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EQUATION OF STATE OF STATIC AND DYNAMIC  
PROBLEMS OF THERMOPLASTICITY UNDER  
COMPLEX LOADING

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Foreign Technology Division  
Wright-Patterson Air Force Base, Ohio

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EQUATION OF STATE OF STATIC AND DYNAMIC PROBLEMS  
OF THERMOPLASTICITY UNDER COMPLEX LOADING

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| Block | Italic     | Transliteration | Block | Italic     | Transliteration |
|-------|------------|-----------------|-------|------------|-----------------|
| А а   | <i>А а</i> | A, a            | Р р   | <i>Р р</i> | R, r            |
| Б б   | <i>Б б</i> | B, b            | С с   | <i>С с</i> | S, s            |
| В в   | <i>В в</i> | V, v            | Т т   | <i>Т т</i> | T, t            |
| Г г   | <i>Г г</i> | G, g            | У у   | <i>У у</i> | U, u            |
| Д д   | <i>Д д</i> | D, d            | Ф ф   | <i>Ф ф</i> | F, f            |
| Е е   | <i>Е е</i> | Ye, ye; E, e*   | Х х   | <i>Х х</i> | Kh, kh          |
| Ж ж   | <i>Ж ж</i> | Zh, zh          | Ц ц   | <i>Ц ц</i> | Ts, ts          |
| З э   | <i>З э</i> | Z, z            | Ч ч   | <i>Ч ч</i> | Ch, ch          |
| И и   | <i>И и</i> | I, i            | Ш ш   | <i>Ш ш</i> | Sh, sh          |
| Й й   | <i>Й й</i> | Y, y            | Щ щ   | <i>Щ щ</i> | Shch, shch      |
| К к   | <i>К к</i> | K, k            | Ъ ъ   | <i>Ъ ъ</i> | "               |
| Л л   | <i>Л л</i> | L, l            | Ы ы   | <i>Ы ы</i> | Y, y            |
| М м   | <i>М м</i> | M, m            | Ь ь   | <i>Ь ь</i> | '               |
| Н н   | <i>Н н</i> | N, n            | Э э   | <i>Э э</i> | E, e            |
| О о   | <i>О о</i> | O, o            | Ю ю   | <i>Ю ю</i> | Yu, yu          |
| П п   | <i>П п</i> | P, p            | Я я   | <i>Я я</i> | Ya, ya          |

\*ye initially, after vowels, and after ъ, ь; e elsewhere.  
 When written as ë in Russian, transliterate as yë or ë.  
 The use of diacritical marks is preferred, but such marks  
 may be omitted when expediency dictates.

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# RUSSIAN AND ENGLISH TRIGONOMETRIC FUNCTIONS

| Russian   | English                    |
|-----------|----------------------------|
| sin       | sin                        |
| cos       | cos                        |
| tg        | tan                        |
| ctg       | cot                        |
| sec       | sec                        |
| cosec     | csc                        |
| sh        | sinh                       |
| ch        | cosh                       |
| th        | tanh                       |
| cth       | coth                       |
| sch       | sech                       |
| csch      | csch                       |
| arc sin   | $\sin^{-1}$                |
| arc cos   | $\cos^{-1}$                |
| arc tg    | $\tan^{-1}$                |
| arc ctg   | $\cot^{-1}$                |
| arc sec   | $\sec^{-1}$                |
| arc cosec | $\csc^{-1}$                |
| arc sh    | $\sinh^{-1}$               |
| arc ch    | $\cosh^{-1}$               |
| arc th    | $\tanh^{-1}$               |
| arc cth   | $\coth^{-1}$               |
| arc sch   | $\operatorname{sech}^{-1}$ |
| arc csch  | $\operatorname{csch}^{-1}$ |
| —         |                            |
| rot       | curl                       |
| lg        | log                        |

## GREEK ALPHABET

|         |   |   |     |         |   |     |
|---------|---|---|-----|---------|---|-----|
| Alpha   | A | α | •   | Nu      | N | ν   |
| Beta    | B | β |     | Xi      | ξ | Ξ   |
| Gamma   | Γ | γ |     | Omicron | Ο | ο   |
| Delta   | Δ | δ |     | Pi      | Π | π   |
| Epsilon | E | ε | •   | Rho     | Ρ | ρ ϑ |
| Zeta    | Z | ζ |     | Sigma   | Σ | σ ς |
| Eta     | H | η |     | Tau     | Τ | τ   |
| Theta   | Θ | θ | •   | Upsilon | Υ | υ   |
| Iota    | I | ι |     | Phi     | Φ | φ ϕ |
| Kappa   | K | κ | κ • | Chi     | Χ | χ   |
| Lambda  | Λ | λ |     | Psi     | Ψ | ψ   |
| Mu      | M | μ |     | Omega   | Ω | ω   |

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References [1]-[8] examine the coupling equations between stresses and strains in differential form, i.e., the connection between the differentials of the stress and strain tensors. In this case it is assumed that neutral loading (loading tangentially to the surface of the loading) does not cause plastic deformation, but the hyperspherical surface of loading is replaced at the point of loading by a hyperplane. The latter is exhibited in the fact that the condition that the new surface of loading passes through the terminus of vector of the final load is replaced by the condition of its passage through the terminus of the vector which is the projection of the vector of final load on the direction of the gradient to the surface of loading at the point of loading.

Experimental determination of the yield surface established the specific tolerance to plastic strain ( $\sim 0.002$ ). A decrease in this tolerance means that the fundamental positions of the phenomenological theory of plasticity cease to be valid. This fact leads to this position that, on the one hand the experimentally obtained increases in the stress and strain tensors cannot be

considered as differentials, but on the other hand, in practical calculations there is no sense to determine plastic deformations to within less than tolerance, i.e., also it is not possible to consider them as infinitesimal. Furthermore, in practical calculations, especially during the solution of dynamic problems, in many instances it is difficult to regulate the size of the tensors of stress and strain increments, and at isolated points they can reach a significant magnitude.

This work attempts to derive the relationships between increases in the stress and strain tensors without relying on the assumption of their smallness, and gives the "physical" algorithm of practical realization of these relationships when solving problems of thermoplasticity.

#### DERIVATION OF FUNDAMENTAL PRINCIPLES

During derivation of fundamental principles, as usual, the stress-strain tensors and their increase are broken down into spherical and deviator parts, and the connection is established separately for each part.

The complete stress tensor  $e_{ij}$  is broken down into elastic  $e_{ij}^v$  and plastic  $e_{ij}^p$  parts:

$$e_{ij} = e_{ij}^v + e_{ij}^p. \quad (1)$$

At a certain point in time let the stress and strain state of the material be characterized by the fields of the stress-strain tensors  $\sigma_{ij}^{cm}$  and  $e_{ij}^{cm}$ , and at the following point in time by tensors  $\sigma_{ij}^{HOB}$  and  $e_{ij}^{HOB}$ .

Then it is possible to write the relationships:

$$\sigma_{ij}^{cr} = 2Ge_{ij}^{cr}, \quad (2)$$

$$\sigma'_{ij}{}^y{}_{\text{нов}} = 2(G + \Delta G)e'_{ij}{}^y{}_{\text{нов}}, \quad (3)$$

where  $G=G(T, \sigma)$  is the shear modulus;

$\Delta G$  - change in the modulus due to change in the temperature  $T$  of the medium and the hydrostatic pressure  $\sigma = \frac{\sigma_{11}}{3}$ .

By deducting from equality (2) equality (3) and carrying out some transformations, we will obtain

$$\Delta\sigma'_{ij}{}^y - \Delta\sigma'_{ij}{}^{**} = 2GP\Delta e'_{ij} + \frac{\Delta GP}{G + \Delta G} \sigma'_{ij}{}^y{}_{\text{нов}},$$

where (see Fig. 1)

$$\Delta\sigma'_{ij}{}^y = 2G\Delta e'_{ij} + \frac{\Delta G}{G + \Delta G} \sigma'_{ij}{}^y{}_{\text{нов}}, \quad (4)$$

$$\Delta\sigma'_{ij}{}^{**} = 2G\Delta e'_{ij}{}^y + \frac{\Delta G}{G + \Delta G} \sigma'_{ij}{}^y{}_{\text{нов}},$$

$$\sigma'_{ij}{}^y{}_{\text{нов}} = \sigma'_{ij}{}^y + \Delta\sigma'_{ij}{}^y, \quad (4)^*$$

$\Delta e_{ij} = \Delta e_{ij}{}^y + \Delta e_{ij}{}^p$  - strain increment tensor;

$$P = \frac{k\Delta\sigma'_{ij}{}^y \{ \{-k\Delta\sigma'_{ij}{}^{**}\} \}}{\{ \{k\Delta\sigma'_{ij}{}^y\} \}}$$

$$\Delta\sigma'_{ij}{}^{**} = \Delta\sigma'_{ij}{}^y - \Delta\sigma'_{ij}{}^{**} \quad (\text{for case a } 1);$$

$$\Delta\sigma'_{ij}{}^{**} = \Delta\sigma'_{ij}{}^y + \Delta\sigma'_{ij}{}^{**} \quad (\text{for case b } 2);$$

Let us assume that the tensor of increase in the plastic strain is coaxial to a certain tensor  $S_{ij}^*$ , whose sense will be explained below:

$$\Delta e_{ij}^p = \lambda S_{ij}^* \quad (5)$$

By utilizing (4) and (5), we will obtain:

$$\Delta \sigma_{ij}^* - \Delta \sigma_{ij}^p = 2(G + \Delta G) \lambda S_{ij}^* \quad (\text{see Fig. 1}). \quad (6)$$

Contracting in (6):

$$(\Delta \sigma_{ii}^* - \Delta \sigma_{ii}^p) S_{ii}^* = 2(G + \Delta G) \lambda S_{ii}^* S_{ii}^*$$

we will obtain for  $\lambda$ :

$$\lambda = \frac{\Delta \sigma_{ii}^* S_{ii}^* - \Delta \sigma_{ii}^p S_{ii}^*}{2(G + \Delta G) S_{ii}^* S_{ii}^*} \quad (7)$$

Let us explain the sense of tensor  $S_{ij}^*$ .

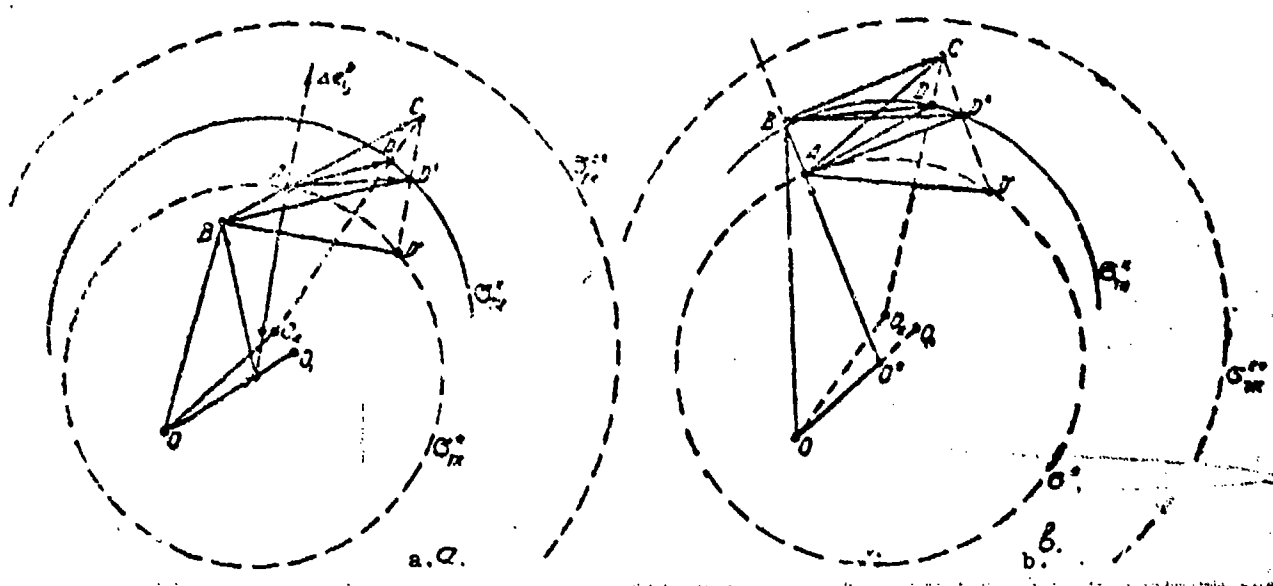


Figure 1

If we reject the previous surface, i.e., consider that the tensor of increase in plastic strain is normal to the previous surface of loading at the point of its intersection with vector  $\Delta\sigma_{ij}^y$ , then by  $S_{ij}^*$  it is necessary to understand  $S_{ij}^0$  (see Fig. 1). In this case the terminus of vector  $\Delta\sigma_{ij}^D$  will lie at the point of intersection of the new surface with the line passing through point c (terminus of vector  $\Delta\sigma_{ij}^y$ ) parallel to vector  $S_{ij}^0$  (point D). This assumption can give considerable error with large values of increase in the strain tensor  $\Delta e_{ij}(\Delta\sigma_{ij}^y)$ .

In this case it is expedient to accept as  $S_{ij}^*$  the vector:

$$S_{ij}^* = [\theta\sigma_{ij}^* - \theta_1\rho_{ij}^{ct} + (1-\theta)\sigma_{ij}^{HOB} - (1-\theta_1)\rho_{ij}^{HOB}],$$

where  $\theta$  and  $\theta_1$  are certain constants:

$$0 \leq \theta \leq 1, \quad 0 \leq \theta_1 \leq 1.$$

when  $\theta=\theta_1=0$  the vector of increase in the plastic strain will be normal to the new surface of loading at point D. With  $\theta=\theta_1=1$  we will obtain the first case.

The connection between the tensor of increase in permanent microstrain and the tensor of increase in the plastic strain is given by the expression:

$$\Delta\rho_{ij} = g \Delta e_{ij}^p + c \rho_{ij}^{ct}, \quad (9)$$

where

$$g = g(T, |\rho_{ij}|, |\dot{e}_{ij}|, \sigma), \quad c = c(|\dot{e}_{ij}|, \sigma)$$

is determined as follows.

If for different materials we use experimental data to graph  $\frac{|\rho_{1j}|}{|\rho_{ij}|}$  as a function of temperature of T, where  $\rho_{ij}$  are coordinates of the center of the yield surface with simple elongation at different values of  $e^p$  and temperature  $T=0^\circ\text{C}$ .

$\rho_{ij}$  are coordinates of the center of the yield surface at certain temperature T (for example, see Fig. 2), then accurate to within experimental values it is possible to consider the ratio

$\frac{|\rho_{1j}|}{|\rho_{ij}|}$  as independent of the degree of plastic strain. This graph

has the following features:

up to a certain value  $T=T^* \frac{|\rho_{1j}|}{|\rho_{ij}|}$  const.=1, then it sharply

decreases to 0 in a narrow temperature range. By having the experimental curve  $\frac{|\rho_{1j}|}{|\rho_{ij}|}$  as a function of T, it is possible to

determine c from the formula

$$c = \frac{\theta_{T_2} - \theta_{T_1}}{\theta_{T_1}} \quad \text{when } \Delta T > 0, \quad (9^*)$$

where  $c=0$  when  $\Delta T < 0$ .

The first and second terms in the right side of formula (8) reflect the two really existing coexisting processes: the process of formation of microheterogeneities with plastic strain and the process of recrystallization. Temperature  $T^*$  is the temperature of recrystallization. Here we do not consider the fact that the process of recrystallization occurs with holding at constant temperature (annealing).

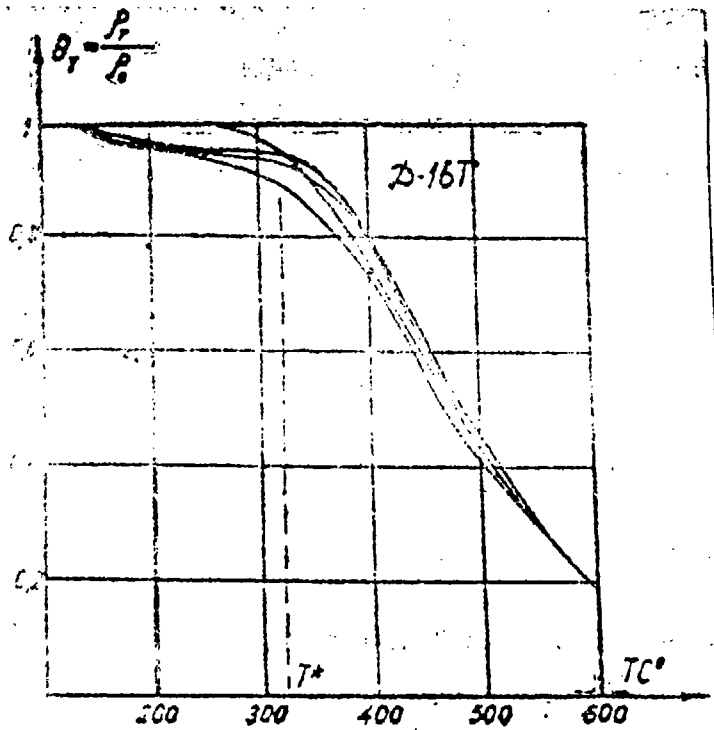
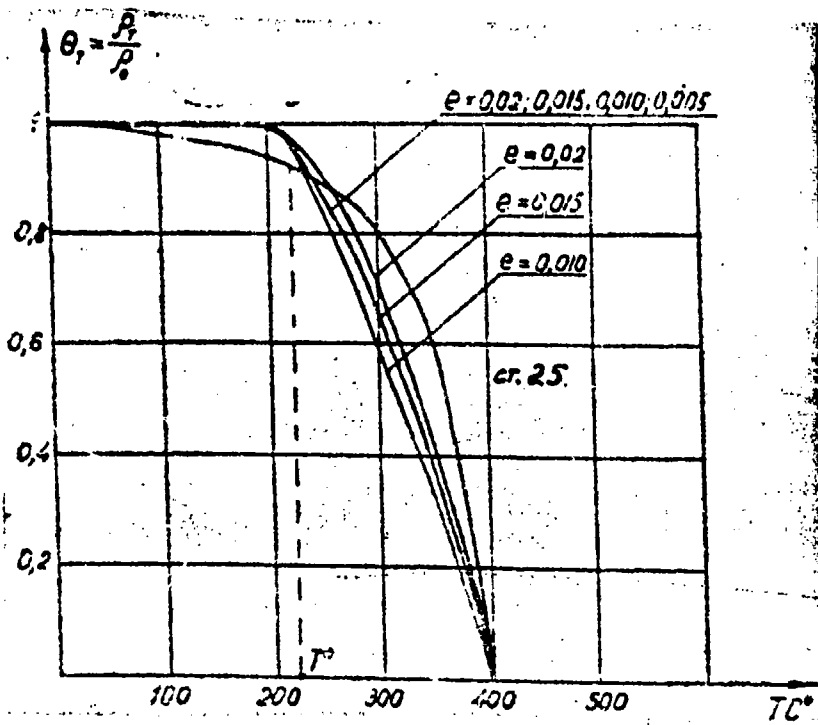


Figure 2

"PHYSICAL" ALGORITHM

Cases a and b Fig. 1 are not distinguished below. Case b differs from case a in that in case a:

$$AC = \{\Delta\sigma'_{ij}\} = \{\Delta\sigma'_{ij}\} - \{\Delta\sigma'_{ij}\},$$

and in case b:  $AC = \{\Delta\sigma'_{ij}\} = \{\Delta\sigma'_{ij}\} + \{\Delta\sigma'_{ij}\}.$

The iterative process of determination of the true value of increase in the tensor of the stress deviator  $\{\Delta\sigma'_{ij}\}=BD$  can be represented by the following algorithm:

1) In the beginning we take  $\theta=\theta_1=1$ . Through the terminus of vector  $\{\Delta\sigma'_{ij}\}$  (point c) we draw a straight line parallel to  $\{S^*_{ij}\}$  up to the intersection with the previous yield surface, shifted and changed because of  $\Delta T, \sigma_{ij}, e$  (point D'). If there is no intersection, in any manner select point D' (deflection along normal or tangentially). The vector of stress tensor increment  $(\Delta\sigma^p_{ij})$  in the 1st approximation is defined as the half-sum of vectors  $\overline{AD'}$  and  $\overline{AC}$ .

Knowing  $(\Delta\sigma^p_{ij})$  in the 1st approximation, we compute  $\lambda$  according to formula (7), and then according to formulas (5) we compute  $(\Delta e^p_{ij})$  and  $(\Delta\rho^**_{ij})_1 = g (\Delta e^p_{ij})_1$ .

We compute  $(S^*_{ij})$  according to formula (8) at some values  $\theta$  and  $\theta_1$ . We compute the new value of the radius of the loading surface  $(\sigma_T^{HOB})_1$  as a function of the new value of the strengthening parameter, calculated in terms of value  $(\Delta e^p_{ij})_1$  according to the formula:

$$(\Delta\sigma'_{ij})_2 = 2G[\Delta e_{ij} - (\Delta e^p_{ij})_1] + \frac{G}{G + \Delta G} (\sigma_{ij}^{HOB}), \quad (10)$$

we find the value of the vector of the stress increment, which corresponds to this value  $(\Delta \epsilon_{ij}^p)_1$ . We test to see whether the terminus of vector  $(\sigma_{ij}^{HOB'})_2 = \sigma_{ij}^{CT'} + (\Delta \sigma_{ij}')_2$  lies on the new loading surface with center

$$(\rho_{ij}^{HOB})_1 = \rho_{ij}^{CT} + (\Delta \rho_{ij}^{*})_1 + c \rho_{ij}^{CT} \quad (11)$$

radius equal to:  $\sqrt{\frac{2}{3} [\sigma_T^{CT} + (\Delta \sigma_T)_1]}$ . This condition must be fulfilled accurate to a certain  $\epsilon > 0$ . If this condition is not satisfied, we find the intersection of the loading surface obtained in this approximation with the straight line passing through point c parallel to  $(S_{ij}^*)$  (point  $D^{HOB1}$ ). For vector  $(\Delta \sigma_{ij}')_2$  in the second approximation we select the vector equal to the half-sum of vectors  $(\Delta \sigma_{ij}')_1$  and vector  $\overline{BD}^{HOB1}$  etc.

The procedure for the correction of the stress tensor proposed in work [9] is a particular case of the given method ( $\rho \equiv 0$ ,  $\sigma_T \equiv \text{const.}$ ,  $\theta \equiv \theta_1 \equiv 0$ ) and does not require the iterative process. If there is a yield delay effect, the given algorithm is put into use only after fulfillment of the conditions:

$$\sum_k \frac{\Delta t^k}{t_k} \geq 1, \quad (11^*)$$

where  $\Delta t^k$  is the k-th time step, and  $t^k$  is the yield delay time [10], which corresponds to an excess of  $\frac{\sigma_u^k - \sigma_T}{\sigma_T}$  at this step, and

$\sigma_u^k$  is the value of stress intensity on this step. In this case, after fulfillment of the conditions (11), it is possible to use different laws governing change in the dynamic yield point.

The above relationships between the strain increment tensor and the stress increment tensor can be presented in anisotropic

form. By transforming (5), (7) and (4)\*, it is possible to obtain:

$$\Delta e_{kl}^{ep} = \frac{\Delta \sigma_{ij}^* S_{ij}^*}{(2G + \Delta G) S_{ij}^* S_{ij}^*} S_{kl}^* - \frac{S_{ij}^* S_{kl}^*}{2(G + \Delta G) S_{ij}^* S_{ij}^*} \Delta \sigma_{ij}^{*p}, \quad (12)$$

$$\Delta e_{kl}^{*y} = \frac{\Delta \sigma_{kl}^*}{2(G + \Delta G)} - \frac{\Delta G}{2G(G + \Delta G)} \sigma_{kl}^{*cr}. \quad (13)$$

By utilizing then the equality  $\Delta e_{ij} = \Delta e_{ij}^y + \Delta e_{ij}^p$ , it is possible to obtain the unknown connection between the increase in the stress tensor and strain tensor in anisotropic form. By converting dependences (12) and (13), it is possible to obtain the expressions of stress tensor increment through the strain tensor increment.

The connection between the spherical stress-strain tensor components in static problems can be presented in the form:

$$\Delta e = \frac{\Delta \sigma}{3k} - \frac{\Delta k}{3k^2} \sigma + \Delta(\alpha T), \quad (14)$$

where  $k=k(T)$  - the bulk modulus,  $\alpha$  - the coefficient of linear expansion,  $\Delta e = \frac{\Delta e_{ij}}{3}$ .

In the case of dynamic problems with the large hydrodynamic pressures which occur during shock wave propagation, it is necessary to utilize the equations of state of solid bodies at high pressures and temperatures in order to know the connection between spherical components.

As shown in work [11], a large role in these equations belongs to the dependence of pressure on internal energy. Subsequently, the connection between the spherical stress-strain

tensor components is established on the basis of equations of state of Mie-Grueneisen [12-13]

$$P = P_n(v) + \frac{\gamma}{v} [E - E_v], \quad (15)$$

where  $P_n(v)$  - the potential part of the pressure, caused by compression of crystal lattice at 0° Kelvin. This dependence can be obtained from the experimental data [14] in the form:

$$P = \sum_i a_i \left( \frac{\rho}{\rho_0} \right)^{i/3-1}, \quad (16)$$

where  $a_i$  - certain experimentally determined coefficients, and  $\rho$  - density of element  $(\rho/\rho_0) = (v_0/v)$ ,  $\frac{\gamma}{v}(E - E_v) = P_T$  - thermal part of pressure.

$\gamma = \gamma(v)$  - Grueneisen's coefficient. It is given in the form:

$$\gamma = \gamma_0 + A \left( \frac{\rho}{\rho_0} - 1 \right) + B \left( \frac{\rho}{\rho_0} - 1 \right)^2 + C \left( \frac{\rho}{\rho_0} - 1 \right)^3, \quad (17)$$

where A, B, C are experimentally determined coefficients.

If the shock adiabat is known for just one material with normal and low density (porous), then  $\gamma(v)$  can be determined by the formula:

$$\gamma = \frac{2v(P_0 - P)}{P_0 v_{00} - P v_0}, \quad (18)$$

or calculated from certain approximation formulas given in [16].

E - specific internal energy.

$$E = E_0(v) + E_T = - \int_{v_0}^v p(v) dv + \Delta z^{cAB} + \int_{T_0}^T c_v dT + E_0, \quad (19)$$

where  $\Delta z^{cAB}$  - energy of deviator strain, and

$c_v$  - specific heat of lattice at constant pressure.

To equation (15) it is necessary to add a certain thermodynamic identity, which characterizes the strain condition. This can be either condition on the discontinuity - the Rankine-Hugoniot equation

$$E - E_0 = \frac{1}{2} (p + p_0) (v_0 - v) + \Delta z^{cAB} \quad (20)$$

or the equation of the adiabatic curve

$$dS = 0, \quad (21)$$

where  $dS$  is the increase in the entropy, or any other thermodynamic relationship, which expresses the condition of conservation of energy. The use of the indicated relationships makes it possible to consider the fact that the shock loading of a material passes along the shock adiabat, and unloading passes along the isentrope.

Utilizing equations (15), (20) or (21), it is possible to calculate the change in temperature at a particular point of the body [12]; temperature is determined from the integral relationship.

$$E_T = E_0 + \int_{T_0}^T c_v dT, \quad (22)$$

where  $E_0$  is initial thermal energy:  $E_0 = \int_0^{T_0} c_v dT$ , and  $c_v$  is determined from formula [12]:

$$c_v = \frac{3R}{\mu} \left[ 4D\left(\frac{\theta}{T}\right) - \frac{3\theta T}{e^{\theta/T} - 1} \right] \quad (23)$$

R - the universal gas constant,

$\mu$  - molecular weight,

D - Debye function

$\theta$  - Debye temperature.

$$D\left(\frac{\theta}{T}\right) = \frac{\pi^4}{15} \left(\frac{T}{\theta}\right)^3 - 3 \sum_{k=1}^{\infty} \frac{1}{k} \left[ 1 + \frac{3}{k} \left(\frac{T}{\theta}\right) + \frac{6}{k^2} \left(\frac{T}{\theta}\right)^2 + \frac{6}{k^3} \left(\frac{T}{\theta}\right)^3 \right] e^{-\frac{k\theta}{T}} \quad \text{when } T < \theta. \quad (24)$$

Since at  $T=\theta$ ,  $c_v$  reaches 96% of its maximum value, when  $T \geq \theta$   $c_v$  is considered constant, and Debye function D will take the form

$$D(\theta) = \frac{\pi^4}{15} - 3 \sum_{k=1}^{\infty} \frac{1}{k} \left[ 1 + \frac{3}{k} + \frac{6}{k^2} + \frac{6}{k^3} \right] e^{-k} \quad (25)$$

Energy sources of neutron irradiation type are considered through the increase in internal energy of the element. After determining the connection between the spherical and deviator stress-strain tensor components, we obtain the complete relationships between stresses and strains, which determine the behavior of the medium with nonisothermal elasto-plastic deformations.

## BIBLIOGRAPHY

1. Вакуленко А. А. О связях между напряжениями и деформациями в изотропных неупругих средах. В сб.: Исследования по упругости и пластичности, № 1, № 2, изд. ЛГУ, 1963.
2. Биргер И. А. Расчет конструкций с учетом пластичности и ползучести. Изд. АН СССР, Механика, № 2, 1965.
3. Кадашевич Ю. И., Новожилов В. В. Теория пластичности, учитывающая остаточные микронапряжения. ПММ, 22, № 1 (1958).
4. Новожилов В. В. О сложном нагружении и перспективах феноменологического подхода к исследованию микронапряжений. ПММ, 28, вып. 3, (1964).
5. Паллей И. З. Приложение теории остаточных микронапряжений к неизотермическому деформированию, изд. АН СССР, Механика, № 2, 1965.
6. Коротких Ю. Г., Белевич С. М. Основные уравнения термопластичности при сложном нагружении. Уч. зап. ГГУ, Серия мех.-мат., вып. 89, 1967.
7. Коротких Ю. Г., Белевич С. М. Плоская задача термопластичности при сложном нагружении. В сб.: Тепловые напряжения в элементах конструкций, Киев, 1968.
8. Белевич С. М. Расчет плоской задачи термопластичности при сложном нагружении. В настоящем сборнике.
9. Уилкинс. Расчет упруго-пластических течений. Вычислительные методы в гидродинамике, 1967.
10. Волошенко-Климовицкий Ю. Я. Динамический предел текучести, 1965.
11. Hoffman R., Andren D., Maxwell D. Computed shock response of porous aluminum "J. Appl Phys", 1968, 39, № 10.
12. Жарков В. И., Калинин В. А. Уравнения состояния твердых тел при высоких давлениях, 1968.
13. Ман-Кунг. Сжатие твердых тел сильными ударными волнами. Из книги: Динамические исследования твердых тел при высоких давлениях, 1965.
14. Альтшулер Л. В., Кормер С. Б., Баканова А. А., Трунин Р. Ф. Уравнения состояния алюминия, меди и свинца для области высоких давлений, ЖЭ и ТФ, 38, вып. 3 (1968).
15. Огибалов П. М., Кийко И. А. Поведение вещества под давлением, изд. МГУ, 1962.
16. Зельдович Б. Я. Физика ударных волн и гидродинамических высокотемпературных явлений. 1966.