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IRRADIANCE SCINTILLATIONS: COMPARISON OF THEORY WITH EXPERIMENT

Ronald L. Fante

Air Force Cambridge Research Laboratories
Hanscom Air Force Base, Massachusetts

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Calculations are made using a new theory for the covariance of the irradiance scintillations of an optical wave in a turbulent medium. These are compared with recent experimental data and found to be in good agreement.

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Preface

The author is grateful to Richard Taylor for his assistance in the calculation of $f_3(S)$ and $g(W)$.

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Irradiance Scintillations: Comparison of Theory With Experiment

I. INTRODUCTION

Considerable effort has been devoted recently to the study of the variance and covariance of the irradiance fluctuations of a wave propagating in a random medium. The impetus for these studies was provided by the experimental measurements of Gracheva et al,[1] who found that the variance σ_I^2 of the irradiance fluctuations saturated for very strong turbulence, in disagreement with theoretical predictions.[2,3] In the last year, several physical models have been postulated to calculate the variance and covariance of the irradiance fluctuations,[4,5] but both rely on the introduction of an arbitrary parameter which cannot be specified

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[1] Gracheva, M., Gurvich, A., and Kallistrova, M. (1970) Dispersion of strong atmospheric fluctuations in the intensity of laser radiation, Radiophys. and Quantum Electronics **13**:40-42.

[2] Tatarski, V. (1971) The Effects of the Turbulent Atmosphere on Wave Propagation (National Technical Information Service, U.S. Dept. of Commerce, Springfield, Va.

[3] Lawrence, R. and Strohbehn, J. (1970) A survey of clean-air propagation effects relevant to optical communications, Proc. IEEE **58**:1523-1545.

[4] Clifford, S., Ochs, G., and Lawrence, R. (1974) Saturation of optical scintillation by strong turbulence, J. Opt. Soc. Am. **64**:148-154.

[5] Yura, H. (1974) Physical model for strong optical-amplitude fluctuations in a turbulent medium, J. Opt. Soc. Am. **64**:59-67.

except by comparison with experimental data. In contrast to these other theories, we have recently[6] developed a new model which is free from these limitations, but agrees with the theories in references 4 and 5 on the physical mechanisms which produce saturation. To outline and amplify on our theory, let us consider the fourth moment of the electric field

$$\Gamma_4 = < e(L,\underline{\rho}_1)\, e(L,\underline{\rho}_2)\, e^*(L,\underline{\rho}_1')\, e^*(L,\underline{\rho}_2') > , \qquad (1)$$

where L is the distance measured along the direction of propagation and $\underline{\rho}$ is the distance measured transverse to the propagation path. Tatarski[7] has derived the equation satisfied by Γ_4, but it has not proved possible to obtain solutions, except in the limit of weak turbulence. Our technique is to obtain an iterative solution to this equation starting with the solution for infinitely strong turbulence. For $\sigma_1^2 = \infty$, where $\sigma_1^2 = 1.23\, k_o^{7/6}\, C_n^2\, L^{11/6}$, k_o is the signal wavenumber and C_n^2 is the index of refraction structure constant, it can be seen[8] that the statistics of the field are gaussian, since the field at (L, 0) consists of the independent contributions from very many off-axis eddies. We emphasize that the statistics are exactly gaussian only for $\sigma_1^2 = \infty$. For finite σ_1^2, we have shown that the statistics of the field cannot be gaussian. In fact, for $0 < \sigma_1^2 < 100$, it appears that the field statistics are very nearly log-normal.[9] Consequently, for $\sigma_1^2 = \infty$ we have from Eq. (1):

$$\Gamma_4 \xrightarrow[\sigma_1^2 \to \infty]{} |\Gamma_2(L,\underline{r}_1)|^2 + |\Gamma_2(L,\underline{r}_2)|^2 , \qquad (2)$$

where Γ_2 is the mutual coherence function and $\underline{r}_1 = (\underline{\rho}_1 - \underline{\rho}_2 + \underline{\rho}_1' - \underline{\rho}_2'/2$, $\underline{r}_2 = (\underline{\rho}_1 - \underline{\rho}_2 - \underline{\rho}_1' + \underline{\rho}_2')/2$.

---

[6] Fante, R. (1975) Electric field spectrum and intensity covariance of a wave in a random medium, Radio Sci. 10:

[7] Tatarski, V. (1969) Light propagation in a medium with random refractive index inhomogenieties in the Markov random process approximation, Soviet Physics JETP 29:1133-1138.

[8] DeWolf, D. (1974) Waves in turbulent air; a phenomenological model, Proc. IEEE 62:1523-1529.

[9] Gracheva, M., Gurvich, A., Kashkarov, S., and Pokesov, V. (1974) Similarity Correlations and their Experimental Verification in the Case of Strong Intensity Fluctuations of Laser Radiation (in Russian). English translation available from Aerospace Corp., Library Services, P.O. Box 92957, Los Angeles, Calif. 90009, Translation No. LRG-73-T-28.

If we use Eq. (2) as a first iteration in the integral equation, given in Eq. (72. 1) of the book by Tatarski,[2] for the Fourier transform of Γ_4, we can show a Kolmogorov spectrum that the covariance,

$$B_I(L, \underline{r}) \equiv \frac{\langle I(L, \frac{\underline{r}}{2})\; I(L, -\frac{\underline{r}}{2})\rangle - \langle I\rangle^2}{\langle I\rangle^2} \tag{3}$$

of the irradiance fluctuations of a plane wave in homogeneous turbulence is given by[6]

$$B_I(L, R) = \exp\left\{-11.2\,\sigma_1^2\, R^{5/3}\right\} + \frac{1}{(\sigma_1^2)^{2/5}} \left\{ f_3\left[\frac{R}{(\sigma_1^2)^{3/5}}\right] + g\left[(\sigma_1^2)^{3/11}\, R\right]\right\}, \tag{4}$$

where it is assumed that $\sigma_1^2 >> 1$. Also $R = r/(\lambda_o L)^{1/2}$, λ_o is the signal wavenumber and

$$f_3(S) = 1.43 \int_0^1 y^{-1/3}\, dy \int_0^\infty dt\, t^{2/5}\, e^{-t(4.26 - 2.66\, y)}\, J_0(3.54\, t^{3/5} y^{-1} S) \tag{5}$$

$$g(W) = 0.27 \int_0^\infty dt\, t^{-8/3}\, (1 - \cos t) \int_0^\infty ds\, e^{-s}\, J_0(2.43\, t^{8/3}\, s^{-3/11}\, W)\,. \tag{6}$$

The first two terms in Eq. (4) are the result of one iteration, while the last term arises from a second iteration of the solution. The functions f_3 and g were not computed in reference 6, but have since been evaluated, and are plotted in Figures 1 and 2. We note from Eq. (4) that for $\sigma_1^2 >> 1$, the covariance varies on three different scale lengths. The first term in Eq. (4) dominates over scales R of order $(11.2\,\sigma_1^2)^{-3/5} << 1$, and corresponds to the rapid dropoff in the covariance function in strong turbulence which has been observed experimentally.[9,10] The second term is important over separations R of order $(\sigma_1^2)^{3/5} >> 1$, and corresponds to the long tail seen in experimental measurements. The last term in Eq. (4) varies over separations R of order $(\sigma_1^2)^{-3/11}$ and is important in the transition region between the first and second terms.

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[10] Dunphy, J. and Kerr, J. (1973) Scintillation measurements for large integrated path turbulence, J. Opt. Soc. Am. **63**:981-986.

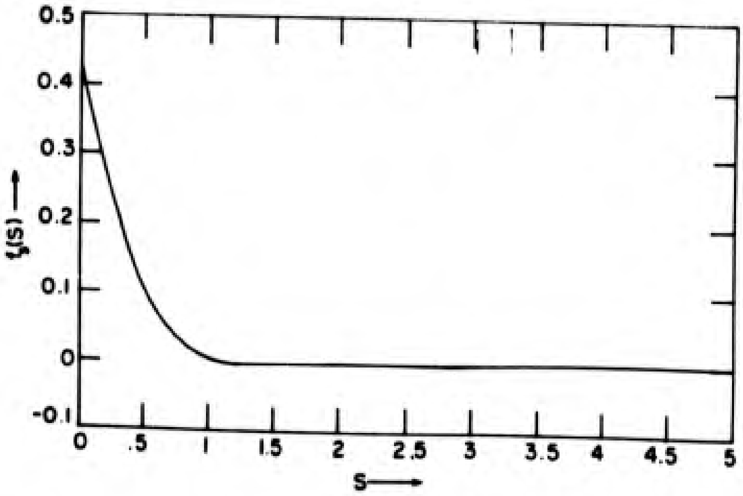

Figure 1. Plot of the Function $f_3(S)$ Defined in Eq. (5)

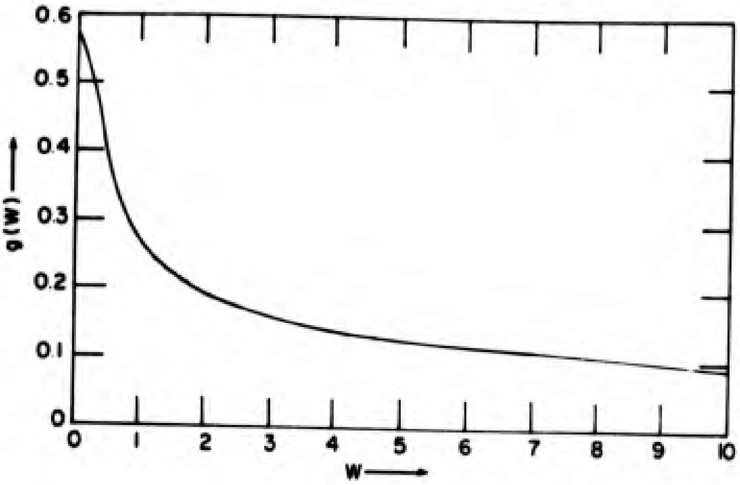

Figure 2. Plot of the Function g(W) Defined in Eq. (6)

2. COMPARISON WITH EXPERIMENTAL DATA

Recently Gracheva[9] and his co-workers have made careful measurements of the irradiance covariance of laser beams propagating in strong turbulence. These measurements were performed for both collimated and diverging beams. Unfortunately, none of these experimental results are valid for strictly plane waves, as is the case with our theoretical results; therefore, we shall not be able to make a one-to-one comparison of our theory with their data. However, it can be shown that except possibly in the focal plane, the beam results should be the same order as the plane wave measurements. Therefore, in what follows we shall compare our plane wave theory with the experimental results for a finite beam, keeping in mind that the two may be slightly different in magnitude, but should display the same trends.

In Figure 3 we present the theoretical and experimental results for the standard deviation $\sigma_I = [B_I(L, 0)]^{1/2}$ of the irradiance fluctuations. The solid curve is the average of the experimental points; the actual[9] experimental data points were considerably dispersed about this curve, lying within the envelope indicated by the pair of dashed curves. We note that the theoretical results display the same trend as the experimental ones, but are on the average somewhat lower. At present we are uncertain whether this is so because the theory is for plane waves while the experiments are for a finite beam, or because we have failed to include the next higher order iteration to Eq. (4) in our theoretical solution. These points are currently under investigation.

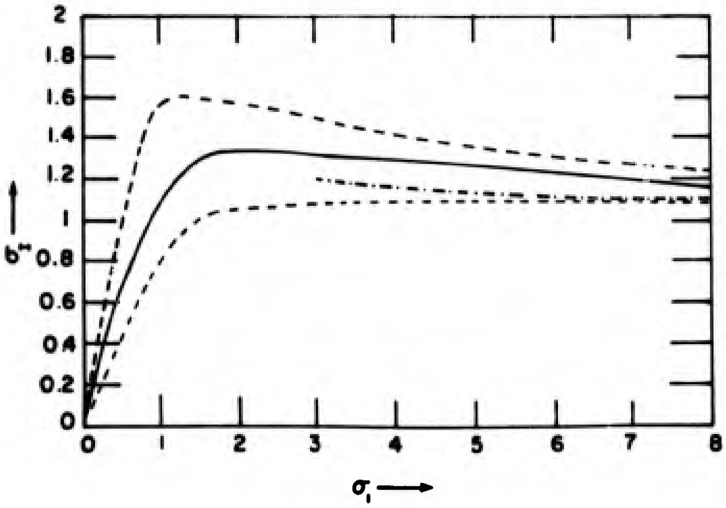

Figure 3. Standard Deviation of the Irradiance Fluctuations. The experimental[9] data points for a collimated beam fall within the envelope given by the dashed (---) curves, while the solid curve (——) is the average of the experimental data points. The theoretical plane wave result calculated from Eq. (4) is given by the (– · – ·) curve

In Figures 4 through 6, we compare the theoretical predictions from Eq. (4) with the experimental data[9] for the normalized covariance function,

$$b_I(R) = \frac{B_I(L, R)}{B_I(L, 0)} .$$

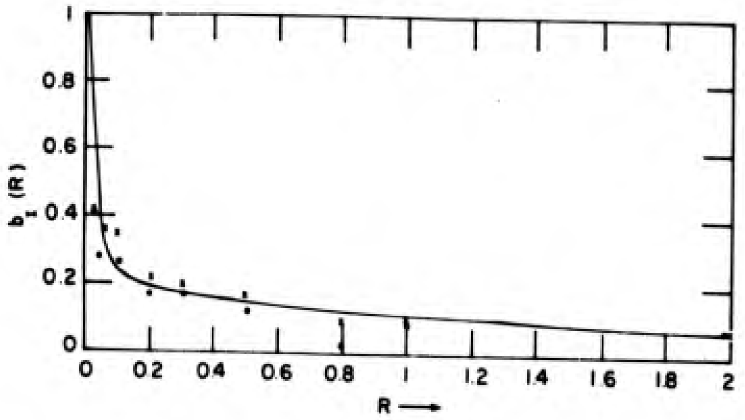

Figure 4. Covariance of the Irradiance Fluctuations for $7.9 < \sigma_1^2 < 27$. The solid curve is the theoretical plane wave result calculated using Eq. (4) The points (x) are the average experimental[9] results for a collimated beam, and the points (o) are the averages for a divergent beam

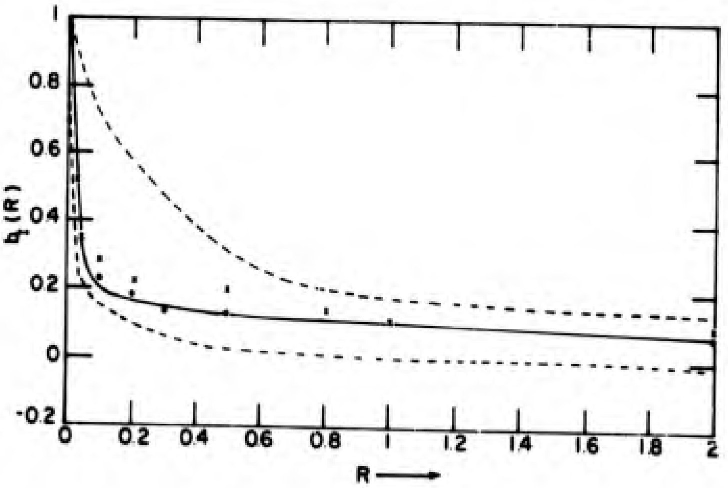

Figure 5. Covariance of the Irradiance Fluctuations for $\sigma_1^2 > 27$. The solid curve is calculated from Eq. (4), and the dashed curves are the envelope of the band over which data points were found experimentally.[9] The averages of the experimental data points for a collimated beam are indicated by x, while the averages for a divergent beam are indicated by o

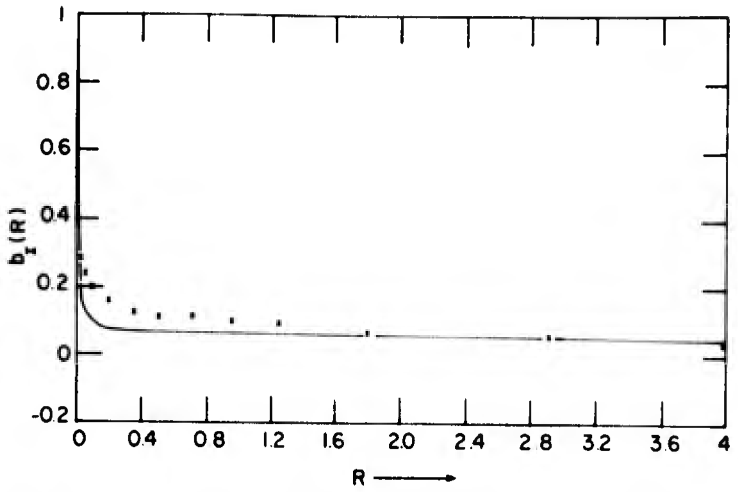

Figure 6. Covariance of the Irradiance Fluctuations for $200 \le \sigma_1^2 < 1600$. The solid curve is calculated from Eq. (4) and the averages of the experimental data points for a collimated beam are indicated by x

We note that the agreement is quite good. In the figures, the points denoted by (x) are the average of the experimental[9] data points for a collimated beam, while the points marked (o) are the average for a divergent beam. In order to indicate the dispersion of the measured data points about these averages, we have shown by a pair of dashed curves on Figure 5 the envelope within which the actual data points lie.

3. CONCLUSIONS AND DISCUSSION

The theoretical result of Eq. (4) for the covariance of the irradiance fluctuations of a plane wave has been compared with available experimental data. The agreement for the normalized covariance is quite good, but the agreement between Eq. (4) and experimental data for the standard deviation is only reasonably good.

We are presently extending the results of Eq. (4) to spherical waves and finite beams. The former solution can be obtained by applying the iterative solution technique described in Section 1 to the integral equation for the covariance of a spherical wave derived by Shishov,[11] whereas the latter can be obtained by an iterative solution of Eq. (71.35) in Tatarski's[2] book.

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[11] Shishov, V. (1972) Strong fluctuations of the intensity of a spherical wave propagating in a randomly refractive medium, *Radiophys. and Quantum Electronics* **15**:689-695.

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5. Yura, H. (1974) Physical model for strong optical-amplitude fluctuations in a turbulent medium, *J. Opt. Soc. Am.* **64**:59-67.
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