AD-A008 038

DETECTION, SUPPRESSION AND FRACTIONAL SUPPRESSION. A PRELIMINARY EXAMINATION

Timothy J. Horrigan

Horrigan Analytics

Prepared for:

Army Strategy and Tactics Analysis Group

31 August 1973

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DETECTION, SUPPRESSION
AND FRACTIONAL SUPPRESSION

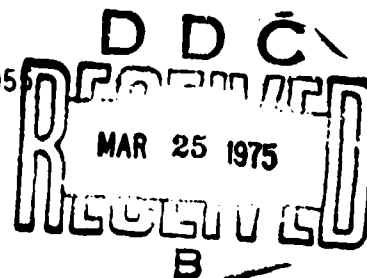A Preliminary Examination

Timothy J. Horrigan

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Prepared For

U. S. Army Strategy and Tactics Analysis Grcup
8120 Woodmont Avenue
Bethesda, Maryland 20014

In Accordance With

Contract   DAAB09-71-C-005

31 August 1973

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MAR 25 1975

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document HA 73-3

ADA008038

DOCUMENT CONTROL DATA - R & D

(Security classification of title, body of abstract and indexing annotation must be entered when the overall report is classified)

1. ORIGINATING ACTIVITY (Corporate author)	2a. REPORT SECURITY CLASSIFICATION
HORRIGAN ANALYTICS 1460 North Sandburg Terrace Chicago, Illinois 60610	unclassified 2b.

3. REPORT TITLE

DETECTION, SUPPRESSION, AND FRACTIONAL SUPPRESSION

4. DESCRIPTIVE NOTES (Type of report and inclusive dates)

FINAL REPORT

5. AUTHOR(S) (First name, middle initial, last name)

Timothy J. Horrigan

6. REPORT DATE	7a. TOTAL NO. OF PAGES	7b. NO. OF REFS
31 August 1973	42	two

8a. CONTRACT OR GRANT NO. DAAB09-71-C-0055 b. PROJECT NO. c. d.	9a. ORIGINATOR'S REPORT NUMBER(S) HA 73-3 9b. OTHER REPORT NO(S) (Any other numbers that may be assigned this report) none

10. DISTRIBUTION STATEMENT

Approved for public release; distribution unlimited.

11. SUPPLEMENTARY NOTES	12. SPONSORING MILITARY ACTIVITY U.S. Army Strategy and Tactics Analysis Group 8120 Woodmont Avenue Bethesda, Maryland 20014

13. ABSTRACT

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PRICES SUBJECT TO CHANGE

14. KEY WORDS	LINK A		LINK B		LINK C	
	ROLE	WT	ROLE	WT	ROLE	WT
BOMBARDMENT						
CASUALTIES						
COMBAT MODELING						
DETECTION						
MUNITION SUPPRESSIVE EFFECTS						
NEUTRALIZATION BOMBARDMENT						
POISSON PROCESSES						
PROBABILITY THEORY						
SEARCH						
STOCHASTIC PROCESSES						
SUPPRESSION						
SUPPRESSION-DETECTION INTERACTION						
TACTICS						
VULNERABILITY						

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suppression is established.

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I. INTRODUCTION AND CONCLUSIONS

Suppression, as a concept, is given a simple operational explication through a 'single-round period of suppressive effect' which is associated with each projectile impacting in the vicinity of a combatant. During each such single-round period of suppressive effect, which commences at some indicator event, the affected combatant is *suppressed*; at all other times the combatant is *unsuppressed*. A *period of suppression* for a combatant that is unsuppressed begins with an event that produces a nonzero, single-round period of suppressive effect; and it ends when the affected combatant first thereafter becomes unsuppressed. Arbitrarily long random periods of suppression for the affected combatant may thus arise from overlap between consecutive single-round periods of suppressive effect.
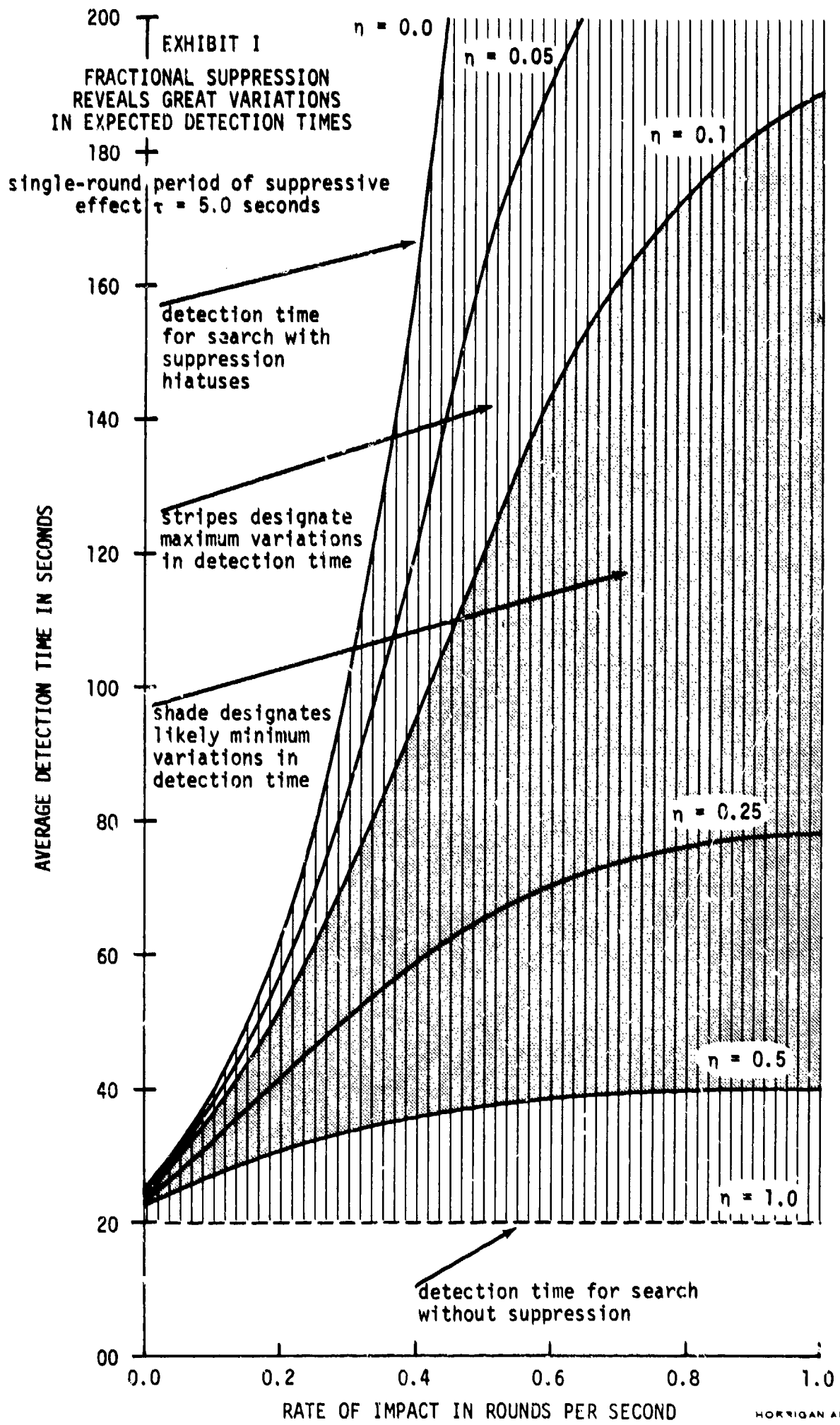
By proceeding from this definition, expected durations of periods of suppression are deduced under very general conditions for situations in which the impact times of the associated projectiles are adequately represented by independent Poisson processes with constant intensities. The resulting model is mathematically exact, and it includes:

- Arbitrary, random durations for individual single-round periods of suppressive effect that stochastically depend on the miss-distance of the associated projectile

- An arbitrary number of different, nonuniform impact distributions for each type of projectile

- Different distributional characteristics for the single-round period of suppressive effect associated with each distinct pair of projectile-target types

The formulas which result are remarkably simple; they depend only on the average durations of the random single-round periods of suppressive effect and the average arrival rates for the associated rounds. Expected detection times for search processes in which the search activity is suspended during periods of suppression retain the same simplicity.

In those situations the expected durations of a period of suppression and of a period to a detection grow exponentially both with the rates at which projectiles impact and with the average durations of the probabilistically different, single-round periods of suppressive effect. When the detection rate during suppression is small but not identically zero, the corresponding expected detection times can be much smaller than they are when that rate is identically zero. Indeed, they can become small enough to make the usual all-or-nothing representations of suppressive effect unsatisfactory for many typical applications. Fractional suppression, a more satisfactory concept, is introduced to accommodate nonzero activity rates during suppression.

Exhibit I, following this page, summarizes well our theoretical findings. The expected times needed by a suppressed combatant to make a detection are plotted as a function of the impact rate in its vicinity for a selection of 'suppression fractions', fractional rates at which activities can proceed

EXHIBIT I

FRACTIONAL SUPPRESSION
REVEALS GREAT VARIATIONS
IN EXPECTED DETECTION TIMES

single-round period of suppressive
effect τ = 5.0 seconds

detection time
for search with
suppression
hiatuses

Stripes designate
maximum variations
in detection time

shade designates
likely minimum
variations in
detection time

n = 0.0
n = 0.05
n = 0.1
n = 0.25
n = 0.5
n = 1.0

AVERAGE DETECTION TIME IN SECONDS

RATE OF IMPACT IN ROUNDS PER SECOND

detection time for search
without suppression

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during periods of suppressive effect. The combat situation is such that
the expected detection time in the absence of suppressive fire is twenty
seconds, a situation which, in terms of fractional suppression, provides
the same expected detection time as one with fire that has no suppressive
effect whatsoever. Such ineffectual fire is characterized by a unit
suppression fraction ($\eta = 1$); and the associated expected detection time
is constant with respect to the impact rate, as the dashed line in the
exhibit indicates. More effectual fire of course forces the combatant
into concealment or into cover and tends to degrade progressively the
rate, scope, and quality of its activity.

Suppression models typically assume that the resulting degradation is
total: suppression introduces a hiatus in whatever activity the combatant
was pursuing. In the case of a search activity the search rate becomes
zero, and therefore the detection rate becomes zero as well. Because the
analytical formulas devised in the course of the subject investigation
permit the direct computation of expected detection times in the presence
of periods of total and fractional suppression, the consequences of that
assumption can be examined in relation to what happens when the detection
rate, although diminished by suppression, remains greater than zero.

When a single-round period of suppressive effect produces a hiatus with
a five-second duration, for example, expected detection times increase
substantially even for very low impact rates. They increase from the
dashed line ($\eta = 1$) at the bottom of the vertical stripes in the exhibit,
where the effect of the suppressive fire on detection is nil, to the curve
($\eta = 0$) at the top of the stripes, where the detection rate is forced to
zero during periods of suppression. The divergence between the dashed line
and that curve, marked by the vertical stripes, emphatically illustrates
the suppression-induced extremes in expected detection times for rates of
impact up to one round per second. A rate of impact producing only one
impact every ten seconds doubles the expected detection time. Suppressive
fires with single-round periods of suppressive effect that force activity
rates to zero are thus overwhelming. For that reason whether the rate is
forced *exactly* to zero becomes a crucial question: if not, then repre-
senting suppression as a hiatus leads to a major misrepresentation of an
important combat factor and, in the case of search activities, to a major
overestimation of expected detection times.

Suppressive effect resulting in search rates that are one-half ($\eta = 0.5$)
to one-tenth ($\eta = 0.1$) of what they would be in the absence of suppressive
fires is prima facie much more plausible than that producing only an exactly
zero suppression fraction. The shaded region in Exhibit I shows the corre-
sponding range of expected detection times. Although it is less extensive
than the maximum range, the extremes it covers are indeed great.

Small misestimates in the suppression fraction or its equivalent tend to
produce large errors in the resulting expected detection times. Small
variations in the impact rate can similarly produce large changes in the
resulting expected detection times, once a moderate or better rate has
been achieved. For all these situations the respective coefficients of
variation (ratios of the standard deviations to the corresponding expected

detection times) are approximately unity; hence, when the expected detection times are moderate or larger, the random fluctuations about them will be great. Both the expected detection times and the random fluctuations are strongly amplified by even slight additional increases in the expected detection time in the absence of suppression. Additional increases in the duration of a single-round period of suppressive effect, however, have little additional effect, except for very low rates of fire.

Fractional suppression thus shows that the overall suppressive effect of increased rates of fire reaches a point of diminishing return, after which additional increase in the rate of fire produce no appreciable increases in expected detection times. Casualty production further limits such increases in expected detection times, because a combatant must survive in order to detect. The greatest changes in predicted expected detection times are introduced by departures from zero-rate, all-or-nothing suppression; hence, for all but the lowest impact rates, the typical idealization of suppression as a hiatus is ill-advised.

Beyond that noteworthy, cautionary fact the understanding and quantification of suppression achieved during the subject investigation focus attention on suppression's inherently voluntary nature, the singular importance of which is emphatically illustrated by the major changes made in the expected detection times by variations in the suppression fraction, not to mention additional, large changes made in response to variations in the duration of a single-round period of suppressive effect. As suppression is voluntary, a combatant need never be suppressed, provided survival is of no importance. Somewhere, however, between everyone's charging into the sustained fire of machine guns -- a discipline entailing essentially zero survival prospects -- and hiding permanently, head first in a foxhole with similar survival prospects, there exists a degree and a duration for a single-round period of suppressive effect, which may vary from combatant to combatant in accordance with their tactical roles, that optimizes survival prospects.

Suppression, in summary, cannot be quantified in the sense that weight, distance, or even ballistic errors can be. It is different from them because it is mainly voluntary and therefore admits degrees as well as systematic and unsystematic variations. Examination of analytical results, of which Exhibit I is representative, and reflection thereon suggest two major conclusions:

- suppression idealized as a hiatus in combat activities is analytically unsatisfactory and counterproductive

- the degree and duration of single-round periods of suppressive effect can be chosen to maximize tactical advantage.

Jointly these conclusions provide a much improved vantage for considering suppression: they suggest that the more important question about suppressive effect is not what it *was* in past situations, but rather what it *should be* in a given combat situation to provide that balance between immediate survival and ability to return fire which yields the maximum tactical advantage.

II. A SIMPLE MODEL FOR SUPPRESSION AND DETECTION

Suppression is initially idealized herein as a hiatus introduced into a combatant's activity by the nearby impact of a round. Such a hiatus, when associated with a single round, is defined to start at the time of the impact or other indicator and to continue for a positive duration thereafter. It is termed a *single-round period of suppressive effect*; 'volley' or 'burst' may, of course, be substituted for 'round' when appropriate.

The duration of a single-round period of suppressive effect is inherently voluntary and accordingly may vary widely from combatant to combatant and even from one combatant at a given time to that same combatant at another time. Miss-distance, environment, and round type are additional, important sources of variations. However, because the duration is voluntary, speaking of a constant duration is meaningful notwithstanding what may be its actual, probably great variation from instance to instance.

So long as all inter-round impact times exceed the duration of a single-round period of suppressive effect, the total time during which a combatant is suppressed is defined to be the sum of the individual durations. When additional rounds impact during an existing period of suppressive effect, that period will be prolonged, at least until cessation of the single-round period of suppressive effect associated with the last of the additional rounds. A period of suppression for a combatant is consequently defined to terminate when an inter-impact time first exceeds the duration of a single-round period of suppressive effect. The discipline thus prescribed for the idealized combatant is that its combat activities are to be resumed at the expiration of the single-round period of suppressive effect associated with the last impact in its proximity.

(1) Suppression at Its Simplest

Together these concepts determine a nearly irreducibly simple mathematical model of suppression. It requires only

- a region of suppressive affect associated with each combatant

- a constant duration τ for the single-round period of suppressive effect caused by an impact in the affect region

- a Poisson process $N^*(t)$ with constant intensity λ for the impact stream within the affect region

so that λ and τ, two parameters, alone need quantification. $N^*(t)$ is of course the impact point process, the number of impacts in the affect region in a duration t. Define S^* to be the random duration of a resulting period of suppression.

Without loss of generality the combatant may be assumed to be suppressed initially by an impact in its region of suppressive affect at time zero. It will thus remain suppressed at least until τ ; whether it continues to

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be suppressed at some time t depends on whether an appropriate number of timely additional impacts occur. On the hypothesis that $N^*(t) = n$, the impact times of the n rounds in the affect region are uniformly and independently distributed on the interval $[0,t]$ because $N^*(t)$ is Poisson. If T_i^* for $i = 1, 2, \ldots, n$ respectively designate the random inter-impact times for those rounds, and if T_{n+1}^* is the duration between the last impact and t, it follows from a theorem[1] of De Finetti (with Λ designating the extended 'and' operation) that

$$\Pr\{ \overset{n+1}{\underset{1}{\Lambda_i}} \ T_i^* > t_i \} = (1 - \frac{1}{t} \sum_{1}^{n+1} {}_i \ t_i)_+^n$$

when $(x)_+$ designates the positive part of x : $(x)_+ = 0$ for $x<0$, and $(x)_+ = x$ otherwise. By virtue of an identity[2] for the realization of none out of m events (with $m = n+1$), the probability that all the T_i^* are equal to or less than τ is:

$$\Pr\{ \overset{n+1}{\underset{1}{\Lambda_i}} \ T_i^* \leq \tau \} = \sum_{0}^{n+1} {}_k \binom{n+1}{k} (-1)^k (1-k\tau/t)_+^n .$$

Since the duration S^* of the period of suppression exceeds t if and only if all the T_i^* are equal to or less than τ, it follows that the right member of the preceding equation is in fact $\Pr\{S^* > t | N^*(t) = n\}$. Therefore, the unconditional probability that $S^* > t$ is

$$\Pr\{S^* > t\} = \sum_{0}^{\infty} {}_k \ (-1)^k [\lambda(t-k\tau)_+]^{k-1} \left[\frac{\lambda(t-k\tau)_+ + k}{\lfloor k} \right] e^{-\lambda[t-(t-k\tau)_+]}$$

after the resultant order of summations is exchanged and the inner extended summation is put into closed form.

The right member of this equation is not convenient for the determination of the expected value of S^* or its variance. Its Laplace transform, however, is both convenient and intrinsically useful, as later considerations will illustrate. Let £ be the Laplace transformation operator, and let s be the transform variable. Termwise application of the fundamental transformation

for powers of t , which may be written

$$\mathcal{L}(t^{m-1}/\lfloor m-1 \rfloor) = s^{-m}$$

for positive, integral m , together with the shift theorem yields

$$\mathcal{L}[Pr\{S^*>t\}] = [1-e^{-(\lambda+s)\tau}]/[s+\lambda e^{-(\lambda+s)\tau}] .$$

Since the moments of S^* can be obtained from $Pr\{S^*>t\}$ in the following manner

$$E(S^{*n}) = n\int_0^\infty t^{n-1} Pr\{S^*>t\}dt ,$$

it follows directly that $E(S^*)$ is merely the value of $\mathcal{L}[Pr\{S^*>t\}]$ at s = 0 ; therefore,

$$E(S^*) = (e^{\lambda\tau}-1)/\lambda .$$

The second moment similarly follows from the derivative of $\mathcal{L}[Pr\{S^*>t\}]$ with respect to s as evaluated at s = 0 ; and the variance clearly follows therefrom as

$$Var(S^*) = (e^{2\lambda\tau}-2\lambda\tau e^{\lambda\tau}-1)/\lambda^2 ,$$

after the appropriate algebra is performed.

The exponential dependency of $E(S^*)$ on λ and τ implies that small increases in the impact rate in the course of an engagement can induce large, sudden increases in the average duration of suppression periods, once a moderate impact rate has been achieved. The similar growth in the variance suggests very substantial fluctuations in those durations. In fact the coefficient of variation for S^* is asymptotically one.

Just how rapidly $E(S^*)$ can change is shown by Exhibit II, following this page. For selected durations τ of the single-round period of suppressive effect, $E(S^*)$ is graphed as a function of the impact rate λ in the region of suppressive affect. When τ is as small as two seconds, slight changes in the impact rate can produce great changes in $E(S^*)$, the average duration of a period of suppression. As Exhibit III shows, those great changes in the average duration of suppression in response to slight variations in the impact rate are matched by the correspondingly great changes caused by slight variations in the duration of a single-round period of suppressive effect. Consequently small discrepancies between assumed durations of suppressive effect and actual durations can introduce great variations in any durations of suppression periods extrapolated therefrom.

EXHIBIT II
SIMPLE SUPPRESSION FORMULA
AS A FUNCTION OF
RATE OF IMPACT λ

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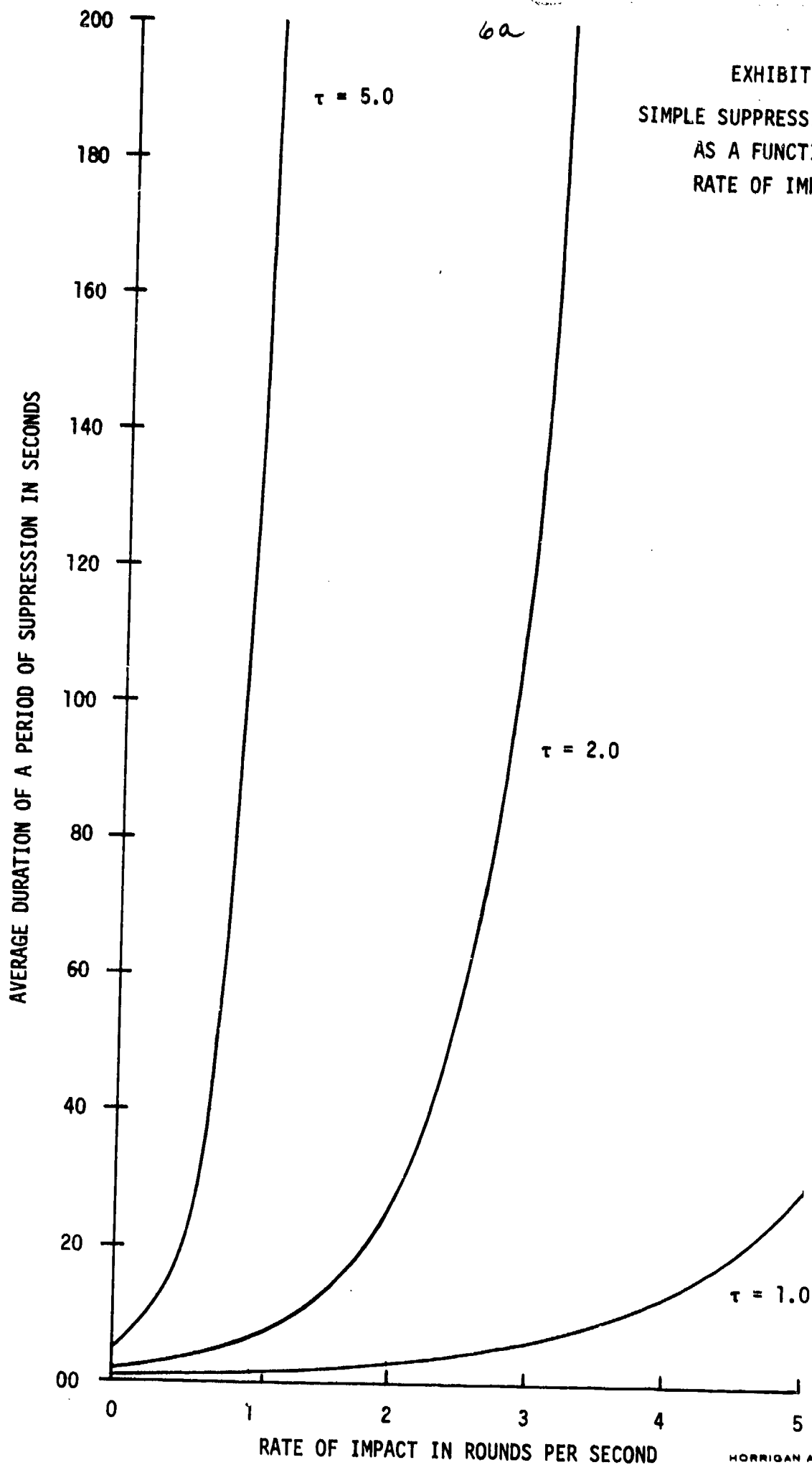
EXHIBIT III

SIMPLE SUPPRESSION FORMULA
AS A FUNCTION OF SINGLE-ROUND
SUPPRESSION PERIOD τ

$\lambda = 5.0$

$\lambda = 2.0$

$\lambda = 1.0$

AVERAGE DURATION OF A PERIOD OF SUPPRESSION IN SECONDS

SINGLE-ROUND PERIOD OF SUPPRESSIVE EFFECT IN SECONDS   HORRIGAN ANALYTICS
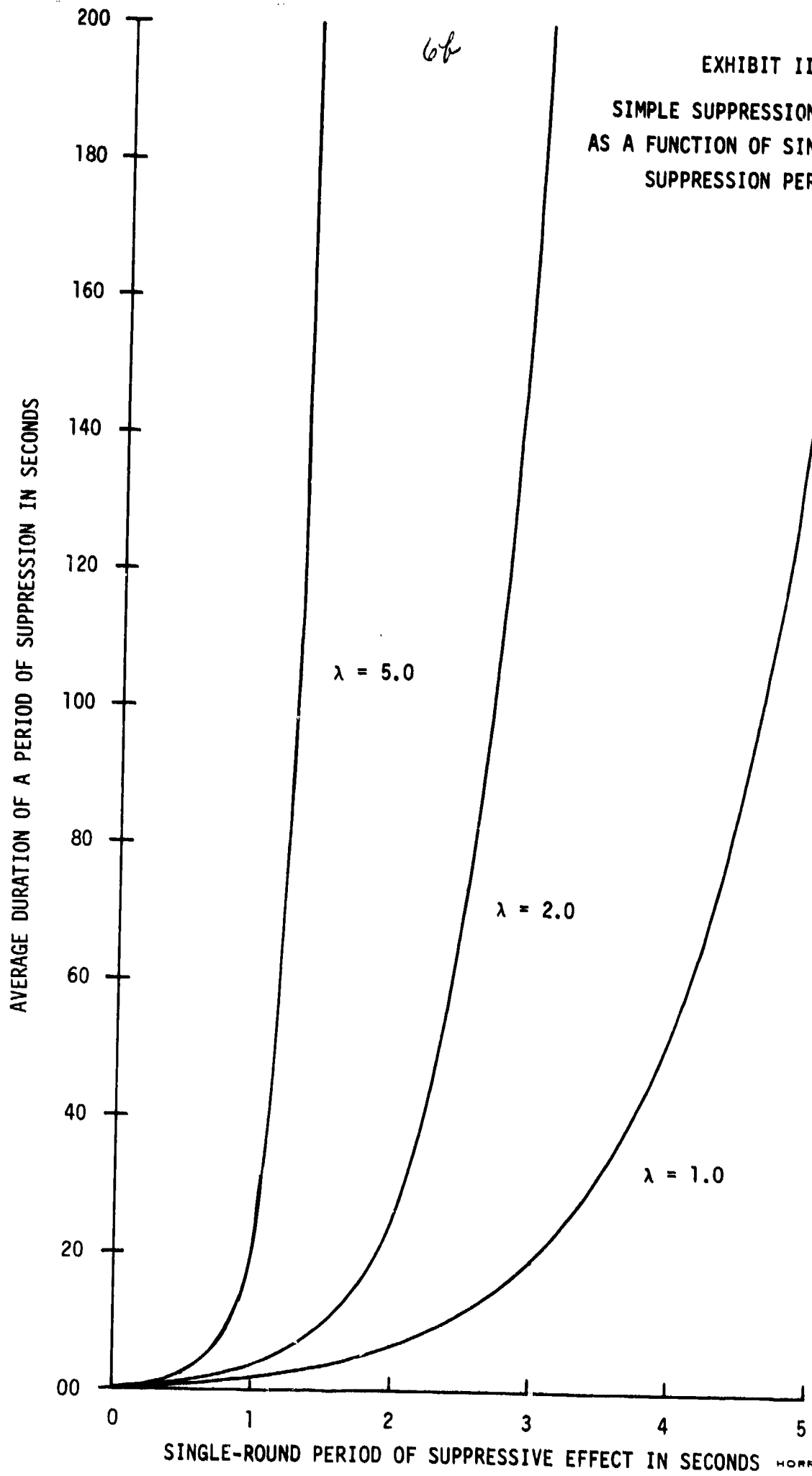
Although these formulas appear new in the context of suppression, they are well-known in other applications[3]. They may also be derived by more general means than those herein employed, notably by the methods of renewal theory. The derivation just outlined is, however, direct and is the one that led to the formulas in the context of suppression.

(2) The Simplest Suppression and Detection Interaction

Many search activities in the combat environment are characterized by an exponentially distributed detection time. Any such activity consequently possesses the Markov property for the exponential distribution[4] and is therefore easily adjusted to account for being suspended during periods of suppression. Indeed, a detection may occur only between periods of suppression because the hiatuses they create block all such events while they last; in other respects the search and bombardment activities are presumed independent. The Markov property then insures that the random detection time retains the *same* exponential distribution regardless of the number and duration of preceding periods of suppression and fruitless search. Since $N^*(t)$ is Poisson, it similarly insures that the duration between the end of one period of suppression and the start of the next defines a family of independent, identically distributed random variables.

Accordingly a basic suppression-search cycle exists. It begins with the onset of a period of suppression and ends either with the onset of another period of suppression or a detection, whichever first follows the initial period of suppression. All cycles are identically and independently distributed in duration. The first part of a cycle of course has the duration S^*, that of a simple period of suppression. The last part is the period between the cessation of suppression and either a detection or an impact, whichever occurs first. Since the search activity and the bombardment activity are independent aside from periods of suppression, the probability distribution for the duration from the end of the period of suppression to the end of the cycle follows directly.

Designate the duration by T^*. Since T^* is the minimum of the time to the first detection and the time to the next impact, which are independent, exponentially distributed random variables, it follows that

$$Pr\{T^*>t\} = e^{-(\lambda+\gamma)t}, \quad t\geq 0$$

when γ is the detection rate in the absence of suppression. A cycle thus has the duration $S^* + T^*$; and the probability that it ends with a detection, an event which is independent of both S^* and T^*, is easily shown to be $\gamma/(\gamma+\lambda)$.

A combatant that is initially suppressed at the time zero may or may not end its first cycle with a detection. The random number of cycles up to and including that on which its first detection occurs has a geometric

distribution. Designate that random number by N^*; it then has the geometric probability density

$$Pr\{N^* = n\} = \frac{\gamma}{\gamma+\lambda}\left[\frac{\lambda}{\gamma+\lambda}\right]^{n-1} .$$

Further, designate the random duration of the i-th cycle by C_i^*. Then the time D^* to the first detection is simply

$$D^* = \sum_1^{N^*} {}_n C_n^*$$

a sum of independent, identically distributed random variables. The average time $E(D^*)$ to the first detection following the onset of a period of suppression is

$$E(D^*) = (1/\lambda + 1/\gamma)e^{\lambda\tau} - 1/\lambda ,$$

which is an immediate consequence of the preceding expression for D^*.

Suppression, when taken as a hiatus, is thus seen to have a great effect on detection times. They grow at a rate even greater than the simple suppression periods previously examined. Exhibit IV, following this page, illustrates that rapid growth when the average detection time in the absence of suppression is 20 seconds. The average detection time in the presence of suppression is displayed as a function of the impact rate for a single-round period of suppressive effect of unit duration in comparison with the average duration of a single period of suppression under the same circumstances. The strong effect that all-or-nothing periods of suppressive effect have on detection times is manifest.

Because the random duration of a suppression-search cycle is $S^* + T^*$, a sum of two independent random variables, the Laplace transformation $C(\cdot)$ of its frequency function is the product of those for S^* and T^*. Since that for S^* follows directly from that of its tail, which is already established, and that for T^* is immediate, their respective product

$$C(s) = \frac{\lambda+s}{\lambda+se^{(\lambda+s)\tau}} \cdot \frac{\lambda+\gamma}{\lambda+\gamma+s} ,$$

with s as the transform variable, gives the Laplace transform of the frequency function for the duration of a suppression-search cycle. As D^* is the sum of N^* such variables, which are identically distributed and are independent both of each other and of N^*, a simple calculation

8a

EXHIBIT IV

BRIEF, SIMPLE SUPPRESSION
STRONGLY DEGRADES DETECTION

detection time for search
with simple suppression

detection time for search
without suppression

average duration of suppression period
arising from unit single-round period

AVERAGE SUPPRESSION OR DETECTION TIME IN SECONDS

RATE OF IMPACT IN ROUNDS PER SECOND

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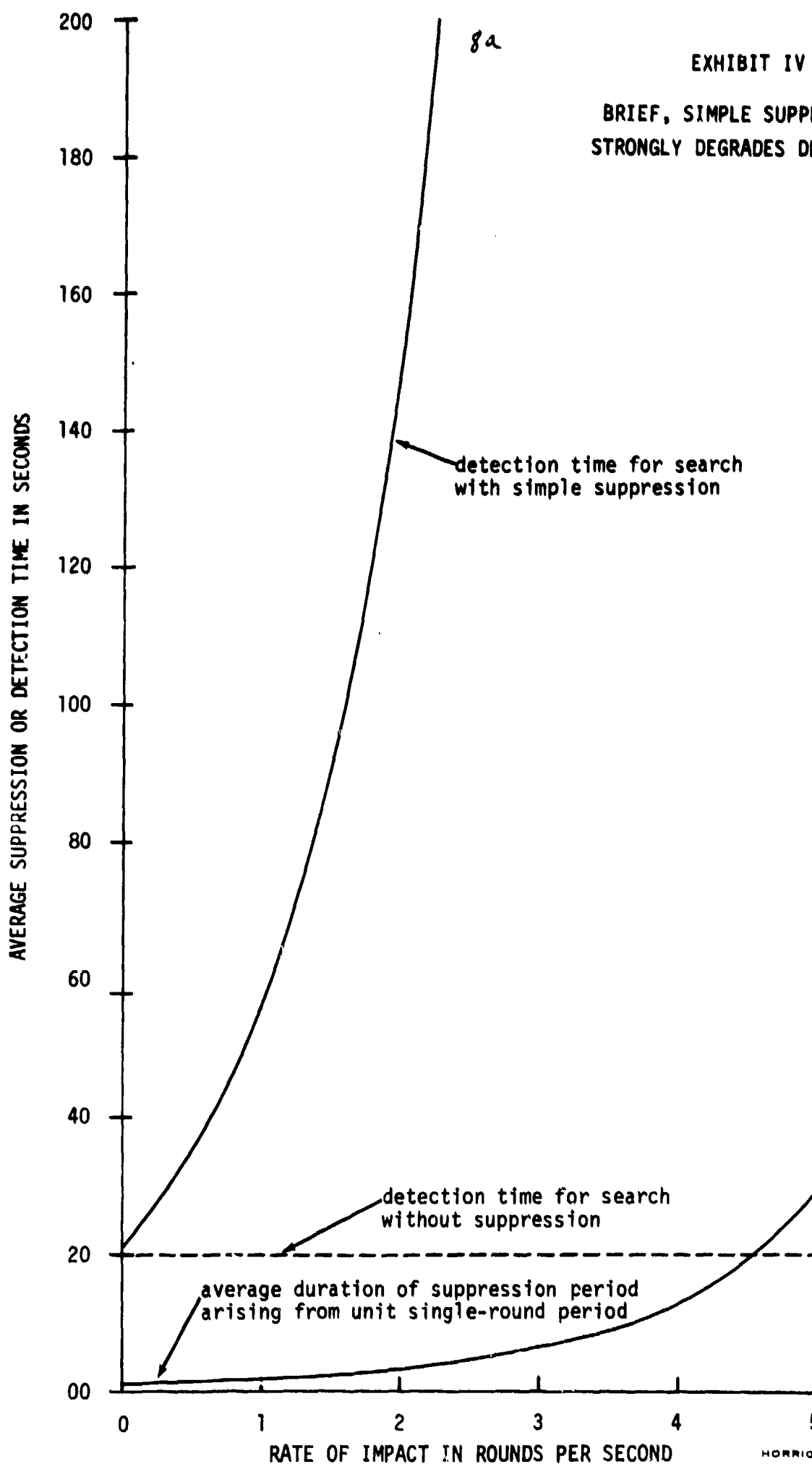
shows that the Laplace transformation of the tail of D^* may be written

$$\mathcal{L}[Pr\{D^*>t\}] = \frac{\lambda+\gamma}{s}[1-C(s)]/[\lambda+\gamma-\lambda C(s)]$$

in which s remains the transform variable. The expected value of D^*, already derived, as well as the higher moments can of course be easily and directly obtained from this equation. Later a more significant use will emerge in the context of fractional suppression.

III. MISS-DISTANCES, WEAPON MIXES, AND GENERALIZED SUPPRESSION

No doubt the most apparent unsatisfactory assumption underlying these
formulas is the idealized constant duration of the single-round period
of suppressive effect. Further, that duration is required to be inde-
pendent of miss-distance, and it must be the same for each type of
round. Ignoring casualties is of course a shortcoming, but the suppres-
sion process itself is not thereby grossly restricted, as it is by the
aforementioned assumptions.

Several avenues of generalization for the simple model are thus suggested;
and they lead to broadly applicable formulas of remarkable simplicity.
The generalized suppression model established therefrom permits:

- Random durations for single-round periods of
  suppressive effect

- Durations for single-round periods of suppres-
  sive effect that depend on miss-distance

- Distinct characteristics for the periods of
  suppressive effect associated with each ord-
  nance or projectile type

- Segregated, nonuniform delivery of any mixture
  of projectile types

The general model thus encompasses a substantial number of factors that
affect suppression. Durations of suppression for each round type are not
only permitted to be distinct, but also they may be random variables with
different probability distributions, which may be functions of miss-distance.

Random durations for single-round periods of suppressive effect allow
differences in judgment of an individual combatant to be reflected as
variations in the single-round suppressive effect of even identical
rounds impacting at the same distance. Durations of single-round periods
of suppressive effect that deterministically depend on miss-distance are
thereby randomized regardless and thus illustrate another variation in
the suppressive effect of identical rounds. Permitting single-round
periods of suppressive effect to depend on miss-distance also allows
local nonuniformities in projectile delivery to be faithfully represented.

(1) Random Single-Round Periods of Suppressive Effect

In the simple model all impacts in the region of suppressive affect produce
a single-round period of suppressive effect of fixed duration τ ; in the
general model a projectile of the i-th type fired from the j-th source
produces a single-round period of suppressive effect with the random
duration T_{ij}^{*} , all of which are independently distributed. In the simple

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model there is only one impact rate in the region of suppressive affect; in the general model there is one such rate λ_{ij} for each projectile type from each source. The respective impact times of projectiles of each type from each source are assumed to follow independent Poisson processes with the respective intensities λ_{ij}.

Presumably the duration of a single-round period of suppressive effect depends on miss-distance. Given a particular combatant and situation, a particular projectile type, and a fixed miss-distance x , there is a random variable $T^*(x)$ which is the duration of the single-round period of suppressive effect that results from an impact a distance x from the combatant. Of course, the duration of such a suppression period may be taken as a function of the miss-distance. In either event, because the miss-distance itself is a random variable, the resulting single-round period of suppressive effect has a random duration.

As indicated above the random duration of this suppression period for a projectile of the i-th type from the j-th source is T^*_{ij} in which dependency on miss-distance is implicit. If the function $s_i(t,x)$ is the probability density for a single-round period of duration t arising from the impact of the i-th projectile type a distance x from the combatant, and if $f_{ij}(x)$ is the probability density governing impacts at x by a projectile of the i-th type from the j-th source, then the expected (average) duration of a single-round period of suppressive effect is

$$E(T^*_{ij}) = \int_0^\infty \int_0^\infty t s_i(t,x) f_{ij}(x) dx dt .$$

The remarkable aspect of the generalized model is that these expected values together with the average impact rates λ_{ij} determine the expected durations of suppression periods as well as expected detection times.

As in the simple model, for an entity to be suppressed for a duration t , there must be an unbroken chain of overlapped, single-round suppression periods which together, from the beginning of the first to begin, to the

and of the last to cease, constitute a duration t . Unlike the simple
model, the durations of the single-round periods of suppressive effect
are no longer the same in duration; short ones and long ones are haphaz-
ardly mixed, and many gaps between short ones may be filled by a single
long one. Despite this great increase in physical complexity and a
comparable increase in mathematical difficulty, there is little change
in the formula for the expected duration of a suppression period. The
cumbersome but necessary mathematical details are treated in Appendix A;
in the following only the major results are described.

For R round types and N fire sources define λ , the combined impact rate
of projectiles in the region of suppressive affect, as follows:

$$\lambda = \sum_{i}^{R} \sum_{j}^{N} \lambda_{ij}$$

As in the simple model, designate the random duration of an overall
suppression period by S^* . Then the expected duration of an overall
suppression period in the generalized model is

$$E(S^*) = \frac{1}{\lambda} \left\{ \exp\left[\sum_{i}^{R} \sum_{j}^{N} \lambda_{ij} E(T_{ij}^*) \right] - 1 \right\}$$

a remarkably simple formula, which involves only the expected durations
of single-round periods of suppressive effect.

When each round type is represented by a distinct single-round period of
suppressive effect which is a constant independent of miss-distance, the
formula simplifies further. In that case there are no random variations
in the duration of a single-round period of suppressive effect. For a
fixed round type all such periods are of identical duration. For the
i-th round type designate the duration of a single-round period of sup-
pressive effect by τ_i . Because the τ_i are functionally independent of
miss-distance, they are consequently independent of the source of fire.
Hence, the segregation of rates of impact by the source of fire is not

necessary in this case. Accordingly, if λ_i is defined by

$$\lambda_i = \sum_{j}^{N} \lambda_{ij}$$

then it designates the impact rate of the i-th type of projectile in the region of suppressive affect. The expected duration of an overall period of suppression is accordingly given by

$$E(S^*) = \frac{1}{\lambda} \left\{ \exp \left[\sum_{i}^{R} \lambda_i \tau_i \right] - 1 \right\}$$

in which S^* again designates the random duration of an overall suppression period and λ the combined impact rate.

(2) Expected Detection Times in the Generalized Model

Detection in the generalized model is conceptualized just as it is in the simple model. A combatant cycles between suppression and search until it first makes a detection before the onset of the next suppression period. Despite the greatly increased physical complexity encompassed by the general model there is no proportionate increase in the complexity of the formula for expected detection times. With D^* again designating the random time to a detection by an initially suppressed combatant, it can be shown, using the argument advanced for treating detection in the presence of simple suppression, that

$$E(D^*) = (1/\gamma + 1/\lambda) \exp \left[\sum_{i}^{R} \sum_{j}^{N} \lambda_{ij} E(T^*_{ij}) \right] - 1/\lambda$$

when γ remains the parameter in the exponential distribution of detection time in the absence of suppressive fires. Thus a simple, general, and convenient formula is available for connecting the effect of suppressive fires with the ability to return fires.

When the durations of single-round periods of suppressive effect are assumed constant for a given projectile type a somewhat simpler formula governs:

$$E(D^*) = (1/\gamma + 1/\lambda) \exp \left[\sum_{i}^{R} \lambda_i \tau_i \right] - 1/\lambda$$

in which the τ_i again represents the single-round suppression duration assigned to the i-th projectile type, and λ_i designates the corresponding impact rate.

IV. DURATIONS OF SUPPRESSION FOR UNDAMAGED COMBATANTS

These formulas neglect casualties. While that is a minor omission relative to the simple model, it is still a flaw. It tends to lengthen erroneously the expected detection times, because it implicitly ignores the fact that an entity must survive in order to detect. When a combatant survives the rounds impacting in its region of suppressive affect during the suppression periods preceding a detection, the expected number of such impacts is consequently reduced; hence, the expected time it is suppressed is reduced and therewith its expected detection time.

How the duration of a period of suppression is affected is easily seen in terms of the simple model. With δ designating the single-round damage probability and $u(t)$ designating the event that the combatant is undamaged during the time t , it is shown in Appendix B that the formula

$$E[S^*|u(S^*)] = [(1+\delta\lambda\tau)/(\delta + \overline{1-\delta}e^{-\lambda\tau})-1]/\lambda$$

gives the expected duration of those periods of suppression during which the combatant is undamaged. In situations in which no damage is possible δ is zero, and $E[S^*|u(S^*)]$ then equals $E(S^*)$. For positive δ it is always less than $E(S^*)$; and it strictly decreases with increasing δ until finally, when δ is one, it becomes τ , the smallest possible period of suppression in the simple model.

Survival prospects during a period of suppression are best given by the probability $Pr\{u(S^*)\}$ that the combatant is undamaged during a period of suppression. It is related to the expected duration $E(S^*)$ of a period of suppression by the illuminating formula

$$Pr\{u(S^*)\} = (1-\delta)/[1+\lambda\delta E(S^*)] ,$$

which makes very clear how the probability of surviving a period of suppression depends on the expected duration of those periods. As that duration increases, $Pr\{u(S^*)\}$ decreases. Consequently periods during which the combatant is undamaged should have shorter durations.

Whether the quantitative consequences of using $E(S^*)$ vice $E[S^*|u(S^*)]$ are major or minor obviously depends strongly on the single-round casualty probability δ . When it is small and the impact rate is small to moderate, the consequences appear to be negligible. However, whenever it is not small or the impact rate is high, the consequences are major. In such cases the consequences are greater for damaged combatants; for instance, if δ is small and λ moderate, then $E[S^*|u(S^*)]$ can be about ten percent less than $E(S^*)$, while $E[S^*|\bar{u}(S^*)]$ can be twice $E(S^*)$. On the other hand, when δ is moderate and λ high, the reverse can easily obtain; $E[S^*|u(S^*)]$ can be about half $E(S^*)$, while $E[S^*|\bar{u}(S^*)]$ exceeds it by no more than ten percent or so. In either case, those periods of suppression during which casualties occur are much longer than those during which there are none. Combatants, in effect, are pinned down by suppressive fires for much longer times when damage occurs -- a possibly surprising fact considering the assumed total randomness of the fires.

V. FRACTIONAL SUPPRESSION AND EXPECTED DETECTION TIMES

Neglect of casualties is not the only flaw in the generalized suppression model. A more fundamental one is the idealization of suppression as a hiatus in the activity of the suppressed combatant. Although that handy idealization is commonly used in modeling suppression, it is nonetheless counterfactual. Suppressive fires slow down activities, but they do not necessarily stop them. Idealizing suppression as a hiatus is adequate only insofar as periods of suppression are considered in abstraction -- without any interaction with combat activities.

Search activities are a case in point. Expected detection times in the presence of suppressive fires can very easily become very long, as Exhibit IV illustrates. Simply because those times can be so long, the difference between suppression as a stopping of all activity and suppression as a slowing of it is important. If suppression is truly a hiatus in combat activities, then detection cannot be made during periods of suppression, regardless of their durations. If suppression is anything less total, however, detections will then frequently be made during periods of suppression, particularly when their expected durations are long.

Suppression that is less than total is herein termed *fractional suppression*; during periods of fractional suppression combat activities proceed at a fraction of their unsuppressed rates. Search activities of the type previously defined, that develop a detection rate γ in the absence of suppression, proceed with the reduced, fractional rate $\eta\gamma$ (for an appropriate η in the unit interval) during periods of suppression. Expected detection times therefore can never exceed $1/(\eta\gamma)$ regardless of the duration of periods of suppression. Fractional suppression and casualty production thus both operate to decrease the duration of detection times.

Idealizing a single-round period of suppressive effect not as a hiatus in a search activity but as reduction in some major factor, for example, the solid angle available to the combatant for search, captures a vital characteristic of the interaction of search and suppression. A limit on the efficacy of suppressive fires to inhibit detection is imposed; a point of diminishing return is established. Increasing rates of fire no longer produces progressively greater increases in expected detection times. Instead, the increases in those times reach a maximum and then become progressively smaller. Never can the expected detection time be forced beyond $1/(\eta\gamma)$. A necessary logical boundary is thus incorporated without which the suppression process itself is compromised.

What fractional suppression means is easily visualized in terms of the example. An upright combatant, for example, typically has a field of view that is much greater than that available from a crouching or a prone position. In keeping with maximum simplicity no new region of intervisibility is permitted to be introduced in shifting from upright to prone. Nearby impacts which result in that combatant's taking temporarily a position other than erect thereby introduce fractional suppression by

reducing the solid angle available for search activity from that available in an upright position to some smaller portion. As a result the detection rate is decreased, and the expected detection time is increased. Impacts in the vicinity of a crouching combatant similarly can cause the solid angle available for search activities to be reduced to that portion available from a prone position. Thus conceived, fractional suppression makes the counterfactuality of suppression as a status obvious.

Quantifying fractional suppression is straightforward in concept. The fraction η itself, in terms of the example, is merely the ratio of the steradian of the solid angle available to the search activity in the presence of suppression to that available in the absence of suppression. The search activity can accordingly be represented by two independent processes, one characterized by the detection rate $(1-\eta)\gamma$ and the other by the detection rate $\eta\gamma$. The first process arises from search in the solid angle that is unavailable during periods of suppression; the second process arises from search in the solid angle that is always available. Suppression always suspends the first process, but it never affects the second.

Such straightforward quantification notwithstanding, an interesting question develops when commonplace approximations in search modeling are introduced. If targets are uniformly distributed over the solid angle and a glimpse model or a constant search rate model is employed, for example, then the expected time to a (first) detection has the same value for all solid angles within the initial one. Consequently, for such representations, search through an arbitrarily small, ill-situated knothole is as effective as search with an unrestricted field of view. However, when intermittent intervisibility is the dominant factor and the model reflects that dominance, then the fraction by which the solid angle is reduced indeed directly and similarly modifies the corresponding detection rate. The suppression-detection model herein explored is of course directed to the latter situation.

Consequently, the random detection time D_1^* associated with the first process behaves exactly the same as the random detection time in the presence of simple suppression previously examined after the associated detection rate γ is replaced by $(1-\eta)\gamma$. That random detection time D_2^* associated with the second process of course follows an exponential distribution in which the parameter is the appropriately diminished detection rate $\eta\gamma$. The random time $D^*(\eta)$ at which the combatant, cycling between fractional suppression and search, makes its next detection is clearly just the minimum of those two random times. The tail of the distribution of $D^*(\eta)$ is thus

$$\Pr\{D^*(\eta) > t\} = \Pr\{D_1^* > t, D_2^* > t\} = \Pr\{D_1^* > t\}e^{-\eta\gamma t},$$

in which the rightmost member follows from the assumed independence of the underlying processes. That form of the tail $\Pr\{D^*(\eta) > t\}$ is handy for

establishing the moments of $D^*(n)$ as those of D^* were established initially. Indeed, using that form the n-th moment of $D^*(n)$ is given by

$$E[D^*(n)]^n = n \int_0^\infty t^{n-1} Pr\{D_1^*>t\} e^{-n\gamma t} dt \ ,$$

which is essentially nothing other than the (n-1)-th derivative of the previously established Laplace transform $\pounds[Pr\{D^*>t\}]$ of the tail of D^*, after $n\gamma$ is substituted for the transform variable and $(1-n)\gamma$ is substituted for the original search rate. Consequently, if the variable z is defined by

$$z = \frac{(\lambda+n\gamma)[\lambda + (1-n)\gamma]}{(\lambda+\gamma)[\lambda + n\gamma e^{(\lambda+n\gamma)\tau}]} \ ,$$

then the expected detection time $E[D^*(n)]$ in the presence of fractional suppression is

$$E[D^*(n)] = \frac{[\lambda + (1-n)\gamma](1-z)}{n\gamma[(1-n)\gamma + \lambda(1-z)]} \ .$$

Regrettably, the algebraic simplicity of the expected detection time $E(D^*)$ in simple suppression is lost, but a vital recognition of diminishing returns, which is much more than compensatory, is acquired.

Although the variance is slightly more cumbersome, it too follows easily from the Laplace transform of the tail of D^*, after the second moment is obtained by differentiation. Define the intermediate variable z' by

$$z' = \left[\frac{1}{\lambda + n\gamma} - \frac{1}{\lambda+\gamma} - \frac{1+n\gamma\tau}{n\gamma + \lambda e^{-(\lambda+n\gamma)\tau}}\right] z$$
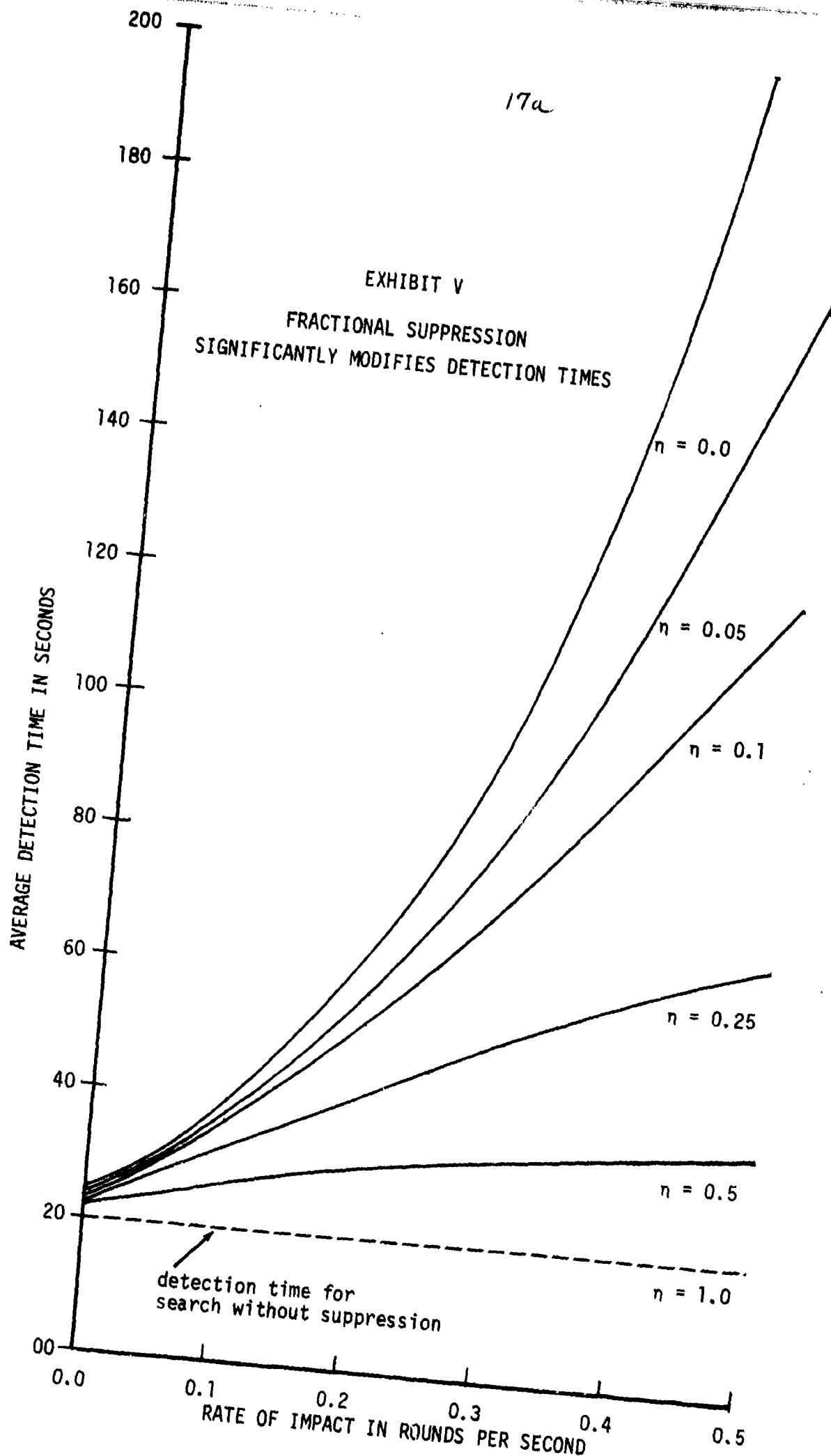
and the variance can then be written

$$Var[D^*(n)] = 2\left[\frac{(1-n)\gamma z'}{[(1-n)\gamma + \lambda(1-z)](1-z)} + \frac{1}{n\gamma}\right] E[D^*(n)] - E^2[D^*(n)]$$
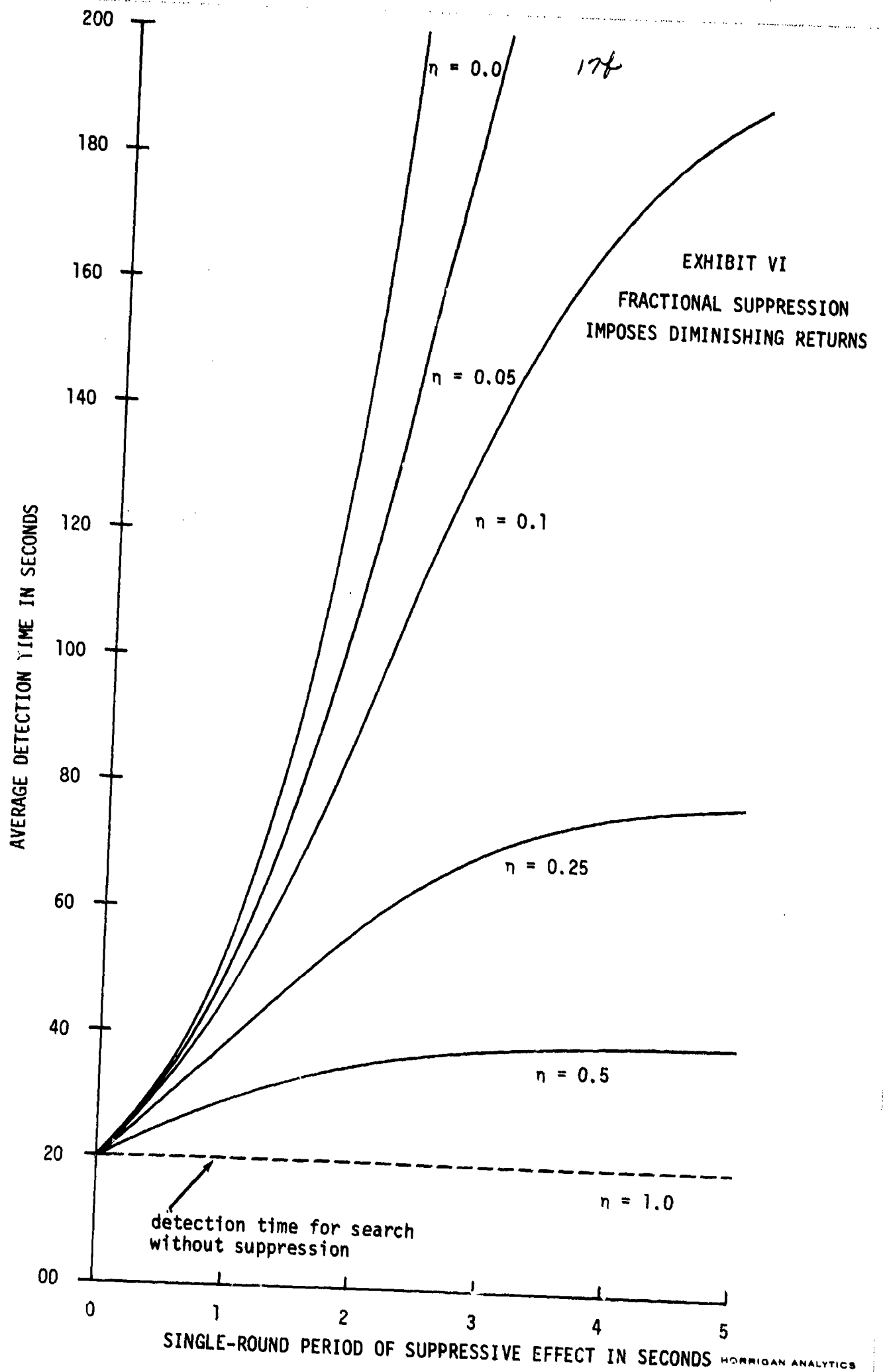
Despite its more cumbersome form, the variance is closely related to the expected value, because the coefficient of variation is very nearly unity for a wide range of situations, including all those herein considered.

How fractional suppression affects expected detection times is shown in Exhibits V and VI, which follow this page. In both those exhibits the suppression fraction n takes the values: 0.0, 0.05, 0.1, 0.25, and 0.5 . The first value, of course, corresponds to the usual idealization of suppression as a hiatus; the values from 0.1 to 0.5 are perhaps more representative. In each exhibit the expected detection time in the absence of suppression is 20 seconds.

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EXHIBIT V

FRACTIONAL SUPPRESSION
SIGNIFICANTLY MODIFIES DETECTION TIMES

$\eta = 0.0$

$\eta = 0.05$

$\eta = 0.1$

$\eta = 0.25$

$\eta = 0.5$

detection time for
search without suppression

$\eta = 1.0$

AVERAGE DETECTION TIME IN SECONDS

RATE OF IMPACT IN ROUNDS PER SECOND

HORRIGAN ANALYTICS

EXHIBIT VI

FRACTIONAL SUPPRESSION
IMPOSES DIMINISHING RETURNS

In Exhibit V the single-round period of suppressive effect is 5 seconds, and the rates of impact λ are small to moderate, yet variations in the expected detection times are great. When λ is about 0.1 the range is already significant, and it increases substantially with increases in λ. When λ equals 0.5 the slight difference in the suppression fraction η between total suppression ($\eta = 0$) and nearly total suppression ($\eta = 0.05$) results in an almost 40 percent reduction in the expected detection time. The difference in detection times arising from total suppression and the next level of reduced activity ($\eta = 0.1$) exceeds 50 percent. Thus, if η is about 0.1 instead of 0, then the expected detection time is overestimated by 120 percent. The percentage differences increase slightly with smaller expected detection times for detection in the absence of suppression and decrease slightly with larger ones.

A single, high impact rate ($\lambda = 1$) is used in Exhibit VI, and the expected detection times for the selected suppression fractions are graphed as functions of the single-round period of suppressive effect τ. The effect of the high impact rate is plain. When τ is about 2.5 seconds, the range of detection time variations matches the maximum encountered in Exhibit V. For values of τ larger than 2.5 seconds, that range, which is already more than substantial, becomes gross. When τ is about 5 seconds, the expected detection time for all-or-nothing suppression ($\eta = 0$) is nearly ten times greater than that with a suppression fraction η of only 0.05.

For moderate and higher impact rates and moderate single-round periods of suppressive effect, small variations in the suppression fraction thus produce large to gross changes in the expected detection times. As the exhibits show, particularly Exhibit VI, fractional suppression strongly limits the increases in expected detection times that can be obtained by increases in the single-round period of suppressive effect; diminishing returns from the longer periods are most apparent. Fractional suppression similarly limits the increases in detection times that can be obtained from increases in the rate of impact, and the diminishing returns it imposes are equally impressive, as Exhibit I indicates. Casualty production further limits such increases in expected detection times. The greatest changes in expected detection times occur relative to departures from all-or-nothing suppression; hence, for all but the lowest impact rates, idealizing suppression as a hiatus is ill-advised.

Obviously fractional suppression can be extended to allow return of fire following a detection. Thus suppression can be more closely related to combat activities, and its role in optimizing tactical response can be better identified and understood.

APPENDIX A

EXPECTED SUPPRESSION DURATIONS ARISING FROM
RANDOM SINGLE-ROUND PERIODS OF SUPPRESSIVE EFFECT

Various formulas have been given in the preceding pages for the relationship between the expected duration of an overall period of suppression and the expected durations of the individual single-round periods of suppressive effect that together make up the overall period. The objective of this appendix is to establish a simple, general formula of which the ones previously employed are immediate consequences. In the general situation the rounds impacting in the region of suppressive affect have impact times that are governed by a Poisson process of intensity λ ; and the associated individual single-round periods of suppressive effect are independently and identically distributed random variables, a particular but unspecified one of which is designated T^*. Specifically, for an overall random duration S^* of a period of suppression arising from single-round periods of suppressive effect that have random durations distributed as T^* this appendix demonstrates that the remarkably simple formula

$$E(S^*) = [\exp\lambda E(T^*) - 1]/\lambda$$

expresses the expected duration of an overall period of suppression in terms of the expected duration of a representative constituent single-round period of suppressive effect and λ , the rate of impact in the region of suppressive affect. In accordance with the previous considerations T^*, which must be positive with probability one, is independent of the impact time or indicator event associated with the arrival of the corresponding projectile, although it can stochastically depend on the associated miss-distance. The details of that dependency, however, do not enter into the necessary derivations and are therefore not further explicitly considered.

Consider first the case in which T^* takes discrete values. Designate by A^* the random duration of a suppression period produced by any mixture of all the different-valued, individual single-round periods of suppressive effect except the smallest. Designate by B^* the duration of a period of suppression created by the smallest single-round period of suppressive effect only. An overall period of suppressive effect thus consists of a sequence of such a-periods and b-periods. As the impacts in the region of suppressive affect are governed by a Poisson process with intensity λ , impacts entailing single-round periods of suppressive effect in excess of the minimum -- those periods the mixture of which create A^* -- are also governed by a Poisson process; similarly, those rounds giving rise to the minimum single-round period of suppressive effect also constitute a Poisson process, which is independent of the previous one. Designate the intensity of the process giving rise to A^* by α and the intensity of the one giving rise to B^* by β ; of course, $\lambda = \alpha+\beta$.

In the following paragraph a formula expressing $E(S^*)$ in terms of $E(A^*)$, the indicated rates of impact, and the duration B of the shortest single-round period of suppressive effect is established. As will be shown, that formula inductively generates the explicit formula for $E(S^*)$ in all cases in which the random single-round period of suppressive effect is a mixture of a finite number of different nonrandom durations. The situation in which the individual single-round periods of suppressive effect give rise to random variables with continuous distributions follows therefrom by a limit argument.

An overall period of suppression S^* must naturally begin with an a-period or a b-period. Since all the durations of single-round periods of suppressive effect which constitute a-periods exceed the duration B of that single smallest period which constitutes the b-periods, the end of any a-period can be considered to be the beginning of a b-period because A^* is greater than B with probability one. An overall period of suppression that begins with an a-period consists of one or more independently distributed a-periods that are joined together by b-periods. So long as the gaps between a-periods are covered by b-periods the overall period of suppression continues. When, for the first time, a gap between a-periods is not filled by a b-period, the overall period of suppression terminates. Designate by F^* that portion of a gap between a-periods which is filled by a b-period. As the last B units portion of an a-period can be taken as the beginning of a b-period, F^* is the smaller of $B^* - B$ and H_a^*, the time of the next impact of a round with a single-round period of suppressive effect of the type appropriate to an a-period; thus F^* is defined by

$$F^* = \min(B^* - B, H_a^*) .$$

With F designating the event that a gap is filled, that is,

$$F = \{B^* - B \geq H_a^*\}$$

it follows that the gaps between a-periods that are indeed filled by b-periods have a random duration the distribution of which is the conditional distribution of F^* given F. When an overall period of suppression that began with an a-period is terminated by an unfilled gap between a-periods, the portion that may be filled has a duration the distribution of which is that of F^* given that $\tilde{F}$. An overall period of suppression that begins with an a-period thus consists of that a-period, zero or more filled gaps followed by a-periods, and one unfilled gap. Obviously the number of filled gaps has a geometric distribution with parameter $f = Pr\{F\}$ so that the expected number $E(N^*)$ of filled gaps followed by a-periods is $f/(1-f)$. Therefore the expected duration of an overall period of suppression $E(S^*|A)$ that begins with an a-period is given by

$$E(S^*|A) = E(A^*) + E(N^*)[E(A^*) + E(F^*|F)] + E(F^*|\tilde{F}) ,$$

which may be simplified by replacing $E(N^*)$ by $f/(1-f)$ and then replacing

the sum of the two suitably weighted conditional expectations of F^* by its unconditional expectation, which is equal thereto. Thus, it follows that

$$E(S^*|A) = \frac{E(A^*) + E(F^*)}{1-f}$$

after the simplifying operations are performed.

When an overall period of suppression begins with a b-period, there is again a gap that must be filled for the overall period to continue. In this case it extends from the beginning of the initiating period to the onset of the first a-period. If the b-period is sufficiently persistent to meet the first a-period, then the additional duration of the overall period is $E(S^*|A)$, for which a formula is already established; otherwise the end of the a-period is the end of the overall period. Represent by R^* the portion of the gap that is filled by the b-period. R^* is thus either the end of the b-period or the first impact of a round with a single-round period of suppressive effect that belongs to an a-period, whichever occurs first, that is

$$R^* = \min(B^*, H_a^*) .$$

Because the connection with an a-period that is necessary to sustain the overall period occurs if and only if the event R that $B^* \geq H_a^*$ occurs, the expected duration of the overall period given R occurs is clearly

$$E(R^*|R) + E(S^*|A) .$$

On the other hand, when R does not occur the overall process is terminated; hence, its expected duration is only

$$E(R^*|\tilde{R})$$

in that case. With r equal to the probability $Pr\{R\}$ that the initiating b-period persists sufficiently long to meet the first a-period, the expected duration $E(S^*|B)$ of an overall period of suppression initiated by a b-period is therefore

$$E(S^*|B) = E(R^*) + rE(S^*|A) ,$$

after the suitably weighted conditional expectations of R^* are replaced by its unconditional expectation.

This result and the previous one for $E(S^*|A)$ permit the unconditional expectation $E(S^*)$ for the duration of the overall period of suppression to be expressed in terms already established. First note that it may be written

$$E(S^*) = \frac{1}{\lambda} [\alpha E(S^*|A) + \beta E(S^*|B)]$$

with α and β continuing to designate the impact rates of rounds with

single-round periods of suppressive effect appropriate to a-periods and to b-periods respectively. Substituting the previously established relationship for $E(S^*|B)$ permits $E(S^*)$ to be expressed in the form

$$E(S^*) = \frac{1}{\lambda} [(\alpha+\beta r)E(S^*|A)+\beta E(R^*)] \ ,$$

after the necessary simplifications are made. Observing that $r = \alpha E(R^*)$, which is obvious when the right member is expressed in tail form (as is done further on), permits r to be eliminated. After simplifying the result is

$$E(S^*) = \frac{1}{\lambda} [1+\beta E(R^*)][1+\alpha E(S^*|A)] - \frac{1}{\lambda} \ ,$$

in which only the conditional expectation $E(S^*|A)$ cannot yet be directly evaluated. Using the previously established relationship

$$E(S^*|A) = [E(A^*) + E(F^*)]/(1-f)$$

and observing that $f = \alpha E(F^*)$, which like the similar preceding expression for r is also obvious when the right member is expressed in tail form, permits the substitution

$$1 + \alpha E(S^*|A) = \frac{1 + \alpha E(A^*)}{1 - \alpha E(F^*)}$$

to be made in the right member of the preceding equation for $E(S^*)$. The result is that

$$E(S^*) = \frac{1}{\lambda} [1 + \alpha E(A^*)] \frac{1 + \beta E(R^*)}{1 - \alpha E(F^*)} - \frac{1}{\lambda}$$

in which only $E(A^*)$ cannot yet be directly evaluated. The factors having $E(F^*)$ and $E(R^*)$ as terms are easily treated, as the following shows.

Evaluating $E(F^*)$ and $E(R^*)$ is facilitated by using the tails of their respective probability distributions and noting that since

$$F^* = \min(B^* - B , H_a^*)$$

and

$$R^* = \min(B^*, H_a^*) \ ,$$

the latter is essentially a particular case of the former. By noting $F^* > t$ if and only if both $B^* - B > t$ and $H_a^* > t$ and by recalling the

representation of the expected value of a random variable in terms of its tail the expected value of F^* may be written

$$E(F^*) = \int_0^\infty Pr\{B^*>B+t, \ H_a^*>t\}dt \ .$$

Since B^* is independent of H_a^* , which is exponentially distributed with parameter α , it follows that

$$E(F^*) = \int_0^\infty Pr\{B^*>B+t\}e^{-\alpha t}dt \ ,$$

the right member of which is easily expressed in terms of the Laplace transform of $P(t)$, the previously established tail function (cf. Section II) for the duration of a simple period of suppression. After the necessary substitutions are performed $E(F^*)$ is given explicitly by

$$E(F^*) = e^{\alpha B}\left[\frac{1-e^{-(\alpha+\beta)B}}{\alpha+\beta e^{-(\alpha+\beta)B}} - \frac{1-e^{-\alpha B}}{\alpha}\right]$$

from which the explicit result

$$1-\alpha E(F^*) = \frac{\lambda e^{-\beta B}}{\alpha+\beta e^{-(\alpha+\beta)B}}$$

follows by successive simplification. An almost identical, simpler derivation shows that

$$1+\beta E(R^*) = \frac{\lambda}{\alpha+\beta e^{-(\alpha+\beta)B}} \ .$$

Accordingly, after substitution in the previous formula for $E(S^*)$, the relationship

$$E(S^*) = \frac{1}{\lambda}\ [1+\alpha E(A^*)]e^{\beta B} - \frac{1}{\lambda}$$

is demonstrated. Thus $E(S^*)$, the expected duration of the overall period, is inductively expressed in terms of $E(A^*)$, the expected duration of the period formed when the impacts associated with the minimum single-round period of suppressive effect are ignored, and the impact rate β and expected duration B of those minimum periods.

A simple notational change makes the formula's scope apparent. Recalling that T^* is the random duration of a single-round period of suppressive

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effect, which has been assumed temporarily to be discrete, define its probability density to be

$$Pr\{T^* = \tau_i\} = \rho_i \quad i = 1, 2, \ldots, n$$

for $\tau_1 > 0$ and $\tau_i < \tau_{i+1}$. Because the overall impact process is Poisson, the impact times of those rounds producing only a single-round period of suppressive effect with duration τ_i themselves form a Poisson process with intensity $\lambda_i = \lambda \rho_i$. The n thus segregated impact processes are obviously independent, and each permits only a single nonrandom single-round period of suppressive effect. Collectively they form the overall impact process with intensity λ and the *random* single-round period of suppressive effect with duration T^*.

Define S_1^* to be the duration of a period of suppression produced by the impact process associated with τ_n , the longest single-round period of suppressive effect; S_2^* to be that associated with τ_{n-1} and τ_n , the two longest single-round period of suppressive effect; ...; and, finally, S_n^* to be that associated with all the τ_i . Applying the formula for $E(S^*)$ first to S_1^* , which plays the role of A^*, and the impact process associated with τ_{n-1} as the b-period process yields $E(S_2^*)$. Using the resulting formula for $E(S_2^*)$ as the new $E(A^*)$ and the impact process associated with τ_{n-2} as the new b-period process yields $E(S_3^*)$. Finally, by continuing in this manner, $E(S_n^*)$ is reached. Consequently, as these successive evaluations show, the simple formula

$$E(S_n^*) = \frac{1}{\lambda}\left[\exp\sum_1^n \lambda_i\tau_i - 1\right]$$

gives the expected value of S_n^* for any n arising from a discrete valued T^*. As λ_i is by definition $\lambda Pr\{T^* = \tau_i\}$, the summation in the exponential function may be written $\lambda E(T^*)$; therefore,

$$E(S^*) = \frac{1}{\lambda}\left[\exp\lambda E(T^*) - 1\right]$$

is established for all random single-round periods of suppressive effect that have discrete durations.

The generalization to continously distributed random durations for the single-round periods of suppressive effect is straightforward. Let $s(\cdot)$ designate the probability density for a single-round period of suppressive effect, and define

$$p_i = s(\tau_i)\Delta\tau$$

for $i = 1, 2, \ldots, n$. The probability that T^* takes a value in the neighborhood of τ_i accordingly is p_i. Defining λ_i to be λp_i and associating it with the impact process producing single-round periods of suppressive effect with durations in the neighborhood of τ_i produce an overall process that approximates the actual process as closely as desired. Furthermore, for all n the expected duration $E(S_n^*)$ of a period of suppression is given by a formula containing the sum

$$\sum_1^n \lambda_i \tau_i$$

which, in terms of the probability density $s(\cdot)$, is in fact

$$\lambda \sum_1^n \tau_i s(\tau_i)\Delta\tau \, ,$$

the Riemann sum approximating the integral

$$\lambda \int_0^\infty t s(t) dt \, ,$$

which is of course just $\lambda E(T^*)$. Therefore, after the appropriate limits are taken, the expected duration $E(S^*)$ of a period of suppression is

$$E(S^*) = \frac{1}{\lambda} [\exp \lambda E(T^*) - 1]$$

for a single-round period of suppressive effect with the arbitrarily randomly distributed duration T^*. The formulas discussed in the body of this report all follow from this equation by merely particularizing the distribution of T^*. Nonuniform fires, for instance, are expressed by merely taking T^* to be governed by an appropriately mixed distribution.

APPENDIX B

CASUALTY PRODUCTION AND
DURATIONS OF PERIODS OF SUPPRESSION

Casualty production and suppression are probabilistically interrelated.
Combatants that are undamaged after exposure to fires tend to have
experienced fewer impacts in regions of nonzero vulnerability. If the
typical round that impacts in a combatant's vicinity has a probability
δ of damaging the combatant, then, on the average, the number of rounds
that impact in the vicinity of an undamaged combatant decreases with
increasing δ even though the areal impact probability density is uni-
formly random. Whether a combatant is damaged by suppressive fires
consequently affects the number and duration of the periods of suppres-
sion experienced by that combatant. This appendix quantifies the prob-
abilistic (but not interactive) interrelation between simple suppression
and casualty production and derives the formulas discussed previously.

In the case of simple suppression only one round type is considered,
and it impacts in the region of suppressive affect with the rate λ ; as
before, the impact times or indicator events are governed by a Poisson
process with λ as its intensity. For simplicity the combatant's vulner-
able region is limited to being a subregion of its region of suppressive
affect, and their relative disposition is characterized by the probability
δ that an impact in the region of suppressive affect damages the combatant.
Let $U(t)$ designate the event that the combatant is undamaged by the rounds
impacting in its region of suppressive affect between time zero and t .
In keeping with the structure of the simple suppression model, an initi-
ating impact takes place at time zero and the random additional number of
rounds $N^*(t)$ impacting between zero and t has a Poisson distribution with
expectancy λt . If $N^*(t) = n$ and the combatant is to be undamaged at t ,
it must not be damaged by the n+1 impacts. Hence, since the rounds are
independent, the probability $Pr\{U(t)|N^*(t) = n\}$ that a combatant experi-
encing an impact at time zero is undamaged at time t after an additional n
impacts is of course $(1-\delta)^{n+1}$, the probability that none of the n+1
rounds produces damage. As $N^*(t)$ follows a Poisson distribution with
expectancy λt , the equation

$$Pr\{U(t)\} = (1-\delta)e^{-\delta\lambda t}$$

gives the probability $Pr\{U(t)\}$ that an undamaged combatant initially
suppressed at time zero remains undamaged during the time t. Consequently,
the conditional probability $Pr\{N^*(t) = n|U(t)\}$ that n additional impacts
occur between zero and t when the combatant remains undamaged is given by

$$Pr\{N^*(t) = n|U(t)\} = \frac{[(1-\delta)\lambda t]^n}{\underline{|n}}e^{-(1-\delta)\lambda t} \; ;$$

and $N^*(t)$ remains a Poisson process when there is no damage, but it then

has a new intensity $(1-\delta)\lambda$, which is simply the average rate of impact of rounds producing no damage.

In order to calculate the expected duration $E[S^*|u(S^*)]$ of a period of suppression during which the incoming fire produces no damage, it is necessary to derive a formula for the probability $Pr\{u(S^*)\}$ that the combatant is undamaged at the end of the period of suppression. Define T^* to be the random time at which an undamaged combatant that is initially suppressed by an impact at time zero first becomes damaged. As the probability $Pr\{T^*>t\}$ that T^* exceeds t is the probability $Pr\{U(t)\}$ that the combatant is undamaged during t , the probability distribution for T^* is simply

$$Pr\{T^* \le t\} = 1-(1-\delta)e^{-\delta\lambda t} ,$$

which has an atom of mass δ at $t=0$. The probability $Pr\{u(S^*)\}$ that the combatant survives the suppression period is conversely simply the probability $Pr\{T^* > S^*\}$ that the first damage occurs after the period of suppression. Consequently, by expressing $Pr\{T^* > S^*\}$ in terms of the condition $T^* = t$ it follows that

$$Pr\{u(S^*)\} = \int_0^\infty Pr\{S^*<t|T^* = t\}dPr\{T^*\le t\}$$

$$= 1-\delta-(1-\delta)\delta\lambda \int_0^\infty Pr\{S^*>t|T^* = t\}e^{-\delta\lambda t}dt ,$$

after the integrand is expressed in tail notation, and adjustment for the saltus at zero is made. For the duration S^* of a period of suppression to exceed t given that damage first occurs at t , suppression must be maintained through the time t , during which the impact rate in the region of suppressive affect is only $(1-\delta)\lambda$. Hence, the latter integrand is just the probability $P(t)=Pr\{S^*>t\}$ that the duration of a period of simple suppression exceeds t , which is already established (cf. Section II), when the impact rate is $(1-\delta)\lambda$ instead of λ . The integral itself is similarly merely the also already established Laplace transform of the P(t) so modified, with $\delta\lambda$ replacing the transform variable. Therefore the probability $Pr\{u(S^*)\}$ that the combatant is undamaged during a suppression period is given by the formula

$$Pr\{u(S^*)\} = \frac{(1-\delta)e^{-\lambda\tau}}{\delta+(1-\delta)e^{-\lambda\tau}} ,$$

after the necessary substitutions and simplifications are made.

Similar reasoning is applicable to the joint probability $Pr\{S^*>t, u(S^*)\}$ that the duration of a period of suppression exceeds t and the combatant

is undamaged for the entire period. Expressing $u(S^*)$ in terms of T^* as before yields

$$Pr\{S^*>t, u(S^*)\} = Pr\{t < S^* < T^*\} \ ;$$

and representing the right member in terms of the condition that $T^* = x$ yields

$$Pr\{S^*>t, u(S^*)\} = (1-\delta)\delta\lambda \int_t^\infty [Pr\{S^*>t|T^*=x\} - Pr\{S^*>x|T^*=x\}]e^{-\delta\lambda x}dx \ ,$$

when the conditionalized version of the right member is written in tail notation. Integrating this expression from zero to infinity with respect to t results in

$$\int_0^\infty Pr\{S^*>t, u(S^*)\}dt = (1-\delta) \int_0^\infty (1-\delta\lambda t)Pr\{S^*>t|T^*=t\}e^{-\delta\lambda t}dt \ ,$$

after the order of integration is reversed. As the conditional probability in the integrand is the same as that previously encountered in the evaluation of $Pr\{u(S^*)\}$, it also may be expressed in terms of $P(t)$ with λ replaced by $(1-\delta)\lambda$ and, consequently, the integral itself in terms of its Laplace transform, with $\delta\lambda$ again replacing the transform variable. The only significant difference is the occurrence of t in the first factor of the integrand; it introduces the first derivative of the modified transform as well. Dividing both members of the preceding equation by $Pr\{u(S^*)\}$, of course makes the left member the expected duration $E[S^*|u(S^*)]$ of a period of suppression during which the combatant is undamaged. After the resulting right member is expressed in terms of the previously established Laplace transform results with the indicated substitutions and of the previously derived formula for $Pr\{u(S^*)\}$, the very simple formula

$$E[S^*|u(S^*)] = \frac{1}{\lambda}\left[\frac{1+\delta\lambda\tau}{\delta+(1-\delta)e^{-\lambda\tau}} - 1\right]$$

emerges as a consequence of algebraic simplification. Of course, when δ is zero it becomes $E(S^*)$, the unconditional expected duration, as it should.

REFERENCES

[1] William Feller, *An Introduction to Probability Theory and Its Applications*, Vol.II, Second Edition, John Wiley & Sons, Inc. (1971), p.42

[2] _____, *An Introduction to Probability Theory and Its Applications*, Vol.I, Third Edition, John Wiley & Sons, Inc. (1968), p.99

[3] _____, *An Introduction to Probability Theory and Its Applications*, Vol.II, Second Edition, John Wiley & Sons, Inc. (1971), pp.468-470

[4] _____, ibid., p.9