Best
Available
Copy

AD/A-001 122

PARAMETRIC SCALING LAWS. PART II. A COMPUTER MODEL FOR DERIVING THE PERFORMANCE CHARACTERISTICS OF SATURATION-LIMITED PARAMETRIC SONARS

F. H. Fenlon, et al

Westinghouse Research Laboratories

Prepared for:

Office of Naval Research
Advanced Research Projects Agency

26 August 1974

DISTRIBUTED BY:

NTIS
National Technical Information Service
U. S. DEPARTMENT OF COMMERCE

AD A001122

The views and conclusions contained in this document are those of the authors and should not be interpreted as necessarily representing the official policies, either expressed or implied, of the Advanced Research Projects Agency or the U. S. Government.

PARAMETRIC SCALING LAWS

F. H. Fenlon and J. W. Kesner

Final Report, Part II - A003
"A Computer Model for Deriving the Performance Characteristics of Saturation-Limited Parametric Sonars"

ARPA Order Number:	2655/11-30-73
Program Code Number:	4E20
Effective Date of Contract:	February 1, 1974
Contract Expiration Date:	July 31, 1974
Amount of Contract:	$29,591
Contract Number:	N00014-74-C-0214
Principle Investigator:	F. H. Fenlon (412) 256-3676
Scientific Officer:	R. Obrochta

Sponsored by Advanced Research Projects Agency
ARPA Order No. 2655/11-30-73

This research was supported by the Advanced Research Projects Agency of the Department of Defense and was monitored by ONR under Contract No. N00014-74-C-0214

August 26, 1974

NATIONAL TECHNICAL INFORMATION SERVICE
U.S. Department of Commerce
Springfield VA 22151

DISTRIBUTION STATEMENT A
Approved for public release;
Distribution Unlimited

D D C
RECEIVED
OCT 8 1974
B

WESTINGHOUSE RESEARCH LABORATORIES PITTSBURGH, PA. 15235

Unclassified

SECURITY CLASSIFICATION OF THIS PAGE (When Data Entered)

REPORT DOCUMENTATION PAGE		READ INSTRUCTIONS BEFORE COMPLETING FORM
1. REPORT NUMBER	2. GOVT ACCESSION NO.	3. RECIPIENT'S CATALOG NUMBER
4. TITLE (and Subtitle) PARAMETRIC SCALING LAWS - Part II - A Computer Model for Deriving the Performance Characteristics of Saturation-Limited Parametric Sonars		5. TYPE OF REPORT & PERIOD COVERED Final Report (Part II) 1 Feb. 1974 to 31 July 1974
		6. PERFORMING ORG. REPORT NUMBER 72-9M7-NONLN-R3
7. AUTHOR(s) F. H. Fenlon and J. W. Kesner		8. CONTRACT OR GRANT NUMBER(s) N00014-74-C-0214
9. PERFORMING ORGANIZATION NAME AND ADDRESS Westinghouse Electric Corporation Research & Development Center Beulah Road, Pittsburgh, PA 15235		10. PROGRAM ELEMENT, PROJECT, TASK AREA & WORK UNIT NUMBERS ARPA Order No. 2655/11-30-73 Program Code No. 4E20
11. CONTROLLING OFFICE NAME AND ADDRESS Procuring Contracting Officer Office of Naval Research Department of the Navy Arlington, VA 22217		12. REPORT DATE August 26, 1974
		13. NUMBER OF PAGES 124
14. MONITORING AGENCY NAME & ADDRESS(if different from Controlling Office) Commander, Defense Contract Administration Services District 1610-S Federal Building, 1000 Liberty Ave. Pittsburgh, PA 15222		15. SECURITY CLASS. (of this report) Unclassified
		15a. DECLASSIFICATION DOWNGRADING SCHEDULE

16. DISTRIBUTION STATEMENT (of this Report)

[redacted]

DISTRIBUTION STATEMENT A
Approved for public release;
Distribution Unlimited

17. DISTRIBUTION STATEMENT (of the abstract entered in Block 20, if different from Report)

NA

18. SUPPLEMENTARY NOTES

19. KEY WORDS (Continue on reverse side if necessary and identify by block number)

Nonlinear, saturation, sonar

20. ABSTRACT (Continue on reverse side if necessary and identify by block number)

This report outlines the procedures used in a computer program for calculating the performance characteristics of saturation-limited parametric transmitters and receivers. The program enables the amplitude and frequency response of such sonars to be evaluated including the amplitude dependence of their far field beam patterns and directivity indices.

DD FORM 1 JAN 73 1473 EDITION OF 1 NOV 65 IS OBSOLETE

Unclassified

SECURITY CLASSIFICATION OF THIS PAGE (When Data Entered)

PARAMETRIC SCALING LAWS

Part II: A Computer Model for Deriving the Performance Characteristics of Saturation-Limited Parametric Sonars

by

F. H. Fenlon and J. W. Kesner

SUMMARY

The purpose and significance of this ARPA sponsored report is to provide a detailed description and listing of the computer programs developed to compute the amplitude, frequency, and angular response characteristics of parametric sonars, from an analytical model developed in Part I. Technical results include examples of the performance characteristics and beam patterns generated by the program. Implications for further research would be to work backwards from the asymptotic far-field solutions derived in this report to compute the range dependence of the entire spectrum throughout the nonlinear interaction zone via the paraxial wave equation.

TABLE OF CONTENTS

LIST OF SYMBOLS i

THEORY 1

PROGRAM INPUT 7

PROGRAM OUTPUT 10

REFERENCES 11

APPENDIX A 13

FIGURE 1 (Pages 37-85) 37

FIGURE 2 (Pages 87-99) 87

FIGURE 3 (Pages 101-104) 101

PROGRAM LISTING (Pages 105-118) 105

LIST OF SYMBOLS

ρ_o	density of the medium
c_o	small signal speed of sound
$\beta = 1 + (B/2A)$	nonlinear parameter for liquids
$= \frac{1}{2}(1 + C_p/C_v)$	in gases
A, B	first and second-order coefficients in the equation of state
C_p, C_v	specific heats at constant pressure and constant volume
$r_o, r_o' = r_o(f_o/f_-)$	Rayleigh distance, and end-fire truncation distance
$f_i, (i=1,2)$	primary wave frequencies
$f_o = \frac{1}{2}(f_1+f_2)$	mean primary wave frequency
$f_\pm = f_1 \pm f_2$	sum or difference frequency
$k_i = 2\pi f_i/c_o, (i=1,2)$	primary wave numbers
$k_o = 2\pi f_o/c_o$	mean primary wave number
$\bar{p}_{oi}, (i=1,2)$	rms primary wave amplitudes at the source
$\bar{\delta} = \alpha/f^2$	nondispersive thermo viscous attenuation parameter
α	attenuation coefficient
$\alpha_i, (i=1,2)$	primary wave attenuation coefficients
$\alpha_\pm$	sum or difference-frequency attenuation coefficient
$\alpha_{T_\pm} = (\alpha_1+\alpha_2-\alpha_\pm)$	combined attenuation coefficient
$\alpha_T \equiv \alpha_{T_-}$	
α_o	mean primary wave attenuation coefficient
$\Gamma_o = \sigma_o/\alpha_o r_o$	mean primary wave acoustic Reynolds number

$x_i = \sigma_i E_1(\alpha_T r_o), (i=1,2)$

$x_o = \sigma_o E_1(\alpha_T r_o)$

$\sigma_i = \sqrt{2}\beta\bar{p}_{oi} k_i r_o / \rho_o c_o^2$

$\sigma_o = \beta\bar{p}_o r_o k_o / \rho_o c_o^2$

$\Delta = E_1(\alpha_T r_o)\, e^{\alpha_T r_o}$

$\Delta' = E_1(\alpha_T r_o')\, e^{\alpha_T r_o'}$

$$n = 1 + \frac{\log_{10}(\Delta/\Delta')}{\log_{10}(f_o/f_-)}$$

$E_1(z) = \int_z^\infty \frac{e^{-x}}{x}\, dx$ exponential integral

$D_i(\theta), (i=1,2)$ far-field primary wave directivity functions

$D_\pm(\theta)$ far-field sum or difference-frequency directivity function for a "spreading-loss-limited" parametric array

$\delta_\pm(\theta)$ far-field sum or difference-frequency directivity function for an "absorption-limited" parametric array

$b_\pm(\theta)$ convolved beam pattern

$d_\pm(\theta)$ $D_\pm(\theta)$, $\delta_\pm(\theta)$, or $b_\pm(\theta)$

DI directivity index

SL_o combined rms primary wave source level in dB//1μPam

SL_∞ equivalent mean rms primary wave source level at 1 m in dB//1μPam

$SL_\pm$ equivalent sum or difference-frequency rms source level at 1 m in dB//1μPam

$SL_o^* = SL_o + 20 \log_{10} f_o$, ($f_o$ in kHz) scaled combined rms primary wave source level in dB//1μPam kHz

$SL_\infty^* = SL_\infty + 20 \log_{10} f_o$ scaled equivalent mean rms primary wave source level at 1 m in dB//1μPam kHz

$SL_\pm^* = SL_- + 20 \log_{10} f_o$ scaled equivalent sum or difference frequency rms source level in dB//1μPa kHz

$20 \log_{10} \eta_\pm = SL_\pm - SL_o$ parametric (sum or difference) conversion efficiency

THEORY

In Part I of this report an analytical solution for the far-field pressure of a parametric array was derived from Burgers' equation. Section 2 of Part I gave this solution in terms of scaled source levels. Since these scaled expressions form the basis of the computer program outlined in this part of the report, they are reexpressed here for the benefit of the reader as follows:

Let SL_o = the combined primary wave source level at 1m, in dB//1μPam

and $SL_\pm$ = the equivalent sum or difference-frequency source level referred to 1m, in dB//1μPam.

Then the scaled source levels SL_o^* and $SL_\pm^*$ are defined as,

$$\left.\begin{aligned} SL_o^* &= SL_o + 20\log_{10} f_o \quad (1a) \\ SL_\pm^* &= SL_\pm + 20\log_{10} f_o \quad (1b) \end{aligned}\right\} \quad f_o \text{ in kHz}$$

where $$f_o = \tfrac{1}{2}(f_1 + f_2), \quad (2)$$

f_1 and f_2 being the operating frequencies of the parametric sonar. f_o is thus the mean primary wave frequency.

Parametric Transmitter:

From Section 2 of Part I the conversion efficiency η_- of a parametric transmitter can be expressed in logarithmic form as,

$$\begin{aligned} 20\log_{10}\eta_- &= (SL_- - SL_o) \\ &\equiv (SL_-^* - SL_o^*), \text{ from Eqs. (1a) and (1b)} \\ &= 20\log_{10}F - 20n\log_{10}(f_o/f_-) \quad (3) \end{aligned}$$

where $$F = (2/\chi_o)\,\frac{I_1(\chi_1)I_1(\chi_2)}{I_o(\chi_1)I_o(\chi_2)}\ ; \quad \chi_o = \sqrt{2}\,\bar{\chi}_o \tag{4}$$

and $$20\log_{10}\chi_o = SL_o^* + 20\log_{10}\Delta + 20\log_{10}N \tag{5a}$$

$$20\log_{10}\chi_i = SL_i^* + 20\log_{10}\Delta + 20\log_{10}N,\ (i=1,2) \tag{5b}$$

with $$N = (2\sqrt{2}\pi\ \beta/\rho_o c_o^3)(10^3) \tag{6}$$

and $$\Delta = E_1(\alpha_T r_o)\exp(\alpha_T r_o);\quad \Delta' = E_1(\alpha_T r_o')\exp(\alpha_T r_o');$$

$$\alpha_T r_o' = \alpha_T r_o(f_o/f_-) \tag{7}$$

$$n = 1 + \frac{\log_{10}(\Delta/\Delta')}{\log_{10}(f_o/f_-)}. \tag{8}$$

The program computes, plots, and tabulates the conversion efficiency $20\log\eta_-$ as a function of the scaled combined primary wave source level SL_o^* for fixed values of $\alpha_T r_o$ and f_o/f_-. It also plots and tabulates the scaled difference-frequency source level SL_-^* as a function of SL_o^* for fixed values of $\alpha_T r_o$ and f_o/f_-. In addition the frequency response index n, as defined by Eq. 8 is tabulated. By reading in the relative amplitude $\bar{p}_{o1}/\bar{p}_{o2}$ of the rms primary wave amplitudes at the source the program will provide results for cases where the drive amplitudes are unequal.

Parametric Receiver:

For a limited class of problems such as that of a plane wave low frequency signal interacting with a plane wave "absorption limited" finite-amplitude source, or a spherical wave signal interacting with a spherical pump wave, the same expressions used to define the performance of a parametric transmitter will also define the performance of a parametric receiver if f_o/f_- is replaced by f_o/f_+ in Eq. 3 and if

$\alpha_T r_o$ is replaced by $-\alpha_T r_o$ in Eq. 7. The reason for this can be seen by writing $\alpha_T r_o$ as $\alpha_{T_\pm} r_o$ where,

$$\begin{aligned}\alpha_{T_\pm} r_o &= (\alpha_1 + \alpha_2 - \alpha_\pm) r_o \\ &= \bar{\delta}\{f_1^2 + f_2^2 - (f_1 \pm f_2)^2\} r_o \\ &= \mp 2\bar{\delta} f_1 f_2 r_o. \end{aligned} \tag{9}$$

Thus $\alpha_{T_-} r_o = 2\bar{\delta} f_1 f_2 r_o$, for the difference-frequency signal (10a)

and $\alpha_{T_+} r_o = -2\bar{\delta} f_1 f_2 r_o$

$= -\alpha_{T_-} r_o$, for the sum-frequency signal. (10b)

In the computer output $20 \log_{10} \eta_-$ becomes $20 \log_{10} \eta_+$ and SL_-^* becomes SL_+^*. The substitution of $z = -\alpha_T r_o$ in the exponential integral $E_1(z)$ transforms this to $E_1(z)$, as defined by Abramowitz and Segun.[1]

It should be noted in the case of a parametric receiver that the index n is still calculated by Eq. 8 since the sum frequency component must have the same frequency response as the difference-frequency component.

Evaluation of $E_1(z)\exp(z)$ for $z \geqslant 10.0$:

For $z \geqslant 10.0$, the function $E_1(z)\exp(z)$ which is used in calculating ℓ, ℓ', and ℓ_o as defined by Eqs. 7 and 22, respectively, is determined from an approximate expression given by Abramowitz and Segun[1] as,

$$E_1(z)\exp(z) = \frac{z^2 + A_1 z + A_2}{z^3 + B_1 z^2 + B_2 z}$$

where $A_1 = 4.03640$

$A_2 = 1.15198$

$B_1 = 5.03637$

$B_2 = 4.19160$

Far-Field Directivity Function:

For a "spreading-loss-limited" parametric array ($\alpha_T r_o' \leq 10^{-2}$ dB) the program gives the far-field directivity function $D_\pm(\theta)$ as,

$$D_\pm(\theta) = \left\{\frac{I_1[\chi_1 D_1(\theta)]\, I_1[\chi_2 D_2(\theta)]}{I_o[\chi_1 D_1(\theta)]\, I_o[\chi_2 D_2(\theta)]}\right\} \Big/ \left\{\frac{I_1(\chi_1) I_1(\chi_2)}{I_o(\chi_1) I_o(\chi_2)}\right\}, \tag{11}$$

where $D_i(\theta)$, $(i=1,2)$ are the far-field primary wave directivity functions given in the program as,

$$D_i(\theta) = \frac{2J_1(k_i a \sin\theta)}{k_i a \sin\theta}; \quad (i=1,2), \text{ for a circular piston of radius a} \tag{12}$$

$$= \frac{\sin(k_i a \sin\theta)}{k_i a \sin\theta}; \quad (i=1,2), \text{ for a line array of length 2a.} \tag{13}$$

It should be noted that although Eq. 11 is only applicable for $\alpha_T r_o' \leq 10^{-2}$ dB, the program permits it to be evaluated for all values of $\alpha_T r_o'$. For values of $\alpha_T r_o' > 10^{-2}$ dB therefore, the "absorption-limited," "virtual-end-fire-array" directivity function $\delta_\pm(\theta)$ should be used instead. This function is given in the program by the expression,

$$\delta_\pm(\theta) = \left\{\sum_{n=o}^{\infty} \frac{a_n}{(n+1)\alpha_{T_\pm} r_o + j2k_\pm r_o \sin^2(\theta/2)}\right\} \Big/ \left\{\sum_{n=o}^{\infty} \frac{a_n}{(n+1)\alpha_{T_\pm} r_o}\right\} \tag{14}$$

where $$a_n = (\Gamma_o/r)^n \frac{\sin[(n+1)\tan^{-1}(4/\Gamma_o)]}{[\sqrt{1 + (\Gamma_o/4)^2}]^{n+1}} \tag{15}$$

and $$20 \log_{10} \Gamma_o = SL_o^* - 20 \log_{10} \left| \frac{\alpha_T r_o}{2} \right| + 20 \log_{10} N; \; \bar{p}_{o1} = \bar{p}_{o2}. \quad (16)$$

It should be noted that unlike Eq. 11, Eq. 14 only holds for equal primary wave amplitudes. Although it resembles Bartram's[2] end-fire-directivity function, Eq. 14 is a new and more general form of the directivity function described by Childs.[3] It was derived from Kuznetsov's equation[4] via Burgers' equation and will be the subject of a later article.

For $10^{-2} \lesssim \alpha_T r_o' \lesssim 10$ the convolution of Eqs. 11 and 14, which is available in the program, should be used to obtain the far-field parametric array beam pattern $b_\pm(\theta)$ where,

$$b_\pm(\theta) = \int_{-\pi/2}^{\pi/2} D_\pm(\theta')\delta_\pm(\theta - \theta')\cos\theta' \, d\theta' \quad (17)$$

where $D_\pm(\theta)$ is defined by Eq. 11 and $\delta_\pm(\theta)$ is given in Eq. 14.

Directivity Index:

The directivity index DI is calculated in the program as a function of the scaled source level SL_o^* for fixed values of $\alpha_T r_o$ and f_o/f_-, from the expression,

$$DI = 10 \log_{10} \frac{2}{\int_o^\pi |d_\pm(\theta')d_\pm^*(\theta')| \sin\theta' \, d\theta'} \quad (18)$$

where $d_\pm(\theta)$ stands for $D_\pm(\theta)$, $\delta_\pm(\theta)$, or $b_\pm(\theta)$ whichever is appropriate and $d_\pm^*(\theta)$ is its complex conjugate.

Far-Field Monofrequency Source Level:

The program can also be used to obtain the referred far-field scaled source level at 1 m, SL_∞^* of a monofrequency source of frequency f_o, as a function of the scaled "input" source level SL_o^*, for fixed values of $\alpha_o r_o$,

where $$SL^*_\infty = SL^*_o + 20 \log_{10} F' \tag{19}$$

and $$F' = (2/\chi_o)\frac{I_1(\chi_o)}{I_o(\chi_o)} \tag{20}$$

with $$20 \log_{10}\chi_o = SL^*_o + 20 \log_{10}\ell_o + 20 \log_{10} N \tag{21}$$

$$\Delta_o = E_1(2\alpha_o r_o)\exp(2\alpha_o r_o). \tag{22}$$

Eqs. 19 to 22 will be the subject of a later article. Note that

$$SL^*_\infty = SL_\infty + 20 \log_{10} f_o; \quad (f_o \text{ in kHz}), \tag{23}$$

where SL_∞ will differ from SL_o as the latter increases, due to the transfer of energy from f_o into self-generated harmonics in the medium. In the case of a parametric source the program gives SL^*_∞ vs SL^*_o as an "input-output" relationship for the mean primary wave frequency.

PROGRAM INPUT

The data for the program is in free field format which means that the data is not restricted to any particular columns of the data card. Input variables which begin with the letters I, J, K, L, M, N are integers (no decimal point). All others are real variables and must have a decimal point. The data values on a card must be separated from each other by commas.

Card 1 NOC = the number of separate cases being run.

Card 2 IOP, IOE, IOS, IOD, IOO

Each of these integer indicators must be either 0 or 1. A zero indicates that that particular part of the program can be skipped. A one indicates that that section is to be done. IOP is for the pattern section. IOE is for the 20 log (ETA) section. IOS is for the SPL-* section. IOD is for the directivity index section. IOO is for the SPL (OUT) section.

Card 3 ALTRO, P1P2

ALTRO = $\alpha_T r_o$ in dB. ALTRO is positive for difference-frequency cases and is negative for sum frequency cases. P1P2 is the ratio (P1/P2) of the primary wave pressure amplitudes of the two drive frequencies F1 and F2 at the source.

Card 4 NFM, NFMRD

NFM = the number of FO/F- values. NFM must be 5 or less.

NFMRD = 0 or 1. A zero will generate a set of FO/F- cases according to the formula FO/F- = $5.0*2^{K-1}$, K=1,..., NFM.

If NFMRD = 1 then NFM values for FO/F- must be read in.

Card 5 FFM(K), K = 1, . . . , NFM

FFM(K) = the Kth value of FO/F-. If NFMRD = 0 this card must be omitted.

Card 6 LIN, IOC

LIN = 0 for a piston, = 1 for a rectangle.

IOC = 1 for the "I" Bessel function formula for the beam pattern.

= 2 for the beam pattern of the delta formula.

= 3 for the beam pattern of the convolution of the delta formula with the "I" Bessel function formula. If IOP = 0 and IOD = 0 then this card must be omitted.

Card 7 AKOR if LIN = 0

AKOL, AKOW if LIN = 1

AKOR = $k_o a$ = 2.0 * π * piston radius/wavelength.

AKOL = $k_o \ell$ = 2.0 * π * rectangle length/wavelength.

AKOW = $k_o w$ = 2.0 * π * rectangle width/wavelength.

The wavelength is of FO (FO = (F1+F2)/2.0). If IOP = 0 and IOD = 0 then this card must be omitted.

Card 8 SLS, PFFM, DB, NP, TD

SLS = the value in dB of SLO* at which a pattern is desired.

PFFM = the value of FO/F- at which a pattern is desired.

DB = the lower limit of the pattern plot (e.g. DB = -80.0)

NF = the number of theta values at which the beam pattern is evaluated.

TD = $\Delta\theta$ = the step size between theta values in degrees. If IOP = 0 this card must be omitted.

Some of the sections have graphs with multiple curves. The program presently limits the number of curves per graph to five.

The graphs that have SLO* as the X-axis are presently limited to twenty-one (21) separate values of SLO*. These are generated in the program by the formula SLO* = 180.0 + 10.0 * (J-1), J = 1, 2, 3, . . . , 21.

The maximum number of theta values for patterns is restricted to 101 because of the size of the plotting grid.

If a pattern is desired from a rectangular source the pattern is assumed to be in the plane of the length axis of the rectangle.

For the directivity index of a piston the pattern is assumed to be zero behind the piston.

Three subroutines are used in the program--BMPAT, DLPAT, CNPAT.

BMPAT is used to calculate the beam pattern for IOC = 1.

DLPAT is used to calculate the beam pattern for IOC = 2.

CNPAT is used to calculate the beam pattern for IOC = 3.

The program calls on several packaged routines to evaluate one-dimensional definite integrals, N-dimensional definite integrals, "I" Bessel functions, "J" Bessel functions, and exponential integrals. Packaged plotting routines are used as well as a routine to determine the amount of CPU time used by the program. All of these routines are described in Appendix A.

PROGRAM OUTPUT

Figure 1 shows an example of the program output for 20 log(ETA), SPL-*, and SPL(OUT) for a range of $\alpha_T r_o$ values from 10^{-5} dB to 10^{2} dB. Five values of FO/F- were used varying from 5.0 to 80.0.

Figure 2 shows an example of the program output for the patterns of a piston with a $k_o a$ of 10.0 where FO/F- = 10.0 and SPLO* is varied from 240 dB to 330 dB.

Figure 3 shows the program output for the directivity index of a piston with a $k_o a$ of 10.0 and an FO/F- of 10.0.

REFERENCES

1. M. Abramowitz and I. A. Segun, "Handbook of Mathematical Functions," (Dover Publications, Inc. 1965) pp. 228-237.
2. J. F. Bartram, "A Useful Analytical Model for the Parametric Acoustic Array," J. Acoust. Soc. Am. 52, 1042-1044 (1972).
3. D. R. Childs, "Beam Patterns and Directivity Indices of Parametric Acoustic Arrays," Proc. Symp. Finite-Amplitude Wave Effects in Fluids, Technical University of Denmark, Copenhagen (August 1973).

(This page intentionally left blank.)

APPENDIX A

Included in this appendix are descriptions of the packaged routines that are used in the program. The routines ROMBS, ROM2, RMB1, RMB2, RMB3, BJYO1, BESI, and DEI are from the JPL Fortran V Subroutine Directory, Edition No. 4, October 1970. The routines PSCALE, PSETUP, PPLOT, and SECOND are from the Westinghouse Research Laboratories' Fortran library.

11.1. ONE-DIMENSIONAL

11.1.1. QUADRATURE, ONE-DIMENSION, S.P.

11.1.1.1. IDENTIFICATION

QUADRATURE, ONE-DIMENSIONAL, SINGLE PRECISION

LANG	FILE	ELT/VERS	SIZE	ENTRY NAMES
F-V	LIB*JPL$	ROMBS/JPL	1251(8)=681(10)	ROMBS,ROM2

SUBROUTINES USED: #NONE#

COGNIZANT PERSONS: W. R. BUNTON AND M. DIETHELM,
JPL, SECTION 314, 1969 SEPT 30

11.1.1.2. PURPOSE

OBTAIN APPROXIMATE EVALUATION OF A ONE-DIMENSIONAL DEFINITE INTEGRAL BY NUMERICAL QUADRATURE.

ANS = THE INTEGRAL FROM A TO B OF F(X)*DX

11.1.1.3. REFERENCES:

FOR A COMPLETE DESCRIPTION OF THIS SUBROUTINE, INCLUDING A DISCUSSION OF A VARIETY OF TEST CASES, SEE:

WILEY R. BUNTON, MICHAEL DIETHELM, AND KAREN HAIGLER, 'ROMBERG QUADRATURE SUBROUTINE FOR SINGLE AND MULTIPLE INTEGRALS', JPL

W. BUNTON, M. DIETHELM, G. WINUE, 'MODIFIED ROMBERG QUADRATURE: A SUBROUTINE TO SUPPORT GENERAL SCIENTIFIC COMPUTING', JPL INTERNAL MEMORANDUM TM 314-258, APRIL 1, 1970.

W. BUNTON, M. DIETHELM, 'MODIFICATIONS TO THE JPL ROMBERG SUBROUTINES', TM 314-247, 1 SEPT. 1970.

11.1.1.4. METHOD

THIS SUBROUTINE COMBINES TECHNIQUES FROM 'ROMBERG' AND 'ADAPTIVE STEP' QUADRATURE METHODS. THE SUBROUTINE INITIALLY PICKS A SUBINTERVAL [A,B1] OF THE TOTAL INTERVAL [A,B] AND ATTEMPTS TO APPROXIMATE THE INTEGRAL OVER THIS SUBINTERVAL BY USING THREE

Reproduced from best available copy.

STAGES OF ROMBERG QUADRATURE. THIS REQUIRES EVALUATION OF THE INTEGRAND AT FIVE EQUALLY SPACED POINTS.

IF THIS QUADRATURE IS REGARDED AS SUCCESSFUL (SEE: ERROR CONTROL) THE SUBROUTINE WILL ADD THE VALUE OBTAINED TO A RUNNING SUM AND PROCEED TO TREAT A NEW DISJOINT SUBINTERVAL OF THE SAME OR GREATER LENGTH.

IF THIS QUADRATURE IS NOT REGARDED AS SUCCESSFUL AND THE CURRENT STEP LENGTH IS GREATER THAN HMIN, THEN THE SUBROUTINE WILL REJECT THE RESULT FOR THE CURRENT SUBINTERVAL AND TAKE THE LEFT HALF OF THE SUBINTERVAL AS THE NEW SUBINTERVAL TO BE TREATED.

IF THE CURRENT STEP LENGTH IS HMIN OR SMALLER, THEN THE SUBROUTINE WILL ACCEPT THE CURRENT RESULT AND PROCEED TO THE NEXT SUBINTERVAL, WRITING A MESSAGE ON FORTRAN UNIT 6 IDENTIFYING THE SUBINTERVAL ON WHICH THE ACCURACY TEST WAS NOT SATISFIED.

11.1.1.5. ERROR CONTROL

THE QUADRATURE IS REGARDED AS SATISFACTORY OVER A PARTICULAR SUBINTERVAL IF

1. THE ESTIMATED RELATIVE ERROR OF THE QUADRATURE OVER THAT SUBINTERVAL IS AT MOST ERMAX, OR

2. THE ESTIMATED ERROR IN THAT SUBINTERVAL RELATIVE TO THE ACCUMULATED VALUE OF THE INTEGRAL UP TO AND INCLUDING THAT SUBINTERVAL IS AT MOST ERMAX.

3. THE ESTIMATED ABSOLUTE ERROR OVER THAT SUBINTERVAL IS AT MOST .1*ABS(ERMAX).

11.1.1.6. PARAMETER CHECKING

THE SUBROUTINE WILL TERMINATE EXECUTION WITH A PRINTED MESSAGE AND A STANDARD EXEC 8 WALK-BACK (RETURN C) IF THE GIVEN PARAMETERS DO NOT SATISFY

A<B, OR A>B, BUT NOT A=B
0.<HMIN.LE.HSTAR.LE.HMAX AND

RELATIVE - 0.<ERMAX<1.0, BUT NOT ERMAX=0.
ABSOLUTE - ANY NEGATIVE NUMBER, BUT NOT =0.

IF A.GT.B OR IF HSTAR.GE.(B-A)/4, ONLY A WARNING MESSAGE IS PRINTED.

11.1.1.7. USAGE

```
      REAL     A, B, X, FOFX, HSTAR, HMIN, HMAX, ERMAX, ANS
      INTEGER  K, KEY

      [ASSIGN VALUES TO A, B, HSTAR, HMIN, HMAX, ERMAX, AND KEY]

      CALL  ROMBS(A,B,X,FOFX,HSTAR,HMIN,HMAX,ERMAX,ANS,K,KEY)

   10 [EVALUATE THE INTEGRAND USING THE CURRENT VALUE OF  X  AND
      STORE THE RESULT IN  FOFX.]

      CALL  ROM2

      IF(K.EQ.1) GO TO 10
      [QUADRATURE IS COMPLETED.  RESULT IS IN  ANS]
```

THE SUBROUTINE PARAMETERS ARE DEFINED AS FOLLOWS:

A,B — LIMITS OF INTEGRATION. REQUIRE A NOT=B.

X — VARIABLE SET BY THE SUBROUTINE FOR INTEGRAND EVALUATION IN THE USER'S PROGRAM.

FOFX — VALUE OF INTEGRAND COMPUTED BY USER'S PROGRAM USING THE ARGUMENT X.

HSTAR — SUGGESTED INITIAL STEP SIZE. THE INITIAL STEP SIZE, H, WILL BE SET AT H=.01*(B-A) IF HSTAR.GE.(B-A)/4. OR AT H=HSTAR IF HSTAR.LT.(B-A)/4. THE FIRST SUBINTERVAL WILL BE OF LENGTH 4.*H AND WILL REQUIRE FIVE EVALUATIONS OF THE INTEGRAND AT X=A,A+H,...,A+4*H. SUGGEST HSTAR.GE.(B-A)/4.

REQUIRE HMIN .LE. HSTAR .LE. HMAX.

HMIN — MINIMUM ALLOWABLE STEP SIZE. REQUIRE 0. < HMIN .LE. HSTAR

HMAX — MAXIMUM ALLOWABLE STEP SIZE. REQUIRE HSTAR .LE. HMAX.

ERMAX — TOLERANCE ON RELATIVE OF ABSOLUTE ERROR. SEE DISCUSSION ABOVE UNDER 'ERROR CONTROL'. REASONABLE SETTINGS FOR ERMAX WOULD BE IN THE RANGE FROM 1.E-4 TO 1.E-7. IF GREATER ACCURACY IS REQUIRED, SEE THE WRITE-UP ON ROMBD. IT IS REQUIRED THAT 0.LT.ERMX.LT.1. FOR THE RELATIVE ERROR TEST. ONE SHOULD KNOW THE RANGE OF THE ANSWER

BEFORE HE USES ABSOLUTE ERROR.

ANS THE FINAL VALUE OF THE INTEGRAL. AVAILABLE WHEN ROM2 RETURNS WITH K=2.

K BRANCHING FLAG SET BY THE SUBROUTINE FOR USE IN THE USER'S PROGRAM. K=1 MEANS THE USER SHOULD EVALUATE THE INTEGRAND AT X, STORE THE VALUE IN FOFX, AND RE-ENTER ROM2. K=2 MEANS THE COMPUTATION IS COMPLETED AND THE VALUE IS IN ANS.

KEY FLAG TO CONTROL PRINTING OF ERROR MESSAGES. PREVIOUSLY, ANY VALUE OF KEY NOT = 7 WOULD WRITE A DIAGNOSTIC MESSAGE WHEN H BECAME.LT.HMIN. THE INPUT VALUES HAVE BEEN CHANGED SO THAT WHEN

ACTION

KEY=5 PRINT INTERMEDIATE T AND Y VALUES; PRINT THE HMIN DIAGNOSTIC IF DETECTED.

=6 PRINT INTERMEDIATE T AND Y VALUES; DO NOT PRINT THE HMIN DIAGNOSTIC.

=7 DO NOT PRINT THE T AND Y VALUES OR HMIN DIAGNOSTIC.

=ANY OTHER VALUE PRINT THE HMIN DIAGNOSTIC IF DETECTED.

THE T VALUES PRINTED ARE T(1,0), T(1,1), AND T(2,0). THE PRINTED Y VALUES ARE THE FUNCTIONAL EVALUATIONS Y(1) THRU Y(5) AT THE POINTS X,X+H,...,X+4H. SEE REFERENCES.

11.1.1.8. REMARKS:

THE FOLLOWING NOTES MAY BE APPLICABLE FOR DIFFICULT INTEGRANDS.

1. IF IN DOUBT, ONE SHOULD USE A SMALL VALUE OF HSTAR. THE STEP SIZE H CAN DOUBLE QUICKLY AND THE USER IS PENALIZED ONLY A SMALL NUMBER ON FUNCTIONAL EVALUATIONS WHILE HE INCREASES HIS CHANCES OF GETTING AN ACCURATE APPROXIMATION MANY FOLD.

2. BE CAUTIOUS WHEN RELATIVE ERMAX IS .GT. 10-3. IF HSTAR.LT.(B-A)/4, BUT NOT SMALL ENOUGH, AND THE FUNCTION IS OCCILATORY, VERY DIFFICULT, ETC., ROMBS CAN RETURN A WRONG ANSWER. EXAMPLE: F(X)=X*SIN30X+COSX ON THE INTERVAL (0,2PI), HSTAR=1.57, TRUE ANSWER=-.20967248, A RELATIVE TOLERANCE OF .1 GAVE -.25E-5, AND A RELATIVE TOLERANCE OF .01 GAVE 4.188; BAD ANSWERS.

3. ALSO FOR RELATIVE ERMAX .LT.10**-6. IF THE RELATIVE ERROR IS ASKING FAR GREATER THAN 6 SIGNIFICANT DIGITS ONE IS PUSHING THE ACCURACY OF THE UNIVAC 1108. ON A HIGHLY OCCILATORY, VERY DIFFICULT PROBLEM. ROMBC MAY BE TAKING THOUSANDS MORE FUNCTIONAL EVALUATIONS AND NOT REACHING THE ACCURACY IT DID AT 10**-6. EXAMPLE: SAME F(X) AS ABOVE WHEN GIVEN TOLERANCE WAS 10**-6, ANS=.20967380 WITH A 2009 FUNCTIONAL EVAUATIONS; BUT WHEN WHEN ERMAX=10**-8, ANS=.20967765 AND 10,267 FUNCTIONAL EVALUATIONS.

4. IF ONE WANTS A ROUGH APPROXIMATION OF THE INTRGRAL, HE CAN SET HMIN=HSTAR=HMAX. A FIXED STEP INTRGRATION OF THE FUNCTION WILL TAKE PLACE. BE SURE THAT KEY=7, OR MANY DIAGNOSTICS MAY BE PRINTED.

Reproduced from best available copy.

11.2. MULTI-DIMENSIONAL

11.2.1. QUADRATURE, MULTI-DIMENSION, S.P.

11.2.1.1. IDENTIFICATION

QUADRATURE, MULTI-DIMENSIONAL, SINGLE PRECISION

LANG	FILE	ELT/VERS	SIZE	ENTRY NAME
F-V	LIB*JPLS	RMB1/JPL	361(8)=241(10)	RMB1
F-V	LIB*JPLS	RMB/JPL	2267(8)=1207(10)	RMB1A,RMB2 RMB3

SUBROUTINES USED: #NONE#

COGNIZANT PERSONS: W. R. BUNTON AND M. DIETHELM,
JPL, SECTION 314, 1969 SEPT 30

```
- - - - - - - - - - - - - - - - - - - - - - - -
-  CALLING SEQUENCE TO THIS SUBROUTINE       -
-  HAS BEEN CHANGED SINCE ED. 3 1 APRIL 70-
-  VERSION OF THIS DIRECTORY.                -
- - - - - - - - - - - - - - - - - - - - - - - -
```

11.2.1.2. PURPOSE

OBTAIN APPROXIMATE EVALUATION OF AN N-DIMENSIONAL DEFINITE INTEGRAL BY NUMERICAL QUADRATURE. THE LIMITS OF INNER INTEGRALS CAN BE FUNCTIONS OF THE VARIABLES OF THE OUTER INTEGRALS.

```
     RESULT = THE INTEGRAL FROM  A1  TO  B1
     WITH RESPECT TO  X(1)  OF

[THE INTEGRAL FROM  A2  TO  B2  WITH RESPECT TO  X(2)  OF
[...[THE INTEGRAL FROM  AN  TO  BN  WITH RESPECT TO  X(N) OF  F]
...]]
```

WHERE

1. A1 AND B1 ARE CONSTANTS

2. FOR J=2,...,N· AJ AND BJ MAY BE FUNCTIONS OF X(1),...,X(J-1).

3. THE INTEGRAND F. IS A FUNCTION OF X(1),...,X(N).

11.2.1.3. REFERENCES

FOR A COMPLETE DESCRIPTION OF THIS SUBROUTINE, INCLUDING A DISCUSSION OF A VARIETY OF TEST CASES, SEE:

WILEY R. BUNTON, MICHAEL DIETHELM, AND KAREN HAIGLER, 'ROMBERG QUADRATURE SUBROUTINE FOR SINGLE AND MULTIPLE INTEGRALS', JPL INTERNAL MEMORANDUM TM 314-221, JULY 1, 1969. SEE OTHER REFERENCES UNDER ROMBS.

11.2.1.4. METHOD

THE SUBROUTINE PERFORMS A NESTED SEQUENCE OF ONE-DIMENSIONAL INTEGRATIONS. THE SUBROUTINE USED FOR THE ONE-DIMENSIONAL INTEGRATIONS IS ESSENTIALLY ROMBS WITH CHANGES IN THE WAY STORAGE IS MANAGED. SEE THE WRITE-UP ON ROMBS.

11.2.1.5. USAGE

THE USER MUST PROGRAM AN INITIALIZATION CALL TO RMB1, THEN N SEPARATE CALLS TO RMB2, ONE FOR EACH LEVEL OF INTEGRATION, AND FINALLY A CALL TO RMB3. SOME OF THESE CALL STATEMENTS WILL BE EXECUTED MORE THAN ONCE.

THE CODING SPECIFICATIONS ARE AS FOLLOWS:

```
INTEGER N,KGO,KEY
REAL      X(NN1N), F, RESULT, ERMAX, W(NN2N)
REAL      A1, B1, HSTAR1, HMIN1, HMAX1
  .       .
  .       .
  .       .
REAL      AN, BN, HSTARN, HMINN, HMAXN
```

WARNING: CALLING SEQUENCE TO THIS SUBROUTINE HAS BEEN CHANGED SINCE ED. 3, 1 APRIL 1970 VERSION OF THIS DIRECTORY.

```
      [ASSIGN VALUES TO  N  AND  ERMAX.]
   CALL RMB1(N,X,F,RESULT,ERMAX,KGO,W,KEY)

1     [ASSIGN VALUES TO ALL PARAMETERS IN THE FOLLOWING
       CALL  STATEMENT.]
   CALL  RMB2(A1,B1,HSTAR1,HMIN1,HMAX1)

2     [ASSIGN VALUES TO ALL PARAMETERS IN THE FOLLOWING
       CALL  STATEMENT.  SOME OF THESE VALUES MAY BE COMPUTED
       AS FUNCTIONS OF  X(1).]
```

```
      CALL  RMB2(A2,B2,HSTAR2,HMIN2,HMAX2)
      .
      .
      .
 #N#     [ASSIGN VALUES TO ALL PARAMETERS IN THE FOLLOWING
          CALL  STATEMENT.  SOME OF THE VALUES MAY BE COMPUTED AS
          FUNCTIONS OF  X(1),...,X(N-1).]
      CALL  RMB2(AN,BN,HSTARN,HMINN,HMAXN)

#N+1#    [COMPUTE  F  AS A FUNCTION OF  X(1,...,X(N).]
      CALL  RMB3
      GO TO  (1,2,...,#N#,#N+1#,#N+2#),KGO

#N+2#    [THE COMPUTATION IS COMPLETED.  THE VALUE OF THE INTEGRAL
          IS IN  RESULT.]
```

THE DIMENSIONING PARAMETERS MUST SATISFY:

```
        #N1# .GE. N
        #N2# .GE. 29*N
```

THE PARAMETERS FOR RMB1 ARE DEFINED AS FOLLOWS:

N NUMBER OF LEVELS OF INTEGRATION REQUIRED. (N .GE. 1). EXAMPLE: FOR A TRIPLE INTEGRAL, SET N=3.

(X(I)I=1,N) CURRENT VALUES OF THE INTEGRATION VARIABLES SET BY THE SUBROUTINE. X(1) IS ASSOCIATED WITH THE OUTERMOST INTEGRATION AND X(N) WITH THE INNERMOST.

F VALUE OF INTEGRAND, TO BE COMPUTED BY THE USER AS A FUNCTION OF X(1),...,X(N) AFTER THE N-TH CALL TO RMB2 AND WHEN RMB3 RETURNS WITH KGO = N+1.

RESULT VALUE OF INTEGRAL COMPUTED BY THE SUBROUTINE. AVAILABLE WHEN RMB3 RETURNS WITH KGO = N+2.

ERMAX RELATIVE OR ABSOLUTE ERROR TOLERANCE SET BY THE USER. REASONABLE VALUES WOULD BE IN THE RANGE 1.E-2 TO 1.E-4 FOR RELATIVE. REQUESTING MORE ACCURACY MAY GREATLY INCREASE THE EXECUTION TIME. ERMAX MUST SATISFY 0.LT.ERMAX.LT.1. FOR RELATIVE, ANY NEGATIVE NUMBER NOT = 0 FOR ABSOLUTE.

KGO BRANCHING FLAG SET BY THE SUBROUTINE RMB3 FOR USE IN THE USER'S PROGRAM.

KGO = 1,...,N MEANS THE USER SHOULD SET THE PARAMETERS FOR THE KGO-TH CALL TO RMB2 AND THEN EXECUTE THAT

CALL.

KGO = N+1 MEANS THE USER SHOULD COMPUTE THE INTEGRAND, F, AND THEN CALL RMB3.

KGO = N+2 MEANS THE COMPUTATION IS COMPLETED AND THE VALUE OF THE INTEGRAL IS IN RESULT.

(W(I),I=1,29*N) WORKING SPACE NEEDED BY THE SUBROUTINE.

KEY FLAG TO CONTROL PRINTING OF ERROR MESSAGE WHEN ERROR TOLERANCE IS NOT MET WITH MINIMUM STEP SIZE.

KEY=7 NO ERROR MESSAGE IS PRINTED
KEY=ANY OTHER VALUE ERROR MESSAGE IS PRINTED.

THE PARAMETERS FOR THE J-TH CALL TO RMB2 ARE AS FOLLOWS:

AJ,BJ LOWER AND UPPER LIMITS OF INTEGRATION FOR THE J-TH INTEGRAL. EITHER AJ > BJ OR AJ .LE. BJ IS PERMITTED.

HSTARJ,HMINJ,HMAXJ QUADRATURE STEP PARAMETERS FOR THE J-TH INTEGRAL. THESE PARAMETERS MUST BE POSITIVE, REGARDLESS OF THE SIGN OF (BJ-AJ), AND MUST SATISFY 0. < HMINJ .LE. HSTARJ .LE. HMAXJ. THE J-TH QUADRATURE WILL START WITH A STEP WHOSE MAGNITUDE IS

HJ=MIN(HSTARJ,ABS(BJ-AJ)/4.) EXCEPT FOR OUTER INTEGRAL. SEE REMARK 4. BELOW.

AND THE MAGNITUDE OF THE STEP WILL BE KEPT BETWEEN HMINJ AND HMAXJ.

11.2.1.6. REMARKS

1. THE VALUE OF KGO WILL NEVER BE 1, THUS THE FIRST CALL TO RMB2 WILL ONLY BE EXECUTED ONCE.

2. THE PARAMETERS N, ERMAX, AND ALL PARAMETERS IN THE CALLS TO RMB2 WHICH ARE TO REMAIN CONSTANT DURING THE QUADRATURE CAN BE SPECIFIED LITERALLY IN THE CALL STATEMENTS.

3. PARAMETERS IN THE SECOND THROUGH THE N-TH CALL TO RMB2 WHICH ARE NOT CONSTANT CAN BE WRITTEN IN THE CALL STATEMENTS AS ARITHMETIC EXPRESSIONS. THE PARAMETER F, HOWEVER, CANNOT BE REPLACED IN THE CALL TO RMB1 BY AN ARITHMETIC EXPRESSION EXCEPT FOR THE SPECIAL CASE IN WHICH F IS CONSTANT.

4. IF THE STARTING STEP SIZE FOR THE OUTER INTEGRAL IS GREATER

Reproduced from best available copy.

THAN OR EQUAL TO (B-A)/4., A WALK-BACK DIAGNOSTIC (RETURN 0) IS PRINTED AND EXECUTION IS TERMINATED.

5. SUGGESTED VALUE FOR KEY IS 6.

11.2.1.7. EXAMPLE

CASE 11, PAGE 48, OF THE REFERENCE CITED ABOVE CAN BE CODED AS FOLLOWS :

```
      REAL  X(3), WORK(87)

      CALL RMB1(3,X,F,RESULT,1.E-4,KGO,WORK,0)
   10 CALL  RMB2(0.        , 1.        , 1.E-2, 1.E-4, 1.)
   20 CALL  RMB2(X(1)      , X(1)**2   , 1.E-2, 1.E-4, 1.)
   30 CALL  RMB2(X(1)*X(2), X(1)+X(2), 1.E-2, 1.E-4, 1.)
   40 F = X(1)*X(2)*X(3)
      CALL  RMB3
      GO TO (10, 20, 30, 40, 50), KGO
   50 CONTINUE
```

THE TRUE VALUE OF THIS INTEGRAL IS -0.032060185 . THE COMPUTED VALUE HAD AN ABSOLUTE ERROR OF 5.1E-9 AND A RELATIVE ERROR OF 1.6E-7 . THE SUBROUTINE WROTE TWO DIAGNOSTIC MESSAGES INDICATING THAT THE REQUESTED ACCURACY COULD NOT BE ATTAINED WITH THE MINIMUM STEP SIZE ON THE INTERVALS 0.0 .LE. X(1) .LE. 4.E-4 AND 4.E-4 .LE. X(1) .LE. 8.E-4 . THIS EXAMPLE USED 10361 EVALUATIONS OF THE INTEGRAND.

THIS EXAMPLE WAS RUN 20 TIMES AND TIMED USING PRTIM1/PRTIM2. THE TIME VARIED FROM 2.611 SECONDS TO 2.943 SECONDS.

4.2.6. BESSEL J0,J1,Y0,Y1, D.P.

4.2.6.1. IDENTIFICATION

BJY01/DOUBLE PRECISION BESSEL FUNCTIONS J0, J1, Y0, AND Y1

LANG	FILE	ELT/VERS	SIZE	ENTRY NAME
F-V	LIB*JPL$	BJY01/JPL	1010(8)=520(10)	BJY01

SUBROUTINES USED: DSQRT, DSIN, DCOS, DLOG, AND CHBPOL

COGNIZANT PERSON: E. W. NG, JPL, SECTION 314, 1969 AUGUST 1

4.2.6.2. PURPOSE

TO COMPUTE THE VALUES OF THE BESSEL FUNCTIONS J0, J1, Y0, AND Y1.

4.2.6.3. ACCURACY

FOR X .LE. N, AT LEAST 15 SIGNIFICANT DIGITS,

FOR X > N, AT LEAST 15 DECIMAL PLACES.

4.2.6.4. USAGE

```
INTEGER              N0, N1

DOUBLE PRECISION     X, B0(2), B1(2)

CALL                 BJY01(X,N0,N1,B0,B1)
```

THE PARAMETERS ARE DEFINED AS FOLLOWS:

X — ARGUMENT AT WHICH FUNCTION EVALUATION IS DESIRED. REQUIRE X > 0. IF X .LE. 0 THE SUBROUTINE WILL PRINT ON FORTRAN UNIT 6 THE MESSAGE:

FOR X=0., J0=1, J1=0., Y0=Y1=-INFINITY
AND WILL SET B0(1)=1., B1(1)=0., B0(2)=B1(2)=-1.D38.

N0
- =0 DO NOT COMPUTE J0 OR Y0
- =1 COMPUTE J0 BUT NOT Y0
- =2 COMPUTE J0 AND Y0

N1
- =0 DO NOT COMPUTE J1 OR Y1

```
         =1   COMPUTE  J1  BUT NOT  Y1
         =2   COMPUTE  J1  AND  Y1

B0(1)    COMPUTED VALUE OF  J0
B0(2)    COMPUTED VALUE OF  Y0
B1(1)    COMPUTED VALUE OF  J1
B1(2)    COMPUTED VALUE OF  Y1
```

4.2.6.3. REMARKS

THIS SUBROUTINE WAS ORIGINALLY DESIGNED FOR POSITIVE X. IF IT IS DESIRED TO INCLUDE ITS UTILITY FOR NEGATIVE X AS WELL, THE FOLLOWING IDENTITIES SHOULD BE USED:

```
J0(-X) = J0(X),     J1(-X) = -J1(X)
Y0(-X) = Y0(X) + 2*SQRT(-1)*J0(X)
Y1(-X) = -Y1(X) + 2*SQRT(-1)*J1(X)
```

NOTE THAT Y OF NEGATIVE ARGUMENT IS IN GENERAL COMPLEX.

(REFERENCE: NBS HANDBOOK OF MATH'L FUNCTIONS, APPLIED MATH SERIES NO. 55, 1964, P. 361)

4.2.9. BESSEL I AND K OF GENERAL ORDER, D.P.

4.2.9.1. IDENTIFICATION

DOUBLE PRECISION BESSEL FUNCTIONS I AND K OF GENERAL ORDER NU AND ARGUMENT X.

LANG	FILE	ELT/VERS	SIZE	ENTRY NAMES
F-V	LIB*JPL$	BESI/JPL	1774(8)=1020(10)	BESI, BESK

SUBROUTINES USED: DSIN, DLOG, DEXP

COGNIZANT PERSON: E. W. NG, JPL, SECTION 314, 1969 JULY 23

4.2.9.2. PURPOSE

TO COMPUTE IN DOUBLE PRECISION ARITHMETIC THE VALUES OF BESSEL FUNCTIONS I AND K OF THE GENERAL ORDER NU AND ARGUMENT X.

4.2.9.3. METHOD

RECURRENCE TECHNIQUES AND THE PHASE-AMPLITUDE METHOD ARE USED. (SEE REFERENCES)

4.2.9.4. ACCURACY

FOR NU .GE. X, A MINIMUM ACCURACY OF 15 SIGNIFICANT FIGURES IS FOUND; FOR NU < X, A MINIMUM ACCURACY OF 15 DECIMAL PLACES IS INSURED.

4.2.9.5. USAGE

```
      DOUBLE PRECISION  X, V, A(LDIMA), AK(LDIMA)
      INTEGER           N, LDIMA

      CALL  BESI(X,V,N,A,LDIMA)
OR    CALL  BESK(X,V,N,A,AK,LDIMA)
```

THE ENTRY BESI COMPUTES THE FUNCTION DEXP(-X) * I OF ARGUMENT X AND OF ORDERS V, V+1,...,V+N.

THE ENTRY BESK COMPUTES BOTH THE FUNCTIONS DEXP(-X) * I AND DEXP(X) * K OF ARGUMENT X AND OF THE ORDERS V, V+1,...,V+N.

THE SUBROUTINE PARAMETERS ARE DEFINED AS FOLLOWS:

X THE ARGUMENT AT WHICH FUNCTION EVALUATION IS DESIRED; X .GE. 0.D0

V THE NON-INTEGER PART OF THE ORDER OF THE BESSEL FUNCTIONS? 0.D0 .LE. V .LE. 1.D0

N N+V IS THE HIGHEST ORDER OF THE BESSEL FUNCTIONS DESIRED; N MUST BE A POSITIVE INTEGER.

A(LDIMA) A(K), K=1,...,(N+1) ARE THE LOCATIONS IN WHICH THE VALUES FOR THE BESSEL FUNCTION DEXP(-X) * I OF ORDER V, (V+1),..., (V+N) ARE RETURNED. A( ) MUST HAVE DIMENSION .GE. LDIMA

AK(LDIMA) AK(K), K=1,..., (N+1) ARE THE LOCATIONS IN WHICH THE VALUES FOR THE BESSEL FUNCTION DEXP(X) * K OF THE ORDERS V, (V+1),..., (V+N) ARE RETURNED. AK( ) MUST HAVE DIMENSION .GE. LDIMA

LDIMA DIMENSION OF A AND AK; IT MUST BE AT LEAST MAX(IDINT(X),N) + 20 OR MAX(IDINT(2*X),N) FOR X > 20.

4.2.9.6. REMARKS

FOR X, V, OR N BEYOND ITS REQUIRED RANGE OR LDIMA NOT LARGE ENOUGH, THE MESSAGE 'ERROR IN BESSEL' AND THE VALUES OF X, V, N, NU, AND LDIMA ARE PRINTED. THE NUMBER (N+2) IS THE DIMENSION OF A NEEDED. IN THE CASE X OR V IS OUT OF RANGE, THE PARAMETER NU IS IRRELEVANT.

4.2.9.7. REFERENCES

M. GOLDSTEIN AND R.M. THALER, 'RECURRENCE TECHNIQUES FOR THE CALCULATION OF BESSEL FUNCTIONS', MTAC, VOL. XIII, PP. 102-108.

M. GOLDSTEIN AND R. M. THALER, 'BESSEL FUNCTIONS FOR LARGE ARGUMENTS', MTAC, VOL. XII, 1958, PP. 18-26.

D. JORDAN, ARGONNE NATIONAL LAB., PROGRAM WRITE-UP NO. C3715, NOV. 1967.

4.2.13. EXPONENTIAL INTEGRAL

4.2.13.1. IDENTIFICATION

EXPONENTIAL INTEGRAL

LANG	FILE	ELT/VERS	SIZE	ENTRY NAMES
F-V	LIB*JPL$	DEI/JPL		DEI

SUBROUTINES USED: DEXP,DLOG

COGNIZANT PERSON: E.W. NG, JPL, SECTION 314, 1970, SEPT. 14

4.2.13.2. PURPOSE

COMPUTE THE EXPONENTIAL INTEGRAL IN DOUBLE PRECISION ARITHMETIC. FOR X .GT. 0, THE EXPONENTIAL INTEGRAL, EI, IS DEFINED AS

```
EI(X) = INTEGRAL FROM T=-INFINITY TO T=X OF
(EXP(T)/T)*DT
```

WHERE THE INTEGRAL IS TO BE INTERPRETED AS THE CAUCHY PRINCIPAL VALUE. FOR X .LT. 0, EI(X) =-E1(-X), WHERE

```
E1(Z)=INTEGRAL FROM T=Z TO T=INFINITY OF
(EXP(-T)/T)*DT
```

4.2.13.3. METHOD

THIS SUBROUTINE COMPUTES THE EXPONENTIAL INTEGRAL BY CHEBYSHEV RATIONAL APPROXIMATIONS FROM W.J. CODY AND H.C. THACHER, JR., MATH. COMP. VOL. 22, PP. 641-650, AND VOL. 23, PP. 289-303.

4.2.13.4. ACCURACY

EXTENSIVE TESTS WERE PERFORMED ON THE UNIVAC 1108 AND THE FOLLOWING ACCURACY STATISTICS WERE FOUND:

INTERVAL OF X	MAXIMUM RELATIVE	RMS RELATIVE
(-150, -4)	2.2D-16	5.1D-17

(-4, -1)	7.5D-17	1.2D-17
(-1, 0)	8.7D-18	1.4D-18
(0, 0.5)	1.8D-16*	3.2D-17*
(0.5, 6)	5.5D-17	1.0D-17
(6, 12)	1.6D-17	3.0D-18
(12, 24)	2.9D-17	7.1D-18
(24, 100)	8.9D-17	1.9D-17

CF. E.W. NG, COMM. ACM VOL. 13, #7, PP. 448-449.

4.2.13.5. USAGE

```
DOUBLE PRECISION DEI, X
```

USE THE FOLLOWING FUNCTION IN A FORTRAN ARITHMETIC STATEMENT:

```
DEI(X)
```

WHERE DEI IS THE COMPUTED VALUE OF THE EXPONENTIAL INTEGRAL.

4.2.13.6. LIMITATIONS

EI(0)=-INFINITY IS APPROXIMATED BY -7.2D75 AND EI(X .GT. 174.673) IS APPROXIMATED BY +7.2D75.

THESE LIMITATIONS WERE ORIGINALLY IMPOSED FOR THE IBM/360 AND HAVE NOT BEEN MODIFIED FOR THE UNIVAC 1108.

4.2.13.7. ERROR EXITS

NONE

PPLOT

Reference:

D. P. Wei, "A Printer Plot Package for the Univac U1106", Research Memo 72-1K4-COMPS-M16 (9-8-72).

Purpose:

This package of subroutines is used to generate graphs on the line printer. It produces an 8 by 10 grid plot; a grid is 6 print characters high and 10 characters wide. Data points are plotted as a symbol (e.g., *, A, 3) which is specified by the user. One or more plots can be included in a single graph.

Usage:

The package consists of three Fortran V subroutines.

1. PSCALE scans data for the minimum and maximum values, and establishes a range of values which produces a rational scale for the axis. The scale is chosen such that each grid is 1, 2, or 3 times an appropriate power of 10.

2. PSETUP sets up the work area for the plot. The scales for the abscissa and ordinate, and the labelling information for the axes are stored. PSETUP must be called to initialize each graph.

3. PPLOT stores plot data and generates the plot. An option controls initiation of plotting. Thus, multiple calls on PPLOT can be made to store data for several plots before a request for plotting is made.

The parameter specifications for each subroutine follow.

PSCALE

Subroutine Specification:

```
SUBROUTINE PSCALE (V, NPTS, VLOW, VHIGH, FIRST, IXORY)
    DIMENSION V(1)
    INTEGER NPTS, IXORY
    LOGICAL FIRST
    REAL VLOW, VHIGH
```

Purpose:

To scan the data in vector V, and obtain the minimum and maximum values. These values are then used to establish a range which produces a rational scale for the plot axis.

Usage:

The Fortran calling sequence is:

```
CALL PSCALE(V, NPTS, VLOW, VHIGH, FIRST, IXORY)
```

where:

V = REAL vector containing data to be scanned.

NPTS = number of points in V to be scanned.

VLOW,VHIGH = output variables into which the routine returns the lower and upper values for the range of V.

FIRST = LOGICAL variable or value indicating when it is the first set of values to be scanned. The value is true (.TRUE.) if there is a single vector to be scanned. For multiple vectors (when more than one plot is to be included in a graph), a call on PSCALE must be made for each plot (vector). The first call will have the value .TRUE.; for subsequent calls, the value will be .FALSE. The range defined by the final values of VLOW and VHIGH will include all the values scanned.

IXORY = indicator whether abscissa range (x) or ordinate range (y) is to be established. Specify the value 0 for abscissa and 1 for ordinate.

Note:

The abscissa axis is 10 grid wide. The ordinate axis is 8 grid high. PSCALE need not be used if the range of values for the abscissa (or ordinate) is known. The number of grid positions should be taken into consideration for proper annotation of the axis.

PSETUP

Subroutine Specification:

```
SUBROUTINE (XLOW, XHIGH, YLOW, YHIGH, IEOF, IGRID, LBLX,
            LBLY)
      REAL XLOW, XHIGH, YLOW, YHIGH
      INTEGER IEOF, IGRID
      DIMENSION LBLX(1), LBLY(1)
```

Purpose:

To setup work area before data values to be plotted are stored (via PPLOT routine). Labelling information, ranges of values for the abscissa and ordinate variables, and options for grid and annotation are supplied as input parameters.

Usage:

The Fortran calling sequence is:

```
CALL PSETUP (XLOW, XHIGH, YLOW, YHIGH, IEOF, IGRID,
LBLX, LBLY)
```

where:

XLOW,XHIGH = the lower and upper values for the range of the abscissa variable (x).

YLOW,YHIGH = the lower and upper values for the range of the ordinate variable (y).

IEOF = Hollerith option to set either E or F format for annotating the axes. Specify the Hollerith constant 1HE for E-format (1PE10.3), or 1HF for F-format (F10.3).

IGRID = Hollerith option to select grid. Specify the Hollerith constant 1HG for grid; otherwise, specify 1Hb (b denotes a blank or space).

LBLX = INTEGER vector containing information for labelling the x-axis. The first element LBLX(1) contains the number of characters in the label. The labelling information is stored, six characters per word, starting in the second element LBLX(2). The maximum label length is 30 characters.

LBLY = INTEGER vector containing information for labelling the y-axis. The rules for storing the information is the same as described for LBLX. The maximum label length is 24 characters.

Restrictions:

1. The value of XHIGH must be equal to or greater than XLOW.

2. The value of YHIGH must be equal to or greater than YLOW.

3. Labelling information will be truncated if the maximum label length is exceeded; i.e., 30 characters for x and 24 for y.

PPLOT

Subroutine Specification:

```
SUBROUTINE PPLOT (IPLOT, ICHR, NPTS, XV, YV)
    LOGICAL IPLOT
    INTEGER ICHR, NPTS
    DIMENSION XV(1), YV(1)
```

Purpose:

To store plot data and initiate plotting. The IPLOT parameter controls plot initiation.

Usage:

The Fortran calling sequence is:

```
CALL PPLOT (IPLOT, ICHR, NPTS, XV, YV)
```

where:

IPLOT = LOGICAL control variable for plotting. Specify .FALSE. if data values are to be stored and plotting is to be deferred; (used for multiple plots per graph). Specify .TRUE. if data values are to be stored and plotted.

ICHR = a single Hollerith character to be used as the plot symbol; (e.g., 1H*).

NPTS = number of data points to be plotted.

XV,YV = vectors containing coordinate values (x_i, y_i) to be plotted.

Restrictions:

1. If more than one data point occupies the same plot position, the most recent value processed will be used.

2. Data values outside the range specified by PSETUP will not be plotted. They will be listed below the plot.

SECOND

Subroutine Specification:

```
SUBROUTINE SECOND(ISEC)
      INTEGER ISEC
```

Programmed by: L. C. Lintner

Purpose:

This routine provides the user with a means of determining the amount of central processor (CPU) time that he has used. This time is accumulated from the beginning of the run; i.e., it includes the CPU time accrued for all preceding tasks in the run setup.

Usage:

SECOND is an assembly language routine in "FORTRAN callable" form.

The FORTRAN calling sequence is:

```
CALL SECOND(ISEC)
```

The parameter ISEC must be an integer variable. The result returned in ISEC is the integer number of CPU seconds used since the start of the run.

This routine may also be called as an integer function:

```
L = NSEC(ISEC)
```

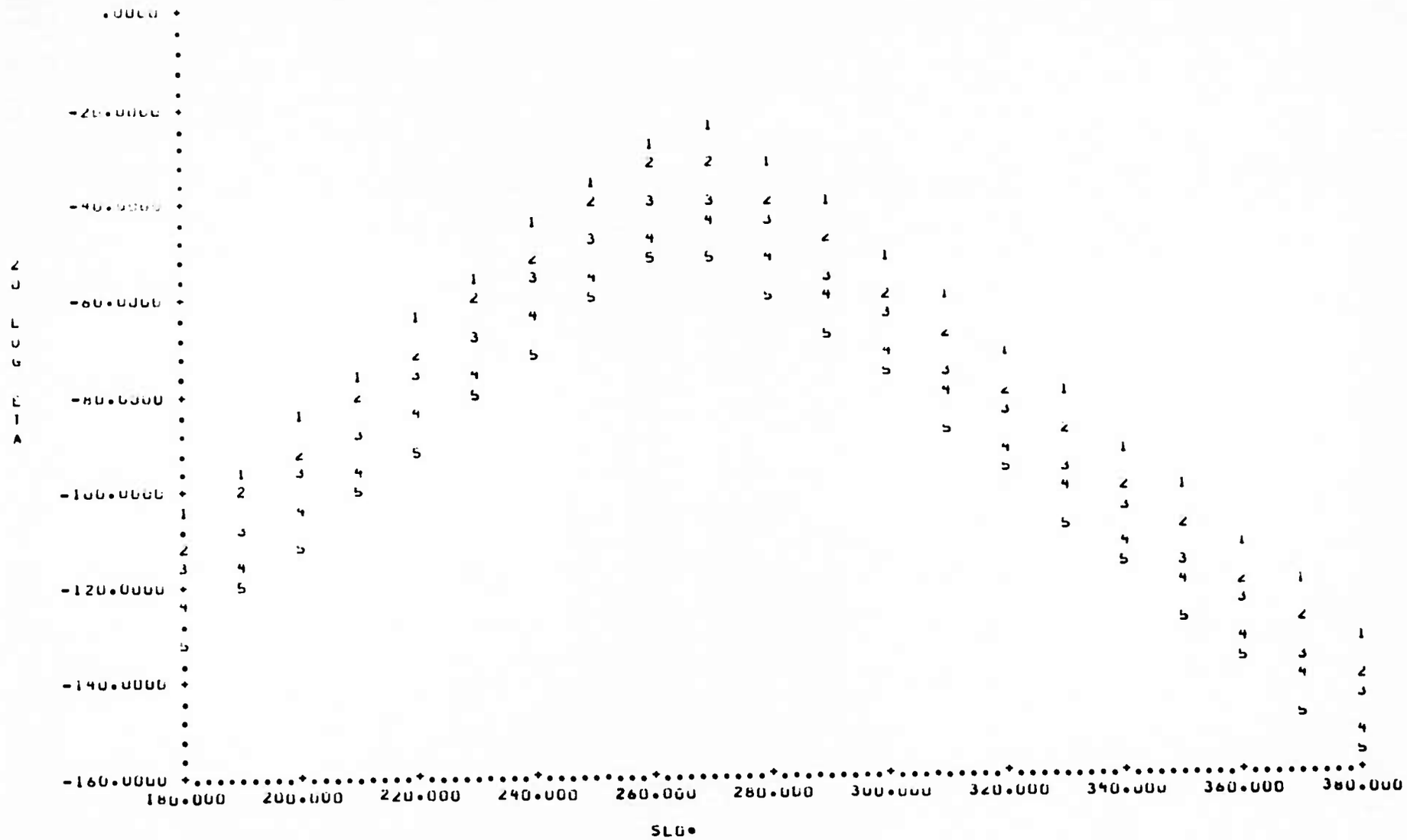
In this case, the integer number of seconds will be returned as the value of the function in addition to being stored in ISEC.

(This page intentionally left blank.)

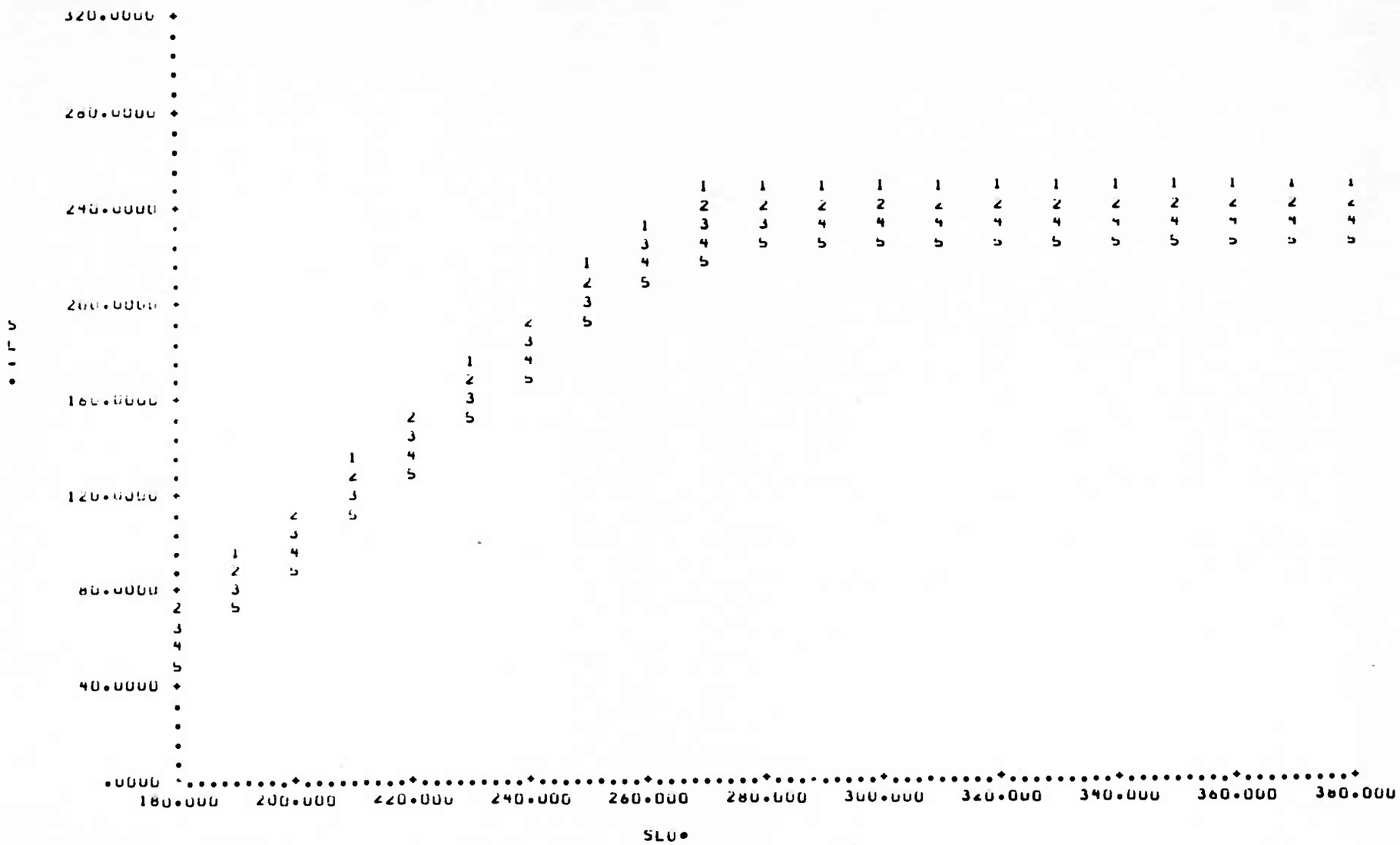
FIGURE 1

(Pages 37 to 85)

```
*XQT SONIC*NONLN.BEAM
 NOC=  8

     IOP          IOE          IOS          IOD          IOO
      0            1            1            0            1
    ALTRO        PIP2
   .101-04      .100+01
 NFM=  5     NEMRD=  0


 FO/F- =    .500+01     EN=    .108+01
 FO/F- =    .100+02     EN=    .108+01
 FO/F- =    .200+02     EN=    .109+01
 FO/F- =    .400+02     EN=    .109+01
 FO/F- =    .800+02     EN=    .109+01
```

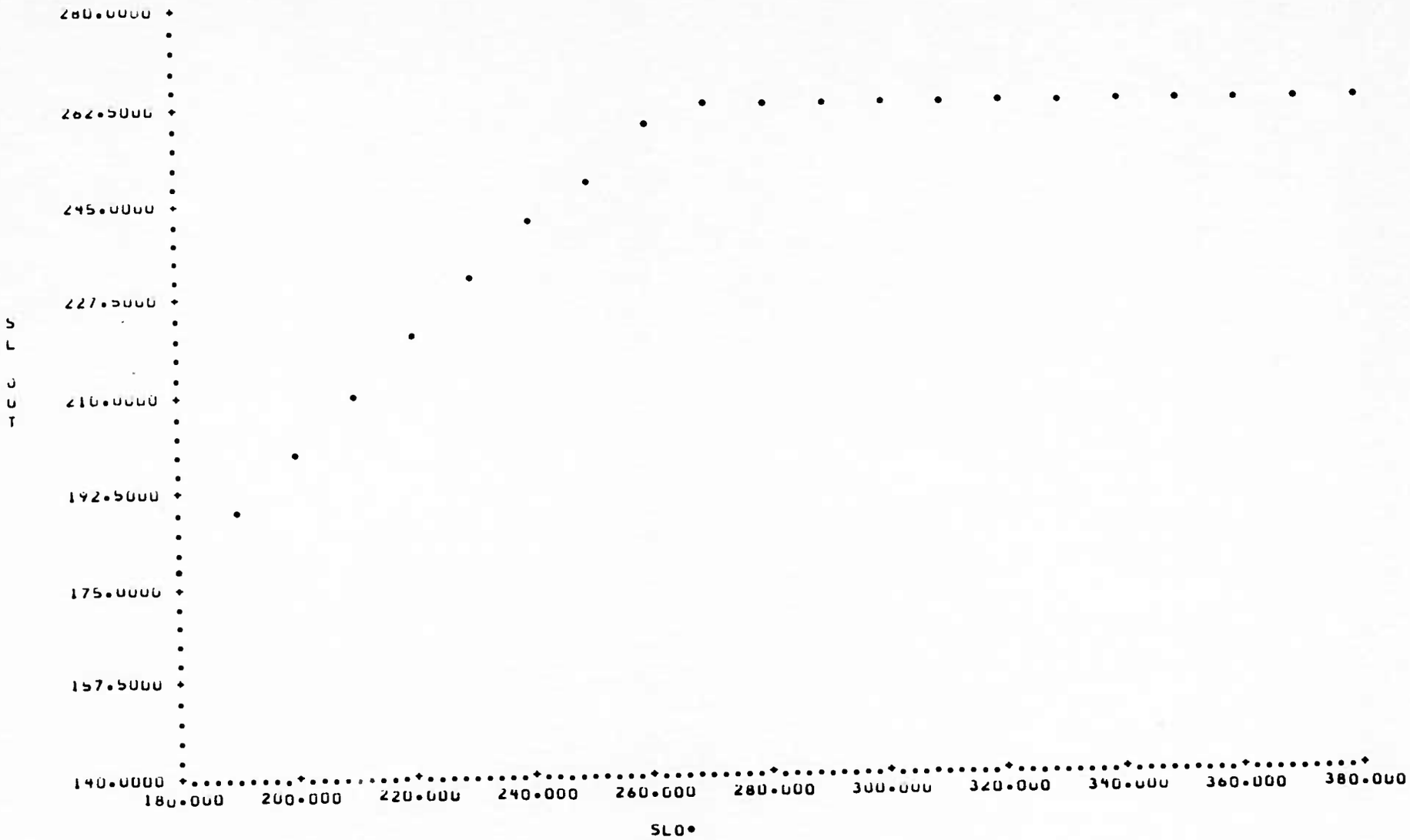


ALPHA(T)*RO= .100-04 DB
CURVE 1 IS FOR FO/F-= .500+01
CURVE 2 IS FOR FO/F-= .100+02
CURVE 3 IS FOR FO/F-= .200+02
CURVE 4 IS FOR FO/F-= .400+02
CURVE 5 IS FOR FO/F-= .800+02
PRESSURE UNITS DB/1 MICRO PASCAL (KHZ) RMS



ALPHA(T)*RO= .100-04 DB
CURVE 1 IS FOR FO/F-= .500+01
CURVE 2 IS FOR FO/F-= .100+02
CURVE 3 IS FOR FO/F-= .200+02
CURVE 4 IS FOR FO/F-= .400+02
CURVE 5 IS FOR FO/F-= .800+02
PRESSURE UNITS DB/1 MICRO PASCAL (KHZ) RMS

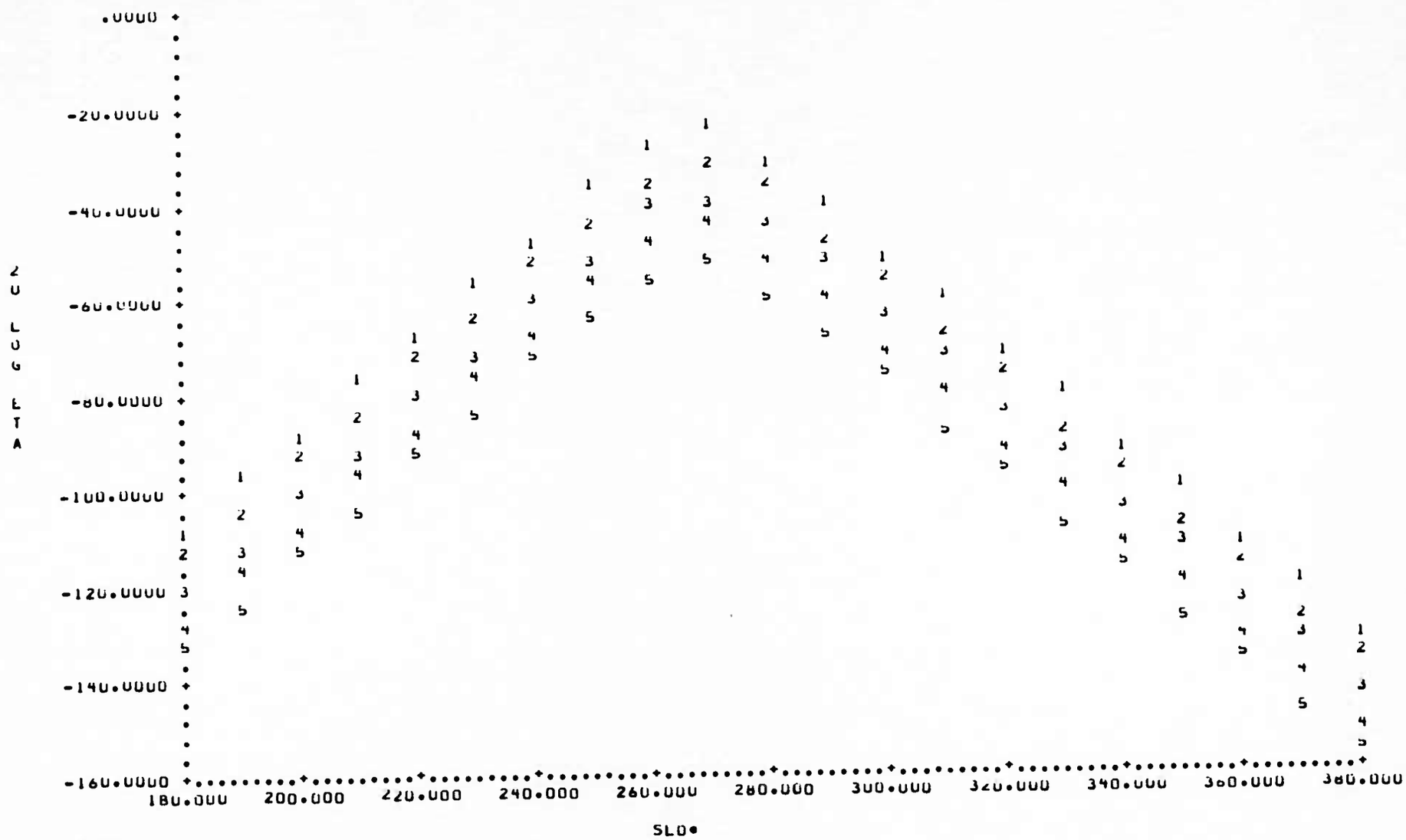
SL0.	SL-(1).	SL-(2).	SL-(3).	SL-(4).	SL-(5).
.180+03	.752+02	.666+02	.620+02	.554+02	.487+02
.190+03	.952+02	.886+02	.820+02	.754+02	.687+02
.200+03	.115+03	.109+03	.102+03	.954+02	.887+02
.210+03	.135+03	.129+03	.122+03	.115+03	.109+03
.220+03	.155+03	.149+03	.142+03	.135+03	.129+03
.230+03	.175+03	.169+03	.162+03	.155+03	.149+03
.240+03	.195+03	.189+03	.182+03	.175+03	.169+03
.250+03	.215+03	.208+03	.202+03	.195+03	.189+03
.260+03	.233+03	.227+03	.220+03	.214+03	.207+03
.270+03	.245+03	.238+03	.232+03	.225+03	.218+03
.280+03	.248+03	.241+03	.234+03	.228+03	.221+03
.290+03	.248+03	.242+03	.235+03	.228+03	.222+03
.300+03	.249+03	.242+03	.235+03	.229+03	.222+03
.310+03	.249+03	.242+03	.235+03	.229+03	.222+03
.320+03	.249+03	.242+03	.235+03	.229+03	.222+03
.330+03	.249+03	.242+03	.235+03	.229+03	.222+03
.340+03	.249+03	.242+03	.235+03	.229+03	.222+03
.350+03	.249+03	.242+03	.235+03	.229+03	.222+03
.360+03	.249+03	.242+03	.235+03	.229+03	.222+03
.370+03	.249+03	.242+03	.235+03	.229+03	.222+03
.380+03	.249+03	.242+03	.235+03	.229+03	.222+03



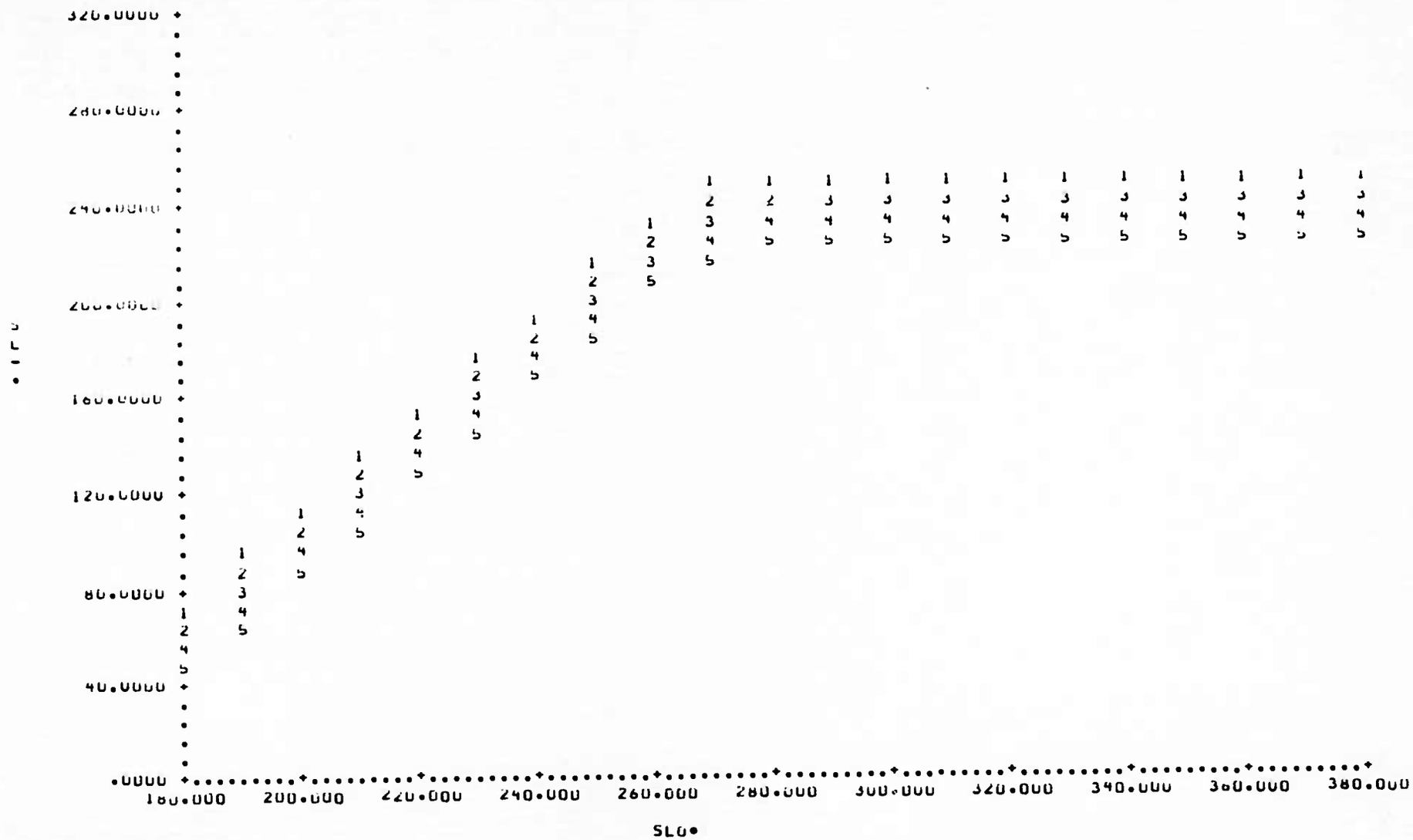
ALPHA(0)*RO= .57/-06 NEPERS
PRESSURE UNITS DB/1 MICRO PASCAL (KHZ) RMS

SL0*	SL OUT
.180+03	.180+03
.190+03	.190+03
.200+03	.200+03
.210+03	.210+03
.220+03	.220+03
.230+03	.230+03
.240+03	.240+03
.250+03	.250+03
.260+03	.258+03
.270+03	.262+03
.280+03	.263+03
.290+03	.263+03
.300+03	.263+03
.310+03	.263+03
.320+03	.263+03
.330+03	.263+03
.340+03	.263+03
.350+03	.263+03
.360+03	.263+03
.370+03	.263+03
.380+03	.263+03

```
   IOP          IOE          IOS          IOD          IOO
    0            1            1            0            1
  ALTRD        PIP2
 .100-03      .100+01
NFM=  5     NFMKD=  0


FO/F- =    .500+01     EN=    .110+01
FO/F- =    .100+02     EN=    .110+01
FO/F- =    .200+02     EN=    .111+01
FO/F- =    .400+02     EN=    .111+01
FO/F- =    .800+02     EN=    .112+01
```

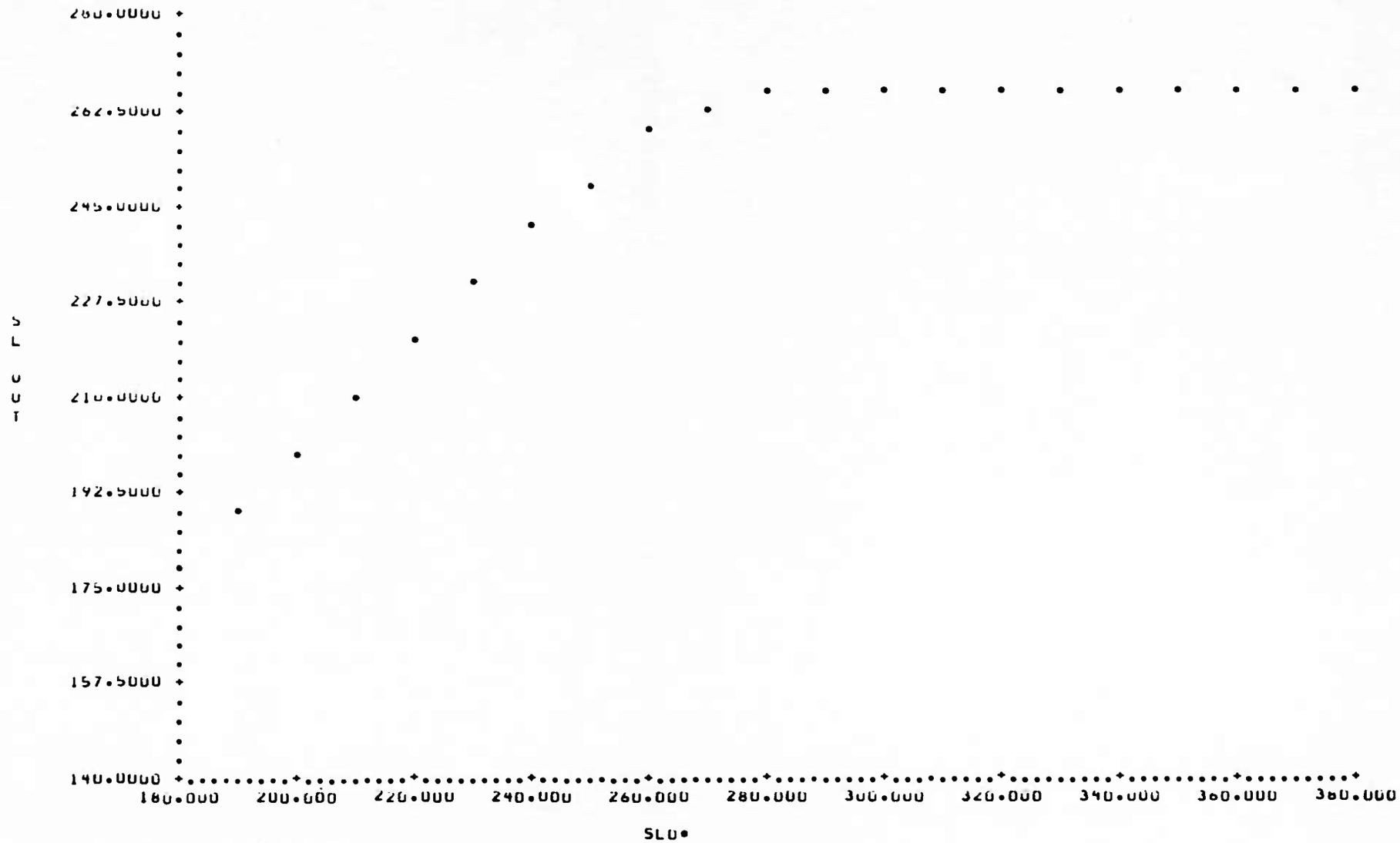


ALPHA(T)*RO= .100-03 DB
CURVE 1 IS FOR FO/F-= .500+01
CURVE 2 IS FOR FO/F-= .100+02
CURVE 3 IS FOR FO/F-= .200+02
CURVE 4 IS FOR FO/F-= .400+02
CURVE 5 IS FOR FO/F-= .800+02
PRESSURE UNITS DB/1 MICRO PASCAL (KHZ) RMS



ALPHA(T)*RO= .100-03 DB
CURVE 1 IS FOR FO/F-= .500+01
CURVE 2 IS FOR FO/F-= .100+02
CURVE 3 IS FOR FO/F-= .200+02
CURVE 4 IS FOR FO/F-= .400+02
CURVE 5 IS FOR FO/F-= .800+02
PRESSURE UNITS DB/1 MICRO PASCAL (KHZ) RMS

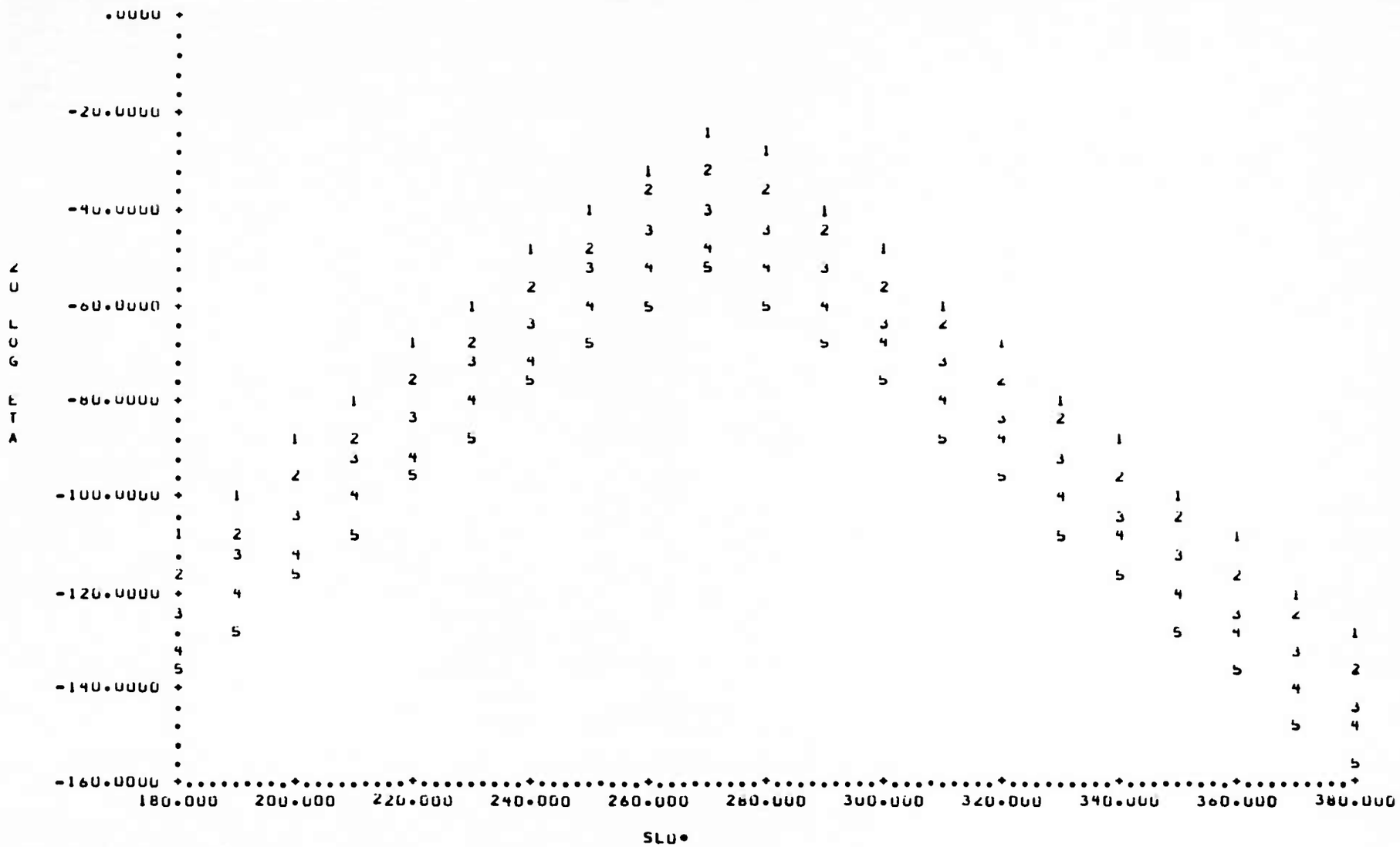
SL0*	SL-(1)*	SL-(2)*	SL-(3)*	SL-(4)*	SL-(5)*
.180+03	.732+02	.665+02	.598+02	.530+02	.460+02
.190+03	.932+02	.865+02	.796+02	.730+02	.660+02
.200+03	.113+03	.107+03	.996+02	.930+02	.860+02
.210+03	.133+03	.127+03	.120+03	.113+03	.106+03
.220+03	.153+03	.147+03	.140+03	.133+03	.126+03
.230+03	.173+03	.167+03	.160+03	.153+03	.146+03
.240+03	.193+03	.187+03	.180+03	.173+03	.166+03
.250+03	.213+03	.206+03	.200+03	.193+03	.186+03
.260+03	.232+03	.225+03	.219+03	.212+03	.205+03
.270+03	.245+03	.238+03	.232+03	.225+03	.218+03
.280+03	.249+03	.242+03	.235+03	.228+03	.222+03
.290+03	.250+03	.243+03	.236+03	.229+03	.222+03
.300+03	.250+03	.243+03	.237+03	.230+03	.223+03
.310+03	.250+03	.243+03	.237+03	.230+03	.223+03
.320+03	.250+03	.243+03	.237+03	.230+03	.223+03
.330+03	.250+03	.243+03	.237+03	.230+03	.223+03
.340+03	.250+03	.243+03	.237+03	.230+03	.223+03
.350+03	.250+03	.243+03	.237+03	.230+03	.223+03
.360+03	.250+03	.243+03	.237+03	.230+03	.223+03
.370+03	.250+03	.243+03	.237+03	.230+03	.223+03
.380+03	.250+03	.243+03	.237+03	.230+03	.223+03



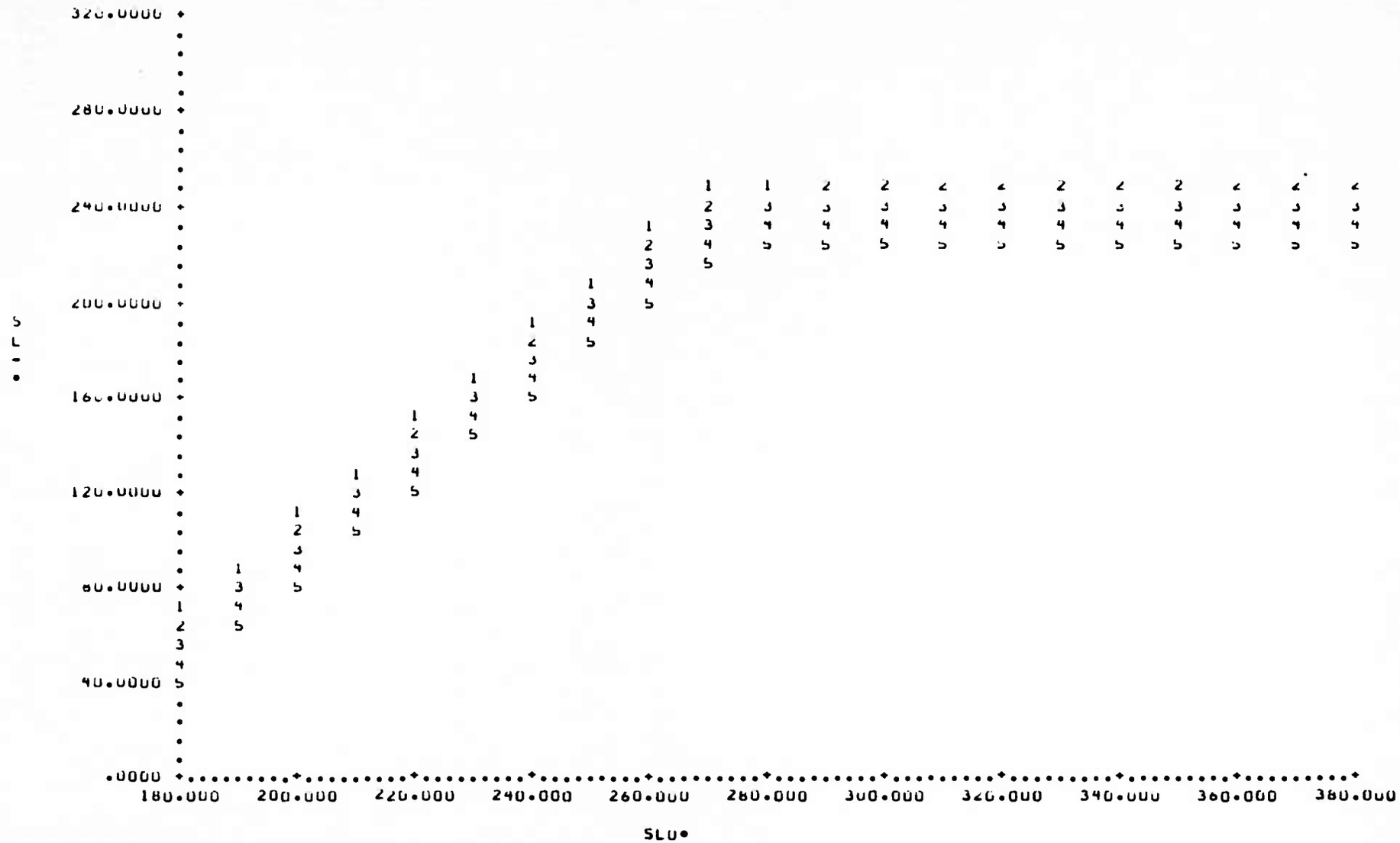
ALPHA(0)*KO= .577-05 NEPERS
PRESSURE UNITS DB/1 MICRO PASCAL (KHZ) RMS

SL0*	SL OUT
.180+03	.180+03
.190+03	.190+03
.200+03	.200+03
.210+03	.210+03
.220+03	.220+03
.230+03	.230+03
.240+03	.240+03
.250+03	.250+03
.260+03	.259+03
.270+03	.263+03
.280+03	.264+03
.290+03	.265+03
.300+03	.265+03
.310+03	.265+03
.320+03	.265+03
.330+03	.265+03
.340+03	.265+03
.350+03	.265+03
.360+03	.265+03
.370+03	.265+03
.380+03	.265+03

```
   IOP          IOC          IOS          IOD          IOO
    0            1            1            0            1
  ALTRO        PIPZ
 .100-02      .100+01
NFM=  5     NFMRD=  0


FO/F- =    .500+01     EN=    .113+01
FO/F- =    .100+02     EN=    .114+01
FO/F- =    .200+02     EN=    .114+01
FO/F- =    .400+02     EN=    .115+01
FO/F- =    .800+02     EN=    .116+01
```

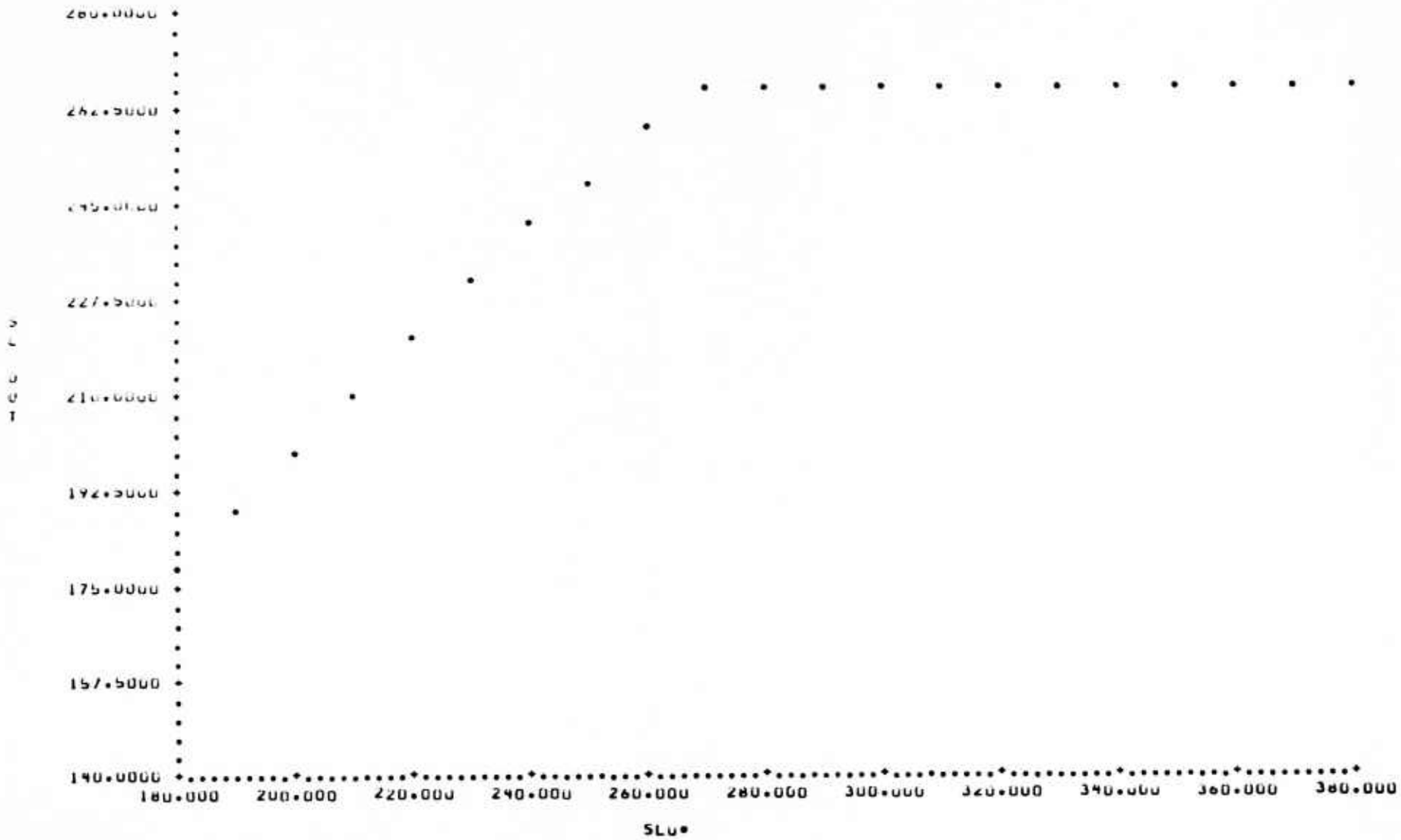


ALPHA(T)*RO= .100-02 DB
CURVE 1 IS FOR FO/F-= .500+01
CURVE 2 IS FOR FO/F-= .100+02
CURVE 3 IS FOR FO/F-= .200+02
CURVE 4 IS FOR FO/F-= .400+02
CURVE 5 IS FOR FO/F-= .800+02
PRESSURE UNITS DB/1 MICRO PASCAL (KHZ) RMS



ALPHA(T)*RO= .100-02 DB
CURVE 1 IS FOR FO/F-= .500+01
CURVE 2 IS FOR FO/F-= .100+02
CURVE 3 IS FOR FO/F-= .200+02
CURVE 4 IS FOR FO/F-= .400+02
CURVE 5 IS FOR FO/F-= .800+02
PRESSURE UNITS DB/1 MICRO PASCAL (KHZ) RMS

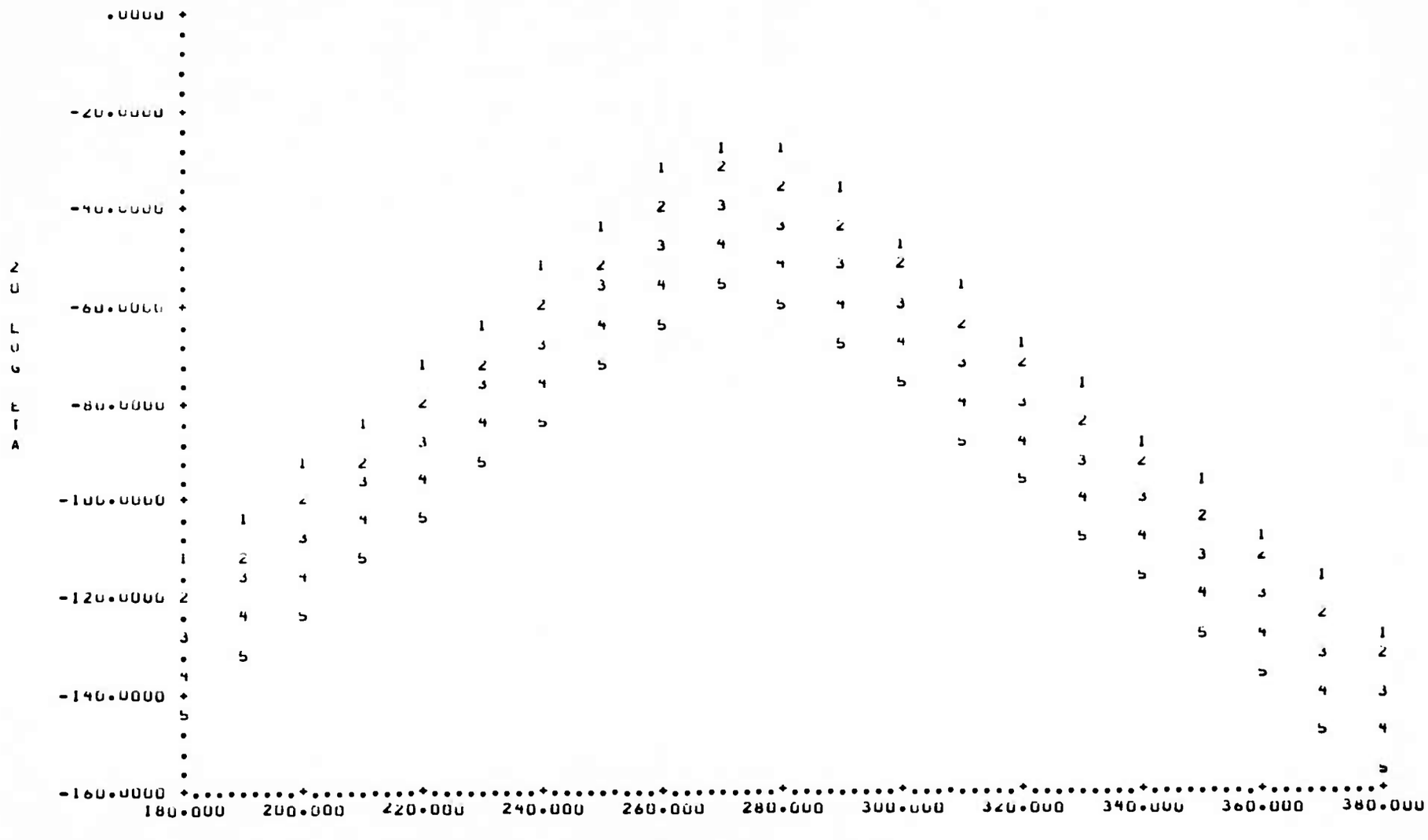
SL0*	SL-(1)*	SL-(2)*	SL-(3)*	SL-(4)*	SL-(5)*
.180+03	.707+02	.638+02	.568+02	.496+02	.423+02
.190+03	.907+02	.838+02	.768+02	.696+02	.623+02
.200+03	.111+03	.104+03	.968+02	.896+02	.823+02
.210+03	.131+03	.124+03	.117+03	.110+03	.102+03
.220+03	.151+03	.144+03	.137+03	.130+03	.122+03
.230+03	.171+03	.164+03	.157+03	.150+03	.142+03
.240+03	.191+03	.184+03	.177+03	.170+03	.162+03
.250+03	.211+03	.204+03	.197+03	.190+03	.182+03
.260+03	.230+03	.223+03	.216+03	.209+03	.202+03
.270+03	.245+03	.236+03	.231+03	.224+03	.216+03
.280+03	.250+03	.243+03	.236+03	.229+03	.222+03
.290+03	.251+03	.244+03	.237+03	.230+03	.223+03
.300+03	.252+03	.245+03	.238+03	.230+03	.223+03
.310+03	.252+03	.245+03	.238+03	.230+03	.223+03
.320+03	.252+03	.245+03	.238+03	.230+03	.223+03
.330+03	.252+03	.245+03	.238+03	.230+03	.223+03
.340+03	.252+03	.245+03	.238+03	.230+03	.223+03
.350+03	.252+03	.245+03	.238+03	.230+03	.223+03
.360+03	.252+03	.245+03	.238+03	.230+03	.223+03
.370+03	.252+03	.245+03	.238+03	.230+03	.223+03
.380+03	.252+03	.245+03	.238+03	.230+03	.223+03



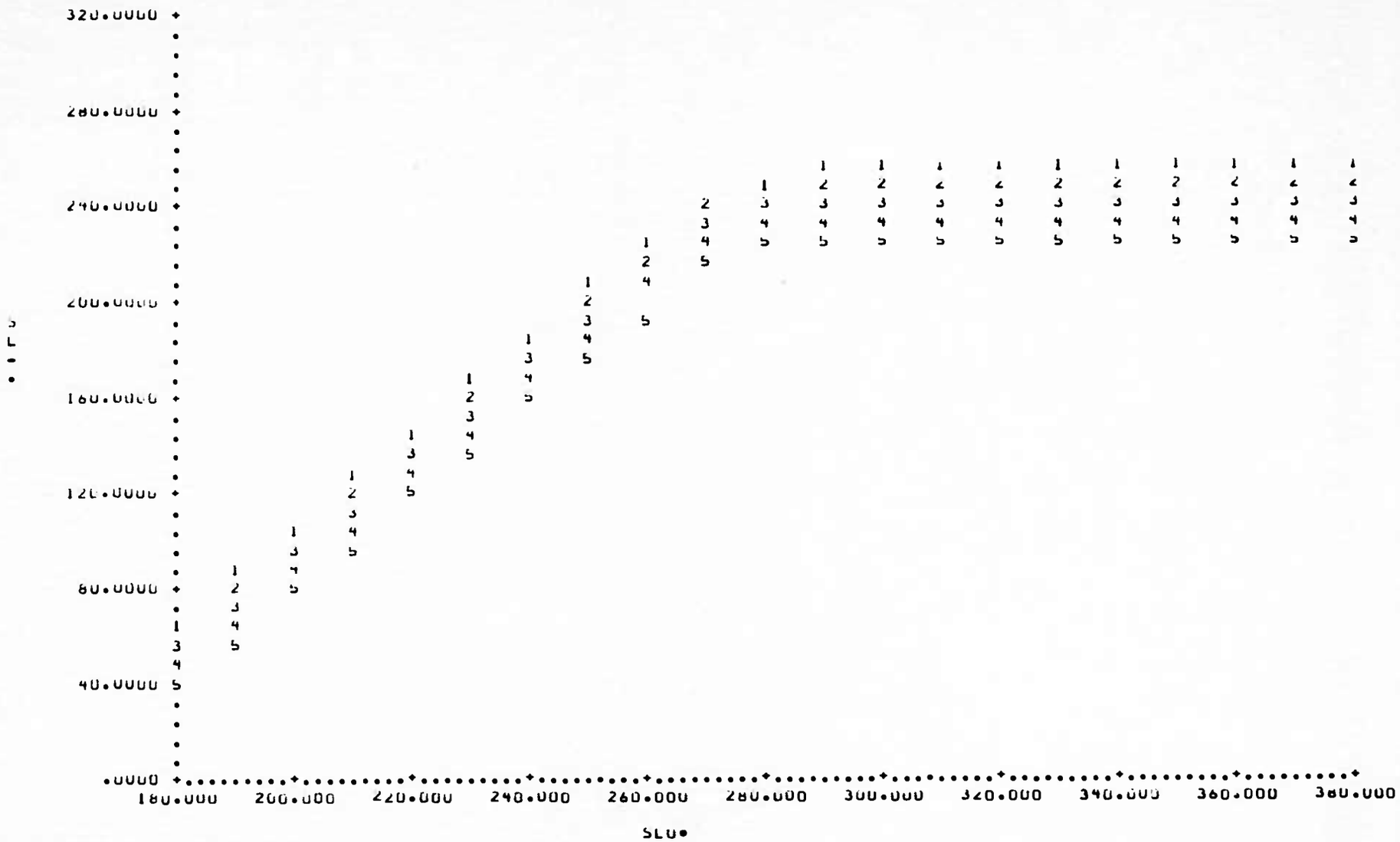
ALPHA(U)*KO= .577-04 NEPERS
PRESSURE UNITS DB/1 MICRO PASCAL (KHZ) RMS

SL0*	SL 001
.180+03	.180+03
.190+03	.190+03
.200+03	.200+03
.210+03	.210+03
.220+03	.220+03
.230+03	.230+03
.240+03	.240+03
.250+03	.250+03
.260+03	.259+03
.270+03	.265+03
.280+03	.266+03
.290+03	.267+03
.300+03	.267+03
.310+03	.267+03
.320+03	.267+03
.330+03	.267+03
.340+03	.267+03
.350+03	.267+03
.360+03	.267+03
.370+03	.267+03
.380+03	.267+03

```
   IOP          IOE          IOS          IOO          IOO
    0            1            1            0            1
  ALTRO        PIP2
 .100-01      .100+01
NFM=  5     NFMRD=  0


FO/F- =    .500+01     EN=    .118+01
FO/F- =    .100+02     EN=    .120+01
FO/F- =    .200+02     EN=    .121+01
FO/F- =    .400+02     EN=    .123+01
FO/F- =    .800+02     EN=    .125+01
```

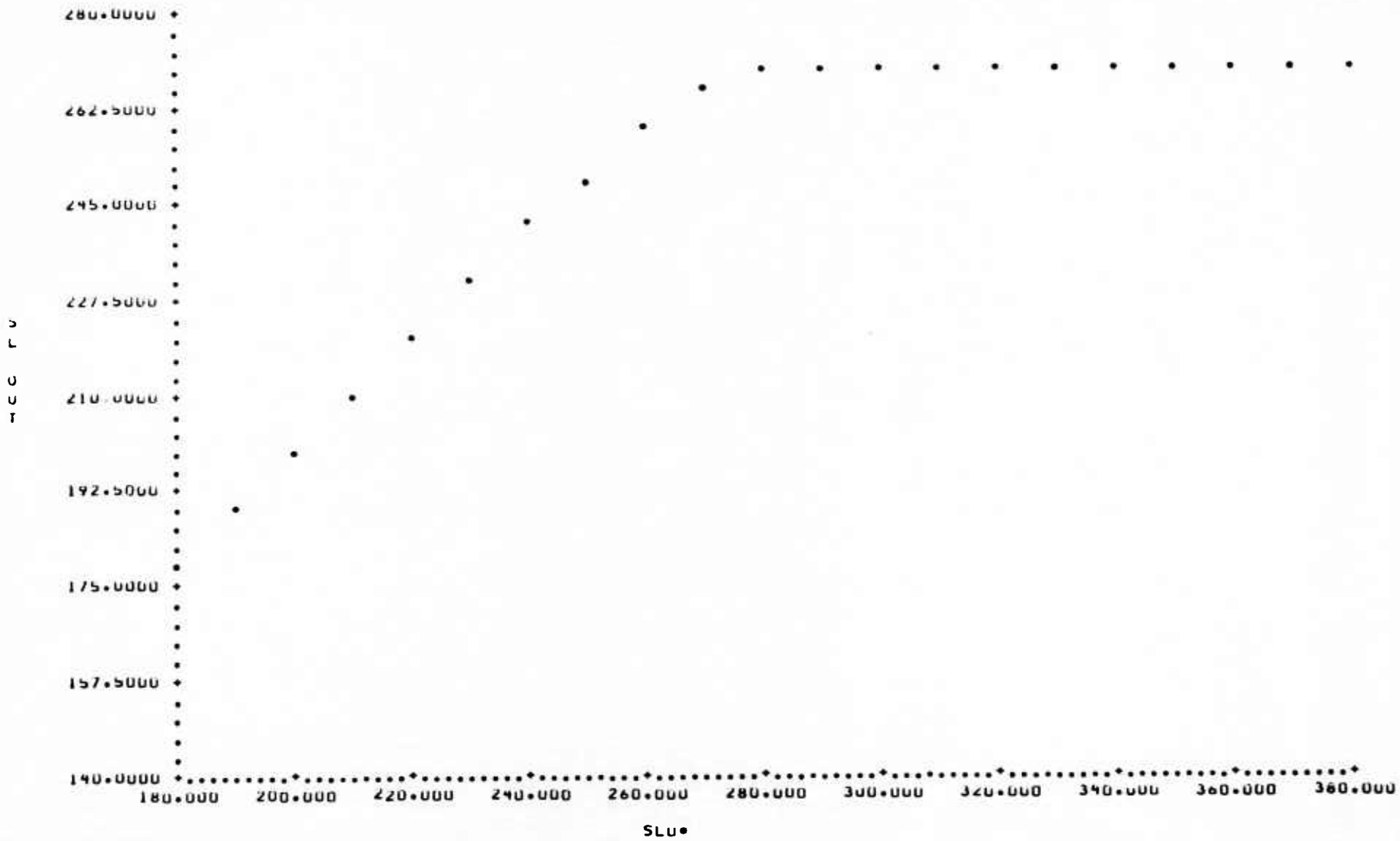


ALPHA(T)*KO= .100-01 DB
CURVE 1 IS FOR FO/F-= .500+01
CURVE 2 IS FOR FO/F-= .100+02
CURVE 3 IS FOR FO/F-= .200+02
CURVE 4 IS FOR FO/F-= .400+02
CURVE 5 IS FOR FO/F-= .800+02
PRESSURE UNITS DB/1 MICRO PASCAL (KHZ) RMS



ALPHA(T)•RO= .100-01 DB
CURVE 1 IS FOR FO/F-= .500+01
CURVE 2 IS FOR FO/F-= .100+02
CURVE 3 IS FOR FO/F-= .200+02
CURVE 4 IS FOR FO/F-= .400+02
CURVE 5 IS FOR FO/F-= .800+02
PRESSURE UNITS DB/1 MICRO PA: AL (KHZ) RMS

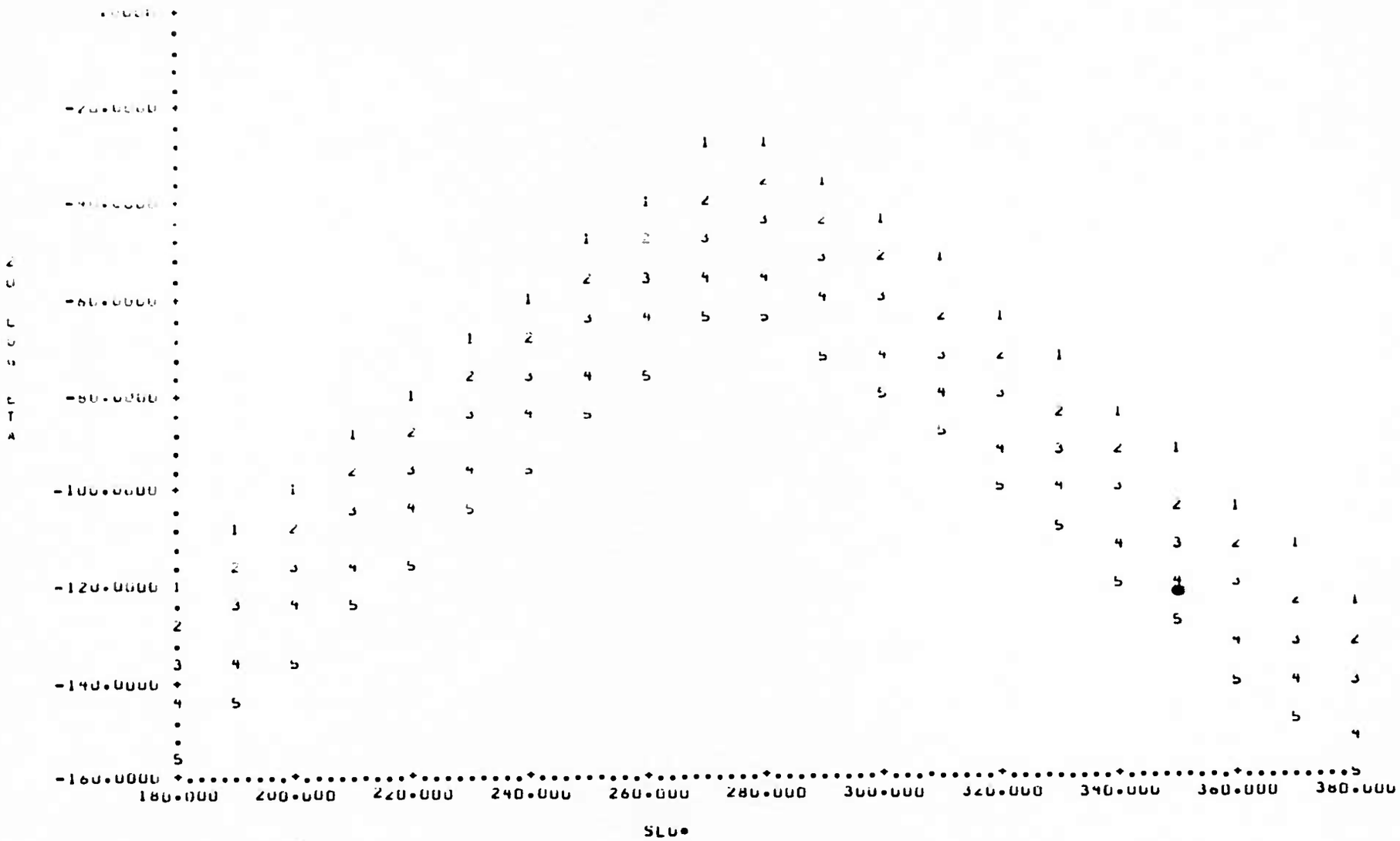
SLO•	SL-(1)•	SL-(2)•	SL-(3)•	SL-(4)•	SL-(5)•
.180+03	.673+02	.599+02	.523+02	.444+02	.363+02
.190+03	.873+02	.799+02	.723+02	.644+02	.563+02
.200+03	.107+03	.999+02	.923+02	.844+02	.763+02
.210+03	.127+03	.120+03	.112+03	.104+03	.963+02
.220+03	.147+03	.140+03	.132+03	.124+03	.116+03
.230+03	.167+03	.160+03	.152+03	.144+03	.136+03
.240+03	.187+03	.180+03	.172+03	.164+03	.156+03
.250+03	.207+03	.200+03	.192+03	.184+03	.176+03
.260+03	.227+03	.219+03	.212+03	.204+03	.196+03
.270+03	.244+03	.236+03	.229+03	.221+03	.213+03
.280+03	.251+03	.244+03	.236+03	.229+03	.220+03
.290+03	.253+03	.246+03	.238+03	.230+03	.222+03
.300+03	.254+03	.246+03	.239+03	.231+03	.223+03
.310+03	.254+03	.246+03	.239+03	.231+03	.223+03
.320+03	.254+03	.246+03	.239+03	.231+03	.223+03
.330+03	.254+03	.246+03	.239+03	.231+03	.223+03
.340+03	.254+03	.246+03	.239+03	.231+03	.223+03
.350+03	.254+03	.246+03	.239+03	.231+03	.223+03
.360+03	.254+03	.246+03	.239+03	.231+03	.223+03
.370+03	.254+03	.246+03	.239+03	.231+03	.223+03
.380+03	.254+03	.246+03	.239+03	.231+03	.223+03



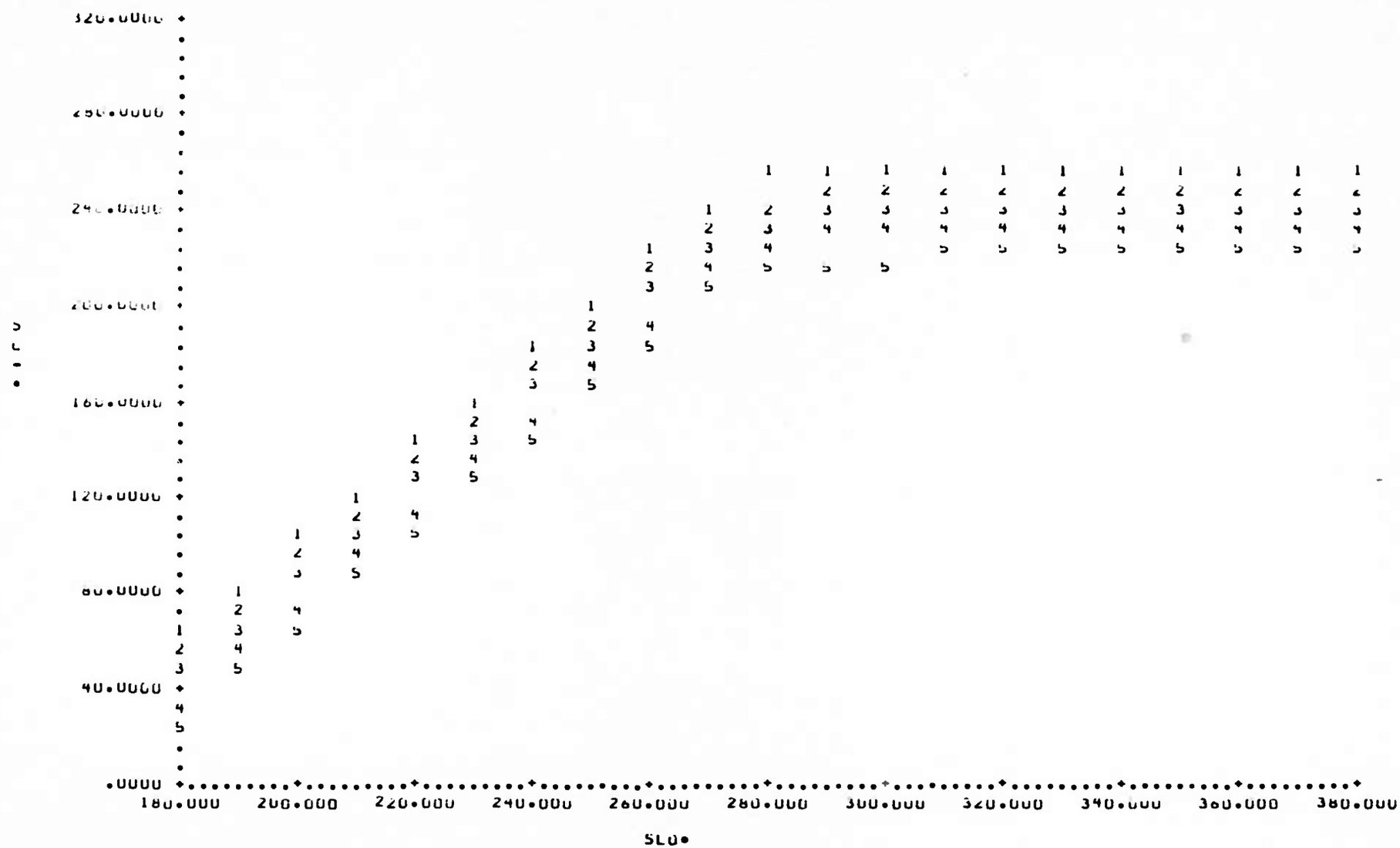
ALPHA(U)*RO= .577-03 NEPERS
PRESSURE UNITS DB/1 MICRO PASCAL (KHZ) RMS

SL0*	SL 00T
.180+03	.180+03
.190+03	.190+03
.200+03	.200+03
.210+03	.210+03
.220+03	.220+03
.230+03	.230+03
.240+03	.240+03
.250+03	.250+03
.260+03	.259+03
.270+03	.266+03
.280+03	.269+03
.290+03	.269+03
.300+03	.269+03
.310+03	.269+03
.320+03	.269+03
.330+03	.269+03
.340+03	.269+03
.350+03	.269+03
.360+03	.269+03
.370+03	.269+03
.380+03	.269+03

```
   IOP        IOR        IOS        IOD        IOO
    0          1          1          0          1
  ALTRD      PIP2
 .100+00    .100+01
NFM=  5   NFMRD=  0


FO/F= =   .500+01    EN=   .129+01
FO/F= =   .100+02    EN=   .132+01
FO/F= =   .200+02    EN=   .135+01
FO/F= =   .400+02    EN=   .138+01
FO/F= =   .800+02    EN=   .142+01
```

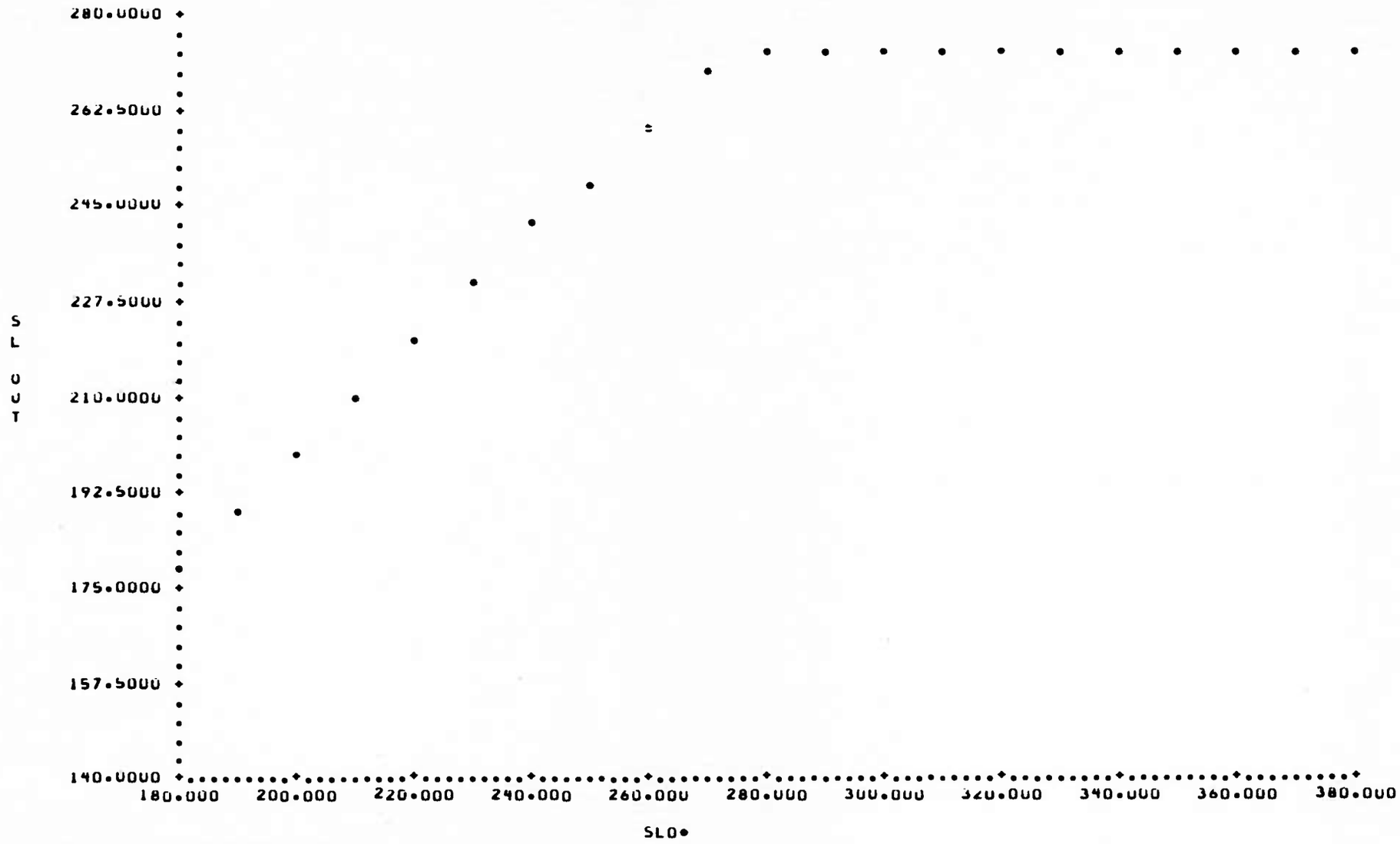


ALPHA(T)*KO= .100+00 DB
CURVE 1 IS FOR FO/F-= .500+01
CURVE 2 IS FOR FO/F-= .100+02
CURVE 3 IS FOR FO/F-= .200+02
CURVE 4 IS FOR FO/F-= .400+02
CURVE 5 IS FOR FO/F-= .800+02
PRESSURE UNITS DB/1 MICRO PASCAL (KHZ) RMS



ALPHA(T)*RO= .100+00 DB
CURVE 1 IS FOR FO/F-= .500+01
CURVE 2 IS FOR FO/F-= .100+02
CURVE 3 IS FOR FO/F-= .200+02
CURVE 4 IS FOR FO/F-= .400+02
CURVE 5 IS FOR FO/F-= .800+02
PRESSURE UNITS DB/1 MICRO PASCAL (KHZ) RMS

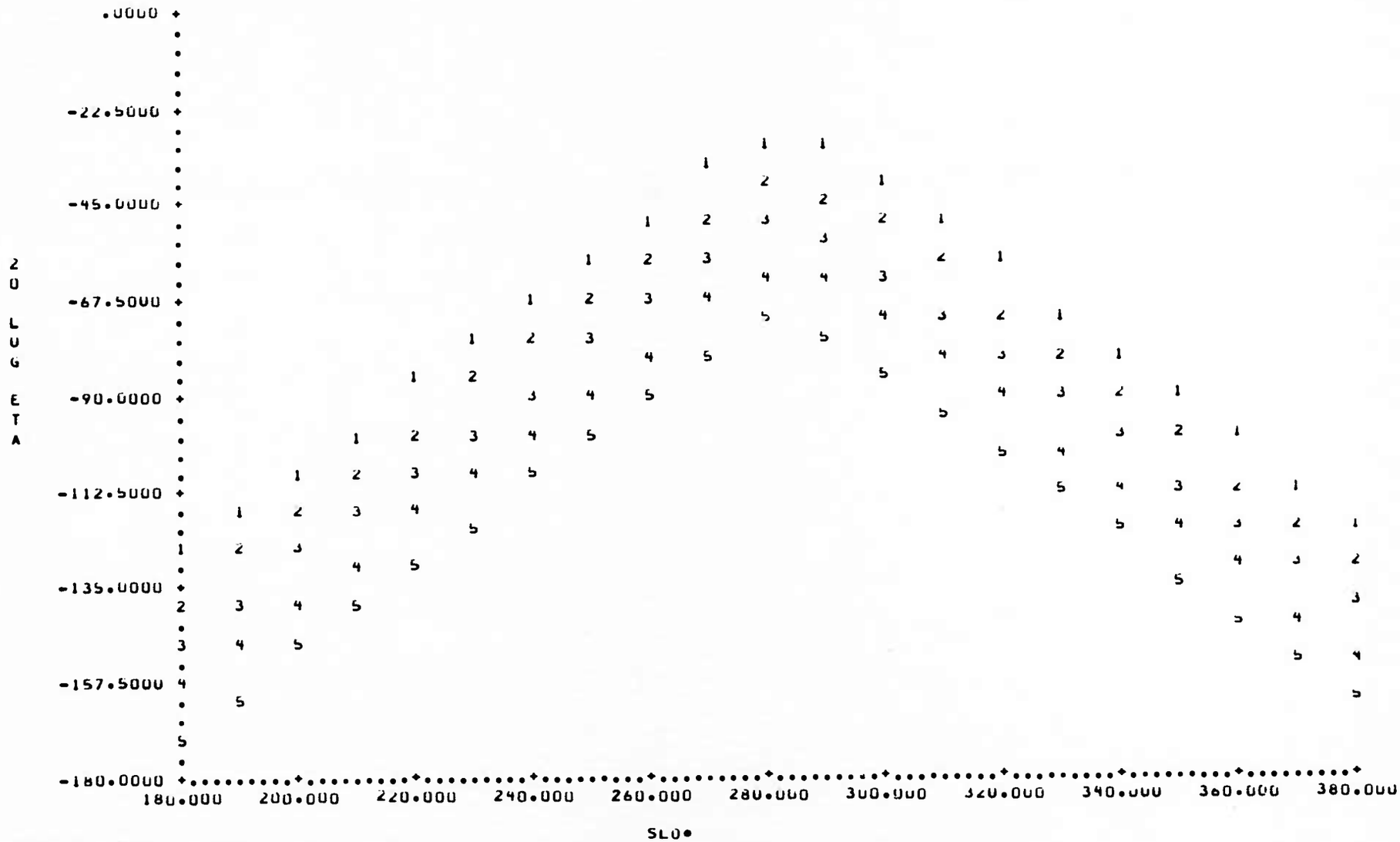
SL0.	SL-(1).	SL-(2).	SL-(3).	SL-(4).	SL-(5).
.180+03	.616+02	.535+02	.446+02	.356+02	.259+02
.190+03	.818+02	.735+02	.646+02	.556+02	.459+02
.200+03	.102+03	.935+02	.846+02	.756+02	.659+02
.210+03	.122+03	.114+03	.105+03	.956+02	.859+02
.220+03	.142+03	.134+03	.125+03	.116+03	.106+03
.230+03	.162+03	.154+03	.145+03	.136+03	.126+03
.240+03	.182+03	.174+03	.165+03	.156+03	.146+03
.250+03	.202+03	.194+03	.185+03	.176+03	.166+03
.260+03	.222+03	.213+03	.205+03	.195+03	.186+03
.270+03	.240+03	.232+03	.223+03	.214+03	.204+03
.280+03	.252+03	.244+03	.235+03	.226+03	.216+03
.290+03	.255+03	.247+03	.238+03	.229+03	.219+03
.300+03	.256+03	.247+03	.239+03	.230+03	.220+03
.310+03	.256+03	.248+03	.239+03	.230+03	.220+03
.320+03	.256+03	.248+03	.239+03	.230+03	.220+03
.330+03	.256+03	.248+03	.239+03	.230+03	.220+03
.340+03	.256+03	.248+03	.239+03	.230+03	.220+03
.350+03	.256+03	.248+03	.239+03	.230+03	.220+03
.360+03	.256+03	.248+03	.239+03	.230+03	.220+03
.370+03	.256+03	.248+03	.239+03	.230+03	.220+03
.380+03	.256+03	.248+03	.239+03	.230+03	.220+03



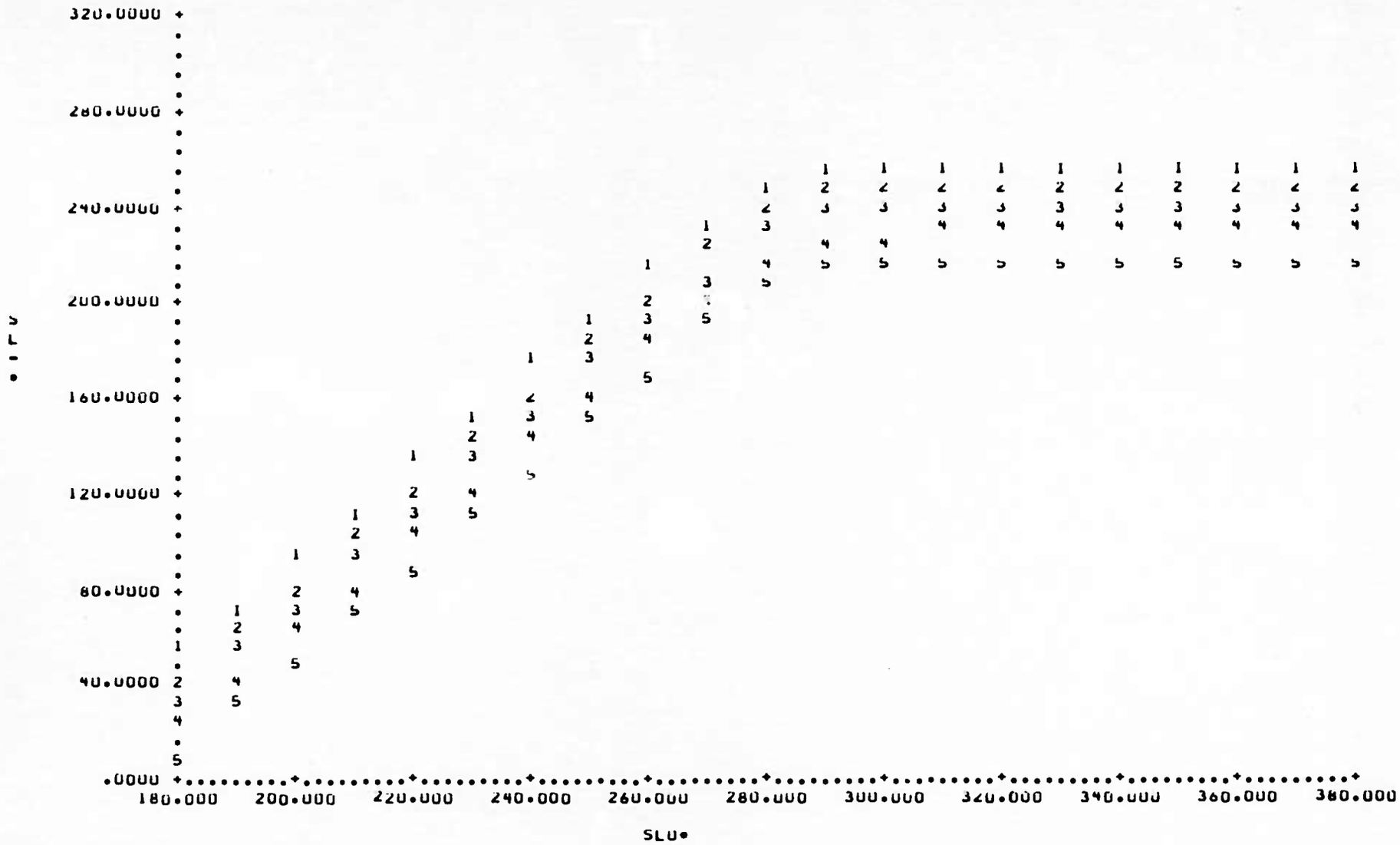
ALPHA(0)*RO= .577-02 NEPERS
PRESSURE UNITS DB/1 MICRO PASCAL (KHZ) RMS

SLO*	SL OUT
.180+03	.180+03
.190+03	.190+03
.200+03	.200+03
.210+03	.210+03
.220+03	.220+03
.230+03	.230+03
.240+03	.240+03
.250+03	.250+03
.260+03	.260+03
.270+03	.268+03
.280+03	.272+03
.290+03	.272+03
.300+03	.273+03
.310+03	.273+03
.320+03	.273+03
.330+03	.273+03
.340+03	.273+03
.350+03	.273+03
.360+03	.273+03
.370+03	.273+03
.380+03	.273+03

```
   IOP        IOE        IOS        IOD        IOO
    0          1          1          0          1
  ALTRO      P1P2
 .100+01    .100+01
NFM=  5   NFMRD=  0


FO/F- =   .500+01    EN=   .150+01
FO/F- =   .100+02    EN=   .155+01
FO/F- =   .200+02    EN=   .159+01
FO/F- =   .400+02    EN=   .164+01
FO/F- =   .800+02    EN=   .168+01
```

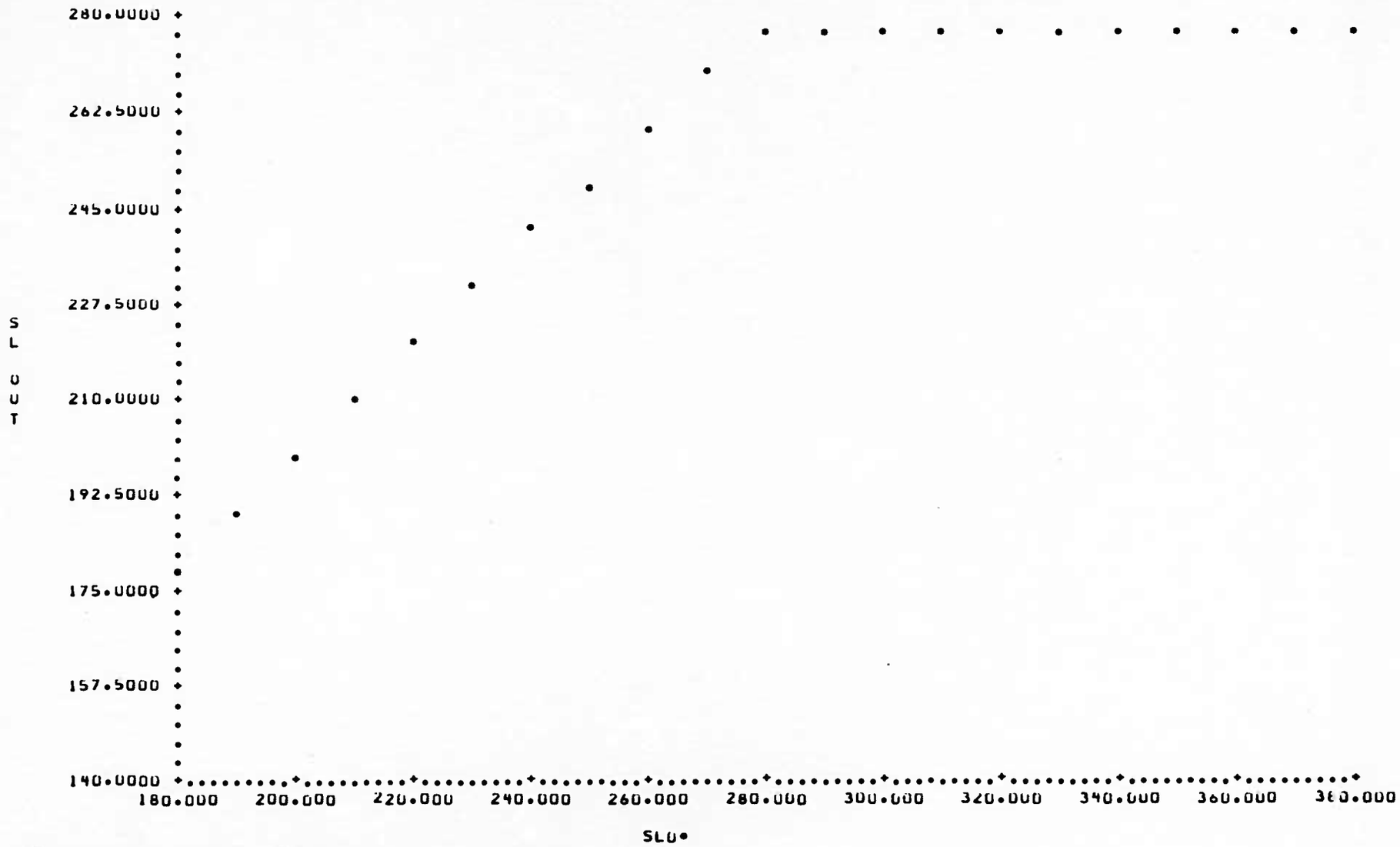


ALPHA(T)*RO= .100+01 DB
CURVE 1 IS FOR FO/F-= .500+01
CURVE 2 IS FOR FO/F-= .100+02
CURVE 3 IS FOR FO/F-= .200+02
CURVE 4 IS FOR FO/F-= .400+02
CURVE 5 IS FOR FO/F-= .800+02
PRESSURE UNITS DB/1 MICRO PASCAL (KHZ) RMS



ALPHA(T)*RU= .100+01 DB
CURVE 1 IS FOR FO/F-= .500+01
CURVE 2 IS FOR FO/F-= .100+02
CURVE 3 IS FOR FO/F-= .200+02
CURVE 4 IS FOR FO/F-= .400+02
CURVE 5 IS FOR FO/F-= .800+02
PRESSURE UNITS DB/1 MICRO PASCAL (KHZ) RMS

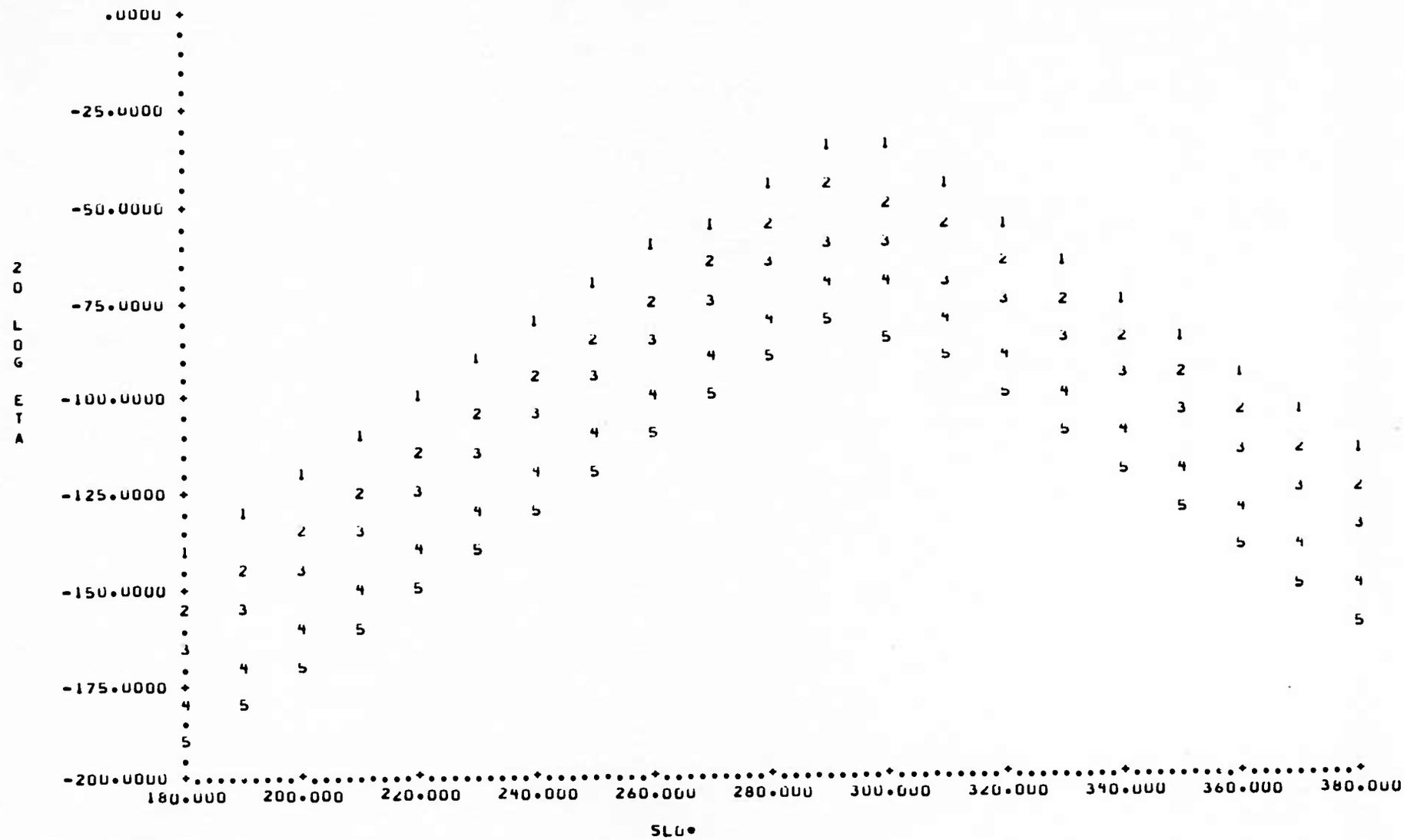
SL0*	SL-(1)*	SL-(2)*	SL-(3)*	SL-(4)*	SL-(5)*
.180+03	.525+02	.426+02	.321+02	.212+02	.977+01
.190+03	.725+02	.626+02	.521+02	.412+02	.296+02
.200+03	.925+02	.826+02	.721+02	.612+02	.496+02
.210+03	.113+03	.103+03	.921+02	.812+02	.696+02
.220+03	.133+03	.123+03	.112+03	.101+03	.896+02
.230+03	.153+03	.143+03	.132+03	.121+03	.110+03
.240+03	.173+03	.163+03	.152+03	.141+03	.130+03
.250+03	.193+03	.183+03	.172+03	.161+03	.150+03
.260+03	.213+03	.203+03	.192+03	.181+03	.170+03
.270+03	.232+03	.222+03	.212+03	.201+03	.189+03
.280+03	.249+03	.239+03	.229+03	.218+03	.206+03
.290+03	.257+03	.247+03	.237+03	.226+03	.214+03
.300+03	.259+03	.249+03	.238+03	.227+03	.216+03
.310+03	.259+03	.250+03	.239+03	.228+03	.217+03
.320+03	.259+03	.250+03	.239+03	.228+03	.217+03
.330+03	.259+03	.250+03	.239+03	.228+03	.217+03
.340+03	.259+03	.250+03	.239+03	.228+03	.217+03
.350+03	.259+03	.250+03	.239+03	.228+03	.217+03
.360+03	.259+03	.250+03	.239+03	.228+03	.217+03
.370+03	.259+03	.250+03	.239+03	.228+03	.217+03
.380+03	.259+03	.250+03	.239+03	.228+03	.217+03



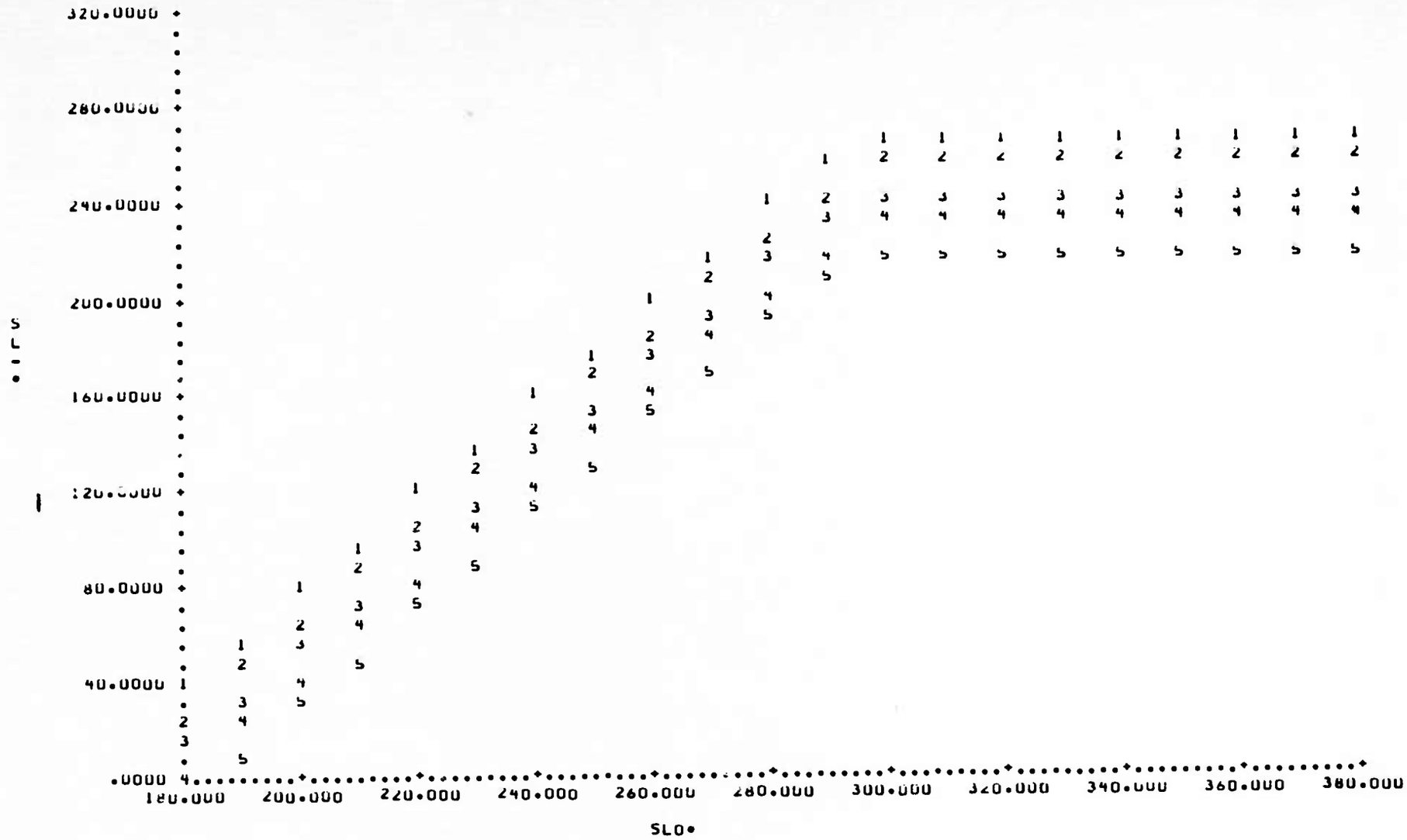
ALPHA(0)•RO= .577-01 NEPERS
PRESSURE UNITS DB/1 MICRO PASCAL (KHZ) RMS

SL0*	SL OUT
.180+03	.180+03
.190+03	.190+03
.200+03	.200+03
.210+03	.210+03
.220+03	.220+03
.230+03	.230+03
.240+03	.240+03
.250+03	.250+03
.260+03	.260+03
.270+03	.269+03
.280+03	.276+03
.290+03	.278+03
.300+03	.278+03
.310+03	.278+03
.320+03	.278+03
.330+03	.278+03
.340+03	.278+03
.350+03	.278+03
.360+03	.278+03
.370+03	.278+03
.380+03	.278+03

```
   IOP        IOE        IOS        IOD        IOO
    0          1          1          0          1
  ALTRO      PIP2
 .100+02    .100+01
NFM=  5   NFMKD=  0


FO/F- =   .500+01    EN=   .179+01
FO/F- =   .100+02    EN=   .183+01
FO/F- =   .200+02    EN=   .186+01
FO/F- =   .400+02    EN=   .188+01
FO/F- =   .800+02    EN=   .189+01
```



ALPHA(T)*RO= .100+02 DB
CURVE 1 IS FOR FO/F-= .500+01
CURVE 2 IS FOR FO/F-= .100+02
CURVE 3 IS FOR FO/F-= .200+02
CURVE 4 IS FOR FO/F-= .400+02
CURVE 5 IS FOR FO/F-= .800+02
PRESSURE UNITS DB/1 MICRO PASCAL (KHZ) RMS

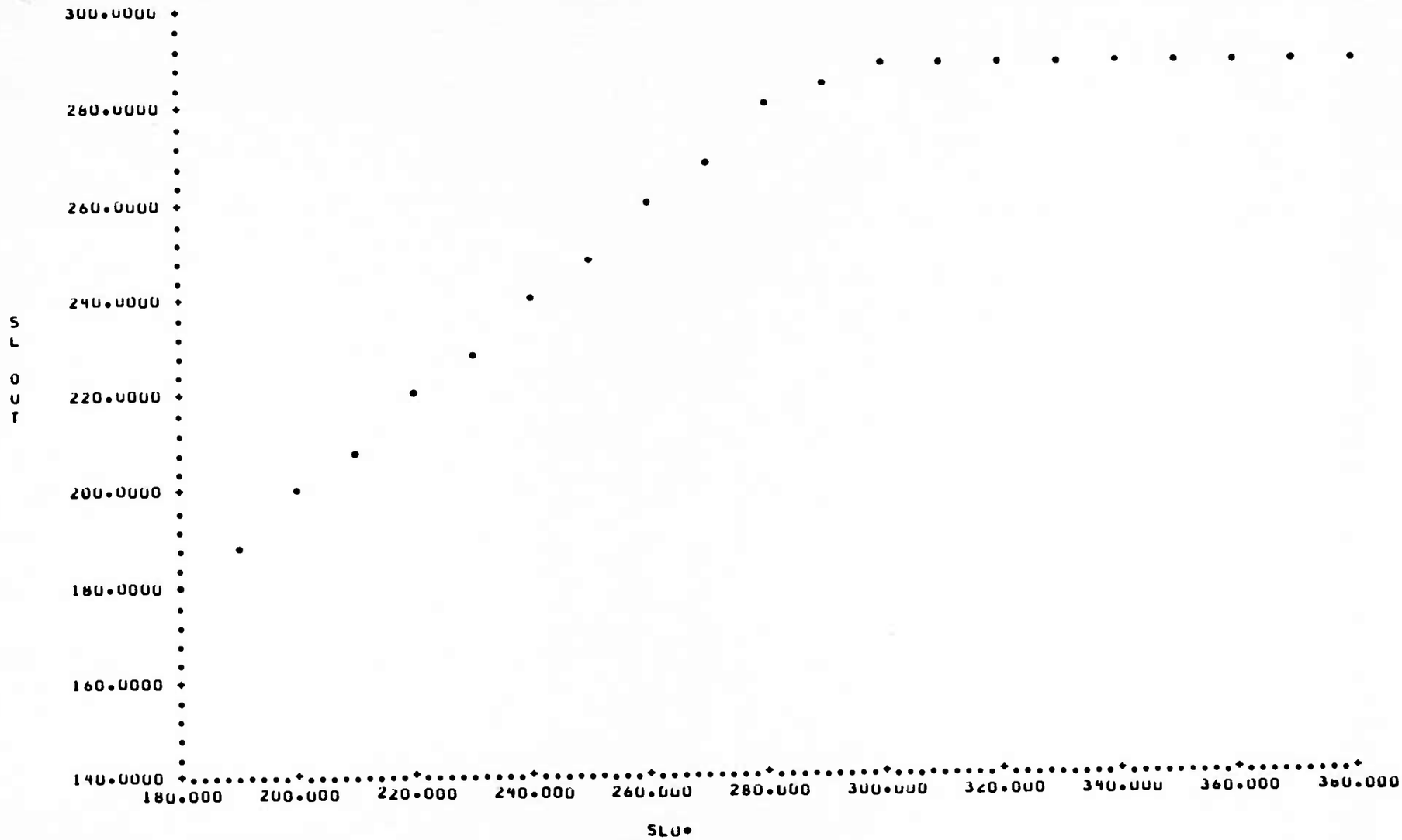


***** THE FOLLOWING 1 POINTS WERE OUT OF RANGE *****

X= 180.000 Y= -9.497

ALPHA(T)*RO= .100+02 DB
CURVE 1 IS FOR FO/F-= .500+01
CURVE 2 IS FOR FO/F-= .100+02
CURVE 3 IS FOR FO/F-= .200+02
CURVE 4 IS FOR FO/F-= .400+02
CURVE 5 IS FOR FO/F-= .800+02
PRESSURE UNITS DB/1 MICRO PASCAL (KHZ) RMS

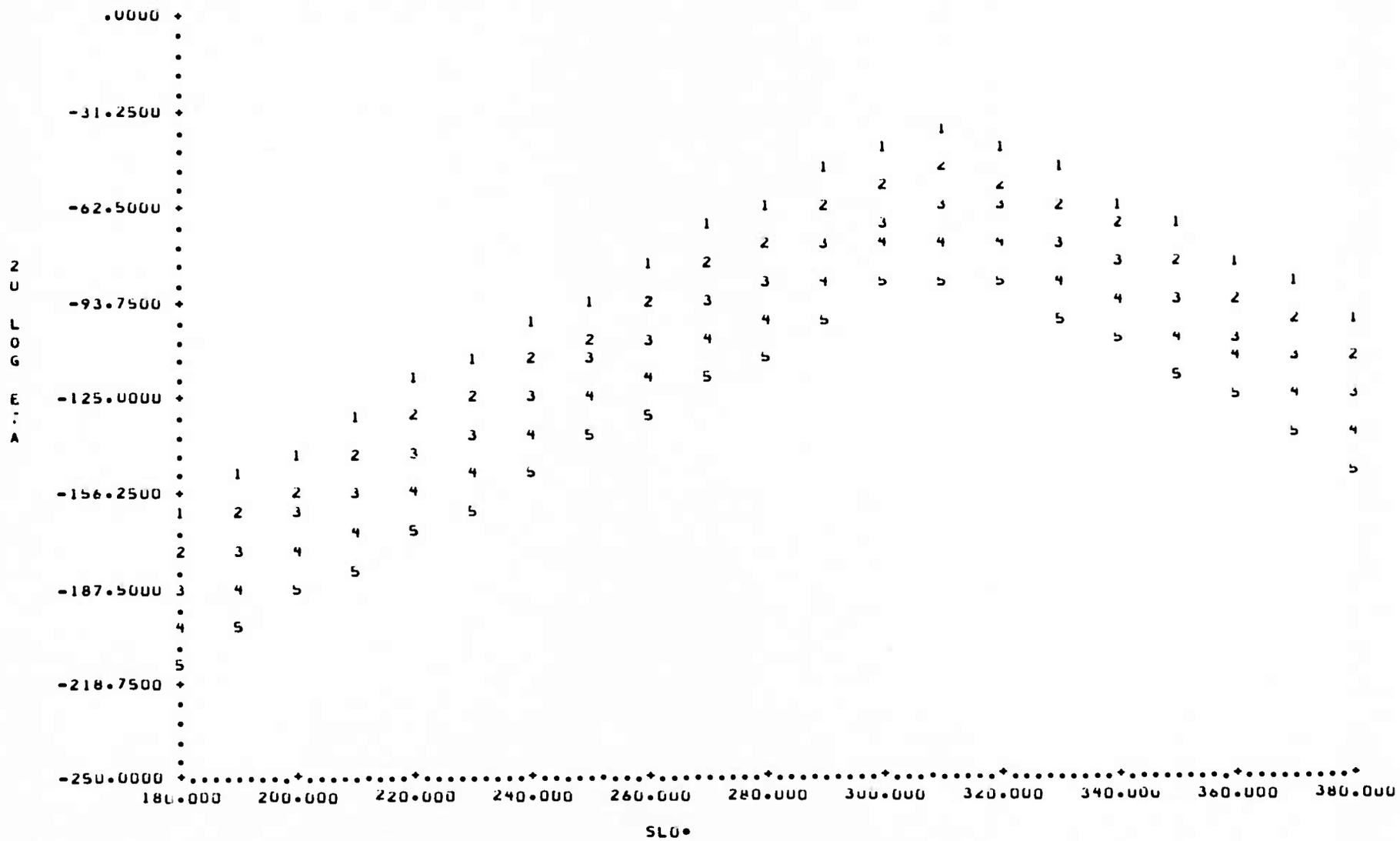
SL0*	SL-(1)*	SL-(2)*	SL-(3)*	SL-(4)*	SL-(5)*
.180+03	.375+02	.260+02	.143+02	.245+01	-.950+01
.190+03	.575+02	.460+02	.343+02	.225+02	.105+02
.200+03	.775+02	.660+02	.543+02	.425+02	.305+02
.210+03	.975+02	.860+02	.743+02	.625+02	.505+02
.220+03	.118+03	.106+03	.943+02	.825+02	.705+02
.230+03	.138+03	.126+03	.114+03	.102+03	.905+02
.240+03	.158+03	.146+03	.134+03	.122+03	.111+03
.250+03	.178+03	.166+03	.154+03	.142+03	.131+03
.260+03	.198+03	.186+03	.174+03	.162+03	.150+03
.270+03	.217+03	.206+03	.194+03	.182+03	.170+03
.280+03	.237+03	.226+03	.214+03	.202+03	.190+03
.290+03	.255+03	.243+03	.232+03	.220+03	.208+03
.300+03	.264+03	.252+03	.240+03	.229+03	.217+03
.310+03	.266+03	.254+03	.242+03	.230+03	.218+03
.320+03	.266+03	.255+03	.243+03	.231+03	.219+03
.330+03	.266+03	.255+03	.243+03	.231+03	.219+03
.340+03	.266+03	.255+03	.243+03	.231+03	.219+03
.350+03	.266+03	.255+03	.243+03	.231+03	.219+03
.360+03	.266+03	.255+03	.243+03	.231+03	.219+03
.370+03	.266+03	.255+03	.243+03	.231+03	.219+03
.380+03	-.266+03	.255+03	.243+03	.231+03	.219+03



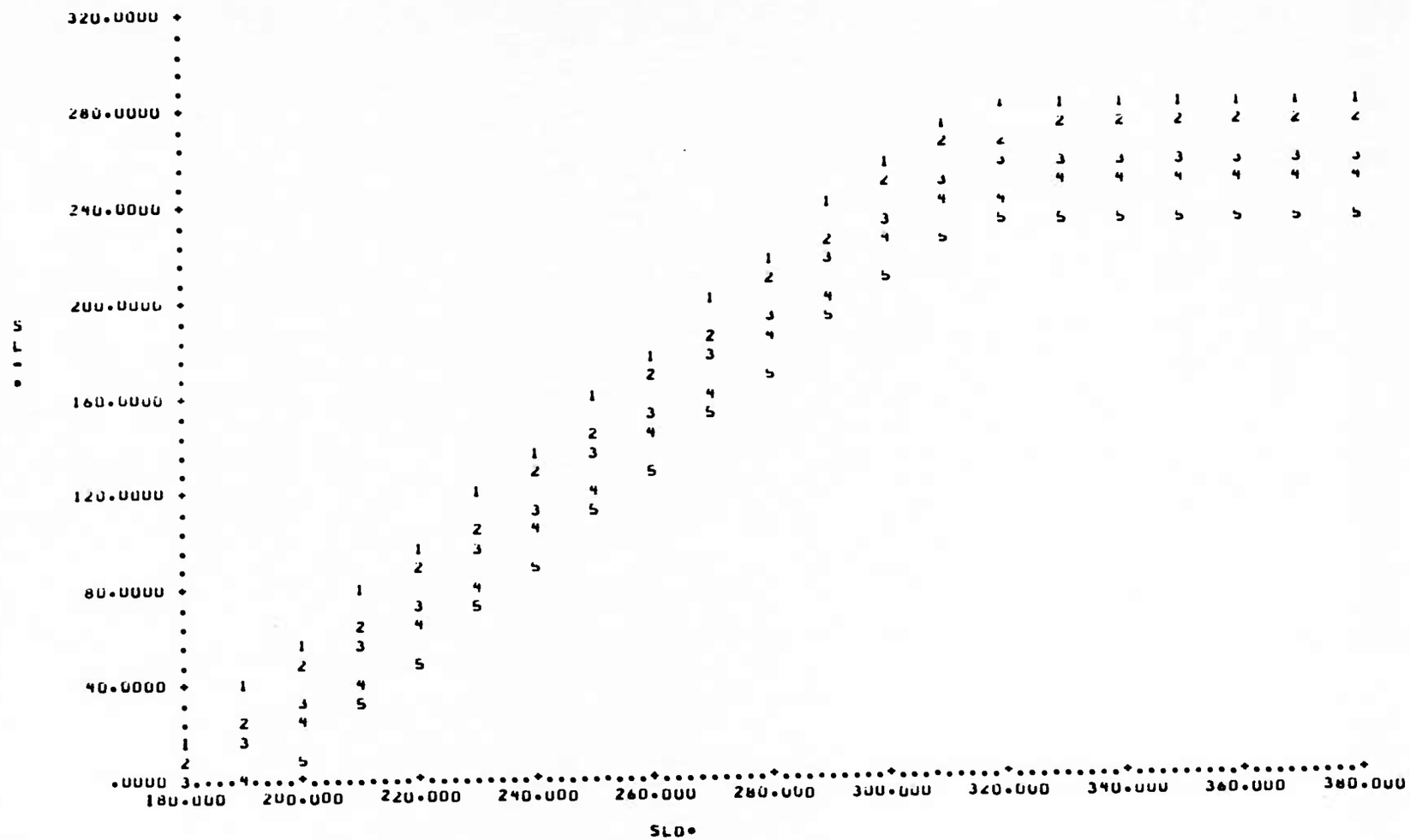
ALPHA(0)*RO= .577+00 NEPERS
PRESSURE UNITS DB/1 MICRO PASCAL (KHZ) RMS

SLU*	SL OUT
.180+03	.180+03
.190+03	.190+03
.200+03	.200+03
.210+03	.210+03
.220+03	.220+03
.230+03	.230+03
.240+03	.240+03
.250+03	.250+03
.260+03	.260+03
.270+03	.270+03
.280+03	.279+03
.290+03	.285+03
.300+03	.287+03
.310+03	.287+03
.320+03	.287+03
.330+03	.287+03
.340+03	.287+03
.350+03	.287+03
.360+03	.287+03
.370+03	.287+03
.380+03	.287+03

```
    IOP          IOE          IOS          IOD          IOO
     0            1            1            0            1
   ALTRU        P1P2
  .100+03      .100+01
NFM=  5     NFMRD=  0


FO/F- =    .500+01     EN=    .196+01
FO/F- =    .100+02     EN=    .197+01
FO/F- =    .200+02     EN=    .198+01
FO/F- =    .400+02     EN=    .198+01
FO/F- =    .600+02     EN=    .198+01
```



ALPHA(T)*RO= .100+03 DB
CURVE 1 IS FOR FO/F-= .500+01
CURVE 2 IS FOR FO/F-= .100+02
CURVE 3 IS FOR FO/F-= .200+02
CURVE 4 IS FOR FO/F-= .400+02
CURVE 5 IS FOR FO/F-= .800+02
PRESSURE UNITS DB/1 MICRO PASCAL (KHZ) RMS

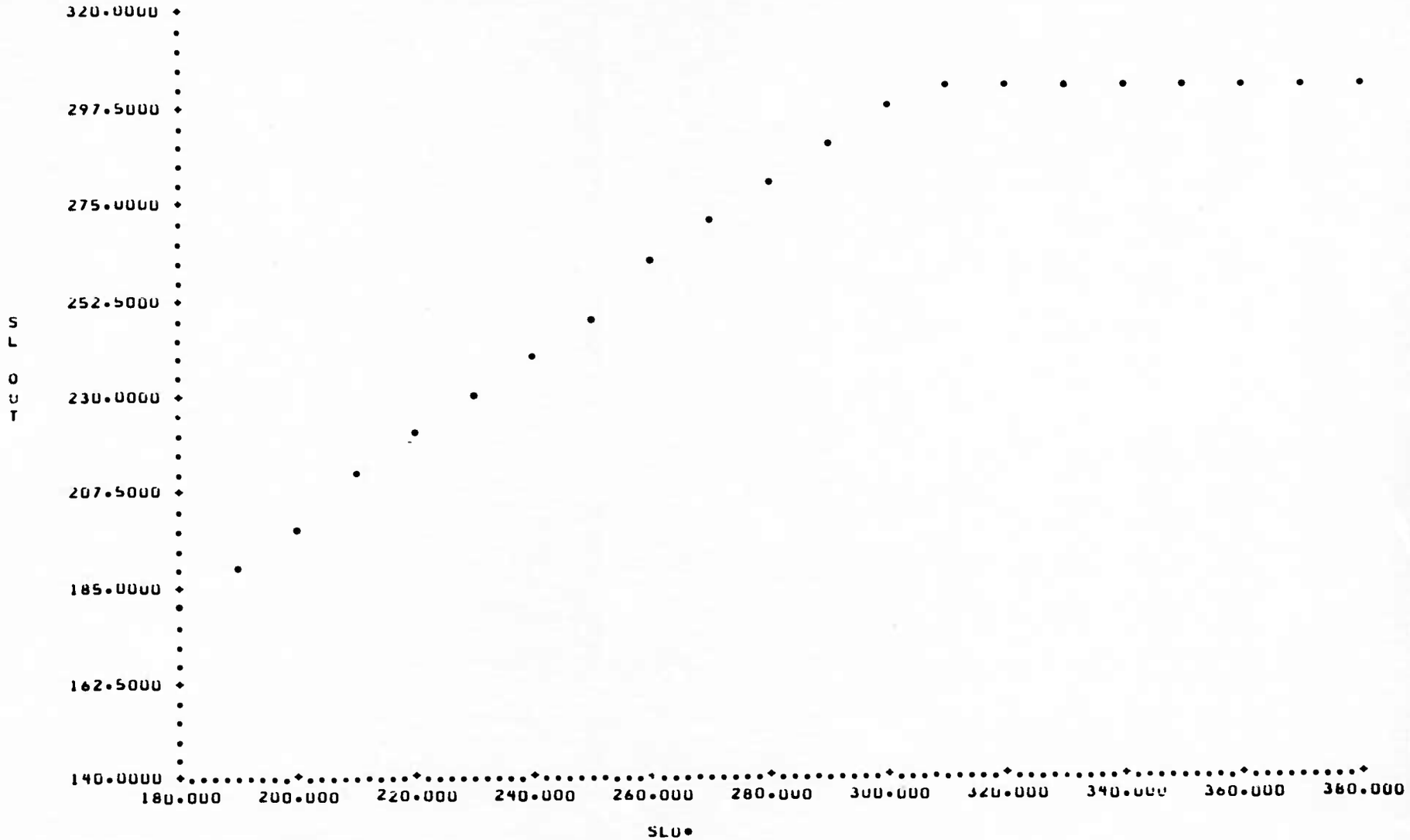


***** THE FOLLOWING 3 POINTS WERE OUT OF RANGE *****

X=	180.000	Y=	-17.362
X=	180.000	Y=	-29.414
X=	190.000	Y=	-9.414

ALPHA(T)*RO= .100+03 DB
CURVE 1 IS FOR FO/F-= .500+01
CURVE 2 IS FOR FO/F-= .100+02
CURVE 3 IS FOR FO/F-= .200+02
CURVE 4 IS FOR FO/F-= .400+02
CURVE 5 IS FOR FO/F-= .800+02
PRESSURE UNITS DB/1 MICRO PASCAL (KHZ) RMS

SL0.	SL-(1).	SL-(2).	SL-(3).	SL-(4).	SL-(5).
.180+03	.186+02	.664+01	-.536+01	-.174+02	-.294+02
.190+03	.386+02	.266+02	.146+02	.262+01	-.941+01
.200+03	.586+02	.466+02	.346+02	.226+02	.106+02
.210+03	.786+02	.666+02	.546+02	.426+02	.306+02
.220+03	.986+02	.866+02	.746+02	.626+02	.506+02
.230+03	.119+03	.107+03	.946+02	.826+02	.706+02
.240+03	.139+03	.127+03	.115+03	.103+03	.906+02
.250+03	.159+03	.147+03	.135+03	.123+03	.111+03
.260+03	.179+03	.167+03	.155+03	.143+03	.131+03
.270+03	.199+03	.187+03	.175+03	.163+03	.151+03
.280+03	.219+03	.207+03	.195+03	.183+03	.171+03
.290+03	.239+03	.227+03	.215+03	.203+03	.191+03
.300+03	.258+03	.246+03	.234+03	.222+03	.210+03
.310+03	.273+03	.261+03	.249+03	.237+03	.225+03
.320+03	.279+03	.267+03	.255+03	.243+03	.231+03
.330+03	.280+03	.268+03	.256+03	.244+03	.232+03
.340+03	.281+03	.269+03	.257+03	.245+03	.232+03
.350+03	.281+03	.269+03	.257+03	.245+03	.232+03
.360+03	.281+03	.269+03	.257+03	.245+03	.232+03
.370+03	.281+03	.269+03	.257+03	.245+03	.232+03
.380+03	.281+03	.269+03	.257+03	.245+03	.232+03



ALPHA(0)*RO= .577+01 NEPERS
PRESSURE UNITS DB/1 MICRO PASCAL (KHZ) RMS

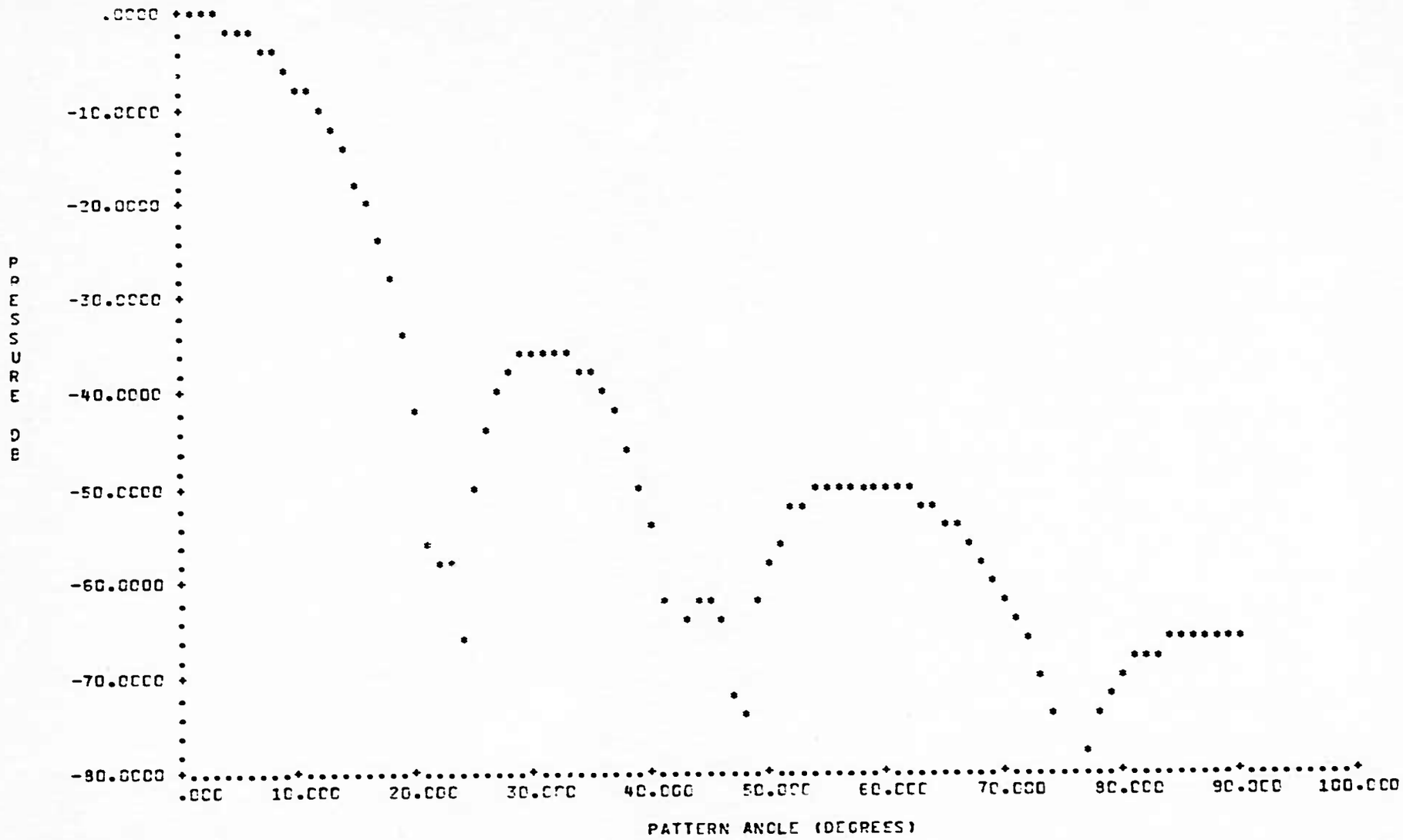
SL0*	SL OUT
.180+03	.180+03
.190+03	.190+03
.200+03	.200+03
.210+03	.210+03
.220+03	.220+03
.230+03	.230+03
.240+03	.240+03
.250+03	.250+03
.260+03	.260+03
.270+03	.270+03
.280+03	.280+03
.290+03	.290+03
.300+03	.298+03
.310+03	.301+03
.320+03	.302+03
.330+03	.302+03
.340+03	.302+03
.350+03	.302+03
.360+03	.302+03
.370+03	.302+03
.380+03	.302+03

$FIN

FIGURE 2

(Pages 87 to 99)

```
@XQT SONIC*NONLN.BEAM
 NOC=  7

    IOP          IOE          IOS          IOD          IOO
     1            C            C            C            C
   ALTRC        P1P2
  .100-04      .100+01
 NFM=  1    NFMRD=  1
   FO/FM=      .100+02
 LIN=  0    IOC=  1
 KO*A=    .100+02
 PATTERN SLS=    .240+03    FO/F-=    .100+02    BB=  -.300+02    NP= 90    TO=    .100+01


 FO/F- =    .100+02    EN=    .103+01
```

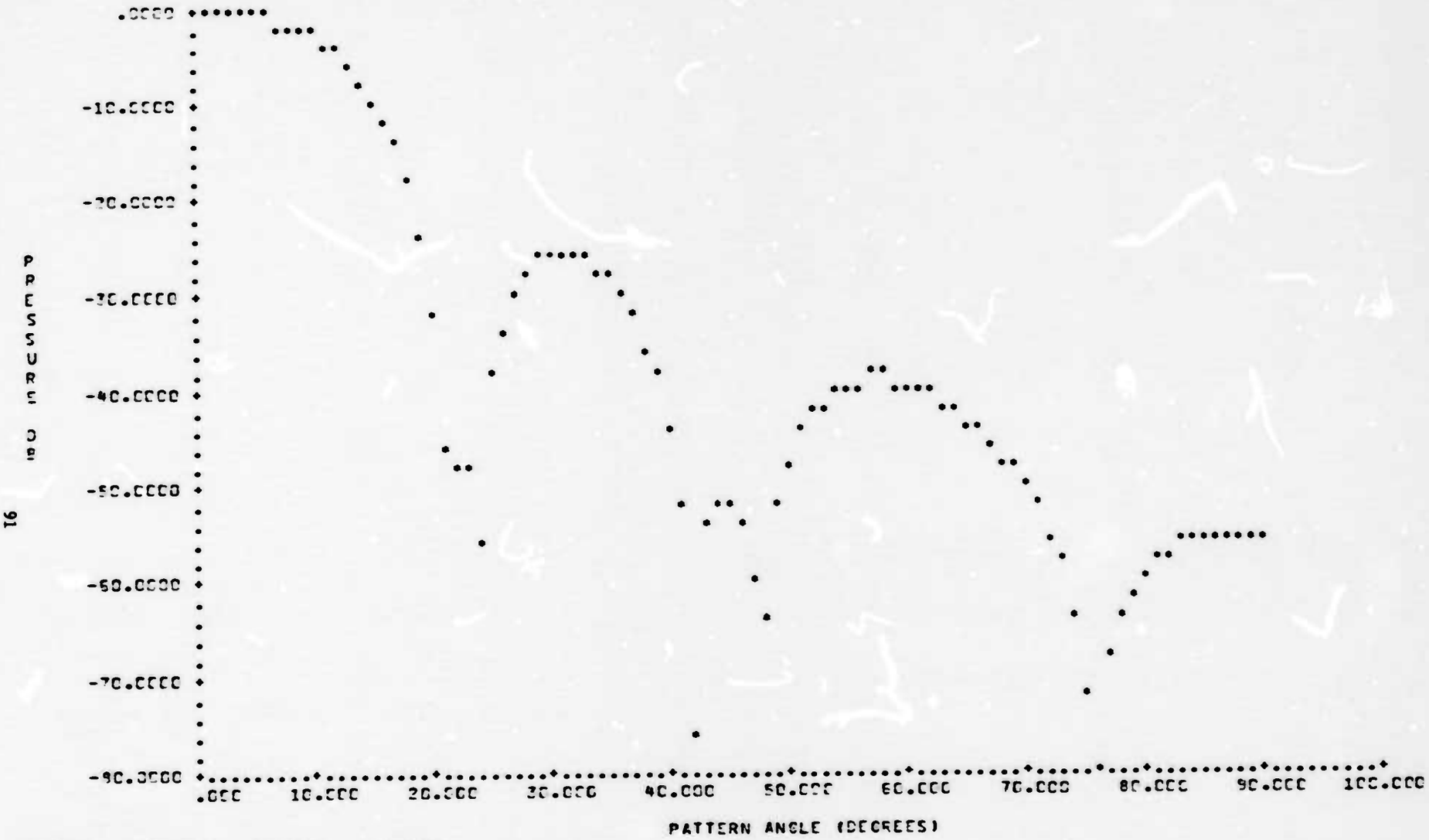


***** THE FOLLOWING 3 POINTS WERE OUT OF RANGE *****

X=	42.000	Y=	-85.437
X=	75.000	Y=	-82.565
X=	76.000	Y=	-89.510

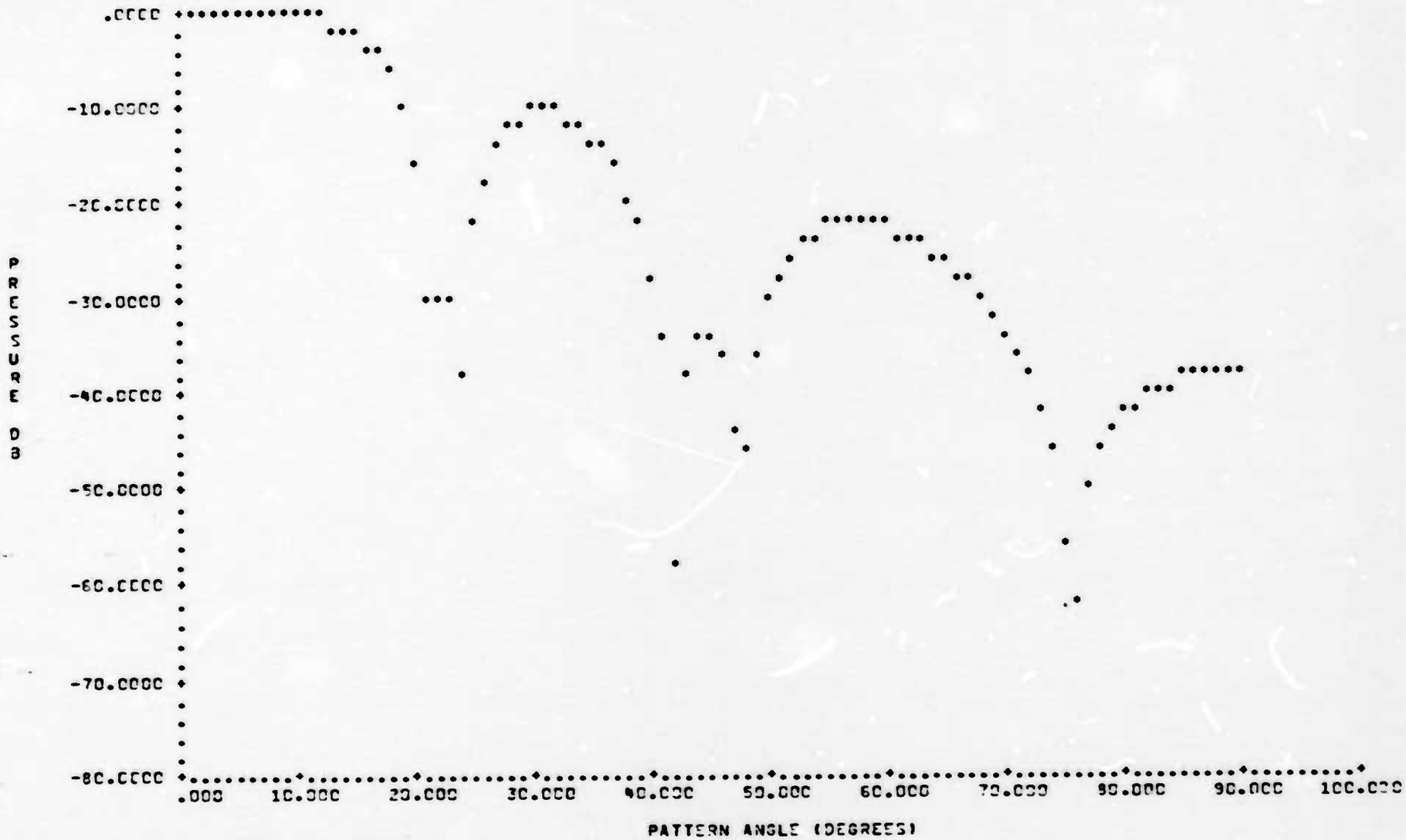
SLO*= .240+03 FO/F-= .100+02
ALPHA(T)*RO= .100-04 DB
PRESSURE UNITS DB/1 MICRO PASCAL (KHZ) RMS

```
    IOP          IOC          IOC          IOC          IOC
     1            0            0            0            0
   ALTRO        P1P2
  .100-04      .100+01
NFM=  1    NFMRO=  1
   FO/FM=      .100+02
LIN=  0    IOC=  1
KO*A=   .100+02
PATTERN SLS=   .270+03    FO/F-=   .100+02    OB=  -.300+02    NP= 90    TO=   .100+01


FO/F- =   .100+02    SN=   .103+01
```



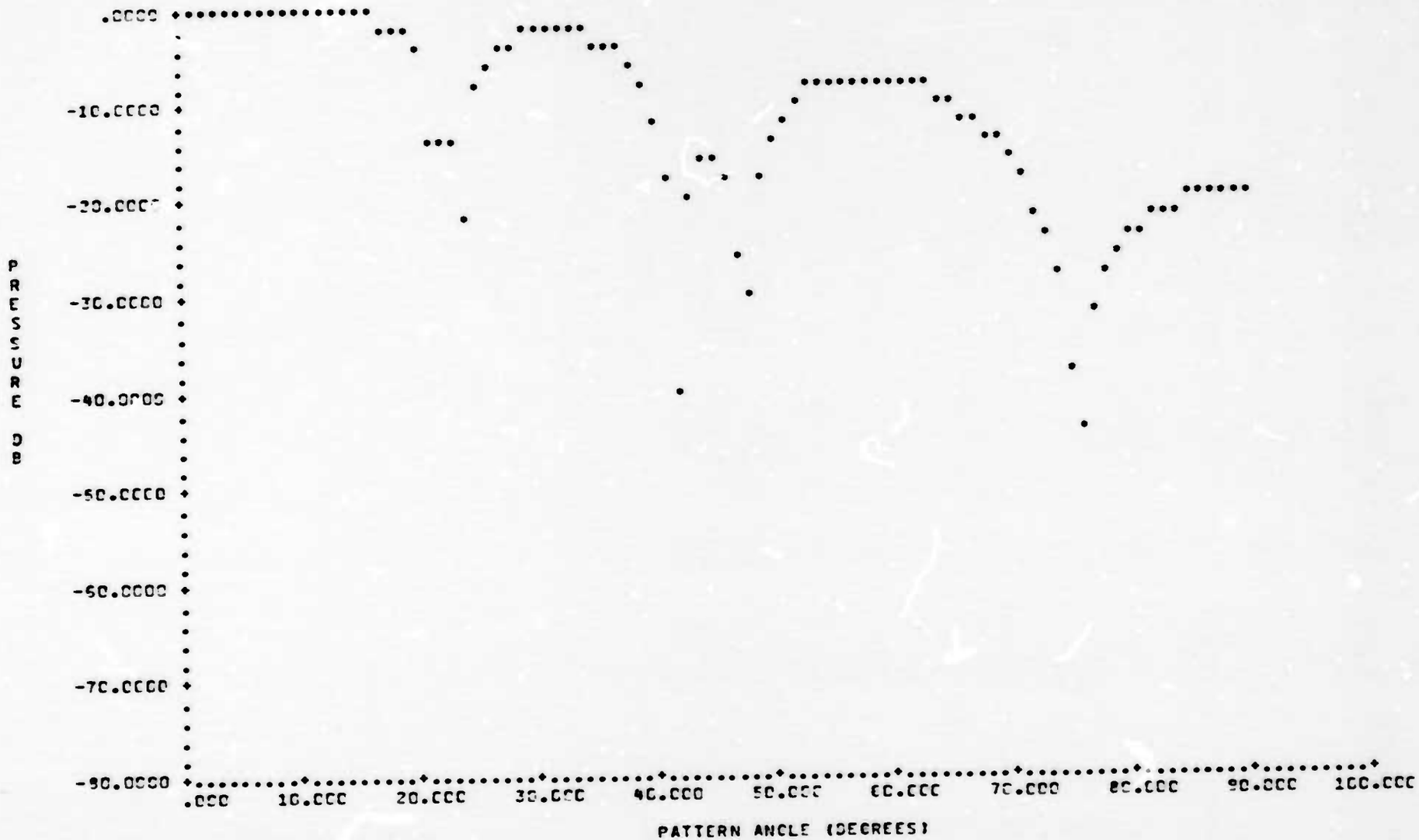
SLO*= .270+03 SO/S-= .100+02
ALPHA(T)*RC= .100-04 DB
PRESSURE UNITS DB/1 MICRO PASCAL (KHZ) RMS

```
    ICP          ICE          ICS          ICD          ICO
     1            0            0            0            0
   ALTRC        PIP2
  .100-04      .100+01
NFM=  1     NFMRD=  1
   FO/FM=      .100+02
LIN=  0     IOC=  1
KO*A=    .100+02
PATTERN SLS=   .280+03    FO/F-=   .100+02    DB=  -.800+02    NP= 90    TO=   .100+01


FO/F- =   .100+02    EN=   .108+01
```



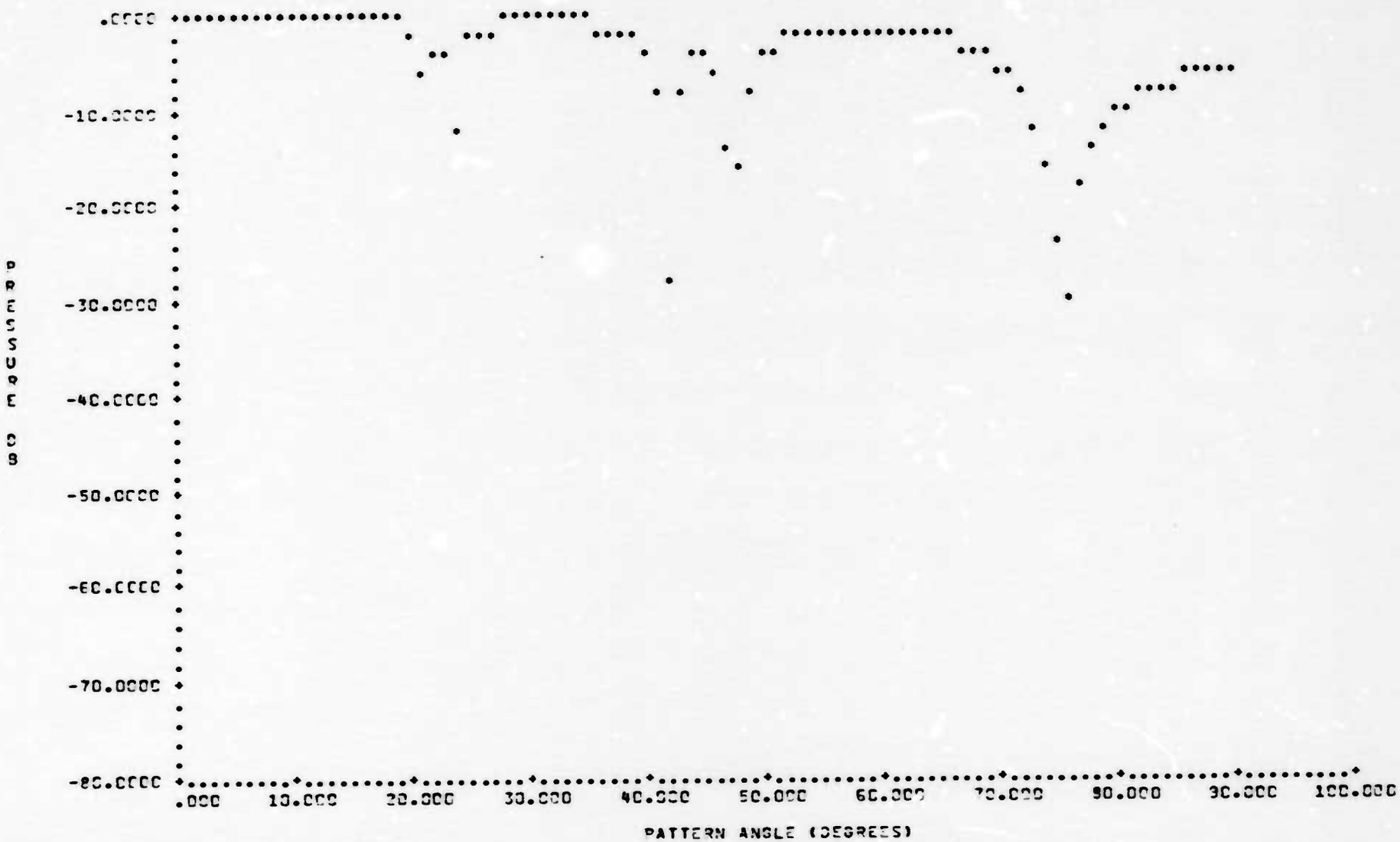
SLC*= .280+03 FC/F-= .100+02
ALPHA(T)*RC= .100-04 DB
PRESSURE UNITS DB/1 MICRO PASCAL (KHZ) RMS

```
   ICP          ICE          IOS          IOD          IOC
    1            0            0            0            0
  ALTRO        P1P2
 .100-04      .100+01
NFM=  1    NFMRO=  1
  FO/FM=      .100+02
LIN=  0    IOC=  1
KO*A=    .100+02
PATTERN SLS=   .290+03    FO/F-=   .100+02    DB=  -.300+02    NP= 90    TO=   .100+01


FO/F- =   .100+02    SN=   .103+01
```



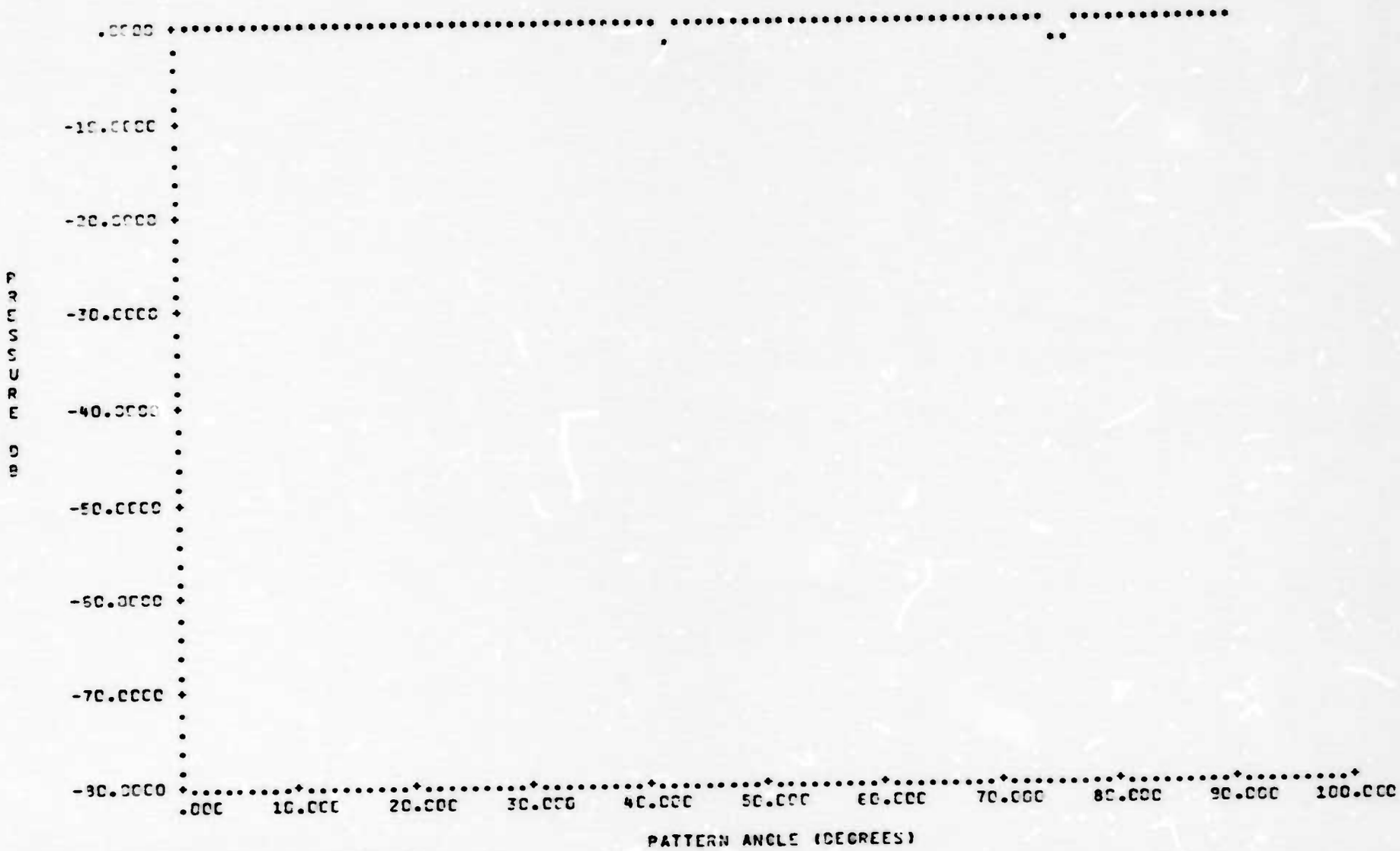
SLO= .290+03 FO/F-= .100+02
ALPHA(T)+RO= .100-04 DB
PRESSURE UNITS DB/1 MICRO PASCAL (KHZ) RMS

```
    IOP          IOE          IOD          IOD          IOC
     1            0            0            0            0
   ALTRC        P1P2
  .100-04      .100+01
NFM=  1     NFMPO=  1
   FO/FM=      .100+02
LIN=  0     IOC=  1
KO*A=    .100+02
PATTERN SLS=    .300+03     FO/F-=    .100+02     OB=  -.800+02     NP= 90     TD=    .100+01


FO/F- =    .100+02     EN=    .108+01
```



SLO*= .300+03 FO/F-= .100+02
ALPHA(T)*RO= .10L-04 DB
PRESSURE UNITS DB/1 MICRO PASCAL (KHZ) RMS

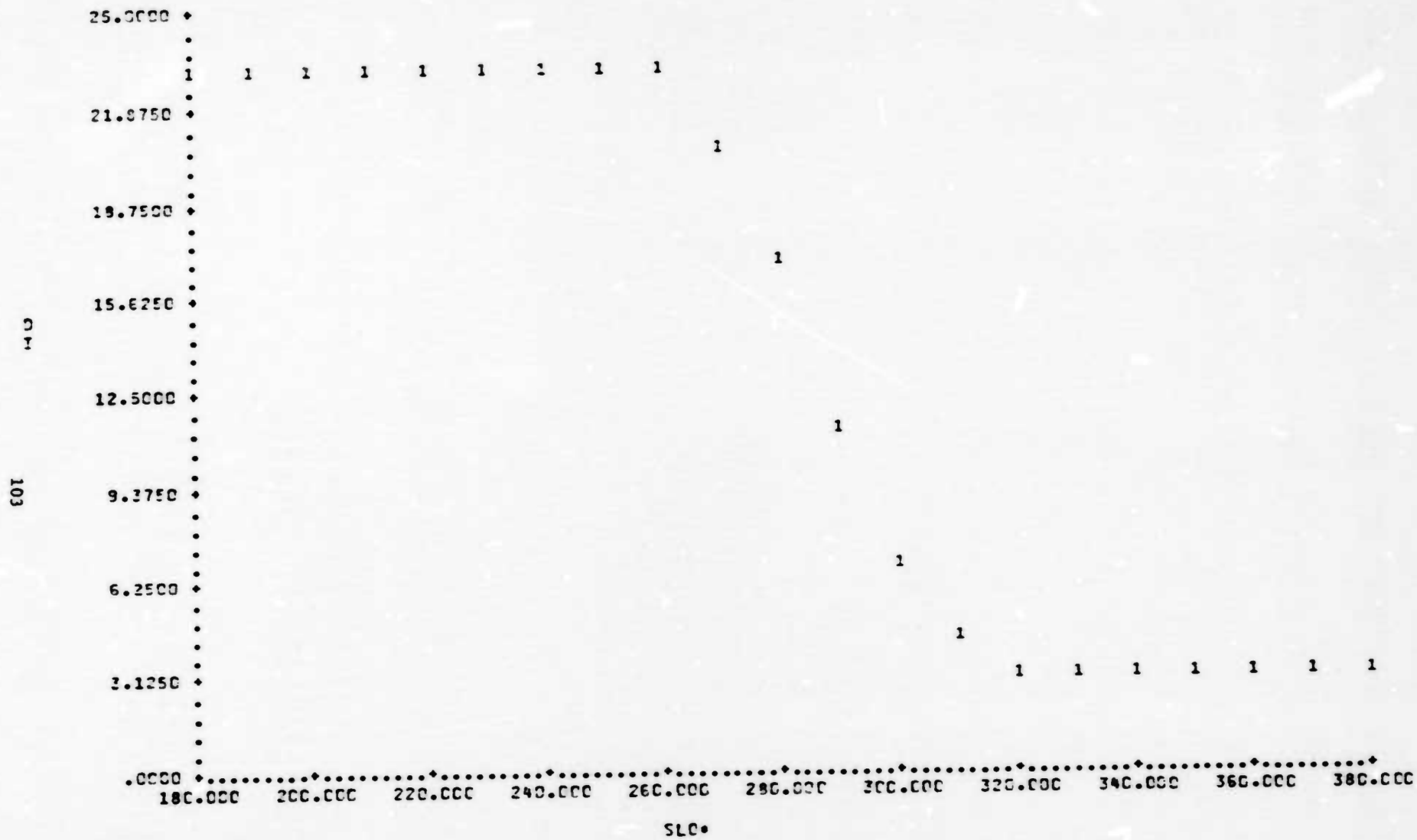
```
    IOP          IOE          IOS          IOD          IOD
     1            0            0            0            0
   ALTRO        P1P2
  .100-04      .100+01
NFM=  1    NFMRO=  1
   FO/FM=      .100+02
LIN=  0    IOC=  1
KO*A=   .100+02
PATTERN SLS=   .330+03    FO/F-=   .100+02    DB=  -.800+02    NP= 90    TO=   .100+01


FO/F- =   .100+02    EN=   .103+01
```



SLO*= .330+03 FO/F-= .100+02
ALPHA(T)*RO= .100-04 DB
PRESSURE UNITS DB/1 MICRO PASCAL (KHZ) RMS

FIGURE 3

(Pages 101 to 104)



ALPHA(T)*RO= .100-04 D3
CURVE 1 IS FOR FC/F-= .100+02
PRESSURE UNITS DB/1 MICRO PASCAL (KHZ) RMS

SLO*	DI(1)	DI(2)	DI(3)	DI(4)	DI(5)
.180+03	.234+02				
.190+03	.234+02				
.200+03	.234+02				
.210+03	.234+02				
.220+03	.234+02				
.230+03	.234+02				
.240+03	.233+02				
.250+03	.233+02				
.260+03	.228+02				
.270+03	.204+02				
.280+03	.166+02				
.290+03	.112+02				
.300+03	.679+01				
.310+03	.436+01				
.320+03	.341+01				
.330+03	.310+01				
.340+03	.299+01				
.350+03	.299+01				
.360+03	.299+01				
.370+03	.299+01				
.380+03	.299+01				

@FIN

PROGRAM LISTING

(Pages 105 to 118)

```
$ELT,ILD
ELTOT7 RL1370 03/25-14:51:07
000001        CCC          COMMON AKOR,AKOL,AKOW,PI,CNVA,DNM,LIN,CH1,CH2,YNM,ATPC,CONWA,
000002        CCC         1 DELP,DELI,DTLA
000003        CCC          INTEGER READER
000004        CCC          DOUBLE PRECISION DEI,ARO1,ARO2,DI(DE),CH1,CH2,XC,AORC,ATPC,ATRCP
000005        CCC          DOUBLE PRECISION ACRC2
000006        CCC    C     DIMENSION OF DI        MUST BE AT LEAST MAX(X,N)+20 OR
000007        CCC    C     MAX(2*X,N) FOR X>20
000008        CCC          DIMENSION VAR(2),*K(58)
000009        CCC          DIMENSION PLTT(101),PLTP(101),PLTX(21),PLTC(21),PLTK(21),
000010        CCC         1 PLTE(21,5),PLTS(21,5),PLTD(21,5),LBX(5),LBY(3),LBXX(2),LBYE(3),
000011        CCC         2 LBYD(2),LBYD(2),LBYD(2),IDHR(5),FFM(5)
000012        CCC    1000 FORMAT( )
000013        CCC    1011 FORMAT (5H NFM=,I3,3X,7H NFMPD=,I3)
000014        CCC    1012 FORMAT (13H PATTERN SLG=,E10.3,3X,7H FO/F-=,E10.3,3X,4H DB=,E10.3,
000015        CCC         1 3X,4H NFE=,I2,3X,4H TDE=,E10.7)
000016        CCC    1013 FORMAT (1H1,7X,4H IOP,8X,4H IOE,8X,4H IOS,8X,4H IOD,8X,4H IOO)
000017        CCC    1014 FORMAT (5(4X,I2,5X))
000018        CCC    1015 FORMAT (3X,7H FO/FM=,5(2X,E10.3,))
000019        CCC    1016 FORMAT (3X,6H ALTRO,6X,5H P1P2)
000020        CCC    1017 FORMAT (2(E10.3,2X))
000021        CCC    1018 FORMAT (5H LIN=,I2,3X,5H IOO=,I2)
000022        CCC    1019 FORMAT (6H KO*A=,E10.3)
000023        CCC    1020 FORMAT (6H KO*L=,E10.3,3X,6H KO*W=,E10.3)
000024        CCC    1030 FORMAT (8H FO/F- =,E10.3,3X,4H CN=,E10.3)
000025        CCC    1040 FORMAT (22H NFM MUST BE 5 OR LESS)
000026        CCC    1050 FORMAT (1H1,4X,5H SLO*,7X,8H SL-(1)*,4X,8H SL-(2)*,4X,8H SL-(3)*,
000027        CCC         1 4X,8H SL-(4)*,4X,8H SL-(5)*)
000028        CCC    1051 FORMAT (1H1,4X,5H SLO*,7X,8H SL+(1)*,4X,8H SL+(2)*,4X,8H SL+(3)*,
000029        CCC         1 4X,8H SL+(4)*,4X,8H SL+(5)*)
000030        CCC    1060 FORMAT (1X,6(2X,E10.3))
000031        CCC    1070 FORMAT (///)
000032        CCC    1080 FORMAT (10H CPU TIME=,I4,8H SECONDS)
000033        CCC    1100 FORMAT (13H ALPHA(T)*RO=,E10.3,2H DB)
000034        CCC    1110 FORMAT (7H CURVE ,I2,14H IS FOR FO/F-=,E10.3)
000035        CCC    1120 FORMAT (43H PRESSURE UNITS DB/1 MICRO PASCAL (KHZ) RMS)
000036        CCC    1130 FORMAT (13H ALPHA(O)*RO=,E10.3,7H NEPERS)
000037        CCC    1140 FORMAT (6H SLE*=,E10.3,3X,7H FO/F-=,E10.3)
000038        CCC    1150 FORMAT (5H NOO=,I3)
000039        CCC    1260 FORMAT (1H1,4X,5H SLO*,7X,7H SL OUT)
000040        CCC    1270 FORMAT (10X,77H ***** F1*F2 UP CONVERSION CASE *****///)
000041        CCC    1280 FORMAT (1H1,4X,5H SLO*,7X,6H DI(1),6X,6H DI(2),6X,6H DI(3),6X,
000042        CCC         1 6H DI(4),6X,6H DI(5))
000043        CCC          READER=5
000044        CCC    C     UNSCALED VALUES OF SPL- AND SPLO CAN BE OBTAINED BY SPECIFYING
000045        CCC    C     KOA AND USING THE FOLLOWING EQUATIONS
000046        CCC    C     SPLO   =SPLO*   -40.0*LOG10(KOA)+18.46
000047        CCC    C     SPL-   =SPL-*   -40.0*LOG10(KOA)+18.46
000048        CCC    C     NOTE THAT THE CONSTANT 18.46 DO ONLY PERTAINS TO A WATER MEDIUM.
000049        CCC    C     IN A MEDIUM WITH A DIFFERENT SOUND SPEED THIS CONSTANT CAN BE
000050        CCC    C     CHANGED BY ADDING 20.0*LOG10(1.5/(C OF THE NEW MEDIUM IN
000051        CCC    C     KILOMETERS/SEC)) TO 18.46
000052        CCC    C     CONWA=MEDIUM CONSTANT=290.0 FOR WATER
000053        CCC          CONWA=290.0
000054        CCC          READ(READER,1000) NOO
```

```
000055   000          PRINT 1150,NCC
000056   000          INCC=0
000057   000        5 CONTINUE
000058   000          INCC=INCC+1
000059   000          READ(READER,1000) ICP,ICE,ICS,ICD,ICC
000060   000          PRINT 1013
000061   000          PRINT 1014,ICP,ICE,ICS,ICD,ICC
000062   000          NS=0
000063   000          IC=ICL+ICS+ICD+ICC
000064   000          IF (IC .GT. 0) NS=21
000065   000          READ(READER,1000) ALTRC,F1F2
000066   000          ATRC=ALTRC/3.67
000067   000    C     ALTRC IS IN DB, ATRC IS IN METERS
000068   000    C     A NEGATIVE ALTRC IS FOR F1+F2
000069   000          PRINT 1016
000070   000          PRINT 1017,ALTRC,F1F2
000071   000          READ(READER,1000) NFM,NFMRD
000072   000          PRINT 1011,NFM,NFMRD
000073   000          IF (NFMRD .NE. 1) GO TO 10
000074   000          IF (NFM .GT. 5) GO TO 13
000075   000          READ(READER,1000) (FFM(K),K=1,NFM)
000076   000    C     FFM=FO/F-
000077   000          PRINT 1015, (FFM(K),K=1,NFM)
000078   000          GO TO 10
000079   000       13 PRINT 1040
000080   000          GO TO 990
000081   000       10 CONTINUE
000082   000          IF (ICP .EQ. 0 .AND. ICD .EQ. 0) GO TO 16
000083   000          READ(READER,1000) LIN,ICC
000084   000          PRINT 1018,LIN,ICC
000085   000    C     LIN=0 FOR PISTON,1 FOR RECTANGLE
000086   000    C     ICC=1 FOR F(THETA), =2 FOR DELTA(THETA), =3 FOR CONVOLUTION
000087   000          IF (LIN .EQ. 1) GO TO 11
000088   000          READ(READER,1000) AKOR
000089   000    C     AKOR=KO*A=2.0*PI*(PISTON RADIUS)/WAVELENGTH
000090   000          PRINT 1019, AKOR
000091   000          GO TO 12
000092   000       11 READ(READER,1000) AKOL,AKOW
000093   000    C     AKOL=2.0*PI*(RECTANGLE LENGTH)/WAVELENGTH
000094   000    C     AKOW=2.0*PI*(RECTANGLE WIDTH)/WAVELENGTH
000095   000          PRINT 1020, AKOL,AKOW
000096   000       12 CONTINUE
000097   000          IF (ICP .NE. 1) GO TO 16
000098   000          READ(READER,1000) SLS,PFFM,DB,NP,TO
000099   000          PRINT 1012,SLS,PFFM,DB,NP,TO
000100   000          IF (DB .GT. 0.0) DB=-DB
000101   000       16 CONTINUE
000102   000          PRINT 1070
000103   000          IF (ALTRC .LT. 0.0) PRINT 1270
000104   000          PI=3.14159
000105   000          ICHR(1)='1'
000106   000          ICHR(2)='2'
000107   000          ICHR(3)='3'
000108   000          ICHR(4)='4'
000109   000          ICHR(5)='5'
000110   000          LSXX(1)=4
```

```
000111   000          LBXX(2)='SLO*'
000112   000          LBYS(1)=10
000113   000          LBYS(2)='20 LOO'
000114   000          LBYS(3)=' ETA'
000115   000          LBYS(1)=4
000116   000          LBYS(2)='SL-*'
000117   000          IF (ALTRO .LT. 0.0) LBYS(2)='SL+*'
000118   000          LBYD(1)=2
000119   000          LBYD(2)='DI'
000120   000          LBYD(1)=6
000121   000          LBYD(2)='SL OUT'
000122   000    C     ABRAMOWITZ PAGE 231 FORMULA 5.1.55 FOR EXP(X)*E1(X)
000123   000          A1=4.03340
000124   000          A2=1.15198
000125   000          B1=5.03607
000126   000          B2=4.19160
000127   000          IF (ATRO .GE. 10.000) GO TO 61
000128   000          DEL=-DEXP(ATRO)*DEI(-ATRO)
000129   000          GO TO 62
000130   000       61 DEL=(ATRO**2+A1*ATRO+A2)/(ATRO**3+B1*ATRO**2+B2*ATRO)
000131   000       62 DELL=DEL
000132   000          DLLL=DELL
000133   000          IF (ALTRO .LT. 0.0) DLLL=-DEXP(DABS(ATRO))*DEI(-DABS(ATRO))
000134   000          DO 300 K=1,NFM
000135   000          IF (NFMRO .EQ. 1) GO TO 14
000136   000          FOFM=5.0*(2.0**(K-1))
000137   000          FFM(K)=FOFM
000138   000          GO TO 15
000139   000       14 FOFM=FFM(K)
000140   000       15 CONTINUE
000141   000          F1FO=1.0+0.5/FOFM
000142   000          F2FO=1.0-0.5/FOFM
000143   000          F1F2=F1FO/F2FO
000144   000          IF (ALTRO .LT. 0.0) GO TO 67
000145   000          A1RO=ATRO/2.0*F1F2
000146   000          A2RO=ATRO/(2.0*F1F2)
000147   000          ADRO=(A1RO+A2RO+ATRO)/4.0
000148   000          GO TO 68
000149   000       67 A1RO=-ATRO/2.0*F1F2
000150   000          A2RO=-ATRO/(2.0*F1F2)
000151   000          ADRO=(A1RO+A2RO-ATRO)/4.0
000152   000       68 CONTINUE
000153   000          ADRO2=2.0*ADRO
000154   000          IF (ADRO .GE. 10.000) GO TO 63
000155   000          DELO=-DEXP(ADRO2)*DEI(-ADRO2)
000156   000          GO TO 64
000157   000       63 DELO=(ADRO2**2+A1*ADRO2+A2)/(ADRO2**3+B1*ADRO2**2+B2*ADRO2)
000158   000       64 CONTINUE
000159   000          ATROP=DABS(ATRO)*FOFM
000160   000          IF (ATROP .GE. 10.000) GO TO 65
000161   000          DELP=-DEXP(ATROP)*DEI(-ATROP)
000162   000          GO TO 66
000163   000       65 DELP=(ATROP**2+A1*ATROP+A2)/(ATROP**3+B1*ATROP**2+B2*ATROP)
000164   000       66 CONTINUE
000165   000          EN=1.0+ALOG10(DLLL/DELP)/ALOG10(FOFM)
000166   000          PRINT 1030,FOFM,EN
```

```
000167      000        IF (NS  .EQ. 0) GO TO 800
000168      000        DO 801 J=1,NS
000169      000        SLOS=100.0+10.0*(J-1)
000170      000        PLTX(J)=SLOS
000171      000        SL1S=SLOS-10.0*ALOG10(1.0+(1.0/P1PZ)**2)
000172      000        SL2S=SLOS-10.0*ALOG10(1.0+P1PZ**2)
000173      000        XCDB=SLOS+20.0*ALOG10(DELC)-CONWA
000174      000        CH1DB=SL1S+20.0*ALOG10(ABS(DELC))-CONWA
000175      000        CH2DB=SL2S+20.0*ALOG10(ABS(DELC))-CONWA
000176      000        CHCDB=SLOS+20.0*ALOG10(ABS(DELC))-CONWA
000177      000        XC=10.0**(XCDB/20.0)
000178      000        CH1=10.0**(CH1DB/20.0)
000179      000        CH2=10.0**(CH2DB/20.0)
000180      000        CHC=10.0**(CHCDB/20.0)
000181      000        IF (XC .GE. 73.000) GO TO 20
000182      000        CALL BESI(XC,0.00,1,BI,65)
000183      000        FNC=BI(2)/BI(1)
000184      000        GO TO 21
000185      000     20 FNC=1.0
000186      000     21 CONTINUE
000187      000        SLOUT=(CONWA+6.01-20.0*ALOG10(DELC/FNC)
000188      000        IF (IOD .EQ. 1) PLTO(J)=SLOUT
000189      000        IF (CH1 .GE. 33.000) GO TO 22
000190      000        CALL BESI(CH1,0.00,1,BI,65)
000191      000        FN1=BI(2)/BI(1)
000192      000        GO TO 23
000193      000     22 FN1=1.0
000194      000     23 CONTINUE
000195      000        IF (CH2 .GE. 33.000) GO TO 24
000196      000        CALL BESI(CH2,0.00,1,BI,65)
000197      000        FM1=BI(2)/BI(1)
000198      000        GO TO 25
000199      000     24 FM1=1.0
000200      000     25 CONTINUE
000201      000        FCHI=2.0*FN1*FM1/CHC
000202      000        IF (ALTRC .LT. 0.5) GO TO 71
000203      000        ETADB=20.0*ALOG10(FCHI)-20.0*EN*ALOG10(FCFM)
000204      000        GO TO 72
000205      000     71 ETADB=20.0*ALOG10(FCHI)-20.0*EN*ALOG10(0.5)
000206      000     72 CONTINUE
000207      000        IF (IOD .EQ. 1) PLTE(J,K)=ETADB
000208      000        SLMS=SLOS+ETADB
000209      000        IF (IOD .EQ. 1) PLTS(J,K)=SLMS
000210      000        IF (IOD .NE. 1) GO TO 40
000211      000  C     DI SECTION - - - - - - - - - - - - - - - - - - - - - - - -
000212      000        IF (IOD .NE. 3) GO TO 90
000213      000        VAR(1)=0.0
000214      000        VAR(2)=0.0
000215      000        CALL CMFAT(VAR,SLOS,FCFM)
000216      000        CNVX=CNVA
000217      000     90 CONTINUE
000218      000        IF (IOD .EQ. 2) GO TO 30
000219      000        ARG1=CH1*F1FC
000220      000        ARG2=CH2*F2FC
000221      000        IF (ARG1 .GE. 33.000) GO TO 26
000222      000        CALL BESI(ARG1,0.00,1,BI,65)
```


Reproduced from best available copy.

```
000223      000          YN=BI(2)/BI(1)
000224      000          GO TO 27
000225      000       26 YN=1.0
000226      000       27 CONTINUE
000227      000          IF (ARG2 .GE. 73.0D0) GO TO 28
000228      000          CALL DESI(ARG2,0.D0,1.3I,65)
000229      000          YM=BI(2)/BI(1)
000230      000          GO TO 29
000231      000       28 YM=1.0
000232      000       29 CONTINUE
000233      000          YNM=YN*YM
000234      000       30 CONTINUE
000235      000          IF (LIN .EQ. 1 .AND. IOC .NE. 2) GO TO 700
000236      000          TA=0.0
000237      000          TB=PI
000238      000          HSTAR=PI*1.0D-2
000239      000          HMIN=PI*1.0D-4
000240      000          HMAX=PI/2.0
000241      000          ERMAX=1.0E-3
000242      000          KEY=0
000243      000    C     FOR PISTON, PATTERN ASSUMED ZERO BEHIND PISTON
000244      000          CALL ROM2S(TA,TB,THET,FOFX,HSTAR,HMIN,HMAX,ERMAX,ANS,KSTOP,KEY)
000245      000      700 IF (THET .GT. PI/2.0) GO TO 745
000246      000          VAR(1)=0.0
000247      000          VAR(2)=THET
000248      000          GO TO (710,720,730),IOC
000249      000      710 CALL SMPAT(VAR,SLOS,FOFM)
000250      000          FOFX=DNM**2 *SIN(THET)
000251      000          GO TO 746
000252      000      720 CALL DLPAT(VAR,SLOS,FOFM)
000253      000          FOFX=DELA**2 *SIN(THET)
000254      000          GO TO 746
000255      000      730 CALL CNPAT(VAR,SLOS,FOFM)
000256      000          FOFX=(CNVA/CNVX)**2 *SIN(THET)
000257      000          GO TO 746
000258      000      745 FOFX=0.0
000259      000      746 CONTINUE
000260      000          CALL ROM2
000261      000          IF (KSTOP .EQ. 1) GO TO 700
000262      000          DI=10.0*ALOG10(4.0*PI/(2.0*PI*ANS))
000263      000          PLTD(J,K)=DI
000264      000          GO TO 301
000265      000      750 NINT=2
000266      000          ERMAX=1.0E-3
000267      000          KEY=0
000268      000          CALL RM01(NINT,VAR,FUN,ANS,ERMAX,KCO,WK,KEY)
000269      000      751 TA1=0.0
000270      000          TB1=2.0*PI
000271      000          HSTAR1=PI*1.0E-2
000272      000          HMIN1=PI*1.0E-4
000273      000          HMAX1=PI/2.0
000274      000          CALL RM02(TA1,TB1,HSTAR1,HMIN1,HMAX1)
000275      000      752 CALL RME2(0.0,PI,0.01*PI,PI*1.0E-4,PI/2.0)
000276      000      753 IF (IOC .EQ. 3) GO TO 760
000277      000          CALL SMPAT(VAR,SLOS,FOFM)
000278      000          FUN=DNM**2 *SIN(VAR(2))
```

```
000279   000          GO TO 761
000280   000      760 CALL CNPAT(VAR,SLOS,FOFM)
000281   000          FUN=(CNVA/CNVX)**2 *SIN(VAR(2))
000282   000      761 CALL RM33
000283   000          GO TO (751,752,753,754),KGO
000284   000      754 DI=10.0*ALOG10(4.0*PI/ANS)
000285   000          PLTD(J,K)=DI
000286   000      751 CONTINUE
000287   000    C     END OF DI SECTION - - - - - - - - - - - - - - - - - - - - - - -
000288   000       40 CONTINUE
000289   000      801 CONTINUE
000290   000      800 CONTINUE
000291   000          IF (IOE .NE. 1) GO TO 810
000292   000          DO 802 K=1,NFM
000293   000          DO 803 J=1,NS
000294   000          PLTK(J)=PLTE(J,K)
000295   000      803 CONTINUE
000296   000          IF (K .GT. 1) GO TO 804
000297   000          CALL PSCALE(PLTK,NS,YLO,YHI,.TRUE.,1)
000298   000          GO TO 802
000299   000      804 CALL PSCALE(PLTK,NS,YLO,YHI,.FALSE.,1)
000300   000      802 CONTINUE
000301   000          CALL PSETUP(180.,380.,YLO,YHI,1HF,1H ,LBXX,LBYE)
000302   000          DO 805 K=1,NFM
000303   000          DO 806 J=1,NS
000304   000          PLTK(J)=PLTE(J,K)
000305   000      806 CONTINUE
000306   000          IF (K .EQ. NFM) GO TO 807
000307   000          CALL PPLOT(.FALSE.,ICHR(K),NS,PLTX,PLTK)
000308   000          GO TO 805
000309   000      807 CALL PPLOT(.TRUE.,ICHR(K),NS,PLTX,PLTK)
000310   000      805 CONTINUE
000311   000          PRINT 1100,ALTRC
000312   000          DO 850 K=1,NFM
000313   000      850 PRINT 1110,K,FFM(K)
000314   000          PRINT 1120
000315   000      810 CONTINUE
000316   000          IF (IOS .NE. 1) GO TO 820
000317   000          CALL PSETUP(180.,380.,0.,320.,1HF,1H ,LBXX,LBYS)
000318   000          DO 815 K=1,NFM
000319   000          DO 816 J=1,NS
000320   000          PLTK(J)=PLTS(J,K)
000321   000      816 CONTINUE
000322   000          IF (K .EQ. NFM) GO TO 817
000323   000          CALL PPLOT(.FALSE.,ICHR(K),NS,PLTX,PLTK)
000324   000          GO TO 815
000325   000      817 CALL PPLOT(.TRUE.,ICHR(K),NS,PLTX,PLTK)
000326   000      815 CONTINUE
000327   000          PRINT 1100,ALTRC
000328   000          DO 851 K=1,NFM
000329   000      851 PRINT 1110,K,FFM(K)
000330   000          PRINT 1120
000331   000          IF (ALTRC .LT. 0.0) GO TO 811
000332   000          PRINT 1050
000333   000          GO TO 812
000334   000      811 PRINT 1051
```

```
000335      000      812 CONTINUE
000336      000          DO 818 J=1,NS
000337      000          PRINT 1100,PLTX(J),(PLTC(J,K),K=1,NFM)
000338      000      818 CONTINUE
000339      000      822 CONTINUE
000340      000          IF (ICC .NE. 1) GO TO 830
000341      000          DO 825 K=1,NFM
000342      000          DO 823 J=1,NS
000343      000          PLTK(J)=PLTC(J,K)
000344      000      823 CONTINUE
000345      000          IF (K .GT. 1) GO TO 824
000346      000          CALL PSCALE(PLTK,NS,YLO,YHI,.TRUE.,1)
000347      000          GO TO 822
000348      000      824 CALL PSCALE(PLTK,NS,YLO,YHI,.FALSE.,1)
000349      000      822 CONTINUE
000350      000          CALL PSETUP(130.,330.,YLO,YHI,1HF,1H ,LSXX,LSYC)
000351      000          DO 825 K=1,NFM
000352      000          DO 826 J=1,NS
000353      000          PLTK(J)=PLTC(J,K)
000354      000      826 CONTINUE
000355      000          IF (K .EQ. NFM) GO TO 827
000356      000          CALL PPLOT(.FALSE.,ICHR(K),NS,PLTX,PLTK)
000357      000          GO TO 825
000358      000      827 CALL PPLOT(.TRUE.,ICHR(K),NS,PLTX,PLTK)
000359      000      825 CONTINUE
000360      000          PRINT 1100,ALTPC
000361      000          DO 852 K=1,NFM
000362      000      852 PRINT 1110,K,FFM(K)
000363      000          PRINT 1120
000364      000          PRINT 1060
000365      000          DO 826 J=1,NS
000366      000          PRINT 1060,PLTX(J),(PLTC(J,K),K=1,NFM)
000367      000      826 CONTINUE
000368      000      830 CONTINUE
000369      000          IF (ICC .NE. 1) GO TO 840
000370      000          CALL PSCALE(PLTC,NS,YLO,YHI,.TRUE.,1)
000371      000          CALL PSETUP(180.,370.,YLO,YHI,1HF,1H ,LSXX,LSYC)
000372      000          CALL PPLOT(.TRUE.,1H*,NS,PLTX,PLTC)
000373      000          PRINT 1130,ACRC
000374      000          PRINT 1120
000375      000          PRINT 1060
000376      000          DO 831 J=1,NS
000377      000      831 PRINT 1060,PLTX(J),PLTC(J)
000378      000      840 CONTINUE
000379      000          IF (ICF .NE. 1) GO TO 860
000380      000          IF (ICC .EQ. 2) GO TO 130
000381      000          SL1S=SLS -10.0*ALOG10(1.0+(1.0/P1P2)**2)
000382      000          SL2S=SLS -10.0*ALOG10(1.0+P1P2**2)
000383      000          CH1DB=SL1S+20.0*ALOG10(ABS(DELL))-CONWA
000384      000          CH2DB=SL2S+20.0*ALOG10(ABS(DELL))-CONWA
000385      000          CH1=10.0**(CH1DB/20.0)
000386      000          CH2=10.0**(CH2DB/20.0)
000387      000          ARC1=CH1*F1FC
000388      000          ARC2=CH2*F2FC
000389      000          IF (ARC1 .GE. 33.000) GO TO 120
000390      000          CALL SECI(ARC1,0.00,1,BI,65)
```

```
000391      000          YM=DI(I)/DI(1)
000392      000          GO TO 127
000393      000      126 YM=1.0
000394      000      127 CONTINUE
000395      000          IF (ABS2 .GE. 33.0001) GO TO 128
000396      000          CALL SECI(ABS2,0.00,1,BI,65)
000397      000          YM=DI(I)/DI(1)
000398      000          GO TO 129
000399      000      128 YM=1.0
000400      000      129 CONTINUE
000401      000          YNM=YN*YM
000402      000      130 CONTINUE
000403      000          IF (TCC .NE. 3) GO TO 190
000404      000          VAR(1)=0.0
000405      000          VAR(2)=0.0
000406      000          CALL CNPAT(VAR,SLS,PFFM)
000407      000          CNVX=CNVA
000408      000      190 CONTINUE
000409      000          DO 300 I=1,NP
000410      000          THET= TD*I*PI/180.0
000411      000          THETD= TD*I
000412      000          PLTT(I)=THETD
000413      000          VAR(1)=0.0
000414      000          VAR(2)=THET
000415      000    C     PATTERN ASSUMED IN PLANE OF LENGTH AXIS FOR RECTANGLE
000416      000          GO TO (862,863,864),ICC
000417      000      862 CALL BMPAT(VAR,SLS,PFFM)
000418      000          PLTP(I)=20.0*ALOG10(ABS(CNM))
000419      000          GO TO 300
000420      000      863 CALL DLPAT(VAR,SLS,PFFM)
000421      000          PLTP(I)=20.0*ALOG10(DELA)
000422      000          GO TO 300
000423      000      864 CALL CNPAT(VAR,SLS,PFFM)
000424      000          PLTP(I)=20.0*ALOG10(CNVA/CNVX)
000425      000          CALL SECOND(TSEC)
000426      000          PRINT 1090,TSEC
000427      000      300 CONTINUE
000428      000          LBX(1)=33
000429      000          LBX(2)='PATTER'
000430      000          LBX(3)='N ANGL'
000431      000          LBX(4)='E (DEG'
000432      000          LBX(5)='REES)'
000433      000          LBY(1)=11
000434      000          LBY(2)='PRESSU'
000435      000          LBY(3)='RE DB'
000436      000          TF=100.0*TC
000437      000          CALL PSETUP(0.0,TF,DD,0.0,1HF,1H ,LBX,LBY)
000438      000          CALL PPLOT(.TRUE.,1H+,NP,PLTT,PLTP)
000439      000          PRINT 1140,SLS,PFFM
000440      000          PRINT 1100,ALTRD
000441      000          PRINT 1120
000442      000      360 CONTINUE
000443      000      990 IF (INCC .LT. NCC) GO TO 5
000444      000      999 END

END ELT.
```


Reproduced from best available copy.

```
BELT.ILS
ELTOT7 RL1370 03/25-14:51:10
000001        SUBROUTINE IMPAT(VAR,SL,FF)
000002        COMMON AKCP,AKCL,AKCW,PI,CNW1,CNM,LIN,CH1,CH2,YNM,ATPD,CONWA,
000003       1 DELP,DELI,DELA
000004        DOUBLE PRECISION ARG1,ARG2,TERM1,TERM2,CH1,CH2,BC(2),B1(2),B2(65)
000005        DIMENSION VAR(2)
000006   C    PATTERN CALCULATION - - - - - - - - - - - - - - - - - - - - - -
000007        THET=VAR(2)
000008        F1FC=1.0+0.5/FF
000009        F2FC=1.0-0.5/FF
000010        IF (LIN .EQ. 1) GO TO 100
000011        TERM1=DABS(AKCP*F1FC*SIN(THET))
000012        TERM2=DABS(AKCP*F2FC*SIN(THET))
000013        IF (TERM1 .EQ. 0.00) GO TO 710
000014        CALL BJYO1(TERM1,1,1,BC,B1)
000015        D1=2.0*B1(1)/TERM1
000016        GO TO 720
000017    710 D1=1.0
000018    720 IF (TERM2 .EQ. 0.00) GO TO 730
000019        CALL BJYO1(TERM2,1,1,BC,B1)
000020        D2=2.0*B1(1)/TERM2
000021        GO TO 771
000022    730 D2=1.0
000023        GO TO 771
000024    100 CX=SIN(VAR(2))*COS(VAR(1))
000025        CY=SIN(VAR(2))*SIN(VAR(1))
000026        CZ=COS(VAR(2))
000027        IF (CX .EQ. 0.0) GO TO 761
000028        TNL=SQRT(CZ**2+CY**2)/CX
000029        THL=PI/2.0-ATAN(TNL)
000030        GO TO 762
000031    761 THL=0.0
000032    762 IF (CY .EQ. 0.0) GO TO 763
000033        TNW=SQRT(CZ**2+CX**2)/CY
000034        THW=PI/2.0-ATAN(TNW)
000035        GO TO 764
000036    763 THW=0.0
000037    764 ANGL1=(AKCL*F1FC/2.0)*SIN(THL)
000038        ANGL2=(AKCL*F2FC/2.0)*SIN(THL)
000039        ANGW1=(AKCW*F1FC/2.0)*SIN(THW)
000040        ANGW2=(AKCW*F2FC/2.0)*SIN(THW)
000041        IF (ANGL1 .EQ. 0.0) GO TO 765
000042        IF (ANGW1 .EQ. 0.0) GO TO 766
000043        D1=SIN(ANGL1)/ANGL1*SIN(ANGW1)/ANGW1
000044        GO TO 767
000045    766 D1=SIN(ANGL1)/ANGL1
000046        GO TO 767
000047    765 IF (ANGW1 .EQ. 0.0) GO TO 768
000048        D1=SIN(ANGW1)/ANGW1
000049        GO TO 767
000050    768 D1=1.0
000051    767 IF (ANGL2 .EQ. 0.0) GO TO 769
000052        IF (ANGW2 .EQ. 0.0) GO TO 770
000053        D2=SIN(ANGL2)/ANGL2*SIN(ANGW2)/ANGW2
000054        GO TO 771
```

```
000055        000        770 D2=SIN(ANGL2)/ANGL2
000056        000            GO TO 771
000057        000        769 IF (ANGW2 .EQ. 0.0) GO TO 772
000058        000            D2=SIN(ANGW2)/ANGW2
000059        000            GO TO 771
000060        000        772 D2=1.0
000061        000        771 CONTINUE
000062        000            ARG1=CH1*D1*FIFC
000063        000            ARG2=CH2*D2*FIFC
000064        000            ARG1=DABS(ARG1)
000065        000            ARG2=DABS(ARG2)
000066        000            IF (ARG1 .GE. 33.000) GO TO 773
000067        000            CALL BESI(ARG1,0.00,1,BI,65)
000068        000            DN=BI(2)/BI(1)
000069        000            GO TO 774
000070        000        773 DN=1.0
000071        000        774 CONTINUE
000072        000            IF (D1 .LT. 0.0) DN=DN*(-1.0)**1
000073        000            IF (ARG2 .GE. 33.000) GO TO 775
000074        000            CALL BESI(ARG2,0.00,1,BI,65)
000075        000            DM=BI(2)/BI(1)
000076        000            GO TO 776
000077        000        775 DM=1.0
000078        000        776 CONTINUE
000079        000            IF (D2 .LT. 0.0) DM=DM*(-1.0)**1
000080        000            DNM=DN*DM/YNM
000081        000            GO TO 200
000082        000        200 RETURN
000083        000            END

END ELT.
```

```
SELT.ILD
ELTOT7 RL370 09/25-14:51:11
000001      CCC          SUBROUTINE DLPAT(VAR,SL,FF)
000002      CCC          COMMON AKOR,AKOL,AKOW,PI,CNV1,DNM,LIN,CH1,CH2,YNY,ATRC,CONWA,
000003      CCC        1 DELT,DELI,DELA
000004      CCC          DOUBLE PRECISION CH1,CH2,ATRC
000005      CCC          DIMENSION VAR(2)
000006      CCC          THET=VAR(2)
000007      CCC          INF=20
000008      CCC          SCDB=SL  -CONWA-20.0*ALOG10(ABS(ATRC/2.0))
000009      CCC          XK=(10.0**(SCDB/20.0))/4.0
000010      CCC          D=ATAN(1.0/XK)
000011      CCC          SQ1PX2=SQRT(1.0+XK**2)
000012      CCC          SQ1MX2=SQRT(1.0+(1.0/XK)**2)
000013      CCC          IF (LIN .EQ. 1) GO TO 13
000014      CCC          AKMRC=(AKOR**2)/(2.0*FF)
000015      CCC          IF (ATRC .LT. 0.00) AKMRC=AKOR**2
000016      CCC          GO TO 14
000017      CCC       13 AKMRC=AKOL*AKOW/(2.0*PI*FF)
000018      CCC          IF (ATRC .LT. 0.00) AKMRC=AKOL*AKOW/PI
000019      CCC       14 CONTINUE
000020      CCC          CF=2.0*AKMRC/ATRC
000021      CCC          Y=CF*(SIN(THET/2.0))**2
000022      CCC          SMR1=0.0
000023      CCC          SMI1=0.0
000024      CCC          SMX1=0.0
000025      CCC          DO 10 N=1,INF
000026      CCC          IF (XK .LT. 1.0) GO TO 11
000027      CCC          AN=SIN(N*D)/(XK*(SQ1MX2)**N)
000028      CCC          GO TO 12
000029      CCC       11 AN=SIN(N*D)*((XK)**(N-1))/(SQ1PX2**N)
000030      CCC       12 CONTINUE
000031      CCC          TEMP1=AN/N
000032      CCC          TEMP2=1.0+(Y/N)**2
000033      CCC          SMX1=SMX1+TEMP1
000034      CCC          SMR1=SMR1+TEMP1/TEMP2
000035      CCC          SMI1=SMI1-TEMP1*(Y/N)/TEMP2
000036      CCC       10 CONTINUE
000037      CCC          DELR=SMR1/SMX1
000038      CCC          DELI=SMI1/SMX1
000039      CCC          DELA=SQRT(DELR**2+DELI**2)
000040      CCC          RETURN
000041      CCC          END

END ELT.
```

```
$ELT,ILC
ELT077 RL1070 03/25-14:51:12
000001      CCC          SUBROUTINE CMPAT(VAR,SL,FF)
000002      CCC          COMMON AKO2,AKCL,AKCW,PI,CNVA,CNM,LIN,CH1,CH2,YNM,ATRC,CONWA,
000003      CCC        1 DELP,DELI,DELA
000004      CCC          DOUBLE PRECISION CH1,CH2,ATRC
000005      CCC          DIMENSION VAR(2),WK(53),VP(2),VP1(2)
000006      CCC          IF (LIN .EQ. 1) GO TO 300
000007      CCC          TA=-PI/2.0
000008      CCC          TB=PI/2.0
000009      CCC          HSTAR=PI*1.0E-2
000010      CCC          HMIN=PI*1.0E-4
000011      CCC          HMAX=PI/2.0
000012      CCC          ERMAX=1.0E-3
000013      CCC          KEY=0
000014      CCC          CALL ROMBS(TA,TB,THETP,FOFX,HSTAR,HMIN,HMAX,ERMAX,ANS,KSTOP,KEY)
000015      CCC      100 VP(1)=0.0
000016      CCC          VP(2)=THETP
000017      CCC          CALL CMPAT(VP,SL,FF)
000018      CCC          VP(1)=0.0
000019      CCC          VP(2)=VAR(2)-THETP
000020      CCC          CALL DLPAT(VP,SL,FF)
000021      CCC          FOFX=CNM*DELP*COS(THETP)
000022      CCC          CALL ROM2
000023      CCC          IF (KSTOP .EQ. 1) GO TO 100
000024      CCC          CNVR=2.0*PI*ANS
000025      CCC          CALL ROMBS(TA,TB,THETP,FOFX,HSTAR,HMIN,HMAX,ERMAX,ANS,KSTOP,KEY)
000026      CCC      110 VP(1)=0.0
000027      CCC          VP(2)=THETP
000028      CCC          CALL CMPAT(VP,SL,FF)
000029      CCC          VP(1)=0.0
000030      CCC          VP(2)=VAR(2)-THETP
000031      CCC          CALL DLPAT(VP,SL,FF)
000032      CCC          FOFX=CNM*DELI*COS(THETP)
000033      CCC          CALL ROM2
000034      CCC          IF (KSTOP .EQ. 1) GO TO 110
000035      CCC          CNVI=2.0*PI*ANS
000036      CCC          CNVA=SQRT(CNVR**2+CNVI**2)
000037      CCC          GO TO 300
000038      CCC      200 NINT=2
000039      CCC          ERMAX=1.0E-3
000040      CCC          KEY=0
000041      CCC          CALL RME1(NINT,VP ,FUN,ANS,ERMAX,KSTO,WK,KEY)
000042      CCC      210 TA1=0.0
000043      CCC          TB1=2.0*PI
000044      CCC          HSTAR1=PI*1.0E-2
000045      CCC          HMIN1=PI*1.0E-4
000046      CCC          HMAX1=PI/2.0
000047      CCC          CALL RME2(TA1,TB1,HSTAR1,HMIN1,HMAX1)
000048      CCC      220 TA2=-PI/2.0
000049      CCC          TB2=PI/2.0
000050      CCC          CALL RME2(TA2,TB2,0.01*PI,PI*1.0E-4,PI/2.0)
000051      CCC      230 CALL CMPAT(VP,SL,FF)
000052      CCC          VP1(1)=0.0
000053      CCC          VP1(2)=VAR(2)-VP(2)
000054      CCC          CALL DLPAT(VP1,SL,FF)
```

```
000055      000          FUN=DNM*DELR*COS(VP(2))
000056      000          CALL RMD3
000057      000          GO TO (210,220,230,240),KGO
000058      000      240 CNVR=ANS
000059      000          CALL RMD1(NINT,VP ,FUN,ANS,ETMAX,KGO,WK,KEY)
000060      000      250 CALL RMD2(TA1,TB1,HSTAR1,HMIN1,HMAX1)
000061      000      260 CALL RMD2(TA2,TB2,0.D1*PI,PI*1.0E-4,PI/2.0)
000062      000      270 CALL SMPAT(VP,SL,FF)
000063      000          VPI(1)=0.0
000064      000          VPI(2)=VAR(2)-VP(2)
000065      000          CALL SLPAT(VPI,SL,FF)
000066      000          FUN=DNM*DELI*COS(VP(2))
000067      000          CALL RMD3
000068      000          GO TO (250,260,270,280),KGO
000069      000      280 CNVI=ANS
000070      000          CNVA=SQRT(CNVR**2+CNVI**2)
000071      000      300 RETURN
000072      000          END

END SLT.


BFIN
```