

AD-A009 139

A MODEL FOR TARGET SCINTILLATION

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Aerospace Corporation

Prepared for:

Space and Missile Systems Organization

20 December 1974

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REPORT DOCUMENTATION PAGE		READ INSTRUCTIONS BEFORE COMPLETING FORM
1. REPORT NUMBER SAMSO-TR-75-107	2. GOVT ACCESSION NO.	3. RECIPIENT'S CATALOG NUMBER AD-A009139
4. TITLE (and Subtitle) A MODEL FOR TARGET SCINTILLATION		5. TYPE OF REPORT & PERIOD COVERED Interim Report
7. AUTHOR(s) D. J. Reifer		6. PERFORMING ORG. REPORT NUMBER TR-0075(5112)-1
9. PERFORMING ORGANIZATION NAME AND ADDRESS The Aerospace Corporation El Segundo, California 90245		8. CONTRACT OR GRANT NUMBER(s) F04701-74-C-0075
11. CONTROLLING OFFICE NAME AND ADDRESS Space and Missile Systems Organization Air Force Systems Command Los Angeles Air Force Station, CA 90045		10. PROGRAM ELEMENT, PROJECT, TASK AREA & WORK UNIT NUMBERS
14. MONITORING AGENCY NAME & ADDRESS (if different from Controlling Office)		12. REPORT DATE 20 December 1974
		13. NUMBER OF PAGES 22 24
		15. SECURITY CLASS. (of this report) UNCLASSIFIED
		15a. DECLASSIFICATION/DOWNGRADING SCHEDULE
16. DISTRIBUTION STATEMENT (of this Report) Approved for public release; distribution unlimited.		
17. DISTRIBUTION STATEMENT (of the abstract entered in Block 20, if different from Report)		
18. SUPPLEMENTARY NOTES Reproduced by NATIONAL TECHNICAL INFORMATION SERVICE US Department of Commerce Springfield, VA 22151		DDC DECLASSIFIED MAY 5 1975 RECEIVED D
19. KEY WORDS (Continue on reverse side if necessary and identify by block number) radar scintillation target scintillation target scintillation simulation target scintillation algorithm target strength estimates		
20. ABSTRACT (Continue on reverse side if necessary and identify by block number) This report discusses the implementation of an algorithm used to simulate target scintillation. The algorithm takes uniform random noise and modifies it using a simulated low-pass filter to produce estimates of target strength. The actual implementation statistics are compared with the theoretical distribution to ascertain the usefulness of the approach.		

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1. INTRODUCTION

Target scintillation is the variation of angular location of the apparent source of the echo signal and is dependent upon relative phase and amplitude of the component echo signals and their apparent angular locations. The motion of the target causes the apparent source to wander back and forth about the physical center of the target. This wander is called angle noise or scintillation [Ref. 1]. Since the noise is random, it is usually calculated using experimental observation data. Few articles have appeared to propose models of the phenomenon.

Recently, we implemented a simulation of target scintillation. The objective of this report is to document this implementation and to define its theoretical basis. These objectives are realized by answering the following questions:

1. What algorithm was actually implemented.
2. What are the statistical characteristics of simulated scintillation?
3. What are the theoretical statistical characteristics of our model?

2. ALGORITHM

The scintillation implementation is based on the model illustrated in Figure 1. Random noise is generated by the algorithm derived in Appendix A. The noise is passed through a filter to simulate the correlation of scintillation. The filtered noise then exponentially modifies the nominal target strength which is our output. This arrangement allows for simulation of virtually all types of scintillation because it allows control of both the statistics of the noise and the characteristics of the filter.

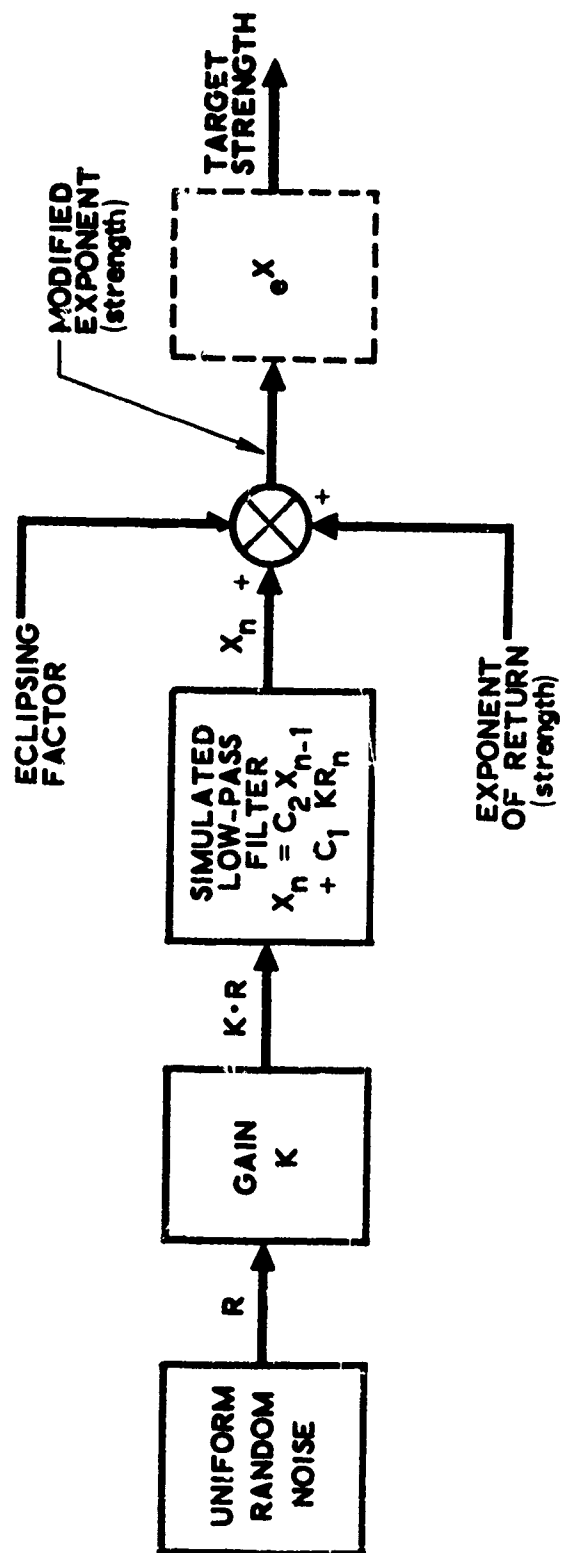


Figure 1. Filter Model

Pseudo-random noise is generated by the algorithm:

$$R_{i(n)} = 65539 R_{i(n-1)} \text{ (modulo } 2^{31}) \quad (1)$$

$$R_{i(0)} = 12345$$

$$R_n = 2(R_{i(n)} - (0.5 \times 2^{31})) \quad (2)$$

This noise, magnified by a scale factor K , is filtered by a digital one-pole filter:

$$X_n = C_2 X_{n-1} + C_1 K R_n \quad (3)$$

where:

$$C_1 = 1 - e^{-\frac{\Delta t}{\tau}} \sim \frac{\Delta t}{\tau}$$

$$C_2 = e^{-\frac{\Delta t}{\tau}} \sim 1 - \frac{\Delta t}{\tau}$$

Thus:

$$X_n = (1 - \frac{\Delta t}{\tau}) \cdot X_{n-1} + \frac{\Delta t}{\tau} \cdot K \cdot R_n \quad (4)$$

The values selected for implementation were

$$\tau = 1.024 \text{ second}$$

$$\Delta t = 8 \text{ milliseconds}$$

$$\Delta t / \tau = 2/256$$

$$K = 128$$

Therefore:

$$X_n = \frac{254}{256} X_{n-1} + R_n \quad (5)$$

The value of both X_n and the eclipsing constant are added to the exponent of the normal target strength. The range for this exponent is limited to 0 through +63.

3. STATISTICS OF THE ALGORITHM

The random numbers generated in equations (1) and (2) have the following uniform distribution with mean of 0 and variance equal to 1/3.

$$p(R_n) = 1/2 \text{ for } -1 \leq R_n < +1$$

where $p(\cdot)$ is the probability of $(\cdot)$.

$$\text{mean} = \mu_R = \int_{-\infty}^{+\infty} R \cdot p(R) dR = 0$$

$$\text{variance} = \sigma_R^2 = \int_{-\infty}^{+\infty} (R - \mu_R)^2 p(R) dR = 1/3$$

After passing through the filter, the mean, variance, and autocorrelation functions of any one sequence $[X_0, X_1, \dots, X_n]$ as derived in Appendix B are as follows:

$$\mu_X = \mu_R = 0 \quad (6)$$

$$\sigma_X^2 = \frac{K^2 C_1^2}{1 - C_2} \sigma_R^2 = \frac{K^2 C_1}{2 + C_1} \sigma_R^2 = \frac{K^2 C_1}{2 + C_1} \cdot \frac{1}{3} \quad (7)$$

$$\rho(k) = \frac{E(X_n, X_{n-k})}{\sigma_n \cdot \sigma_{n-k}} = C_2^k \quad (8)$$

Substituting values:

$$\mu_X = 0$$

$$\sigma_X^2 = 21.4$$

$$p(k) = (0.9921875)^k \quad (9)$$

$$\text{Time} = k \cdot \Delta t = 8 \cdot k \text{ milliseconds}$$

where

k = number of samples

The amplitude distribution of X is Gaussian with the above mean and variance. These perturbations are applied to the exponent of target strength so that, in effect, the logarithm of strength is varied. Applying an inverse log transformation, we find:

$$p(X) = \frac{1}{\sqrt{2\pi} \sigma_X} e^{-\frac{X^2}{2\sigma_X^2}} \quad (10)$$

$$p(S) = \frac{1}{\sqrt{2\pi} \cdot \sigma_X \cdot S} e^{-\frac{(\ln S - \ln \bar{S})^2}{2\sigma_X^2}} \quad (11)$$

for $-\infty < S < +\infty$

where S = target strength

$\bar{S}$ = mean of S or nominal target strength

This is a log-normal distribution whose

$$\text{mean} = \bar{S} e^{\frac{\sigma_X^2}{2}} = \bar{S} e^{\frac{K^2 C_1^2}{1 - C_2^2} \frac{\sigma_R^2}{2}} \quad (12)$$

$$\text{and variance} = \bar{S}^2 \cdot e^{\sigma_X^2} \left(e^{\sigma_X^2} - 1 \right) \quad (13)$$

Note that the average target strength changes from its nominal value $\bar{S}$.

Qualitatively, this can be understood by examining a case where the exponent of target strength is alternately changed by +1 and -1. The average value for the exponent for this case is unchanged. But when one inspects the corresponding strengths, they become 1-1/4 rather than 1.

4. THEORETICAL MODEL

Radar scintillation discussions invariably refer back to Swerling's classic paper [Ref. 2]. For his cases I or II, which apply to aircraft, the distribution of reflected energy is:

$$p(S, \bar{S}) = \frac{S}{\bar{S}} e^{-\frac{S}{\bar{S}}} \quad \text{for } x \geq 0 \quad (14)$$

$$= 0 \quad \text{elsewhere}$$

where S = input signal-to-noise ratio (target cross-section)

$\bar{S}$ = average of S over all fluctuations

We must therefore generate a distribution which, when applied to the logarithm of target strength, will result in Eq. (14). If $p(x)$ is this distribution, then

$$x = \ln S \text{ or } S = e^x$$

$$dx = \frac{dS}{S} = e^{-x} dS$$

$$p(x) = \left. \frac{p(S)}{\left| \frac{dx}{dS} \right|} \right|_{S=e^x} = \frac{\frac{1}{S} e^{-\frac{S}{\bar{S}}}}{1/S} = \frac{S}{\bar{S}} e^{-\frac{S}{\bar{S}}}$$

$$p(x) = \frac{e^x}{e^{\bar{x}}} e^{-\frac{e^x}{e^{\bar{x}}}} \text{ for } -\infty < x < +\infty \quad (15)$$

The mean and variance of this distribution $p(x)$ as derived in Appendix C are:

$$\text{mean} = \mu_x = \bar{x} - C = \ln \bar{S} - C$$

where $C = \text{Euler's constant}$

$$\text{variance} = \sigma_x^2 = \frac{\pi}{6}$$

5. COMPARISON OF MODELS

A plot of the theoretical versus the implemented distribution is included as Figure 2. Figure 3 illustrates the input distribution used to generate the distribution in Figure 2. Figure 4 illustrates the transform used to convert theoretically correct noise from the available Gaussian noise. The algorithm closely realizes theoretical scintillation for values greater than 0.6.

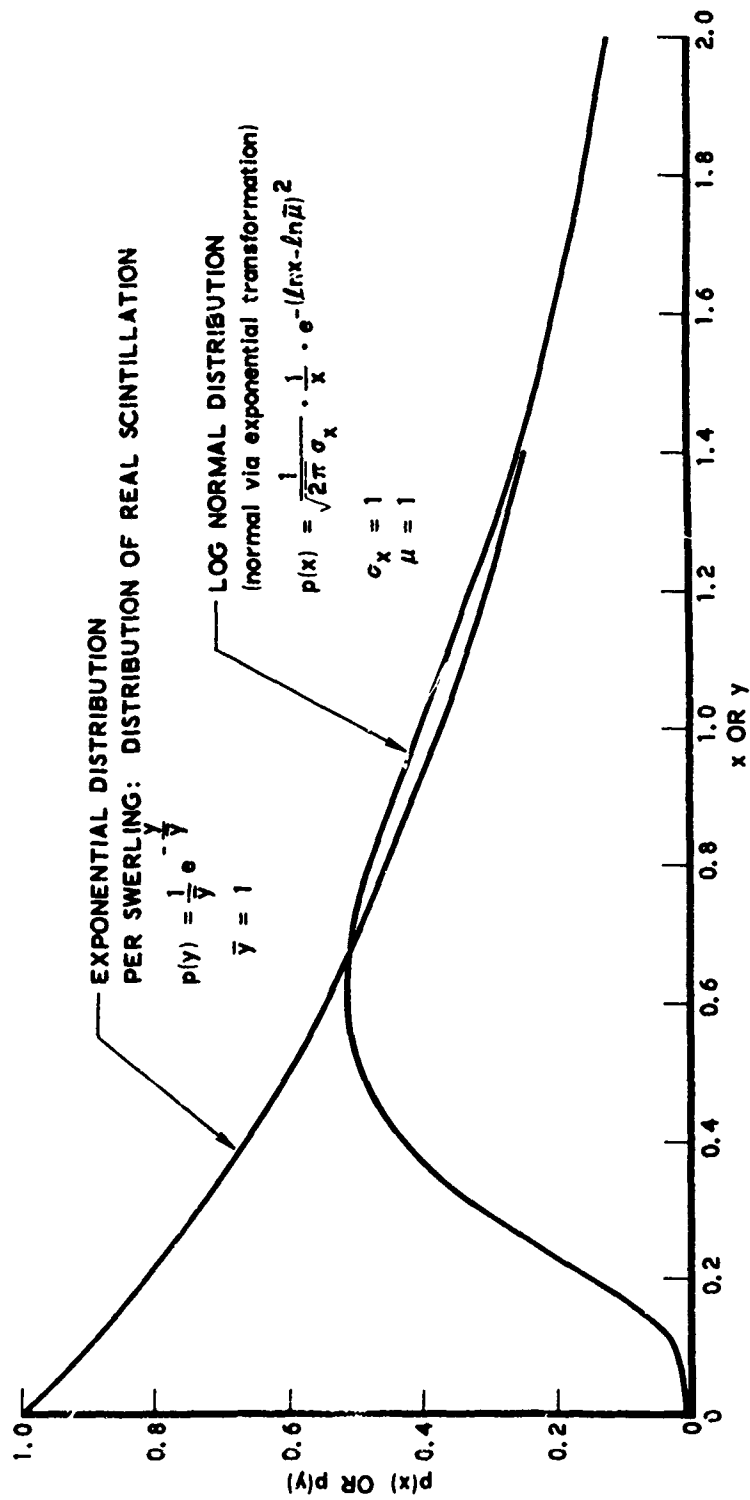


Figure 2. Actual versus Theoretical Distribution of Scintillation

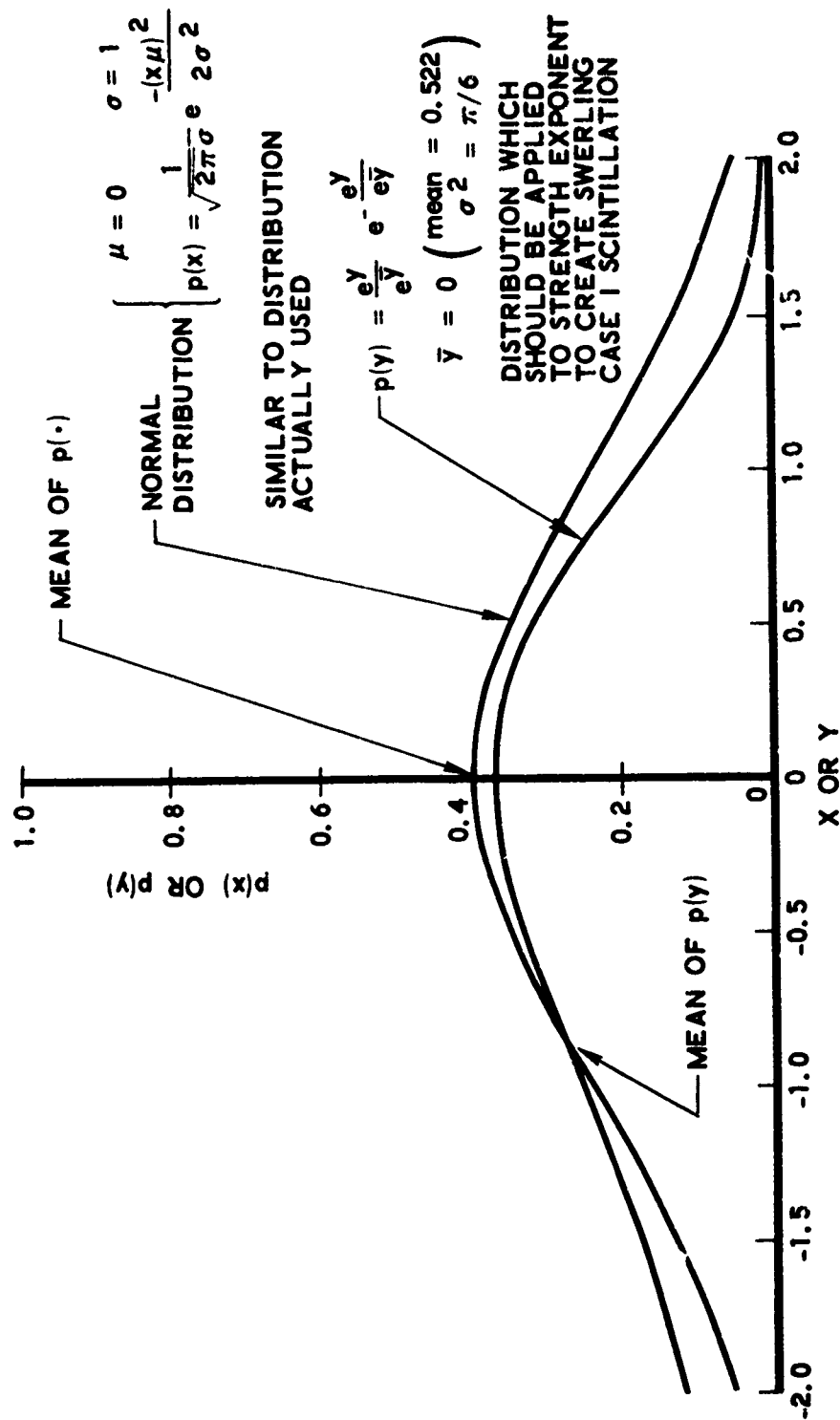


Figure 3. Distributions at Input to Produce Actual or Theoretically Correct Scintillation

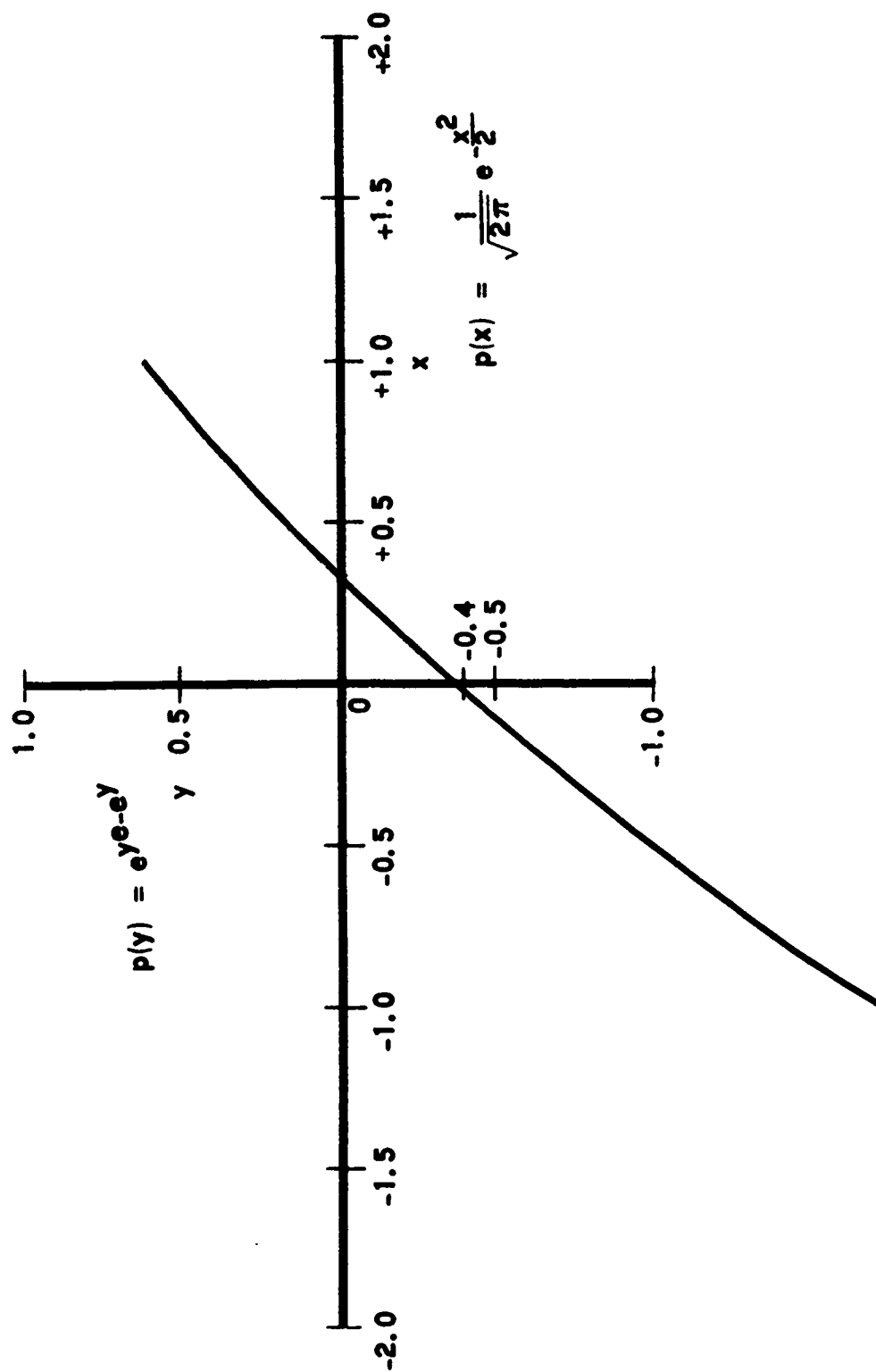


Figure 4. Transformation for Samples from $p(x)$ to Samples from $p(y)$

APPENDIX A

STATISTICS OF NOISE GENERATOR

The basic random number generator is the same one used in the XDS standard FORTRAN Numerical Subroutine Package. It generates pseudo-random numbers uniformly distributed between 1 and positive full scale (X'7FFFFFFF'). Only odd numbers are generated. Treating the generated sequence R_1 as numbers scaled over 2^{31}

$$\begin{aligned} R_1 &= 1 && \text{for } 0 \leq R_1 < +1 \\ &= 0 && \text{elsewhere} \end{aligned} \quad (\text{A1-1})$$

By equation (2), these are converted to a uniform distribution in the region -1 to +1:

$$\begin{aligned} R &= \frac{1}{2} && \text{for } -1 \leq R < +1 \\ &= 0 && \text{elsewhere} \end{aligned} \quad (\text{A1-2})$$

Hence the mean is:

$$\mu_R = \int_{-\infty}^{+\infty} R p(R) dR = \int_{-\infty}^{+\infty} \frac{1}{2} \cdot R dR = \left[\frac{R^2}{4} \right]_{-1}^{+1} = 0 \quad (\text{A1-3})$$

and the variance:

$$\sigma_R^2 = \int_{-\infty}^{+\infty} p(R) (R - \mu_R)^2 dR = \int_{-\infty}^{+\infty} \frac{1}{2} \cdot R^2 dR = \frac{1}{2} \cdot \left[\frac{R^3}{3} \right]_{-1}^{+1} = \frac{1}{3}$$

APPENDIX B

STATISTICS OF SCINTILLATION SEQUENCES

Each sequence of scintillation factors is generated by the recursion relation of Eq.(3). Expanding this equation:

$$\begin{aligned}
 X_{0,j} &= C_1 K R_{0,j} \\
 X_{1,j} &= C_1 K R_{1,j} + C_1 C_2 K R_{0,j} \\
 X_{2,j} &= C_1 K R_{2,j} + C_1 C_2 K R_{1,j} + C_1 C_2^2 K R_{0,j} \\
 X_{3,j} &= C_1 K R_{3,j} + C_1 C_2 K R_{2,j} + C_1 C_2^2 K R_{1,j} + C_1 C_2^3 K R_{0,j} \\
 &\vdots \\
 X_{n,j} &= C_1 K R_{n,j} + C_1 C_2 K R_{n-1,j} + \cdots C_1 C_2^n K R_{0,j}
 \end{aligned}$$

or

$$X_{n,j} = C_1 K \sum_{i=0}^n C_2^{n-i} R_{i,j}$$

The mean of the j^{th} sequence is obtained by summing all terms in the sequence:

$$\begin{aligned}
 \mu_j &= \lim_{n \rightarrow \infty} \frac{1}{n} C_1 K \left[\left(C_2^0 + C_2^1 + C_2^2 + \cdots C_2^n \right) R_{0,j} \right. \\
 &\quad \left. + \left(C_2^0 + C_2^1 + C_2^2 + \cdots C_2^n \right) R_{1,j} \right. \\
 &\quad \left. + \cdots \right] \\
 \mu_j &= \lim_{n \rightarrow \infty} \frac{1}{n} C_1 K \left[C_2^0 + C_2^1 + \cdots C_2^n \right] \left[R_{0,j} + R_{1,j} + R_{2,j} + \cdots R_{n,j} \right]
 \end{aligned}$$

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The mean of all sequences is obtained by summing across all j:

$$\mu = \lim_{\substack{n \rightarrow \infty \\ j \rightarrow \infty}} \frac{1}{n} C_1 K \left[C_2^0 + C_2^1 + C_2^2 + \cdots + C_2^n \right] \left[\frac{1}{j} \sum_{m=0}^j R_{0,m} + \frac{1}{j} \sum_{m=0}^j R_{1,m} + \cdots + \frac{1}{j} \sum_{m=0}^j R_{n,m} \right]$$

But

$$\mu = \lim_{\substack{j \rightarrow \infty \\ n \rightarrow \infty}} \frac{1}{j} \sum_{m=0}^j R_{n,m} = \mu_R \text{ (the mean of the random numbers)}$$

and

$$\lim_{n \rightarrow \infty} C_2^0 + C_2^1 + \cdots + C_2^n = \frac{1}{1 - C_2}$$

Thus

$$\mu_X = \frac{C_1 K}{1 - C_2} \mu_R$$

However, since μ_R is zero,

$$\mu_X = 0$$

Similarly, we can find the variance:

$$\sigma_X^2 = E \left[(X_n - \mu_X)^2 \right] = E(X_n^2) = \lim_{n \rightarrow \infty} \frac{1}{n} \sum_{i=0}^n X_i^2$$

If we expand a typical term, say $X_{3,j}^2$:

$$\begin{aligned}
 X_{3,j}^2 &= \left(C_1 K R_{3,j} + C_1 K C_2 R_{2,j} + C_1 K C_2^2 R_{1,j} + C_1 K C_2^3 R_{0,j} \right)^2 \\
 &= C_1^2 K^2 \left[C_2^6 (R_{0,j}^2) \right. \\
 &\quad + C_2^5 (R_{1,j} R_{0,j} + R_{0,j} R_{1,j}) \\
 &\quad + C_2^4 (R_{2,j} R_{0,j} + R_{1,j}^2 + R_{0,j} \cdot R_{2,j}) \\
 &\quad + C_2^3 (R_{3,j} \cdot R_{0,j} + R_{2,j} \cdot R_{1,j} + R_{1,j} \cdot R_{2,j} + R_{0,j} \cdot R_{3,j}) \\
 &\quad + C_2^2 (R_{3,j} \cdot R_{1,j} + R_{2,j}^2 + R_{1,j} \cdot R_{3,j}) \\
 &\quad + C_2^1 (R_{3,j} \cdot R_{2,j} + R_{2,j} \cdot R_{3,j}) \\
 &\quad \left. + C_2^0 (R_{3,j}^2) \right]
 \end{aligned}$$

Summing across all j and recognizing that:

$$\begin{aligned}
 E(R_{i,j} \cdot R_{m,j}) &= \sigma_x^2 & i = m \\
 &= 0 & i \neq m
 \end{aligned}$$

and that:

$$C_2^0 + C_2^2 + C_2^4 + \dots \rightarrow \frac{1}{1 - C_2^2}$$

We find:

$$\boxed{\sigma_x^2 = \frac{C_1^2 K^2}{1 - C_2^2} \sigma_x^2}$$

In an analogous manner we can find the expected value of displaced samples:

$$E(X_n \cdot X_{n-m}) = \frac{C_1^2 K^2 C_2^m}{1 - C_2^2} \sigma_R^2$$

where m is the displacement.

By definition, the correlation coefficient, ρ , is:

$$\rho(m) = \frac{E(X_n \cdot X_{n-m})}{\sigma_{X_n} \cdot \sigma_{X_{n-m}}} = \frac{E(X_n \cdot X_{n-m})}{\sigma_X^2}$$

which, in this case is:

$$\boxed{\rho(m) = C_2^m}$$

The mean and variance of the transformed sequences may be computed from Eq. (11), which is derived in the following manner. Gaussian perturbations characterized by μ_X and σ_X^2 :

$$p(X) = \frac{1}{\sqrt{2\pi} \sigma_X} e^{-\frac{(X-\mu_X)^2}{2\sigma_X^2}}$$

are transformed by an inverse log transformation:

$$S = e^X \quad \text{or} \quad X = \ln S$$

$$\frac{dS}{dX} = e^X = \frac{1}{S}$$

where

S = transformed target strength

X = log strength = contents of target files

$$p(S) = \left. \frac{p(X)}{\left| \frac{dS}{dX} \right|} \right|_{X=\ln S} = \frac{1}{\sqrt{2\pi} \sigma_X} \frac{1}{S} e^{-\frac{(X-\mu_X)^2}{2\sigma_X^2}}$$

$$p(S) = \frac{1}{\sqrt{2\pi} \sigma_X} \cdot \frac{1}{\bar{S}} \cdot e^{-\frac{(\ln S - \ln \bar{S})^2}{2\sigma_X^2}}$$

note:

$$\bar{S} = e^{\mu_X} \quad \text{or} \quad \mu_X = \ln \bar{S}$$

The mean & variance of this log-normal distribution are given in Reference 3, Section 10.14.2:

$$\text{mean}(S) = \mu_S = e^{\ln \bar{S}} \cdot e^{\frac{\sigma_X^2}{2}} = \bar{S} e^{\frac{\sigma_X^2}{2}}$$

$$\begin{aligned} \text{variance of } S = \sigma_S^2 &= e^{2\ln \bar{S}} \cdot e^{\sigma_X^2} \left(e^{\sigma_X^2} - 1 \right) \\ &= \bar{S}^2 \cdot e^{\sigma_X^2} \left(e^{\sigma_X^2} - 1 \right) \end{aligned}$$

APPENDIX C

STATISTICS OF THEORETICAL DISTRIBUTION

The distribution of the sequence, which, when added to the exponent of target strength will generate theoretically correct scintillation, as derived in Section 4, is:

$$p(x) = \frac{e^x}{e^{\bar{x}}} e^{-\frac{e^x}{\bar{x}}} \quad (4-3)$$

$$\text{for } -\infty < x < +\infty$$

The mean, μ_x , and mean square, μ_2 , of this distribution are:

$$\mu_x = \int_{-\infty}^{+\infty} x \frac{e^x}{e^{\bar{x}}} e^{-\frac{e^x}{\bar{x}}} dx \quad (A3-1)$$

$$\mu_2 = \int_{-\infty}^{+\infty} x^2 \frac{e^x}{e^{\bar{x}}} e^{-\frac{e^x}{\bar{x}}} dx \quad (A3-2)$$

To integrate these, we make the substitutions

$$y = e^x \text{ for } 0 \leq y < +\infty \quad (A3-3)$$

which implies:

$$dy = de^x = e^x dx \quad (A3-4)$$

and

$$x = \ln y \quad (A3-5)$$

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Substituting into (A3-2) and (A3-3)

$$\mu_x = \frac{1}{e^{\bar{x}}} \int_0^{\infty} e^{-\frac{y}{e^{\bar{x}}}} \ln y \, dy \quad (\text{A3-6})$$

$$\mu_2 = \frac{1}{e^{\bar{x}}} \int_0^{\infty} e^{-\frac{y}{e^{\bar{x}}}} (\ln y)^2 \, dy \quad (\text{A3-7})$$

The standard forms of these integrals are [from Ref. 4]:

$$\int_0^{\infty} e^{-px} \ln x \, dx = -\frac{1}{p} (C + \ln p) \quad (3.711, \text{Ref. 3})$$

$$\int_0^{\infty} e^{-px} (\ln x)^2 \, dx = C^2 + \frac{\pi^2}{6} \quad (3.714, \text{Ref. 3})$$

where $C = \text{Euler's constant} = 0.577 \dots\dots\dots$

Using these and:

$$p = \frac{1}{e^{\bar{x}}}$$

it can be shown that:

$$\mu_x = \left(\frac{1}{e^{\bar{x}}} \right) \left(-e^{\bar{x}} \left(C + \ln \frac{1}{e^{\bar{x}}} \right) \right) \quad (\text{A3-8})$$

$$= -(C - \ln e^{\bar{x}}) = \bar{x} - C \quad (\text{A3-9})$$

and

$$\begin{aligned} \mu_2 &= \bar{x}^2 - 2\bar{x}C + C^2 + \frac{\pi}{6} \\ &= (\bar{x} - C)^2 + \frac{\pi}{6} \end{aligned} \quad (\text{A3-10})$$

The variance is therefore:

$$\begin{aligned}\sigma_x^2 &= \mu_2 - \mu_x^2 \\ &= (\bar{x} - C)^2 + \frac{\pi}{6} - (\bar{x} - C)^2 \\ &= \frac{\pi}{6}\end{aligned}\tag{A3-11}$$

In other words, to generate random variations of target strength for Swerling's Case I or Case II distributions, we apply random perturbations to the exponent of target strength with the distribution of (15), mean of (A3-8), and variance of (A3-11).

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