a target of delay τ_0. While these references are sinusoidal and in quadrature in θ, they will, in general, be neither sinusoidal nor in quadrature in time. Use of these references allows directional doppler detection for IF correlation-FM ranging systems without requiring particular modulation waveforms. The directional doppler processor may be either of the sum or the product type. The sum processor is equivalent to a matched filter.

Mathematical formulas are developed which provide the directional range response from either a sum processor or a product processor when the quadrature reference voltages are non-sinusoidal--such as a quadrature square wave estimates of the signal beat pattern--which are useful in systems employing low cost digital circuitry. These formulas are verified experimentally by numerous examples of measured directional doppler responses.



(a) Processor with Sum Circuit

(b) Product Circuit

FIGURE 1: DIRECTIONAL DOPPLER PROCESSORS

2. RANGE RESPONSE OF THE DIRECTIONAL DOPPLER SUM PROCESSOR

A general formula for the range response of the directional doppler sum processor, as shown in Figure 1(a), will be obtained. This formula is valid for any type of FM modulation--deterministic or stochastic. It is also valid for distant as well as close targets. Later, this formula will be simplified for close targets and for periodic modulations.

In Figure 1(a), the signal $y(t,\tau)$ from the R.F. mixer is applied to two mixers which will extract quadrature components of the signal. The reference voltages $r_i(t)$ and $r_q(t)$ are voltages corresponding to the estimated quadrature components of $y(t,\tau)$ evaluated at $\tau = \tau_0$. The methods by which these reference voltages may be generated will be discussed in Section 4. For the present discussion we will assume that they are available.

2.1 General Result

Referring to Figure 1, the transmitted signal is represented by

$$x_t(t) = \sqrt{2}\, \mathrm{Re}\{v(t)e^{j\omega_c t}\} \tag{1}$$

where ω_c is the radian carrier frequency and $v(t)$ is the complex envelope [10]. Then the corresponding received signal is

$$x_r(t) = \sqrt{2}\, \mathrm{Re}\{v(t-\tau)e^{j\omega_c (t-\tau)}\} \tag{2}$$

where τ is the round trip time delay.
To the first approximation, the mixer output is given by the cross-product term

$$y(t,\tau) = \mathrm{Re}\{v(t)v^*(t-\tau)e^{j\omega_c \tau}\} \tag{3}$$

where the output about $2\omega_c$ has been neglected since it is filtered out.

For a frequency modulated radar

$$v(t) = e^{j[D\int_0^t m(t)dt]} \tag{4}$$

where D is the deviation sensitivity (radians/volt-sec) and $m(t)$ is the frequency modulating waveform. Then

$$v(t)v^*(t-\tau) = e^{j\theta_d(t,\tau)} \tag{5}$$

where

$$\theta_d(t,\tau) = D \int_{t-\tau}^{t} m(\sigma)d\sigma \quad . \tag{6}$$

$\theta_d(t,\tau)$ is the mixer output difference phase with τ being a parameter.

Using these equations, the mixer output is given by

$$y(t,\tau) = \mathrm{Re}\{e^{j\theta_d(t,\tau)}e^{j\omega_c\tau}\} \quad . \tag{7}$$

Referring to Figure 1(a), the reference signals are the estimated quadrature components of the complex envelope of the RF mixer output expected from targets at a fixed range corresponding to a delay time τ_0. The exact reference signals are then

$$r_i(t) = A_r \,\mathrm{Re}\{e^{j[\theta_d(t,\tau_0) + \theta_0]}\} \tag{8}$$

$$r_q(t) = A_r \,\mathrm{Im}\{e^{j[\theta_d(t,\tau_0) + \theta_0]}\} = A_r \,\mathrm{Re}\{e^{j[\theta_d(t,\tau_0) + \theta_0]}e^{-j\,\pi/2}\} \tag{9}$$

where A_r is the peak amplitude of the reference signals and θ_0 is the reference phase constant. For perfect reference signals $\theta_0 = \omega_c\tau_0$. In practice it is often more ecomonical to generate quasi - reference signals which will give similar results. (This will be illustrated later by some examples.) Equations (8) and (9) are the exact estimates of the quadrature components of the mixer output at a target delay τ_0. They will be called exact replicas and are given by the real and imaginary components of $e^{j\theta}$, where $\theta = \theta_d(t,\tau_0) + \theta_0$. Quasi-replicas will be defined as other functions of θ--such as square wave and triangular--which are not necessarily periodic in time. These quasi-replicas can be expanded in terms of exact replicas and their harmonics [8] by the Fourier series expansion shown below.

$$f(\theta) = \sum_{-\infty}^{\infty} r_n e^{jn\theta} \tag{10}$$

where

$$r_n = \frac{1}{2\pi}\int_{-\pi}^{\pi} f(\theta)e^{-jn\theta}d\theta \quad . \tag{11}$$

In analog computer technology $f(\theta)$ is the output of a function generator where θ is its input. The r_n are the (complex) Fourier coefficients that describe the waveshape of the output of the function generator *with respect to* θ. Thus, the I and Q reference signals described by (8) and (9) can be generalized to

$$r_i(t) = \sum_{n=-\infty}^{\infty} r_{in}e^{jn[\theta_d(t,\tau_0) + \theta_0]} \tag{12}$$

$$r_q(t) = \sum_{n=-\infty}^{\infty} r_{qn}e^{jn[\theta_d(t,\tau_0) + \theta_0]} \tag{13}$$

where the $\{r_{in}\}$ are the set of Fourier coefficients describing the functional waveshape for the I reference signal and the $\{r_{qn}\}$ are the Fourier coefficients for the Q signal functional waveshape. For the case of exact replicas the Fourier coefficients are $r_{i1} = 1/2$, $r_{i(-1)} = 1/2$, $r_{q1} = -j/2$, $r_{q(-1)} = j/2$ and the other r_{in} and r_{qn} are zero. Thus (12) and (13) reduce to (8) and (9) which are the exact replicas.

Referring to Figure 1(a), the multiplier outputs are

$$y_{1i}(t,\tau) = A_m\, y(t,\tau) r_i(t)$$

$$= A_m \,\mathrm{Re}\{ \sum_{n=-\infty}^{\infty} r_{in} e^{j[\theta_d(t,\tau) + n\theta_d(t,\tau_0) + n\theta_0]} e^{j\omega_c \tau} \} \qquad (14)$$

$$y_{1q}(t,\tau) = A_m\, y(t,\tau) r_q(t)$$

$$= A_m \,\mathrm{Re}\{ \sum_{n=-\infty}^{\infty} r_{qn} e^{j[\theta_d(t,\tau_0) + n\theta_d(t,\tau_0) + n\theta_0]} e^{j\omega_c \tau} \} \qquad (15)$$

where A_m is the multiplier gain constants.

The doppler filters pass only the doppler frequency components. We realize that $\omega_c \tau = \omega_d t + \phi$ where ω_d is the radian doppler frequency and ϕ is the reference doppler phase. Now define $C_n(\tau,\tau_0)$ to be the average values of the complex envelopes of the individual terms in the equation above. That is

$$C_n(\tau,\tau_0) \triangleq \overline{e^{j[\theta_d(t,\tau) + n\theta_d(t,\tau_0) + n\theta_0]}} \qquad (16)$$

where $\overline{[\cdot]}$ denotes the averaging operator, which may be a time average or an ensemble average for either deterministic or stochastic (noise) modulations.

Using (16) in (14) and (15), the outputs of the doppler filters are

$$y_{2i}(t,\tau) = A_m \,\mathrm{Re}\{ \sum_{n=-\infty}^{\infty} r_{in} C_n(\tau,\tau_0) e^{j(\omega_d t + \phi)} \} \qquad (17)$$

and

$$y_{2q}(t,\tau) = A_m \,\mathrm{Re}\{ \sum_{n=-\infty}^{\infty} r_{qn} C_n(\tau,\tau_0) e^{j(\omega_d t + \phi)} \} \quad . \qquad (18)$$

It is noted that (17) or (18) gives the output of a single channel system, that is (17) can be rewritten as

$$y_{2i}(t,\tau) = A_m |z(\tau,\tau_0)| \cos[\omega_d t + \phi + \angle z(\tau,\tau_0)] \qquad (19)$$

where

$$z(\tau,\tau_0) \triangleq \sum_{n=-\infty}^{\infty} r_{in} C_n(\tau,\tau_0) \quad . \qquad (20)$$

The range law for a single channel system is $|z(\tau,\tau_0)|$.

Returning to the calculations for the two channel system of Figure 1(a), the -90° phase shift network, also known as a Hilbert transformer, has the transfer function:

$$H(\omega) = \begin{bmatrix} -j & , & \omega > 0 \\ \\ +j & , & \omega < 0 \end{bmatrix} \quad . \tag{21}$$

Using (18) and (21), the output of the phase shift network is

$$y_{3q}(t,\tau) = A_m \, \mathrm{Re}\{ \sum_{n=-\infty}^{\infty} r_{qn} C_n(\tau,\tau_0) e^{j(\omega_d t + \phi \mp \pi/2)} \} \tag{22}$$

where the upper sign is used when $\omega_d > 0$ and the lower sign is used when $\omega_d < 0$.

Referring to Figure 1(a), the output of the directional doppler sum processor is

$$y_s(t,\tau) = y_{2i}(t,\tau) + y_{3q}(t,\tau) \tag{23}$$

where the s subscript denotes the output for the summing circuit. Using (17) and (22) in (23), the doppler output is

$$y_s(t,\tau) = A_m \, \mathrm{Re}\{[\sum_{n=-\infty}^{\infty} (r_{in} \mp j r_{qn}) C_n(\tau,\tau_0)] e^{j(\omega_d t + \phi)} \} \quad . \tag{24}$$

The output of the sum network reduces to

$$y_s(t,\tau) = \begin{bmatrix} A_m |z_+(\tau,\tau_0)| \cos[\omega_d t + \phi + \angle z_+(\tau,\tau_0)] & , & \omega_d > 0 \\ \\ A_m |z_-(\tau,\tau_0)| \cos[\omega_d t + \phi + \angle z_-(\tau,\tau_0)] & , & \omega_d < 0 \end{bmatrix} \tag{25}$$

where

$$z_+(\tau,\tau_0) \triangleq \sum_{n=-\infty}^{\infty} (r_{in} - j r_{qn}) C_n(\tau,\tau_0) \tag{26}$$

$$z_-(\tau,\tau_0) \triangleq \sum_{n=-\infty}^{\infty} (r_{in} + j r_{qn}) C_n(\tau,\tau_0) \quad . \tag{27}$$

If the two channels are subtracted, the inequalities are reversed.

For the case of exact replica reference signals, the Fourier coefficients are $r_{i1} = 1/2$, $r_{i(-1)} = 1/2$, $r_{q1} = -j/2$, $r_{q(-1)} = j/2$ and the other r's are zero. Thus, for exact (i.e. matched filter type) reference signal (23) becomes

$$y_s(t,\tau) = \begin{bmatrix} A_m|C_d(\tau,\tau_0)| \cos [\omega_d t+\phi-\theta_0 + \underline{/C_d(\tau,\tau_0)}] \ , & \omega_d > 0 \\ A_m|C_s(\tau,\tau_0)| \cos [\omega_d t+\phi+\theta_0 + \underline{/C_s(\tau,\tau_0)}] \ , & \omega_d < 0 \end{bmatrix} \tag{28}$$

where

$$C_d(\tau,\tau_0) \triangleq \overline{e^{j[\theta_d(t,\tau) - \theta_d(t,\tau_0)]}} \tag{29}$$

$$C_s(\tau,\tau_0) \triangleq \overline{e^{j[\theta_d(t,\tau) + \theta_d(t,\tau_0)]}} \tag{30}$$

and the difference phase, for the modulating signal m(t), is

$$\theta_d(t,\tau) = D \int_{t-\tau}^{t} m(\sigma)d\sigma \quad . \tag{31}$$

It is seen from (28), that if $|C_d(\tau,\tau_0)|$ peaks up at $\tau = \tau_0$ and if $|C_s(\tau,\tau_0)|$ is essentially zero for $\tau > 0$; then positive doppler targets are detected for the sum circuit and negative doppler targets are detected if a difference circuit is used. Furthermore $C_d(\tau,\tau_0)$ can be reduced to a complex autocorrelation function as follows. Using (31) into (29)

$$C_d(\tau,\tau_0) = \overline{e^{j\theta_d(t,\tau-\tau_0)}} \quad . \tag{32}$$

Define the complex autocorrelation function of the FM radar as the autocorrelation of the complex envelope; that is,

$$R_c(\tau) \triangleq \overline{v(t)v^*(t-\tau)} = \overline{e^{j\theta_d(t,\tau)}}$$

$$= \overline{\cos \theta_d(t,\tau)} + j \overline{\sin \theta_d(t,\tau)} \quad . \tag{33}$$

If the power spectrum of the transmitted signal is even about the carrier frequency, ω_0, then $R_c(\tau)$ is real; that is, $\overline{\sin \theta_d(t,\tau)} = 0$. For the most practical waveforms the power spectrum is even and $R_c(\tau)$ is real. For any case $|R_c(\tau)| = |R_T(\tau)|$, where $R_T(\tau)$ is the autocorrelation of the transmitted R.F. waveform. Using (33) in (32),

$$\boxed{C_d(\tau,\tau_0) = R_c(\tau-\tau_0)} \quad . \tag{34}$$

The other expressions for $C_n(\tau,\tau_0)$ cannot be reduced unless some further assumptions are made, as in the section presented below.

2.2 Close Targets

For the case of close targets, τ and τ_0 are relatively small quantities. As shown in the Appendix, $\theta_d(t,x)$, may be expanded into a Taylor's series:

$$\theta_d(t,x) = Dm(t)x \text{ , when } |x| < \sqrt{\frac{\pi}{4m'(t)}} \quad . \tag{35}$$

Using (35) and (33), (16) can be reduced to

$$\boxed{C_n(\tau,\tau_0) = R_c(\tau+n\tau_0)e^{jn\theta_0}} \quad . \tag{36}$$

Using (36) in (25), the output of the sum processor for the case of close targets is

$$y_s(t,\tau) = \begin{bmatrix} A_m|z_+(\tau,\tau_0)| \cos\left[\omega_d t+\phi + \angle z_+(\tau,\tau_0)\right] & , \quad \omega_d > 0 \\ \\ A_m|z_-(\tau,\tau_0)| \cos\left[\omega_d t+\theta + \angle z_-(\tau,\tau_0)\right] & , \quad \omega_d < 0 \end{bmatrix} \tag{37}$$

where

$$z_\pm(\tau,\tau_0) = \sum_{n=-\infty}^{+\infty} (r_{in} \mp j r_{qn}) R_c(\tau+n\tau_0) e^{jn\theta_0} \tag{38}$$

For close targets and exact replica reference signals these equations become

$$C_d(\tau,\tau_0) = \overline{e^{jDm(t)(\tau-\tau_0)}} = R_c(\tau-\tau_0) \tag{39}$$

$$C_s(\tau,\tau_0) = \overline{e^{jDm(t)(\tau+\tau_0)}} = R_c(\tau+\tau_0) \quad . \tag{40}$$

$$y_s(t,\tau) = \begin{bmatrix} A_m|R_c(\tau-\tau_0)| \cos\left[\omega_d t+\phi-\theta_0 + \angle R_c(\tau-\tau_0)\right] & , \quad \omega_d > 0 \\ \\ A_m|R_c(\tau+\tau_0)| \cos\left[\omega_d t+\phi+\theta_0 + \angle R_c(\tau+\tau_0)\right] & , \quad \omega_d < 0 \end{bmatrix} \tag{41}$$

where

$$R_c(x) = \overline{v(t)v^*(t-x)} \quad . \tag{42}$$

As discussed previously, for an even power spectrum about ω_0, $R_c(\tau)$ is real.

2.3 Periodic Modulation and Exact Replica References

For periodic FM, $v(t)v^*(t-\tau)$ can be expanded in a Fourier series. That is, if $m(t)$ is periodic, then $\theta_d(t,\tau)$ is periodic with respect to t. Thus, from (5), $v(t)v^*(t-\tau)$ is also periodic and can be expanded in a Fourier series

$$v(t)v^*(t-\tau) = e^{j\theta_d(t,\tau)} = \sum_{k=-\infty}^{k=\infty} c_k e^{jk\omega_m t} \tag{43}$$

where

$$c_k = \frac{1}{T}\int_{-T/2}^{T/2} e^{j\theta_d(t,\tau)} e^{-jk\omega_m t} dt = \frac{1}{T}\int_{-T/2}^{T/2} v(t)v^*(t-\tau)e^{-jk\omega_m t} dt \quad . \tag{44}$$

ω_m is the angular frequency of the modulating waveform and T is the period. Thus, $\omega_m = 2\pi/T$. In terms of the Fourier components (33) becomes

$$R_c(\tau) = c_0(\tau) \quad . \tag{45}$$

Using (45), (29) becomes

$$C_d(\tau,\tau_0) = c_0(\tau-\tau_0) \quad . \tag{46}$$

For $C_s(\tau,\tau_0)$, using (30) and (43)

$$C_s(\tau,\tau_0) = \overline{[e^{j\theta_d(t,\tau)}][e^{j\theta_d(t,\tau_0)}]} \tag{47}$$

$$= \sum_{k=-\infty}^{k=\infty} \sum_{\ell=-\infty}^{+\infty} c_k(\tau)c_\ell(\tau_0)\ \overline{e^{j(k+\ell)\omega_m t}} \quad . \tag{48}$$

The averaged factor is zero unless $\ell=-k$, so (48) reduces to

$$C_s(\tau,\tau_0) = \sum_{k=-\infty}^{k=\infty} c_k(\tau)c_{-k}(\tau_0) \quad . \tag{49}$$

For close targets this reduces to

$$C_s(\tau,\tau_0) = c_0(\tau+\tau_0) \quad . \tag{50}$$

An equation for evaluating $c_0(x)$ by using a piecewise linear approximation to the modulation waveform has been derived in a previous report [7].

3. RANGE RESPONSE OF THE PRODUCT DIRECTIONAL DOPPLER PROCESSOR

Formulas will now be obtained for the range response of the directional doppler processor with product detection as shown in Figure 1(b). The range response for the product processor is obtained by using (17) and (22). The output of the multiplier is

$$
\begin{aligned}
y_4(t) &= y_{2i}(t) y_{3q}(t) \\
&= A_m \mathrm{Re}\ \{ \sum_{m=-\infty}^{\infty} r_{im} C_m(\tau,\tau_0) e^{j(\omega_d t + \phi)} \} \\
&\quad \cdot \mathrm{Re}\ \{ \sum_{n=-\infty}^{\infty} r_{qn} C_n(\tau,\tau_0) e^{j(\omega_c t + \phi)} \} \\
&= \frac{1}{2} A_m \mathrm{Re}\ \{ \sum_m \sum_n r_{im} r_{qn}^* C_m(\tau,\tau_0) C_n^*(\tau,\tau_0) e^{j \pm \pi/2} \} \\
&\quad + \frac{1}{2} A_m \mathrm{Re}\ \{ \sum_m \sum_n r_{in} r_{qn} C_m(\tau,\tau_0) C_n(\tau,\tau_0) e^{j(2\omega_d t + 2\phi \mp \pi/2)} \}
\end{aligned} \tag{51}
$$

The low pass filter eliminates the second-harmonic doppler terms. Reducing (51), the range law at the output of the product processor is a slowly varying voltage given by

$$
\boxed{y_p(t,\tau) = \frac{1}{2} A_m \mathrm{Re}\ \{ \sum_m \sum_n r_{im} r_{qn}^* C_m(\tau,\tau_0) C_n^*(\tau,\tau_0) e^{\pm j\ \pi/2} \}} \tag{52}
$$

For close targets, using (36), this reduces to

$$
y_p(t,\tau) = \frac{1}{2} A_m \mathrm{Re}\ \{ \sum_m \sum_n r_{im} r_{qn}^* R_c(\tau+m\tau_0) R_c^*(\tau+n\tau_0) e^{j[(m-n)\theta_0 \pm j\ \pi/2]} \} . \tag{53}
$$

For exact replica reference signals (52) becomes

$$
y_p(t,\tau) = \begin{bmatrix} \frac{1}{8} (A_m)^2 [|C_d(\tau,\tau_0)|^2 - |C_s(\tau,\tau_0)|^2] , & \omega_d > 0 \\ \\ \frac{1}{8} (A_m)^2 [-|C_d(\tau,\tau_0)|^2 + |C_s(\tau,\tau_0)|^2] , & \omega_d < 0 \end{bmatrix} . \tag{54}
$$

If modulation waveforms are chosen such that the range response sidelobes are relatively small, then $|C_s(\tau,\tau_0)|^2$ is negligible with respect to $|C_d(\tau,\tau_0)|^2$ and (54) can be approximated by

$$y_p(t,\tau) \simeq \begin{bmatrix} \frac{1}{8}(A_m)^2|C_d(\tau,\tau_0)|^2 , & \omega_d > 0 \\ -\frac{1}{8}(A_m)^2|C_d(\tau,\tau_0)|^2 , & \omega_d < 0 \end{bmatrix} . \tag{55}$$

For close targets and using exact replica reference signals (39) and (40) may be used and (54) becomes

$$y_p(t,\tau) = \begin{bmatrix} \frac{1}{8}(A_m)^2[|R_c(\tau-\tau_0)|^2 - |R_c(\tau+\tau_0)|^2] , & \omega_d > 0 \\ \frac{1}{8}(A_m)^2[-|R_c(\tau-\tau_0)|^2 + |R_c(\tau+\tau_0)|^2] , & \omega_d < 0 \end{bmatrix} . \tag{56}$$

For periodic modulation, (46), (49) and (50) may be used.

4. GENERATION OF MODULATION AND QUADRATURE REFERENCE WAVEFORMS

It was shown in Section 2 that if the output of the RF detector, $y(t,\tau)$ is separated into quadrature components by multiplying this waveform by the proper quadrature references voltage waveforms, as given by (8) and (9), directional doppler ranging could be obtained. The directional responses are approximately $|R_c(\tau-\tau_0)|$ or $|R_c(\tau+\tau_0)|$. For FM modulating waveforms which provide good range definition, as given by $|R_c(\tau)|$, the directional responses will be such as to peak-up for one direction and be very low for the other direction.

$R_c(\tau)$ is related to the transmitted power spectrum, shifted to baseband, by the Fourier transform. Desirable range responses can be achieved by controlling the power spectrum. For high index FM, the envelope of the power spectrum is directly proportional to the probability density of the frequency modulation. Thus, for a linear FM modulator, the range response will be determined by the probability density $p(m)$ of the modulating waveform. *Since the results are determined by the probability density, any modulating waveform, periodic or non-periodic, which has the desired probability density and for which the proper references can be generated will produce the same results* [8]. For practical systems, the approach is determined by the economy and availability of hardware in most cases. Power spectra which are approximately triangular or which approximate a raised cosine will have lower range-response sidelobes than a uniform spectrum (which has a sin x/x shaped range response) and are more desirable for many applications.

Simple hybrid circuitry can be used to generate modulations which have the desired power spectra [4,7,8]. The resulting spectrum is generally a digital approximation to the triangular or raised cosine spectrum. Two such modulations are illustrated in the test results.

Generation of IF references as described in the references cited have taken two general approaches. For long time delay systems, the RF bandwidth time-delay product $B\tau$ may be large, and non-linearities in the modulator will significantly affect the IF signal. Also, large propagation-delay effects as discussed in Section 2 become significant because $\theta_d(t) = D \int_{t-\tau}^{t} m(t)dt$ and cannot be estimated by $Dm(t)\tau$. For these cases, it is generally necessary to use an R.F. delay-line/mixer technique to develop the proper IF reference voltage as described in [9]. References generated using this technique are not dependent on the particular modulation used or the RF bandwidth-time delay product, $B\tau_0$.

For many ranging systems, $B\tau_0$ will be small and the propagation delay errors can be ignored. When the modulator is linear, references can be generated directly from the modulation circuitry which match the expected IF signal except for a phase constant which, as illustrated in Section 2, does not affect the range response of directional doppler systems. These systems are more economical since they give acceptable results for low height applications and do not require the RF delay-line/mixer hardware. The IF reference requirements for directional doppler detection is the same as for the systems described in [4,7,8] except that two quadrature references are required. The following sections will describe the two methods of generating quadrature references.

4.1 Delay-Line Quadrature Reference Generation

As given by (1), the transmitted signal can be represented by

$$x_t(t) = \sqrt{2} \,\mathrm{Re}\, v(t)e^{j\omega_c t} \quad . \tag{57}$$

If this signal is applied to the delay-line mixer shown in Figure 2, the reference phase is $\theta_0 = \omega_c\tau_0$ and the reference waveforms generated will be given by

$$r_i(t) = \mathrm{Re}[v(t)v^*(t-\tau)e^{j\omega_c\tau_0}] = \mathrm{Re}[e^{j[\theta_d(t,\tau_0) + \theta_0]}] \tag{58}$$

$$r_q(t) = \mathrm{Im}[v(t)v^*(t-\tau)e^{j\omega_c\tau_0}] = \mathrm{Re}[e^{j[\theta_d(t,\tau_0) + \theta_0]}e^{-j\,\pi/2}] \quad . \tag{59}$$

These are exactly the same waveforms as those required of the quadrature references given by (8) and (9). This type of reference generator is a quadrature version of the reference generator used by Peperone [9], and generates a reference derived from the transmitted RF voltage. The delay-line may be a coiled coaxial transmission line or a non-dispersive acoustical RF delay line.

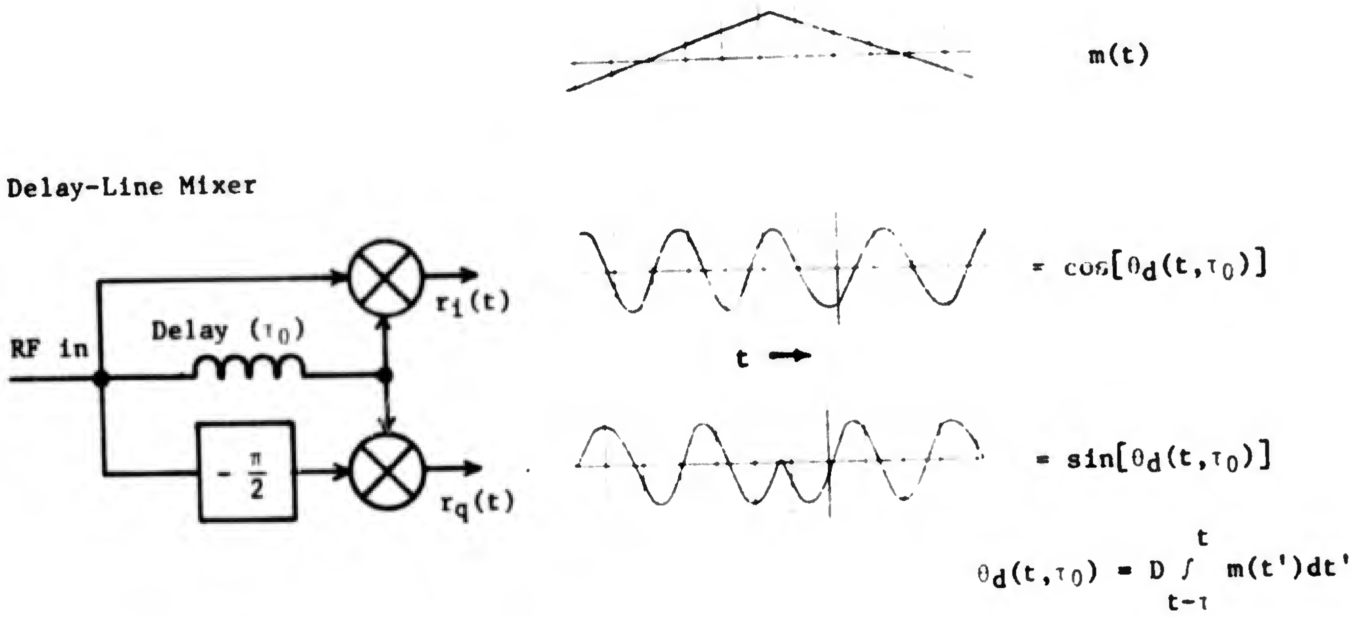


FIGURE 2: DELAY-LINE MIXER AS REFERENCE WAVEFORM GENERATOR

The quadrature reference waveforms shown on Figure 2 are drawn for triangular modulation. If the in-phase reference is described as $\cos[\theta_d(t,\tau_0)+\theta_0]$ then the quadrature reference is described by $\sin[\theta_d(t,\tau_0)+\theta_0]$. Note that these waveforms, in general, are not continuous sine waves but have a spectrum which is broad. For the triangular modulation shown, the 'turn-arounds' as shown in the waveform $r_q(t)$ are the distinguishing characteristic of directional doppler references when compared with single-sideband references (which are continuous sine and cosine waveforms).

Figures 3, 4, and 5, illustrate delay-line quadrature references for triangular, sawtooth, and sinusoidal modulations. The RF bandwidth B and the time delay τ_0 are the same for the three cases. For these illustrations, the $B\tau_0$ product is slightly greater than 4. It is evident from the figures that while the references are sinusoidal and in quadrature with respect to θ, they are not sinusoidal and not in quadrature with respect to time.

4.2 Modulation Circuitry Quadrature Reference Generation

The exact quadrature reference voltages are described by (58) and (59). We require that these voltages depend only on $\theta_d(t,\tau_0)$ since the phase, θ_0, affects only the phase of the doppler and is common to both. Also, from (28), $\theta_d(t,\tau_0) \approx D\tau_0 m(t)$ for small τ_0. If the deviation sensitivity, D, of the RF process is constant, then m(t) may be used to estimate $\theta_d(t,\tau_0)$. By letting $\theta_0 = 0$ the reference voltage waveforms as given by (58) and (59) may be estimated to be

$$r_i(t) = \cos[\theta_d(t,\tau_0)] \simeq \cos[D\tau_0 m(t)] \quad (60)$$

$$r_q(t) = \sin[\theta_d(t,\tau_0)] \simeq \sin[D\tau_0 m(t)] \quad . \quad (61)$$

Thus, the quadrature reference voltages may be generated from m(t).

Hybrid circuit techniques (digital and analog) can be used to economically generate modulation and reference waveforms which are functionally related as described by the previous equations. In many cases, circuit techniques can be used which produce modulations and references whose probability densities and functional relationships are not changed when the generator is frequency modulated. Techniques of this type are useful in analyzing FM/FM systems [8]. Square wave IF references are conveniently generated in digital circuitry but have some undesirable features since range responses corresponding to the square-wave harmonics are produced. Various techniques for reducing the undesired range responses are described in [4, 7, 8]. Square wave references can be written as

$$r_i(t) = \text{sgn}\{\cos[D\tau_0 m(t)]\} = \frac{4}{\pi}\sum_{1}^{\infty}\frac{\sin n\pi/2}{n}\cos\,[nD\tau_0 m(t)] \qquad (62)$$

$$r_q(t) = \text{sgn}\{\sin[D\tau_0 m(t)]\} = \frac{4}{\pi}\sum_{\substack{1 \\ n\ \text{odd}}}^{\infty}\frac{1}{n}\sin\,[nD\tau_0 m(t)] \quad . \qquad (63)$$

For the <u>sum processor</u> the directional doppler response obtained with these references will then be given by (37) and (38).

$$|y_S(t,\tau)| = \begin{bmatrix} G_s\left|R_c(\tau-\tau_0) + \frac{1}{3}R_c(\tau+3\tau_0) + \cdots\right| & ; \quad \omega_d < 0 \\ \\ G_s\left|R_c(\tau+\tau_0) + \frac{1}{3}R_c(\tau-3\tau_0) + \cdots\right| & ; \quad \omega_d > 0 \end{bmatrix} \qquad (64)$$

For the <u>product processor</u> and square wave references, by using (53), we get

$$y_p(t,\tau) = \pm G_p\{[|R_c(\tau-\tau_0)|^2 - |R_c(\tau+\tau_0)|^2] - \frac{1}{9}[|R_c(\tau-3\tau_0)|^2 - |R_c(\tau+3\tau_0)|^2] + \cdots\} \qquad (65)$$

where the upper sign is used for positive doppler and the lower sign for negative doppler and G is the overall gain constant for the system. We see from the above range responses that using square waves for the reference waveforms causes the third harmonic response to peak in the opposite doppler direction from the main range response. The third harmonic response will peak-up to a value of one-third for $\tau = 3\tau_0$. This response, however, will suffer an additional loss factor of 1/3 (due to free space attenuation and reflection from an infinite plane target) at a range of $3\tau_0$ as compared to the desired response at τ_0. This corresponds to an attenuation of 19dB which might be acceptable for many applications. For the product processor, even though the third harmonic term has a factor of 1/9, it corresponds to an input ratio of 1/3 because of square-law detection.

5. TEST RESULTS

The following examples are chosen to illustrate the principles described in the previous sections and to illustrate simple hardware techniques which can be used for implementation. Range responses were measured using a moving reflector on a 300Ω transmission line. The exponential transmission line loss characteristic is illustrated in Figure 18. Both sum processing and product processing directional responses will be demonstrated. Any processing circuitry shown for one processor could, of course, be used for the other processor with results given by (64) or (65).

5.1 Example: Sum Directional Doppler Processing Using RF Delay-line/Mixer Quadrature References

References are generated as illustrated in Figure 2 and the directional processor was implemented as illustrated in Figure 1(a). Figures 6, 7, and 8 illustrate the range responses for triangular, sawtooth, and sinusoidal modulations. Figure 9 illustrates the range response obtained when the function generator which generated the triangle modulation was frequency modulated to give an FM/FM system. The envelopes of the range responses are displaced sin x/x responses for triangular and sawtooth modulations and displaced $J_0(x)$ responses for sinusoidal modulation.

5.2 Example: Sum Directional Doppler Processing Using Quadrature Square Wave References Derived From the Modulation Circuitry

Figure 10 illustrates the block diagram and waveforms of a hybrid modulation/demodulation $B\tau_0 = 3/2$ system which uses square wave references. The functional relationship between the references and the modulation voltage is determined explicity by the circuitry and is illustrated along with the probability density of the modulating waveform. The power spectrum and range response is determined by p(m) as discussed previously. The quadrature doppler signals are detected by frequency modulating the function generator with the IF signal $y(t,\tau)$ and doppler filtering the waveforms which would normally be used as the quadrature references. This technique [11] uses the low index FM effect to achieve mixing and results in the elimination of the two multipliers shown in Figure 1(a). The quadrature signals are denoted $r_i(t)$ and $r_q(t)$ since they differ slightly from $r_i(t)$ and $r_q(t)$ due to frequency modulation by $y(t,\tau)$. The directional doppler responses for this system are shown in Figure 11.

5.3 Example: Sum Directional Doppler Processing Using RF Delay-line/Mixer Quadrature References and Hybrid Modulation

Figure 12 illustrates the modulation waveform and probability density of a modulation which can be conveniently generated using hybrid circuit techniques. The resulting modulation gives reduced range response sidelobes and better directional discrimination.

The test results shown in Figure 13 exhibit somewhat less than the calculated directional discrimination due to circuit imperfections in the directional doppler processor.

5.4 Example: Product Directional Doppler Processing Using RF Delay-line/Mixer Quadrature References

Examples of product directional doppler range responses are shown on Figure 14. The quadrature reference waveforms for these responses were obtained from an RF delay line as before. For the product system the directional responses are of opposite polarity and the range sidelobes are reduced according to $[\sin(x)/x]^2$ for triangular modulation and $J_0{}^2(x)$ for sinusoidal FM.

5.5 Example: Product Directional Doppler Using Square Wave Reference Voltage

The product range law for a $B\tau_0 = 1/2$ directional doppler system is shown on Figure 15. For this system the square wave estimates of $\cos(\theta)$ and $\sin(\theta)$ for the quadrature references, as given by (48) and (49), turn out, in time, to be a square wave at the fundamental frequency of the modulation waveform and a constant. These two time reference waveform will produce an N=1 (square wave reference) doppler range law from the quadrature channel and an N=0 doppler range law from the in-phase channel. The product of these two laws, when filtered, then provides the directional doppler response given on Figure 15. This is not a surprising result when it is recalled that any two adjacent harmonic range responses produce doppler phasings which are in quadrature. The effect of the square wave reference is to add a small reverse directional response at $\tau = 3\tau_0$.

5.6 Example: Product Directional Doppler Using Triangular Wave Reference Voltage

Using triangular modulation and triangular estimates to $\cos(\theta)$ and $\sin(\theta)$, for a $B\tau_0 = 1/2$ system, results in reference voltage time waveforms which are a triangle at double the modulation frequency and a triangle at the modulation frequency. The double frequency triangle can be obtained by full-wave rectification of the single frequency triangle. The range law for this system is shown on Figure 16. Since the reference triangles are good estimates of sinusoidal references, the resulting range law is not appreciably different than that given by the product of an N=1 law with an N=2 law.

5.7 Example: Product Directional Doppler $B\tau_0 = 1/2$ System With Range Sidelobe Suppression

By adding a square wave to the triangular modulation waveform in the ratio of 1:3, the transmitted spectrum becomes a three step approximation to a triangular spectrum. The resulting shape of $R_c(\tau-\tau_0)$, as used in (65), will have low sidelobes. This produces a range law from the product processor as shown on Figure 17. The circuitry used for developing the quadrature reference voltage waveforms again uses the modulation itself as the quadrature reference and the output of a full-wave rectifier as the in-phase reference voltage waveform. These reference waveforms are again triangular estimates of $\cos(\theta)$ and $\sin(\theta)$. This demonstrates that any modulation waveform can be separated into quadrature components by this method and these can be used as quadrature reference voltage waveforms in $B\tau_0 = 1/2$ systems.

6. CONCLUSIONS

This study has demonstrated a new IF processing technique for obtaining directional doppler responses. This technique uses any practical FM waveform. It has been shown, for the first time, that it is not necessary to use an FM waveform which produces a single sideband IF response (such as sawtooth) in order to achieve directional doppler ranging. This is accomplished by developing two quadrature IF reference waveforms which are used to separate the IF signal beat waveform into quadrature components. The analysis provides range-law formulas for use in systems employing either addition or multiplication of the two quadrature channel doppler signals to achieve the directional doppler separation. These results were verified experimentally by recording the directional doppler range responses obtained from many different processing systems.

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APPENDIX

Approximation for $\theta_d(t,\tau)$ for small τ

Assuming that $\theta_d(t,\tau)$ is analytic, it can be expanded into a Taylor's series:

$$\theta_d(t,\tau) = \theta_d(t,\tau)\Big|_{\tau=0} + \frac{d\theta_d(t,\tau)}{d\tau}\Big|_{\tau=0}\tau + \frac{d^2\theta(t,\tau)}{d\tau^2}\frac{\tau^2}{2} + \cdots \qquad \text{(A-1)}$$

Using (6) and Leibnitz's rule for differentiating an integral, we obtain

$$\theta_d(t,\tau)\Big|_{\tau=0} = D\int_{t-\tau}^{t} m(\sigma)d\sigma\Big|_{\tau=0} = 0 \qquad \text{(A-2)}$$

$$\frac{d\theta_d(t,\tau)}{d\tau}\Big|_{\tau=0} = Dm(t-\tau)\Big|_{\tau=0} = Dm(t) \qquad \text{(A-3)}$$

$$\frac{d^2\theta_d(t,\tau)}{d\tau^2} = Dm'(t-\tau)(-1)\Big|_{\tau=0} = -Dm'(t) \qquad \text{(A-4)}$$

and the series is

$$\theta_d(t,\tau) = Dm(t)\tau - \frac{1}{2}Dm'(t)\tau^2 + \cdots \quad . \qquad \text{(A-5)}$$

From (A-5) it is seen that $Dm(t)\tau$ is a good estimate for $\theta_d(t,\tau)$ when $\theta_d(t,\tau)$ is analytic and τ is sufficiently small. Consequently, let $\theta_d(t,\tau)$ be represented by

$$\theta_d(t,\tau) = Dm(t)\tau + \theta_e(t,\tau) \qquad \text{(A-6)}$$

where $\theta_e(t,\tau)$ is defined as the error in the difference phase when $Dm(t)\tau$ is used as an estimator for $\theta_d(t,\tau)$. Using (A-6), (22) and (23)

$$C_d(\tau,\tau_0) = \overline{e^{jDm(t)(\tau-\tau_0)}e^{j[\theta_e(t,\tau) - \theta_e(t,\tau_0)]}} \qquad \text{(A-7)}$$

$$C_s(\tau,\tau_0) = \overline{e^{jDm(t)(\tau+\tau_0)}e^{j[\theta_e(t,\tau) + \theta_e(t,\tau_0)]}} \quad . \qquad \text{(A-8)}$$

These equations show that $\theta_d(t,\tau)$ can be approximated by $Dm(t)\tau$ if the error phase $\theta_e(t,\tau)$ is sufficiently small for the range of t and τ that is of interest. The error factors, $e^{j[\cdot]}$, always have a magnitude of 1. However, if $\theta_e(t,\tau)$ is not small, the filtered functions Cd and C_s may be seriously affected by $\theta_e(t,\tau)$.

For $Dm(t)\tau$ to be a good estimate of $\theta_d(t,\tau)$, a sufficient requirement is that the real part of the error factors, $Re[e^{j[\cdot]}]$, be approximately unity and the imaginary part be almost zero. For example, if $\theta_e(t,\tau) < \pi/8$, for the worst case, the range response would be correct to within 3dB for the values of τ and t that are of interest. If the second term of (A-5) is the significant part of the error, the range response computed by using $\theta_d(t,\tau) = Dm(t)\tau$ would be correct to within 3dB when τ is restricted to small values given by

$$|\tau| < \sqrt{\frac{\pi}{4Dm'(t)}} \quad . \qquad \text{(A-9)}$$

For triangle modulation of frequency fm (Hz) and peak-to-peak frequency deviation of B(Hz), this reduces to

$$|\tau| < 1/(4\sqrt{Bf_m}) \quad . \qquad \text{(A-10)}$$

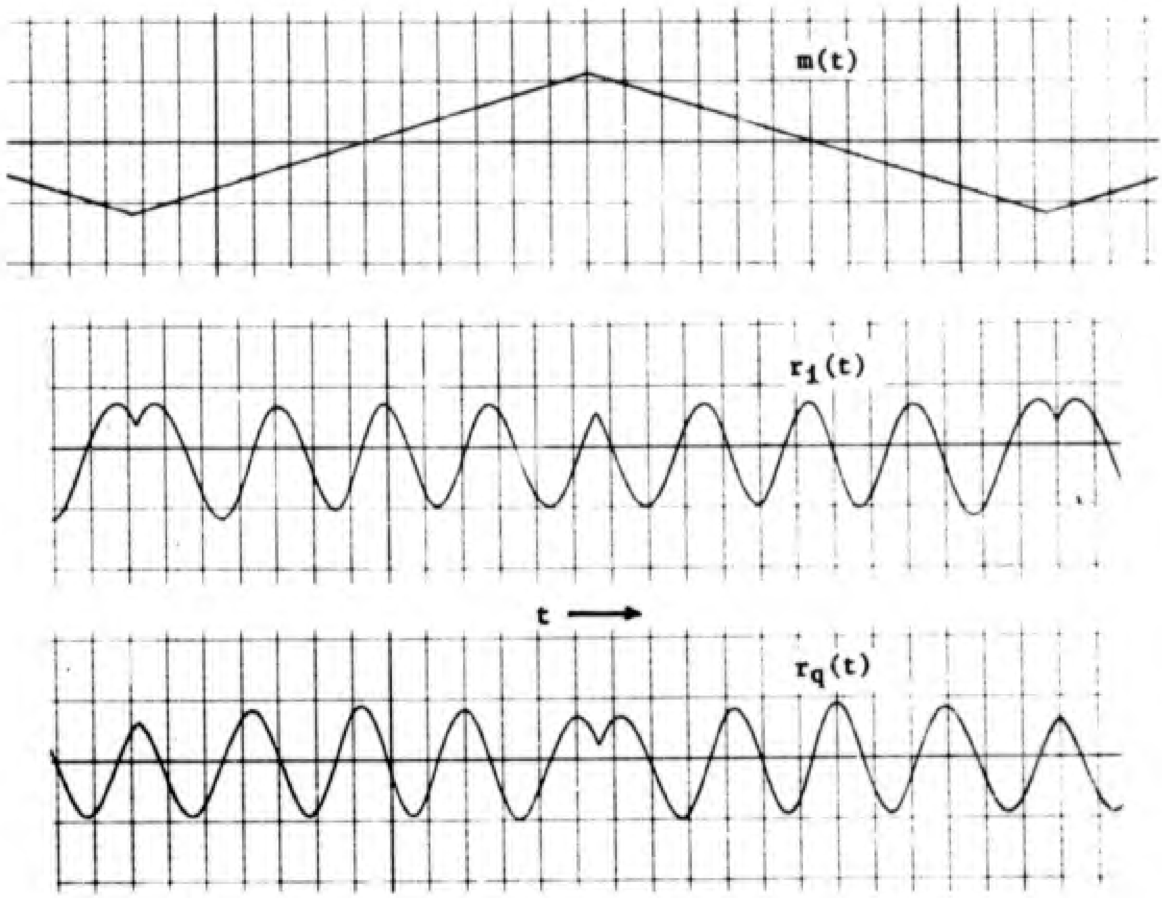


FIGURE 3: RF DELAY-LINE QUADRATURE REFERENCES FOR TRIANGULAR MODULATION

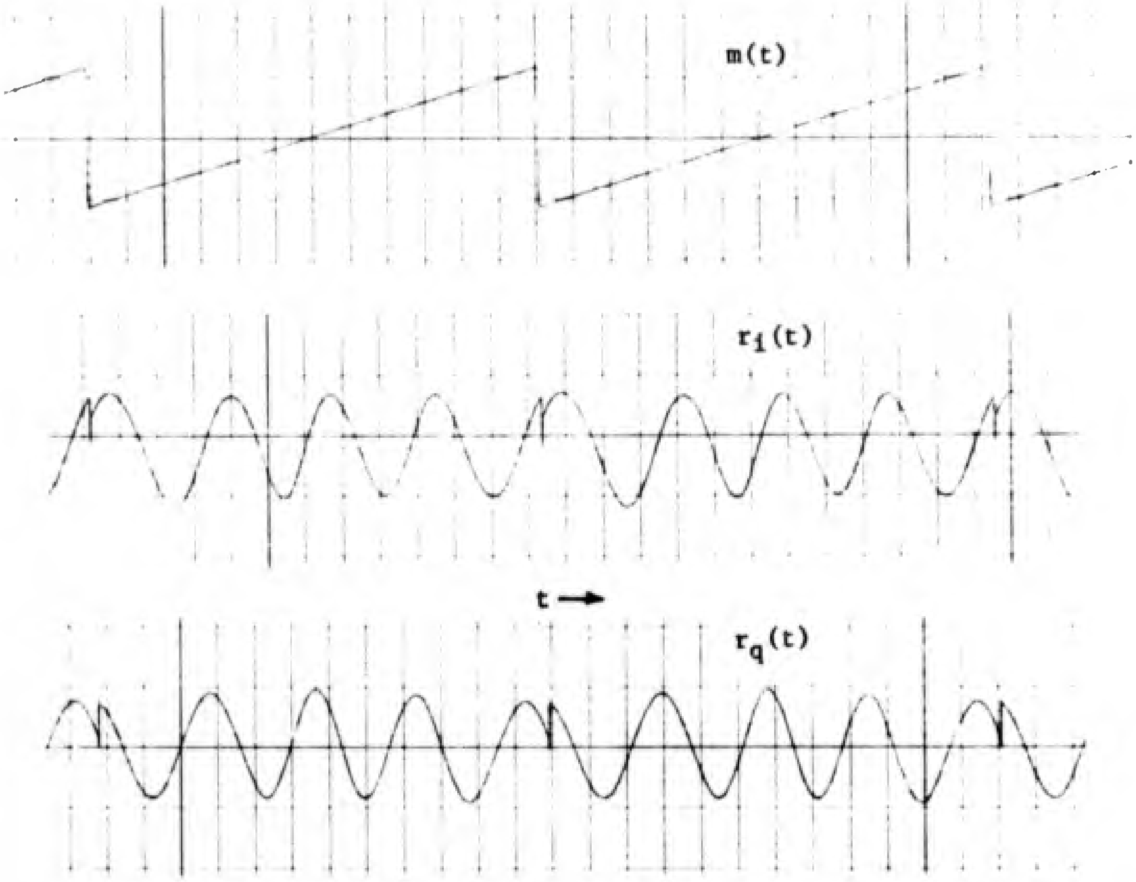


FIGURE 4: RF DELAY-LINE QUADRATURE REFERENCES FOR SAWTOOTH MODULATION

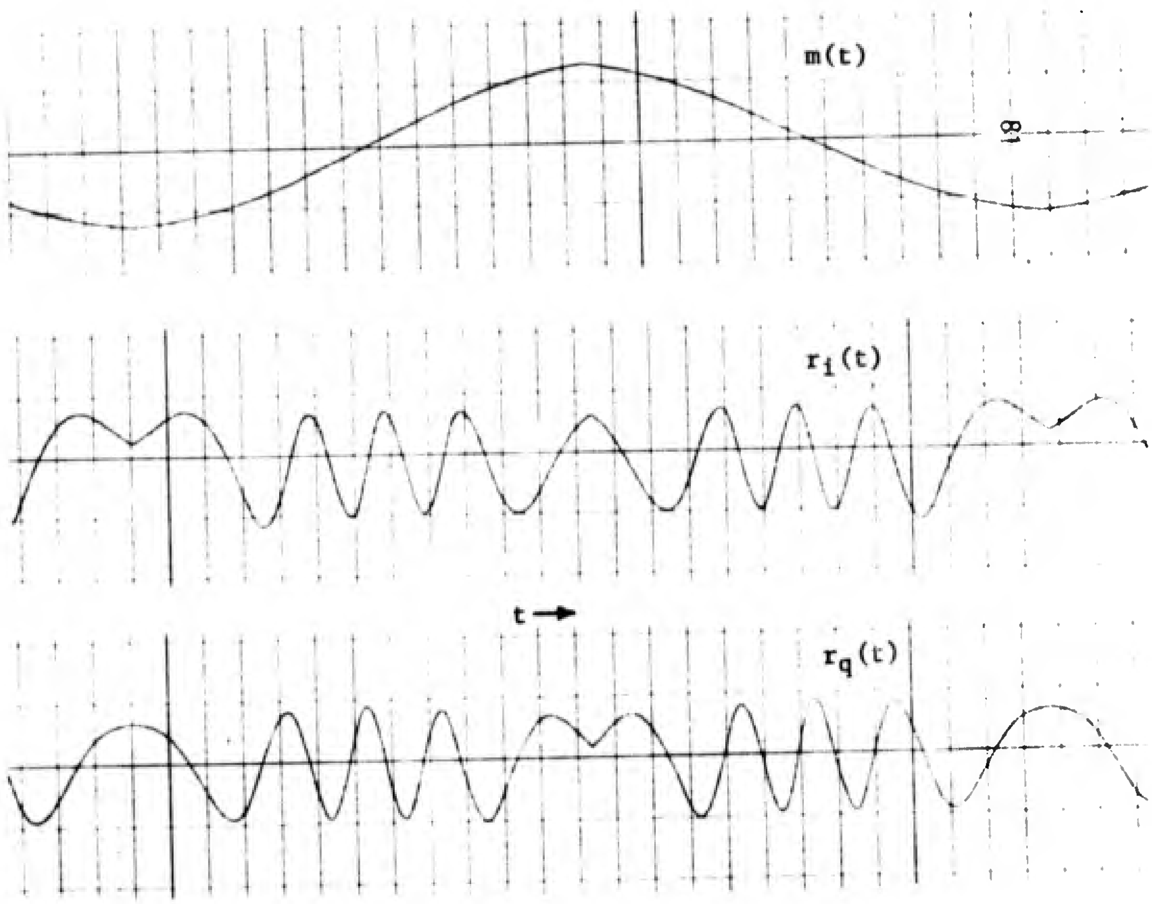


FIGURE 5: RF DELAY-LINE QUADRATURE REFERENCES FOR SINUSOIDAL MODULATION

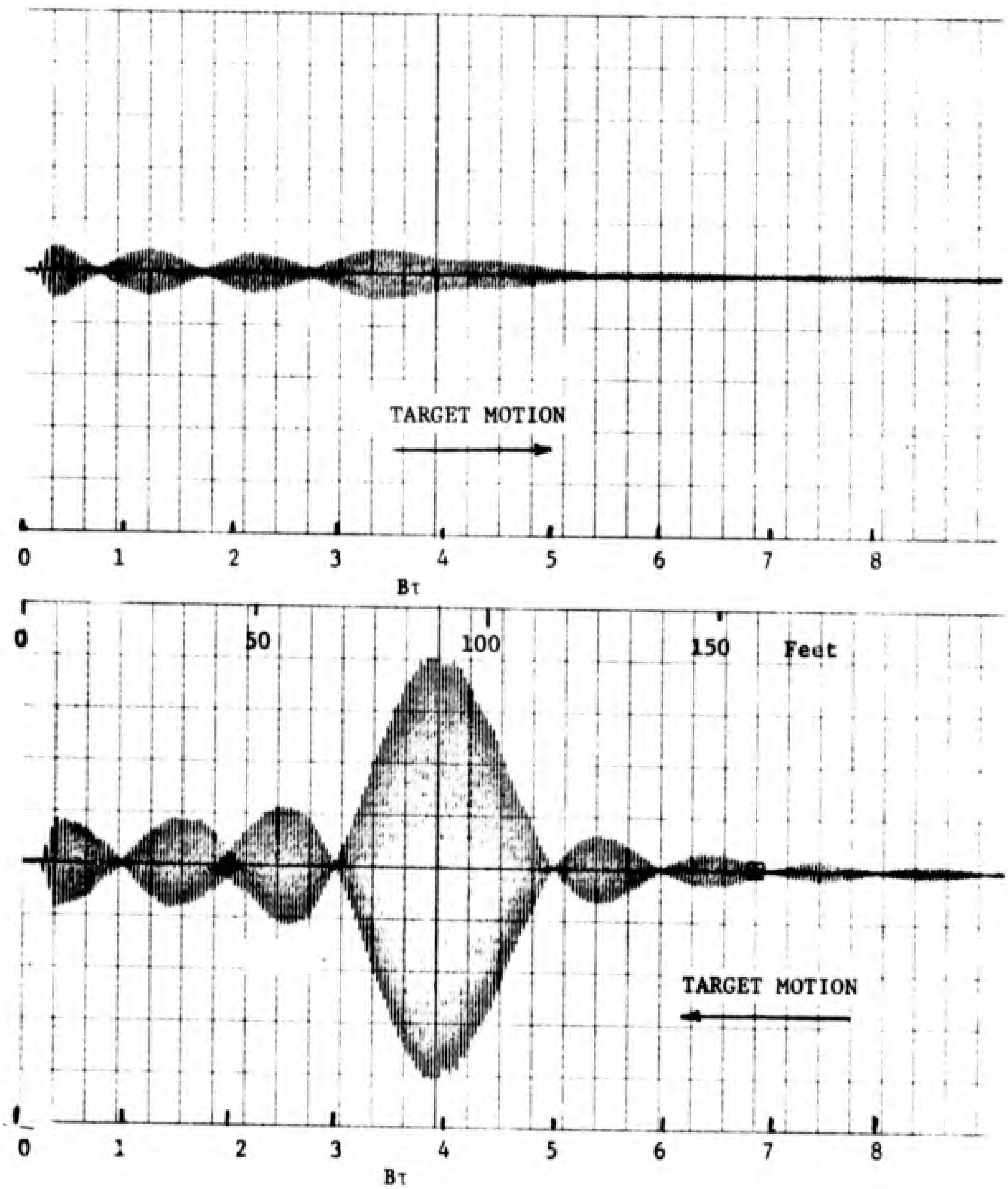


FIGURE 6: DIRECTIONAL DOPPLER RANGE RESPONSE USING DELAY LINE QUADRATURE REFERENCES AND TRIANGULAR MODULATION WITH SUM PROCESSOR

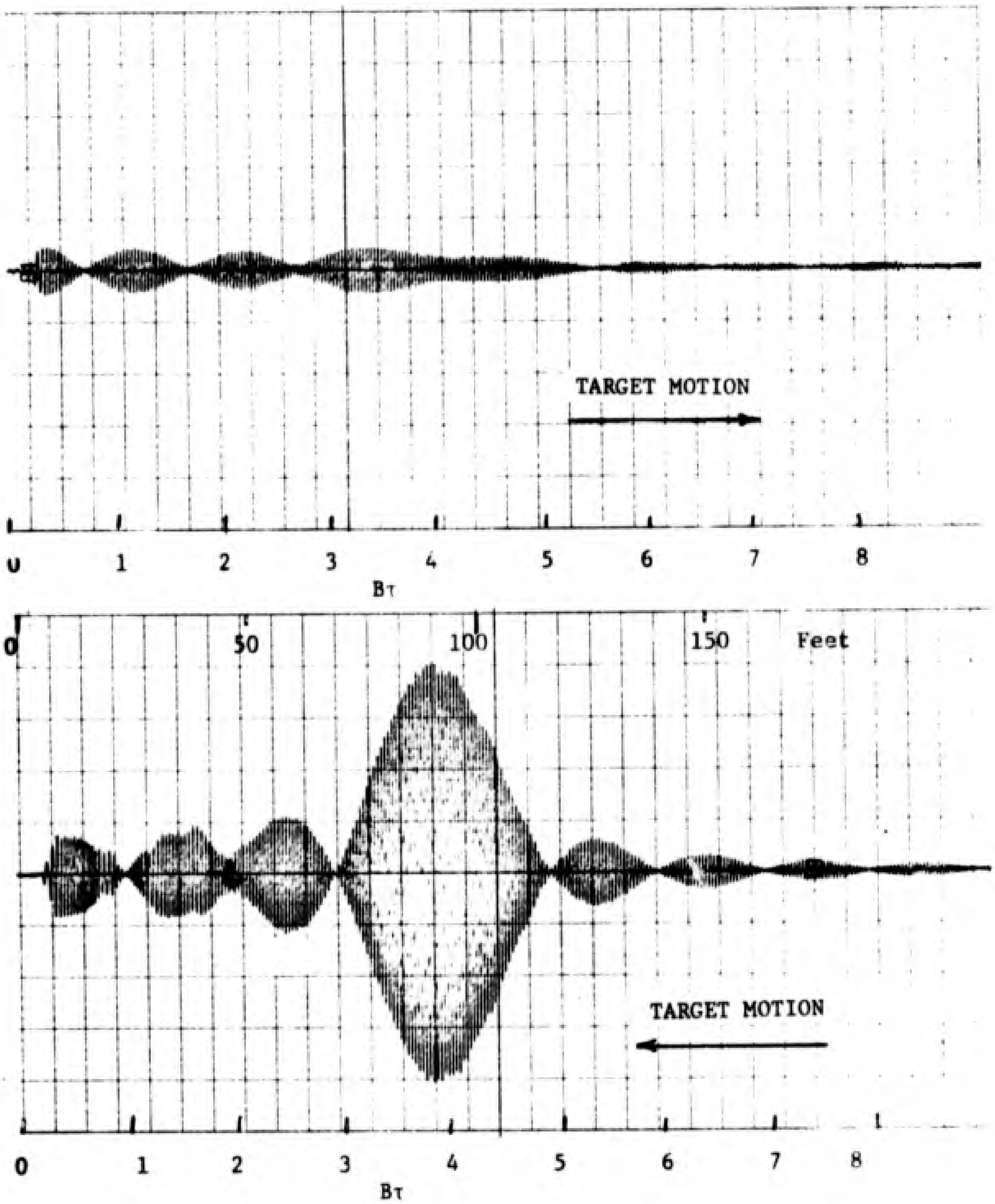


FIGURE 7: DIRECTIONAL DOPPLER RANGE RESPONSE USING DELAY LINE QUADRATURE REFERENCES AND SAWTOOTH MODULATION WITH SUM PROCESSOR

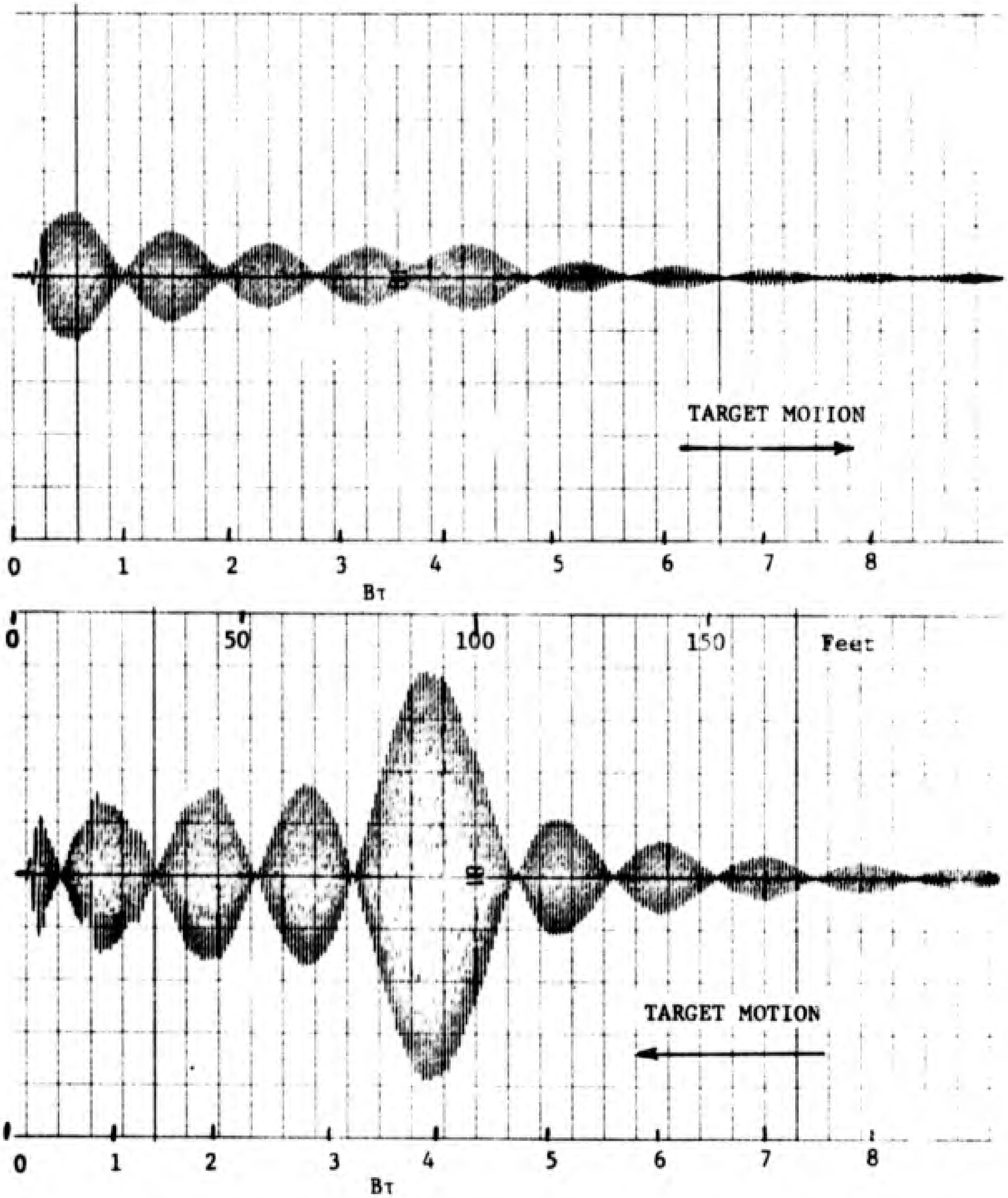


FIGURE 8: DIRECTIONAL DOPPLER RANGE RESPONSE USING DELAY LINE QUADRATURE REFERENCES AND SINUSOIDAL MODULATION WITH SUM PROCESSOR

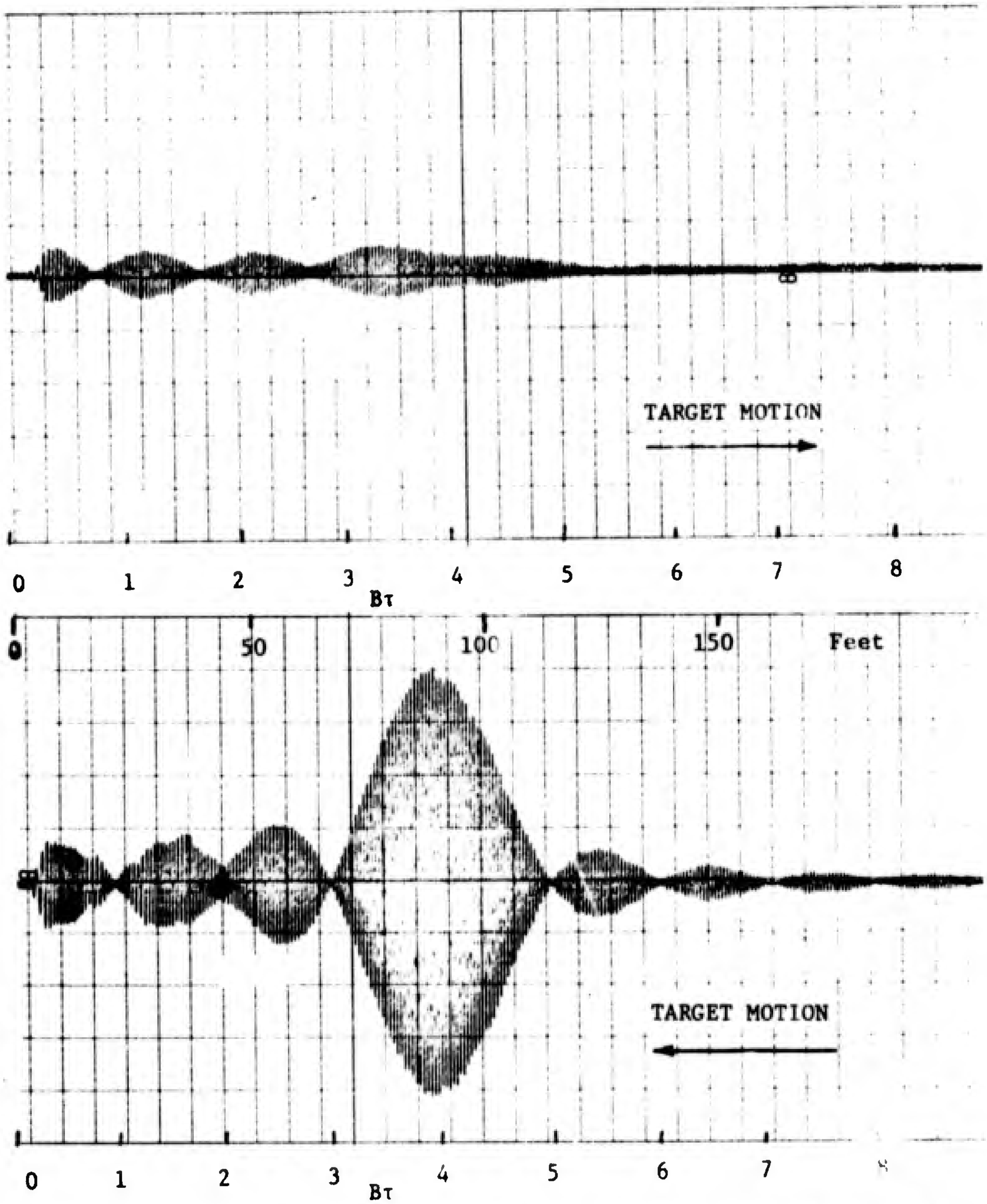


FIGURE 9: DIRECTIONAL DOPPLER RANGE RESPONSE USING DELAY LINE QUADRATURE REFERENCES AND FREQUENCY MODULATED TRIANGULAR MODULATION WITH SUM PROCESSOR

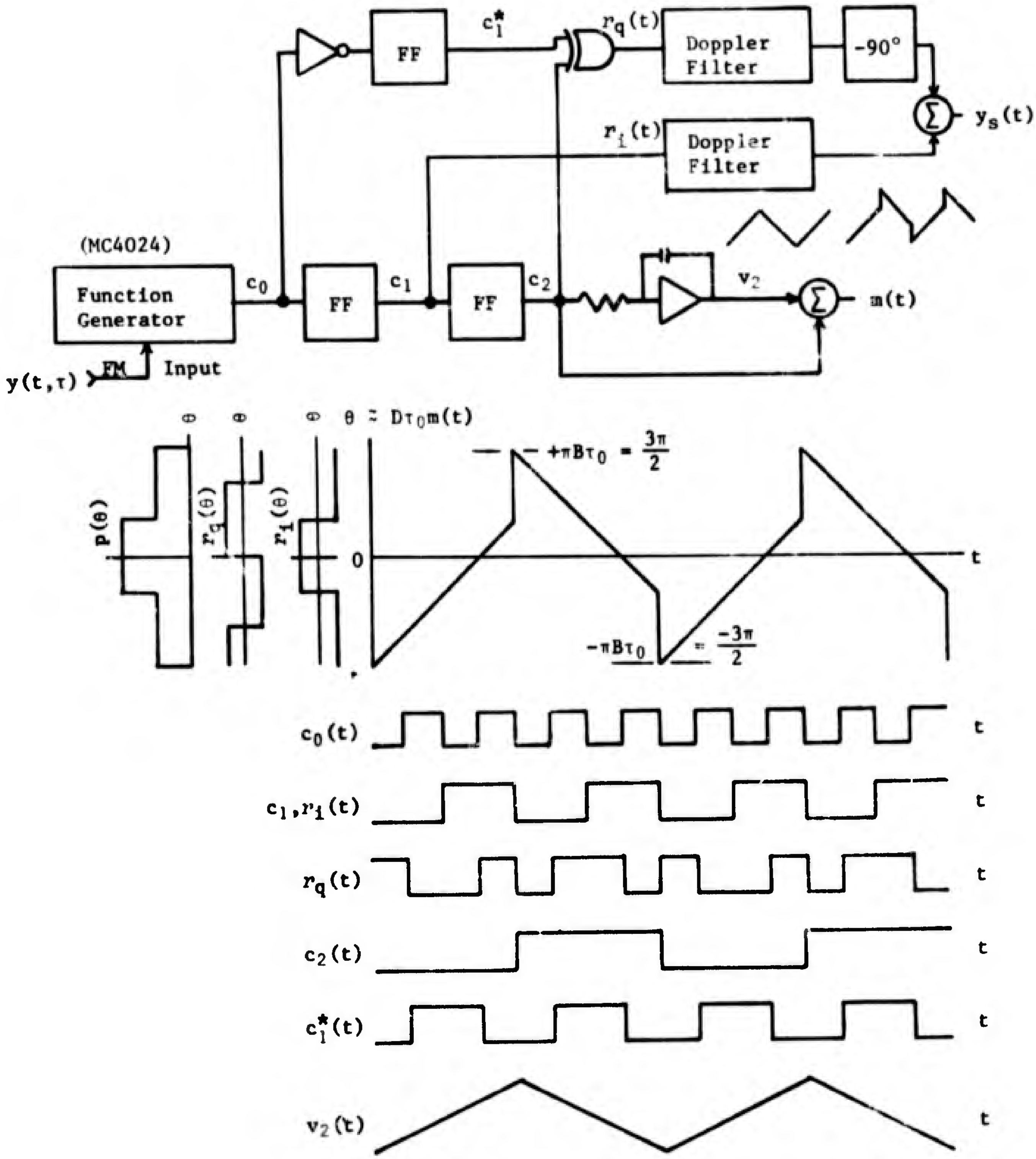


FIGURE 10: HYBRID $B\tau_0$ = 3/2 SYSTEM USING QUADRATURE SQUARE WAVE REFERENCES AND VCO/MIXER IF DETECTION TECHNIQUE WITH SUM PROCESSOR

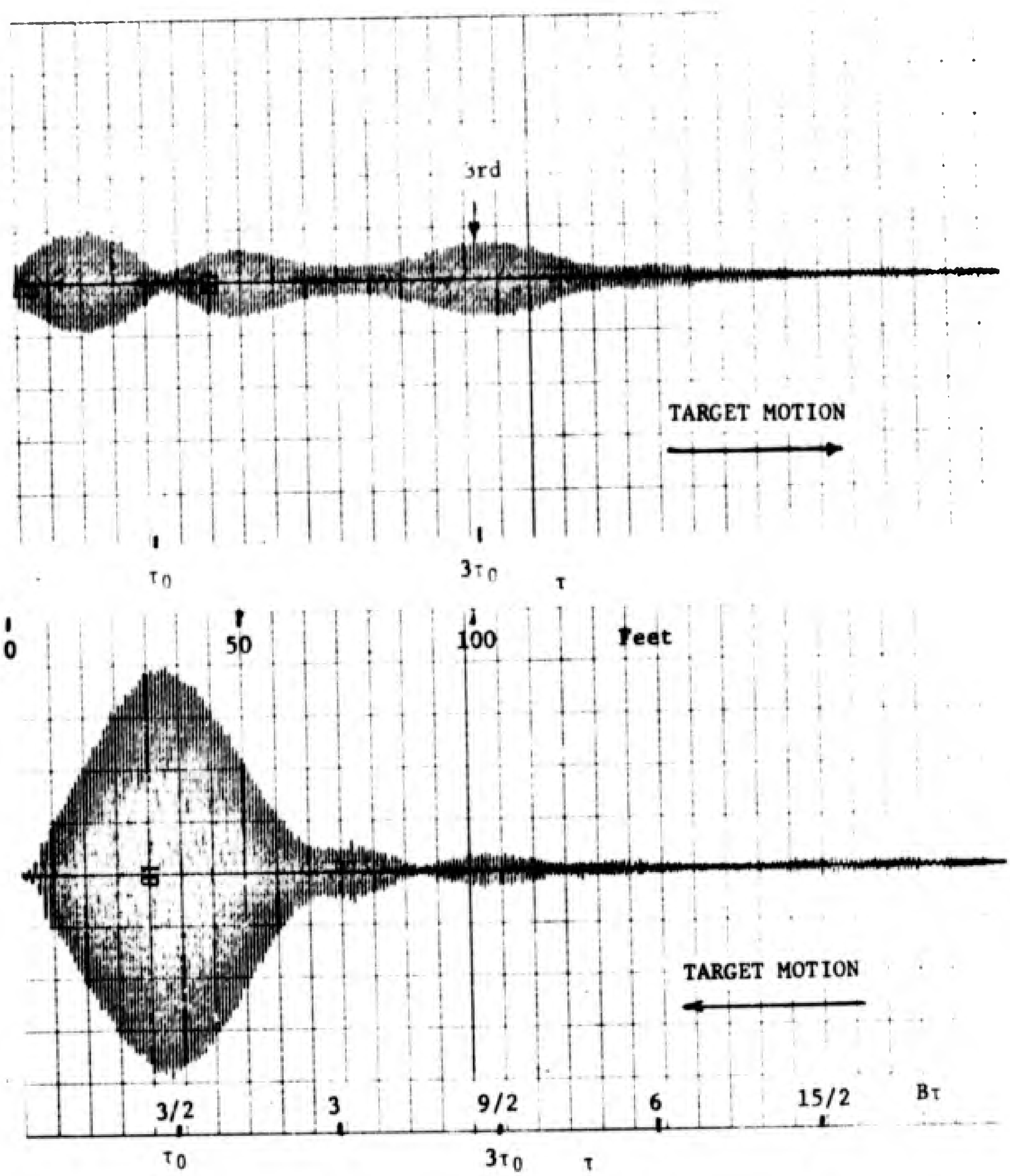


FIGURE 11: DIRECTIONAL DOPPLER RANGE RESPONSE FOR HYBRID $B\tau_0 = 3/2$ SYSTEM USING VCO/MIXER AND SUM PROCESSOR

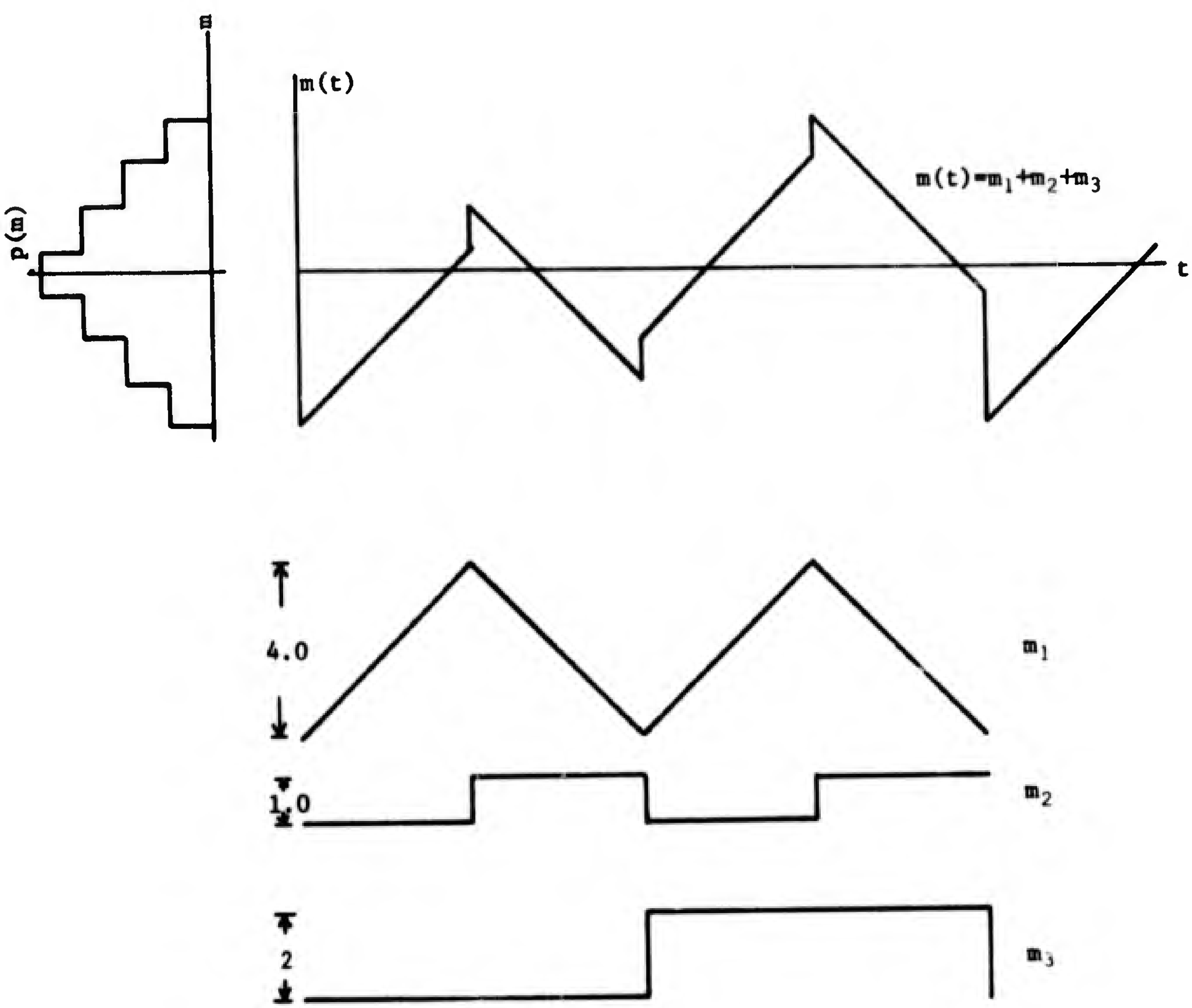


FIGURE 12: HYBRID MODULATION WAVEFORMS FOR APPROXIMATELY TRIANGULAR POWER SPECTRUM

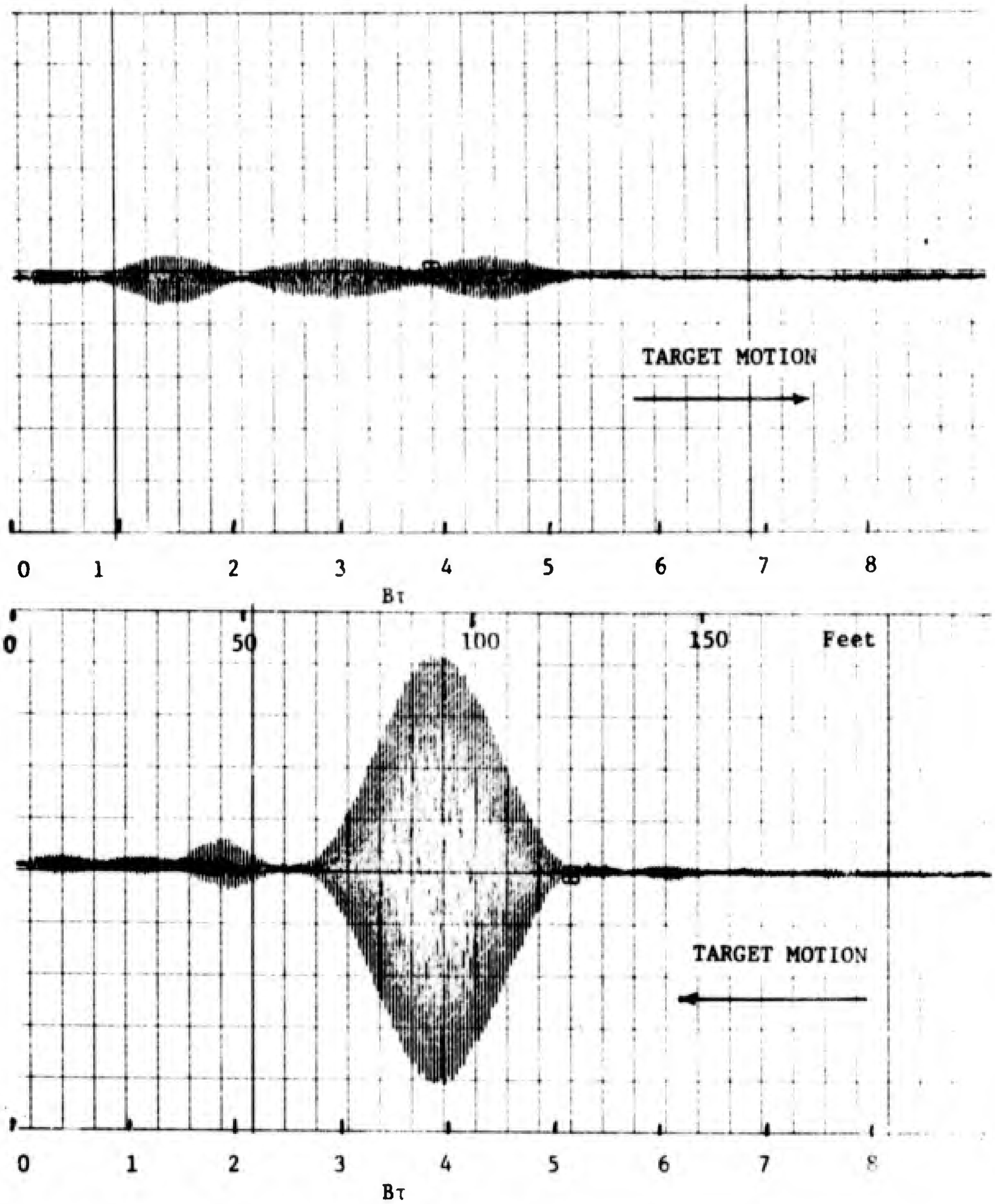


FIGURE 13: DIRECTIONAL DOPPLER RANGE RESPONSE FOR HYBRID MODULATION USING DELAY-LINE QUADRATURE REFERENCES AND SUM PROCESSOR ($B\tau_0 \approx 4$)

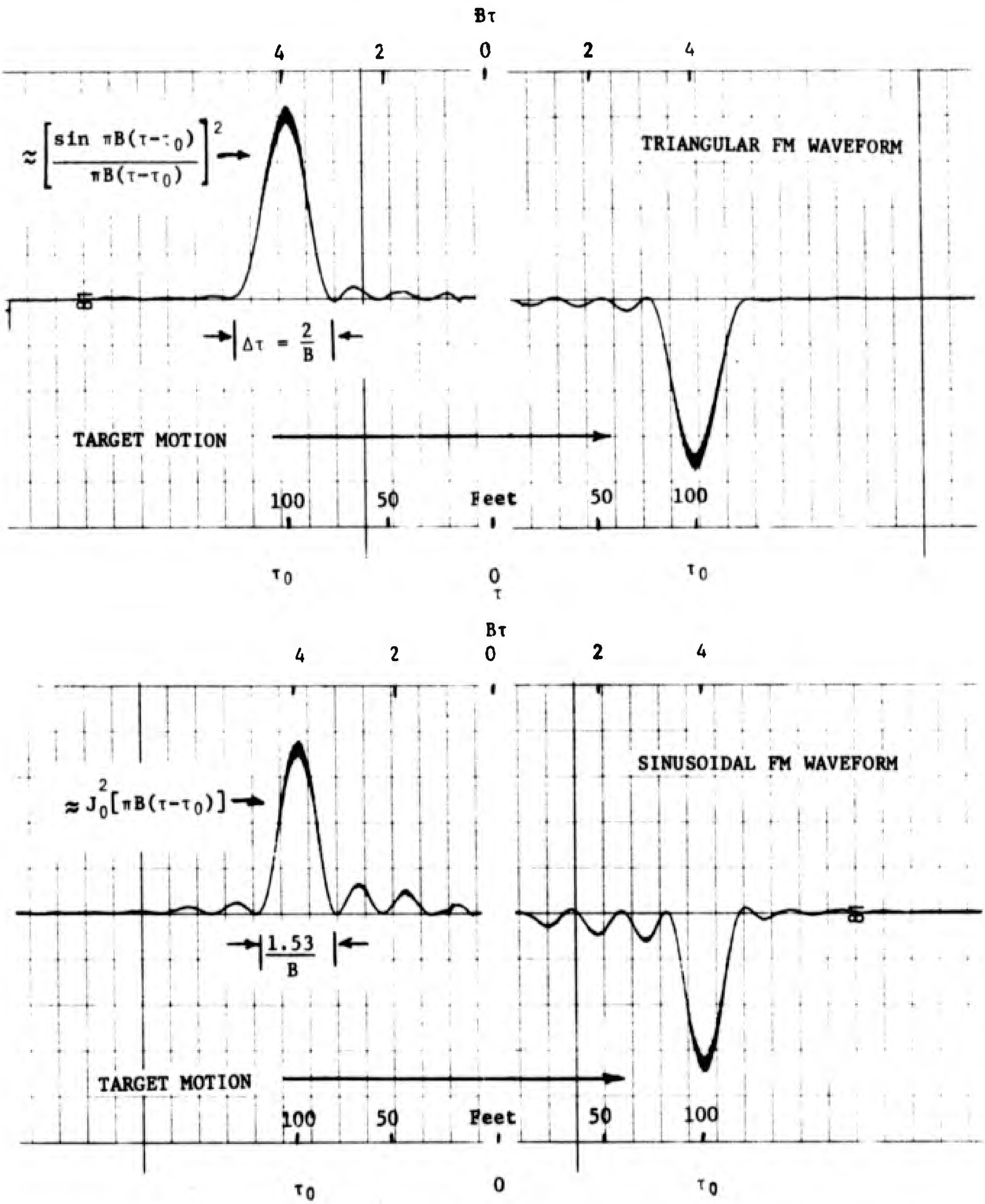
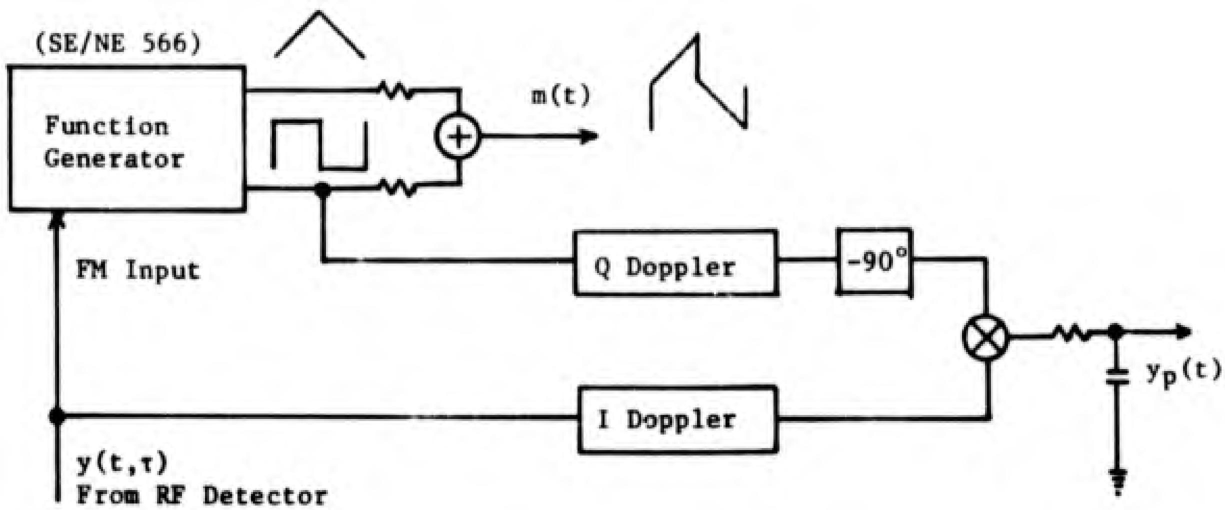


FIGURE 14: DIRECTIONAL DOPPLER RESPONSES USING DELAY LINE QUADRATURE REFERENCES AND PRODUCT CIRCUIT



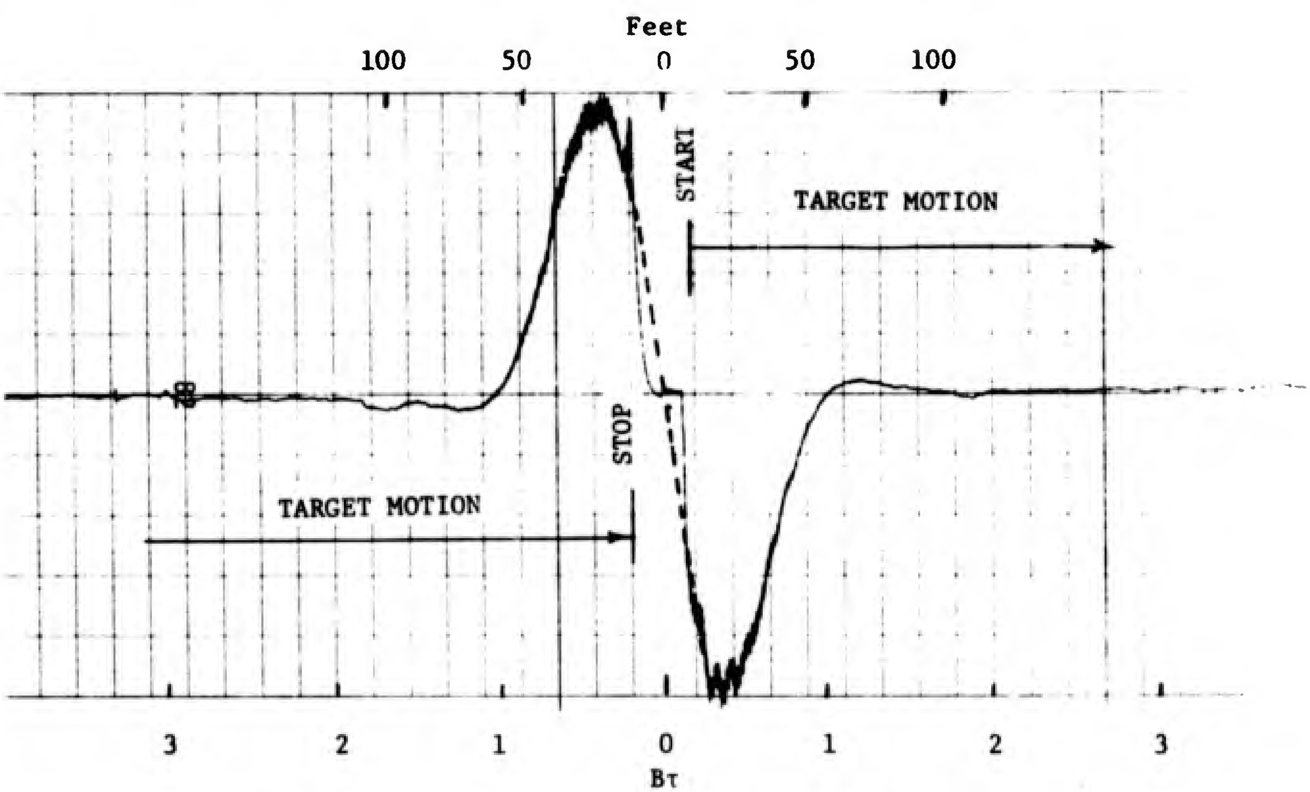
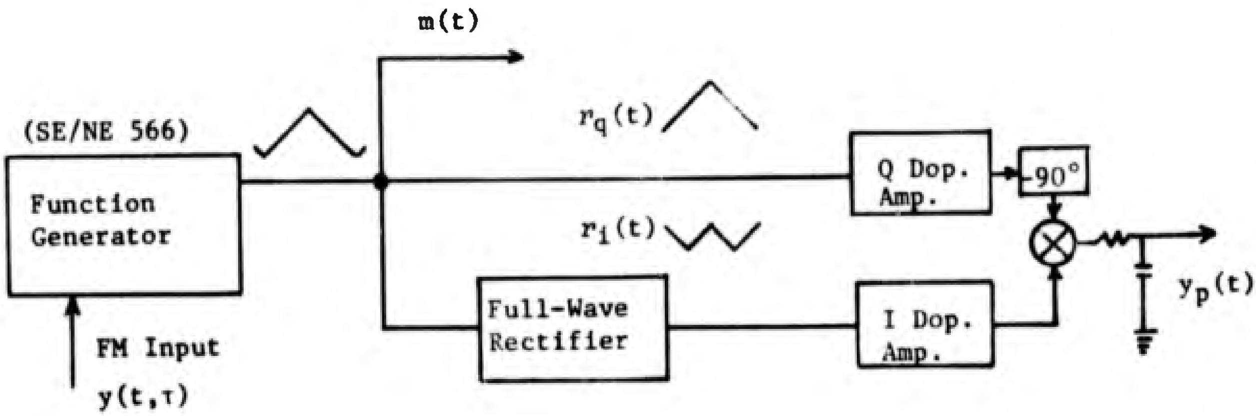


FIGURE 15: DIRECTIONAL DOPPLER RANGE RESPONSE FOR $B\tau_0 = 1/2$ SYSTEM USING SQUARE WAVE REFERENCE VOLTAGES, VCO/MIXER AND PRODUCT CIRCUIT



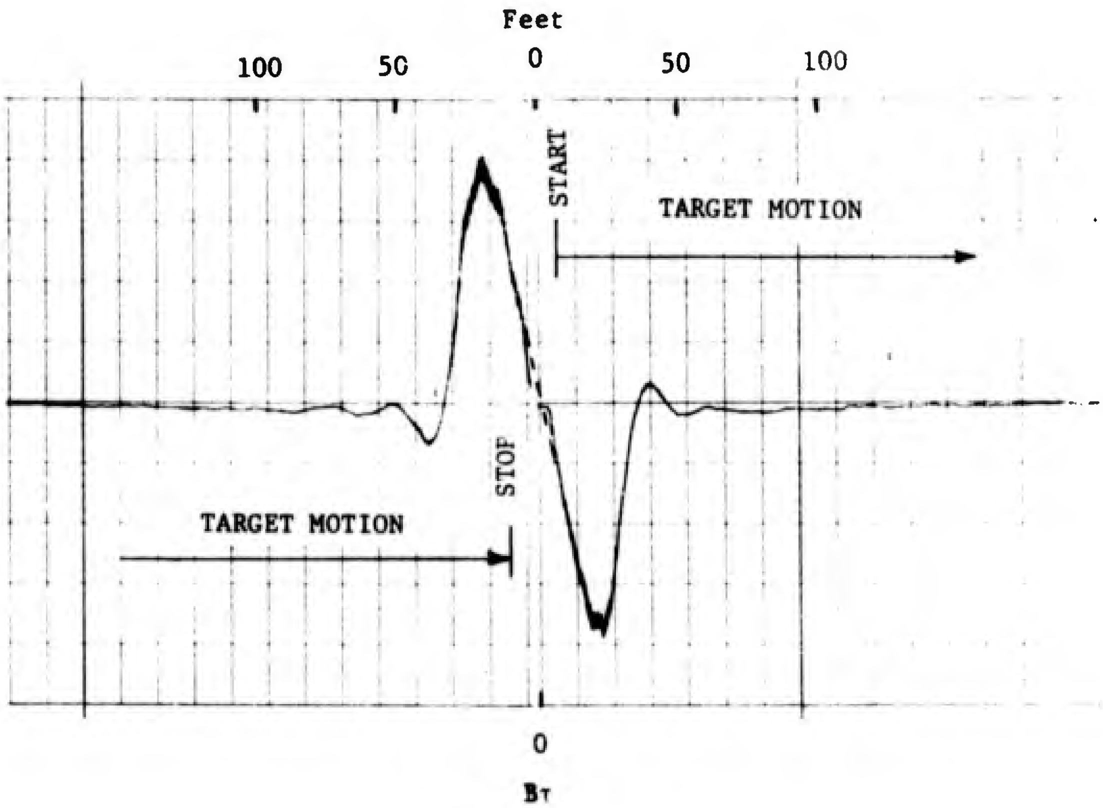


FIGURE 16: DIRECTIONAL DOPPLER RANGE RESPONSE FOR $B\tau_0 = 1/2$ SYSTEM USING TRIANGULAR REFERENCE VOLTAGES, VCO/MIXER AND PRODUCT CIRCUIT

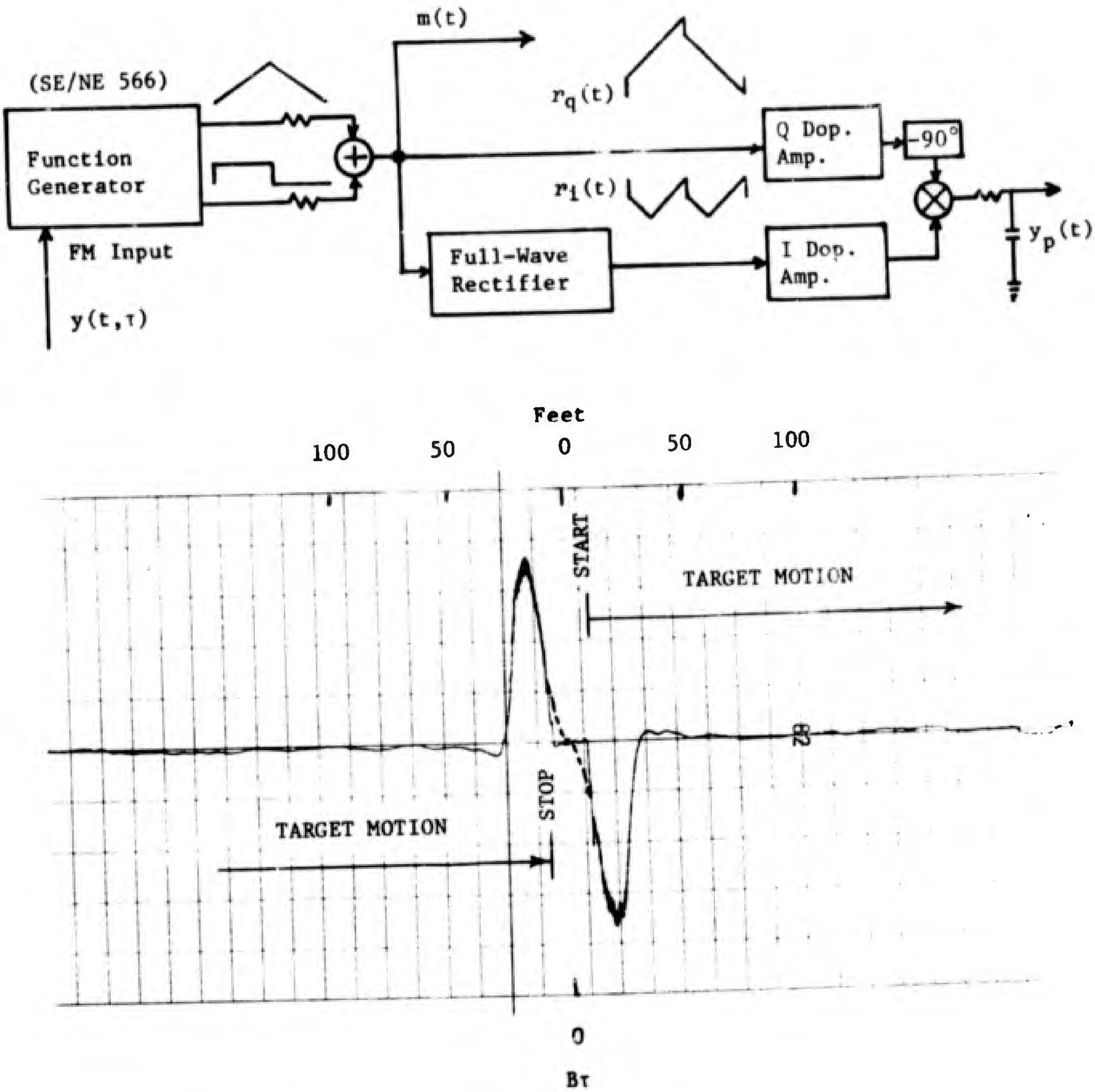


FIGURE 17: DIRECTIONAL DOPPLER RANGE RESPONSE FOR $B\tau_0 = 1/2$ SYSTEM USING TRIANGLE PLUS SQUARE WAVE MODULATION, VCO/MIXER AND PRODUCT CIRCUIT

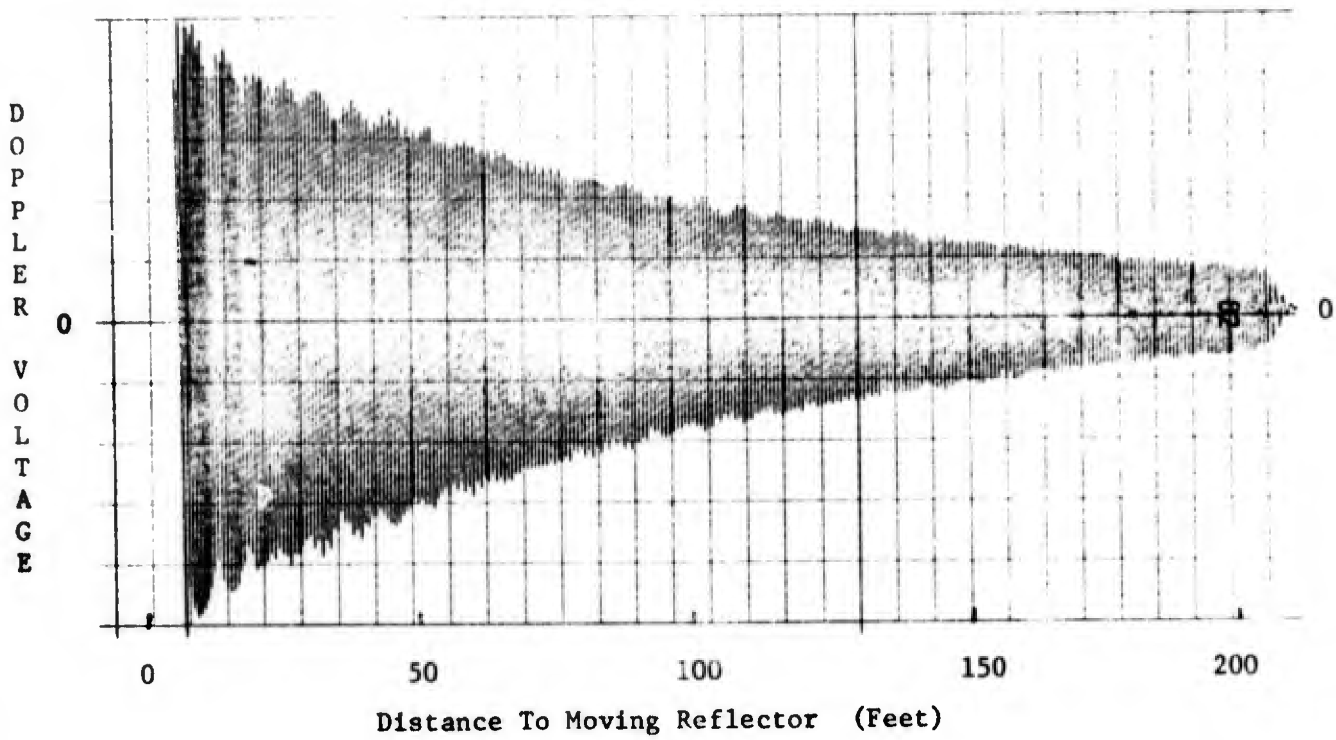


FIGURE 18: UNMODULATED RANGE RESPONSE SHOWING TRANSMISSION LINE LOSS CHARACTERISTIC