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THE IDEAL SEARCH THEOREM

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**STUDY 297**

**THE IDEAL SEARCH THEOREM**

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### THE IDEAL SEARCH THEOREM

It is frequently desired to find the best search plan to detect a given target, in the sense of that plan of a given total track length which gives the maximum probability of detection. It is usually impossible to find an analytic solution to this problem. The usual method of attack is to construct a number of plans, and compute the value of  $P$  for varying parameters to maximize  $P$ .

This method is hampered by a lack of knowledge of any upper bound to  $P$ , and even if the optimum plan were found, it would be difficult to establish the fact that there is no better plan.

The ideal search theorem furnishes such an upper bound. In many cases, it will be found that a trial plan yields a value of  $P$  which is not very much below  $P_I$ . In such cases one knows immediately that not much improvement is possible, and no further efforts need be made.

G.E. Kimball, /s/  
Operations Evaluation Group.

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## APPENDIX

### Definitions and Symbols

We consider here a search conducted with the object of locating a single target known to lie within an area  $T$  (the target area). Our knowledge of the position of the target is expressed by a function  $t(x, y)$ , such that the probability that the target position lies within the area element  $dx dy$  at the point  $x, y$  is  $t(x, y) dx dy$ , and

$$\int_T t(x, y) dx dy = 1 \quad (1)$$

It is assumed that the search plan is to consist of a series of straight tracks, and that the detection curve, that is, the probability  $p(Z)$  of detecting a target at a distance  $Z$  from a given track, is the same for all tracks. Effects due to turns where the tracks join are assumed to be negligible. The sweep width,  $W$ , is defined by

$$\int_{-\infty}^{\infty} p(Z) dZ = W \quad (2)$$

For any given search plan there exists a function  $s(x, y)$  which expresses the probability of detecting the target if the target is at the point  $x, y$ . If  $Z_j$  is the distance of the point  $x, y$  from the  $j^{\text{th}}$  track of the search plan,

$$1 - s(x, y) = \pi_j (1 - p(Z_j)) \quad (3)$$

If  $L$  is the total track length of the search plan, it is easily seen that

$$\int_T s(x, y) dx dy \leq LW \quad (4)$$

The total probability of detection of the target by the given plan is

$$P = \int_T s(x, y) t(x, y) dx dy \quad (5)$$

### The Ideal Search

It is assumed that the function  $t(x, y)$  is of such a character that the points of  $T$  at which  $t(x, y) \geq t'$ , where  $t'$  is any positive number, form a finite number of regions possessing a definite area  $A(t')$ . As  $t'$  increases from 0 to infinity,  $A(t')$  decreases monotonically from a value  $T$  to 0. It is then possible to choose an area  $I$ , the ideal search area, such that

- (a) the area of  $I$  is  $LW$
- (b) within  $I$ ,  $t(x, y) \geq t_0$ , and outside  $I$ ,  $t(x, y) \leq t_0$ ,  
where  $t_0$  is some positive number.

An ideal search is defined as a search carried out in such a way that

$$\begin{aligned} s(x, y) &= 1 \quad \text{inside } I \\ s(x, y) &= 0 \quad \text{outside } I . \end{aligned}$$

It goes without saying that such a search is ordinarily impossible.

The probability of detection for the ideal search is

$$P_I = \int_I t(x, y) dx dy . \quad (6)$$

### The Ideal Search Theorem

The importance of the ideal search concept arises from the following theorem: The probability of detection of a target by any given search plan is not greater than that of the corresponding ideal search. To prove this, we begin with equation (5).

$$\begin{aligned}
P &= \int_T s(x, y) t(x, y) dx dy \\
&= \int_I s(x, y) t(x, y) dx dy + \int_{T-I} s(x, y) t(x, y) dx dy \\
&= \int_I t(x, y) dx dy - \int_I (1-s(x, y)) t(x, y) dx dy \\
&\quad + \int_{T-I} s(x, y) t(x, y) dx dy \\
&\leq \int_I t(x, y) dx dy - \int_I (1-S(x, y)) t_0 dx dy \\
&\quad + \int_{T-I} s(x, y) t_0 dx dy \\
&\leq \int_I t(x, y) dx dy - \int_I t_0 dx dy + \int_T S(x, y) t_0 dx dy \\
&\leq P_I - L W t_0 + L W t_0 \\
&\leq P_I .
\end{aligned}$$