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A SHALLOW WATER ACOUSTIC MODEL FOR AN OCEAN STRATIFIED IN RANGE AND DEPTH. VOLUME I

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25 March 1975

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PREFACE

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In shallow water the stratification of the medium quite often varies significantly with range as well as with depth, and thus affects the nature of acoustic transmission. Hence, a model of acoustic propagation should be able to accommodate the effect of such changes in the stratification of the ocean. A computer-oriented procedure with its foundations in normal mode theory has been developed for problems of this type. Arbitrary changes in velocity profile, water depth (including

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20. ABSTRACT

a sloping bottom), and bottom composition with range can be accommodated. The effect of mode conversion is included in the analysis. The theoretical procedure is discussed and the mechanics of the resulting computer program are described in volume I. The results of the validation procedure for the model are described in volume II.

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A SHALLOW WATER ACOUSTIC MODEL FOR AN OCEAN STRATIFIED IN RANGE AND DEPTH VOLUME I

INTRODUCTION

The Block Island — Fishers Island (BIFI) acoustic program was undertaken with the objective of formulating an analytical model suitable for the prediction of sonar performance in shallow water. This report describes an acoustic model, with its foundations in normal mode theory, which has been developed for this purpose. The model is applicable to low frequency, long range propagation and can accommodate changes in stratification with range as well as with depth. The rate of change in the stratification with range is not limited to be "slowly varying" because the effect of mode conversion is included in the analysis. Hence, large changes in velocity profile, water depth, and bottom composition with range can be accommodated.

Volume I describes the theoretical basis for the model and the mechanics of the computer program which can be used to perform the necessary calculations. Volume II deals with the validation of the model.

The following outlines the most important input parameters and output information obtained from the model via a Fortran V computer program.

Description of Medium

a. Ocean Surface — flat

b. Bottom Profile — can be approximated in any one of three ways: (1) flat, (2) segmented into range intervals of constant depth, and (3) segmented into range intervals characterized by a mean depth and a constant slope.

c. Bottom Composition — semi-infinite bottom characterized by constant density ρ_B and sound velocity C_B. The bottom composition is constant within any one segment but may differ in various segments.

d. Velocity Profile — the velocity profile is constant within any one segment but may differ in various segments.

Description of Source and Receiver

a. Source Characteristics — the source may be either a point source or a vertical array of point sources. In a given computer run, the source field can be calculated for as many as 45 specified source depths or for an array containing as many as 45 elements at specified depths. For an array of sources, the relative amplitudes and phases of the individual elements must be specified and the wave form can be either CW or pulsed CW of a specified length.

b. Receiver Characteristics — the receiver may be either a point receiver or a vertical array of point receivers. In a given computer run, the sound field can be calculated for as many as 45 specified receiver depths or for an array containing as many as 45 elements at specified depths. When there is an array of receivers, the relative amplitude and phase shading of the individual elements must be specified. (One restriction is that both source and receiver cannot be arrays.)

Output Information

a. Propagation loss as a function of range is calculated for any specified set of modes. The loss is listed and plotted on the Calcomp plotter.

b. Propagation loss as a function of depth at any given range can be calculated for any specified set of modes. The loss is listed and can be stored on tape. These values can later be plotted by using another computer program.

c. In the case of a source which transmits a pulsed CW signal, the time history of the pulse can be determined at any range or depth. The time history can be stored on tape and later plotted by using another computer program. In addition, the power spectrum of the signal can be obtained.

d. For the case of an array of point sources or receivers, the theoretical optimum array shading for transmitting or receiving any single mode can be obtained. A listing of the array shading necessary for the selection of individual modes using either amplitude matching or phase matching (defined later) can be obtained.

e. For any mode, the group velocity, phase velocity, excitation pressure and the angle of incidence of sound waves striking the boundaries are listed.

f. For any mode, a listing and Calcomp plots can be obtained for the amplitude as a function of depth and the ray equivalent.

THEORY

In this section, the normal mode theory and the numerical procedures supporting the acoustic model described herein are presented. First, acoustic propagation over a flat, homogeneous ocean bottom in a medium whose stratification is constant with range is considered. Next, propagation in a medium whose stratification varies with range is described, but mode conversion is neglected. Then the effect of mode conversion and a sloping bottom is described. This is followed by a consideration of the propagation of time-limited signals. Finally, transmission and reception of energy by means of arrays and the use of such arrays matched to individual modes are described.

PROPAGATION OVER A FLAT, HOMOGENEOUS BOTTOM IN A MEDIUM IN WHICH STRATIFICATION IS CONSTANT WITH RANGE

Normal mode theory is based on the assumption that at large distances from a source, the main part of the sound field consists of standing waves formed by energy that is striking the boundaries in certain preferred directions.

The preferred directions and the amplitude distribution of the standing waves may be determined by solving the wave equation

$$\nabla^2 \Phi = \frac{1}{c^2} \frac{\partial^2 \Phi}{\partial t^2} , \qquad (1)$$

where Φ is a displacement, or a velocity potential (herein Φ is the displacement potential) and c is the velocity of sound in the medium considered.

One may obtain the solution of equation (1) by direct numerical integration, as done by A. D. Little, Incorporated, by means of the computer program described in references 1 and 2. The basis of this program is the rapid evaluation afforded by the use of a high-speed computer.

When equation (1) is solved for Φ, the incremental pressure p is found by definition,

$$p = -\rho \frac{\partial^2 \Phi}{\partial t^2} , \qquad (2)$$

where Φ is the displacement potential and ρ is the material density at the point considered.

We find that equation (1) is separable in terms of range r and depth z, so that the pressure can be written as

$$p = F(r)\, u(z)\, e^{j\omega t} , \qquad (3)$$

where $F(r)$ gives pressure as a function of range, and a differential equation in terms of u(z) may be obtained

$$\frac{d^2u}{dz^2} + \left(\frac{\omega^2}{c^2} - k_r^2\right) u = 0 \ , \qquad (4)$$

where

ω is the angular frequency,

c is the sound velocity (a function of z),*

u = u(z) gives the pressure amplitude distribution as a function of depth,

k_r is the horizontal wave number

$$\frac{\omega}{c} \sin \theta \ , \qquad (5)$$

in which θ is measured relative to the normal to the ocean bottom and c and θ are measured at the same depth.

The physical picture presented by equation (3) is that of a standing wave that has a particular pressure amplitude distribution as a function of depth, u(z), and travels unchanged in shape as it progresses in the r direction.

Equation (4) may be written as

$$\frac{d^2u}{dz^2} + f(z)u = 0 \ , \qquad (6)$$

where

$$f(z) = \frac{\omega^2}{c^2} - k_r^2 \ . \qquad (7)$$

In solving equation (4), we must impose two constraints on any solution: the boundary conditions at the interface between the water and the bottom material, and at the water surface. First, because there must be continuity of pressure and particle velocity in the vertical direction at the bottom interface, it is necessary that

*The z-dependence will henceforth be dropped in the notation c.

$$\frac{1}{u_1}\frac{du_1}{dz} = \frac{\rho_1}{\rho_b}\sqrt{k_r^2 - \frac{\omega^2}{c_b^2}} \quad , \tag{8a}$$

where the terms with subscript b refer to quantities in the bottom material, whereas those with subscript 1 refer to the water side of the interface, and z is measured upward relative to the ocean bottom. Second, since we assume a pressure-release surface at the air-water boundary,

$$u = 0 \tag{8b}$$

at this surface.

Also, we assume that only "unattenuated" modes compose the sound field. These modes, by definition, involve propagation with energy that strikes the bottom at angles larger than the critical angle so that "total reflection" occurs. Since the critical angle θ_c measured relative to the normal to the interface is given by

$$\sin\theta_c = \frac{c_1}{c_b} \quad ,$$

then

$$\sin\theta > \frac{c_1}{c_b}$$

must hold for all energy striking the bottom. (Naturally, c_b must be greater than c_1 to ensure the existence of a critical angle.)

Since

$$k_r = \frac{\omega}{c}\sin\theta \quad ,$$

by using equation (7) and the above inequality, we obtain

$$\left| f(z) \right| < \frac{\omega^2}{c_1^2} - \frac{\omega^2}{c_b^2} \tag{8c}$$

at the bottom interface.

Upon examining equation (6), we can see that if f(z) is positive, u(z) is in the form of a sinusoid. This form of solution is obtained for the interference pattern between upgoing and downgoing waves in the water. If $f(z)$ is negative, $u(z)$ is in the form of an exponential. This is a consequence of Sturm's comparison theorem (reference 3).

The value $f(z)$ is negative everywhere in the bottom and in the water at depths that correspond to shadow zones caused by the vertexing of rays that form a mode. Both distributions are a result of the condition of continuity of pressure in the medium. Thus, when there is total reflection at a level (either by vertexing or reflection from a boundary) and the pressure is finite at that level, then at adjacent levels there is a decay (whose rate is determined by the boundary conditions) but not a discontinuous step to zero pressure. Examples of a distribution involving both exponentials are shown in figure 1, which presents the amplitude distribution as a function of depth, and in figure 2, which presents the ray equivalent of the mode. This concept of the ray equivalent is discussed in more detail in the next subsection of this report. In figure 2, it is seen that the ray equivalent of the normal mode vertexes at a depth of 48 ft; therefore, in figure 1, the pressure amplitude distribution is in the form of an exponential between the surface of the ocean and a depth of 48 ft. It can also be seen in figure 1 that the pressure amplitude distribution in the bottom is in the form of an exponential.

To integrate equation (6) numerically, formulas relating u_{n+1} and its derivative $(du/dz)_{n+1}$ with the quantities u_n and $(du/dz)_n$ must be obtained (the subscripts signify the depth at which the quantities are calculated). This is done by writing a Taylor series for u_{n+1} and $(du/dz)_{n+1}$ and retaining all terms of the third order or less. The details of this procedure are given in references 1 and 2; the results are summarized in equations (9) and (10):

$$u_{n+1} = \frac{\left(1 + \frac{h}{3} f_n\right) u_n + h \left(\frac{du}{dz}\right)_n}{1 + \frac{h^2}{6} f_{n+1}} \tag{9}$$

$$\left(\frac{du}{dz}\right)_{n+1} = \frac{\left(1 - \frac{h}{3} f_{n+1}\right)\left(\frac{du}{dz}\right)_n - \frac{h}{2}\left(f_n + f_{n+1} - \frac{h^2}{6} f_n f_{n+1}\right) u_n}{1 + \frac{h^2}{6} f_{n+1}} \tag{10}$$

where h is the increment between level n and level $n + 1$, and f_n and f_{n+1} are the values of $f(z)$ at n and $n + 1$, respectively.

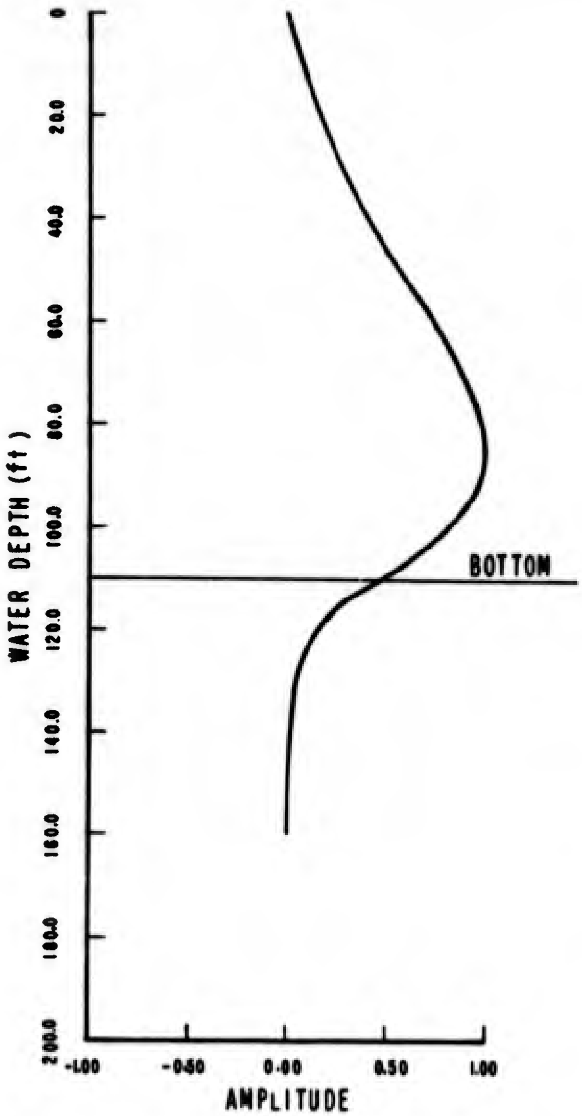


Figure 1. Amplitude Versus Depth, Frequency 282 Hz, Mode 1

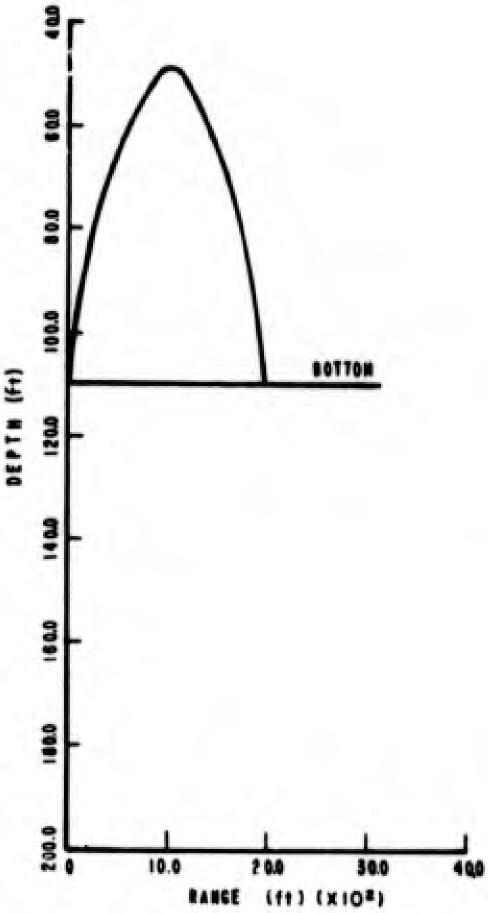


Figure 2. Ray Equivalent, Frequency 282 Hz, Mode 1

Equations (9) and (10) are recursion relationships that permit calculation of u and du/dz at all levels in the water if the given values for u_1 and du_1/dz are the values of u and du/dz just above the bottom.

Since we are interested in normalized values of u over the water column, we can select u_1 to be any arbitrary value ($u_1 = 1$ is convenient). For an arbitrary value of the horizontal wave number k_r that is restricted by equation (8c), we can evaluate (du_1/dz) by equation (8a). Then we can determine u for all levels by using equations (9) and (10) repeatedly. If the value of k_r corresponds to a mode, equation (8b) will be satisfied at the surface of the water. Each mode has, at most, one such solution for a given frequency. There is a low cutoff frequency for each mode, so that at frequencies below the cutoff frequency, equation (8b) cannot be satisfied.

After finding the amplitude distribution of a mode, we must define the mode number. For a finite frequency, the mode number is equal to the number of nodes in the amplitude distribution. Thus, the first mode has a node only at the surface. A representation of the amplitude distribution of the first mode is shown in figure 1.

The Ray Equivalent

Corresponding to the definition of a mode discussed above is a more physical approach in which the ray equivalent of the solution to the wave equation is considered. Additional information concerning this approach is contained in references 4, 5, 6, 7, and 8.

For simplicity, let us consider a two-layer medium of constant water depth H, density ρ_1, and sound velocity c_1 lying over an infinite bottom of density ρ_2 and sound velocity c_2, as shown in figure 3. At large ranges from a point source, we may consider sound to be propagated by plane waves. It is clear from figure 3 that for certain waves whose direction is defined by an angle θ, there will be constructive interference between that wave and a plane wave undergoing one more bottom and surface reflection. For constructive interference to exist, the phase difference between points A and B of figure 3 must be $2(n - 1)\pi$ degrees; i.e., the phase difference must satisfy the equation

$$\frac{2\pi}{\lambda_n}\left[\frac{H}{\cos\theta} + \frac{H}{\cos\theta}\cos 2\theta\right] - \epsilon - \pi = 2(n-1)\pi$$

or

$$\frac{2\pi}{\lambda_n}\left[2H\cos\theta\right] - \epsilon - \pi = 2(n-1)\pi , \qquad (11)$$

where λ_n is the wavelength of the preferred mode, ϵ is the phase change undergone by a plane wave upon bottom reflection, n is the mode number, and the phase change upon reflection from the water surface is assumed to be $-\pi$. If the sound velocity in the water layer varies with depth, the first term of equation (11) would be different from that given above; however, the discussion below applies to either case.

If, for a given wavelength λ_n and angle θ, there is constructive interference between plane waves suffering different numbers of bottom and surface reflections, then propagation consists of a series of upgoing and downgoing waves, as shown in figure 4. The left term of equation (11) is the phase change, 2Δ, undergone in the z direction when a ray makes a surface-bottom-surface cycle. For finite frequencies, $\pi > \epsilon \geq 0$ as $\pi/2 > \theta \geq \theta_c$. Therefore, the phase change, Δ, undergone in the z direction over the water depth is limited by

$$\Delta < n\pi \; . \qquad (12)$$

The pressure is zero at the surface, the phase change upon reflection is $-\pi$, and the direction of propagation is reversed upon reflection from the surface. Therefore, the sound field in the vertical direction is the sum of two sine waves representing the upgoing and downgoing waves in the z direction. These waves are shown for the first two modes in figure 5. Because of equation (12), the number of nodes in this amplitude distribution is equal to the mode number. Thus, there is a correspondence between the definitions of mode number in the solution of the wave equation and the ray equivalent solution.

Now let us consider the procedure actually used in the computer program to determine the ray equivalent. When the wave equation is solved numerically, values of f(z) are obtained; f(z) is given in equation (7) by

$$f(z) = \frac{\omega^2}{c^2} - k_r^2 \; , \qquad (13)$$

where k_r is given in equation (5) by

$$k_r = \frac{\omega}{c} \sin\theta \; . \qquad (14)$$

Therefore, given positive f(z), one can determine, from equations (13) and (14), the cosine of the angle of inclination of the equivalent ray as a function of z

$$\cos\theta = \frac{c}{\omega} \sqrt{f(z)} \; . \qquad (15)$$

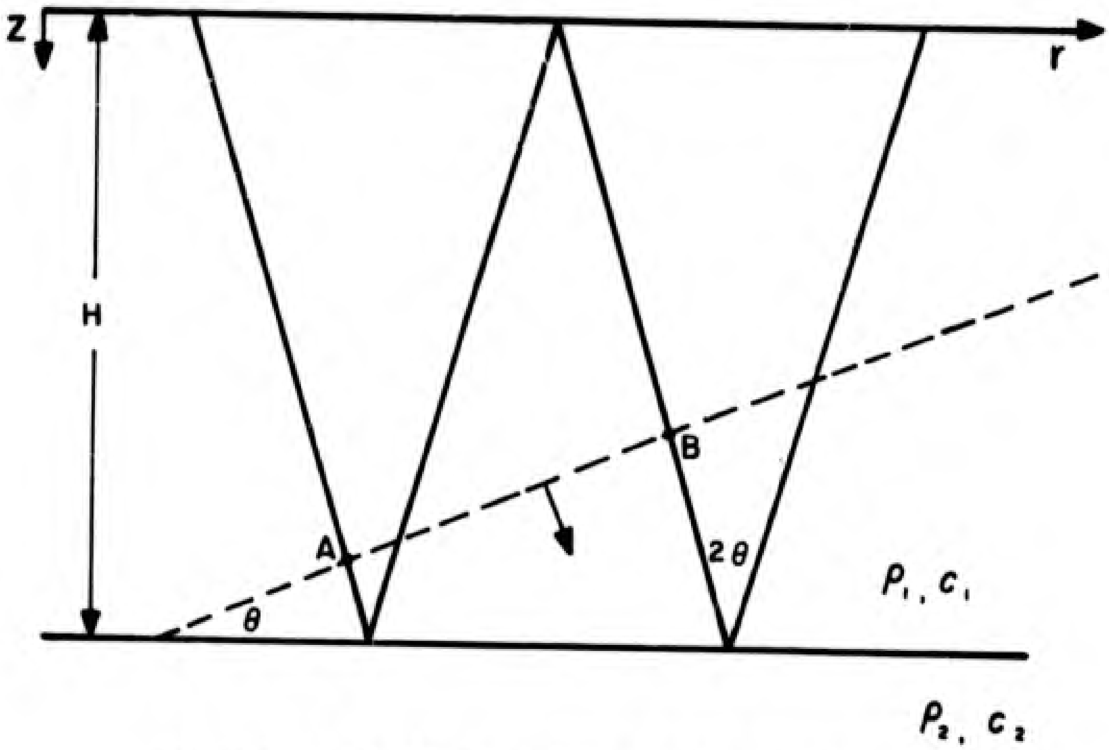


Figure 3. Path of Reflected Waves That Interfere

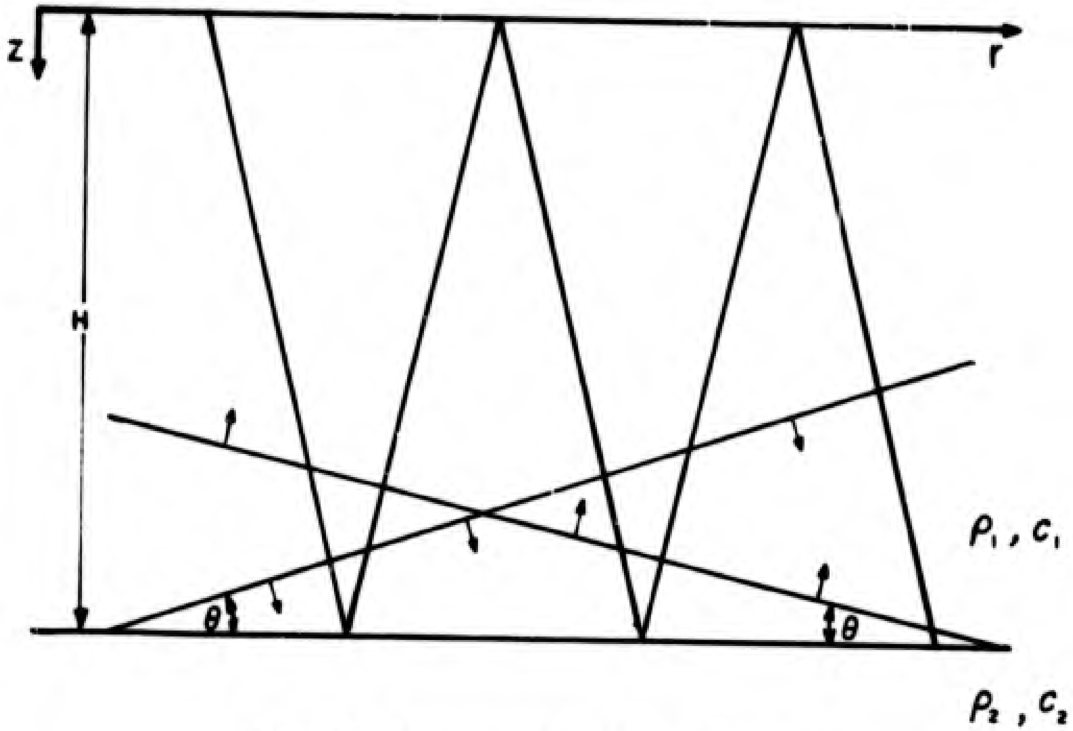


Figure 4. Downgoing and Upgoing Waves

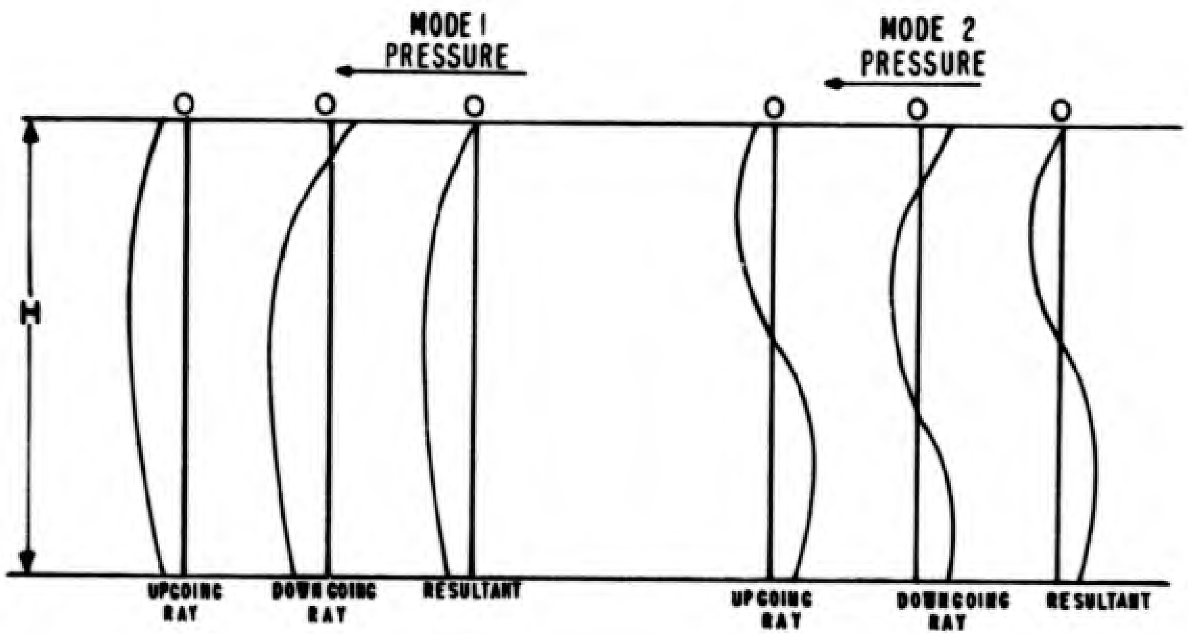


Figure 5. Pressure of Upgoing and Downgoing Rays and the Resultant

If the ray between two points z_1 and z_2 is continuous, then $\Delta z = z_1 - z_2$ may be given in terms of the horizontal distance $\Delta R = R_1 - R_2$ and one particular value of the $\tan\theta$ over the path. This relationship is

$$\frac{\Delta R}{\Delta z} = \tan\theta = \frac{c \sin\theta}{c \cos\theta} = \frac{c}{c_v \cos\theta} ,$$

where c_v, the vertex velocity, equals $c/\sin\theta$.

If the value of θ does not vary appreciably between z_1 and z_2, then we can approximate $\tan\theta$ by

$$\tan\theta = \frac{\tan\theta_1 + \tan\theta_2}{2} ,$$

where θ_1 and θ_2 are taken at $z = z_1$ and z_2, respectively. Thus (reference 9),

$$\frac{\Delta R}{\Delta z} \approx \frac{1}{c_v} \frac{c_1 + c_2}{\cos\theta_1 + \cos\theta_2} , \qquad (16)$$

where

c_v is the vertex velocity,

c_1, c_2 are the sound velocities at z_1 and z_2, respectively,

θ_1, θ_2 are the angles relative to the normal of the ray at z_1 and z_2, respectively.

Thus, given values of $f(z)$, one can construct a ray equivalent. If vertexing takes place, the depth z_v at which it occurs is the level whose value of sound velocity equals c_v.

The ray equivalent has a counterpart in the analysis of sound propagation using ray theory. In ray theory, rays corresponding to a continuum of angles of propagation are summed at the receiver. Most groups of rays effectively cancel each other and contribute little to the sound field. This leaves a discrete number of rays that sum constructively to form the sound field. These rays correspond to the ray equivalent of modes that compose the sound field. According to ray theory, the rays that contribute to the sound field are determined by interference effects, and hence, as in normal mode theory, the geometry of the dominant rays changes as a function of frequency. Also as in normal mode theory, the sum of these rays is influenced by the source-receiver geometry. However, there are two factors that limit the correspondence between normal mode and ray theory. First, in ray calculations the sound field is often obtained by simply adding all rays without regard to their phase. Second, the geometrical approximation limits the validity of ray tracing. This limitation can produce differences not only in the theoretical pressure field but also in the ray representations in ray tracing and normal mode methods.

If the velocity profile assumed in the calculations contains only a monotonic variation in sound velocity with depth (as, for example, in figure 6) or describes a simple sound channel, a relatively simple ray equivalent obtains (as in figure 2). However, let us consider the ray equivalent associated with the more complicated velocity profile shown in figure 7. This profile describes two sound channels at depths of about 60 and 120 ft, respectively. The associated ray equivalent of mode 1 at a frequency of 200 Hz is shown in figure 8. It can be seen that the ray equivalent consists of two traveling waves, one for each of the two sound channels in the water column. This phenomenon of wave "leakage" through a layer is described in reference 10.

Phase Velocity and Group Velocity

The phase velocity V_p is given by the relationship

$$V_p = \frac{c}{\sin\theta} \quad , \tag{17}$$

where θ is the direction of propagation of a plane wave where the sound velocity has a value c. Equation (17) can also be written

$$V_p = c_v \quad , \tag{18}$$

where c_v is the vertex velocity.

The group velocity V_g can be considered from two points of view. First, V_g may be considered as a measure of the speed of propagation in the horizontal direction of a number of frequencies in a band $\Delta\omega$ centered about ω. V_g may be given by (reference 11)

$$V_g = \frac{\Delta\omega}{\Delta\kappa} \tag{19}$$

or by (reference 11)

$$\frac{V_g}{c} = \frac{V_p}{c} + (\kappa H)\frac{d\left(\frac{V_p}{c}\right)}{d(\kappa H)} \quad , \tag{20}$$

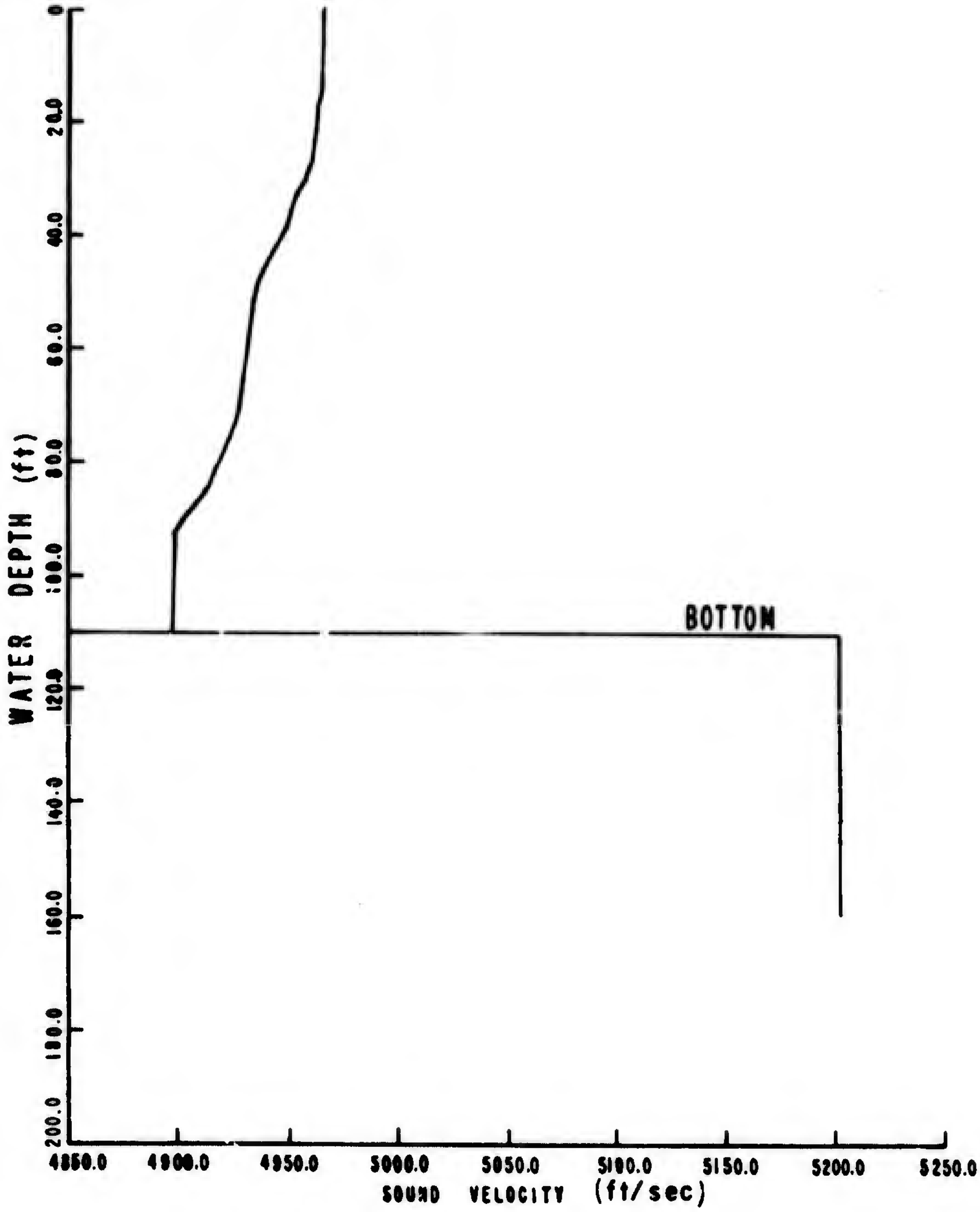


Figure 6. Velocity Profile, Simple Variation

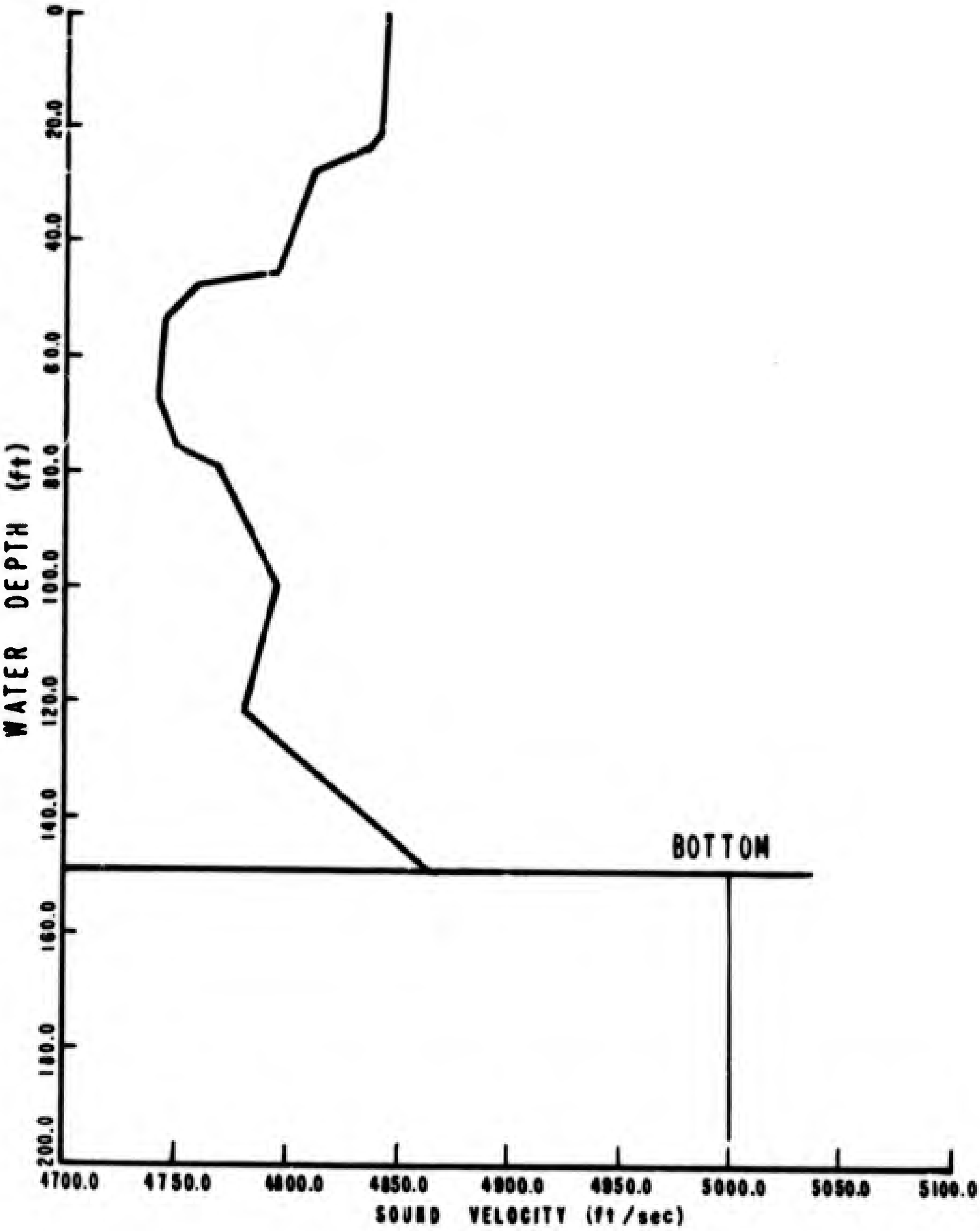


Figure 7. Velocity Profile, Complex Variation

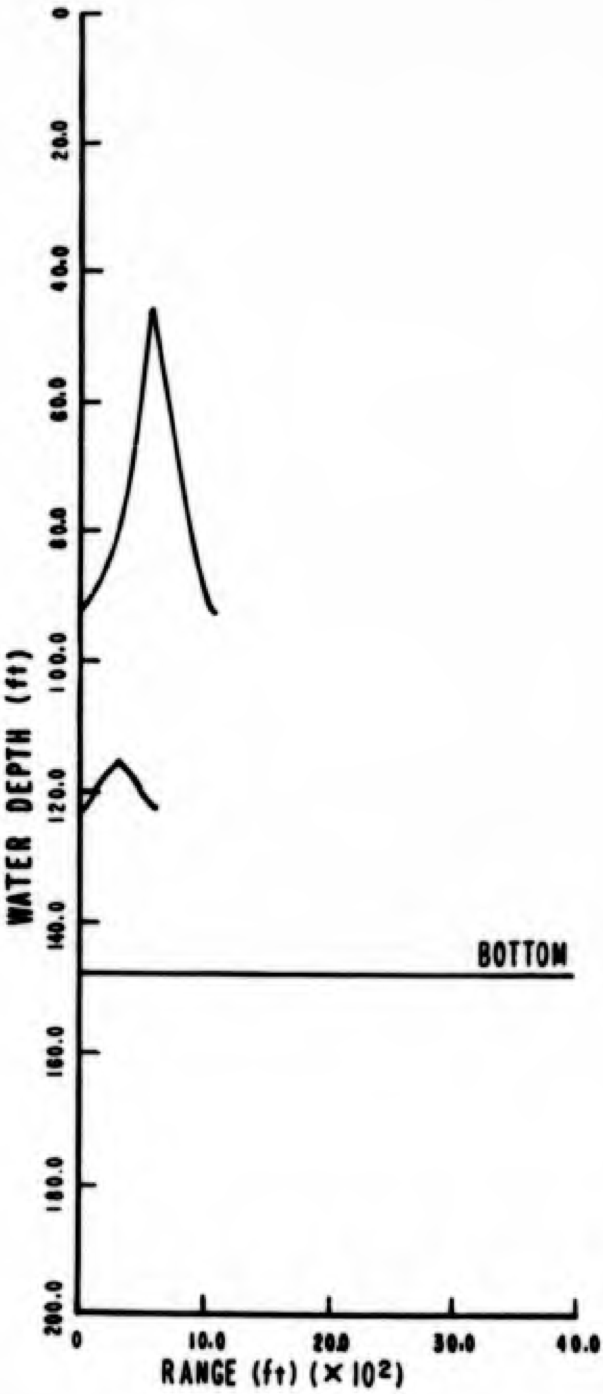


Figure 8. Ray Equivalent, Frequency 200 Hz, Mode 1

where H is the water depth, and κ is wave number.

It can be seen that this approach to the calculation of group velocities involves the calculation of derivatives, which is not desirable in a computer program since it produces inaccuracies and makes it necessary to obtain an unnecessarily large number of values of phase velocity as a function of frequency.

Second, Tolstoy (reference 12) has used a general theorem by Biot (reference 13) to show the equivalence of V_g in equations (19) and (20) to the rate of energy transport in the horizontal direction. The group velocity is given by

$$V_g = \frac{1}{V_p} \frac{\nu}{\sigma}, \tag{21}$$

where

$$\nu = \int_{-\infty}^{+\infty} \rho \phi^2 \, dz \tag{22}$$

$$\sigma = \int_{-\infty}^{+\infty} \frac{\rho}{c^2} \phi^2 \, dz , \tag{23}$$

in which ρ and c are, respectively, the density and sound velocity at z, and ϕ is given in the equation

$$\Phi = \phi(z)\, e^{j(\pm \alpha x - \omega t)} , \tag{24}$$

in which Φ is the displacement potential in equation (1).

Since, by equation (2),

$$p = -\rho \frac{\partial^2 \Phi}{\partial t^2}$$

and u is the value of pressure (normalized to maximum amplitude) as a function of z; then, by equations (2) and (24),

$$u \propto \rho \varphi , \tag{25}$$

where φ is the normalized value of ϕ. Thus, by equation (25) we can obtain φ once u is known, since we are interested only in the normalized values of u and φ for given ω.

Given ρ_1 in the water, ρ_b in the bottom, and u normalized to maximum pressure amplitude; to obtain φ, the normalization of ϕ, we first obtain

$$\varphi_{1,\,u_u(z)} = \frac{u(z)}{\rho_1}$$

$$\varphi_{b,\,u_u(z)} = \frac{u(z)}{\rho_b} \quad , \tag{25a}$$

where 1 and b signify water and bottom, respectively, and u_u signifies unnormalized. Since the maximum value of u lies in the water and ρ_b is greater than ρ_1 and because u(z) is normalized with respect to maximum amplitude, we multiply both expressions in equation (25a) by ρ_1 to obtain φ normalized with respect to maximum amplitude, so that

$$\varphi_1 = u(z)$$

$$\varphi_b = u(z)\ \rho_1/\rho_b \quad . \tag{25b}$$

The normalized value for ϕ is represented by φ and may be used instead of ϕ in equations (22) and (23).

Excitation Pressure and Propagation Loss

The sound field produced by a simple harmonic source in a two-layered half-space (figure 9) with a free surface at z = 0 and the boundary between two fluids at z = H is given by the solution of equation (1). The solution is given in reference 11 by

$$\phi = -j\ \frac{1}{\sqrt{r}}\ \frac{1}{\omega^2} \sum_m P_m e^{-j(\kappa_m r - \omega t - \pi/4)} \quad , \tag{26}$$

where

r is the horizontal range,

ω is the angular frequency,

m is the mode number,

κ_m is the horizontal wave number k_r for mode m, and

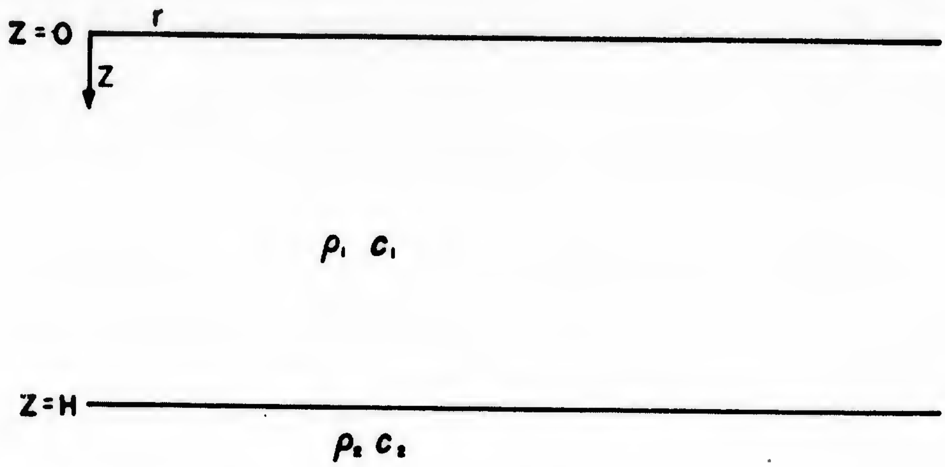


Figure 9. Two-Layered Half-Space

$$P_m = p_m \frac{1}{\rho_s} \varphi_m(z)\varphi_m(z_o) \;, \tag{27}$$

in which

ρ_s is the water density at the source,

$\varphi_m(z)$ is the normalized displacement potential, a function of depth,

z_o is the source depth,

z is the receiver depth,

m is the mode number.

The notation p_m is the excitation function given by

$$p_m = 2\pi(\rho_o c_o S)^{1/2} \frac{\rho_o}{\nu_m \sqrt{\kappa_m}} \;, \tag{28}$$

where

ρ_o is the water density at the source,

c_o is sound velocity at the source,

S is the power output of an omnidirectional source,*

κ_m is the horizontal wave number, and

$$\nu_m = \int_{-\infty}^{+\infty} \rho \varphi_m^2 \, dz \ , \qquad (29)$$

where φ_m is the normalized displacement potential.

If the source produces a unit sound pressure level, then the following relationship (reference 11) must be satisfied:

$$4\pi \left(\frac{S c_o \rho_o}{2\pi} \right)^{1/2} = 1 \ . \qquad (30)$$

Substituting equation (30) into equation (28), we obtain for the excitation pressure

$$p_m = \frac{\rho_o (2\pi)^{1/2}}{2 \nu_m \sqrt{\kappa_m}} \ . \qquad (31)$$

The excitation pressure is the sound pressure amplitude produced by a source that generates a unit pressure level at unit range when both source and receiver are at antinodes. It is essentially a measure of the source level of mode m for a unit source.

From equation (26) we determine the pressure amplitude characteristics of the sound field, and this amplitude, p_a, is given by

$$p_a = \frac{\rho_o}{\sqrt{r}} \left\{ \left[\sum_m P_m \cos\left(\kappa_m r - \frac{\pi}{4}\right) \right]^2 + \left[\sum_m P_m \sin\left(\kappa_m r - \frac{\pi}{4}\right) \right]^2 \right\}^{1/2} \ . \qquad (32)$$

Since p_a is the sound pressure amplitude at range r from a generator with unit source level, the value of propagation loss L_r at range r for given depths of source and receiver is

$$L_r = -20 \log p_a \ . \qquad (33)$$

* This quantity is represented by Π in reference 11.

It can be seen from equations (27) and (32) that, once φ_m is known, one can easily determine the effect of the source and receiver depths on the sound field at a given range. If the source depth, z_o, is such that $\varphi_m(z_o)$ is a node, the mode m will be suppressed in the sound field; conversely, if z_o is such that $\varphi_m(z_o)$ is an antinode, the sound field of mode m will be greater than it would be at depths for which $\varphi_m(z)$ is less than $\varphi_m(z_o)$. The same relationships apply to the effect of the receiver depth upon the sound field.

It can also be seen from equations (27), (28), and (29) that the pressure amplitude does not depend upon the normalization of $\phi(z)$.

For small attenuations of individual modes, equation (32) may be rewritten to include, for mode m, losses at the boundaries as a function of range and losses caused by absorption of sound energy in the water, so that

$$p_a = \frac{\rho_o}{\sqrt{r}}\left\{\left[\sum_m P_m 10^{(-D_m r/20 - ar/20)} \cos\left(\kappa_m r - \frac{\pi}{4}\right)\right]^2 + \left[\sum_m P_m 10^{(-D_m r/20 - ar/20)} \sin\left(\kappa_m r - \frac{\pi}{4}\right)\right]^2\right\}^{1/2}, \quad (34)$$

where D_m (which must be specified by the user of the program) is a measure of bottom loss in decibels of loss per unit increment of range and a is the attenuation coefficient given in reference 14 by

$$a \frac{\mathrm{dB}}{\mathrm{kyd}} = \frac{0.1f^2}{1+f^2} + \frac{40f^2}{4100+f^2}, \quad (35)$$

where f is the frequency in kilohertz.

PROPAGATION IN A MEDIUM IN WHICH STRATIFICATION IS A FUNCTION OF RANGE

In computing the sound field for a medium in which stratification is a function of position, the following three assumptions (reference 11) are made:

a. The values of $\varphi_m(z)$ correspond to local stratification.

b. The stratification varies slowly from one region to another so that there is no appreciable scattering of energy from one mode to another, when sound propagates through the medium.

c. If the latter assumption holds, then the bottom topography can be approximated by a number of segments of different depth parallel to the surface as shown in figure 10. (Assumptions b and c will be removed in the following section.)

The power in a given mode is determined by $\varphi(z_0)$, where z_o is the source depth. The pressure level at the receiver is determined by $\varphi(z)$, where z is the receiver depth. When there is little scattering between modes, we can approximate (reference 11) the pressure level for a given mode by substituting $\bar{P}_m$ for P_m in equation (34), with

$$\bar{P}_m = \left[P_{source}\, P_{receiver}\right]^{1/2} , \tag{36}$$

where "source" and "receiver" refer to the values of P obtained by taking into account the local stratification at the source or receiver.

Thus, P_{source} is found by calculating P_m in equation (27), using the depth, velocity profile, and bottom composition present in the segment of the range containing the source. In addition, $\varphi_m(z_o)$ should be substituted for $\varphi_m(z)$. Similarly, to calculate $P_{receiver}$, the local stratification at the receiver should be used in equation (27) and $\varphi_m(z)$ should be substituted for $\varphi_m(z_o)$.

In equation (34) the phase of the signal depends upon $\kappa_m r$. In a medium not horizontally stratified, κ_m is a function of range, since the variation in water depth and bottom composition causes κ_m to vary. The term κ_m can be expressed by (reference 11)

$$\kappa_m(r) = \bar{\kappa}_m + \epsilon_m(r) ,$$

where $\bar{\kappa}_m$ is the average of κ_m over r and $\kappa_m(r)$ is the value of κ_m at a range r. Equation (36) can be used to form a perturbation solution of the plane wave equation

$$\nabla^2\Phi + \frac{\omega^2}{c^2}\Phi = 0 ;$$

in this case, one obtains (reference 11) an approximate solution so that in equation (34), $\kappa_m(r)$ can be replaced by

$$\bar{\kappa}_m r + \Delta S_m ,$$

where

$$\Delta S_m = \int_o^r \epsilon_m(\lambda)\, d\lambda . \tag{37}$$

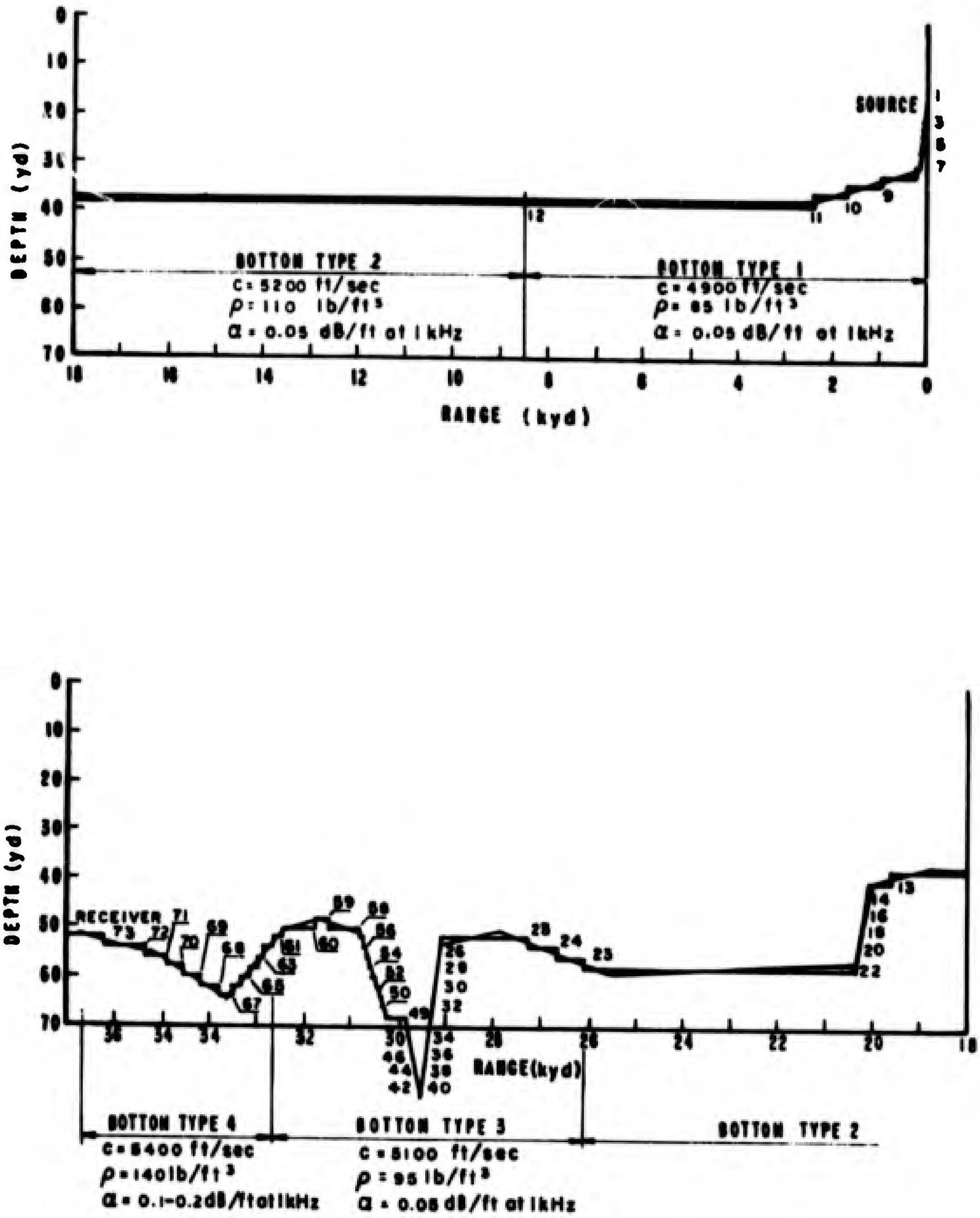


Figure 10. Model of BIFI Range

If the bottom properties vary with range from the source, then the attenuation per unit range due to losses at the boundaries D_m varies as a function of range, and $D_m(r)$ in equation (34) can be replaced by

$$\int_0^r D_m(\rho)\, d\rho \ .$$

Thus equation (34) becomes

$$p_a = \frac{\rho_o}{\sqrt{r}} \left\{ \left[\sum_m \bar{P}_m \, 10^{-ar/20 - \int_0^r D_m(\rho)\, d\rho/20} \cos(\bar{\kappa}_m r + \Delta S - \pi/4) \right]^2 + \left[\sum_m \bar{P}_m \, 10^{-ar/20 - \int_0^r D_m(\rho)\, d\rho/20} \sin(\bar{\kappa}_m r + \Delta S - \pi/4) \right]^2 \right\}^{1/2} \quad (38)$$

and

$$L_r = -20 \log p_a \ , \quad (39)$$

where L_r is the propagation loss at range r .

The amplitude distribution as a function of depth and the ray equivalent can be calculated for a particular segment of the range by a method identical to that used for stratification constant with range, using the parameters determined by the local stratification.

The above formulation corresponds to a simple physical picture of the propagation of sound in a medium whose stratification changes with range. Let us consider two plane waves propagating in a medium segmented as described above. Let the first wave correspond to a given order mode in one segment and the second wave correspond to the same order mode in an adjacent segment. If the change in stratification between segments is small, the difference in direction of the two waves is small; hence the waves are excited at approximately the same level at the source. This level is determined by the excitation pressure of the mode. However, the field at the receiver will correspond to the stratification of the segment in which the receiver is located, and the plane wave that corresponds to the mode considered will dominate the contribution by that mode to the field in the segment. Thus the field in the segments is determined by the plane wave corresponding to the modes that compose the field. By extending the concept to several segments and several plane waves, we can similarly picture a given mode changing shape in conformance to the local stratification, as it moves from segment to segment.

Let us consider, for a specific case, the effect upon sound propagation of differences in c_B, the velocity of sound in the bottom, found in segments of a range. This effect can be seen in table 1, in which θ_1, the angle of incidence at the bottom for the wave corresponding to mode 1 at 127 Hz, is given as a function of c_B for the velocity profile shown in figure 11. Consider the case where c_P varies between 5200 and 6000 ft/sec over a range. The decrease in θ_1 is rather small, 0.9°. Thus it appears that the formulation presented above in equations (38) and (39) can be used for such a variation in c_B, with $D_1(r)$, the bottom loss for mode 1 as a function of range, chosen to correspond to c_B in each segment.

Table 1. Angle of Incidence at Bottom for Mode 1, θ_1, and Critical Angle θ_c as a Function of Velocity of Sound in Bottom c_B, at 127 Hz

c_B (ft/sec)	θ_1	θ_c
5000	83°30'	82°22'
5200	82°0'	72°22'
5400	81°30'	66°36'
5500	81°24'	64°18'
5600	81°18'	62°15'
5800	81°12'	58°42'
6000	81°06'	55°41'

Let us consider the result when the value of c_B varies from 5000 to 5200 ft/sec. The change in θ_1 is larger (1.5°) than for the case considered previously. Furthermore, the θ_c corresponding to a c_B of 5000 ft/sec is greater than the θ_1 corresponding to 5200 ft/sec. In the situation where c_B increases with increasing range from 5000 to 5200 ft/sec, the ray corresponding to mode 1 for the harder bottom (c_B = 5200 ft/sec) can be highly attenuated because the angle of incidence is less than the critical angle as the ray passes through the segment that contains the softer bottom. If the softer segment is long, there is a high probability that the ray considered cannot contribute significantly to the sound field over the hard bottom. Thus, if the ray is highly attenuated, one would expect a dramatic drop in signal level and an unstable field. If, on the other hand, the transition is from a harder to a softer bottom, the value of θ_1 increases, and hence the sound field should be stronger over the softer bottom because of the transition alone. (This subject and pertinent experimental results are discussed in reference 15.)

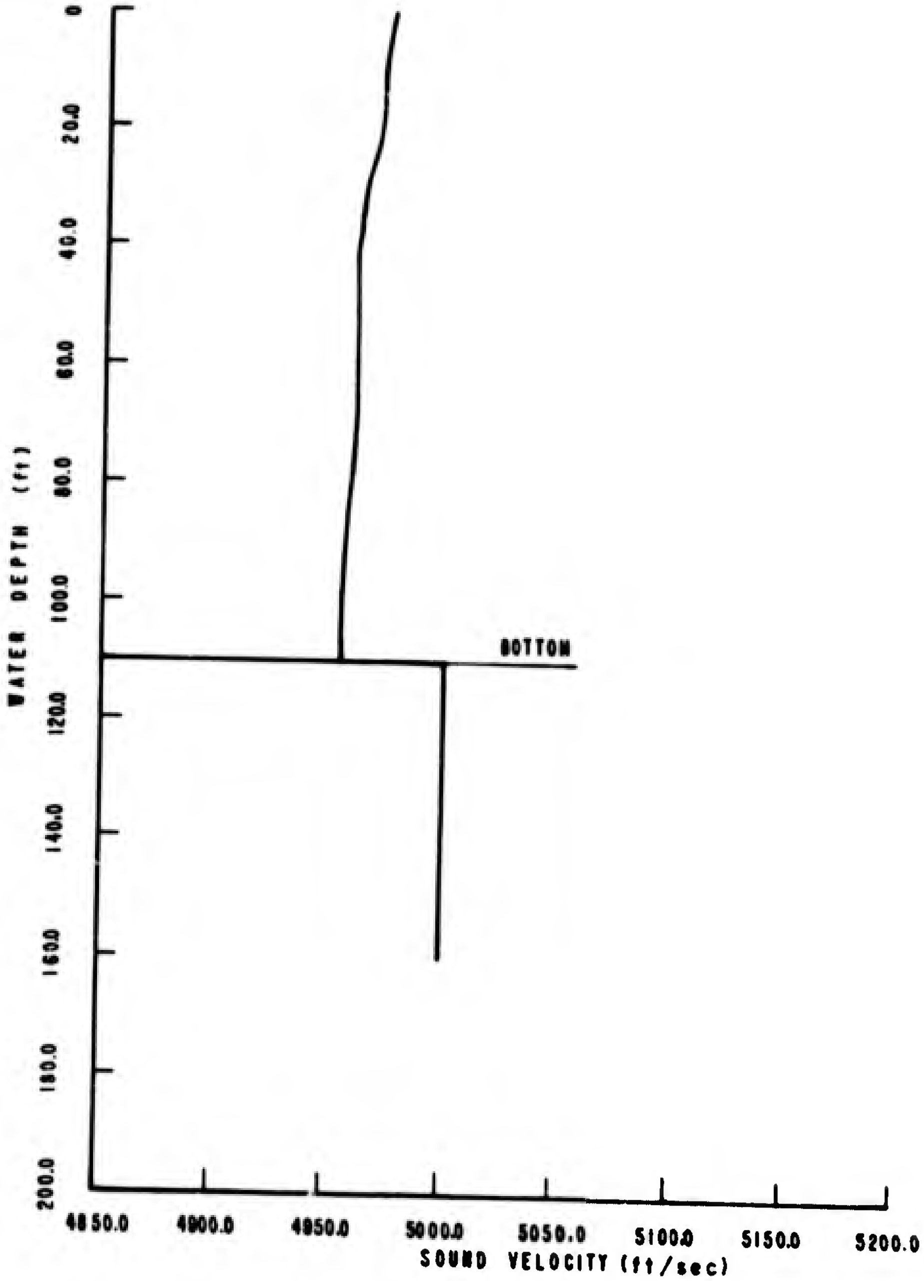


Figure 11. Velocity Profile Input to Program

A change in stratification due to an increase in water depth results in an increase in the θ_1 associated with a given mode. Hence, the result of such a change in stratification is similar to the result of a decrease in c_B. (The effect of a change in depth is discussed in detail in reference 16 in the context of the consequences of a change in water depth due to tide.)

Finally, a change in stratification that can be described by a change in velocity profile with range has two effects. First, like the changes in stratification discussed above, the angle θ_1 associated with a given mode is changed. Second, the angle of propagation of a given wave corresponding to a given mode in one segment is diverted toward the direction corresponding to the same mode in the following segment entered by the wave.

MODE CONVERSION AND A SLOPING BOTTOM

One method of determining the sound field in a medium such as that shown in figure 10, while accounting for the effect of mode conversion, is to assume that the field at the interface between any two segments is produced by an equivalent line source. The purpose of synthesizing such an equivalent source is to determine the sound field in terms of modes immediately to the right of the interface when the field to the left is known. Once the sound field is determined in this manner, it is a trivial matter to determine the sound field throughout the entire right segment. The equivalent source can be located anywhere in the left segment. It is convenient to consider the source at unit distance from the interface but it can be placed arbitrarily close to the interface after reducing its amplitude in the appropriate manner. Since the equivalent array can be placed arbitrarily close to the interface, it follows that the field immediately to the right of the interface can be determined by calculating the field produced by this source as it radiates energy into the right segment. The amplitude and phase of such a line source may be determined uniquely for each mode by taking advantage of the fact that a set of functions closely related to the eigenfunctions $\varphi_m(z)$ of the modes are orthogonal. Thus (reference 17)

$$\int_{-\infty}^{+\infty} \rho \varphi_m(z) \varphi_n(z) \, dz = \begin{cases} k & m = n \\ 0 & m \neq n \end{cases} , \qquad (40)$$

where ρ is the density at depth z and

$$\rho = \begin{cases} \rho_o & \text{in the water column} \\ \rho_B & \text{in the bottom} \end{cases} ,$$

and k is a constant.

Examination of equations (34) and (27) shows that, for a given mode, the pressure amplitude p_a is dependent on the source depth only through $\varphi_m(z_o)$.

Hence

$$p_a = k\varphi_m(z_o) \tag{41}$$

for a given mode m and source depth z_o. Here k, a constant independent of source depth, is obtained from equation (34), with a modification of the D_m term to reflect the changing stratification with range, and is given by

$$k = \frac{\rho_r}{\sqrt{r}} \frac{P_m}{\varphi_m(z_o)} 10^{-ar/20 - \int_o^r D(\gamma)\, d\gamma/20} , \tag{42}$$

where r is the range to the interface.

Given a continuous transmitting array, we have from superposition

$$p_a = k\int_{-\infty}^{+\infty} w(z)\, \varphi_m(z)\, dz , \tag{43}$$

where w(z) is the amplitude of the array measured at unit distance and depth z.

The phase of the array will be independent of depth and can be considered separately. We wish to determine an equivalent array corresponding to mode n and the orthogonality property given by equation (40) dictates the form of the amplitude distribution of the array. Hence

$$w(z) = k_n \rho\varphi_n(z) \tag{44}$$

where k_n is a constant independent of z for mode n.

Because of orthogonality (equation (40)) and the condition that the array duplicate the sound field given by equation (41), we obtain

$$p_a = k\int_{-\infty}^{+\infty} k_n \rho\varphi_n(z)\, \varphi_m(z)\, dz = \begin{cases} k\varphi_n(z) & m = n \\ o & m \neq n \end{cases} . \tag{45}$$

Hence

$$k_n = \frac{\varphi_n(z_o)}{\int_{-\infty}^{+\infty} \rho\varphi_n^2(z)\, dz} , \tag{46}$$

and from equation (44)

$$w_n(z) = \frac{\varphi_n(z_o)}{\int_{-\infty}^{\infty} \rho\, \varphi_n^2(z)\, dz}\, \rho\, \varphi_n(z) \quad . \tag{47}$$

The phase of the array measured at unit distance is obtained directly from the phase of the mode given by equation (38). The phase Φ is independent of depth and is indicated by

$$e^{j\Phi_n} = e^{j(\bar{\kappa}_m r + \Delta S - \pi/4)} \quad , \tag{48}$$

where r is the range to the interface.

This procedure can be followed for all modes in the sound field and the pressure amplitude of equivalent sources obtained as shown on the left of figure 12. These sources can then be added, taking into account the phases of the arrays, so that the sources are combined to obtain one equivalent line source, such as the one sketched on the right of figure 12, so that the amplitude and phase are given by

$$w(z)\, e^{j\Phi} = \sum_m w_n(z)\, e^{j\Phi_n} \quad . \tag{49}$$

The resultant pressure amplitudes of the individual modes immediately to the right of the interface are obtained in a manner similar to equation (43), so that

$$p_a = \left| k' \int_{-\infty}^{\infty} w(z)\, e^{j\Phi}\, \varphi_m'(z)\, dz \right| \quad , \tag{50}$$

where the primes indicate that the parameters are determined in the right segment. The phase of a given mode may be obtained from equation (50) before the absolute value operation is performed. Naturally the total field follows from a knowledge of the amplitude and phase of the individual modes.

In the model, the pressure amplitude distribution as a function of depth for the individual modes is determined by numerical integration at a discrete number of depths. Hence, a continuous equivalent source must be approximated by a limited number of discrete sources. Hence the orthogonality property is only approximate and

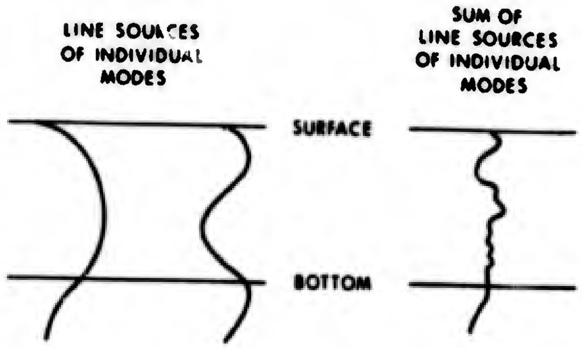


Figure 12. Representations of Equivalent Arrays

$$\sum_{i=1}^{N} \rho \varphi_m(i)\, \varphi_n(i)\, \Delta z \approx \begin{cases} k & m = n \\ o & m \neq n \end{cases} , \qquad (51)$$

where N is the number of discrete depths and $\varphi(i)$ is $\varphi(z)$ evaluated at the i-th discrete depth. It is believed that the vertical separation between values of the pressure amplitude distribution necessary to obtain convergence in the numerical integration over the water column is sufficient for this approximation. This has been borne out by test cases where the stratification was assumed to be identical in the adjacent segments. Assuming orthogonality, the level of a given mode produced is the same immediately to the left or right of the interface and the levels of all other modes are zero. For sample cases run, for a few simple velocity profiles, the level of the approximated mode was virtually the same on both sides of the interface, and the levels of all other modes were at least 50 dB below this level. Likewise, when identical stratification was assumed in eight adjacent segments, the level of the approximated mode was virtually the same on both sides of all interfaces, and the levels of all other modes were at least 50 dB below this level at all ranges. Assuming orthogonality for arrays of point sources, the integrals in the preceding equations must be replaced by sums so that equations (43), (45), (46), (47), (49), and (50) become

$$p_a = k \sum_{i=1}^{N} w(i) \varphi_m(i) \, \Delta z \tag{52}$$

$$p_a = k \sum_{i=1}^{N} k_n \rho \varphi_n(i) \, \varphi_m(i) \, \Delta z = \begin{cases} k \varphi_n(z_o) & m = n \\ o & m \neq n \end{cases} \tag{53}$$

$$k_n = \frac{\varphi_n(z_o)}{\sum_{i=1}^{N} \rho \varphi_n^2(i) \, \Delta z} \tag{54}$$

$$w_n(i) = \frac{\varphi_n(z_o)}{\sum_{i=1}^{N} \rho \varphi_n^2(i) \, \Delta z} \rho \varphi_n(i) \tag{55}$$

$$w(i) e^{j\Phi} = \sum_m w_n(i) \, e^{j\Phi_n} \tag{56}$$

$$p_a = \left| k' \sum_{i=1}^{N} w(i) \, e^{j\Phi} \, \varphi'_m(i) \, \Delta z \right| . \tag{57}$$

Another way of looking at the same problem is to consider the consequences of continuity of pressure at the interface and orthogonality. Assume for the sake of simplicity that functions $u_m(z)$ and $u'_m(z)$ are both orthonormal (the unprimed symbol indicates a function applicable in the left segment and the primed symbol indicates applicability in the right segment) so that

$$\int_{-\infty}^{+\infty} u_m(z)\, u_n(z)\, dz = \begin{cases} 1 & m = n \\ 0 & m \neq n \end{cases} \tag{58}$$

$$\int_{-\infty}^{+\infty} u'_{m'}(z)\, u'_{n'}(z)\, dz = \begin{cases} 1 & m' = n' \\ 0 & m' \neq n' \end{cases} \tag{59}$$

and by a reasoning similar to that applied above that the pressure be expressed by $\sum_m k u_m(z)$ and $\sum_{m'} k' u'_{m'}(z)$ (k and k' are complex constants) so that by continuity of pressure across the interface

$$\sum_m k_m u_m(z) = \sum_{m'} k'_{m'} u'_{m'}(z) \; . \tag{60}$$

Generally, we know values of k_m, $u_m(z)$ and $u'_{m'}(z)$ and wish to determine $k'_{m'}$. If we are interested in the field to the left corresponding to mode n and the resultant field to the right after mode conversion, we multiply both sides of the equation (60) by $u_n(z)$ and integrate over all z and get

$$\int_{-\infty}^{+\infty} \sum_m k_m u_m(z)\, u_n(z)\, dz = \int_{-\infty}^{+\infty} \sum_{m'} k'_{m'} u'_{m'}(z)\, u_n(z)\, dz \; . \tag{61}$$

By using orthonormality

$$\int_{-\infty}^{+\infty} k_n u_n(z)\, u_n(z)\, dz = \int_{-\infty}^{+\infty} \sum_{m'} k'_{m'} u'_{m'}(z)\, u_n(z)\, dz \; . \tag{62}$$

Equation (62) tells us that an equivalent array proportional to $u_n(z)$, transmitting energy into mode n in the left segment represented by

$$\int_{-\infty}^{+\infty} u_n(z)\, u_n(z)\, dz$$

on the left in the equation, produces energy in all modes in the right segment as indicated by the additional $\sum_{m'}$ on the right in the equation. For $m' \neq n$ in the sum, the term corresponds to the level of the mode produced by mode conversion.

If we wish to determine the extent of this coupling, the coefficient k'_p of a given mode p must be determined. This can be done by multiplying equation (62) by $u'_p(z)/u_n(z)$ so that

$$\int_{-\infty}^{\infty} k_n u_n(z) u'_p(z) dz = \int_{-\infty}^{\infty} \sum_{m'} k'_{m'} u'_{m'}(z) u'_p(z) dz \quad . \tag{63}$$

By orthogonality of $u'_{m'}(z)$,

$$\int_{-\infty}^{\infty} k_n u_n(z) u'_p(z) dz = \int_{-\infty}^{\infty} k'_p u'_p(z) u'_p(z) dz \quad . \tag{64}$$

or

$$k'_p = \int_{-\infty}^{\infty} k_n u_n(z) u'_p(z) dz \quad . \tag{65}$$

Equation (65) tells us that an equivalent array proportional to $u_n(z)$, transmitting energy into mode p in the right segment, produces the energy associated with mode p in the right segment. This mathematically corresponds to the physical ideas inherent in the conception of equivalent arrays. However, in utilizing equivalent arrays, the field may be found directly and it is not necessary to solve explicitly for coupling coefficients.

In extending the scope of the model to include the effect of a sloping bottom, the following idealization is made. The total range interval is divided, as shown in figure 13, into segments in which the bottom is approximated as being smooth and having a constant slope. The bottom composition and velocity profile are constant within any one segment but may differ in various segments.

The effect of a sloping bottom can be accounted for in any segment in the following manner. First, as for the kind of segmentation shown in figure 10, the sound field is obtained at the beginning of the segment as generated by an equivalent array in the previous segment. Within the segment, there is a continuous process of mode conversion due to two processes: (1) mode conversion results from the change in the eigenfunctions of the modes due to the changing depth, and (2) mode conversion results from the deflection of energy by the sloping bottom.

The effect of the change in depth can be determined in a manner identical to that used in treating the stratification shown in figure 10. An equivalent array can be determined at any range where the sound field is known. This array can be used

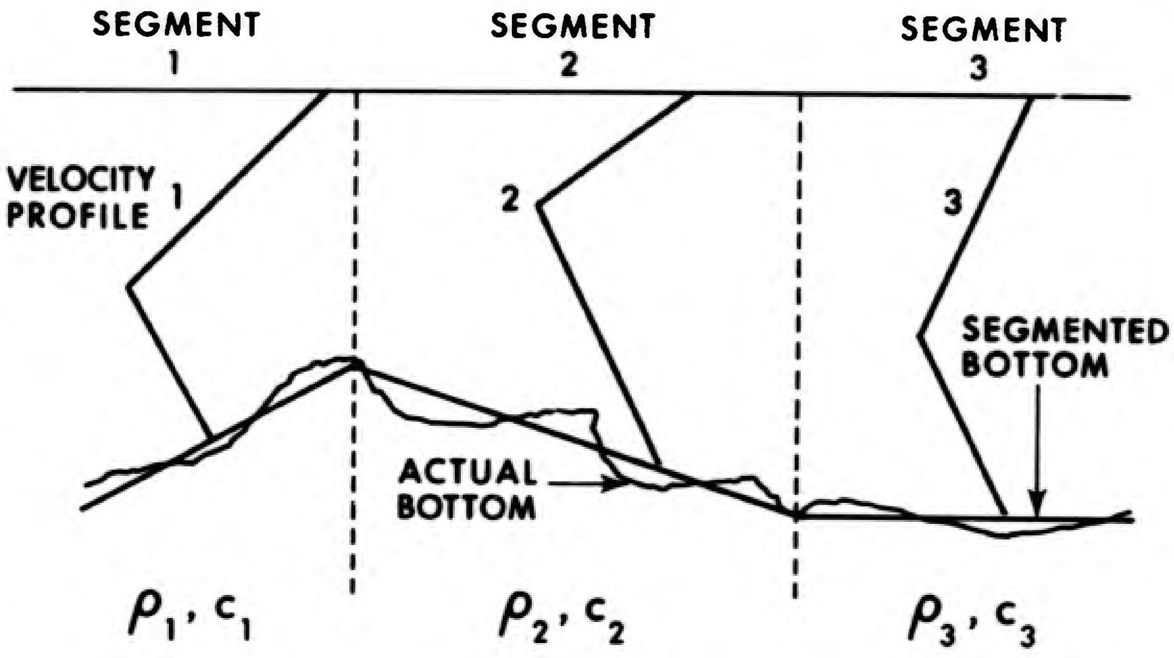


Figure 13. Typical Segmentation of a Medium Which Varies with Range

to determine the sound field immediately to the right. This process is continued until the sound field has been determined throughout the segment.

The deflection of energy by the sloping bottom is accounted for by considering the ray equivalent of an individual mode as shown in figure 14. As can be seen by observing the ray equivalent, the extent to which energy associated with a mode is deflected, at a given range, is determined by the skip distance of the mode and the angle of inclination of the bottom. In a so-called conversion distance D, all energy is deflected by an angle $2\theta_B$, where θ_B is the angle of inclination of the bottom. Knowing the ray equivalent and θ_B, one can determine D "graphically" on a computer. At the beginning of the segment, all energy progresses in a direction determined by the local stratification. Hence one equivalent array can be used to represent the mode. At greater ranges, part of the energy is deflected and the remaining energy retains its original direction. Hence the one equivalent array must be divided into two subarrays, which represent, respectively, the undeflected energy and the energy deflected by the sloping bottom. Since the skip distance is a function of range, the conversion distance is also a function of range, and the composition of these subarrays is determined by the number of bounces $N_c(r)$ at any given range, given by

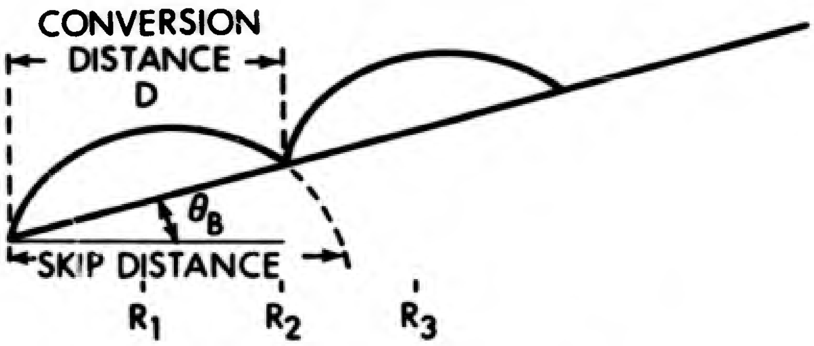
$$N_c(r) = \int_0^r \frac{dr}{D(r)} , \qquad (66)$$

where r is the range measured relative to the beginning of the segment.

$N_c(r)$ is not generally an integer and the fractional portion of $N_c(r)$ determines the division of energy between the two subarrays. For example at range R_1 shown in figure 14, $N_c(r)$ equals 0.5 which indicates a range of one half the conversion distance D. The two subarrays have equal energy. Subarray 1 has no time delays and subarray 2 has time delays corresponding to angle of inclination $2\theta_B$ which is

$$\text{Subarray 2:} \quad \frac{2\pi z \sin(2\theta_B)}{\lambda} \quad \text{radians,}$$

where z is the vertical coordinate measured down from the ocean surface and λ is the acoustic wavelength measured at the ocean bottom. At range R_2, $N_c(r)$ equals 1 conversion distance and all energy is deflected and is contained in subarray 2 with time delay corresponding to angle of inclination $2\theta_B$. At range R_3, $N_c(r)$ equals 1.5 conversion distances and subarrays 1 and 2 have time delays corresponding to angles $2\theta_B$, and $4\theta_B$, respectively, and they have equal energy. The time delays are given by



	SUB ARRAY 1	SUB ARRAY 2
TIME DELAY :	$\frac{2\pi Z \sin(2(n-1)\theta_B)}{\lambda}$	$\frac{2\pi Z \sin(2n\theta_B)}{\lambda}$
ENERGY PROPORTIONAL TO :	$nD-R$	$R-(n-1)D$

Figure 14. Mode Conversion Due to Sloping Bottom

Subarray 1: $\dfrac{2\pi z \sin(2\,\theta_B)}{\lambda}$ radians

Subarray 2: $\dfrac{2\pi z \sin(4\,\theta_B)}{\lambda}$ radians.

In general, given $N_c(r)$, the energy of the subarrays E_1 and E_2 and the time delays Φ_1 and Φ_2 can be determined in the following manner

$$E_1 \propto I\left[N_c(r) + 1\right] - N_c(r)$$

$$E_2 \propto N_c(r) - I\left[N_c(r)\right] \quad ,$$

where $I[x]$ represents the largest integer less than x. Therefore, the pressure amplitudes w_1 and w_2 are given by

$$w_1 \propto \mathrm{SQRT}\left(I\left[N_c(r) + 1\right] - N_c(r)\right)$$

$$w_2 \propto \mathrm{SQRT}\left(N_c(r) - I\left[N_c(r)\right]\right) .$$

Since the total energy in the subarrays must equal that in the original array, the amplitude, w, of the undivided array must be such that

$$w = w_1 + w_2 \tag{67}$$

or

$$k\ \mathrm{SQRT}\left(I\left[N_c(r) + 1\right] - N_c(r)\right) + k\ \mathrm{SQRT}\left(N_c(r) - I\left[N_c(r)\right]\right) = w$$

$$k = \frac{w}{\mathrm{SQRT}\left(I\left[N_c(r) + 1\right] - N_c(r)\right) + \mathrm{SQRT}\left(N_c(r) - I\left[N_c(r)\right]\right)} \tag{68}$$

$$w_1 = \frac{\mathrm{SQRT}\left(\left(I\left[N_c(r) + 1\right] - N_c(r)\right)w\right.}{\mathrm{SQRT}\left(I\left[N_c(r) + 1\right] - N_c(r)\right) - \mathrm{SQRT}\left(N_c(r) - I\left[N_c(r)\right]\right)} \tag{69}$$

$$w_2 = \frac{\mathrm{SQRT}\left(N_c(r) - I[N_c(r)]\right) w}{\mathrm{SQRT}\left(I[N_c(r) + 1] - N_c(r)\right) + \mathrm{SQRT}\left(N_c(r) - I[N_c(r)]\right)} \; . \tag{70}$$

The phase delays of the two subarrays due to reflections are added to those associated with the unperturbed array. The delays are given by

$$\text{Array 1:} \quad \Phi_1 = \frac{2\pi z \sin\left(2\, I[N_c(r)]\, \theta_B\right)}{\lambda} \tag{71}$$

$$\text{Array 2:} \quad \Phi_2 = \frac{2\pi z \sin\left(2\, I[N_c(r) + 1]\theta_B\right)}{\lambda} \; . \tag{72}$$

Such subarrays can be determined for all modes at any given range of interest and then they are combined to give one equivalent array at this range. The sound field at any depth at the range is obtained from the field produced by this array.

As energy progresses upslope, backscattering will occur when $2\, I[N_c(r) + 1]\theta_B$ exceeds 90 degrees. When this occurs, the reflected field is calculated separately from the field produced by energy propagating forward. In a manner similar to that described when the energy is progressing forward, the reflected field is determined at ranges of interest while considering the effect of the sloping bottom on sound propagating backward.

In practice, certain assumptions are made in the implementation of the procedures described above. Mode conversion due to the changing eigenfunction in the section is neglected. It is assumed that modes unperturbed by the sloping bottom are constant in any section. As a result, the conversion distance D is constant within any one segment and equation (66) becomes

$$N_c = \frac{R}{D} \; . \tag{73}$$

Neglect of this source of mode conversion may be difficult to justify when large slopes occur in the bottom. However, this problem may be reduced by segmenting the range more finely. In future versions of the model, it may prove advantageous to calculate the eigenfunctions corresponding to the modes at more than one depth within any segment. This additional calculation of eigenfunctions can be accomplished with little increase in computation time by making the following modification in the numerical procedure used. Presently the wave equation is integrated numerically from bottom to surface, until the boundary condition ($u = 0$) is satisfied to an acceptable tolerance at the surface. The wave equation could be integrated numerically from surface to bottom, until the boundary condition for du/dz is satisfied to an acceptable tolerance at the bottom. If, in the trial and error process, the slope was tested at all depths of interest as bottom depths, we would obtain the eigenfunctions for all depths of interest. In essence, we could increase dramatically the information obtained from virtually the same procedure.

When the ray equivalent is used to determine mode conversion effects, we necessarily use some of the approximations inherent in ray theory. For instance, if the ray equivalent of a mode vertexes above the bottom, it is assumed that the sloping bottom does not affect the mode. This assumption is justified provided the ray vertexes well above the bottom. In addition, ray theory poses problems when angles of incidence are near the critical angle, and the ray equivalent, strictly speaking, does not truly represent the physical picture.

TIME-LIMITED SIGNALS

In previous discussions, we have considered only CW signals produced by a harmonic source. It is also of interest to consider pulsed CW signals. Knowledge of such signals can serve two purposes. First, such signals are often transmitted in ocean experiments, hence it is desirable to predict the propagation of such signals. Second, such signals can be considered as approximations to other more complex time-limited signals used in ocean experiments, such as filtered explosive signals, and can be used to predict propagation of the more complex signals.

The procedure used in the model described herein is similar to that described in reference 18 but is extended to accommodate a range-dependent environment. This procedure assumes that the propagation of a CW pulse can be described by modes of the same duration as the pulse. These modes are assumed to propagate in accordance with the group velocities of the modes corresponding to the center frequency of the transmitted pulse. An example of such a received signal is shown in figure 15 where each arrival corresponds to a mode and where some of the modes overlap, causing distortion of the received signal. The assumptions which allow the calculation of such a signal are good, provided the CW pulse is long enough so that the bandwidth of the signal is small, or provided the medium is not very dispersive over the bandwidth of the signal.

Under these assumptions, it is extremely easy to determine the characteristics of CW pulses at any point in the water column. The field is given directly by equations (34) and (38) (where the medium is either unstratified or stratified with range, respectively) with a few small modifications. First, we must consider the instantaneous pressure p(t) a function of time and hence the term ωt, suppressed previously must be obtained by adding terms of the form $\sum_m \cos$. In addition, the number of modes included in the total field is a function of time, since each mode arrives at a time consistent with the group velocity and lasts only the length of the transmitted signal. Hence, when the medium is unstratified in range, the signal duration for a given mode is given by

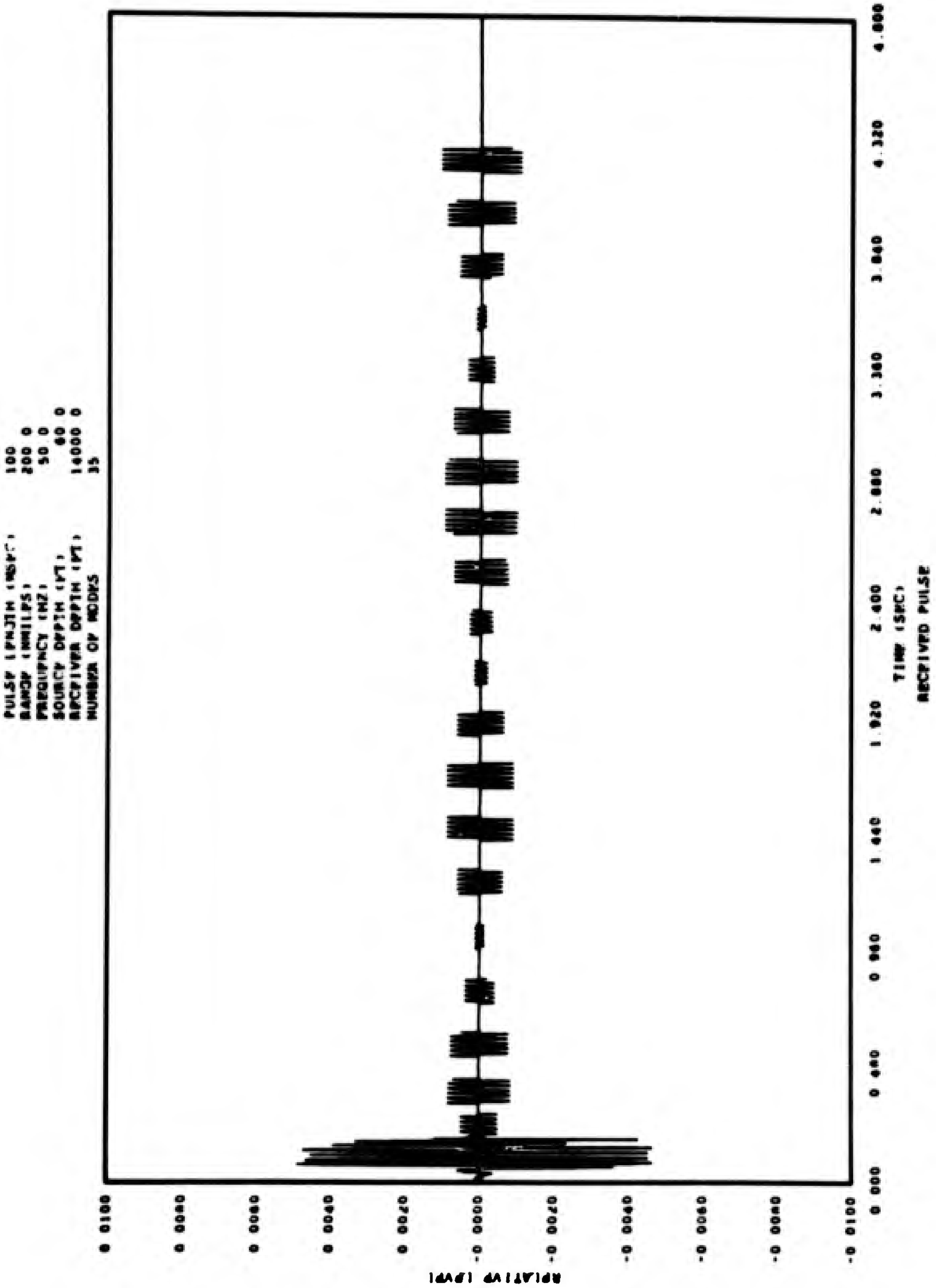


Figure 15. Time History of a Received Signal

$$t_s = \frac{r}{V_g}$$

$$t_e = \frac{r}{V_g} + T \ , \tag{74}$$

where

t_s is the time of arrival of the signal corresponding to the mode,

t_e is the time at the end of the signal corresponding to the mode

r is the range at which the received signal is calculated,

V_g is the group velocity of the mode,

T is the duration of the transmitted CW pulse.

When the medium is stratified in range, the signal duration for a given mode is given by

$$t_s = \sum_{i=1}^{N} \frac{r_i}{V_{gi}}$$

$$t_e = \sum_{i=1}^{N} \frac{r_i}{V_{gi}} + T \ , \tag{75}$$

where

N is the number of segments into which the range is divided,

r_i is the length of segment i ,

V_{gi} is the group velocity of the mode in segment i .

(It may be noted that in equations (74) and (75) t_s and t_e are independent of source or receiver depth. Hence, the basic structure of a received signal is independent of the source and receiver depths. The source and receiver depth only affect the amplitudes of the arrivals.)

As a result of the previous discussion, for a medium unstratified in range, equation (34) yields for the instantaneous pressure $p(t)$

$$p(t) = \frac{\rho_o}{\sqrt{r}} \sum_{m=m(t)} P_m 10^{(-D_m r/20 - ar/20)} \cos(\kappa_m r - \omega t - \pi/4) \qquad (76)$$

and, for a medium stratified in range, equation (38) yields

$$p(t) = \frac{\rho_o}{\sqrt{r}} \sum_{m=m(t)} \bar{P}_m 10^{- ar/20 - \int_o^r D_m(\eta)\, d\eta/20} \cos(\bar{\kappa}_m r + \Delta S - \omega t - \pi/4) \qquad (77)$$

where $\sum_{m=m(t)}$ is used to indicate that at any given time t, a limited number of modes determined by equations (74) and (75), respectively, are included in the summation.

Propagation loss L_r is determined from the ratio of the energy in the received signal to that of the transmitted signal so that

$$L_r = -10 \log \frac{E_1}{E_o} , \qquad (78)$$

where E_1 is the energy of the received signal, E_o is the energy of the transmitted signal at unit distance. Hence,

$$L_r = -10 \log \frac{\int_o^{T'} p^2(t)\, dt}{\int_o^{T} p_o^2(t)\, dt} , \qquad (79)$$

where

T' is the length of the received signal,

T is the length of the transmitted signal,

t is time measured relative to the beginning of the signal,

$p(t)$ is pressure of the received signal as a function of time,

$p_o(t)$ is pressure of the transmitted signal at unit distance as a function of time.

If the pressure amplitude of $p_o(t)$, the transmitted signal, is unity, the lower integral is evaluated so that

$$\int_o^T p_o^2(t)\,dt = \int_o^T \sin^2(\omega t)\,dt = \frac{T}{2} \tag{80}$$

provided the signal contains an integral number of cycles or $T >> \omega$. Thus

$$L_r = -10 \log \frac{\int_o^{T'} p^2(t)\,dt}{T/2} \, . \tag{81}$$

Since, in the actual computer computations, p(t) is calculated only at a discrete number of instants of time, the integral in equation (81) must be replaced by a sum so that

$$L_r = -10 \log \frac{\sum_{i=0}^{T'/\Delta t} p^2(i\Delta t)\Delta t}{T/2} \, , \tag{82}$$

where Δt is the time interval between samples of p(t) .

At present the propagation of time-limited signals is not considered when calculating mode conversion. Mode conversion is not included in the analysis because of the complexities introduced. The situation becomes complicated because each pulse corresponding to a mode, after mode conversion, is transformed into a number of pulses, one corresponding to each mode, displaced in time from all other pulses. Hence, after $n + 1$ segments with each segment containing m modes, there are $n \times m^2$ pulses to keep track of.

ARRAYS AND SELECTION OF MODES

A study (reference 1) conducted by A. D. Little, Inc., for the Naval Underwater Systems Center (NUSC) demonstrated the feasibility of exciting single modes in the Block Island-Fishers Island (BIFI) shallow acoustic range by means of a vertical array of sources. The excitation of single modes in the range has several attractive uses, among them the study of individual modes, reduction of the effects of interference between modes, and the selection of modes to maximize the level of the sound field or to provide information about a particular portion of the water column.

Effective utilization of such an array in acoustic experiments requires: (1) knowledge of the theoretical sound field produced at any range and depth for any amplitude and phase of the individual array elements, and (2) the theoretical optimum array shading for transmitting any single mode. In order to obtain this information here, the propagation of sound generated by a vertical array is discussed in terms of normal mode theory. To facilitate understanding of the physical mechanisms involved, in each of the following subsections, the initial discussion assumes a very idealized description of the medium. A description of the calculations performed in the computer program, which of course employs more realistic assumptions regarding the medium, then follows.

Propagation of Array-Generated Sound

Previously, p_a, the pressure amplitude of the sound field produced by a point source with unit source level in an ocean with a flat, semi-infinite, homogeneous bottom, was described by a function dependent on range:

$$p_a = \frac{\rho_0}{\sqrt{r}} \left\{ \left[\sum_m P_m 10^{(-D_m r/20 - \alpha r/20)} \cos\left(\kappa_m r - \frac{\pi}{4}\right) \right]^2 + \left[\sum_m P_m 10^{(-D_m r/20 - \alpha r/20)} \sin\left(\kappa_m r - \frac{\pi}{4}\right) \right]^2 \right\}^{1/2} \quad (83)$$

where

α	is the attenuation coefficient caused by absorption
ρ_o	is the water density
r	is the horizontal range
m	is the mode number
κ_m	is the horizontal wave number of mode m
D_m	is the attenuation per unit range due to losses at the boundaries

$$P_m = p_m \frac{1}{\rho_s} \varphi_m(z) \varphi_m(z_o) , \quad (84)$$

where

ρ_s	is the water density at the source
$\varphi_m(z)$	is the normalized displacement potential as a function of depth
z_o, z	are the source and receiver depths, respectively.

$$p_m = \frac{\rho_o (2\pi)^{1/2}}{2 \nu_m \sqrt{\kappa_m}} , \tag{85}$$

where

$$\nu_m = \int_{-\infty}^{+\infty} \rho \varphi_m^2 (z)\, dz . \tag{86}$$

Examination of equations (83) and (84) shows that, for a given mode, the pressure amplitude p_a is dependent on the source depth only through $\varphi_m(z_o)$. Hence,

$$p_a \propto \left| \varphi_m(z_o) \right| \tag{87}$$

for a given mode m and source depth z_o. Given a vertical transmitting array of N point sources in phase and with unit amplitude, we have from superposition

$$p_a \propto \left| \sum_{i=1}^{N} \varphi_m(z_i) \right| . \tag{88}$$

The general nature of the sound field produced by such an array can be deduced from figure 16 and equation (88). In the figure, φ is sketched as a function of depth

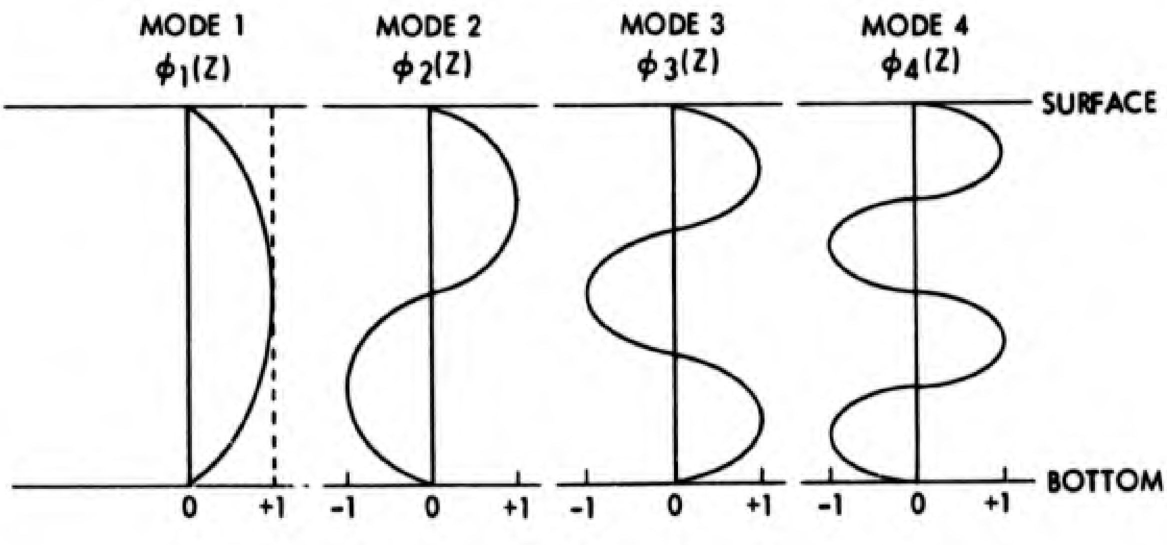


Figure 16. Displacement Potentials of Modes 1 to 4

for modes 1 to 4 in an idealized case of two pressure release surfaces. For mode 1, φ_1 is positive at all depths; hence, all terms in equation (88) for mode 1 are positive and add constructively. For mode 2, the sign of φ changes with source depth; φ_2 is positive in the upper half of the water column and negative in the lower half. Hence, the second mode produced by an element of the upper half of the array combines destructively with the second mode produced by an element in the lower half. A similar discussion can be applied to the higher order modes. (This should suggest to the reader one method of emphasizing selected modes, and this method is described later.)

If the elements of the array are amplitude- and phase-shaded, equation (88) becomes

$$p_a \alpha \left| \sum_{i=1}^{N} w_i \varphi_m(z_i)\, e^{j\Phi_i} \right| , \qquad (89)$$

where

w_i is the amplitude of element i of the array measured at unit distance

Φ_i is the phase of element i of the array measured at unit distance.

The pressure amplitude p_a as a function of range and depth can be obtained for a shaded array by substituting the summation contained in equation (89) for $\varphi_m(z_o)$ in equation (84). If, as in reference 11, we also extend the solution to a medium in which the stratification is a slowly changing function of range, we obtain

$$p_a = \frac{\rho_o}{\sqrt{r}} \left\{ \left[\sum_m \sum_{i=1}^{N} \bar{P}_m 10^{-\alpha r/20 - \int_o^r D_m(\tau)\, d\tau/20} \cos\left(\bar{\kappa}_m r + \Delta S_m + \Phi_i - \frac{\pi}{4}\right) \right]^2 \right.$$

$$\left. + \left[\sum_m \sum_{i=1}^{N} \bar{P}_m 10^{-\alpha r/20 - \int_o^r D_m(\tau)\, d\tau/20} \sin\left(\bar{\kappa}_m r + \Delta S_m + \Phi_i - \frac{\pi}{4}\right) \right]^2 \right\}^{1/2} \qquad (90)$$

where

N is the number of elements in the transmitting array

i is the i-th element of the array

Φ_i is the phase at unit distance associated with the i-th element

$D_m(r)$ is the attenuation per unit range for mode m as a function of range

$\bar{\kappa}_m$ is the average of κ_m over r ,

so that

$$\bar{\kappa}_m = \kappa_m(r) - \epsilon_m(r) \tag{91}$$

$$\Delta S_m = \int_o^r \epsilon_m(r)\, dr \tag{92}$$

$$\bar{P}_m = \left[P_{source}\ P_{receiver}\right]^{1/2} , \tag{93}$$

where

$$P_{source} = p_m \frac{w_i^2}{\rho_s}\, \varphi_m^2(z_i) \tag{94}$$

$$P_{receiver} = p_m \frac{1}{\rho_s}\, \varphi_m^2(z) , \tag{95}$$

where z_i is the depth of the i-th element of the transmitting array, w_i is the amplitude of element i at unit distance, and the other symbols in equations (94) and (95) are given in equations (84), (85), and (86).

Before propagation loss can be considered, a unit source level must be defined. In the case of an array of sources, we can define a unit source level in a manner analogous to that corresponding to a point source. Thus we define a unit source level for the array in such a way that there is a unit sum of the squared amplitudes of each source at unit distance. If the amplitudes of the individual elements are such that

$$\sum_{i=1}^{N} w_i^2 = 1 , \tag{96}$$

it follows that

$$L_r = -20 \log p_a , \tag{97}$$

where L_r is the propagation loss at range r.

Similarly, one can take into account mode conversion by forming equivalent arrays after the superposition of the fields of the point sources.

Amplitude Matching

One approach to emphasizing one mode in the sound field is to match the vertical amplitude distribution of an array to that of the desired mode. Let us consider the medium and the corresponding displacement potential distribution $\varphi_m(z)$ of modes 1 to 4 shown in figure 16. We assume constant ρ and c in the water column and pressure release surfaces at the two boundaries. If $\varphi_m(z)$ is normalized so that

$$\int_{-\infty}^{\infty} \varphi_m^2(z)\, dz = 1 \tag{98}$$

for all modes, it can be shown (references 17, 19, 20) that $\varphi_m(z)$ is orthogonal for this case, with the result that

$$\int_{-\infty}^{\infty} \varphi_m(z)\, \varphi_n(z)\, dz = \begin{cases} 1 & m = n \\ 0 & m \neq n\ . \end{cases} \tag{99}$$

The effect of matching an array to the vertical amplitude distribution of a mode can be determined from equation (98) and from equation (89), which is rewritten below:

$$p_a \propto \left| \sum_{i=1}^{N} w_i\, \varphi_m(z_i)\, e^{j\Phi_i} \right| \ . \tag{100}$$

If we assume a continuous distribution of sources so that the sum in equation (100) may be replaced by an integral and if we match the array shading to $\varphi_n(z_i)$ associated with mode n so that

$$w_i\, e^{j\Phi_i} = \varphi_n(z_i)\ , \tag{101}$$

the pressure amplitude is such that

$$p_a \, \alpha \int_{-\infty}^{\infty} \varphi_n(z_i)\, \varphi_m(z_i)\, dz_i = \begin{cases} 1 & m = n \\ 0 & m \neq n \; . \end{cases} \tag{102}$$

The significance of equation (102) is apparent. Consider a continuous line of point sources matched to $\varphi_n(z)$ (or equivalently matched to the vertical pressure amplitude distribution of mode n). At any arbitrary range and receiver depth, the contributions corresponding to mode n produced at all source depths will add in phase, and the level of all other modes will be zero because of destructive interference.

In figure 16, φ_m is sketched for modes 1 to 4. Let us consider, for definiteness, the case where a continuous array of point sources is matched to mode 1. In this case, proper matching requires all array elements to be in phase and to have amplitudes proportional to $\varphi_1(z_i)$ at each element depth z_i. Thus the first modes generated by all elements are in phase and add constructively at all ranges and depths. However, the second modes generated by elements in the upper half of the water column are in phase opposition to those generated by elements in the lower half. This phase opposition corresponds to the change in sign of $\varphi_2(z)$ at mid depth. Since $\varphi_1(z)$ is symmetrical about mid depth, it follows that the arrivals corresponding to mode 2 cancel exactly. Similarly, by considering the symmetry of $\varphi_m(z)$, we can see that all even modes undergo complete cancellation. In the case of mode 3, the orthogonality property is not as evident; for this matching procedure, the third modes produced by elements in the upper and lower third of the channel are in phase at all ranges and depths but are in phase opposition to those produced by elements in the middle third of the channel. However, in the mode 1 matching, energy is so concentrated in the middle third of the channel (as shown in figure 16) that there is exact cancellation of the third mode. Similar analysis shows that there is exact cancellation for all odd modes.

Thus far we have considered mode matching in a medium bounded by two pressure release surfaces. It is more realistic to assume a semi-infinite, liquid bottom, where there is a sharp discontinuity in ρ and c at the water-bottom interface. For such a bottom, it can be shown (reference 4) that $\varphi_m(z)$ is no longer orthogonal, because of the discontinuity in ρ. Hence matching to a mode will emphasize the mode in the sound field but will not completely eliminate all other modes. The results of matching can be calculated by using the model described herein. Such theoretical results for a 25-element array and typical velocity profiles, presented in reference 21, showed that modes could, in theory, be emphasized under such conditions.

In reference 1, a simple method of amplitude matching was proposed. The method consists of weighting an array in such a way that w_i, the weight of element i, is given by

$$w_i = \begin{cases} 1 & \text{if } \varphi_m(z_i) \geq (\max |\varphi_m(z)|)/2 \\ 0 & \text{if } |\varphi_m(z_i)| < (\max |\varphi_m(z_i)|)/2 \\ -1 & \text{if } \varphi_m(z_i) \leq -(\max |\varphi_m(z_i)|)/2 \end{cases} , \qquad (103)$$

where

m is the number of the mode being matched

z_i is the depth of element i

$\varphi_m(z_i)$ is the value of $\varphi_m(z)$ at z_i

$\max|\varphi_m(z)|$ is the maximum value of $\varphi_m(z)$ over the water column

Φ_i is constant.

Phase Matching

Another approach to emphasizing one mode in the sound field is to match the phase across a vertical array to that of the desired mode. Let us consider the medium and one of the two plane waves associated with an arbitrary mode m, shown in figure 17. We assume constant ρ and c in the water column and pressure release surfaces at the boundaries; hence, the phase change ϵ undergone upon reflection from each of the boundaries is $-\pi$. The direction of propagation of the wave corresponding to a mode must be such that the wave is (reference 22) "self-consistent." Hence, as successive waves progress from the top to the bottom of a vertical segment of the water column, the waves must be in phase at all water depths. This occurs (references 16 and 11) if Δ, the advance in phase that occurs as the wave progresses from top to bottom, is such that

$$2\Delta + \epsilon_1 + \epsilon_2 = 2\pi(m - 1) , \qquad (104)$$

where

m is the mode number

ϵ_1, ϵ_2 are phase changes upon reflection from the boundaries.

Since $\epsilon_1 = \epsilon_2 = -\pi$ at the boundaries in the case considered, it is required that

$$\Delta = m\pi . \qquad (105)$$

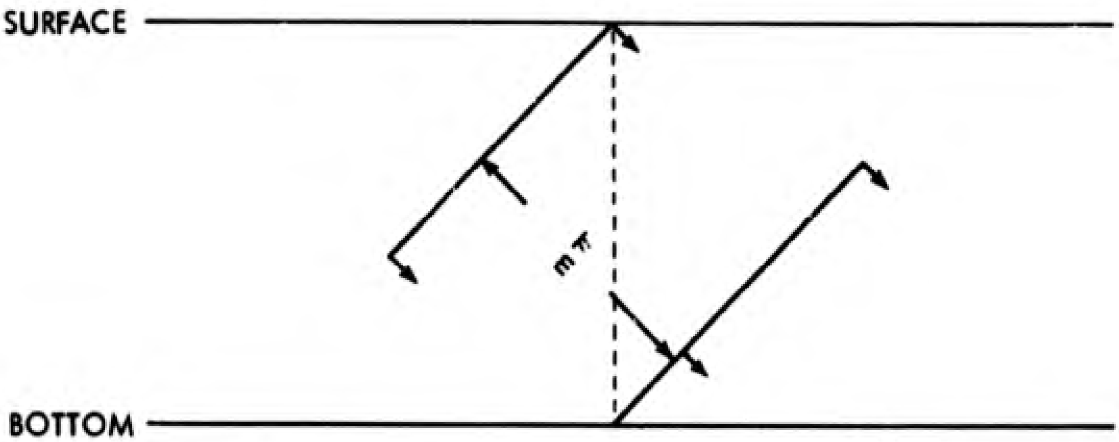


Figure 17. Plane Wave Corresponding to Mode m

The pressure amplitude of a mode as a function of depth can be determined by adding upgoing and downgoing waves corresponding to the mode. Such waves are shown in figure 18. At the ocean surface, these waves are out of phase by $-\pi$ radians. Let us assume, for definiteness, that at the surface the phase of the downgoing wave Φ_1 is $-\pi$ radians and the phase of the upgoing wave Φ_2 is 0 radian. The phase of the downgoing wave increases uniformly with depth, so that, considering equation (105), we obtain

$$\Phi_1(z) = -\pi + \frac{z}{H} m \pi \quad . \tag{106}$$

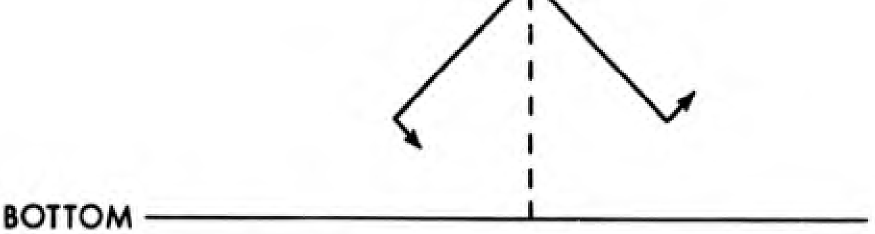


Figure 18. Upgoing and Downgoing Waves Corresponding to a Mode

The phase of the upgoing wave decreases uniformly with depth, so that

$$\Phi_2(z) = -\frac{z}{H} m\pi \quad , \qquad (107)$$

where H is the water depth and $0 \leq z \leq H$.

The waves can be added by using the procedure illustrated in figure 19. The two waves are represented by vectors whose amplitudes are $A_1 = A_2 = 1/2$ and whose phases are given by equations (106) and (107). The vectors are rotated, in the directions indicated, through $m\pi$ radians. Since the vectors are symmetrical about the ordinate the sum of the vectors is given by $\sin \Phi$, where Φ is given by equation (106) or (107). The sums for modes 1 to 4 are sketched in figure 16.

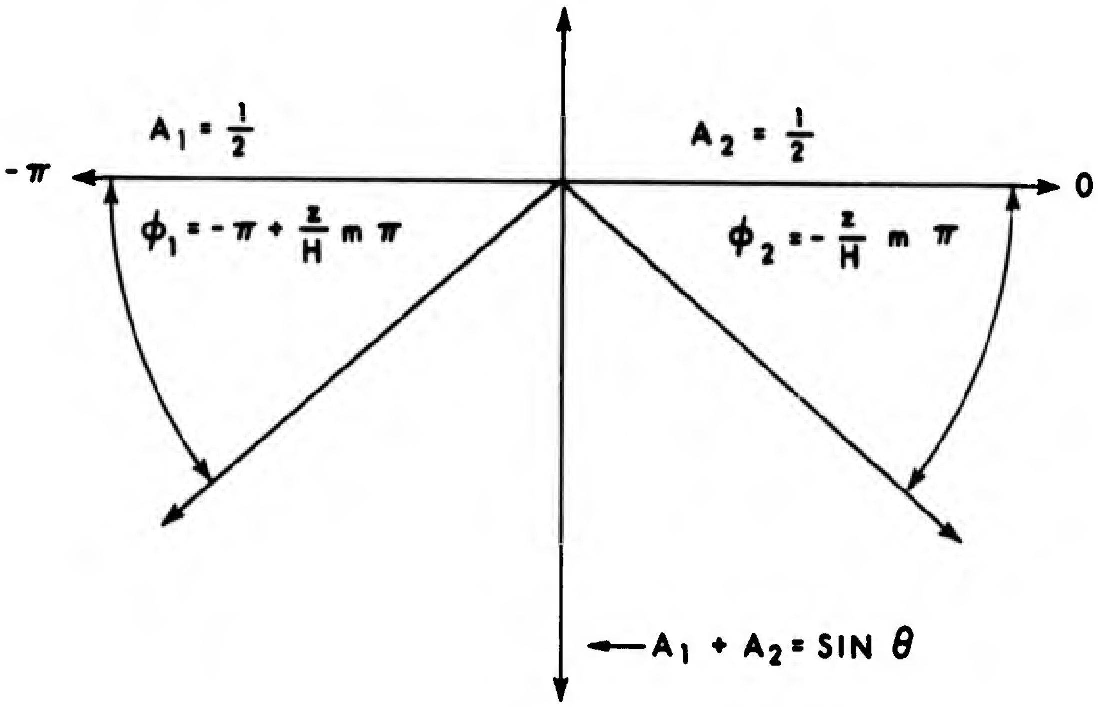


Figure 19. Vector Representation of Sum of Upgoing and Downgoing Waves Corresponding to Mode m

Similarly, the phase of either of the waves at any particular depth can be determined from the amplitude distribution as a function of depth. The phase of a wave corresponding to mode m at depth z_i can be determined simply by calculating the arc sine of $\varphi_m(z_i)$. Thus,

$$\Phi(z_i) = \arc \sin (\varphi_m(z_i)) + (J - 1)\pi \quad , \tag{108}$$

where J is the number of zero crossings of $\varphi_m(z)$ for $0 \leq z \leq z_i$. Hence it can be seen by inspection of figure 16 that, considering equation (108),[1] the phase of mode 1 varies linearly with depth from 0 to π radians, mode 2 varies linearly from 0 to 2π radians, mode 3 from 0 to 3π radians, etc. This is in agreement with equation (106).

For an arbitrary velocity profile, the procedure illustrated in figure 19 must be modified. The angles Φ_1 and Φ_2 do not vary linearly with depth but increase more rapidly for depths corresponding to large sound velocities (large curvature of rays) than for depths corresponding to smaller velocities. Moreover, if one introduces a liquid, semi-infinite bottom, the vectors are rotated through $(\pi - \epsilon_2)/2 + (m - 1)\pi$ radians (ϵ_2 is the phase change upon reflection from the bottom). In all cases, however, the phase at any depth can be more easily calculated by using $\varphi_m(z)$ and equation (108), in the model. Hence, a vertical array is matched to the phase of a mode by setting Φ_i, given in equation (89), equal to $\Phi(z_i)$, the phase of the mode at the depth of array element i. This matching has the effect of steering energy in the direction corresponding to the desired mode.

Given Φ_i, the phases of the array elements, we should choose the amplitude weighting w_i to obtain, for a specific power, the maximum pressure amplitude of the sound field produced by mode m at any given range and depth. Since $\varphi_m(z)$ is the only parameter of the sound field dependent on source depth, we would like to obtain

$$\max \left| \sum_{i=1}^{N} w_i \varphi_m(z_i) e^{j\Phi_i} \right| \tag{109}$$

under the constraints that

$$\sum_{i=1}^{N} w_i^2 = \text{constant}$$

$$w_i \geq 0 \quad i = 1, 2, \ldots, N \, . \tag{110}$$

The maximization of expression (109) subject to constraints (110) is a problem in optimizing a nonlinear objective function subject to nonlinear constraints. A rather complex algorithm serving this purpose is described in references 23 and 24. However, we shall adopt a simpler, albeit less optimum, procedure that utilizes a form of cosine weighting whereby w_i, the amplitude of the i-th array element at unit distance, is given by

$$w_i \propto \varphi_m(z_i) \cos \theta , \tag{111}$$

where

z_i is the depth of element i

θ is the phase difference $\Phi_i - \Phi_{max}$ for mode m ,

in which

Φ_i is the phase of element i

Φ_{max} is the phase corresponding to the maximum value of $|\varphi_m(z)|$ in the lobe of $\varphi_m(z)$ that contains the depth of element i .

In the discussion concerning arrays it has been assumed that the aperture of the array is such that the array extends over the entire water column. Perhaps a few words concerning mode enhancement using limited aperture arrays may be of interest. The capability of a limited aperture array to enhance modes can be demonstrated qualitatively by observing the displacement potentials shown in figure 16. Assume that the array covers the upper half of the water column, and we would like to match to mode 1. Hence, by using the amplitude matching described in equation (101), all arrivals corresponding to mode 1 add in phase. However, all arrivals corresponding to mode 2 also add in phase, but these arrivals will not emphasize mode 2 as much as mode 1 since, for example, the depth of the maximum weighting for the array corresponds to a zero for $\varphi_2(z)$. It can be seen that, for higher-order modes, there is some cancellation and that this is a generally increasing function of mode number. Hence we can conclude that the effect of going to a limited aperture array is effectively, as expected, a broadening of the beam, and as a result the tendency is to reach a state in which groups of modes, rather than individual modes, can be enhanced.

DESCRIPTION OF COMPUTER PROGRAM

It is necessary to use a high-speed computer to perform the computations which correspond to the acoustic model described herein. The mechanics of using the associated computer program and the output which eventuates are described in this section, together with the following aspects of the program:

- Input parameters specifying the source and receiver, and the medium over which sound is transmitted
- Amplitude versus depth and the ray equivalent
- Propagation loss versus range
- Propagation loss versus depth
- Time-limited signals
- Mode matching, using amplitude and phase matching.

INPUT PARAMETERS SPECIFYING SOURCE, RECEIVER, AND MEDIUM

Certain input parameters must be specified by the user of the program. The number of segments into which a given range is divided is specified by input NPCS (table 2, card group 2). If NPCS = 1 and the bottom is flat (IRH = 0, card group 16) then the stratification does not vary with range. If NPCS > 1 and the bottom segments are flat (IRH = 0), the ocean is divided into flat segments such as is shown in figure 10. If IRH $\neq$ 0 for all segments, the ocean is divided into segments with constant slope as in figure 13. The depth, location, size, and bottom properties of each segment are inputs of card group 16, and the quantity of velocity profiles necessary for a full description of the range is given by NVPC (card group 2). The number of the first segment described by a particular velocity profile is given by ICVP(J) (card group 3; this profile describes the velocity conditions from segment ICVP(J) to segment ICVP(J + 1). The velocity profiles themselves are specified in card group 13.

If information is desired concerning propagation from a point source to a point receiver, the source depth (ZS) and receiver depth (ZRC) are specified in card group 12. Such information may be obtained for NDPZ (group 5) additional source — receiver configurations specified in card group 6.

If information is desired concerning propagation from a vertical array of point sources, the number of sources is given by NSE (card group 5). The depths of the elements, ZSD(J), and their relative amplitudes, SMA(J), and phases, SMP(J), at unit distance are given in card group 9. The sound field may be obtained at NDPR (card group 5) receiver depths, which are given by ZRD(J) (card group 8).

If information is desired concerning propagation to a vertical array of point receivers, the number of receivers is given by NRE (card group 5). The depths of the elements, ZSD(J), and their relative amplitudes, SMA(J), and phases, SMP(J), at unit distance are given in card group 10. The sound field may be obtained for NDPS (card group 5) individual source depths, which are given by ZRD(J) (card group 7). At present, propagation from an array of point sources to an array of point receivers cannot be determined by using the present computer program.

For any given source, the frequency FQ is specified in card group 14.

A time-limited CW pulse at length IPL (card group 14) milliseconds may be specified. If IPL = 0, an infinite CW pulse is specified.

If IMA = 1 (card group 5), the sound field is calculated at a depth for which any given mode is a maximum. Thus, in effect, a point receiver is moved to the depth of the maximum amplitude of any given mode.

AMPLITUDE DISTRIBUTION AND RAY EQUIVALENT

Each segment of a range has its own amplitude distribution as a function of depth for each mode. For a range divided into 73 segments as in figure 10, it is doubtful that one would want a plot of the amplitude distribution and ray equivalent of the calculated modes for each segment. Therefore, NADS (card group 2, table 2) is the number of segments for which Calcomp plots of these functions are desired, and IADS(J) (card group 4) gives the segment numbers for which these plots are generated. NMOD (card group 2) is the number of modes calculated. Sample plots are shown in figures 20 and 21. The increment in range in feet in figure 20 is given by SRE (card group 13). The maximum water depth plotted in figure 21 is given by FSC (card group 12) times 200 ft.

In determining u(z), the amplitude distribution as a function of depth, the equation is integrated over the water column and u(z) determined at a discrete number of depths. The depth increment between values of u(z) is specified by FH ft (card group 14). The depth increment in feet between values of u(z) can be decreased by a factor of IMS (card group 14) for MNS (card group 14) intervals. These intervals are given by IAMS(J) (card group 15), where the intervals are numbered from bottom to surface. In the numerical procedure, k_r is varied systematically until numerical conversion is obtained to satisfy the boundary conditions. The increment of k_r is proportional to the factor 10^{-IEX} where IEX is specified in card group 13. The value of u at the bottom, u(1), is determined by the relationship $u(1) = UBOT^{(IX)}$ where UBOT and IX are in card group 13. The numerical integration is terminated when u(z) exceeds the value UM^{IXX}, where UM is specified in card group 14, IXX in card group 13, and u(z) is determined to a depth of ZBOT (card group 13) into the bottom.

Table 2. Input Data for Program

Card Group	Amount of Cards	Format	Input Parameter	Columns	Data
1	1	20A4		1-80	Heading on each page of output
2	1	7I10	NPCS	1-10	Number of segments into which range is divided
			NVPC	11-20	Number of velocity profiles in the range
			NADS	21-30	Number of segments for which amplitude distributions and ray equivalents are plotted
			NCOR	31-40	Segment number considered to be effective water depth at source
			NMOD	41-50	Number of modes calculated
			ISM	51-60	First ISM modes are ignored in calculations
			IBL	61-70	IBL = 0
		F10.5	BK	71-80	BK = 0.0
3	The smallest integer $\geq \frac{NVPC}{8}$	6I10	ICVP(J)	1-60	ICVP(J) is the number of the bottom segment at which velocity profile J is first used. This profile is used in calculations between segments ICVP(J) and ICVP(J+1).
4	The smallest integer $\geq \frac{NADS}{8}$	6I10	IADS(J)	1-60	IADS(J) is the number of the bottom segment for which the pressure amplitude distribution and ray equivalent are plotted

Table 2 (Cont'd). Input Data for Program

Card Group	Amount of Cards	Format	Input Parameter	Columns	Data
5	1	10I5	NDPS	1-5	If the receiver is a vertical array (NRE>0), the field is calculated for NDPS different source depths
			NDPR	6-10	If the source is a vertical array (NSE>0), the field is calculated for NDPR different receiver depths
			IML IMU	11-15 16-20	Mode matching information is printed out for all range segments between the IML-th and the IMU-th segments
			IMA	21-25	If IMA = 1, the sound field is calculated at depth for which pressure amplitude of any given mode is a maximum
			NSE	26-30	If (NSE>0), the source is a vertical array with NSE elements
			NRE	31-35	If (NRE>0) the receiver is a vertical array with NRE elements
			INEU	36-40	INEU = 0
			NDPZ	41-45	If (NSE = 0 and NRE = 0) the sound field is calculated for NDPZ + 1, different point-source point-receiver configurations
			NRT	46-50	If the source transmits a CW pulse (IPL>0), the time history of the received signal at NRT ranges are recorded on digital tape (unit 8)

Table 2 (Cont'd). Input Data for Program

Card Group	Amount of Cards	Format	Input Parameter	Columns	Data
6	If NDPZ>0, the smallest integer ≥ NDPZ/3. If NDPZ=0, card is omitted	6F10.5	ZSD(1) ZRD(1) . . . ZSD(NDPZ) ZRD(NDPZ)	1-60	ZSD(J), ZRD(J) is the J-th point-source point-receiver configuration for which the field is calculated
7	If NDPS>0, the smallest integer ≥NDPS/3. If NDPS=0, card is omitted	6F10.5	ZRD(J)	1-60	ZRD(J) is the J-th source depth for which the sound field at a receiving array is calculated
8	If NDPR>0, the smallest integer ≥ NDPR/3. If NDPR=0, card is omitted	6F10.5	ZRD(J)	1-60	ZRD(J) is the J-th receiver depth for which the sound field of any array of sources is calculated
9	If NSE>0, card(s) a = the smallest integer ≥ NSE/6	6F10.5	ZSD(J)	1-60	ZSD(J) is the depth of the J-th element of an array of NSE sources
	card(s) b = the smallest integer ≥ NSE/6	6F10.5	SMA(J)	1-60	SMA(J) is the relative amplitude at unit range of the J-th element of an array of NSE sources
	card(s) c = the smallest integer ≥ NSE/6 If NSE=0, all cards are omitted	6F10.5	SMP(J)	1-60	SMP(J) is the phase, at unit range, in degrees of the J-th element of an array of NSE sources

Table 2 (Cont'd). Input Data for Program

Card Group	Amount of Cards	Format	Input Parameter	Columns	Data
10	If NRE>0 card(s) a = the smallest integer ≥ NRE/6	6F10.5	ZSD(J)	1-60	ZSD(J) is the depth of the J-th element of a receiving array of NRE elements
	card(s) b = the smallest integer ≥ NRE/6	6F10.5	SMA(J)	1-60	SMA(J) is the relative amplitude at unit distance from the J-th element of a receiving array of NRE elements
	card(s) c = the smallest integer ≥ NRE/6 If NRE = 0, all cards are omitted	6F10.5	SMP(J)	1-60	SMP(J) is the phase, at unit distance, in degrees of the J-th element of an array of NRE elements
11	If NRT>0, the smallest integer ≥ NRT/6 If NRT = 0, cards are omitted	6I10	IRT(J)	1-60	If the source transmits a CW pulse (IPL>0), the time history of the received signal is recorded on digital tape (unit 8) at ranges of IRT(J) ft where J = 1, 2, . . . , NRT
12	1	2I10	IRIC	1-10	Range increments (ft) in propagation loss versus range plot
			NPS	11-20	Number of propagation loss versus range plots for any given source receiver configuration
		4F10.3	ZS	21-30	Source depth (ft)
			ZRC	31-40	Receiver depth (ft)
			FMI	41-50	Increment in range in nmi/in. on propagation loss curves

Table 2 (Cont'd). Input Data for Program

Card Group	Amount of Cards	Format	Input Parameter	Columns	Data
12 (Cont'd)	1	4F10.3	FSC	51-60	200 x FSC is maximum depth (ft) plotted on curves that have depth as a parameter
		2F10.3	FMJ	61-70	Value of propagation loss (dB) at origin of propagation loss curves
			FMK	71-80	Increment in propagation loss in dB/in. on propagation loss curves
13	Card a = 1	I 10	NUMV	1-10	Number of values in velocity profile
		F10.0	VEL 1	11-20	Velocity at origin of plot of velocity profile
	Card a = 1	2 I 10	IVPL	21-30	If IVPL = 1, velocity profile not plotted
			IEX	31-40	Changes increment of k_r by a factor of 10^{-IEX}. Values from 0 to 10
	Card b = 1	4F10.3	ZMM	1-10	Water depth (ft) at location of velocity profile
			CB	11-20	Velocity (ft/sec) of sound in bottom at location of velocity profile
			ZBOT	21-30	Depth into bottom (ft) for which sound field is calculated for all modes
			UBOT	31-40	Value of u(z) at bottom $u(1) = UBOT^{(IX)}$
		2 I 10	IX	41-50	
			IXX	51-60	The maximum value of u(z) $= UM^{(IXX)}$

Table 2 (Cont'd). Input Data for Program

Card Group	Amount of Cards	Format	Input Parameter	Columns	Data
13 (Cont'd)	Card b = 1	F10.3	SRE	61-70	Increment in range in feet in ray equivalent plots
	Card(s) c = NUMV	2F10.3	ZZ(I)	1-10	Height (ft) above bottom at which sound velocity is CC(I)
			CC(I)	11-20	Velocity (ft/sec) of sound at ZZ(I) I = 1, 2, . . . , NUMV in order of increasing z
(Other groups of cards representing a velocity profile should be inserted in front of Group 15 cards that represent the bottom segments ICVP(J), J = 1, NVPC.)					
14	1	F10.0	FH	1-10	Depth increment in feet betwen values of u(z)
		D20.1	UM	11-30	The maximum value of u(z) = UM$^{(IXX)}$
		F10.3	FQ	31-40	Frequency (Hz)
		7I5	MNS IMS	41-45 46-50	Depth increment in feet between values of u(z) can be decreased to FH/IMS for MNS intervals
			IMC	51-55	If IMC = 0, mode conversion effect calculated. NDPZ must equal 0, NDPS and NDPR must equal 1. If IMC = 1, mode conversion neglected
			IPL	56-60	If IPL = 0, CW transmitted. If IPL>0, pulsed CW transmitted with pulse length of IPL msec
			IDL	61-65	If IPL>0, pulse is sampled at rate of 4096/(IBL/1000) samples/second

Table 2 (Cont'd). Input Data for Program

Card Group	Amount of Cards	Format	Input Parameter	Columns	Data
14 (Cont'd)	1	7I5	NFS	66-70	If NFS>0, NFS files are skipped on unit 8
			NFT	71-75	If NFT>0, NFT files are skipped on unit 7
15	If MNS>0, the smallest integer ≥ MNS/6	6I10	IAMS(J)	1-60	IAMS(J), J = 1, 2, . . . , MNS are depth intervals for which the depth increment is diminished IAMS(1) represents the depth of the bottom
	If MNS = 0, card(s) are omitted				IAMS [(ZM/FH) + 1] represents the surface and a linear relationship holds at all other depths
16	Card a = 1	F10.0	ZM	1-10	Mean depth (ft) of water in a given segment
		7I10	IRST	11-20	Distance (ft) of a given segment from the source
			IREN	21-30	Length (ft) of a given segment
			IRH	31-40	Change in depth (ft) over the length of the segment If IRH>0, there is an upslope If IRH<0, there is a downslope
			IRIB	41-50	If IBS = 1 the length (ft) of the subsections of the segment which produce backscattering
			IRI	51-60	If IMC = 0, IRI modes will be neglected in computing equivalent arrays

Table 2 (Cont'd). Input Data for Program

Card Group	Amount of Cards	Format	Input Parameter	Columns	Data
16 (Cont'd)	Card a = 1	7 I 10	IBS	61-70	If IBS = 1, backscattering included in calculations. If IBS = 0, backscattering ignored
			IPB	71-80	If IBS = 1 and IRH = 0, If IPB = 1, the backscattered field is calculated from a steplike bottom
	Card(s) b If IRI>0, the smallest integer ≥ IRI/6 If IRI = 0, cards are omitted	6 I 10	IRM(J)	1-60	Modes IRM(J), J = 1, . . . , IRI are not considered in the sound field
	Card c = 1	4 F10. 3	CB	1-10	Velocity (ft/sec) of sound in a given bottom segment
			RO	11-20	Density (grams/cm^3) of water above a given bottom segment
			RB	21-30	Density (grams/cm^3) of bottom in a given bottom segment
			CMIN	31-40	The minimum velocity (ft/sec) of sound in the water column in a given segment
	Card(s) d = the smallest integer ≥ NMOD/6	6 F10. 5	DD(J)	1-60	DD(J) is the loss at the boundaries per unit range for a given bottom segment (dB/ft for mode J) J = 1, 2, . . . , NMOD

Table 2 (Cont'd). Input Data for Program

Card Group	Amount of Cards	Format	Input Parameter	Columns	Data
17	Card a = 1	2I10	NMS	1-10	Number of modes to be summed in plot of propagation loss versus range NMS $\leq$ NMOD
			IPV	11-20	If IPV = 1, plot of modes specified in card(s) b of card group 17 will be superimposed on previous plot
	Card(s) b = the smallest integer $\geq$ NMS/6	6I10	MDS(J)	1-60	J = 1, 2, . . . , NMS The values of MDS(J) are the mode numbers of the modes to be summed in the propagation loss versus range plots MDS(J) $\leq$ NMOD
There are (NPS + NDPS + NDPR + NDPZ + 1) sets of card group 17.					

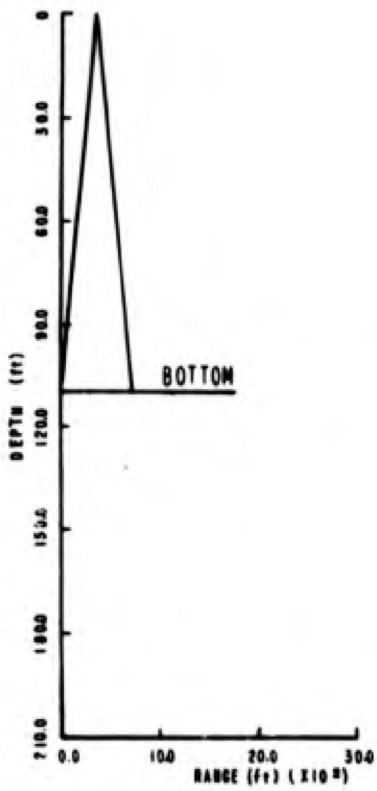


Figure 20. Ray Equivalent, Frequency 46 Hz, Mode 1

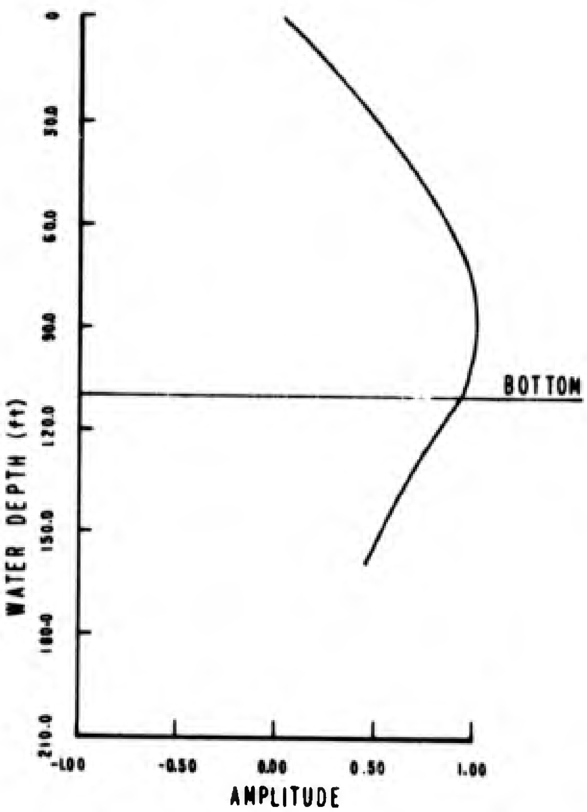


Figure 21. Amplitude Versus Depth, Frequency 46 Hz, Mode 1

PROPAGATION LOSS VERSUS RANGE

Propagation loss for a source level (at one yard reference) as a function of range is calculated and plotted for any given frequency and combination of modes. NMOD (card group 2) gives the number of modes calculated, commencing with mode ISM + 1 (card group 2). IRI (card group 16) of these modes may be neglected, and the neglected modes are specified by IRM(J) (card group 16). Propagation loss plots can be obtained for every specified source-receiver configuration and, for each source-receiver configuration, NPS (card group 12) combinations of modes. Mode conversion is specified by IMC (card group 14), and backscattering by IBS, IPB, and IRIB(card group 16). The actual combination of modes plotted are given in card group 17, and the attenuations of the modes are given by DD(J) in card group 16. The scales of the plots are given by FMI, FMJ, and FMK (card group 12), and the increment in range in feet in the calculated values of propagation loss is given by IRIC (card group 12). If IPV (card group 17) = 1, the plot specified will be superimposed on the previous plot. A sample plot is shown in figure 22.

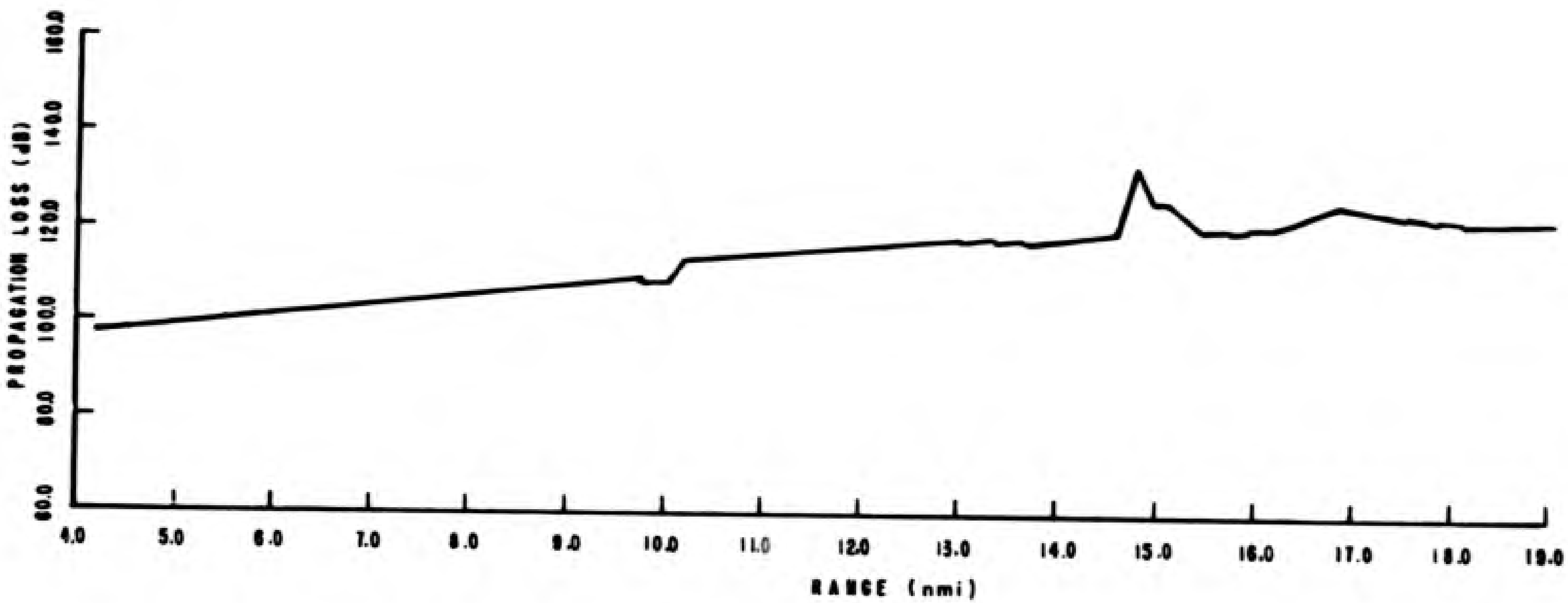


Figure 22. Propagation Loss Versus Range, Frequency 500 Hz, Source Depth 50 Ft, Receiver Depth 104 Ft, Mode 1

TIME-LIMITED PULSES

Time-limited pulsed CW signals are specified by IPL (card group 14) which specifies the pulse length in milliseconds. The sampling rate of the calculated receiver signal is given by IDL (card group 14). The calculated received pulses can be listed on digital tape (unit 8). The pulses at NRT (card group 5) different ranges, given by IRT(J)(card group 11), are recorded. NFS (card group 14) specifies that NFS files should be skipped on unit 8. These recorded values can later be plotted by using an appropriate computer program. A sample plot is shown in figure 15. In addition, the power spectrum of the received pulse can be obtained by a subroutine described in reference 25. A sample plot is shown in figure 23.

PROPAGATION LOSS VERSUS DEPTH

The values of propagation loss as a function of depth are listed on digital tape (unit 7). NFT (card group 14) specifies that NFT files should be skipped on unit 7. Later propagation can be plotted as a function of depth by using an appropriate computer program. Sample plots are shown in figures 24 through 26. In figure 24, propagation loss is plotted as a function of depth. In figure 25, the relative signal level in decibels is plotted as a function of depth. In figure 26, the relative signal level is plotted linearly as a function of depth. Measured values can also be plotted in all three figures for the purpose of comparison.

AMPLITUDE MATCHING AND PHASE MATCHING

The output of the program lists the array element amplitude and phases necessary to match any desired mode. This information is listed at ranges beyond segment IML (group 5), and at ranges less than that corresponding to segment IMU (group 5). Such a list for mode 2 is shown in table 3. The depth, amplitude weighting, and phase of each element are listed first for amplitude matching and then for phase matching.

CONCLUSIONS

A model which can accommodate changes in stratification with range and depth has been described. The model is flexible enough to accommodate many different situations. Propagation can be determined either for point sources, or receivers, or transmitting, or receiving arrays. The stratification may be either constant with range, or may vary either because of changing velocity profile, or bottom characteristics, or slope. The transmission of either CW or pulsed CW signals can be accommodated. Mode conversion may be either included or neglected in the calculations. The amplitude distribution as a function of depth and ray equivalent of individual modes may be determined. Propagation loss can be determined as a function of range and depth for any combination of modes. The level of backscattered energy may be determined. Finally, the proper utilization of arrays to enhance modes by amplitude or phase matching can be determined for any specified array.

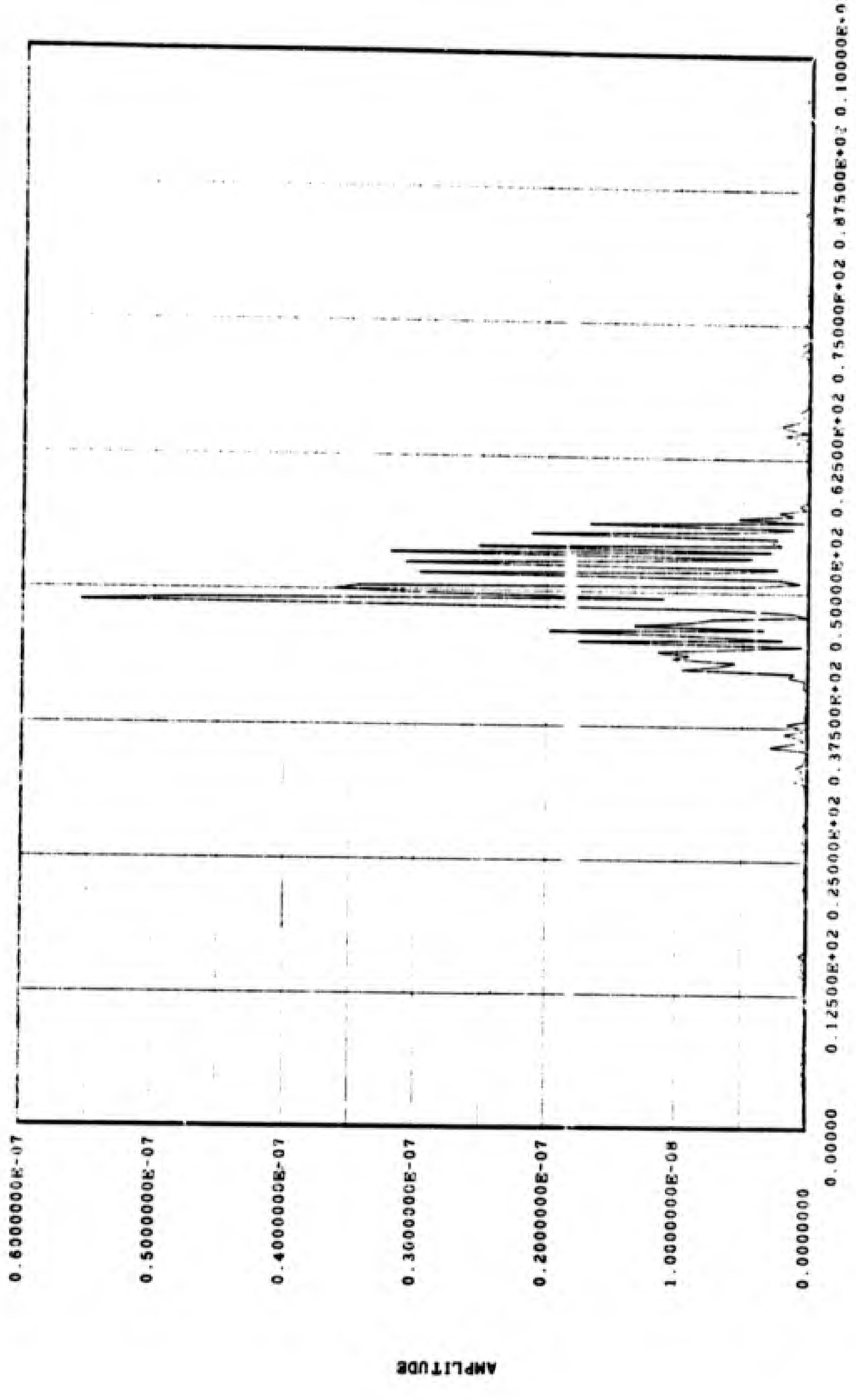


Figure 23. Power Spectrum

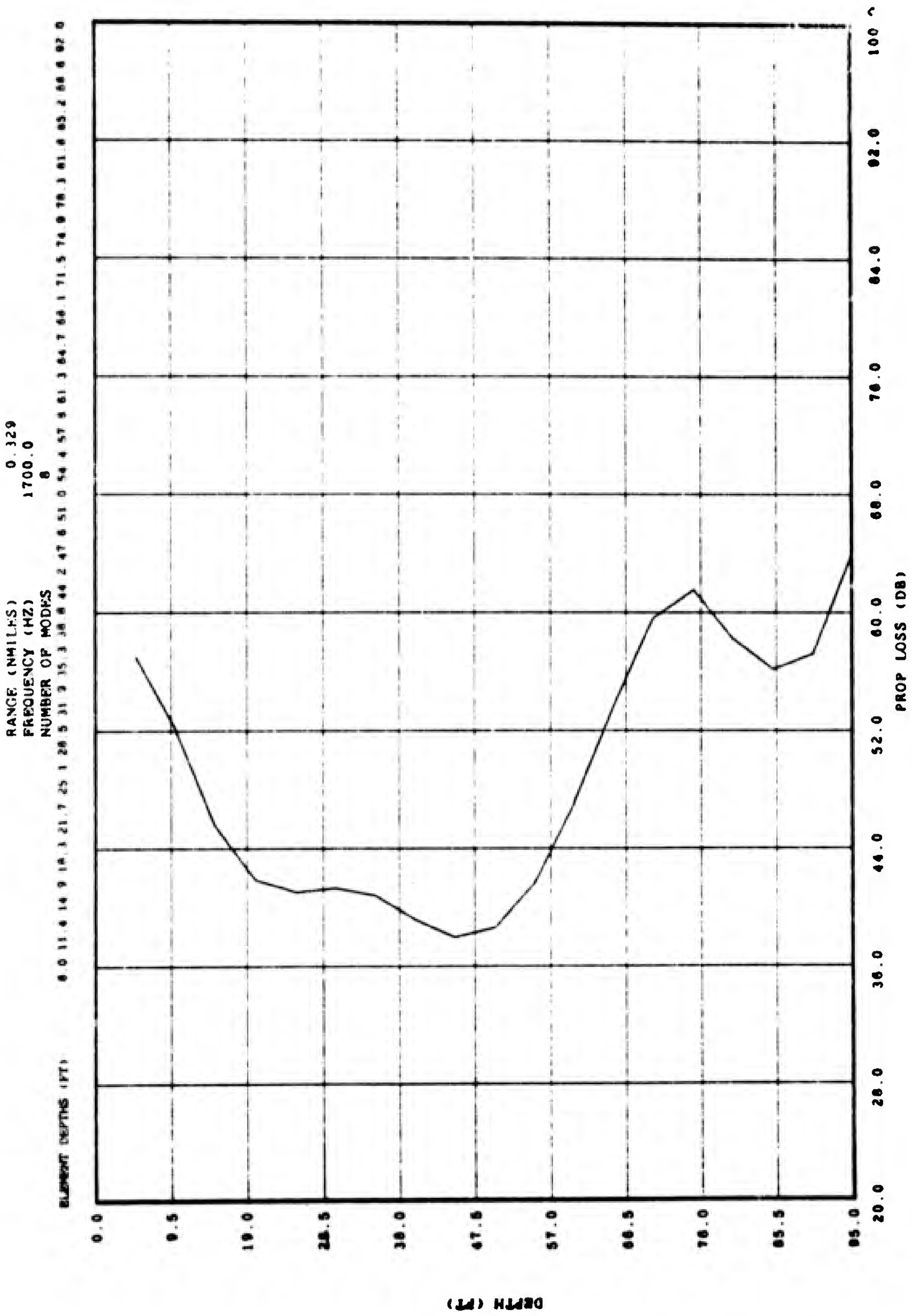


Figure 24. Loss Profile

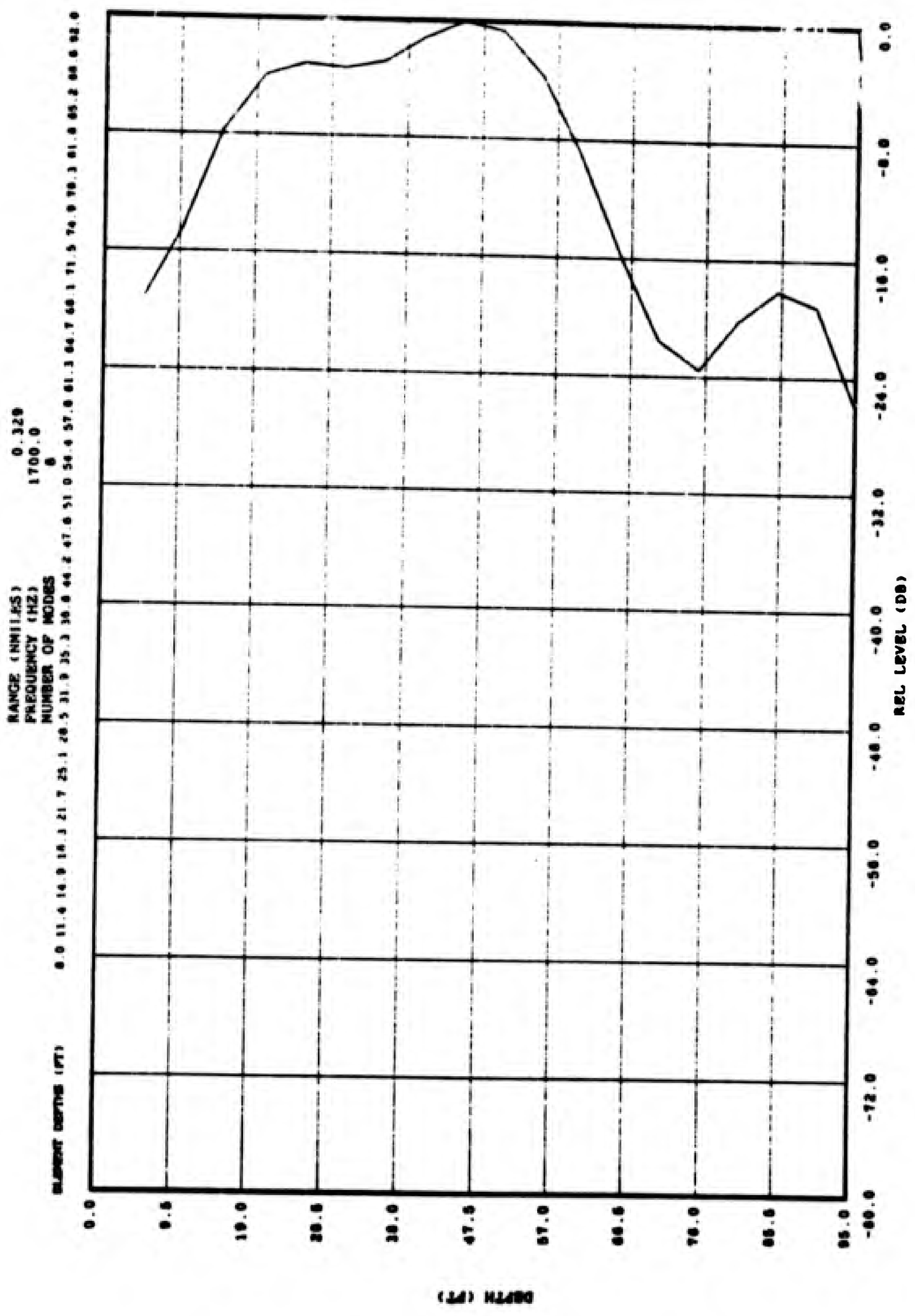


Figure 25. Sound Level Profile (Log Scale)

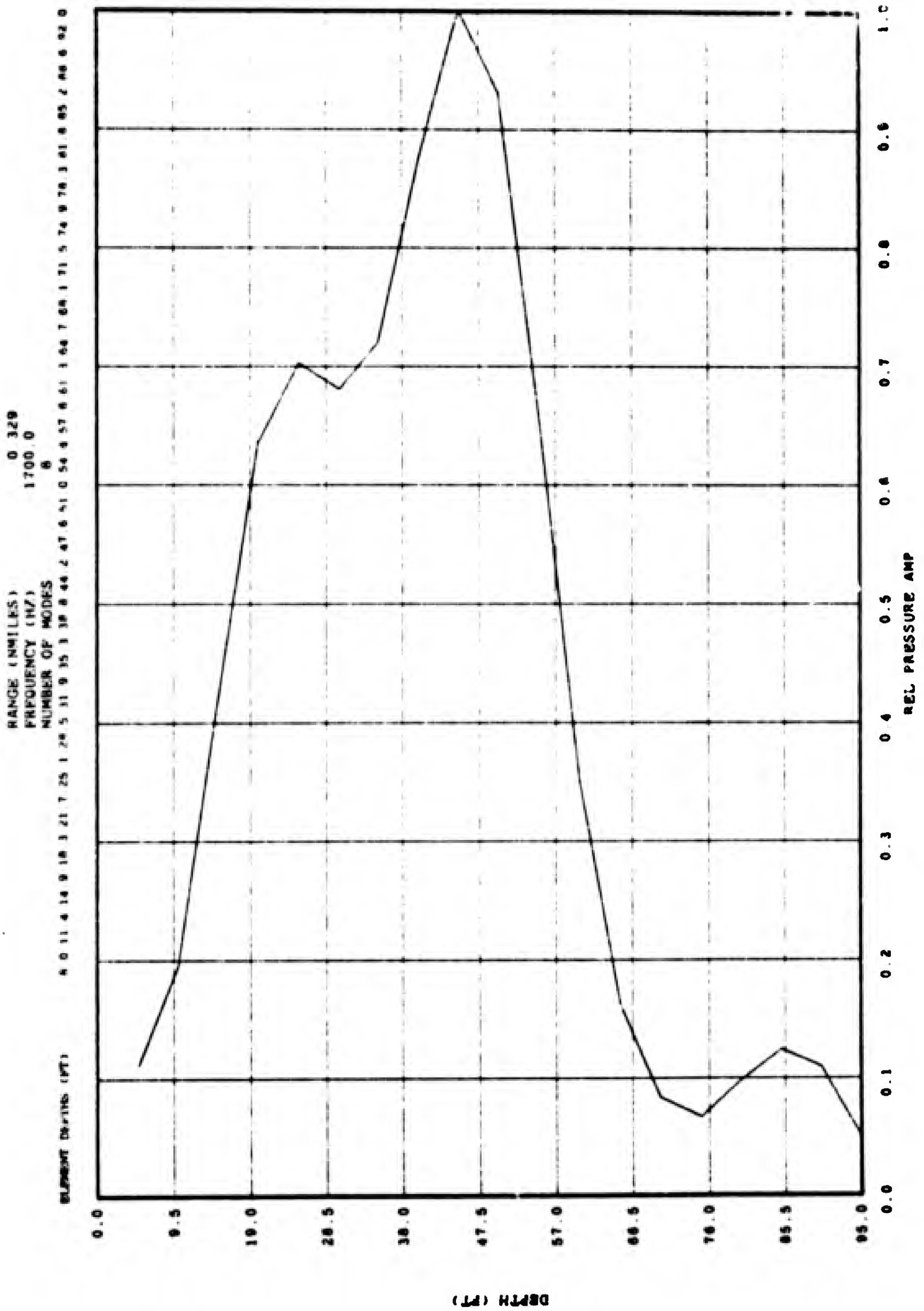


Figure 26. Sound Level Profile (Linear Scale)

Table 3. Element Amplitudes and Phases for Mode 2

	HYDROPHONE NO.	DEPTH (FT)	AMPLITUDE	PHASE (DEG)
AMPLITUDE MATCHING	1	4.0	.217	180.0
	2	8.0	.422	180.0
	3	12.0	.605	180.0
	4	16.0	.755	180.0
	5	20.0	.866	180.0
	6	24.0	.932	180.0
	7	28.0	.951	180.0
	8	32.0	.921	180.0
	9	36.0	.845	180.0
	10	40.0	.728	180.0
	11	44.0	.575	180.0
	12	48.0	.395	180.0
	13	52.0	.196	180.0
	14	56.0	.013	.0
	15	60.0	.220	.0
	16	64.0	.418	.0
	17	68.0	.597	.0
	18	72.0	.750	.0
	19	76.0	.870	.0
	20	80.0	.952	.0
	21	84.0	.994	.0
	22	88.0	.995	.0
	23	92.0	.955	.0
	24	96.0	.876	.0
	25	100.0	.762	.0
PHASE MATCHING	1	4.0	.050	13.2
	2	8.0	.188	26.4
	3	12.0	.385	39.5
	4	16.0	.600	52.6
	5	20.0	.789	65.7
	6	24.0	.914	78.7
	7	28.0	.951	90.0
	8	32.0	.892	104.4
	9	36.0	.752	117.2
	10	40.0	.557	130.0
	11	44.0	.348	142.8
	12	48.0	.164	155.5
	13	52.0	.040	168.1
	14	56.0	.000	180.8
	15	60.0	.049	192.8
	16	64.0	.176	204.8
	17	68.0	.358	216.9
	18	72.0	.565	228.9
	19	76.0	.760	240.9
	20	80.0	.911	253.1
	21	84.0	.994	267.8
	22	88.0	.995	270.0
	23	92.0	.916	286.3
	24	96.0	.771	298.3
	25	100.0	.584	310.0

SUMMARY

In this volume, we have described the theory underlying the model, and the mechanics of its implementation by means of a computer program. Volume II will deal with the validation procedure in testing the model against other models and against experimental data. Finally, in volume II an evaluation of the model's strengths and weaknesses, and recommendations for future work will be given.

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Kanabis, W. G., "A Computer Program to Calculate Normal Mode Propagation in a Medium in Which Stratification Is a Function of Position," NUSL Technical Memorandum 2211-11-71, 14 January 1971.

Kanabis, W. G., "Computer Programs to Calculate Normal Mode Propagation and Applications to Analysis of Explosive Sound Data in the BIFI Range," NUSC Technical Report 4319, 6 November 1972.

Kanabis, W. G., and F. R. DiNapoli, "Application of Normal Mode Theory to an Ocean Stratified in Depth and Range," NUSC Technical Memorandum TA11-305-73, 19 October 1973.

Kanabis, W. G., "Utilization of an Array Matched to Individual Modes; A Computer Program to Calculate Normal Mode Propagation Using a Transmitting Array," NUSC Technical Report 4605, 20 October 1973.

APPENDIX A

THE COMPUTER PROGRAM

```
      DOUBLE PRECISION     F,U,UP ,UM,S2  ,SS,SSS
      DOUBLE PRECISION            K2,    W2,P2,H,ZP,    CZ,G,H2,DEN
      DOUBLE PRECISION S,K20,DK2,GAM,BA   ,K21
      DOUBLE PRECISION E,V
      DOUBLE PRECISION  UMAX
      DOUBLE PRECISION  COS2 , CCC  , CV
      DOUBLE PRECISION  FP, UG, LPG ,ZQ
      DIMENSION HED(20),Z(100),C(100),F( 700),UP( 700),U( 700),
     1  ZMP( 700), XMP( 700), COS2( 700),CCC( 700)
     1  , ZZP( 700), ZPF( 700), VV( 500), DATA( 512) , UU( 700)
     1  ,PM( 700)   ,  ICVP(100)   , IADS(100)   ,CC( 100),ZZ( 100)
     1  , ZR( 104), CR( 104),RP( 700),ZBT(5) , ZBF(5)
     1,DD(100),PFT( 700),FCT( 700),PST( 700),MDS(100),USR(100),URR(100),
     1  PMM(100),FMR(100)   ,FDS(100),DDS(100), DDT(100)
     1 ,GVEL(100 ), PVEL(100 ), FGP(100 ), T1(100 ), T2(100   )
     1,FKB(100),ERS(100),GKB(100),ER(100),PMI(100) ,GMR(100)
      DIMENSION FC1(700)     , FC2(700)   ,FC3(700)   ,FC4(700)
      DIMENSION  BL(100), BB(100)
      DIMENSION  FP( 53,100),IAMS( 53),UG(100) ,LPG(100)
      DIMENSION      TM(100) , TH(4096)
      DIMENSION  ZSD( 45) , ZRD( 45), ZSR( 45) ,IRT( 50)   ,ERSS(100)
      DIMENSION  ZSZ(45)
      DIMENSION  SMA(45) , SMP(45) , IRM(100)
      DIMENSION   FC5(700), SKIP(100), FC6(700), TH1(100),
     1              PMMB(100), IRB(100), PSTB(700), PCTB(700), PPTB(700)
     1           ,FKC(100)
      DIMENSION  FCP(100), FCBP(100)
      EQUIVALENCE  (TH(1), PCT(1) ), (TH(701) ,PST(1))
      EQUIVALENCE   (TH(1401), COS2(1) )
      COMMON MS,JM,U
      COMMON JMS
      CALL PLOTS  ( DATA(1),  512,  2  )
      CALL PLOT (0,0,850)
      P2 = 6.283185306
      DO 256 J = 1,700
      FC6(J) = 0.0
  256 FC3(J) = 0.0
      READ(5,1)HED
```

```
    1 FORMAT (20A4)
      READ (3,121)  NPCS, NVFC , NADS ,NCOR, NMOD,ISM ,IBL, BK
  121 FORMAT (7I10, F10.5)
      BK = 10.0**(BK/10.0)
      READ (3,126)   (ICVP(J),   J= 1, NVPC)
      READ (3,126)   (IADS(J),   J= 1, NADS)
      READ (3,328) NDPS, NDPF, IML, IMU, IMA, NSE, NRE, INEU, NDPZ
     1 ,NRT
  328 FORMAT (16I5)
      NA = 0
      IF(NDPZ.GT.0)  READ(3,122)  (ZSD(J) , ZRD(J) , J = 1,NDPZ)
      IF(NDPZ.GT.0)   ILP= NDPZ
      IF(NDPZ.GT.0)   ILQ =1
      IF(NDPZ.GT.0)  GO TO 341
      IF(NDPS.GT.0)  READ (3,122)  (ZRD(J) , J = 1, NDPS)
      IF(NDPR.GT.0)  READ (3,122)  (ZRD(J) , J = 1, NDPR)
      IF(NSE.EQ.0)   GO TO 340
      READ(3,122)  (ZSD(J) , J = 1, NSE)
      READ(3,122)  (SMA(J) , J = 1, NSE)
      READ(3,122)  (SMP(J) , J = 1, NSE)
      NA = 1
      ILP = NSE
      ILQ = NDPR
      SUM = 0.0
      DO 546 J = 1, NSE
      SUM = SUM +SMA(J)*SMA(J)
  546 SMP(J) = SMP(J)*P2/360.0
      DO 547  J = 1, NSE
  547 SMA(J) = SMA(J)/SQRT(SUM)
  340 IF(NRE.EQ.0)   GO TO 341
      READ (3,122)      (ZSD(J) , J = 1, NRE)
      READ (3,122)      (SMA(J) , J = 1, NRE)
      READ (3,122)      (SMP(J) , J = 1, NRE)
      NA = 2
      ILP = NRE
      ILQ = NDPS
      SUM = 0.0
      DO 548  J = 1, NRE
      SUM = SUM +SMA(J)*SMA(J)
```

```
348 SMP(J) = SMP(J)*P2/360.0
    DO 349  J = 1, NRE
349 SMA(J) = SMA(J)/SQRT(SUM)
341 CONTINUE
    IF(NRT.GT.0)    READ (3,126)   (IRT(J) , J = 1,NRT)
    READ (3,196)  IRIC, NPS, ZS, ZRC, FMI ,FSC ,FMJ, FMK
196 FORMAT (2I10, 6F10.3)
    IF(IADS(1).EQ.1000)       GO TO 147
    FINK = 30*(NADS*NMOD +NVPC)
147 DO 111  L= 1,NPCS
    NMF = 0
    NPZ = 0
    DO 189 J= 1,NVPC
    IF (ICVP(J).EQ.L)    GO TO 4
189 CONTINUE
    GO TO 14
  4 JPM = 25
    IF(IADS(1).EQ.1000)       GO TO 146
    IF(L.GT.1)  FINK = 0.0
    FINC =30*NADS*NMOD+(NVPC -J +1)*30
    FINK = FINC -FINK
    CALL PLOT (-FINK,0.0,-3)
146 READ (3,112)           NUMV, VEL1   ,   IVPL   ,IEX
112 FORMAT ( I10,F10.0,3I10)
    NUM  =  0
    READ (3,2) ZMM, CB, ZBOT ,UBOT ,IX ,IXX  ,SRE
  2 FORMAT (4F10.3,2I10,2F10.3)
    ZME = (200.0*FSC -ZMM)/(20.0*FSC)
    ZBT(1) = ZMM
    ZBT(2) = ZMM
    ZBT(3) = 200.0*FSC
    ZBT(4) = -20.0*FSC
    ZBF(1) =  0.0
    ZBF(2) = 10.0
    ZBF(3) =  0.0
    ZBF(4) =  1.0
    DO 3 I=1,100
    ZZ(I)  =  0.0
```

```
  3 CC(1)  =  0.0
    I=1
  5 READ (3,30) ZZ(I),  CC(I)
 30 FORMAT(2F10.3)
    IF(DABS(ZZ(I) - ZMM) - .01) 6,6,7
  7 I = I+1
    GO TO 5
  6 I = 1
    IF (IVPL.EQ.1)  GO TO 6
    DO 130  K  =  1, NUMV
    CR(NUMV -K +1)  =  CC(K)
130 ZR(NUMV -K +1)  =  ZZ(K)
    ZR(NUMV +1) = ZMM - 0.01
    ZR(NUMV +2) = ZMM + 50.0
    CR(NUMV  +1)  =  CB
    CR(NUMV  +2)  =  CB
    DO 185   NINY  = 1, NUMV
185 ZR(NINY) = ZMM - ZR(NINY)
    ZR(NUMV +3) = 200.0*FSC
    ZR(NUMV +4) = -20.0*FSC
    CR(NUMV  +3)  =  VEL1
    CR(NUMV  +4)  =  50.0
    CALL LINE (CR,ZR,NUMV +2, 1,0,0  )
    CALL LINE (ZBF,ZBT,2,1,0,0)
    CALL SYMBOL (10.25,ZMB,0.14,6HBOTTOM,0.0,6 )
    CALL AXIS (0.0,0.0,14HSOUND VELOCITY,14,10.,0.0,CR(NUMV  +3) ,
   1CR(NUMV+4),10.0  )
    CALL AXIS (0.0,0.0,11HWATER DEPTH,11,10.0,90.0,ZR(NUMV +3)  ,
   1ZR(NUMV+4),10.0  )
    CALL PLOT (15.0,0.0,-3)
    ZR(NUMV +1) = 200.0*FSC
    ZR(NUMV +2) = -20.0*FSC
    CR(NUMV +1) =VEL1
    CR(NUMV +2) =10.0
    CALL LINE (CR,ZR,NUMV,1,0,0)
    CALL LINE (ZBF,ZBT,2,1,0,0)
    CALL SYMBOL (10.25,ZMB,0.14,6HBOTTOM,0.0,6 )
    CALL AXIS (0.0,0.0,14HSOUND VELOCITY,14,10.,0.0,CR(NUMV  +1) ,
```

```
    1CR(NUMV+2),10.0   )
     CALL AXIS (0.0,0.0,11HWATER DEPTH,11,10.0,90.0,ZR(NUMV +1)   ,
    1ZR(NUMV+2),10.0   )
     CALL PLOT (-15.,0.0,-3)
     CALL PLOT (FINC ,0.0,-3)
     CALL PLOT (40.0,0.0,-3)
   8 JP = 3
 188 WRITE(4,9)HED
   9 FORMAT (1H1,9X,20A4)
     WRITE(4,10)
  10 FORMAT(1H0,15X,4HZ,FT,5X,8HC,FT/SEC)
  11 WRITE(4,12)ZR(I),CR(I)
  12 FORMAT(1H0,9X,F10.1,F12.1)
     IF(DABS(ZZ(I) -ZMM) - 0.01)  14,14,13
  13 I = I+1
     JP = JP+1
     IF(JP-JPM)11,8,8
  14 IF (L.NE.1)  GO TO 197
     READ (3,15) FH,UM, FQ, MNS, IMS ,IMC ,IPL  ,IDL ,NFS ,NFT
     W2 = (P2*FQ)*(P2*FQ)
     FR = FQ/1000.0
     A = ( 0.1*FR**2/(1.0 +FR**2)) + (40.0*FR**2/(4100.0 +FR**2))
    1 + (.000275*FR**2)
  15 FORMAT(F10.0,D20.1, F10.3, 8I5)
     IF(NFT.GT.0)  CALL NTRAN(7,8,NFT)
     IF(NFS.GT.0)   CALL NTRAN (8,8,NFS)
     IF(MNS.GT.0) READ(3,126) (IAMS(J) , J= 1,MNS)
     UM = UM**IXX
     H = FH
     IF (L.NE.1)  GO TO 197
     DO 204  J= 1, NM
     FRB(M) = 0.0
     DDS(M) = 0.0
 204 ERS(M) = 0.0
 197 READ (3,195)  ZM, IRST, IREN, IPH, IRIB, IRI ,IBS ,IPE
```

```
195 FORMAT  (F10.0, 7I10)
    THB = ATAN(FLOAT(IRH)/FLOAT(IREN) )
    IF(IRI.GT.0)   READ (3,126)   (IRM(J), J= 1, IRI)
    N = ZM/H
    NNN  =  N
    READ (3,2)  CB , RO , RB ,CMIN
    READ (3,122)  (DD(J), J= 1,NMOD)
122 FORMAT (6F10.5)
    IPOS = 0
    IF (ZRC.GT.ZM)  IPOS = 1
    ZS = ZM - ZS
    ZRC = ZM - ZRC
    ZMB = (200.0*FSC -ZM )/(20.0*FSC)
    DO 200 J= 1,NUMV
    C(J) = CC(J)
200 Z(J) = ZZ(J)  -  (ZMM -ZM)
    DO 201 J= 1,NUMV
    IF(Z(J).LT.0.1)  GO TO 201
    GO TO 202
201 CONTINUE
202 IF (J.EQ.1)  GO TO 120
    C(J -1) = C(J   ) +(C(J) -C(J -1))*-Z(J)/(Z(J) - Z(J -1))
    Z(J -1) = 0.0
    JJJ = NUMV -J +2
    DO 203  JJ= 1,JJJ
    C(JJ) = C(JJ +J  -2)
203 Z(JJ) = Z(JJ +J  -2)
    JJJ= JJJ  +1
    DO 205  JK = JJJ,NUMV
    C(JK) = 0.0
205 Z(JK) = 0.0
120 DO 154  M=  1,NMOD
162 N  =  NNN
    FMOD = M
    IF(N)29,4,29
 29 K20 = ((W2*(CB-CMIN)*(CB+CMIN))/(CMIN*CMIN*CB*CB))
    K21 = ((W2*(CB-C(1))*(CB+C(1)))/(C(1)*C(1)*CB*CB))
    DK2= .099*K20
    K2 = -K20
```

```
 31 K2 = K2 + DK2
    IF(K2-K20)16,32,32
 32 WRITE(4,33)
 33 FORMAT(1H0,13HNO MODE FOUND)
    IF(M.EQ.1)    NMF = 1
    IF(IMC.EQ.1)    NMF = 1
    IC2 = IB2
    ID2 = IA2
    DO 295  J= M, NMOD
295 DDS(J) = DDS(J) +DD(J)*FLOAT(IREN)
    GO TO 123
 16 FLB  =  FLOAT(N)
    H = ZM/ DBLE(FLB)
    ZP = 0.
    J = 1
    I = 1
 17 IF (ZP- Z(I+1)) 18,18,19
 19 I  =  I  +1
    GO TO 17
 18 CZ = (C(I)*(Z(I+1)-ZP)+C(I+1)*(ZP-Z(I)))/(Z(I+1)-Z(I))
    CCC(J)  =  CZ
    G =-(W2*(C(1)-CZ)*(C(1)+CZ))/(CZ*CZ*C(1)*C(1))
    F(J) = K2 - G
    IF(MNS.EQ.0) GO TO 176
    DO 174  KM = 1,MNS
    IF(IAMS(KM).EQ.J)  GO TO 175
174 CONTINUE
    GO TO 176
175 DO 177  KZ = 1,IMS
    ZQ = ZP +H*FLOAT(KZ)/FLOAT(IMS)
178 IF (ZQ- Z(I+1)) 139,139,138
138 I  =  I  +1
    GO TO 178
139 CZ = (C(I)*(Z(I+1)-ZQ)+C(I+1)*(ZQ-Z(I)))/(Z(I+1)-Z(I))
    G =-(W2*(C(1)-CZ)*(C(1)+CZ))/(CZ*CZ*C(1)*C(1))
177 FP(KM,KZ ) = K2 -G
176 IF(ZP-ZM) 20,21,21
```

```
 20 J=J+1
    FLA  =  FLOAT(J -1)
    ZP   =  ZM*DBLE(FLA)/DBLE(FLB)
    GO TO 17
 21 U(1) = 1.0*UBOT**IX
    UP(1) =(RO/RB)*DSQRT ((W2*(CB-C(1))*(CB+C(1)))/(C(1)*(C(1)*CB*CB))
   1-K2)
    UP(1) = U(1)*UP(1)
    J = 1
 22 JP = 3
 25 FLA  =  FLOAT(J -1)
    FLB  =  FLOAT(N)
    ZP   =  ZM*DBLE(FLA)/DBLE(FLB)
    IF(J-N-1)27,34,27
 27 IF(DABS(U(J))-UM)28,34,34
 28 J = J+1
    IF(MNS.EQ.0) GO TO 135
    DO 133  KM = 1,MNS
    IF (IAMS(KM).EQ.J -1)  GO TO 134
133 CONTINUE
    GO TO 135
134 B = H/FLOAT(IMS)
    H3 = H*H/(FLOAT(IMS)*FLOAT(IMS))
    DEN = 1.0 + (H3*FP(KM,1))/6.0
    UG(1)=((1.0-(H3/3.0)*F(J-1))*U(J-1)+B*UP(J-1))/DEN
    UPG(1) =((1.0-(H3*FP(KM,1))/3.0)*UP(J-1)-(F(J-1)+FP(KM,1)-(H3*
   1 FP(KM,1)*F(J-1))/6.0)*B*U(J-1)*0.5)/DEN
    IMX = IMS -1
    DO 136  KZ = 1,IMX
    DEN = 1.0 + (H3*FP(KM,KZ +1))/6.0
    UG(KZ +1) =((1.0-(H3/3.0)*FP(KM,KZ))*UG(KZ) +B*UPG(KZ))/DEN
    UPG(KZ +1) = ((1.0-(H3*FP(KM,KZ+1))/3.0)*UPG(KZ) -(FP(KM,KZ)  +
   1 FP(KM,KZ +1)-(H3*FP(KM,KZ +1)*FP(KM,KZ))/6.0)*B*UG(KZ)*0.5)/DEN
136 CONTINUE
    U(J) = UG(IMX+1)
    UP(J) = UPG(IMX+1)
    GO TO 137
```

```
135 JP= JP+1
    H2= H*H
    IF(INEU.EQ.0)    GO TO 316
    IF(J.GT.2)    GO TO 318
    GO TO 316
318 IF(J.EQ.3)    PST(1) = U(1)
    IF(J.EQ.3)    PST(2) = U(2)
    PST(J) = ((2.0 +10.0*H2*  F(J -1)/12.0)*PST(J -1) -(1.0 -H2*
   1    F(J -2)/12.0)*PST(J -2))/(1.0 -H2*  F(J)/12.0)
316 DEN = 1.0 + (H2*F(J))/6.0
    U(J)= ((1.0-(H2/3.0)*F(J-1))*U(J-1)+H*UP(J-1))/DEN
    UP(J)=((1.0-(H2*F(J))/3.0)*UP(J-1)-(F(J-1)+F(J)-(H2*F(J)*F(J-1))/
   16.0)*H*U(J-1)* 0.5)/DEN
137 IF (JP-JPM)25,22,25
 34 JM = J
    CALL COUNT
    IF(MS -M -ISM)    31,35,35
 35 DO 289 J= 1,N
    IF (F(J).GT.0.0) GO TO 290
289 CONTINUE
290 IJ1 = J +1
    DO 291  J= IJ1,  N
    IF(F(J).LT.0.0)  GO TO 292
291 CONTINUE
    GO TO 294
292 IJ2 = J +1
    DO 293  J= IJ2, N
    IF(F(J).GT.0.0)  GO TO 31
293 CONTINUE
294 K2 = K2 - DK2
    DK2= DK2 / 10.0
    IF(DK2- .000001*K20*10**(-IEX)) 36,36,31
 36 J = 2
501 S = H*U(1)*U(1)/2.0
    S2= H*U(1)*U(1)/(2.0*CCC(1)*CCC(1))
 37 IF(J-JMS)38,39,38
 38 S = H*U(J)*U(J)+S
    S2= H*U(J)*U(J)/(CCC(J)*CCC(J))   +S2
    J=  J+1
    GO TO 37
```

```
 39 S =(H*U(J)*U(J)/2.0) + S
    S2= H*U(J)*U(J)/(2.0*CCC(J)*CCC(J))   +S2
    J = 1
 42 JP = 3
    DO 206 KK= 1, NADS
    IF(IADS(KK).EQ.L)  GO TO 208
206 CONTINUE
    GO TO 43
208 MC = M +ISM
    WRITE(4,23)HED,N,K2,UM,FQ,MC
 23 FORMAT(1H1,5X,20A4,/,2X,3HN =,I5,2X,4HK2 =,F15.8,2X,4HUM =,F6.1,
   12X,6HFREQ = ,F6.1,2X,3HM =,I5)
    WRITE(4,47) ZM,CB,RC,RB
 47 FORMAT(3X,6HZMAX = ,F10.3,4HCB = ,F10.3,4HRO = ,F15.8,4HRB = ,
   1F15.8)
    WRITE(4,24)
 24 FORMAT(1H0,17X,4HZ,FT,20X,1HU,30X,5HDU/DZ,41X,4HF(Z))
 43 ZP = ZM*( FLOAT(J-1)/ FLOAT(N))
    ZQ = ZM - ZP
    DO 207 KK= 1, NADS
    IF(IADS(KK).EQ.L)  GO TO 209
207 CONTINUE
    GO TO 44
209 WRITE(4,26)ZQ,U(J),UP(J),F(J)
    IF(INEQ.EQ.1)  WRITE (4,26)   ZQ, PST(J)
    IF(J-N-1)45,44,45
 45 IF(DABS(U(J))-UM)46,44,44
 46 J = J+1
    JP = JP+1
    IF(JP-JPM)43,42,43
 44 WRITE(4,40) S
 40 FORMAT (1H0,10X, 3HS = ,F15.5)
164 UMAX = 0.0
    NM  =  N  +1
    ICX = 0
    DO  164  K=  1, NM
    IF(ICX.EQ.1)  GO TO 465
    IF(U(NM -K +1)*U(NM -K).LT.0.0)   ICX = 1
    U(NM -K +1) = 0.0
    NW = NM -K +1
```

```
465 IF  ( DABS (U(K)) - UMAX)  104,104,103
103 UMAX  =  DABS(U(K) )
104 CONTINUE
    WRITE(4,126)    NW
    IF(IRI.EQ.0)  GO TO 498
    DO 499  K= 1, IRI
    IF(IRM(K).EQ.M)   GO TO 154
499 CONTINUE
498 CONTINUE
    DO  105  K=  1, NM
    IF(IMC.EQ.1) GO TO 105
    FC2(  NM -K +1) = U(K)/(UMAX)
    FC4(  NM -K +1) = U(K)/UMAX
105 UU(K) =  U(K)/UMAX
    DO 106  K= 1, NM
106 ZZP(K)  =  ZM*(FLOAT(K -1)/FLOAT(N)  )
    GAM = DSQRT(K21 - K2)
    BA = (RO*C(1))/(2.0*RB*CB*GAM*S )
    BL(M) = 1.0/(GAM*S)
    WRITE(4,41) BA,GAM ,BL(M)
 41 FORMAT(1H0,10X,5HB/A = ,F15.5,5X,7HGAMMA = ,F15.5 ,8HE1/E0 = ,
   1 F15.5)
    SS=  RO*S/(UMAX*UMAX)+RO* RO*U(1)*U(1)/(UMAX*UMAX*2.0*GAM*RB)
    SSS= RO*S2/(UMAX*UMAX)          +       RO*RO*U(1)*U(1)/(UMAX*UMAX
   1*2.0*GAM*RB*CB*CB)
    E = DEXP(-GAM*H)
    ZP = 0.0
    V = 1.0*UBOT**IX
    NN   =  1
    ZPP(1)  =  0.0
    VV(1)  = UU(1)
    MC = M +ISM
    WRITE(4,23)HED,N,K2,UM,FQ,MC
    WRITE(4,47) ZM,CB,RO,RB
212 DO 210  KK= 1,NADS
    IF(IADS(KK).EQ.L)  GO TO 211
210 CONTINUE
    GO TO 50
211 WRITE(4,48)
```

```
48 FORMAT(1H0,17X,4HZ,FT,10X,1HU )
49 WRITE(4,26)ZP,V
26 FORMAT (1H0,F22.2,E33.10,E33.10,F36.10)
50 V = V*E
   NN  = NN  +1
   VV(NN)  =  VV(NN -1)*E
   IF(IMC.EQ.1) GO TO 191
   FC2(  NM +NN -1) = VV(NN)
   FC4(  NM +NN -1) = VV(NN)*RO/RB
191 ZPP(NN)  =  ZP -       H
   ZP = ZP-H
   IF(ZPP(NN) +ZBOT)  110,110,212
110 IA2 = NM
   IB2 = NM +NN -1
   DO 190 J= 1, NADS
   IF ( IADS(J).EQ.L)  GO TO 193
190 CONTINUE
   GO TO 192
193 FINC = 30*(NADS*NMOD -J +1)  -(NMOD -M)*30
   CALL PLOT (-FINC,0.0,-3)
   DO 184   NINX  = 1, NN
184 ZPP(NINX) =  ZM - ZPP(NINX)
   NINV  =  N + 1
   DO 183   NINW  = 1, NINV
183 ZZP(NINW) =  ZM - ZZP(NINW)
   VV(NN +1)  =  -1.0
   VV(NN +2)  =  0.5
   ZPP(NN +1) = 200.0*FSC
   ZPP(NN +2) = -20.0*FSC
   ZZP(N +2)  = 200.0*FSC
   ZZP(N +3)  = -20.0*FSC
   ZBT(1) = ZM
   ZBT(2) = ZM
   UU(N+2)  =  -1.0
   UU(N+3)  =  0.5
   CALL  AXIS (0.0,0.0, 9HAMPLITUDE,  9,4.0,0.0,UU(N+2),UU(N+3),10.0)
   CALL AXIS (0.0,0.0,16HWATER DEPTH (FT),16,10.0,90.0,ZZP(N+2),
```

```
    1  ZZP(N +3),10.0)
       CALL LINE  (UU, ZZP,  N+1, 1,0,0)
       CALL LINE (VV, ZPF,  NM,  1,0,0  )
       CALL LINE (ZBF,ZBT,2,1,0,0)
       CALL SYMBOL (10.25,ZMB,0.14,6HBOTTOM,0,0,6 )
       CALL SYMBOL(1.3,9.50,0.14,9HAMPLITUDE,0.0,9)
       CALL SYMBOL(1.7,9.25,0.10,6HVERSUS,0.0,6)
       CALL SYMBOL(1.6,9.00,0.14,5HDEPTH,0.0,5)
       CALL SYMBOL(0.6,8.75,0.14,19HFREQUENCY         HZ,0.0,19)
       CALL NUMBER(2.2,8.75,0.14,FQ,0.0,-1)
       CALL SYMBOL(1.4,8.50,0.14,4HMODE,0.0,4)
       CALL NUMBER(2.1,8.50,0.14,FMOD,0.0,-1)
       CALL PLOT (15.0,0.0,-3)
  192  NUM  =  NUM  +1
  153  XMP(1)  =  0.0
       ZMP(1)  =  0.0
       NM  =  N  +1
       NSTR = 1
       NEND = N +1
       IUZ = 0
       IF(F(1).GT.0.0)   GO TO 179
  681  DO 170 K=  NSTR,NEND
       IF(F(K).GT.0.0)    GO TO 171
  170  IRTB = K
       GO TO 689
  171  COS2(IRTB +1) = F(IRTB +1)*CCC(IRTB +1)*CCC(IRTB +1)/W2
       CV  =  CCC(IRTB +1)/DSQRT(1.0 - COS2(IRTB +1))
       ZMP(2) = H*(CV - CCC(IRTB +1))/(CCC(IRTB)-CCC(IRTB +1))
       XMP(2) =  (ZMP(2) - ZMP(1))*(CCC(IRTB +1) + CV)/(CV*(DSQRT(COS2
      1 (IRTB +1))))
       ZMP(1) = FLOAT(IRTB)*H - ZMP(2)
       ZMP(2) = ZMP(1) + ZMP(2)
       NM = NM -IRTB +1
       DO 172  K = 3,NM
       TH1(M) = 90.0
       KO = IRTB -1 +K
       COS2(KO-1) = F(KO -1)*CCC(KO -1)*CCC(KO -1)/W2
       COS2(KO)=F(KO)*CCC(KO)*CCC(KO)/W2
       IF(CCC(KO).GE.CV) ZMP(K) = ZMP(K  -1) +H*(CV- CCC(KO -1))/(CCC(KO)
```

```
    1 -CCC(KO -1))
      IF(CCC(KO).GE.CV)    GO TO 152
      XMP(K) = XMP(K  -1) +H*(CCC(KO -1) + CCC(KO))/(CV*(DSQRT(COS2(KO-1
    1)) + DSQRT(COS2(KO))))
  172 ZMP(K)  =  (KO -1)*H
      TH2 = (ACOS(SQRT(COS2(NM +IRTB -1))))*57.2958
      N =  NM -1
      GO TO 143
  179 COS2(1)  =  ABS(F(1))*CCC(1)*CCC(1)/W2
      CV  =  CCC(1)/DSQRT(1.0 - COS2(1) )
      DO 140  K=  2,NM
      COS2(K -1) = ABS(F(K -1))*CCC(K -1)*CCC(K -1)/W2
      COS2(K)  =  ABS(F(K))*CCC(K)*CCC(K)/W2
      TH1(M) =   (ACOS(SQRT(COS2(1))))*57.2958
      TH2 =(ACOS(SQRT(COS2(NM))))*57.2958
      IF( CCC(K).GE.CV)  GO TO 142
      XMP(K) = XMP(K -1) + H*(CCC(K-1) + CCC(K))/(CV*(DSQRT(COS2(K-1))+
    1 DSQRT(COS2(K))))
  140 ZMP(K)  =  (K-1)*H
      GO TO 143
  142 ZMP(K) = ZMP(K -1)  +  H*(CV - CCC(K-1))/(CCC(K) - CCC(K -1))
  152 TH2  =  90.0
      XMP(K) = XMP(K -1) + (ZMP(K) - ZMP(K-1))*(CCC(K-1) + CV)/(CV*(
    1  DSQRT(COS2(K -1))))
      N = K -1
      NM  =  K
  143 XMP(NM +1) =  0.0
  182 XMP(NM +2)     =  SRE
      DO 186 NINZ = 1, NM
  186 ZMP(NINZ) = ZM - ZMP(NINZ)
      ZMP(NM +1) =  200.0*FSC
      ZMP(NM +2)     = -20.0*FSC
      SKIP(M) = 2.0*XMP(NM)
      IF(F(1).LT.0.0)    SKIP(M) = -SKIP(M)
      WRITE(4,145)    TH1(M) , TH2 , SKIP(M)
  145 FORMAT (3F10.3)
      IF(ID2.NE.1)  BJ = SKIP(M)
      DO 691 J= 1, NADS
      IF ( IADS(J).EQ.L)  GO TO 194
```

```
691 CONTINUE
    GO TO 157
194 CALL LINE (XMP,ZMP,NM    ,1,0,0)
    DO 141  J = 1,NM
141 XMP(J) = 2.0*XMP(NM) -XMP(J)
    IF(F(1).LT.0.0)   GO TO 504
    IF(IRH.EQ.0)  GO TO 504
    IF(IRH.LT.0)  GO TO 505
    IF(FLOAT(IRH)/FLOAT(IREN).LT.ZMP(NM)/XMP(NM) )  GO TO 506
    SKIP(M) = FLOAT(IREN)*ZMP(NM)/FLOAT(IRH)
    GO TO 504
506 DO 507  J= 1, NM
    IF(FLOAT(IRH)*XMP(J)/FLOAT(IREN).GT.ZMP(J) )  GO TO 507
    SKIP(M) = XMP(J)
    GO TO 504
507 CONTINUE
505 SKIP(M) = SKIP(M)   +FLOAT(-IRH)*SKIP(M)*TAN(TH1(M)/57.2958)/
   1 FLOAT(IREN)    +SKIP(M)*(FLOAT(-IRH)*TAN(TH1(M)/57.2958)/FLOAT
   1 (IREN) )**2
504 CONTINUE
    CALL LINE (XMP,ZMP,NM    ,1,0,0)
    IF (IDZ.EQ.1) GO TO 650
    IDZ = 1
    CALL LINE (ZBF,ZBT,2,1,0,0)
    CALL SYMBOL (10.25,ZMB,0.14,6HBOTTOM,0.0,6 )
    CALL AXIS (0.0,0.0,10HRANGE (FT),10,12.0,0.0,XMP(2*NM)    ,
   1XMP(2*NM +1),10.0)
    CALL AXIS (0.0,0.0,16H      DEPTH (FT),16,10.0,90.0,ZMP(2*NM),
   1 ZMP(2*NM +1),10.0 )
    CALL SYMBOL(5.0,9.50,0.14,14HRAY EQUIVALENT,0.0,14)
    CALL SYMBOL(5.0,9.25,0.14,19HFREQUENCY         HZ,0.0,19)
    CALL NUMBER(6.4,9.25,0.14,FQ,0.0,-1)
    CALL SYMBOL(5.5,9.00,0.14,4HMODE,0.0,4)
    CALL NUMBER(6.2,9.00,0.14,FMOD,0.0,-1)
    NM = NEND
650 NSTR =        IRTB + NM -1
    ZMP(1) = 0.0
    IF(NSTR.GE.NEND) GO TO 689
    GO TO 681
```

```
689 CALL PLOT (-15.0,0.0,-3)
    CALL PLOT (FINC ,0.0,-3)
157 T1(L) = TH1(M)
    T2(L) = TH2
    FMH = SQRT(P2*FQ /(30.48*CV) )
    PM(L) = SQRT(P2)*      RO/(FMH*2.0*(SS) )
    GVEL(M) = SS/(CV*SSS)
    CMIN = CV
    IF(CMIN.GT.C(1))    CMIN = C(1)
    FQP(L) = L
    PM(M)  =  PM(L)
    FMR(M) = P2*FQ/CV
    WRITE ( 4,166)  PM(L) , FMH, SS  , GVEL(M)  ,CV, FMR(M)
166 FORMAT(5E25.10)
    IF(L.EQ.1)   FKB(M) = FMR(M)*IRST
    FKB(M) = FKB(M) + FMR(M)*FLOAT(IREN)
    IF(L.EQ.1)   FKC(M) = FMR(M)*FLOAT(IRST +IREN)
    IF(L.GT.1)   FKC(M) = FMR(M)*FLOAT(IREN)
    DDS(M) = DDS(M) +DD(M)*FLOAT(IREN)
    IF  (L.EQ.NCOR)PMI(M)= PM(L)
    IF(INC.NE.0)  GO TO 476
    PMI(M) = PM(M)
476 CONTINUE
    IZR =ZRC*FLB/ZM +1.0
    FZR =ZRC*FLB/ZM +1.0
    UTA = U(IZR)  +(FZR -IZR)*(U(IZR +1) -U(IZR))
    IZS = ZS*FLB/ZM   +1.0
    UTB = U(IZS) +(FZS -IZS)*(U(IZS +1) -U(IZS))
    FZS = ZS*FLB/ZM   +1.0
    IF(L.EQ.NCOR)USR(M)= UTB/UMAX
    IF(IPOS.EQ.1) GO TO 155
    URR(M) = UTA/UMAX
    IF(NA.GT.0)   GO TO 319
    IF(NDPZ.LT.1)  GO TO 309
    DO  303   J = 1, NDPZ
    IZRZ = (ZM -ZRD(J))*FLB/ZM  +1.0
    FZRZ = (ZM -ZRD(J))*FLB/ZM  +1.0
    UTA =  U(IZRZ) +(FZRZ -IZRZ)*(U(IZRZ +1) -U(IZRZ))
    IF(L.GT.NCOR)   GO TO 304
```

```
      IZSZ = (ZM -ZSD(J))*FLB/ZM  +1.0
      FZSZ = (ZM -ZSD(J))*FLB/ZM  +1.0
      UTB = U(IZSZ) +(FZSZ -IZSZ)*(U(IZSZ +1) -U(IZSZ))
      ZSR(J) =          (UTB*UTA)/(USR(M)*URR(M)*UMAX*UMAX)
      GO TO 303
  304 CALL NTRAN (23,10)
      NPT = (M -1)*NDPZ
      CALL NTRAN (23,6,NPT)
      CALL NTRAN (23,2,NDPZ,ZSR,LL)
      CALL NTRAN (23,22)
      NPT = (NMOD -1)*NDPZ
      CALL NTRAN (23,6,NPT)
      CALL NTRAN (23,2,NDPZ,ZSZ,LL)
      CALL NTRAN (23,22)
      CALL NTRAN (23,10)
  303 ZSZ(J) =  UTA/(URR(M)*UMAX)
      ZSR(J)=(ZSR(J)  *UTA     )/(URR(M)*ZSZ(J)*UMAX)
      NPT = (M -1)*NDPZ
      CALL NTRAN (23,6,NPT)
      CALL NTRAN (23,1,NDPZ,ZSR,LL)
      CALL NTRAN (23,22)
      NPT = (NMOD -1)*NDPZ
      CALL NTRAN (23,6,NPT)
      CALL NTRAN (23,1,NDPZ,ZSZ,LL)
      CALL NTRAN (23,22)
      GO TO 309
  319 DO 322     J = 1, ILP
      IZSZ = (ZM -ZSD(J))*FLB/ZM  +1.0
      FZSZ = (ZM -ZSD(J))*FLB/ZM  +1.0
      UTB = U(IZSZ) +(FZSZ -IZSZ)*(U(IZSZ +1) -U(IZSZ))
      IF(NA.EQ.2)   UTX = UTB
      IF(L.EQ.NCOR)   GO TO 345
      CALL NTRAN  (23,10)
      NPT = (M -1)*ILP*ILQ  +(J -1)*ILQ
      CALL NTRAN (23,6, NPT)
      CALL NTRAN (23,2,ILQ,ZSR, LL)
      CALL NTRAN (23,22)
      NPT = NMOD*ILP*ILQ  -ILQ
```

```
      CALL NTRAN (23,6, NPT)
      CALL NTRAN (23,2,ILQ,ZSZ, LL)
      CALL NTRAN (23,22)
  345 DO 320   JZ = 1, ILQ
      IF(IMA.EQ.1)        UTA = 1.0
      IF(IMA.EQ.1)        GO TO 511
      IZRZ = (ZM -ZRD(JZ))*FLB/ZM   +1.0
      FZRZ = (ZM -ZRD(JZ))*FLB/ZM   +1.0
      UTA = U(IZRZ) +(FZRZ -IZRZ)*(U(IZRZ +1) -U(IZRZ))
      IF(NA.EQ.1)     UTX = UTA
  511 IF(L.GT.NCOR)    GO TO 330
      ZSR(JZ) = (UTA*UTB)/(USR(M)*URR(M)*UMAX*UMAX)
      GO TO 320
  330 ZSR(JZ) = (ZSR(JZ)*UTX)/(URR(M)*ZSZ(JZ)*UMAX)
  320 ZSZ(JZ) = UTX/(URR(M)*UMAX)
      CALL NTRAN  (23,10)
      NPT = (M -1)*ILP*ILQ +(J -1)*ILQ
      CALL NTRAN (23,6, NPT)
      CALL NTRAN (23,1,ILQ, ZSR, LL)
      CALL NTRAN (23,22)
      NPT = NMOD*ILP*ILQ  -ILQ
      CALL NTRAN (23,6, NPT)
      CALL NTRAN (23,1,ILQ, ZSZ, LL)
      CALL NTRAN (23,22)
  322 CONTINUE
  309 IF(L.EQ.1)  GO TO 251
      IF(IMC.EQ.1)  GO TO 251
      USR(M) = 0.0
      USRS = 0.0
      USBR = 0.0
      USRB = 0.0
      DO 256  J= 1,IC2
      IF(IC2.LT.IB2)  GO TO 255
      IF(J -1.LT.IA2)  GO TO 255
      IF(J -1.GE.ID2)   GO TO 255
      IF(IBS.EQ.0)   GO TO 256
      IF(J.GT.1)  GO TO 562
```

```
      CALL NTRAN (23,10)
      NPT=2*NMOD*ILP*ILQ +(M -1)*IB2*2   +I32
      CALL NTRAN (23,6,NPT)
      CALL NTRAN (23,2,IB2,FC3, LL)
      CALL NTRAN (23,22)
  562 USBR = USBR + FC1(J)*FC3(J)*SIN(FC5(J) )
      USRB = USRB + FC1(J)*FC3(J)*COS(FC5(J) )
      GO TO 256
  255 USR(M) = USR(M)  + FC1(J)*FC4(J)*SIN(FC5(J) )
      USRS  = USRS     + FC1(J)*FC4(J)*COS(FC5(J) )
  256 CONTINUE
      IF(IBS.EQ.0)    GO TO 470
      FCBP(M) = ATAN(USBR/USRB)
      USRB = SQRT( USRB**2  +  USBR**2 )
      PMMB(M) = SQRT( PM(M)*PM1(M) )*USRB*URR(M)/RO
      WRITE (4,128)    PMMB(M)
  470 FCP(M) = ATAN(USR(M)/USRS)
      USR(M) = SQRT( USR(M)**2 + USRS**2 )
  251 PMM(M) = SQRT(PM(M)*PM1(M))*USR(M)*URR(M)/RO
      IF(IMC.EQ.1)  GO TO 252
      SUM = 0.0
      DO 253   J = 1,IB2
  253 SUM = SUM +FC2(J)   *FC4(J)
      SUM = USR(M)/SUM
      WRITE (4,128)    SUM
      BL(M) = BL(M)*BK/BJ
      WRITE (4,166)  SUM, USR(M)    ,BL(M)
      DO 254  J= 1,IB2
  254 FC2(J)    = FC2(J)   *SUM
      IF(IPB.EQ.1)  GO TO 560
      IF(IRH.EQ.0)  GO TO 561
  560 CALL NTRAN (23,10)
      NPT=2*NMOD*ILP*ILQ  +(M -1)*IB2*2
      CALL NTRAN (23,6,NPT)
      CALL NTRAN (23,1,IB2,FC2, LL)
      CALL NTRAN (23,1,IB2,FC4, LL)
      CALL NTRAN (23,22)
      IF(IRH.EQ.0)  GO TO 561
      GO TO 252
```

```
501 DO 257    J= 1,182
    FC6(J) = FC6(J) + FC2(J)* SIN(FKC(M) +FCP(M) )
257 FC3(J) = FC3(J) + FC2(J)* COS(FKC(M) +FCP(M) )
252 WRITE (4,166)  PMM(M),FMI(M), USR(M), URR(M) ,PM(M)
    WRITE(4,126)  L, NCOR
155 PVEL(L) = CV
    IF(L.LT.IML)    GO TO 423
    IF(L.GT.IMU)    GO TO 423
    WRITE (4,452)
452 FORMAT (10X,10H PHONE NO., 10X,10H   DEPTH   ,10X,10HAMPLITUDE ,10X
   1,10H   PHASE   )
    DO 459  J= 1, ILP
    PPH = 0.0
    IZSZ = (ZM -ZSD(J))*FLB/ZM  +1.0
    FZSZ = (ZM -ZSD(J))*FLB/ZM  +1.0
    UTA = U(IZSZ) +(FZSZ -IZSZ)*(U(IZSZ +1) -U(IZSZ))
    IF(UTA.LT.-0.00001)   PPH = 180.0
    PMS = ABS(UTA/UMAX)
    WRITE (4,456)  J, ZSD(J), PMS       , PPH
456 FORMAT (14X,I2,17X,F5.1,15X,F5.3,15X,F6.1)
459 CONTINUE
    IDX = 0
    AJP = 0.0
    DO 461  J= 1, ILP
    IZRZ = (ZM -ZSD(J))*FLB/ZM  +1.0
    FZRZ = (ZM -ZSD(J))*FLB/ZM  +1.0
    UTA = U(IZRZ) +(FZRZ -IZRZ)*(U(IZRZ +1) -U(IZRZ))
    UMT = ABS(UTA)
404 DO 400   JA = 1, 700
    IF(FZRZ -IZRZ +JA -1.LT.0.01)   GO TO 400
    IF(IZRZ -JA +1.EQ.1)   GO TO 403
    IF(ABS(U(IZRZ -JA+1)).GE.UMT) UMT = ABS(U(IZRZ -JA +1))
    IF(ABS(U(IZRZ -JA+1)).LT.UMT)        GO TO 405
400 CONTINUE
405 DO 406   JA = 1, 700
    IF(ABS(U(IZRZ +JA)).GE.UMT)   UMT = ABS(U(IZRZ +JA))
    IF(ABS(U(IZRZ +JA)).LT.UMT)   GO TO 403
406 CONTINUE
```

```
403 IF(J.GT.1)  GO TO 407
    IZSZ = (ZM -ZSD(J))*FLE/ZM  +1.0
    FZSZ = (ZM -ZSD(J))*FLE/ZM  +1.0
    UTB = U(IZSZ) +(FZSZ -IZSZ)*(U(IZSZ +1) -U(IZSZ))
    PPH = 360.0*ABS(ASIN(UTB/UMT))/P2
    AJJ = U(IZSZ +1)*UTB
    IF(AJJ.LT.0.0)     PPH = 180.0  -PPH
    IF(AJJ.LT.0.0)     GO TO 402
    IF(ABS(U(IZSZ)).LT.ABS(U(IZSZ +1)))   PPH = 180.0  -PPH
    GO TO 402
407 UTA = UTB
    IZRZ = IZSZ
    IZSZ = (ZM -ZSD(J))*FLE/ZM  +1.0
    FZSZ = (ZM -ZSD(J))*FLE/ZM  +1.0
    UTB = U(IZSZ) +(FZSZ -IZSZ)*(U(IZSZ +1) -U(IZSZ))
    IF(IDX.EQ.1)  GO TO 406
    IF(ABS(U(IZRZ   )).LT.ABS(UTA))    AJP = AJP +90.0
    IF(ABS(U(IZRZ   )).LT.ABS(UTA))    IDX = 1
408 AJJ = U(IZRZ   )*UTA
    IF(AJJ.LT.0.0)               AJP = AJP +90.0
    IF(AJJ.LT.0.0)               IDX = 0
409 IZRZ = IZRZ  -1
    IF(IZRZ.EQ.IZSZ)       GO TO 410
    IF(IDX.EQ.1)    GO TO 411
    IF(ABS(U(IZRZ)).LT.ABS(U(IZRZ +1)))    AJP = AJP +90.0
    IF(ABS(U(IZRZ)).LT.ABS(U(IZRZ +1)))    IDX = 1
411 AJJ = U(IZRZ)*U(IZRZ +1)
    IF(AJJ.LT.0.0)           AJP = AJP  +90.0
    IF(AJJ.LT.0.0)           IDX = 0
    GO TO 409
410 IF(IDX.EQ.1)   GO TO 412
    IF(ABS(UTB).LT.ABS(U(IZRZ+1)))     AJP = AJP  +90.0
    IF(ABS(UTB).LT.ABS(U(IZRZ+1)))     IDX = 1
412 AJJ = UTB*U(IZRZ +1)
    IF(AJJ.LT.0.0)           AJP = AJP +90.0
    IF(AJJ.LT.0.0)           IDX = 0
    PPH = 360.0*ABS(ASIN(UTB/UMT))/P2
```

```
402 PMC = ABS( UTB*SIN(PPH*P2/360.0)/UMAX)
    IF(IDX.EQ.0)    PPH = PPH + AJP
    IF(IDX.EQ.1) PPH = 90.0 -PPH + AJP
    WRITE (4,456)   J,ZSD(J) , PMC       , FPH
461 CONTINUE
423 CONTINUE
154 CONTINUE
123 ZS = ZM - ZS
    ZRC = ZM - ZRC
    IC2 = IB2
    IB2 = IA2
    IREN = IRST + IREN
    IF(IMC.EQ.1)    GO TO 475
    IF(IRM.EQ.0)   GO TO 565
    DO 564  J= 1, 700
    FC1(J) = 0.0
564 FC5(J) = 0.0
    DO 565  M= 1, NMOD
    IF(IRI.EQ.0)   GO TO 566
    DO 567  K= 1, IRI
    IF(IRM(K).EQ.M)  GO TO 565
567 CONTINUE
566 CALL NTRAN (23,10)
    NPT=2*NMOD*ILP*ILG  +(M -1)*IB2*2
    CALL NTRAN (23,6, NPT)
    CALL NTRAN (23,2,IB2,FC2,LL)
    CALL NTRAN (23,22)
    IF(SKIP(M).LT.0.0)  GO TO 556
    NB = (IREN -IRST)/SKIP(M)  +1
    FNB = FLOAT(IREN -IRST)/SKIP(M)
    FEP = SQRT(NB -FNB)/(SQRT(NB -FNB) +SQRT(FNB -NB +1) )
    FEC = SQRT(FNB -NB +1)/(SQRT(NB -FNB) +SQRT(FNB -NB +1) )
    WRITE(4,126)  NB
    WRITE(4,128)  FNB
    TH3 = TH1(M) -2.0*57.2958*THB*(NB -1)
    IF(TH3.LT.0.0)  GO TO 565
    TH3 = 2.0*(NB -1)*THB
    DO 566  J= 1, IB2
    FC1(J) = FC1(J) +          FEP*FC2(J)*SIN(FKC(M) +P2*J*H*FQ*
```

```
   1 SIN(TH3)/C(1) +FCP(M) )
566 FC5(J) = FC5(J) +          FEP*FC2(J)*COS(FKC(M) +P2*J*H*FG*
   1 SIN(TH3)/C(1) +FCP(M) )
    TH3 = TH1(M) -2.0*57.2958*THB*NB
    IF(TH3.LT.0.0)  GO TO 565
    TH3 = 2.0*NB*THB
    DO 569 J= 1, IB2
    FC1(J) = FC1(J) +             FEC*FC2(J)*SIN(FKC(M) +P2*J*H*FQ*
   1 SIN(TH3)/C(1) +FCP(M) )
569 FC5(J) = FC5(J) +             FEC*FC2(J)*COS(FKC(M) +P2*J*H*FQ*
   1 SIN(TH3)/C(1) +FCP(M) )
    GO TO 565
556 DO 570 J= 1, IB2
    FC1(J) = FC1(J) + FC2(J)*SIN(FKC(M) +FCP(M) )
570 FC5(J) = FC5(J) + FC2(J)*COS(FKC(M) +FCP(M) )
565 CONTINUE
    DO 571  J= 1, IB2
    FCS = FC1(J)/FC5(J)
    FC1(J) = SQRT(FC1(J)**2 + FC5(J)**2 )
571 FC5(J) = ATAN(FCS)
    DO 573  IR = IRST, IREN, IRIC
    DO 572  J= 1,700
    FC3(J) = 0.0
572 FC6(J) = 0.0
    RS = IR -IRST
    IREC = (IR -IRST)/IRIC  +1
    DO  574  M= 1, NMOD
    IF(IR1.EQ.0)  GO TO 575
    DO 576  K= 1, IR1
    IF(IRM(K).EQ.M)  GO TO 574
576 CONTINUE
575 CALL NTRAN (23,10)
    NPT=2*NMOD*ILP*ILG + (M -1)*IB2*2
    CALL NTRAN (23,6,NPT)
    CALL NTRAN (23,2,IB2,FC2,LL)
    CALL NTRAN (23,22)
    IF(SKIP(M).LT.0.0)  GO TO 577
    NB = (IR -IRST)/SKIP(M)  +1
```

```
      FNB = FLOAT(IR -IRST)/SKIP(M)
      FEP = SQRT(NB -FNB)/(SQRT(NB -FNB) +SQRT(FNB -NB +1) )
      FEC = SQRT(FNB -NB +1)/(SQRT(NB -FNB) +SQRT(FNB -NB +1) )
      WRITE (4,126)  NB
      TH3 = TH1(M) -2.0*57.2958*THB*(NB -1)
      IF(TH3.LT.0.0)  GO TO 574
      TH3 = 2.0*(NB -1)*THB
      DO 578  J= 1, IB2
      FC3(J) = FC3(J) +           FEP*FC2(J)*SIN(FMR(M)*RS +P2*J*H*FQ*
     1 SIN(TH3)/C(1) +FCP(M) )
  578 FC6(J) = FC6(J) +           FEP*FC2(J)*COS(FMR(M)*RS +P2*J*H*FQ*
     1 SIN(TH3)/C(1) +FCP(M) )
      TH3 = TH1(M) -2.0*57.2958*THB*NB
      IF(TH3.LT.0.0)  GO TO 574
      TH3 = 2.0*NB*THB
      DO 579  J= 1, IB2
      FC3(J) = FC3(J) +               FEC*FC2(J)*SIN(FMR(M)*RS +P2*J*H*
     1 FQ*SIN(TH3)/C(1) +FCP(M) )
  579 FC6(J) = FC6(J) +               FEC*FC2(J)*COS(FMR(M)*RS +P2*J*H*
     1 FQ*SIN(TH3)/C(1) +FCP(M) )
      GO TO 574
  577 DO 580  J= 1, IB2
      FC3(J) = FC3(J) + FC2(J)*SIN(FMR(M)*RS +FCP(M) )
  580 FC6(J) = FC6(J) + FC2(J)*COS(FMR(M)*RS +FCP(M) )
  574 CONTINUE
      DO 581  J= 1, IB2
      FCS = FC3(J)/FC6(J)
      FC3(J) = SQRT(FC3(J)**2 + FC6(J)**2 )
  581 FC6(J) = ATAN(FCS)
      CALL NTRAN (23,10)
      NPT=2*NMOD*ILP*ILQ + NMOD*IB2*2 + (IREO -1)*IB2*2
      CALL NTRAN (23,6,NPT)
      CALL NTRAN (23,1,IB2,FC3,LL)
      CALL NTRAN (23,1,IB2,FC6,LL)
      CALL NTRAN (23,22)
  573 CONTINUE
      IF(IBS.EQ.0)  GO TO 475
      IF(IRH.EQ.0)  GO TO 475
      IRMX = 0
```

```
      DO 583  IR = IRST, IREN, IRIC
      DO 582  J= 1,700
      FC3(J) = 0.0
  582 FC6(J) = 0.0
      IREO = (IR -IRST)/IRIC  +1
      IRMY =0
      DO 584  M= 1, NMOD
      IF(IRI.EQ.0)  GO TO 585
      DO 586  K= 1, IRI
      IF(IRM(K).EQ.M)  GO TO 584
  586 CONTINUE
  585 IF(SKIP(M).LT.0.0)  GO TO 584
      DDT(M) = DDS(M) -FLOAT(IREN -IR)*DD(M)
      IF(IR.NE.IRST)  GO TO 587
      DO 586  IB= 1, 1000
      IRB(M) = SKIP(M)*(IB -1)
      IF(IRB(M).GT.IREN -IRST)  GO TO 584
      TH3 = TH1(M) -2.0*57.2958*IB*THB
      WRITE (4,128)   TH3
      IF(TH3.GT.0.0)  GO TO 588
      GO TO 587
  588 CONTINUE
  587 IF(FLOAT(IREN -IRST -IRB(M)).LT.SKIP(M))  GO TO 589
      ISL = SKIP(M)
      IF(IR -IRST.GT.IRB(M))  ISL = ISL -(IR -IRST -IRB(M) )
      IF(ISL.LT.0)  GO TO 584
      GO TO 590
  589 ISL = IREN  -IRST -IRB(M)
      IF(IR -IRST.GT.IRB(M))  ISL = ISL -(IR -IRST -IRB(M) )
  590 ISD = IRB(M) -IR +IRST
      IF(ISD.LT.0)  ISD = 0
      IB3 = ISL/IRIB +1
      CALL NTRAN (23,10)
      NPT=2*NMOD*ILP*ILQ + (M -1)*IB2*2
      CALL NTRAN (23,6,NPT)
      CALL NTRAN (23,2,IB2,FC2, LL)
      CALL NTRAN (23,22)
      IF(IREO.GT.IRMX)  IRMX = IREO
      IF(IREO.GT.IRMY)  IRMY = IREO
```

```
      DO 591  JJ= 1, IB3
      IF(JJ.EQ.IB3)  GO TO 592
      FRS = 2.0*ISD +2.0*IRIB*(JJ -0.5)
      DDR = DDT(M) +(DD(M) +A/3000.0)*FRS
      DDR = 10.0**(-DDR/20.0)
      NB = FRS/SKIP(M)  +1
      FNB = FRS/SKIP(M)
      FEP = SQRT(NB -FNB)/(SQRT(NB -FNB) +SQRT(FNB -NB +1) )
      FEC = SQRT(FNB -NB +1)/(SQRT(NB -FNB) +SQRT(FNB -NB +1) )
      WRITE (4,126)  NB
      TH3 = 2.0*(NB -1)*THB
      FRP = FMR(M)*FRS
      DO 593  J= 1, IB2
      FC3(J) = FC3(J) +           FEP*SQRT(FLOAT(IRIB)/SKIP(M) )*FC2(J)*
     1 DDR*SIN(FRP       +P2*J*H*FQ*SIN(TH3)/C(1) +FCP(M)   )
     1 /SQRT(3.0*(IR +FRS) )
  593 FC6(J) = FC6(J) +           FEP*SQRT(FLOAT(IRIB)/SKIP(M) )*FC2(J)*
     1 DDR*COS(FRP       +P2*J*H*FQ*SIN(TH3)/C(1) +FCP(M) )
     1 /SQRT(3.0*(IR +FRS) )
      TH3 = 2.0*NB*THB
      DO 594  J= 1, IB2
      FC3(J) =FC3(J) +            FEC*SQRT(FLOAT(IRIB)/SKIP(M))*FC2(J)*
     1 DDR*SIN(FRP       +P2*J*H*FQ*SIN(TH3)/C(1) +FCP(M) )
     1 /SQRT(3.0*(IR +FRS) )
  594 FC6(J) =FC6(J) +            FEC*SQRT(FLOAT(IRIB)/SKIP(M))*FC2(J)*
     1 DDR*COS(FRP       +P2*J*H*FQ*SIN(TH3)/C(1) +FCP(M) )
     1 /SQRT(3.0*(IR +FRS) )
      GO TO 591
  592 ISK = ISL -(IB3 -1)*IRIB
      FRS = 2.0*ISD +2.0*IRIB*(JJ-1.0) +ISK
      DDR = DDT(M) +(DD(M) +A/3000.0)*FRS
      DDR = 10.0**(-DDR/20.0)
      NB = FRS/SKIP(M)  +1
      FNB = FRS/SKIP(M)
      FEP = SQRT(NB -FNB)/(SQRT(NB -FNB) +SQRT(FNB -NB +1) )
      FEC = SQRT(FNB -NB +1)/(SQRT(NB -FNB) +SQRT(FNB -NB +1) )
      WRITE (4,126)  NB
      TH3 = 2.0*(NB -1)*THB
      FRP = FMR(M)*FRS     +FCP(M)
```

```
      DO 595  J= 1, IB2
      FC3(J) = FC3(J) +            FEP*SQRT(FLOAT(ISK )/SKIP(M) )*FC2(J)*
     1 DDR*SIN(FRP      +P2*J*H*FQ*SIN(TH3)/C(1) +FCP(M) )
     1 /SQRT(3.0*(IR +FRS) )
  595 FC6(J) = FC6(J) +            FEP*SQRT(FLOAT(ISK )/SKIP(M) )*FC2(J)*
     1 DDR*COS(FRP      +P2*J*H*FQ*SIN(TH3)/C(1) )/SQRT(3.0*(IR +FRS) )
     1 /SQRT(3.0*(IR +FRS) )
      TH3 = 2.0*NB*THB
      DO 596  J= 1, IB2
      FC3(J) =FC3(J) +              FEC*SQRT(FLOAT(ISK )/SKIP(M))*FC2(J)*
     1 DDR*SIN(FRP      +P2*J*H*FG*SIN(TH3)/C(1) +FCP(M) )
     1 /SQRT(3.0*(IR +FRS) )
  596 FC6(J) =FC6(J) +              FEC*SQRT(FLOAT(ISK )/SKIP(M))*FC2(J)*
     1 DDR*COS(FRP      +P2*J*H*FQ*SIN(TH3)/C(1) +FCP(M) )
     1 /SQRT(3.0*(IR +FRS) )
  591 CONTINUE
  584 CONTINUE
      IF(IRMY.EQ.0)  GO TO 597
      DO 598  J= 1, IB2
      FCS = FC3(J)/FC6(J)
      FC3(J) = SQRT(FC3(J)**2 + FC6(J)**2 )
  598 FC6(J) = ATAN(FCS)
      CALL NTRAN (23,10)
      NPT=2*NMOD*ILP*ILQ  +NMOD*IB2*2  +( (IREN -IRST)/IRIC +1)*IB2*2
     1 +(IREO -1)*IB2*2
      CALL NTRAN (23,6, NPT)
      CALL NTRAN(23,1,IB2,FC3,LL)
      CALL NTRAN(23,1,IB2,FC6,LL)
      CALL NTRAN (23,22)
  583 CONTINUE
  597 WRITE(4,126)   IRMX
      GO TO 475
  563 CONTINUE
      DO  259   J = 1, IB2
      FCS = FC6(J)/FC3(J)
      FC1(J) = SQRT( FC3(J)**2 + FC6(J)**2 )
  259 FC5(J) = ATAN(FCS)
      DO 260  J = 1, 700
      FC6(J) = C.0
```

```
260 FCS(J) = 0.0
475 CONTINUE
    IF(NA.EQ.0)    NDPA = NDPZ +1
    IF(NA.NE.0)    NDPA = ILQ
    DO 124  JD = 1, NDPA
    DO 124 IP=  1,NPS
    READ (3,126)  NMS  ,IPV
    READ (3,126)  (MDS(J),  J=1,NMS)
126 FORMAT (6I10)
198 DO 131 IR = IRST, IREN, IRIC
    IREO = (IR -IRST)/IRIC  +1
    PCTB(IREO) = 0.0
    PSTB(IREO) = 0.0
    PCT(IREO)  =  0.0
131 PST(IREO)  =  0.0
    IF (NMF.EQ.1)  GO TO 111
    IF(IPL.GT.0)     GO TO 300
    DO 127  IQ= 1,NMS
    M=  MDS(IQ)
    IF(NA.GT.0)   GO TO 312
    IF(JD.LT.2)         GO TO 312
    NPT = (M -1)*NDPZ
                     CALL NTRAN (23,10)
               CALL NTRAN (23,6,NPT)
               CALL NTRAN (23,2,NDPZ,ZSR,LL)
    CALL NTRAN (23,22)
    WRITE (4,128)   (ZSR(J) , J= 1, NDPZ)
312 DO 129 IR = IRST, IREN, IRIC
    IF (L.EQ.NCOR)RI= FLOAT(IRST)
    GKB(M) = FKB(M) - FLOAT(IREN - IR)*FMR(M)
    DDT(M) = LDS(M) -FLOAT(IREN -IR)*LD(M)
    IF  (GKB(M).LE.0.0)  GO TO 214
    GKB(M) = GKB(M)/ FLOAT(IR)
214 RR  =  IR
    RS = IR -IRST
    IF(L.EQ.NCOR)GKB(M)= FMR(M)
    GMR(M) = GKB(M) + ERS(M)/RR
    ER(M) = FMR(M) - GKB(M)
    IF(IREN -IR.GE.IRIC)  WGT = IRIC
```

```
      IF(IREN -IR.LT.IRIC)  WGT = IREN -IR
      IF(IR.GT.IRST)   GO TO 416
      IF(JD.GT.1)       ERS(M) = ERSS(M)
      IF(JD.GT.1)       GO TO 416
      IF(IP.GT.1)       ERS(M) = ERSS(M)
      IF(IP.GT.1)       GO TO 416
      ERSS(M) = ERS(M)
  416 ERS(M) = ERS(M) + ER(M)*WGT
      IREO = (IR -IRST)/IRIC  +1
      IF(IMC.NE.0)  GO TO 555
      GMR(M) = FMR(M)*RS/RR
      IF(L.EQ.1)       GMR(M) = GMR(M) +FMR(M)*IRST/RR
  555 IF(IRH.NE.0)   GMR(M) = 0.0
      IF(IRH.EQ.0)   GO TO 599
      CALL NTRAN (23,10)
      NPT=2*NMOD*ILP*ILG +NMOD*IB2*2 +(IREO -1)*IB2*2
      CALL NTRAN (23,6, NPT)
      CALL NTRAN (23,2,IB2,FC3, LL)
      CALL NTRAN (23,2,IB2,FC6, LL)
      CALL NTRAN (23,22)
      IF(IREO.GT.1)  GO TO 559
      CALL NTRAN (23,10)
      NPT=2*NMOD*ILP*ILG +(M -1)*IB2*2 +IB2
      CALL NTRAN (23,6, NPT)
      CALL NTRAN (23,2,IB2,FC4, LL)
      CALL NTRAN (23,22)
  559 CONTINUE
      USR(M) = 0.0
      USRS = 0.0
      DO 558  J= 1, IB2
      USR(M) = USR(M) +FC3(J)*FC4(J)*SIN(FC6(J) )
  558 USRS = USRS +FC3(J)*FC4(J)*COS(FC6(J) )
      FCF(M) = ATAN(USR(M)/USRS)
      USR(M) = SQRT(USR(M)**2 + USRS**2  )
      PMM(M) = SQRT(PM(M)*PMI(M) )*USR(M)*URR(M)/RO
      IF(IBS.EQ.0)  GO TO 599
      IF(IRH.EQ.0)  GO TO 599
      IF(IREO.GT.IRMX)  PMMB(M) = 0.0
```

```
      IF(IREO.GT.IRMX)  GO TO 599
      CALL NTRAN (23,10)
      NPT=2*NMOD*ILP*ILQ +NMOD*IB2*2 +((IREN -IRST)/IRIC +1)*IB2*2
     1 +(IREO -1)*IB2*2
      CALL NTRAN (23,6,NPT)
      CALL NTRAN (23,2,IB2,FC3, LL)
      CALL NTRAN (23,2,IB2,FC6, LL)
      CALL NTRAN (23,22)
      USBR = 0.0
      USRB = 0.0
      DO 557  J= 1, IB2
      USBR = USBR +FC3(J)*FC4(J)*SIN(FC6(J) )
  557 USRB = USRB +FC3(J)*FC4(J)*COS(FC6(J) )
      FCBP(M) = ATAN(USBR/USRB)
      USRB = SQRT(USRB**2 +USBR**2)
      PMMB(M) = SQRT(PM(M)*PMI(M) )*USRB*URR(M)/RO
  599 CONTINUE
      IF(IPOS.EQ.1)  GO TO 129
      DDT(M) = 10.0**(-DDT(M)/20.0)
      BB(M) = BB(M) +BL(M)*WGT
      IF(L.NE.NCOR)  GO TO 165
      IF(IR.EQ.IRST)  BB(M) = BL(M)*RI
  165 IF(IBL.EQ.0) GO TO 146
      IF(ALOG10(BB(M)).GT.0.0)   DDT(M) = DDT(M)/SQRT(BB(M))
  146 IF(NA.GT.0)   GO TO 323
  490 IF(JD.EQ.1)   GO TO 495
      PST(IREO) = PST(IREO) +  PMM(M)*ZSR(JD -1)*DDT(M)*SIN(GMR(M)*RR
     1  -P2/8.0  +FCP(M) )
      PCT(IREO) = PCT(IREO) +  PMM(M)*ZSR(JD -1)*DDT(M)*COS(GMR(M)*RR
     1  -P2/8.0  +FCP(M) )
      IF(IBS.EQ.0)   GO TO 129
      IF(IREO.GT.1)  GO TO 129
      PSTB(IREO) = PSTB(IREO) + PMMB(M)*ZSR(JD -1)*SIN(GMR(M)*RR-P2/8.0
     1  +FCBP(M) )
      PCTB(IREO) = PCTB(IREO) + PMMB(M)*ZSR(JD -1)*COS(GMR(M)*RR-P2/8.0
     1  +FCBP(M) )
      WRITE (4,128)   PST(IREO) ,PCT(IREO) ,PSTB(IREO) ,PCTB(IREO)
      GO TO 129
```

```
495 PST(IREO) = PST(IREO) +  PMM(M)*DDT(M)*SIN(GMR(M)*RR -P2/8.0
   1 +FCP(M) )
    PCT(IREO) = PCT(IREO) +  PMM(M)*DDT(M)*COS(GMR(M)*RR -P2/8.0
   1 +FCP(M) )
    IF(IBS.EQ.0)   GO TO 129
    IF(IREO.GT.1)  GO TO 129
    PSTB(IREO) = PSTB(IREO) +             PMMB(M)*SIN(GMR(M)*RR-P2/8.0
   1 +FCBP(M) )
    PCTB(IREO) = PCTB(IREO) +             PMMB(M)*COS(GMR(M)*RR-P2/8.0
   1 +FCBP(M) )
    WRITE (4,128)   PST(IREO) ,PCT(IREO) ,PSTB(IREO) ,PCTB(IREO)
    GO TO 129
323 NPT = (M -1)*ILP*ILG
    CALL NTRAN (23,10)
    CALL NTRAN (23,6, NPT)
    DO 324  IC = 1, ILP
    CALL NTRAN (23,2,ILG, ZSR, LL)
    CALL NTRAN (23,22)
494 PST(IREO) = PST(IREO) +PMM(M)*    (SMA(IC)*ZSR(JD))*DDT(M)*
   1 SIN(GMR(M)*RR -SMP(IC) -P2/8.0 +FCP(M) )
    PCT(IREO) = PCT(IREO) +PMM(M)*    (SMA(IC)*ZSR(JD))*DDT(M)*
   1 COS(GMR(M)*RR -SMP(IC) -P2/8.0 +FCP(M) )
    IF(IBS.EQ.0)  GO TO 324
    IF(IREO.GT.0)  GO TO 324
    PSTB(IREO) = PSTB(IREO) + PMMB(M)*(SMA(IC)*ZSR(JD))*
   1 SIN(GMR(M)*RR -SMP(IC) -P2/8.0 +FCBP(M) )
    PCTB(IREO) = PCTB(IREO) + PMMB(M)*(SMA(IC)*ZSR(JD))*
   1 COS(GMR(M)*RR -SMP(IC) -P2/8.0 +FCBP(M) )
324 CONTINUE
129 CONTINUE
127 CONTINUE
    GO TO 301
300 DO 302  J = 1,700
302 PPT(J) = 0.0
    DO 329 IR = IRST, IREN, IRIC
    THM = 0.0
    INF = 0
    TE = 0.0
    TS = 1000000.0
```

```
      DO 222 J = 1,NMS
      M = MDS(J)
      IF(IREN -IR.GE.IRIC)  WGT = IRIC
      IF(IREN -IR.LT.IRIC)  WGT = IREN -IR
      TM(M) = FLOAT(IR)/GVEL(M)
      IF(TM(M).LT.TS)          TS = TM(M)
      IF(TM(M).GT.TE)  TE = TM(M)
  222 CONTINUE
      DO 327  IG= 1,NMS
      M=  MDS(IG)
      IF  (L.EQ.NCOR)RI= FLOAT(IRST)
      GKB(M) = FKB(M) - FLOAT(IREN - IR)*FMR(M)
      DDT(M) = DDS(M) -FLOAT(IREN -IR)*DD(M)
      IF  (GKB(M).LE.0.0)  GO TO 314
      GKB(M) = GKB(M)/ FLOAT(IR)
  314 RR  =  IR
      IF(L.EQ.NCOR)GKB(M)= FMR(M)
      GMR(M) = GKB(M) + ERS(M)/RR
      ER(M) = FMR(M) - GKB(M)
      IF(IREN -IR.GE.IRIC)  WGT = IRIC
      IF(IREN -IR.LT.IRIC)  WGT = IREN -IR
      IF(IR.GT.IRST)   GO TO 216
      IF(JD.GT.1)        ERS(M) = ERSS(M)
      IF(JD.GT.1)        GO TO 216
      IF(IP.GT.1)        ERS(M) = ERSS(M)
      IF(IP.GT.1)        GO TO 216
      ERSS(M) = ERS(M)
  216 ERS(M) = ERS(M) + ER(M)*WGT
      IREC = (IR -IRST)/IRIC  +1
      IF(IMC.NE.0)  GO TO 554
      GMR(M) = FMR(M)*RS/RR
      IF(L.EQ.1) GMR(M) = GMR(M) +FMR(M)*IRST/RR
  554 IF(IPOS.EQ.1)  GO TO 327
      DDT(M) = 10.0**(-DDT(M)/20.0)
      BB(M) = BB(M) +BL(M)*WGT
      IF(L.NE.NCOR)  GO TO 365
      IF(IR.EQ.IRST)  BB(M) = BL(M)*RI
  365 IF(IBL.EQ.0) GO TO 327
      IF(ALOG10(BB(M)).GT.0.0)   DDT(M) = DDT(M)/SQRT(BB(M))
```

```
327 CONTINUE
    IF(IPL.EQ.0)   GO TO 329
310 DO 215   J = 1,4096
215 TH(J) = 0.0
    DO 317  IQ = 1,NMS
    M = MDS(IQ)
    IF(NA.GT.0)   GO TO 313
    IF(JD.LT.2)        GO TO 313
    NPT = (M -1)*NDPZ
                        CALL NTRAN (23,10)
                  CALL NTRAN (23,6,NPT)
                  CALL NTRAN (23,2,NDPZ,ZSR,LL)
    CALL NTRAN (23,22)
313 IRP = (IPL *4096)/IDL
    DO 217 J = 1,IRP
    IT = 1000.0*(TM(M) -TS)*4095./FLOAT(IDL)  +J
    IF(IT.GT.4096)   GO TO 317
    IF(IT.LT.1)   GO TO 217
    IF(NA.GT.0)   GO TO 326
    IF(JD.GT.1)  TH(IT) = TH(IT) +PMM(M)*ZSR(JD-1)*DDT(M)*COS(GMR(M)*
   1 RR-P2/8.0 -P2*FQ*(TS +FLOAT(IT -1)*FLOAT(IDL)/(4095.*1000.0)))
    IF(JD.GT.1)  GO TO 311
    TH(IT) = TH(IT) +PMM(M)*DDT(M)*COS(GMR(M)*RR -P2/8.0 -P2*FQ*(TS +
   1  FLOAT(IT -1)*FLOAT(IDL)/(4095.*1000.0)))
    GO TO 311
326 NPT = (M -1)*ILP*ILQ
    CALL NTRAN (23,10)
    CALL NTRAN (23,6, NPT)
    DO 335   IC = 1, ILP
    CALL NTRAN (23,2, ILQ, ZSR, LL)
    CALL NTRAN (23,22)
335 TH(IT)= TH(IT) +PMM(M)*    (SMA(IC)*ZSR(JD))*DDT(M)*COS(GMR(M)*RR-
   1 P2/8.0 -SMP(IC) -P2*FQ*(TS +FLOAT(IT -1)*FLOAT(IDL)/(4095.0*
   1 1000.0)))
311 IF(ABS(TH(IT)).GT.THM)   THM = ABS(TH(IT))
217 CONTINUE
317 CONTINUE
    DO 307  J = 1,NRT
    IF(IRT(J).GT.IR)  GO TO 307
    IF(IRT(J).GT.IR -IRIC)  GO TO 308
```

```
307 CONTINUE
    GO TO 360
308 CALL NTRAN (8,1,4096,TH,LL)
    CALL NTRAN ( 8,22)
    INF = INF +1
    WRITE(4,240)    THM, INF
360 CONTINUE
240 FORMAT   (F10.5, I10)
    DO 216   J = 1,4096
216 PPT(IREO) = PPT(IREO) +TH(J)*TH(J)
    TS = TS +FLOAT(IDL)/1000.0
    IF(TS.GT.TE +FLOAT(IPL)/1000.0)   GO TO 329
    GO TO 310
329 CONTINUE
301 IF(IPOS.EQ.1)  GO TO 113
    DO 132 IR = IRST, IREN, IRIC
    RR  =  IR
    IREO = (IR -IRST)/IRIC  +1
    IF(IPL.EQ.0)  GO TO 219
    PPT(IREO) = SQRT(PPT(IREO))*RO/(SQRT(RR/3.0)*SQRT
   1 (IPL*4096.0/(2.0*IDL)))
    GO TO 221
219 PPT(IREO) = SQRT(PCT(IREO)*PCT(IREO) + PST(IREO)*PST(IREO))*RO/
   1SQRT(RR/3.0)
    IF(IBS.EQ.0)   GO TO 221
    IF(IRM.EQ.0)  PPTB(IREO) = SQRT(PCTB(IREO)**2 +PSTB(IREO)**2)*RO
   1  /SQRT(RR/3.0)
    IF(IRM.NE.0)  PPTB(IREO) = SQRT(PCTB(IREO)**2 +PSTB(IREO)**2)*RO
    IF(PPTB(IREO).GT.10.0**-10)      GO TO 503
    PPTB(IREO) = FMJ +10.0*FMK
    GO TO 492
503 PPTB(IREO) = -20.0*ALOG10(PPTB(IREO) )  +A*IR/3000.0  +3.0
492 IF(PPT(IREO).GT.10.0**-10)  GO TO 221
    PPT(IREO)  = FMJ +10.0*FMK
    GO TO 132
221 PPT(IREO        ) =-20.0*ALOG10(PPT(IREO))   +A*IR/3000.0
    PPT(IREO) = PPT(IREO)  +3.0
```

```
132 RP(IREO        )  = FLOAT(IR)/6076.0
    WRITE (4,128) (RP(J) , PPT(J), J = 1, IREC )
    IF(IBS.EQ.0)  GO TO 496
    IF(IRH.NE.0)    WRITE (4,128)  (RP(J) , PPTB(J), J= 1, IREO )
    IF(IRH.EQ.0)  WRITE (4,128)  RP(1) , PPTB(1)
496 CONTINUE
128 FORMAT (10F10.5)
    WRITE(4,126)  IREO
    CALL NTRAN (7,1,IREO, RP, LL)
    CALL NTRAN ( 7,22)
    CALL NTRAN (7,1,IREO,PPT, LL)
    CALL NTRAN ( 7,22)
113 RP(IREO +1)  =  0.0
    RP(IREO +2)  =  FMI
    PPT(IREO +1)  =  FMJ
    PPT(IREO +2)  =  FMK
    IF(IPOS.EQ.1)  GO TO 114
    IF(IPV.EQ.1)   NPZ = NPZ -1
    IF(IPV.EQ.1)  GO TO 444
    IF(JD.NE.1)  GO TO 446
    IF(IP.NE.1)  GO TO 446
    GO TO 444
446 CALL PLOT (20.0,0.0,-3)
444 IF(IPV.EQ.0)   GO TO 445
    IF(IPVC.EQ.0)  GO TO 445
    CALL PLOT (-20.0,0.0,-3)
    IPVC = 0
445 CALL LINE (RP,PPT,IREO,1,1,85)
    IF(IBS.EQ.0)  GO TO 493
    IF(IRH.EQ.0) GO TO 497
    PPTB(IREO +1)  = FMJ
    PPTB(IREO +2)  = FMK
    CALL LINE (RP,PPTB,IREC,1,1,24)
    GO TO 493
497 PPTB(2) = FMJ
    PPTB(3) = FMK
    RP(2) = 0.0
    RP(3) = FMI
    CALL LINE  (RP,PPTB, 1,1,1,24)
```

```
493 IF(IPV.EQ.0) GO TO 114
    IF(IPV.EG.0)  GO TO 114
    GO TO 124
114 IF (L.NE.NCOR) GO TO 199
    CALL AXIS(0.0,0.0,13HRANGL (MILES),13,20.0,0.0,RP(IREO +1),
   1 RF(IREO +2),10.0)
    CALL AXIS(0.0,0.0,21HPROPAGATION LOSS (DB),21,10.0,90.0,
   1 PPT(IREO +1),PPT(IREC +2),10.0)
    CALL SYMBOL(8.9,9.50,0.14,16HPROPAGATION LOSS,0.0,16)
    CALL SYMBOL(9.7,9.25,0.10,6HVERSUS,0.0,6)
    CALL SYMBOL(9.7,9.00,0.14,5HRANGE,0.0,5)
    CALL SYMBOL(8.9,8.75,0.14,18HFREQUENCY        HZ,0.0,18)
    CALL NUMBER(10.3,8.75,.14,FG,0.0,-1)
    CALL SYMBOL(8.6,8.50,0.14,20HSOURCE DEPTH (FT)    ,0.0,20)
    CALL NUMBER(11.0,8.50,0.14,ZS,0.0,-1)
    CALL SYMBOL(8.6,8.25,0.14,22HRECEIVER DEPTH (FT)    ,0.0,22)
    CALL NUMBER(11.1,8.25,0.14,ZRC,0.0,-1)
    CALL SYMBOL (8.6,8.0,0.14,5HMODES,0.0,5)
    DO 187  J=  1,NMS
    FDS(J)  =  MDS(J)     +ISM
187 CALL NUMBER (9.1  +0.5*J,8.0,0.14,FDS(J),0.0,-1)
199 FINC  =  IP*25
    CALL PLOT (20.0,0.0,-3)
    IPVC  =1
124 CONTINUE
    CALL NTRAN (7,9)
    NPZ = NPS +NUPS +NPZ
    DO 220  J = 1,NPZ
220 CALL PLOT (-20.0,0.0,-3)
```

```
111 CONTINUE
    FINC = 20.0*NPZ
    CALL PLOT (FINC,0.0,-3)
    CALL NTRAN (7,10)
    CALL NTRAN (8,9)
    CALL NTRAN (8,10)
    END
    SUBROUTINE COUNT
    DIMENSION U( 700)
    DOUBLE PRECISION U
    COMMON MS,JM,U
    COMMON JMS
    MS=0
    J=1
    IS=1
    JMS=1
  5 IF (U(J)) 1,2,3
  3 IS1=IS
    IS=1
  7 IF (IS-IS1) 4,2,4
  4 MS=MS+1
    JMS=J
  2 J=J+1
    IF (J-JM-1) 5,6,5
  1 IS1=IS
    IS=0
    GO TO 7
  6 RETURN
    END
```