ANALYSIS OF FLOW AND HEAT TRANSFER IN SMALL ARMS

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Mechanical Engineering Department
Villanova University

Abstract

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NOMENCLATURE

A	cross-sectional area of gun bore, ft^2
a	speed of sound, ft/sec
c_p	specific heat of propellant gas at constant pressure, BTU/lbm-^oR
c_v	specific heat of propellant gas at constant volume, BTU/lbm-^oR
D	diameter of gun bore, ft
D_o	initial diameter of propellant grain (weighted average), ft
E_{gas}	energy per unit mass of gas, BTU/slug
F_d	drag force on propellant particles per unit volume, lbf/ft^3
g	acceleration of gravity, ft/sec^2
H	enthalpy, BTU/slug
J	mechanical equivalent of heat, ft-lbf/BTU
ℓ	distance burned normal to propellant grain surface, ft
m_g	rate of mass of gas generated per unit length, slug/ft-sec
p	gas pressure, lbf/ft^2
q	heat loss through barrel wall, BTU/ft^2-sec
R	gas constant for propellant gas, ft-lbf/slug-^oR
$\bar{R}$	disk radius fo an individual grain, ft
R_i	one half of web thickness of individual grain, ft
r	linear burning rate of propellant, ft/sec
s	specific entropy, BTU/lbm-^oR
S_p	surface area of an individual grain, ft^2
T_g	gas temperature, ^oR
t	time, sec
u_g	velocity of gas, ft/sec
u_p	velocity of propellant particles, ft/sec

V_p	volume of an individual grain, ft^3
W	powder potential, $cvT(\ell)$, BTU/lbm
Wt	weight of projectile, lbm
ϵ	particle volume fraction, dimensionless
η	number of particle grains per unit volume, ft^{-3}
ρ_g	gas density, $slug/ft^3$
ρ_p	propellant density, $slug/ft^3$
τ_w	wall shearing stress, lbf/ft^2 or $slug/ft\text{-}sec^2$

Introduction

Investigations into propellant performance, barrel materials and weapon kinematics require accurate description of the weapon interior ballistics. The present study was undertaken to determine the effect of heat transfer on interior ballistics. In examining the interior ballistics of small arms, the effect of a number of parameters must be included. These parameters include chamber geometry, propellant geometry, propellent composition and burning rate.

The heat transfer rates in small arms are very high and are considered to be a major cause of barrel errosion. Accurate description of these heat transfer rates is necessary for analytical predication of pressure and velocity histories in small arms.

The present study uses the method of characteristics to solve the governing equations. The model assumes the problem to be one dimensional and transient. Assuming a fixed propellent description, the effect of various heat transfer models has been examined. The performance standard is test data from reference 1 presented in figure 1.

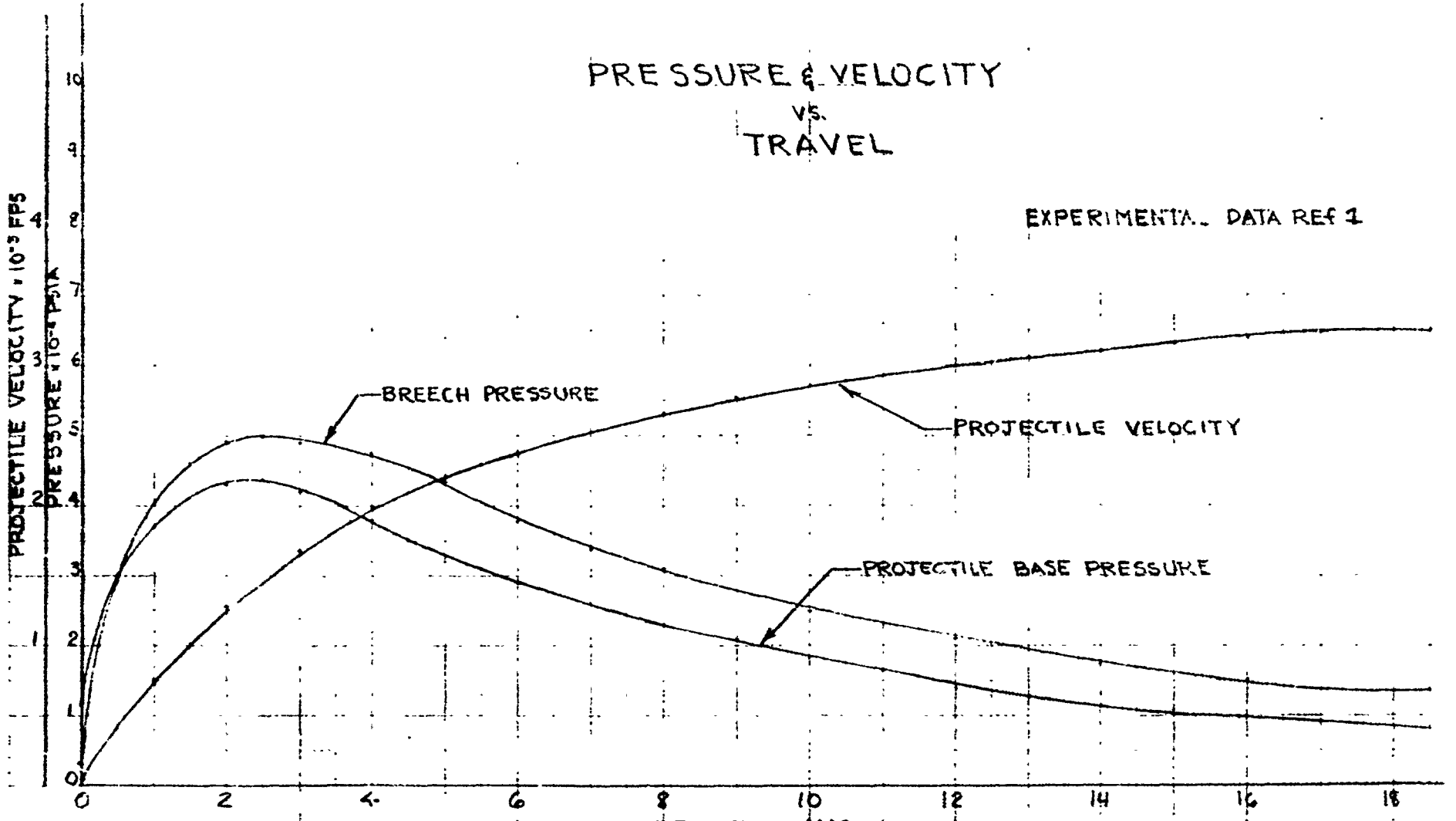
PRESSURE & VELOCITY
VS.
TRAVEL
EXPERIMENTAL DATA REF 1
PROJECTILE VELOCITY, 10^3 FPS
PRESSURE, 10^4 PSIA
BREECH PRESSURE
PROJECTILE VELOCITY
PROJECTILE BASE PRESSURE
0
2
4
6
8
10
12
14
16
18
TRAVEL - INCHES

FIGURE 1

Literature Survey

Reference (2) presents a ballistic model developed from earlier work which was modified to include the effect of non-uniform propellant composition and non-simultaneous ignition. In this reference the propellant is represented by a composite of 9 different composition propellants. This variable propellant array allows the handling of different size grains and different deterrent concentration densities. Sample results are included in reference (2) which shows good agreement with test.

Reference (3) presents an analytical investigation of the flow in gun barrels with particular emphasis on the heat transfer. In addition to the transient nature of the flow, reference (3) considers the problem posed by the lack of a boundary layer at the base of the projectile. This condition occurs because in the projectile based region the gas velocity is equal to the projectile velocity. This condition means that the Reynolds analogy commonly used in heat transfer cannot be applied to this case. However, the reference does propose an expression for heat transfer coefficient in the projectile base region. The resulting convective coefficients are between 5 and 20 Btu/sec ft^2 ^oF. Comparisons between experiement and prediction presented in reference (3) indicate that even these high coefficients underestimate the actual results. However, the results of reference (3) represent the best available procedure for calculating the heating at the base of the projectile.

Reference (4) presented an analytical study of interior ballistics in which the governing equations were solved using a finite difference procedure. The geometric dimensions used in reference (4) did not correspond to standard small arms. In addition, the analysis was limited to stationary propellant and to propellant moving at the same velocity as the gas. In the area of heat transfer reference (4) also obtained very high heat transfer coefficients

($\sim$ 10 Btu/sec ft^2 ^{o}F). It is worthy of note that for cases with and without heat transfer and wall friction the ballistic performance was not changed significantly. The present study has found that heat transfer particularly at the projectile base, is very significant.

Reference (5) presents an extensive study of the flame front movement through a porous bed propellant. The analysis is presented for a venting bomb type device. However, the results for flame propagation can be applied to the small arms case. Indeed, the functional relationship between flame velocity and pressure can be obtained from the data presented in reference (5). Reference (6) considers the same problem as reference (5) but applied to the small arms case. The results of reference (6) have been used in the present study to determine the progress of the flame front through the propellant bed. Reference (7) presents an iterative model for the interior ballistic problem in which parameters affecting the burning rate were changed until the calculated results agreed with test data. The caliber used in reference (7) is the same as the present study, (i.e. 5.56 MM); however, the experimental and analytical results are considerably different. This is most likely due to different charge weight and propellant type. In addition, a number of the loss factors considered in the present study are not included.

Reference (8) is the analytical basis for the present study. This reference presents the model which has been used and also outlines the calculation procedure for the method of characteristics. The nomenclature of reference (8) has been adopted in the present report. The present investigation has expended considerable effort in programming reference (8). Reference (8) describes the interior ballistic problem as a one dimensional transient flow problem. In the model the propellant is assumed to ignite instantaneously and burn at a rate proportional to the gas pressure. The geometry of the propellant is uniform and based upon reference (10).

Although heat transfer between the gas and propellant is neglected, drag between these two is considered. The transformation of the governing equations from the physical domain to the characteristic domain is described in detail in reference (9). The heat transfer to the wall and the wall friction are included in the model but the detailed descriptions are developed in the present study. In addition, the effect of chamberage and projectile resistive forces are also included in the model of reference (9).

Analysis

To determine the effect of heat transfer and to evaluate potential heat transfer relationships, an accurate model of the flow field must be developed. Considerable effort during the present study was devoted to developing a model which could reproduce test data. In addition, during the course of this model development it became apparent that the assumption of uniform ignition was not adequate and therefore a more realistic flame front model had to be developed. This section describes the flow field flame front analysis, and heat transfer models.

The flow is two phase, non-steady, including heat and mass addition. The basic model describing the problem has been developed in reference (8). Because of the geometry present, the problem is quasi-one dimensional. The resulting conservation equations form a set of quasi-linear partial differential equations of the first order and hyperbolic type. For this situation the method of characteristics provides the simplest means of solution. Reference (8) provides the details of the transformation of the governing equations to the characteristic form. In addition, reference (9) provides the theoretical foundation for the applications of the method of characteristic to the present problem.

Flow Field Governing Equations

The governing equations described in these sections are based upon the following assumptions:

1. Flow is quasi-one dimensional
2. The gas is assumed to be ideal
3. The specific heat and molecular weight are dependent upon the amount of deterred and non-deterred propellant burned
4. All propellants grains are initially of uniform surface area, volume, and radius. Weighted mean values based upon reference (10) and (11) are used.
5. Interactions between individual propellant particles and between particles and the boundary are negligible
6. The thermal motion of the propellant particles does not contribute to the pressure
7. The average gas temperature is defined by the heat content of the gas phase
8. The propellant particles are spheres for drag calculations
9. Heat dissipation due to viscosity is negligible
10. Radiation is negligible
11. Gravity is negligible

Overall Gas Continuity Equation

$$\frac{\partial}{\partial t}\left(\rho_g A(1-\epsilon)\right) + \frac{\partial}{\partial x}\left(\rho_g A u_g (1-\epsilon)\right) = \dot{m}_g \tag{1}$$

where $\dot{m}_g$ is the gas generated per unit length

$$\dot{m}_g = \epsilon S_p r \rho_p A / V_p$$

Propellant Particle Continuity Equation

$$\frac{D\epsilon}{Dt} = -\epsilon \frac{\partial u_p}{\partial x} - \frac{\epsilon u_p}{A}\frac{\partial A}{\partial x} - \frac{\dot{m}_g}{A\rho_p} \tag{2}$$

Overall Gas Momentum Equation

$$A\rho_g(1-\epsilon)\frac{\partial u_g}{\partial t} + (1-\epsilon)u_g \rho_g A \frac{\partial u_g}{\partial x} = -u_g \dot{m}_g - \tau_w \pi D - (1-\epsilon)A\frac{\partial P}{\partial x} - F_d A \tag{3}$$

where τ_w is shear stress at the wall and Fd is particle drag per unit volume

Propellant Particle Momentum Equation

$$\frac{Du_p}{Dt} = \frac{u_p \dot{m}_g}{\rho_p A \epsilon} + \frac{F_d}{\rho_p \epsilon} \tag{4}$$

Overall Gas Energy Equation

$$\frac{\partial}{\partial t}\left((1-\epsilon)\rho_g\left(\frac{u_g^2}{2J} + g c_v T_g\right)A\right) + \frac{\partial}{\partial x}\left((1-\epsilon)\rho_g A u_g\left(g c_v T_g + \frac{P}{\rho_g J} + \frac{u_g^2}{2J}\right)\right) = -g\pi D - \frac{F_d A}{J}|u_g - u_p| + g W \dot{m}_g \tag{5}$$

where W is the heat content of the propellant per pound

Overall Gas Entropy Equation

$$\frac{DS}{Dt} = (\dot{E}_{gas} + \frac{U_g F}{J} - R_s)/(A(1-\epsilon)\rho_g T_g g) \qquad (6)$$

where,

$$E_{gas} = -q\pi D - \frac{F_d A}{J}|U_g - U_{pl}| + gW\dot{m}_g$$

$$F = -U_g \dot{m}_g + \tau_w \pi D + F_d A$$

$$R_s = \dot{m}_g\left(gc_v T_g + \frac{U_g^2}{2J}\right) + \frac{PA}{J}\frac{\partial \epsilon}{\partial t} + \frac{P\dot{m}_g}{\rho_g J}$$

In the preceding equations, the dependent gas variables are entropy, gas temperature, pressure, gas density and gas velocity.

The next step in the solution procedure is to apply the method of characteristics. In this approach the dependent variables become the P and Q Riemann variables where:

$$P = \frac{2}{\gamma-1}a + U_g$$

$$Q = \frac{2}{\gamma-1}a - U_g$$

Solutions are determined along these P and Q characteristics. The characteristics represent lines along which infinitesimal disturbances are propagated at the local speed of sound in the gas phase. Along each characteristic the following relationships between X, t are true:

$$\frac{dx}{dt} = U_g + a \qquad \text{(along P characteristic)}$$

$$\frac{dx}{dt} = U_g - a \qquad \text{(along Q characteristic)}$$

Equation (1) and (3) can be combined into two equations for P and Q along the respective characteristic using the ideal gas relations (for details see reference (8)):

$$a^2 = \gamma R T_g$$

$$\ln P = \frac{2\gamma}{\gamma-1}\ln a - g\frac{JS}{R}$$

yielding equations (7) and (8).

$$\frac{\delta_+ P}{\delta t} = -a U_g \frac{\partial}{\partial x} \ln A + \frac{\gamma-1}{\gamma} \frac{a q J}{R} \frac{DS}{Dt} + \frac{a q J}{\gamma R} \frac{\delta_+ S}{\delta t} - \frac{\tau_w \pi D}{A(1-\epsilon)\rho_g} - a \frac{D}{Dt} \ln(1-\epsilon) - \frac{F_d}{\rho_g (1-\epsilon)} - \frac{U_g \dot{m}_g}{A(1-\epsilon)\rho_g} + \frac{a^3 \dot{m}_g}{A(1-\epsilon) P \gamma} \tag{7}$$

$$\frac{\delta_- Q}{\delta t} = -a U_g \frac{\partial}{\partial x} \ln A + \left(\frac{\gamma-1}{\gamma}\right) \frac{a q J}{R} \frac{DS}{Dt} + \frac{a q J}{\gamma R} \frac{\delta_- S}{\delta t} + \frac{\tau_w \pi D}{A(1-\epsilon)\rho_g} - a \frac{D}{Dt} \ln(1-\epsilon) + \frac{F_d}{\rho_g (1-\epsilon)} + \frac{U_g \dot{m}_g}{A(1-\epsilon)\rho_g} + \frac{a^3 \dot{m}_g}{A(1-\epsilon) P \gamma} \tag{8}$$

where:

$$\frac{\delta_\pm}{\delta t} = \frac{\partial}{\partial t} + (U_g \pm a) \frac{\partial}{\partial x}$$

Equation (6), (7) and (8) along with the appropriate ideal gas relations are the governing equations for the gas phase. Equations (2) and (4) describe the propellant particle phase.

Initial and Boundary Condtions

At t=0, the first Q wave leaves the base of the projectile and travels toward the breech. During the time prior to the arrival of the Q wave at the breech, it is assumed that the gas in the chamber is at rest and that the temperature is constant at the flame temperature, T_0. The pressure in the region between the projectile and breech will rise as the Q wave advances

due to the burning of propellant. The slope of this first Q wave is given by equation (9).

$$\frac{dx}{dt} = -a_0 = -\sqrt{\gamma R T_0} \tag{9}$$

The Q wave is divided into a specified number of equally spaced nodes. The nodes are the origin for the P wave. At each node the gas temperature and speed of sound are known, the other parameters are given by the following relations:

$$t_{i+1} = t_i + \frac{(x_{i+1} - x_i)}{a_0}$$

$$\frac{Ds}{Dt} = \left(g_0 W \dot{m}_g - \dot{m}_g\left(g c_v T_g + \frac{P}{\rho_g J}\right) - \frac{P_A}{J}\frac{D\epsilon}{Dt}\right) \Big/ \left(A(1-\epsilon)\rho_g T_g\right)$$

$$\frac{D\epsilon}{Dt} = -\frac{\dot{m}_g}{A\rho_p}$$

$$s_{i+1} = s_i + \frac{Ds}{Dt}(t_{i+1} - t_i)$$

$$P_{i+1} = P_i \, e^{-\frac{gJ}{e}(S_{i+1}-S_i)}$$

Since the speed of sound is constant along this wave, Q is also constant. This procedure is different from that described in reference 8, but it yields the same results.

The spatial boundaries of the problem are the breech (x=0) and the base of the projectile. The breech is at a fixed condition with the gas and propellant velocity equal to zero. The location of the projectile base is determined by the intersection of a P wave and the equation of motion of the projectile. The two equations governing the position of the projectile are:

$$\frac{dx}{dt} = U_g + a \tag{10}$$

$$\frac{d^2x}{dt^2} = \frac{gAP}{W_t} - \frac{gR_1}{W_t} \tag{11}$$

where R_1 is the engraving force and Wt is the weight of the projectile. At the projectile base the gas velocity, propellant velocity and projectile velocity are all equal.

Numerical Procedure

The numerical solutions to the governing equation will be obtained using the finite difference procedure. In this approach, derivatives will be replaced by finite differences (explicit) and these will be used to generate the desired characteristic grid. Fig 1 shows the proposed grid system.

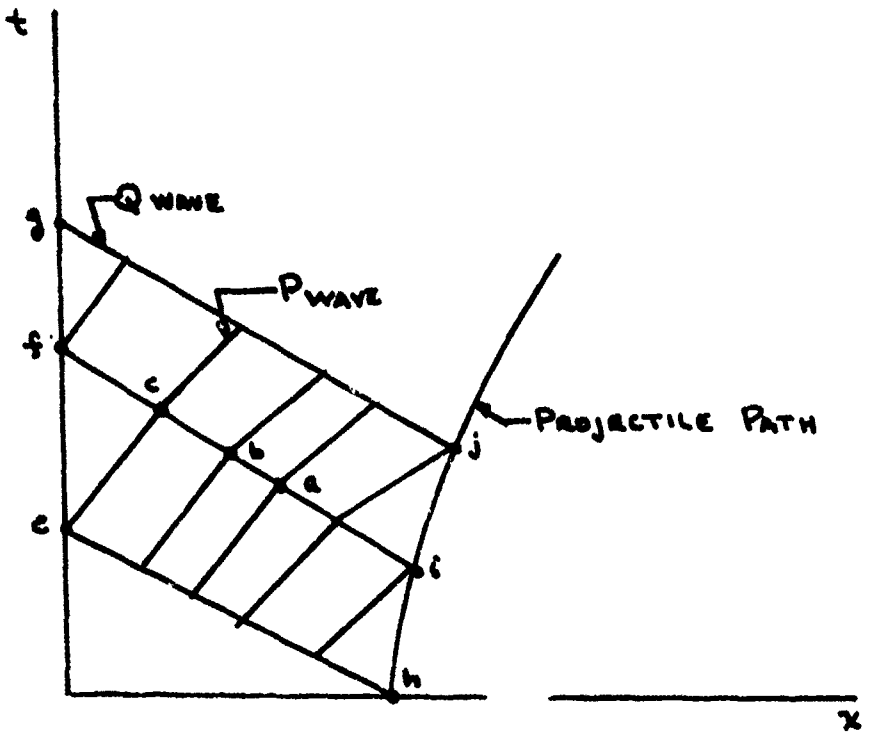

Figure 1: Schematic of Characteristic Grid Points

There are three types of grid points, interior points, (a,b,c), breech points (e,f.g) and projectile base points (h, i, j). The governing equations and solution procedures are different for each of these types of grid points and therefore, each will be handled separately.

Interior Points

Figure 2 shows a typical interior unknown grid point (6 . It is on a P wave from point (4) and a Q wave from point (3).

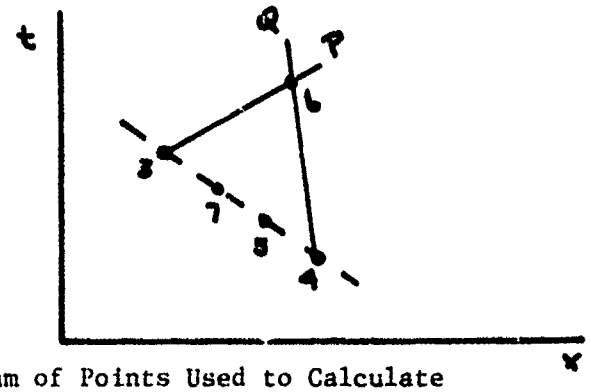

Figure 2: Diagram of Points Used to Calculate Interim Grid Points

Point (7) represents the location of a gas particle which intersects point (6) and the line connecting points (3) and (4). Point (5) is similar to point (7) except it represents a propellant particle. The slope of the P and Q characteristics are given by equations (12) and (13).

$$\frac{X_6 - X_3}{t_6 - t_3} = U_{g_{3,6}} + a_{3,6} \tag{12}$$

$$\frac{X_6 - X_4}{t_6 - t_4} = U_{g_{4,6}} - a_{4,6} \tag{13}$$

where

$$U_{g_{4,6}} = (U_{g_4} + U_{g_6})/2$$
$$a_{3,6} = (a_4 + a_6)/2$$
$$U_{g_{3,6}} = (U_{g_3} + U_{g_6})/2$$
$$a_{3,6} = (a_3 + a_6)/2$$

Solving for t_6 and X_6

$$t_6 = \frac{X_4 - X_3 + t_3(U_{g_{3,6}} + a_{3,6}) - t_4(U_{g_{4,6}} - a_{4,6})}{(U_{g_{3,6}} + a_{3,6}) - (U_{g_{4,6}} - a_{4,6})} \tag{14}$$

$$X_6 = X_3 + (U_{g_{3,6}} + a_{3,6})(t_6 - t_3) \tag{15}$$

Since the values at point 6 are initially unknown, an assumed value of a given parameter is used and then the procedure is repeated until the assumed value and calculated value agree to within a specified tolerance. The initial assumption is an arithmetic average of the values at points (3) and (4).

Knowing the location of point 6, it is possible using the procedure described in reference (8) in pages 23 through 30 to generate the data required for the solution to proceed. For brevity, the details will be omitted; however, certain key points should be mentioned. The average burning rate between points (5) and (6) given by:

$$r_{s,6} = B\, P_{s,6}^{n} \left(1 + k\left(|u_g - u_p|_{s,6} - 500\right)\right)$$

The distance burned normal to a particle surface at (6) is:

$$\ell 6 = \ell 5 + r_{5\text{-}6}\,(t_6 - t_5)$$

Knowing $\ell 6$ and $\ell 5$ the average particle volume and surface area between (5) and (6) can be determined

$$V_{p5,6} = (V_{p5}(\ell 5) + V_{p6}(\ell 6))/2$$

$$S_{p5,6} = (S_{p5}(\ell 5) + S_{p6}(\ell 6))/2$$

The gas per unit length generated along the propellant path (5) and (6) is

$$\dot{m}_{g\,s,6} = \left(\frac{\epsilon_{s,6}}{V_{ps,6}}\right) S_{ps,6}\, r_{s,6}\, \rho_p A$$

The rate of change in propellant particle velocity between (5) and (6) is given by equation (16)

$$\frac{DU_p}{Dt} = \frac{U_{ps,t}\,\dot{m}_{gs,t}}{\rho_p A\,\epsilon_{s,t}} + \frac{F_{ds,t}}{\rho_p\,\epsilon_{s,t}} \tag{16}$$

The rate of change of propellant particle volume fraction is given by equation (17)

$$\frac{D\epsilon}{Dt} = -\frac{\epsilon_{s,t}}{U_{ps,t}}\frac{DU_p}{Dt} - \frac{2\,\epsilon_{s,t}\,U_{ps,t}}{(A_4+A_3)}\frac{(A_4-A_3)}{(X_4-X_3)} - \frac{\dot{m}_{gs,t}}{A\,\rho_p} \tag{17}$$

The rate of entropy change for the gas can now be computed using equation (6). The change in P and Q along the appropriate characteristic is given by equations (18) and (19)

$$P_t = P_3 + \frac{\delta_+ P_{3,t}}{\delta t}(t_t - t_3) \tag{18}$$

$$Q_t = Q_4 + \frac{\delta_- Q_{4,t}}{\delta t}(t_t - t_4) \tag{19}$$

where $\delta_+ P_{3,t}/\delta t$ and $\delta_- Q_{4,t}/\delta t$ are given the equations (20) and (21)

$$\frac{\delta_+ P_{3,t}}{\delta t} = -\frac{2\,a_{3,t}\,U_{g3,t}}{(A_4+A_3)}\frac{(A_4-A_3)}{(X_4-X_3)} + \frac{(\gamma-1)}{\gamma}\frac{a_{3,t}}{R}\,g\,J\,\frac{Ds}{Dt} + \frac{a_{3,t}\,g\,J}{\gamma R}\left[\frac{Ds}{Dt} + a_{3,t}\frac{(s_t-s_3)}{(x_t-x_3)}\right] - \frac{\tau_w\,\pi D}{A(1-\epsilon_{3,t})\,F_{g3,t}} + \frac{a_{3,t}}{(1-\epsilon_{3,t})}\left(\frac{De}{Dt} + \frac{a_{3,t}^2\,\dot{m}_g}{A\,\rho_{3,t}\,\gamma}\right) - \frac{1}{\rho_{g3,t}(1-\epsilon_{3,t})}\left(F_d + \frac{U_{g3,t}\,\dot{m}_g}{A\,\rho_{g3,t}}\right) \tag{20}$$

$$\frac{\delta Q_{4,1}}{\delta t} = -\frac{2\alpha_{4,1} U_{g\alpha_{4,1}} (A_4 - A_3)}{(A_4 + A_3)(X_4 - X_3)} + \frac{(\gamma-1)}{\gamma} \frac{\alpha_{4,1} g J}{R} \frac{DS}{Dt}$$

$$+ \frac{\alpha_{4,1} g J}{TR} \left[\frac{DS}{Dt} - \alpha_{4,1} \frac{(S_4 - S_3)}{(X_4 - X_3)} \right] + \frac{\tau_w \pi D}{A(1-\alpha_{4,1}) \rho_{g4,1}}$$

$$+ \frac{\alpha_{4,1}}{(1-\alpha_{4,1})} \left[\frac{D\epsilon}{Dt} + \frac{\alpha_{4,1}^2 \dot{m}_g}{A \rho_{g4,1} \gamma} \right] + \frac{1}{\rho_{g4,1}(1-\alpha_{4,1})} \left[F_3 + \frac{U_{g\alpha_{4,1}} \dot{m}_g}{A \rho_{g3,1}} \right] \quad (21)$$

Using the calculated values for P_6 and Q_6, new values for the gas velocity and speed of sound can be computed

$$U_{g6} = (P_6 - Q_6)/2 \quad (22)$$

$$a_6 = \left(\frac{\gamma-1}{4}\right) (P_6 + Q_6) \quad (23)$$

The entropy at point 6 can be calculated from equation (24)

$$S_6 = S_7 + \left(\frac{DS}{DT}\right) (t6 - t7) \quad (24)$$

The new values of Ug_6, a_6, and S_6 can be used to calculate the remaining thermodynamic properties based upon the equations on page 34 reference 8.

In addition Ug_6 and a_6 are available to obtain a new estimation of the location of point (6) (i.e., t6, X6). The procedure described above is then repeated until the new values of a_6 and Ug_6 agrees with the previously calculated ones to within a specified tolerance.

Projectile Base Point

The solution at the projectile base is determined by the intersection of a P wave and the path of the projectile. Figure 3 presents a diagram of this intersection. The location and thermodynamic properties at

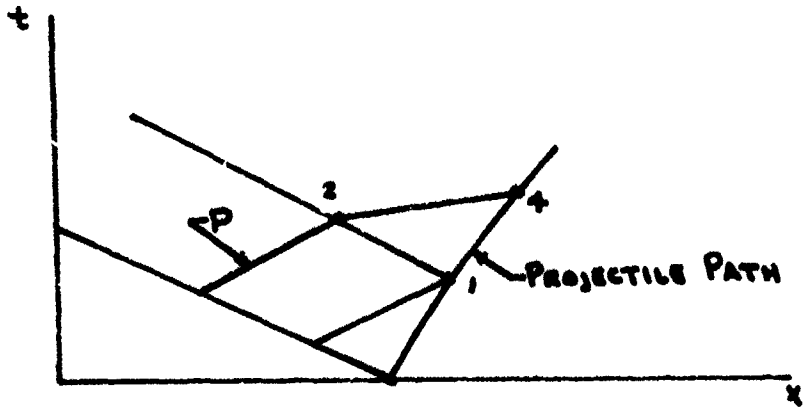

Figure 3: Diagram of Points Used to Calculate a Projectile Base Point

points 1 and 2 are known. This section describes the procedure to find point 4. The slope of the P wave from 2 to 4 is given by equation (25)

$$\frac{x_4 - x_2}{t_4 - t_2} = u_{g_{4,2}} + a_{4,2} \tag{25}$$

In addition integration of the equation of motion of the projectile yields equation (26)

$$x_4 = x_1 + u_{g_1}(t_4 - t_1) + \frac{1}{2}\left[\left(\frac{P_1 + P_4}{2}\right)\frac{gA}{W_t} - \frac{R_{1g}}{W_t}\right](t_4 - t_1)^2 \tag{26}$$

Simultaneous solution of equations (25) and (26) yields equation (27)

$$t_4 = \frac{-B - \sqrt{B^2 - 4AC}}{2A} \tag{27}$$

where

$$C = (X_1 - X_2) + (U_{g_{3,4}} + a_{2,4})t_2 - \left(\frac{(P_1+P_4)}{2W_t}gA - \frac{R_1 g}{W_t}\right)\frac{t_2^2}{2} - t_1\left(U_{g_1} - t_1\left(\frac{gA(P_1+P_4)}{2W_t} - \frac{R_1 g}{W_t}\right)\right)$$

$$B = U_{g_1} - \left(\frac{(P_1+P_4)}{2W_t}gA - \frac{R_1 g}{W_t}\right)t_1 - \left(U_{g_{2,4}} + a_{2,4}\right)$$

$$A = \frac{1}{2}\left(\frac{(P_1+P_4)}{2W_t}gA - \frac{R_1 g}{W_t}\right)$$

Knowing t_4, X_4 can be obtained from equation (25). At the base of the projectile the projectile velocity equals the gas velocity (eq. (28)).

$$U_{g_4} = U_{g_1} + \left(\frac{(P_1+P_4)}{2W_t}gA - \frac{R_1 g}{W_t}\right)(t_4 - t_1) \tag{28}$$

Having established a first approximation for the location and the velocity at point 4, the change in P along the characteristic 2-4 can be determined. The details regarding this procedure are given in reference 8, pages 36 to 40. The rates of change of P ($\delta_4 P_{2,4}/\delta t$) and P4 are given by equations (29) and (30)

$$\frac{\delta_t P_{2,4}}{\delta t} = -\frac{2\, a_{2,4}\, Ug_{2,4}\,(A_4 - A_2)}{(A_2 + A_4)(X_4 - X_2)} + a_{2,4}\frac{gJ}{R}\frac{DS}{Dt}$$

$$-\frac{\tau_w \pi D}{A(1-\epsilon_{2,4})\rho_{g_{2,4}}} + \frac{a_{2,4}}{(1-\epsilon_{2,4})}\frac{D\epsilon}{Dt}$$

$$-\frac{Fd}{\rho_{g_{2,4}}(1-\epsilon_{2,4})} - \frac{Ug_{2,4}\,\dot{m}_{g_{2,4}}}{A(1-\epsilon_{2,4})\rho_{g_{2,4}}}$$

$$+\frac{a^2_{2,4}\,\dot{m}_{g_{2,4}}}{A(1-\epsilon_{2,4})\,P_{2,4}\,\gamma} \qquad (29)$$

$$P_4 = P_2 + \frac{\delta_t P_{2,4}}{\delta t}(t_4 - t_2) \qquad (30)$$

Knowing P_4 and Ug_4, a new velocity of sound, a_4, can be computed from equation (31)

$$a_4 = (P_4 - Ug_4)(\gamma - 1)/2 \qquad (31)$$

Using a_4 and s_4, revised values for the other thermodynamic variable can be computed. The procedure is then repeated until the values of Ug_4 and a_4 for two consecutive calculations agree within a specified tolerance.

After the projectile leaves the barrel the procedure is the same as described on pages 49 and 50 of reference 8. It is assumed that the gas velocity is sonic at muzzle exit and that Q waves after the projectile leaves, originate from the muzzle exit. Under these conditions X_{24} is fixed and the following relations exist:

$$t_4 = t_2 + \frac{(X_4 - X_2)}{(u_{g_{2,4}} + a_{2,4})}$$

$$a_4 = \frac{P_4 (\gamma - 1)}{(\gamma + 1)}$$

$$u_{g_4} = a_4$$

Breech Points

The breech is located at X=0. Figure 4 diagrams the intersection of a Q wave with the breech.

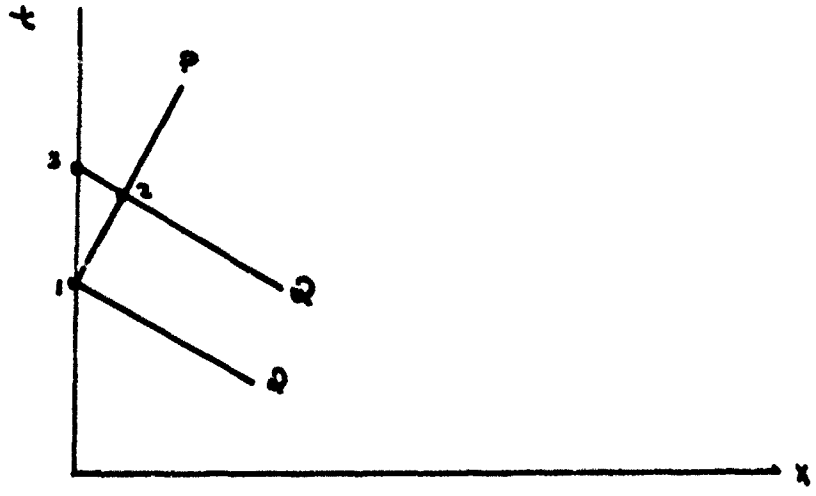

Figure 4: Diagram of Points Used to Calculate Breech Point

The time t_3 is given by equation (32)

$$t_3 = -X_2/Ug_2 - a_{2,3}) + t_2 \tag{32}$$

In equation (32) the initial estimate if a_3 is made by assuming the same values as point (1). The change in Q from 2 to 3 is then computed and a revised a_3 can be obtained from equation (33)

$$Q_3 = \frac{(\gamma - 1)}{2} Q_3 \tag{33}$$

The rate of change of propellant volume fraction and of gas entropy are given by equations (34) and (35)

$$\frac{De}{Dt} = -\frac{\dot{m}_g}{A\rho_p} - \epsilon \frac{\partial U_p}{\partial x} \tag{34}$$

$$\frac{DS}{Dt} = \frac{E_{gas} - R_{s1}}{A(1 - \epsilon_{1,3})\rho_{g1,3} T_{g1,3}} \tag{35}$$

where

$$E_{g_{2,3}} = -g\pi D + gW\dot{m}_g$$

$$R_{2,3} = \dot{m}_g (gC_v T_{g_{1,3}}) + P_{1,3}\frac{A}{J}\frac{D\epsilon}{Dt} + \frac{P_{1,3}\dot{m}_g}{\rho_{g_{1,3}} J}$$

The rate of change in Q along the characteristics from 2 to 3 is given by equation (36)

$$\frac{\delta Q_{2,3}}{\delta t} = a_{1,3}\frac{gJ}{R}\frac{DS}{Dt} + \frac{a_{1,3}}{(1-\epsilon_{1,3})}\frac{D\epsilon}{Dt} + \frac{a_{1,3}^2 \dot{m}_g}{A(1-\epsilon_{1,3})P_{1,3}\gamma} \quad (36)$$

The value of Q and S at point 3 is given by equation (37) and (38)

$$Q_3 = Q_2 + \frac{\delta Q_{2,3}}{\delta t}(t_3 - t_2) \quad (37)$$

$$S_3 = S_1 + \frac{DS}{Dt}(t_3 - t_1) \quad (38)$$

Using equation (33) and (37), a new value of a_3 can be calculated. In addition, a_3 and S_3 can be used to calculate the remaining thermodynamic variables. The procedure is repeated until the value at a_3 from two successive calculations agrees within a specified tolerance.

The preceding procedure has been programmed and the details of the program are presented in Appendix A.

Flame Front Governing Equations

Initially it was assumed that there was instantaneous ignition of the entire propellant bed. This resulted in excessive breech pressure and low projectile base pressure. A better approximation is obtained by using a flame front with a finite velocity. Based upon the results from references (4) and (12), it was determined that the flame front velocity was proportional to the pressure behind the front. Using data from these references the following relationship was developed

$$V_F = \alpha p^{\beta} \tag{39}$$

where

$$\alpha = 31.5 \quad \text{in/sec/(spi)}^{\beta}$$
$$\beta = .48$$

The effect of the flame front progressing through the propellant bed can then be determined by integrating from the breech to the projectile base. The pressure and temperature behind the flame front was assumed uniform. The gas velocity was also assumed to be zero. For a given time increment the distance the flame front moves is given by equation (40).

$$dL = \int_{t_1}^{t_2} V_F \, dt \tag{40}$$

In equation (40) the velocity is based upon the initial pressure(at t_1) and an assumed final pressure (at t_2). The procedure is repeated until the assumed final pressure and calculated final pressure agree to within 5%. The distance burned normal to the propellant surface in the same time period is given by equation (41)

$$d\,XL = (B\,p^{n})\,(t_2 - t_1) \tag{41}$$

Using the results of reference (10) the new volume a propellent grain resulting from the surface recession given in equation (41) is

$$\ell_{i,j} = \ell_{i,j-1} + \Delta x_L$$

$$RL = R_i - \ell_{i,j}$$

$$V_{i,j} = 2\pi (RR^2)(RL) + \pi^2 RL^2 \left(RR + \frac{4RL}{3\pi} \right) \qquad (42)$$

where

$RR = R_o - R_i$

R_o = equivalent mean radius of propellant grain

R_i = one half web

The total gas generated in the cell defined by dL from the time $t = 0$ to $t = t_2$ is given by equation (43).

$$X_{mg} = dL\, \eta\, A (V_{oi} - V_{ij})\, \rho_p \qquad (43)$$

Equation (44) gives the volume fraction.

$$\epsilon_{ij} = (\eta A dL - X_{mg}/\rho_p) / (A\, dL) \qquad (44)$$

The pressure behind the flame front is obtained by applying a modified perfect gas relationship to the total gas generated from every cell.

$$P = \frac{(XTG \cdot W + P_i A z L)}{(A z L - Vols - XTG\, \xi)} \qquad (45)$$

where

XTG = sum of gas generated in each cell at time t2

ZL = distance flame front has progressed from t = 0 to t_2

Vols = total volume of solids behind flame front

η = propellant covolume

The pressure given by equation (45) applies to the entire region behind the flame front. This pressure is averaged with the pressure from the preceding time period to determine the flame front velocity. The procedure is repeated until the pressure in equation (45) for two successive passes agrees to within 5%

The flame front calculation procedure has been programmed and the details of the program as presented in Appendix B.

Heat Transfer Models

Since the analytical model considers the problem in three distinct regions, the breech, the interior, and the projectile base the heat transfer models will be considered in the same manner. The approach in defining the heat rates has been to use the best data either experimental or analytical.

Breech Region

In the breech region, the hot gases stagnate and as such, no boundary layer is formed. Although there may be recirculation flows in this region, analytical description would be extremely difficult. However, there is data available on heating in the projectile case region. Reference (12) developed experimental data on the heating rate at various locations in ammunition case region or a function of time. Figure 5 presents the results from reference 12. To obtain the heating at the breech, the data in figure 5 was extrapolated to x = o and then replotted in figure 6. The peak heating occurs at .25 milsec with a value of 3.7 Btu/ft^{2}sec. After reaching this peak value the breech heating decreases exponentially with time. Rather than trying to match the peak heating, the approach taken was to satisfy the long term heating by fitting the following equation to the data in figure 6.

$$q = K_1 e^{K_2 t}(T_g - T_w)$$

Using 1 and 2 milsec as the matching times, the following values for K_1 and K_2 were obtained:

$$K_1 = 3.24 \text{ Btu/ft}^2\text{sec}$$
$$K_2 = -587.78 \text{ sec}^{-1}$$

The dotted line in figure 6 gives the profile of the fitted curve. To determine the effect of breech heating, the value of K_1 was varied from 0 to 3.24 on a series of computer runs. The results showed that breech heating had a negligible effect of projectile launch velocity and peak pressure.

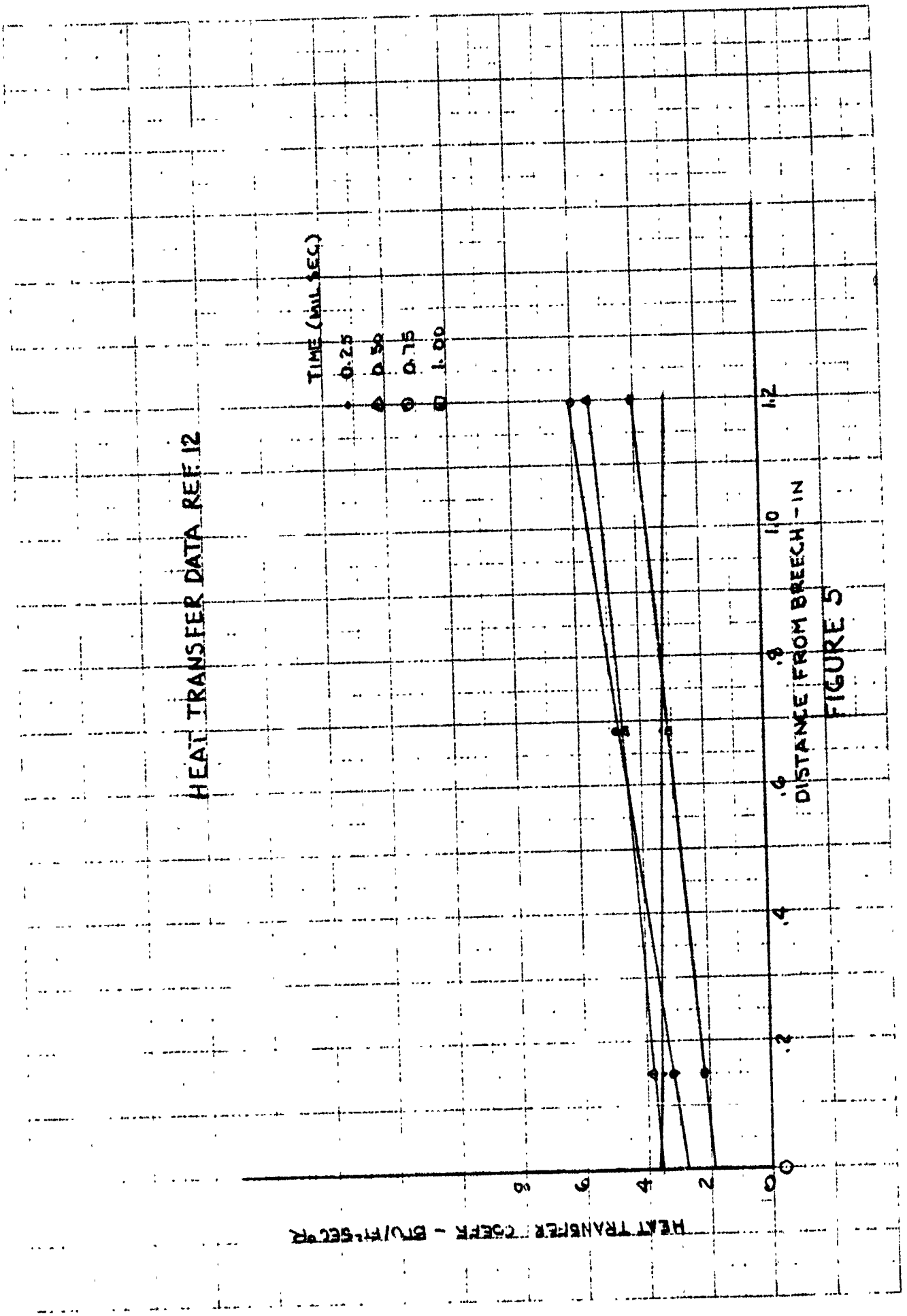
HEAT TRANSFER DATA REF. 12
TIME (MILSEC)
0.25
0.50
0.75
1.00
HEAT TRANSFER COEFF - BTU/FT²-SEC°R
0
2
4
6
8
.2
.4
.6
.8
1.0
1.2
DISTANCE FROM BREECH - IN

FIGURE 5

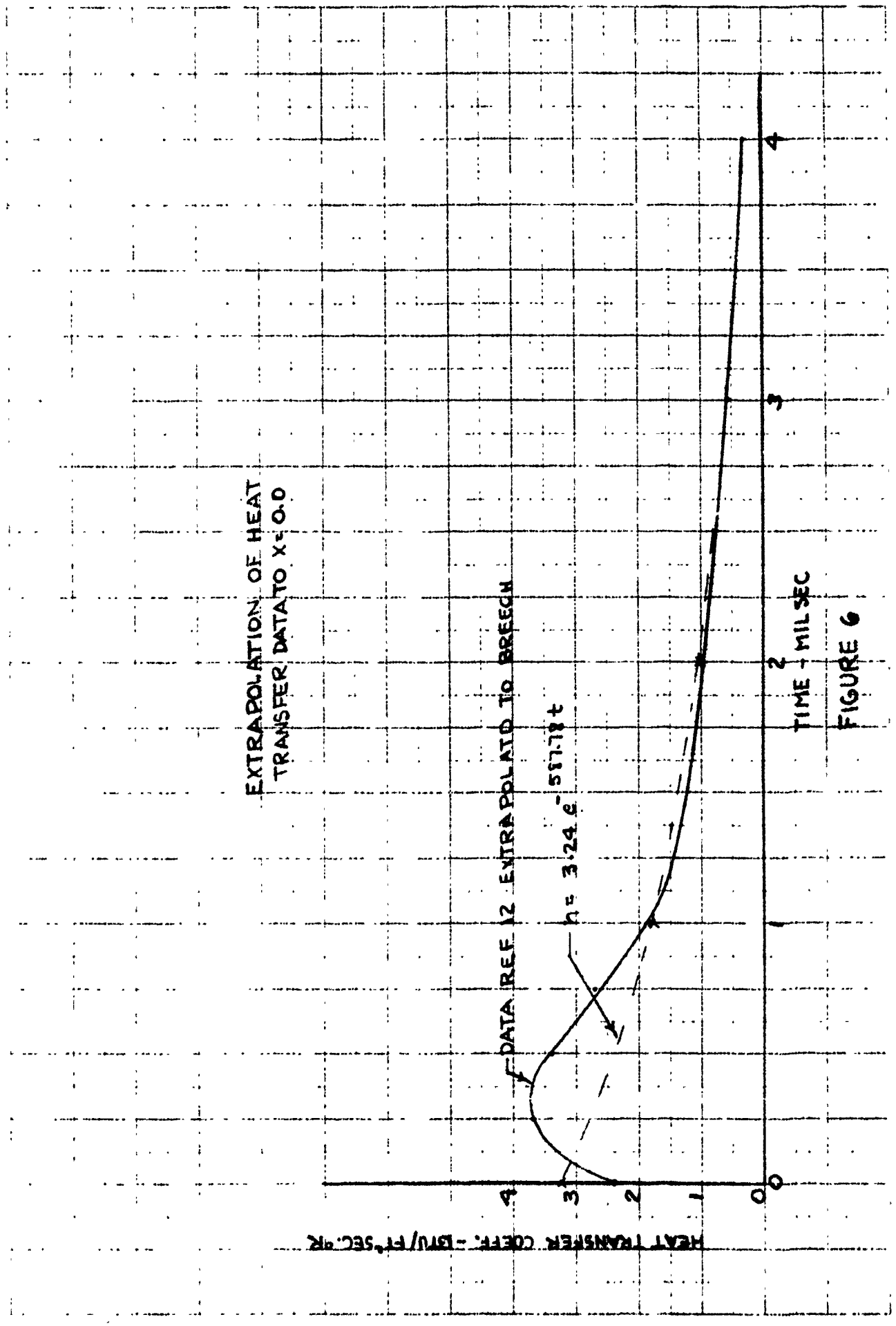
EXTRAPOLATION OF HEAT TRANSFER DATA TO X=0.0
DATA REF 12 EXTRAPOLATD TO BREECH
h = 3.24 e^-551.78t
HEAT TRANSFER COEFF. - BTU/FT²SEC°R
TIME - MILSEC
0
1
2
3
4

FIGURE 6

Interior Region

The interior region refers to those locations not considered to be the breech or projectile base. In the interior region, the boundary layer along the wall is fairly well defined. For this reason the approach has been to use the standard Reynolds Analogy:

$$\frac{h}{\rho V c_p} = \frac{f}{2}$$

The selection of the friction factor, f, is somewhat arbitrary but values between .001 and .006 have been used.

Projectile Base Region

Describing the heat transfer in this region is very difficult because the normal boundary layer approximations are not satisfied. In this region the gas velocity at the wall or base of the projectile equals the projectile velocity which is the highest velocity in the problem. In addition, the level of projectile heating (barrel heat transfer at projectile) has a profound effect upon the peak projectile base pressure and therefore, the launch velocity.

Initially, the approach was to simply use the Reynolds Analogy in this region with the gas velocity equal to the projectile, velocity. However, this approach grossly underestimates the peak pressure at the base. Since the pressure increases with entropy decrease assuming constant gas temperature, this condition is valid as long as the propellant is burning. Entropy can be decreased by higher heat transfer rates. The approach taken therefore, was to arbitrarily increase the heat transfer coefficient. Factors of ten were necessary to significantly increase the base pressure. The requirement of a factor of this size seems unreasonable. In addition the peak heating rate occurs near the barrel exit which does not agree with test data.

The next approach was to use the heating rate based upon a shock tube end condition. This procedure was developed in reference (13). In this formulation the following equation was used:

$$h = \left(\rho_3 \gamma C_v k \right)^{1/2} T_{gas} \left(.5 + .5 \left(\frac{T_w}{T_{gas}} \right)^2 - \frac{T_w}{T_{gas}} \right)^{1/2}$$

In this representation the shape of the heat transfer coefficient as a function of time is correct. However, the overall levels are too low. The constant, K, can vary by a factor of 2 (reference (13)) but this is still inadequate to predict the required levels of heating.

The most promising procedure is the one described in reference (3). In this reference the flow in the barrel is examined qualitatively and quantitatively. The reference notes that away from the projectile the Reynolds analogy could be used. However, in the projectile base region the momentum thickness goes to zero because the gas velocity and projectile velocity are equal. If a boundary layer analysis were attempted this would yield an infinite Nusselt number.

The approach taken in reference (3) was to use the usual boundary layer method away from the projectile and breech. These locations are handled using approximate solutions. The analysis develops the integral momentum equation and solves this equation introducing empirical data when needed. The projectile heat transfer is given by:

$$Nu = \frac{4 a^2 Re_L Pr^{1/3}}{Re_{\theta_L}}$$

where:

Nu = Nusselt Number hcl/k

a = shear law constant = .332

P_r = Prandtl Number

Re_L = Reyrolds Number based upon gas velocity and L

Re_{θ_t} = Reynolds Number based upon momentum thickness at transition

L = Distance from projectile to breech

The transition Reynolds Re_{θ_t} taken as 200. The heat transfer prediction method of reference (3) was used in the computer model because it provided the right profile and heat transfer rates of the right magnitude.

Numerical Results

Using the analysis described in the preceding sections, an analytical model computer program was developed and number of cases were examined. The most important parameter affecting the pressure and velocity history is the surface concentration density (SCD) of deterrent. Figures 7 and 8 present the pressure and velocity histories for an SCD of 0.1 and 0.2 respectively. In these two cases it is assumed that the flame velocity through the propellant bed is infinite. The surface concentration density of deterrent refers to the distribution of deterrent through the propellant grain. For all cases using the same propellant, the total amount of deterrent is the same; however, for an SCD of 0.1 compared to 0.2 the deterrent is spread to a greater grain depth. This means that the effect of the deterrent is less over a greater depth. For instance the SCD = 0.1 deterred flame temperature is 4615^{o}R compared to 3331 ^{o}R for an SCD = 0.2. From figure 7 it can be seen that the launch velocity for an SCD = 0.1 is 2500 fps. Launch occurs 1.27 milsec after ignition. For SCD = 0.2, (figure 8) the launch velocity is 1700 fps at 1.69 milsec. For both these runs the heat transfer rates at the projectile base were defined by the results of reference (3). Comparing the results from figure 7 and 8 with figure 1, the experimental data, it can be seen that considerable differences exist. In figure 7 the launch velocity and peak projectile base pressure are closer to the experimental results than figure 8. However, the launch velocity is only 78% of the measured data and the peak projectile pressure is only 59% of the corresponding experimental results. In contrast the peak breech pressure from figure 7 is 183% of the results in figure 1. It is felt that these differences can be attributed to:

(1) projectile heat transfer

(2) flame front velocity

Based upon earlier heat transfer studies it was determined that projectile heat transfer had a strong influence on projectile velocity and projectile base pressure. For comparison the projectile heat rate for an SCD of 0.1 and 0.2 are presented versus travel in figure 9. The peak projectile heating is 14,500 Btu/ft^2 sec at approximately 7 inches of travel. This is approximately one half the predicted peak from reference (3). In addition, the peak occurs further downstream than anticipated by reference (3) and experimental experience.

The large variation in peak breech pressure was attributed to the high flame speed. To examine this effect, a finite flame program was written and run. The analytical basis for this program was described earlier. Table 1 presents some results from the finite flame velocity analysis.

TABLE 1

	SCD = 0.1	SCD = 0.2	SCD = 0.1*
Max Pressure behine projectile-psia	414081	14,720	16,972
Flame Travel Time - sec	$.595 \times 10^{-3}$	$.756 \times 10^{-3}$	$.731 \times 10^{-3}$

*SCD=0.1 but 1/2 burning rate to simulate ignition delay

Figure 10 presents the volume fraction, ε, distribution versus distance for the same cases on Table 1.

The pressure and velocity history using a finite flame velocity is given in figure 11 for SCD=0.2. The launch velocity is increased by 400 fps and the projectile base pressure is increased slightly. The breech pressure is reduced significantly because the finite flame velocity analysis sets the volume fraction at the breech equal to zero. Figure 12 presents the same results for an SCD=0.1. The launch velocity in figure 12 is 3800 fps. The peak projectile base pressure is 64,500 psia. This represents a substantial change over the results in figure 7. The flat response of the

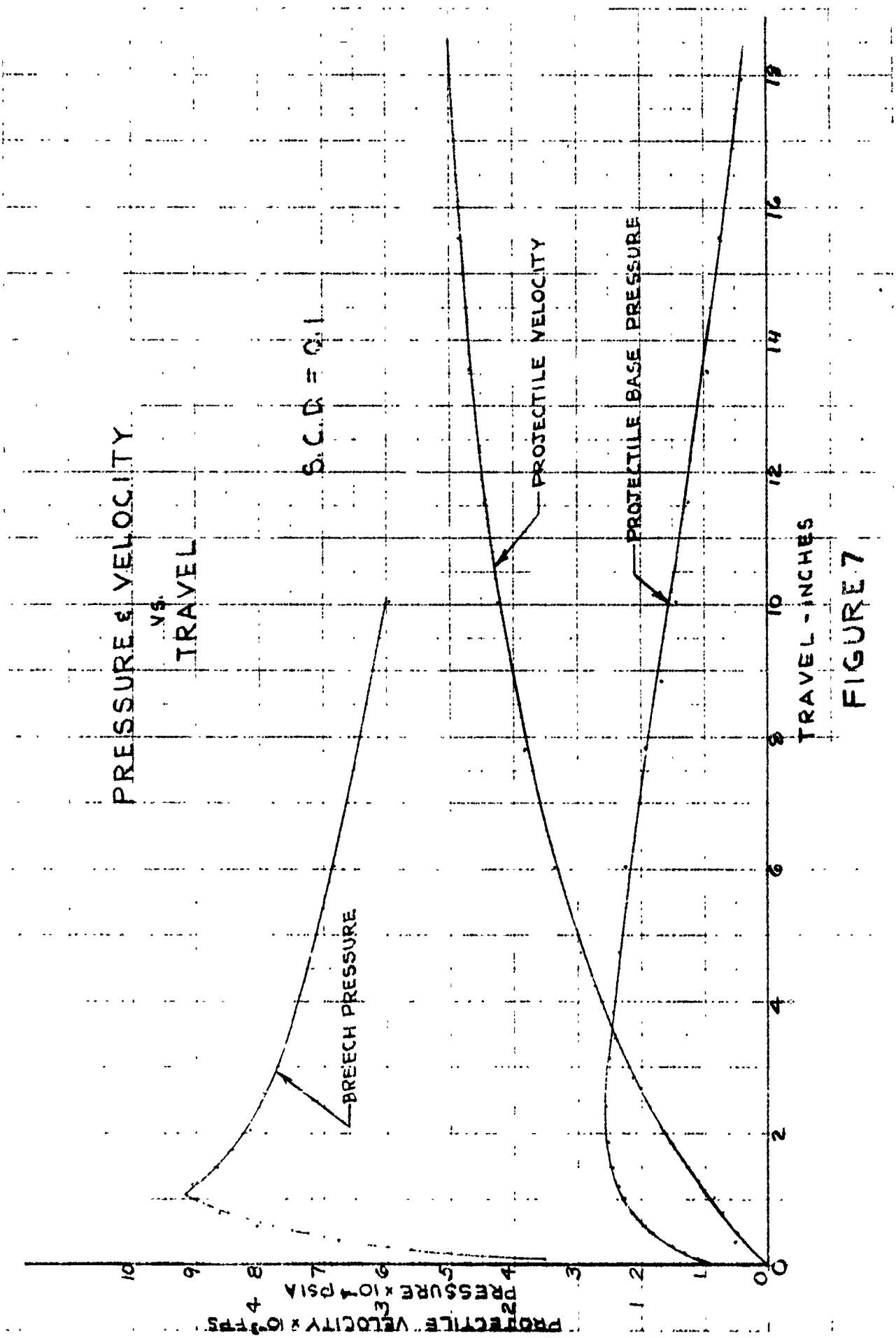
PRESSURE & VELOCITY
VS.
TRAVEL
S.C.D. = 0.1
PROJECTILE VELOCITY
PROJECTILE BASE PRESSURE
BREECH PRESSURE
PROJECTILE VELOCITY x 10⁻³ FPS
PRESSURE x 10⁻⁴ PSIA
TRAVEL - INCHES
0
2
4
6
8
10
12
14
16
18
1
2
3
4
5
6
7
8
9
10

FIGURE 7

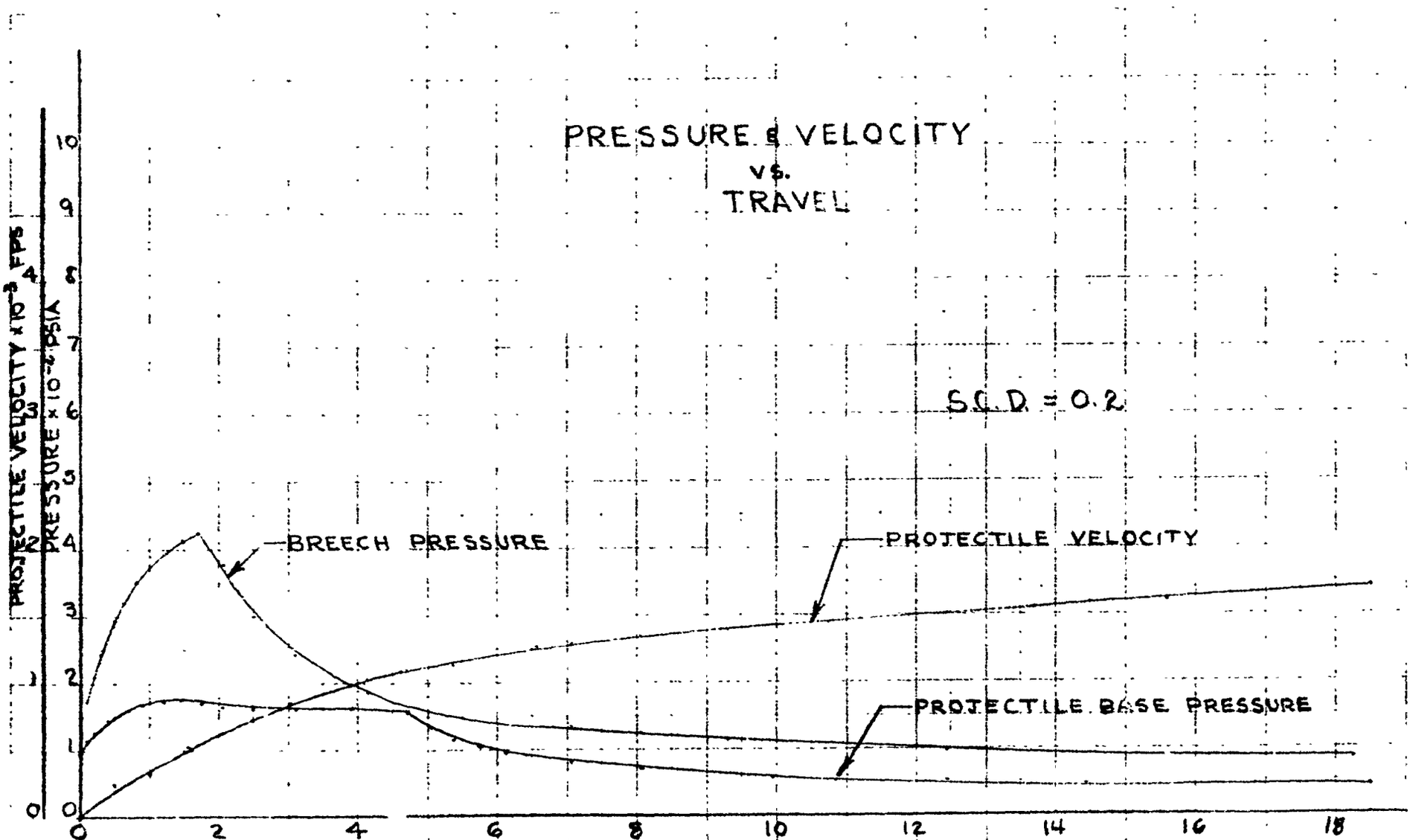
PRESSURE & VELOCITY
vs.
TRAVEL
S.C.D. = 0.2
BREECH PRESSURE
PROJECTILE VELOCITY
PROJECTILE BASE PRESSURE
PROJECTILE VELOCITY x 10⁻³ FPS
PRESSURE x 10⁻⁴ PSIA
TRAVEL - INCHES
0
2
4
6
8
10
12
14
16
18

FIGURE 8

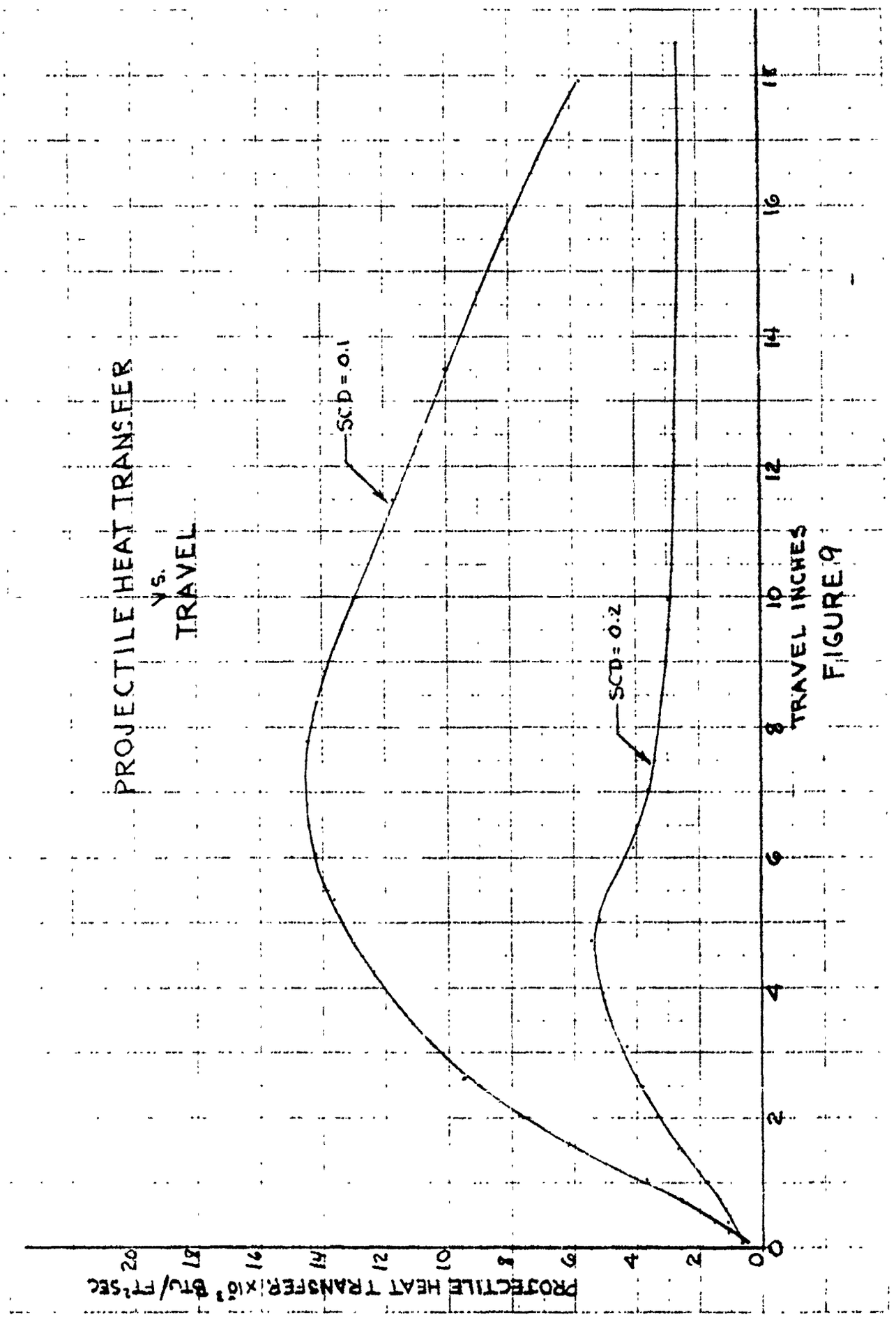
PROJECTILE HEAT TRANSFER
vs.
TRAVEL
SCD = 0.1
SCD = 0.2
PROJECTILE HEAT TRANSFER, x10³ BTU/FT²SEC
TRAVEL INCHES
0
2
4
6
8
10
12
14
16
18
20

FIGURE 9

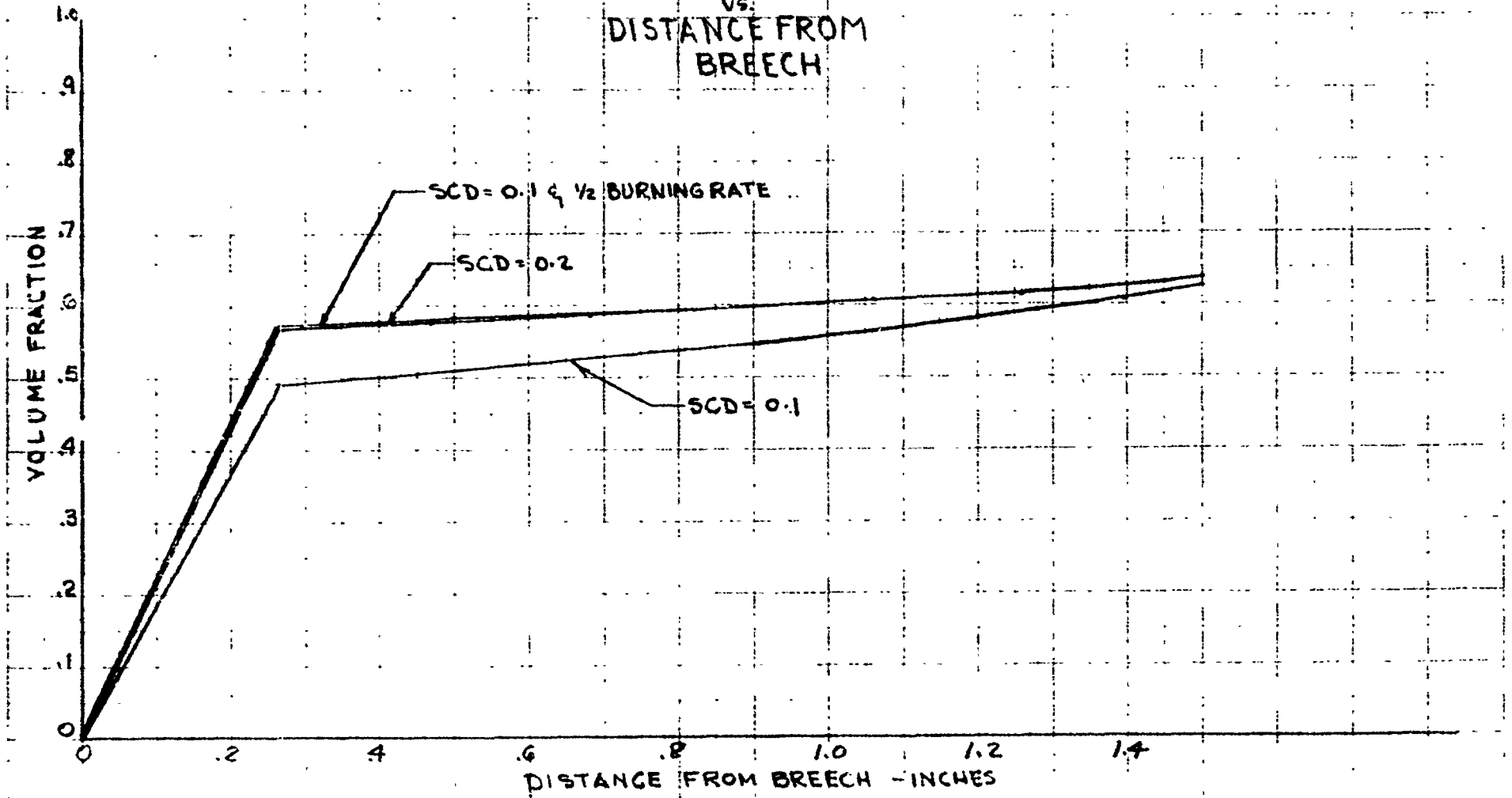
VOLUME FRACTION
VS.
DISTANCE FROM
BREECH
SCD = 0.1 & 1/2 BURNING RATE
SCD = 0.2
SCD = 0.1
VOLUME FRACTION
0
.1
.2
.3
.4
.5
.6
.7
.8
.9
1.0
0
.2
.4
.6
.8
1.0
1.2
1.4
DISTANCE FROM BREECH - INCHES

FIGURE 10

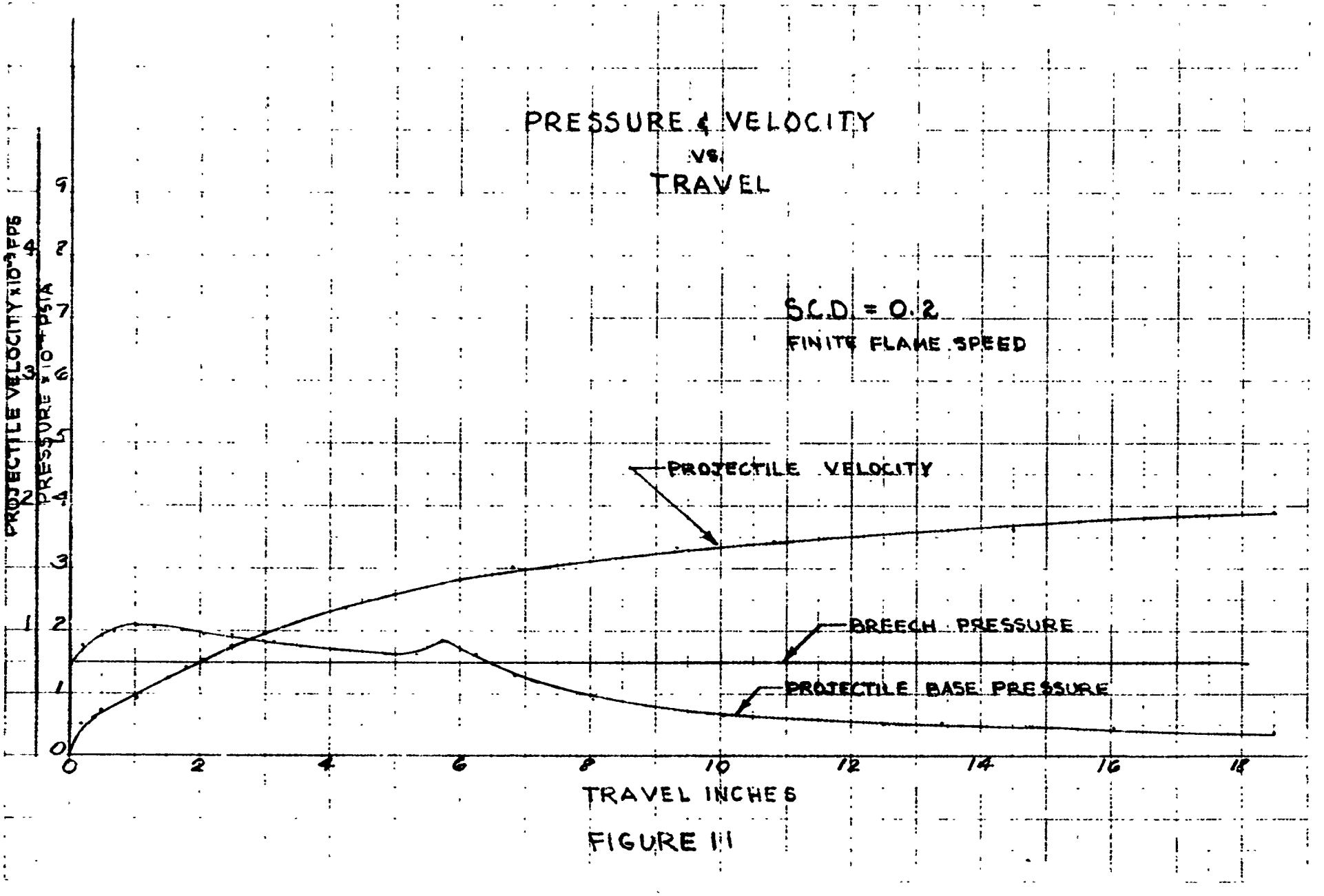
PRESSURE & VELOCITY
VS.
TRAVEL
S.C.D. = 0.2
FINITE FLAME SPEED
PROJECTILE VELOCITY x 10⁻³ FPS
PRESSURE x 10⁻³ PSIA
PROJECTILE VELOCITY
BREECH PRESSURE
PROJECTILE BASE PRESSURE
0
2
4
6
8
10
12
14
16
18
TRAVEL INCHES

FIGURE 11

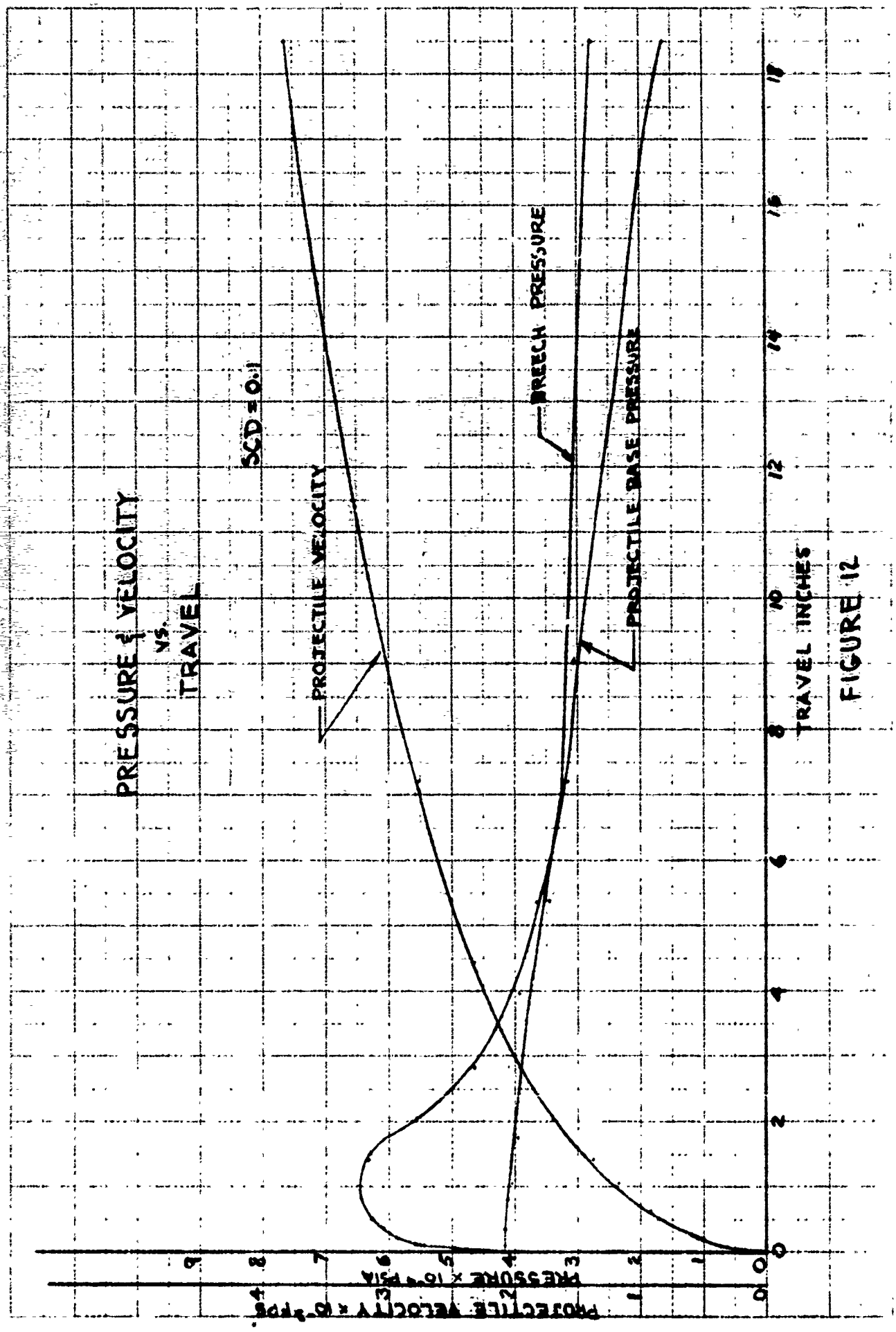
PRESSURE & VELOCITY
VS.
TRAVEL
SCD = 0.1
PROJECTILE VELOCITY
BREECH PRESSURE
PROJECTILE BASE PRESSURE
PROJECTILE VELOCITY × 10⁻³ FPS
PRESSURE × 10⁻⁴ PSIA
TRAVEL INCHES
0
2
4
6
8
10
12
14
16
18
1
2
3
4
5
6
7
8
9

FIGURE 12

breech pressure curve is again attributed to the zero volume fraction at the breech.

To eliminate the flat response at the breech when using the finite flame criteria, it was decided to set the volume fraction at breech equal to the first station in the flame front analysis. This change increases the breech pressure significantly. For an SCD of 0.1, the results are not reasonable. To moderate this effect, the one half burning rate was used in the flame front analysis (see Table 1). With the breech volume fraction equal to zero, the peak breech pressure was approximately 17,000 psia. The peak projectile bore pressure and launch velocity were 57,500 psia and 3640 fps respectively. Figure 13 presents the complete pressure and velocity history for the non zero breech volume fraction. In this figure it can be seen that the peak breech pressure is 130,000 psia. This is approximately twice the experimental results. For this case the starting breech volume fraction was 0.574. A lower value would reduce this pressure.

Regarding the choice of heat transfer model, a number of runs were made using the usual Reynolds analogy. The results from these runs yielded launch velocities and pressure at the projectile well below test data. Some parametric runs were made to determine the necessary increase in heat transfer required to yield agreement with test. These results indicated a factor of 10 or 20 which does not seem reasonable. In addition the general shape of the profile predicted by the Reynolds analogy is incorrect. The Reynold's heating peaks with peak velocity. This means that peak heating would occur near launch. For this reason, the decision was made to use the heat transfer model described in reference (3) exclusively.

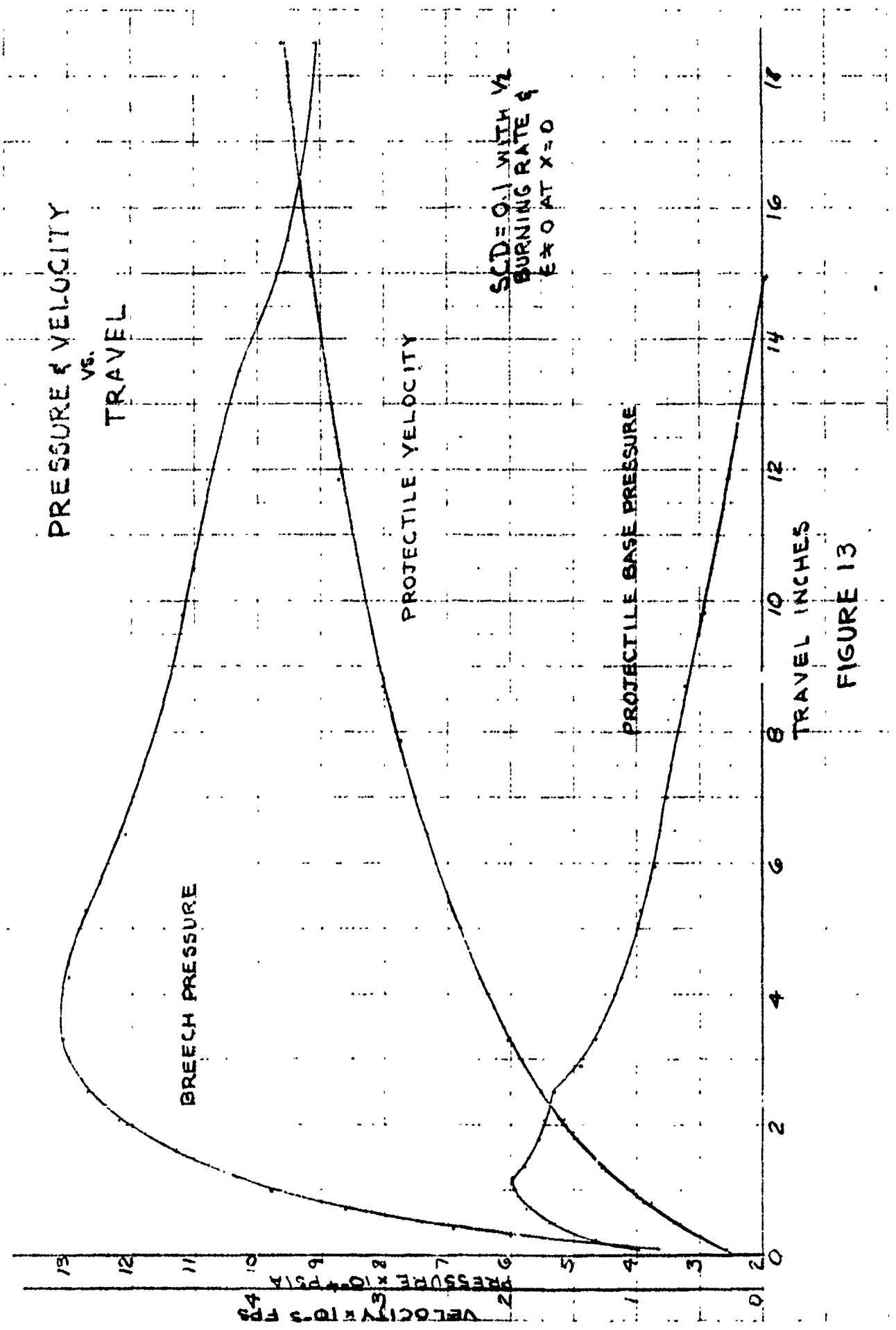
PRESSURE & VELOCITY
VS.
TRAVEL
S.C.D. = 0.1 WITH V2
BURNING RATE ζ
E = 0 AT X = 0
BREECH PRESSURE
PROJECTILE VELOCITY
PROJECTILE BASE PRESSURE
TRAVEL INCHES
PRESSURE × 10⁻⁴ PSIA
VELOCITY × 10⁻³ FPS

FIGURE 13

Conclusions and Recommendations

Computer models have been used to describe the flow field in small arms and also to describe the propagation of a flame front through a porous propellant bed. The model determines the pressure, velocity, propellant volume fraction, and temperature versus time and position.

Numerical results have shown the dependence of projectile velocity and barrel pressure history upon deterrent concentration density and flame velocity. It has also been determined that the heat transfer to the base of the projectile also has a significant effect upon the launch condition.

Unfortunately, it was not possible to simulate the experimental data presented in figure 1. However, with some additional effort, the author feels that the results of figure 1 can be duplicated. Toward this goal, the following recommendations are made:

(1) Vary breech volume fraction to simulate breech pressure.

(2) Parametric study of projectile base heat transfer to obtain a steeper pressure profile.

(3) Parametric study of projectile resistive force.

(4) Development of analytical equations for flow at the base of the projectile.

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APPENDIX A

Description and listing of
Flow Model Computer Program

COMPUTER PROGRAM

The computer program implementing the numerical procedures describer in preceding sections composed of a main program and 10 subprograms. The main program controls the flow of the calculation. The subprogram performs certain specific functions during the calculation. The name and function of each subprogram is given below:

SUBPROGRAM	FUNCTION
DIME	calculates the surface area, Sp, and volume, V_p, of propellant grain as a function of distance normal to the surface.
START	generates the pressure, entropy time history along the first Q wave from the starting point to the breech. The required number of points is specified in the input and program divides the distance from projectile base to breech into equal increments.
INTER	calculates the interior points using value at (3) and (4) (figure 2) as known
BREACH	calculates the breech points
BASEP	calculates the projectile base points
OUTPUT	points the required output information in the specified format
DIAGO	a diagnostic program which prints certain information if the calculations in INTER, BREACH, and BASEP do not converge within 10 tries
RFORCE	calculates the engraving force as to function of X based upon tabular input data
PROP	calculates the specific heat ratio, gas constant and specific heat at constant volume as a function of the amount of deterred and non-deterred propellant burned. The program is called only after the deterred layer is burned through

TAREA	This is a subjunction which calculates the tube cross sectional area as a function of X based on tabulated inputs.

The main program maintains control of the overall program and stores the calculated values of various parameters (i.e., gas velocity, propellant velocity, pressure, etc.) This data is stored in a double subscripted (i,j) array. When i equals j, the points correspond to the base of the projectile. Along a given Q wave the j subscript is constant. For a fixed value of j, the number of i values which determines the grid size is an input parameter. Figure 5 presents an abbreviated flow diagram for the main program.

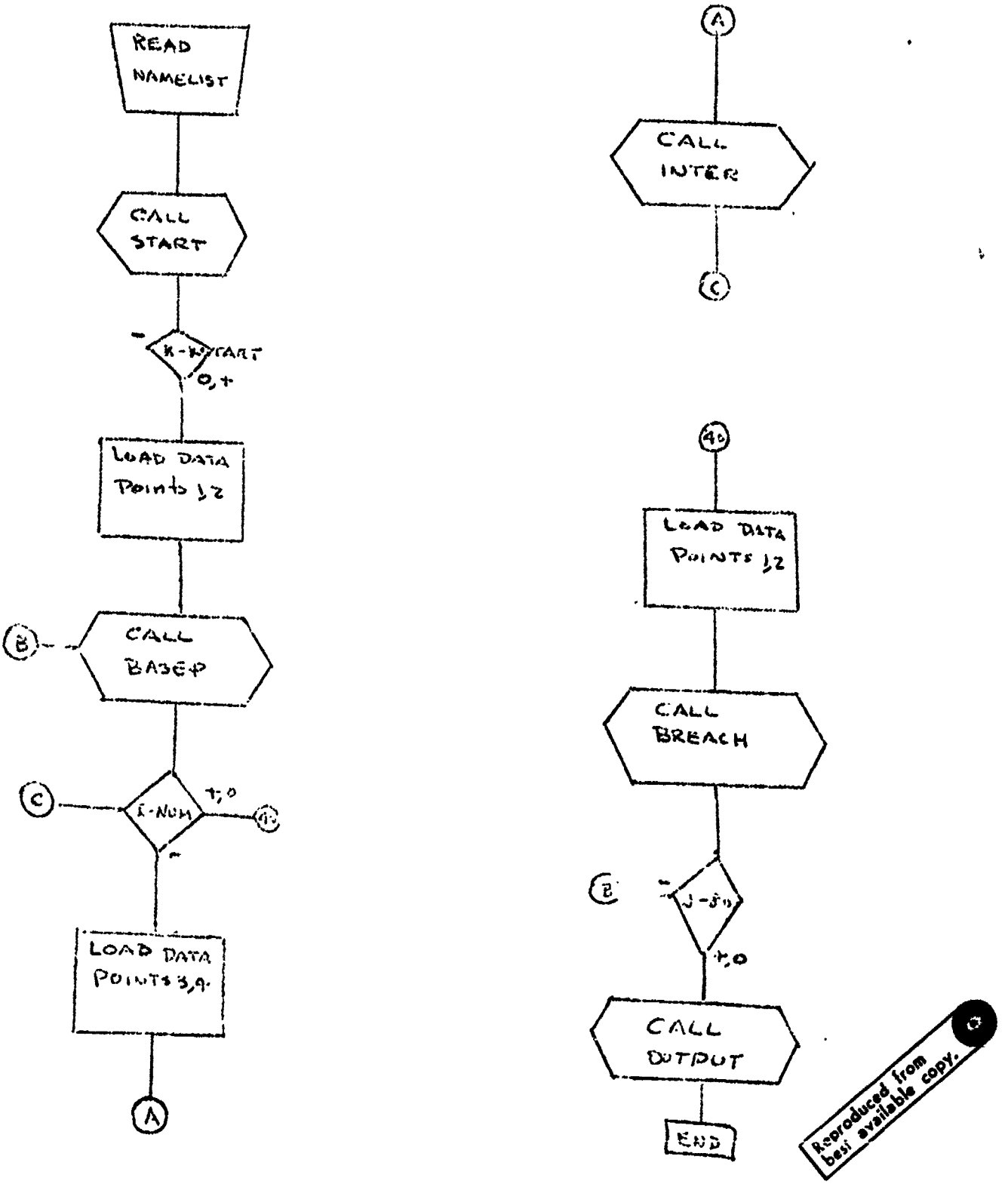

FIGURE 5 : FLOW DIAGRAM for Computer Program

DESCRIPTION OF ARRAYS AND VARIABLES

X(I,J)	X, location of grid points
TIME(I,J)	t, time of grid points
VEL(I,J)	gas velocity at grid points
SS(I,J)	speed of sound at grid points
PRESS(I,J)	pressure at grid points
TEMP(I,J)	gas temperature at grid points
DENS(I,J)	gas density at grid points
VELP(I,J)	propellant velocity at grid points
ENTR(I,J)	entropy of gas at grid points
VOLFRA(I,J)	volume fraction at grid points
AL(I,J)	distance burned normal to propellant grain surface at grid points

The following are used in the main program and subroutine INTER (see Figure 2)

X3, X4, X6	spatial location of points 3, 4 and 6
T3, T4, T6	time location of points 3, 4 and 6
UG3, UG4, UG6	gas velocity at points 3, 4, and 6
A3, A4, A6	speed of sound at points 3, 4 and 6
PSI3, PSI4, PSI6	gas pressure at points 3, 4 and 6
AREA3, AREA4, AREA6	tube cross sectional area at points 3, 4 and 6
TG3, TG4, TG6	gas temperature at points 3, 4 and 6
RHO3, RHO4, RHO6	gas density at points 3, 4 and 6

UP3, UP4, UP6	propellant velocity at points 3, 4 and 6
S3, S4, S6	entropy at points 3, 4 and 6
EPL3, EPL4, EPL6	propellant volume fraction at points 3, 4 and 6
L3, L4, L5, L6	distance burned normal to propellant grain at points 3, 4, 5 and 6

The following are used in the main program and subroutines BREACH and START (see figure 4)

X12	spatial location of point 2
T11, T12, T13	time location of point 1, 2 and 3
UG11, UG12, UG13	gas velocity at points 1, 2 and 3
A11, A12, A13	speed of sound at points 1, 2 and 3
PSI1, PSI2, PSI3	gas pressure at points 1, 2 and 3
TG11, TG12, TG13	gas temperature at points 1, 2 and 3
RHO11, RHO12, RHO13	gas density at points 1, 2 and 3
S11, S12, S13	entropy at points 1, 2 and 3
EPL11, EPL12, EPL13	propellant volume fraction at points 1, 2 and 3
AREA11, AREA12, AREA13	tube cross sectional area at points 1, 2 and 3
L11, L13	distance burned normal to propellant grain at points 1 and 3
UP12	propellant velocity at point 2

The following are used in the main program and subroutine BASEP (see figure 3)

X21, X22, X24	spatial location of points 1, 2 and 4
T21, T22, T24	time location of points 1, 2 and 4

UG21, UG22, UG24	gas velocity at points 1, 2 and 4
A21, A22, A24	speed of sound at points 1, 2 and 4
PSI21, PSI22, PSI24	gas pressure at points 1, 2 and 4
TG21, TG22, TG24	gas temperature at points 1, 2 and 4
RHO21, RHO22, RHO24	gas density at points 1, 2 and 4
S21, S22, S24	entropy at points 1, 2 and 4
EPL21, EPL22, EPL24	propellant volume fraction at points 1, 2 and 4
AREA21, AREA22, AREA24	tube cross sectional area at points 1, 2 and 4
L21, L24	distance burned normal to the propellant surface at points 1 and 4
NSTART	the value of the I index for the Q wave

SUBROUTINES:

Name: INTER

Purpose: To calculate the characteristics and thermodynamic properties at any interior point in the mesh

Input: X3, X4, T3, T4, UG3, UG4, A3, A4, PSI3, PSI4, TG3, TG4, RHO3 RHO4, UP3, UP4, S3, S4, EPL3, EPL4, AREA3, AREA4, L3, L4

Output: X6, T6, UG6, A6, PSI6, TG6, TH06, UP6, S6, EPL6, AREA6,

Method: An iterative shceme is used until convergence to within a given tolerance is obtained. Normally, the convergence check is made on the calculated speed of sound and gas velocity. However, for gas velocities below 300 ft/sec only the speed of sound is checked. This procedure was necessary because below 300 ft/sec the accuracy requirements on P and Q for calculating gas velocity become excessive. At 300 ft/sec, a one percent error in the speed of sound can be translated into approximate a 10 percent in the gas velocity. At higher gas velocities, the percentage tolerance on gas velocity and speed of sound are the same.

Name: BREACH

Purpose: To calculate the characteristics and thermodynamic properties at the location X=0

Input: X12, TI1, TI2, UG11, UG12, A11, A12, PSI11, PSI12, TG11, TG12. RHO11, RHO12, S11, S12, EPL11, EPL12, AREA11, AREA12, L11, UP12

Output: T13, UG13, A13, PSI13, TG13, RHO13, S13, EPL13, L13

Method: An iterative scheme is used until convergence to within a given tolerance is obtained.

Name: BASEP

Purpose: To calculate the characteristics and thermodynamic properties at the base of the projectile and to locate the base of the projectile

Input: X21, X22, T21, T22, UG21, UG22, A21, A22, PSI21, PSI24, TG22, RHO21, RHO22, S21, S22, EPL21, EPL22, AREA21, AREA22, L21, NSTART

Output: X24, T24, UG24, PS124, TG24, RH024, S24, EPL24, AREA24, L24

Method: An iterative scheme is used until convergence to within a given tolerance is obtained

Name: DIME (A, B, SURA, SURB, VOLA, VOLB)

Purpose: To calculate the average propellant grain surface area and volume

Input: A, B corresponding to the distance burned normal to the surface (ℓ) at two points

Output: SURA, SURB surface of grain at respective points

VOLA, VOLB volume of grain at respective points

Method: The following equations are used

$$\text{Volume} = 2\pi(\bar{R} - R_i)^2(R_i - \ell) + \pi^2(R_i - \ell)^2\left((\bar{R} - R_i) + \frac{4}{3\pi}(R_i - \ell)\right)$$

$$\text{Surface area} = 2\pi\left(\pi(R_i - \ell)\left((\bar{R} - R_i) + \frac{2}{\pi}(R_i - \ell)\right) + (\bar{R} - R_i)^2\right)$$

Name: OUTPUT

Purpose: To print the calculated results

Input: Calculated velocities and thermodynamic properties at all of the grid points

Output: Printed results including values of P and Q at each grid point

Method: Format of printed results specified in subroutine

Name: DIAGO (LA, BUG, AA)

Purpose: A diagonostic tool to print results calculated prior to a failure to converge event

Input: LA indication of which subroutine failed LA=1 INTER
LA=2 BASEP
LA=3 BREACH

BUG last calculated gas velocity

AA last calculated speed of sound

Output: Printed output up to and including current value of thermodynamic variables

Method: Format used is specified in the subroutine

Name: RFORCE(A,B)

Purpose: Calculates the engraving force or a function of distance from initial projectile location

Input: A, location of the base of the projectile

Output: B, resistance force in lbf

Method: Resistance force tabulated or function of distance based upon data from Report R-2066, "Study of the Pressure Distribution Behind the M193 Projectile When Fired in the M16 Rifle Barrel"

Name: PROP(A)

Purpose: Calculates the CV, GAMMA, and RGAS as a function of the percentage deterred and non-deterred propellant burned. Subroutine is used only when distance burned normal to the surface is greater than the deterred thickness of 0.000084 ft

Input: A distance burned normal to the propellant surface at a given mesh point

Output: CV, GAMMA, RGAS

Method: Calculation assumes the gas generated to be ideal and that properties are based upon mass fraction of components present

Name: START

Purpose: Calculates the thermodynamic properties along the first Q wave at specified intervals

Input: Same as BREACH but here they refer to data along first Q wave

Output: Same as BREACH but here they refer to data along first Q wave

Method: The subroutine is called a specified number of time (i.e. an input parameter). Each time it calculates the thermodynamic properties along the first Q wave. The results are equally spaced between the projectile initial location and the breech. The gas velocity and heat transfer along this wave are zero

SUBFUNCTION

Name: TAREA(ALOC)

Purpose: Calculates the tube cross sectional area as a function of distance from the breech

Input: ALOC location of the grid point

Output: TAREA tube cross sectional in ft^2

Method: Interpolates data from table which is supplied

```
C THIS PROGRAM CALCULATES THE PRESSURE,TEMPERATURE AND VELOCITY
C TIME HISTORIES BASED UPON A GIVEN PROPELLANT CHARACTERISTIC
C THIS IS THE MAIN PROGRAM
      REAL L5,L6,L3,L4,L21,L24,L11,L13
      COMMON/C/X(99,99),TIME(99,99),VEL(99,99),SS(99,99),PRESS(99,99),TE
     $MP(99,99),DENS(99,99),VELP(99,99),ENTR(99,99),VOLFRA(99,99),AL(99,
     ?99)
      COMMON/COM1/X3,X4,X6,T3,T4,T6,UG3,UG4,UG6,A3,A4,A6,PSI3,PSI4,PSI6,
     $TG3,TG4,TG6,RHO3,RHO4,RHC6,UP3,UP4,UP6,S3,S4,S6,EPL3,EPL4,EPL6,ARE
     >A3,AREA4,AREA6,L3,L4,L5,L6
      COMMON/COM2/X12,T11,T12,T13,UG11,UG12,UG13,A11,A12,A13,PSI11,PSI12
     $,PSI13,TG11,TG12,TG13,RHC11,RHO12,RHO13,S11,S12,S13,EPL11,EPL12,EP
     ^L13,AREA11,AREA12,AREA13,L11,L13,UP12
      COMMON/COM3/X21,X22,X24,T21,T22,T24,UG21,UG22,UG24,A21,A22,A24,PSI
     $21,PSI22,PSI24,TG21,TG22,TG24,RHC21,RHO22,RHO24,S21,S22,S24,EPL21,
     )EPL22,EPL24,AREA21,AREA22,AREA24,L21,L24,NSTART
      COMMON/SAME/RGAS,PSI1,GAMMA,DZERO,PIE,BEE,PODPOT,CV,BASE,GZERO,WT,
     >TOL,AN
      COMMON/STAR/KSTART
      COMMON/OUT/MAXJ,MAXI
      COMMON/EXIT/XLIM,TMUZE
      COMMON/BHEAT/FACT,TWALL,FRICT
      COMMON/PARM/PARIN1,PARIN2,PARIN3,PARBA1,PARBA2,PARBA3,PARBR1,PARBR
     $2,PARBR3
      COMMON/PCWDER/CV1,CV2,RGAS1,RGAS2,GAM1,GAM2,DLAY
      NAMELIST/PAR/PARIN1,PARIN2,PARIN3,PARBA1,PARBA2,PARBA3,PARBR1,PARB
     $R2,PARBR3
      NAMELIST/INPOT/BASE,WT,TCL,XLIM,PSI1,DZERO,TZERO,XZERO,FRAC1,ALL1,
     $FACT,TWALL,AN,FRICT,I,J
      NAMELIST/PRCPEL/PP1,PP2,CV1,CV2,BEE1,BEE2,RGAS1,RGAS2,DLAY,GAM1,GA
     #M2
  999 READ(5,INPOT)
      READ(5,PAR)
      READ(5,PROPEL)
      WRITE(6,PROPEL)
      WRITE(6,INPOT)
      WRITE(6,PAR)
      DO 1 II=1,99
      DO 1 JJ=1,99
      SS(II,JJ)=0.0
    1 CONTINUE
      TMUZE=1000.
      PODPOT=PP1
      CV=CV1
      GAMMA=GAM1
      BEE=BEE1
      RGAS=RGAS1
      PIE=3.141593
      GZERO=32.2
      TATM=530.
      IF(I.GT.0)GO TO 20
C CALCULATION OF FIRST TWO POINTS
      X(1,1)=XZERO
      X(2,1)=0.0
      TIME(1,1)=0.0
      SS(1,1)=SQRT(GAMMA*RGAS*TZERO)
      VOLFRA(1,1)=FRAC1
      VEL(1,1)=0.0
      VELP(1,1)=0.0
      PRESS(1,1)=PSI1
      TEMP(1,1)=TZERO
      DENS(1,1)=PSI1/RGAS/TZERO
      ENTR(1,1)=(RGAS/778.+CV)*ALOG(TZERO/TATM)-RGAS/778.*ALOG(PSI1/2075
```

```
      #.)
       ENTR(1,1)=ENTR(1,1.*778./RGAS/GAMMA
       AL(1,1)=ALL1
       IF(ALL1.GT.CLAY) BEE=BEE1
       IF(ALL1.GT.CLAY) PODPOT=FP2
       IF(ALL1.GT.CLAY)CALL PROP(ALL1)
       KSTART=J
       DO 3 I=1,KSTART
       K=I+1
       X12=X(I,1)
       T12=TIME(I,1)
       EPL12=VOLFRA(I,1)
       A12=SS(I,1)
       UG12=VEL(I,1)
       UP12=VELP(I,1)
       PSI12=PRESS(I,1)
        TG12=TEMP(I,1)
       RHO12=DENS(I,1)
       S12=ENTR(I,1)
       L11=AL(I,1)
       CALL START
       PRPSI=PSI13/144.
       WRITE(6,5002)A13,L13,PRPSI,TG13,RHO13,S13,EPL13,T13
       X(K,1)=X12
       TIME(K,1)=T13
       VOLFRA(K,1)=EPL13
       SS(K,1)=A13
       VEL(K,1)=UG13
       VELP(K,1)=UG13
       PRESS(K,1)=PSI13
       TEMP(K,1)=TG13
       DENS(K,1)=RHO13
       ENTR(K,1)=S13
       AL(K,1)=L13
     3 CONTINUE
       NCHECK=0
       JLAST=1
     5 DO 50 I=2,80
       IF(NCHECK.GT.0)JLAST=JLAST+1
       NCHECK=0
       JQ=JLAST+1
       DO 49 JJ=JQ,I
       J=JJ
       IF(JJ.EQ.I) GO TO 55
       X4=X(I-1,J)
       X3=X(I,J-1)
       T4=TIME(I-1,J)
       T3=TIME(I,J-1)
       UG4=VEL(I-1,J)
       UG3=VEL(I,J-1)
       A4=SS(I-1,J)
       A3=SS(I,J-1)
       PSI4=PRESS(I-1,J)
       PSI3=PRESS(I,J-1)
       TG4=TEMP(I-1,J)
       TG3=TEMP(I,J-1)
       RHO4=DENS(I-1,J)
       RHO3=DENS(I,J-1)
       UP4=VELP(I-1,J)
       UP3=VELP(I,J-1)
       S4=ENTR(I-1,J)
       S3=ENTR(I,J-1)
       EPL4=VOLFRA(I-1,J)
```

```
      EPL3=VOLFRA(I,J-1)
      L4=AL(I-1,J)
      L3=AL(I,J-1)
      TOOL=(L3+L4)/2.
      IF(TCOL.GT.CLAY) GO TO 10
      GO TO 14
   10 BEE=BEE2
      PODPOT=PP2
      CALL PROP(TCOL)
   14 AREA3=TAREA(X3)
      AREA4=TAREA(X4)
      WRITE(6,5003)X3,X4,T3,T4,UG3,UG4,A3,A4,PSI3,PSI4,TG3,TG4,RHO3,RHO4
     $,UP3,UP4,EPL3,EPL4,L3,L4,AREA3,AREA4
      CALL INTER
      RGAS=RGAS1
      BEE=BEE1
      GAMMA=GAM1
      CV=CV1
      PODPCT=PP1
      PRPSI=PSI6/144.
      WRITE(6,5002)A6,L6,PRPSI,TG6,RHO6,S6,EPL6,T6,X6,UG6,UP6
 5002 FORMAT(1X,11E11.3)
      X(I,J)=X6
      TIME(I,J)=T6
      VEL(I,J)=UG6
      PRESS(I,J)=PSI6
      TEMP(I,J)=TG6
      DENS(I,J)=RHO6
      VELP(I,J)=UP6
      ENTR(I,J)=S6
      VOLFRA(I,J)=EPL6
      SS(I,J)=A6
      AL(I,J)=L6
      IF(X3.LE.0.0) GO TO 40
      IF(T6.GE.TMUZE) GO TO 48
      GO TO 49
   55 L=JJ-1
      NSTART=I-1
      K=I
      X21=X(NSTART,L)
      X22=X(K,L)
      T21=TIME(NSTART,L)
      T22=TIME(K,L)
      UG21=VEL(NSTART,L)
      UG22=VEL(K,L)
      A21=SS(NSTART,L)
      A22=SS(K,L)
      PSI21=PRESS(NSTART,L)
      PSI22=PRESS(K,L)
      TG21=TEMP(NSTART,L)
      TG22=TEMP(K,L)
      RHO21=DENS(NSTART,L)
      RHO22=DENS(K,L)
      S21=ENTR(NSTART,L)
      S22=ENTR(K,L)
      EPL21=VOLFRA(NSTART,L)
      EPL22=VOLFRA(K,L)
      L21=AL(NSTART,L)
      IF(L21.GT.DLAY) BEE=BEE2
      IF(L21.GT.DLAY) PODPCT=PP2
      IF(L21.GT.DLAY) CALL PROP(L21)
      AREA21=TAREA(X21)
      AREA22=TAREA(X22)
```

```
      CALL BASEP
      NSTART=I
      PODPOT=PP1
      CV=CV1
      GAMMA=GAM1
      BEE=BEE1
      RGAS=RGAS1
      PRPSI=PSI24/144.
      WRITE(6,5001)A24,L24,PRPSI,TG24,RHO24,S24,EPL24,T24,X24,UG24
 5001 FORMAT(1X,10E11.3)
      X(NSTART,J)=X24
      TIME(NSTART,J)=T24
      VEL(NSTART,J)=UG24
      VELP(NSTART,J)=UG24
      SS(NSTART,J)=A24
      PRESS(NSTART,J)=PSI24
      TEMP(NSTART,J)=TG24
      DENS(NSTART,J)=RHO24
      ENTR(NSTART,J)=S24
      VOLFRA(NSTART,J)=EPL24
      AL(NSTART,J)=L24
      GO TO 49
   40 NCHECK=1
      II=I
      X12=X(II,J)
      T11=T13
      T12=TIME(II,J)
      UG11=0.0
      UG12=VEL(II,J)
      X11=0.0
      A11=A13
      A12=SS(II,J)
      PSI11=PSI13
      PSI12=PRESS(II,J)
      TG11=TG13
      TG12=TEMP(II,J)
      RHO11=RHO13
      RHO12=DENS(II,J)
      S11=S13
      S12=ENTR(II,J)
      EPL11=EPL13
      EPL12=VOLFRA(II,J)
      UP12=VELP(II,J)
      L11=L13
      IF(L11.GT.DLAY) BEE=BEE2
      IF(L11.GT.DLAY) PODPOT=PP2
      IF(L11.GT.DLAY) CALL PROP(L11)
      AREA11=TAREA(X11)
      AREA12=TAREA(X12)
      CALL BREACH
      RGAS=RGAS1
      BEE=BEE1
      GAMMA=GAM1
      CV=CV1
      PODPOT=PP1
      PRPSI=PSI13/144.
      WRITE(6,5002)A13,L13,PRPSI,TG13,RHO13,S13,EPL13,T13,X13
      L=I+1
      K=J
      AL(L,K)=L13
      TIME(L,K)=T13
      SS(L,K)=A13
      PRESS(L,K)=PSI13
```

```
      TEMP(L,K)=TG13
      DENS(L,K)=RHO13
      VEL(L,K)=0.0
       VELP(L,K)=0.0
      ENTR(L,K)=S13
      X(L,K)=0.0
      VOLFRA(L,K)=EPL13
      MAXJ=JJ
      IF(T13.GE.TMUZE) GO TO 48
   49 CONTINUE
   48 MAXI=I
      IF(T13.GE.TMUZE) GO TO 56
   50 CONTINUE
   56 CALL OUTPUT
      MAXI=MAXI+1
      DO 51 I=MAXJ,MAXI
      WRITE(7,5003)X(I,MAXJ),TIME(I,MAXJ),VOLFRA(I,MAXJ),SS(I,MAXJ),VEL(
     $I,MAXJ),VELP(I,MAXJ),PRESS(I,MAXJ),DENS(I,MAXJ),ENTR(I,MAXJ),AL(I,
     #MAXJ),TEMP(I,MAXJ)
 5003 FORMAT(3E20.5)
   51 CONTINUE
      GO TO 999
   20 NSTART=1
      KSTART=J
      JLAST=1
      NCHECK=0
      KK=J+1
      DO 52 I=1,KK
      READ(5,5003)X(I,1),TIME(I,1),VOLFRA(I,1),SS(I,1),VEL(I,1),VELP(I,1
     $),PRESS(I,1),DENS(I,1),ENTR(I,1),AL(I,1),TEMP(I,1)
      WRITE(6,5003)X(I,1),TIME(I,1),VOLFRA(I,1),SS(I,1),VEL(I,1),VELP(I,
     #1),PRESS(I,1),DENS(I,1),ENTR(I,1),AL(I,1),TEMP(I,1)
   52 CONTINUE
      T13=TIME(KK,1)
      A13=SS(KK,1)
      PSI13=PRESS(KK,1)
       TG13=TEMP(KK,1)
      RHO13=DENS(KK,1)
      S13=ENTR(KK,1)
      EPL13=VOLFRA(KK,1)
      L13=AL(KK,1)
      GO TO 5
      END
      SUBROUTINE BASEP
C SUBROUTINE BASEP
      REAL L5,L6,L3,L4,L21,L24,L11,L13
C THIS SUBROUTINE CALCULATES POINTS AT THE BASE OF THE PROJECTILE
      COMMON/COM3/X21,X22,X24,T21,T22,T24,UG21,UG22,UG24,A21,A22,A24,PSI
     $21,PSI22,PSI24,TG21,TG22,TG24,RHO21,RHO22,RHO24,S21,S22,S24,EPL21,
     )EPL22,EPL24,AREA21,AREA22,AREA24,L21,L24,NSTART
      COMMON/SAME/RGAS,PSI1,GAMMA,DZERO,PIE,BEE,PODPOT,CV,BASE,GZERO,WT,
     >TOL,AN
      COMMON/EXIT/XLIM,TMUZE
      COMMON/BHEAT/FACT,TWALL,FRICT
      COMMON/PARM/PARIN1,PARIN2,PARIN3,PARBA1,PARBA2,PARBA3,PARBR1,PARBR
     $2,PARBR3
      DATA RHOP,REC,DELTT/3.1085,0.9,0.000005/
      A1=SQRT(GAMMA*RGAS*530.)
      XTRAV=(X21-0.125)*12.
      WRITE(6,5000)PODPOT,CV,BEE,L21,XTRAV
      ITRY=1
      NSTART=NSTART+1
    5 PSI24=PSI21
```

```
      EPL24=EPL21
      A24=A21
      P22=2.*A22/(GAMMA-1.)+LG22
      RHO24=RHC21
      UG24=UG21
      TG24=TG21
      ABASE=BASE*(PSI24+PSI21)
   10 SLOPEP=(UG22+A22+UG24+A24)/2.
      CALL RFORCE(X21,R1)
      C=X21-X22+SLOPEP*T22
      WORK=(ABASE/2.-R1)*GZERO*T21/WT
      C=C-WORK*T21/2.-T21*(UG21-WORK)
      B=UG21-WORK-SLOPEP
      A=(ABASE/2.-R1)*GZERO/WT/2.
 5000 FORMAT(1X,'*******',//,1X,5E20.5)
      T24=(-B-SQRT(B*B-4.*A*C))/2./A
      X24=X22+SLOPEP*(T24-T22)
      IF(X24.GE.XLIM) GO TO 75
   11 AREA24=TAREA(X24)
      AREA=(AREA24+AREA21)/2.
      UGMN=(UG21+UG24)/2.
      PSIMN=(PSI24+PSI21)/2.
      BR=BEE*((PSIMN/144.)**AN)/12.
      DPDX=0.0
      DL=BR*(T24-T21)
      L24=L21+DL
      IF(L24.GT.0.000625)L24=0.000625
      CALL DIME(L21,L24,SURF21,SURF24,VOL21,VOL24)
      IF(VOL21.LE.0.00)SURF21=0.00
      IF(VOL24.LE.0.00)SURF24=0.00
      DBAR=DZERO-L24-L21
      IF(DBAR.LE.0.000652)DBAR=0.00
      DBAR=DBAR**3
      FOR24=-PIE*DBAR*DPDX/6.
      EPLMN=(EPL21+EPL24)/2.
      VOLMN=(VOL21+VOL24)/2.
      SURFMN=(SURF21+SURF24)/2.
      IF(VOLMN.LE.0.00) GO TO 12
      ETAMN=EPLMN/VOLMN
      GO TO 13
   12 ETAMN=0.00
   13 CONTINUE
      AMGMN=ETAMN*SURFMN*BR*RHOP*AREA
      DUPDX=(UG24-UG21)/(X24-X21)
      DEPLDT=EPL21*(1.-EPL21)*(RHO24-RHC21)/RHO21-(EPL21+(1.-EPL21)*RHO2
     #1/RHOP)*AMGMN*(T24-T21)/RHC21/AREA
      DEPLDT=DEPLDT/(T24-T21)
   14 CONTINUE
      FD24=EPLMN*FOR24/VOLMN
      RHOMN=(RHO24+RHO21)/2.
      TGMN=(TG24+TG21)/2.
      DIA=SQRT(4.*AREA/PIE)
      AMN=(A22+A24)/2.
      TAW=TGMN*(1.+REC*(GAMMA-1.)*UGMN*UGMN/2./AMN/AMN)
      QHEAT1=0.00298*(CV**0.333)*RHOMN*UGMN
      QHEAT=QHEAT1*(TGMN-TWALL)
      EGAS=-QHEAT*PIE*DIA+AMGMN*PODPOT*GZERO
      TAUW=0.0005*RHOMN*UGMN*UGMN
      F1= UGMN*AMGMN-TAUW*PIE*DIA +FD24*AREA
      F1=F1/778.
       RS1=AMGMN*(CV*TGMN+UGMN*UGMN/1556.)+(AREA*DEPLDT
     $+AMGMN/RHOMN)*PSIMN/778.
      DSDT=AMGMN*(PODPOT*GZERO-(CV+RGAS/778.)*TGMN) +(TAUW*UGMN/778. -
```

```
     #QHEAT)*PIE*DIA+FD24*UGMN*AREA/778.-PSIMN*AREA*DEPLDT/778.
      DSDT=DSDT/AREA/RHOMN/TGMN/(1.-EPLMN)
      AREA=(AREA24+AREA22)/2.
      DELPDT=-AMN*UGMN*(AREA24-AREA22)/(X24-X22)/AREA
      DELPDT=DELPDT+778.*AMN*DSDT/RGAS
      WORK=(TAUW*PIE*DIA/AREA-AMN*RHOMN*DEPLDT+FD24+UGMN*AMGMN/AREA)/RHO
     +MN/(1.-EPLMN)
      WORK=WORK-(AMN**3)*AMGMN/AREA/(1.-EPLMN)/PSIMN/GAMMA
      DELPDT=DELPDT-WORK
      P24=P22+DELPDT*(T24-T22)
      ANEW24=(P24-UG24)*(GAMMA-1.)/2.
      TG24=(ANEW24*ANEW24)/GAMMA/RGAS
       S24=S21+DSDT*(T24-T21)*778./RGAS/GAMMA
      EPL24=EPL21+DEPLDT*(T24-T21)
      IF(EPL24.LE.0.00)EPL24=0.0
      WORK= 2.*GAMMA/(GAMMA-1.)
      TG24=(ANEW24*ANEW24)/GAMMA/RGAS
       PSI24=PSI21*((ANEW24/A21)**WORK)
      PSI24=PSI24*EXP(-GAMMA*(S24-S21))
      RHO24=PSI24/RGAS/TG24
      ABASE=BASE*(PSI24+PSI21)
      WORK=(ABASE/2.-R1)*GZERO/WT
      UGNE24=UG21+WORK*(T24-T21)
      CHECK=ABS((ANEW24-A24)/A24)
      IF(CHECK.GT.TOL) GO TO 50
      IF(UG24.LE.0.00) GO TO 50
      CHECK=ABS((UGNE24-UG24)/UG24)
      IF(CHECK.GT.TOL) GO TO 50
      WRITE(6,5000)QHEAT2,QHEAT1,QHEAT,EGAS,RS1
      XTRAV=(X24-0.125)*12.
      DEL=AMGMN*(PODPOT*GZERO-CV*TGMN)
      WRITE(6,5000)L24,DEPLDT,DSDT,AMGMN,DEL,XTRAV
      RETURN
   50 ITRY=ITRY+1
      IF(ITRY.LT.11) GO TO 200
      LOC=2
      CALL DIAGO(LOC,UGNE24,ANEW24)
  200 UG24=UGNE24
      A24=ANEW24
      GO TO 10
   75 X24=XLIM
  300 FORMAT(1X,5E20.5)
      T24=T22+(X24-X22)/SLOPEP
      TMUZE=T24
      GO TO 11
      END
      SUBROUTINE RFORCE(A,B)
      DIMENSION XX(20),RF(20)
      DATA              XX/0.0,0.0333,0.0416,0.0499,0.066,0.100,0.1333,0.
     >1666,0.200,0.2333,0.2666,0.300,0.333,0.500,0.6666,1.00,1.333,1.50,
     $1.666,0.0/
      DATA              RF/0.0,520.,525.,520.,420.,255.,215.,195.,183.,17
     >0.,162.,153.,145.,118.,95.,52.,18.,7.0,0.0,0.0/
      A=A-0.1249
      DO 10 I=1,19
      IF(A.EQ.XX(I)) GO TO 15
      IF(A.GT.XX(I).AND.A.LT.XX(I+1)) GO TO 20
   10 CONTINUE
   15 B=RF(I)
      B=B/2.
      A=A+0.1249
      RETURN
   20 SLOPE=(RF(I+1)-RF(I))/(XX(I+1)-XX(I))
```

```
      C=RF(I)-SLOPE*XX(I)
      B=SLOPE*A+C
      B=B/2.
      A=A+0.1249
      RETURN
      END
      SUBROUTINE BREACH
CSUBROUTINE BREACH
C THIS SUBROUTINE CALCULATES THE POINTS LOCATED ON THE BREECH
      REAL L5,L6,L3,L4,L21,L24,L11,L13
      COMMON/COM2/X12,T11,T12,T13,UG11,UG12,UG13,A11,A12,A13,PSI11,PSI12
     $,PSI13,TG11,TG12,TG13,RHC11,RHO12,RHO13,S11,S12,S13,EPL11,EPL12,EP
     *L13,AREA11,AREA12,AREA13,L11,L13,UP12
      COMMON/SAME/RGAS,PSI1,GAMMA,DZERC,PIE,BEE,PODPOT,CV,BASE,GZERO,WT,
     >TGL,AN
      COMMON/EXIT/XLIM,TMUZE
      COMMON/PARM/PARIN1,PARIN2,PARIN3,PARBA1,PARBA2,PARBA3,PARBR1,PARBR
     $2,PARBR3
      A1=SQRT(GAMMA*RGAS*530.)
      ITRY=1
      CONST=-0.58778E+03
      TWALL=530.
      UG13=0.0
      A13=A11
      RHOP=3.1085
      AREA=AREA11
      Q12=2.*A12/(GAMMA-1.)
      PSI13=PSI11
      EPL13=EPL11
   30 SLOPEQ=(-A13+UG12-A12)/2.
      T13=-X12/SLOPEQ+T12
      IF(T13.GE.TMUZE) T13=TMUZE
      PSIMN=(PSI13+PSI11)/2.
      BR13=BEE*((PSIMN/144.)**AN)/12
      DL13=BR13*(T13-T11)
      L13=L11+DL13
      IF(L13.GT.0.000625)L13=0.000625
      CALL DIME(L11,L13,SURF11,SURF13,VCL11,VOL13)
      IF(VOL11.LE.0.00)SURF11=0.00
      IF(VOL13.LE.0.00)SURF13=0.00
      EPLMN=(EPL13+EPL11)/2.
      VOLMN=(VOL13+VOL11)/2.
      IF(VOLMN.LE.0.00) GO TO 31
      ETAMN=EPLMN/VOLMN
      GO TO 32
   31 ETAMN=0.00
   32 CONTINUE
      AMGMN=ETAMN*BR13*RHOP*AREA
      AMGMN=AMGMN*(SURF13+SURF11)/2.
      DEPLOT=-AMGMN/AREA/RHOP-EPLMN*UP12/X12
  300 FORMAT(1X,5E20.5)
      TG13=A13*A13/GAMMA/RGAS
      TGMN=(TG13+TG11)/2.
      RHC13=PSI13/RGAS/TG13
      RHOMN=(RHO13+RHO11)/2.
      DIA=SQRT(4.*AREA/PIE)
      FILM=3.24*EXP(CONST*T13)*DIA/2./X12
      QHEAT=FILM*(TG13-TWALL)
      QHEAT=QHEAT*PARBR1
      EGAS=-QHEAT*PIE*DIA+PODPOT*AMGMN*GZERO
      RS1=AMGMN*(CV*TGMN+PSIMN/778./RHOMN)+PSIMN*AREA*DEPLOT/778.
      WORK=AREA*(1.-EPLMN)*RHOMN*TGMN
      DSDT=AMGMN*(PODPOT*GZERO-(CV+RGAS/778.)*TGMN) -QHEAT*PIE*DIA  -
```

```
     #PSIMN*AREA*DEPLDT/778.
      DSDT=DSDT/WORK
      IF(T13.GT.PARBR2) GO TO 200
  150 CONTINUE
      DELQDT=((A11+A13)/2.)*(778.*DSDT/RGAS+DEPLDT/(1.-EPLMN))
      DELQDT=DELQDT+((A11/2.+A13/2.)**3)*AMGMN/AREA/PSIMN/(1.-EPLMN)/GAM
     $MA
      Q13= Q12+DELQDT*(T13-T12)
      P13=Q13
      ANEW13=(GAMMA-1.)*Q13/2.
      TG13=ANEW13*ANEW13/GAMMA/RGAS
      S13=DSDT*(T13-T11)*778./RGAS/GAMMA+S11
      PSI13=PSI11*((ANEW13/A11)**(2.*GAMMA/(GAMMA-1.)))
      PSI13=PSI13*EXP(-GAMMA*(S13-S11))
      RHO13=PSI13/RGAS/TG13
      EPL13=EPL11+DEPLDT*(T13-T11)
      IF(EPL13.LE.0.00)EPL13=0.0
      CHECK=ABS((ANEW13-A13)/A13)
      A13=ANEW13
      IF(T13.GE.TMUZE)RETURN
      IF(CHECK.GT.TOL) GO TO 180
      RETURN
  180 ITRY=ITRY+1
      IF(ITRY.LT.11) GO TO 30
      LOC=3
      CALL DIAGO(LOC,UG13,A13)
      RETURN
  200 WRITE(6,300)L11,L13,VOL11,VOL13,SURF11,SURF13,EPLMN,ETAMN,AMGMN,DE
     #PLDT,QHEAT,EGAS,DSDT,AREA
      GO TO 150
      END
      SUBROUTINE DIAGO(LA,BUG,AA)
C THIS IS A DIAGANOSTIC SUBROUTINE
      REAL L5,L6,L3,L4,L21,L24,L11,L13
      COMMON/C/X(99,99),TIME(99,99),VEL(99,99),SS(99,99),PRESS(99,99),TE
     $MP(99,99),DENS(99,99),VELP(99,99),ENTR(99,99),VOLFRA(99,99),AL(99,
     ?99)
      COMMON/COM1/X3,X4,X6,T3,T4,T6,UG3,UG4,UG6,A3,A4,A6,PSI3,PSI4,PSI6,
     $TG3,TG4,TG6,RHO3,RHO4,RHO6,UP3,UP4,UP6,S3,S4,S6,EPL3,EPL4,EPL6,ARE
     >A3,AREA4,AREA6,L3,L4,L5,L6
      COMMON/COM2/X12,T11,T12,T13,UG11,UG12,UG13,A11,A12,A13,PSI11,PSI12
     $,PSI13,TG11,TG12,TG13,RHO11,RHO12,RHO13,S11,S12,S13,EPL11,EPL12,EP
     #L13,AREA11,AREA12,AREA13,L11,L13,UP12
      COMMON/COM3/X21,X22,X24,T21,T22,T24,UG21,UG22,UG24,A21,A22,A24,PSI
     $21,PSI22,PSI24,TG21,TG22,TG24,RHO21,RHO22,RHO24,S21,S22,S24,EPL21,
     )EPL22,EPL24,AREA21,AREA22,AREA24,L21,L24,NSTART
      COMMON/SAME/RGAS,PSI1,GAMMA,DZERO,PIE,BEE,PODPOT,CV,BASE,GZERO,WT,
     >TOL,AN
      COMMON/OUT/MAXJ,MAXI
      GO TO (10,11,12),LA
   10 WRITE(6,100)
  100 FORMAT(1H1,'EXCEEDED ITERATION LIMIT IN SUBROUTINE INTER')
      GO TO 15
   11 WRITE(6,101)
  101 FORMAT(1H1,'EXCEEDED ITERATION LIMIT IN SUBROUTINE BASE')
      GO TO 15
   12 WRITE(6,102)
  102 FORMAT(1H1,'EXCEEDED ITERATION LIMIT IN SUBROUTINE BREACH')
   15 WRITE(6,110)BUG,AA
  110 FORMAT(1HO,'LAST CALCULATED GAS VELOCITY',E12.3,//2X,'LAST CALCULA
     >TED SPEED OF SOUND',E12.3)
  105 FORMAT(1X,I3,1X,I3,E12.3,2X,E12.3,2X,E12.3,2X,E12.3,2X,E12.3,2X,E1
     #2.3,2X,E12.3,2X,F6.3)
```

```
      MAXI=MAXI+1
      DO 20 I=1,MAXI
      DO 20 J=1,95
      IF(SS(I,J).EQ.0.0) GO TO 20
      WRITE(6,105)I,J,X(I,J),TIME(I,J),VEL(I,J),SS(I,J),PRESS(I,J),TEMP(
     $I,J),DENS(I,J),VOLFRA(I,J)
   20 CONTINUE
      STOP
      END
C SUBROUTINE OUTPUT
C THIS SUBROUTINE PRINTS RESULTS
      SUBROUTINE OUTPUT
      COMMON/C/X(99,99),TIME(99,99),VEL(99,99),SS(99,99),PRESS(99,99),TE
     $MP(99,99),DENS(99,99),VELP(99,99),ENTR(99,99),VOLFRA(99,99),AL(99,
     ?99)
      COMMON/SAME/RGAS,PSI1,GAMMA,DZERO,PIE,BEE,PODPOT,CV,BASE,GZERO,WT,
     >TOL,AN
      COMMON/OUT/MAXJ,MAXI
      WRITE(6,100)
  100 FORMAT(1H1,'****** SOLUTION TO GAS FLOW IN TUBE *********')
      WRITE(6,104)
  104 FORMAT(1HO,'FLOW PARAMETERS',//3X,'I',3X,'J',3X,'POSITION',8X,'TIM
     >E',9X,'VELOCITY',3X,'SPEED OF SOUND',3X,'PRESS',3X,'TEMP',3X,'DENS
     $ITY',3X,'FRACTION')
  105 FORMAT(1X,I3,1X,I3,E12.3,2X,E12.3,2X,E12.3,2X,E12.3,2X,E12.3,2X,E1
     "2.3,2X,E12.3,2X,F6.3)
      MAXII=MAXI+1
      DO 20 I=1,MAXII
      DO 20 J=1,95
      IF(SS(I,J).EQ.0.0) GO TO 20
      PRPSI=PRESS(I,J)/144.
      WRITE(6,105)I,J,X(I,J),TIME(I,J),VEL(I,J),SS(I,J),PRPSI,TEMP(I,J),
     $DENS(I,J),VOLFRA(I,J)
   20 CONTINUE
      WRITE(6,110)
      DO 30 I=1,MAXII
      DO 30 J=1,95
      IF(I.NE.J) GO TO 30
      XTRAV=(X(I,J)-0.125)*12.
      PRPSI=PRESS(I,J)/144.
      WRITE(6,111)I,J,X(I,J),TIME(I,J),XTRAV,PRPSI,VEL(I,J)
   30 CONTINUE
  110 FORMAT(1H1,20X,'PROJECTILE DATA',//3X,'I',3X,'J',3X,'POSITION',8X,
     #'TIME',9X,'TRAVEL',8X,'PRESSURE',8X,'VELOCITY')
  111 FORMAT(1X,I3,1X,I3,E12.3,2X,E12.3,2X,F12.2,2X,F12.0,2X,F12.0)
      RETURN
      END
      SUBROUTINE PROP(A)
C SUBROUTINE FOR CALCULATING PROPERTIES AS A FUNCTION OF
C   FRACTION OF DETERRED AND NONDETERRED
      COMMON/SAME/RGAS,PSI1,GAMMA,DZERO,PIE,BEE,PODPOT,CV,BASE,GZERO,WT,
     >TOL,AN
      COMMON/POWDER/CV1,CV2,RGAS1,RGAS2,GAM1,GAM2,DLAY
      A=A*12.
      AMND=2.0175E-07*(A-DLAY*12.)/(0.0075-DLAY*12.)
      AMT=6.815E-08+AMND
      AMFND=AMND/AMT
      CV=CV1*(1.-AMFND)+CV2*AMFND
      RGAS=RGAS1*(1.-AMFND)+RGAS2*AMFND
      GAMMA=1.+RGAS/CV/778.
      A=A/12.
      RETURN
      END
```

```
      SUBROUTINE START
C SUBROUTINE START
      REAL L5,L6,L3,L4,L21,L24,L11,L13
      COMMON/COM2/X12,T11,T12,T13,UG11,UG12,UG13,A11,A12,A13,PSI11,PSI12
     #,PSI13,TG11,TG12,TG13,RHO11,RHO12,RHO13,S11,S12,S13,EPL11,EPL12,EP
     $L13,AREA11,AREA12,AREA13,L11,L13,UP12
      COMMON/SAME/RGAS,PSI1,GAMMA,DZERO,PIE,BEE,PODPOT,CV,BASE,GZERO,WT,
     #TOL,AN
      COMMON/STAR/KSTART
      AK=KSTART
      ITRY=1
      UG13=0.0
      A13=A12
      DELX=-0.125/AK
      PSI13=PSI12
      RHOP=3.1085
      X13=X12+DELX
      AREA12=TAREA(X12)
      AREA13=TAREA(X13)
      AREA=(AREA13+AREA12)/2.
      Q12=2.*A12/(GAMMA-1.)
      EPL13=EPL12
   30 SLOPEQ=-(A13+A12)/2.
      PSIMN=(PSI12+PSI13)/2.
      T13=T12+(X13-X12)/SLOPEQ
      BR13=BEE*((PSIMN/144.)**AN)/12.
      DL13=BR13*(T13-T12)
      L13=L11+DL13
      CALL DIME(L11,L13,SURF12,SURF13,VOL12,VOL13)
      EPLMN=(EPL13+EPL12)/2.
      VOLMN=(VOL13+VOL12)/2.
      ETAMN=EPLMN/VOLMN
      AMGMN=ETAMN*BR13*RHOP*AREA*(SURF13+SURF12)/2.
      DEPLDT=-AMGMN/AREA/RHOP
      TG13=A13*A13/GAMMA/RGAS
      TGMN=(TG13+TG12)/2.
      RHO13=PSI13/RGAS/TG13
      RHOMN=(RHO13+RHO12)/2.
      EGAS=PODPOT*AMGMN*GZERO
      RS1=AMGMN*(CV*TGMN+PSIMN/778./RHOMN)+PSIMN*AREA*DEPLDT/778.
      WORK=AREA*(1.-EPLMN)*RHOMN*TGMN
      DSDT=(EGAS-RS1)/WORK
      DELQDT=((A12+A13)/2.)*(778.*DSDT/RGAS+DEPLDT/(1.-EPLMN))
      DELQDT=DELQDT+(((A12+A13)/2.)**3)*AMGMN/AREA/PSIMN/(1.-EPLMN)/GAMM
     #A
      Q13=Q12+DELQDT*(T13-T12)
      P13=Q13
      ANEW13=(GAMMA-1.)*Q13/2.
      TG13=ANEW13*ANEW13/GAMMA/RGAS
      S13=S12+DSDT*778.*(T13-T12)/RGAS/GAMMA
      PSI13=PSI12*((ANEW13/A12)**(2.*GAMMA/(GAMMA-1.)))
      PSI13=PSI13*EXP(-GAMMA*(S13-S12))
      RHO13=PSI13/RGAS/TG13
      EPL13=EPL12+DEPLDT*(T13-T12)
      CHECK=ABS((ANEW13-A13)/A13)
      A13=ANEW13
      IF(CHECK.GT.TOL) GO TO 180
      X12=X13
      RETURN
  180 ITRY=ITRY+1
      IF(ITRY.LT.11) GO TO 30
      LOC=3
      CALL DIAGO(LOC,UG13,A13)
```

```
      RETURN
      END
      FUNCTION TAREA(ALOC)
C THIS IS A SUB PROGRAM TO CALCULATE THE BORE CROSS SECTIONAL AREA
      DIMENSION BORE(11),POS(11)
      DATA   BORE/0.024900,0.02491,0.02771,0.02746,0.02514,0.01864,0.018
     >64,0.01824,0.01824,0.01824,0.01824/
      DATA       POS/-.10,0.0,0.02792,0.1032,0.1078,0.1135,0.1300,1.542,
     $3.0,5.0,15.0/
      DO 10 I=1,11
      IF(POS(I).EQ.ALOC) GO TO 5
      IF(POS(I).GT.ALOC) GO TO 7
      GO TO 10
    5 TAREA=3.141593*BORE(I)*BORE(I)*0.25
      GO TO 15
    7 AM=(BORE(I-1)-BORE(I))/(POS(I-1)-POS(I))
      B=BORE(I)
      DD=B+AM*(ALOC-POS(I))
      TAREA=3.141593*DD*DD/4.
      GO TO 15
   10 CONTINUE
   15 RETURN
      END
      SUBROUTINE DIME(A,B,SURA,SURB,VOLA,VOLB)
CSUBROUTINE DIME
C THIS IS A SUBROUTINE TO CALCULATE PROPELLANT   SURFACE
C AREA AND VOLUME
      DATA PIE,RBAR,RI/3.141593,0.00095,0.0006249/
      WORK=RI-A
      WORK1=RBAR-RI
      VOLA=2.*PIE*WORK1*WORK1*WORK+PIE*PIE*WORK*WORK*(WORK1+
     >4.*WORK/3./PIE)
      SURA=2.*PIE*(PIE*WORK*(WORK1+2.*WORK/PIE)+WORK1*WORK1)
      WORK=RI-B
      VOLB=2.*PIE*WORK1*WORK1*WORK+PIE*PIE*WORK*WORK*(WORK1+
     >4.*WORK/3./PIE)
      SURB=2.*PIE*(PIE*WORK*(WORK1+2.*WORK/PIE)+WORK1*WORK1)
      RETURN
      END
      SUBROUTINE INTER
C SUBROUTINE INTER
C THIS SUBROUTINE CALCULATES INTERIOR NETWORK POINTS
C
C
      REAL L5,L6,L3,L4,L21,L24,L11,L13
      COMMON/COM1/X3,X4,X6,T3,T4,T6,UG3,UG4,UG6,A3,A4,A6,PSI3,PSI4,PSI6,
     $TG3,TG4,TG6,RHO3,RHO4,RHO6,UP3,UP4,UP6,S3,S4,S6,EPL3,EPL4,EPL6,ARE
     >A3,AREA4,AREA6,L3,L4,L5,L6
      COMMON/SAME/RGAS,PSI1,GAMMA,DZERO,PIE,BEE,PODPOT,CV,BASE,GZERO,WT,
     >TOL,AN
      COMMON/EXIT/XLIP,TMUZE
      COMMON/BHEAT/FACT,TWALL,FRICT
      COMMON/PARM/PARIN1,PARIN2,PARIN3,PARBA1,PARBA2,PARBA3,PARBR1,PARBR
     $2,PARBR3
      DATA AMU,REC,RHOP/1.0445E-06,0.9,3.1085/
      FUG=1.0
      A1=SQRT(GAMMA*RGAS*530.)
      ITRY=1
      UG6=(UG3+UG4)/2.
      A6=(A3+A4)/2.
      TG6=(TG3+TG4)/2.
      RHO6=(RHO3+RHO4)/2.
      PSI6=(PSI3+PSI4)/2.
```

```
      UP6=(UP5+UP6)/2.
      P3=2.*A3/(GAMMA-1.)    +UG3
      Q4=2.*A4/(GAMMA-1.)-UG4
      EPL6=(EPL3+EPL4)/2.
      AREA=(AREA3+AREA4)/2.
      S6=(S3+S4)/2.
C CALCULATION OF THE LOCATION OF THE NEW POINT ON X-T DIAGRAM
   30 SLOPEP=(UG3+A3+UG6+A6)/2.
      SLOPEQ=(UG4-A4+UG6-A6)/2.
      T6=(X4-X3+T3*SLOPEP-T4*SLOPEQ)/(SLOPEP-SLOPEQ)
      IF(T6.GE.TMLZE) GO TO 250
      X6=X3+SLOPEP*(T6-T3)
   31 AREA6=TAREA(X6)
      WORK=X4-X3
      IF(T4.EQ.T3) GO TO 32
      T7=X6-X3+T3*WORK/(T4-T3)-T6*UG6
      T7=T7/(WORK/(T4-T3)-UG6)
      GO TO 33
   32 T7=T3
   33 CONTINUE
      X7=X6-UG6*(T6-T7)
      IF(T4.EQ.T3) GO TO 34
      T5=X6-X3+T3*WORK/(T4-T3)-T6*UP6
      T5=T5/(WORK/(T4-T3)-UP6)
      GO TO 35
   34 T5=T3
   35 CONTINUE
      X5=X6-UP6*(T6-T5)
      IF(X5.LT.0.0) X5=X3
      WORK1=(X7-X3)/WORK
      UG7=UG3+(UG4-UG3)*WORK1
      A7=A3+(A4-A3)*WORK1
      S7=S3+(S4-S3)*WORK1
      TG7=A7*A7/(GAMMA*RGAS)
      WORK2=2.*GAMMA/(GAMMA-1.)
      PSI7=PSI3*((A7/A3)**WORK2)
      PSI7=PSI7*EXP(-GAMMA*(S7-S3))
      RHO7=PSI7/RGAS/TG7
      WORK1=(X5-X3)/WORK
      UG5=UG3+(UG4-UG3)*WORK1
      UP5=UP3+(UP4-UP3)*WORK1
      A5=A3+(A4-A3)*WORK1
      S5=S3+(S4-S3)*WORK1
      TG5=A5*A5/(GAMMA*RGAS)
      PSI5=PSI3*((A5/A3)**WORK2)
      PSI5=PSI5*EXP(-GAMMA*(S5-S3))
      RHO5=PSI5/RGAS/TG5
      EPL5=EPL3+(EPL4-EPL3)*WORK1
      IF(EPL5.LE.0.00)EPL5=0.0
      L5=L3+(L4-L3)*WORK1
      UMN=(UG6+UG7)/2.
      UGMN=UMN
      AMN=(A6+A7)/2.
      TGMN=(TG6+TG7)/2.
      RHOMN=(RHO6+RHO7)/2.
      PSIMN=(PSI6+PSI7)/2.
      UP56=(UP5+UP6)/2.
      RELV5=ABS(UG5-UP5)
      RELV6=ABS(UG6-UP6)
      RELV56=(RELV5+RELV6)/2.
      PEE56=(PSI5+PSI6)/2.
      BR56=BEE*((PEE56/144.)**AN)/12.
      IF(RELV56.GT.500.) BR56=BR56*(1.+0.0004*(RELV56-500.))
```

```
      DL56=BR56*(T6-T5)
      L6=L5+DL56
      IF(L5.GT.0.C00625)L5=0.0C0625
      IF(L6.GT.0.000625)L6=0.0C0625
      CALL DIME(L5,L6,SURF5,SURF6,VOL5,VOL6)
      IF(VOL5.LE.0.00)SURF5=0.C0
      IF(VOL6.LE.0.00)SURF6=C.00
      EPL56=(EPL6+EPL5)/2.
      DBAR=DZERO-L5-L6
      IF(DBAR.LT.0.000652)DBAR=0.00
      REY56=RELV56*DBAR/AMU
      REY56=REY56*(RHO5+RHO6)/2.
      IF(REY56.LE.0.000) GO TO 98
      CDRAG=28./(REY56**(0.85))+0.48
      GO TO 99
   98 CDRAG=0.0
      GO TO 99
C CALCULATION CF VELOCITY ANC PRESSURE GRADIENTS
   99 IF(T3.GT.T4) GO TO 100
      T8=T4
      X8=(UG3+A3)*(T8-T3)+X3
      PSI8=PSI3+(PSI6-PSI3)*(X8-X3)/(X6-X3)
      UP8=UP3+(UP6-UP3)*(X8-X3)/(X6-X3)
      DPDX=0.00
      DUPDX=(UP4-UP8)/(X4-X8)
      GO TO 125
  100 T8=T3
      X8=(UG4-A4)*(T8-T4)+X4
      PSI8=PSI4+(PSI6-PSI4)*(X4-X8)/(X4-X6)
      UP8=UP4+(UP6-UP4)*(X4-X8)/(X4-X6)
      DPDX=0.00
      DUPDX=(UP8-UP3)/(X8-X3)
  125 CONTINUE
C DRAG FORCE ON PROPELLANT PARTICLE
      FOR56=CDRAG*(RHO5+RHO6)*RELV56*RELV56
      FOR56=FOR56*DBAR*DBAR/16.
      FOR56=FOR56-DPDX*(DBAR**3)/6.
      FOR56=FOR56*PIE
      IF(VOL5.LE.C.00.AND.VOL6.LE.0.00)GO TO 1125
      ETA56=2.*EPL56/(VOL5+VOL6)
      GO TO 1126
 1125 ETA56=0.00
 1126 CONTINUE
      FD56=ETA56*FOR56
C MASS OF GAS GENERATED GOING FROM 5 TO 6
      AMG56=ETA56*BR56*RHOP*AREA
      AMG56=AMG56*(SURF5+SURF6)/2.
      AMG56=AMG56*FUG
C CHANGE IN PARTICLE VELOCITY ANC MASS FRACTION
      IF(EPL56.LE.0.000) GO TO 126
      DUPDT =UP56*AMG56/RHOP/AREA/EPL56
      DUPDT=DUPDT+FD56/RHOP/EPL56
      GO TO 127
  126 DUPDT=0.00
  127 CONTINUE
      DEPLDT=-EPL56*DUPDT/UP56
      DEPLDT=DEPLDT-AMG56/RHOP/AREA
      DIA=SQRT(4.*AREA/PIE)
      FILM=FRICT*GAMMA*RGAS*RHOMN*UGMN/2./(GAMMA-1.)/778.
      TAW=TGMN*(1.+REC*(GAMMA-1.)*UGMN*UGMN/2./AMN/AMN)
        QHEAT=FILM*(TAW-TWALL)
      QHEAT=QHEAT*FACT
      EGAS=-QHEAT*PIE*DIA-FD56*AREA*RELV56/778.
```

```
      COMB=COMSTAR*DUDPDT*PODPDT*GZERO
      TAUW=0.5*FRICT*RHOMN*UGMN*UGMN
      F1= UGMN*AMG56+TAUW*PIE*DIA+FD56*AREA
      F1=F1/778.
      RS1=AMG56*(CV*TGMN+UGMN*UGMN/1556.)+PSIMN*AREA*DEPLDT/778.
      RS1=RS1+PSIMN*AMG56/RHOMN/778.
      WORK=AREA*(1.-EPL56)*RHOMN*TGMN
      DSDT=AMG56*(PODPDT*GZERO-(CV+RGAS/778.)*TGMN)+FD56*AREA*(UGMN-RELV
     #56)/778. +(TAUW*UGMN/778.-QHEAT)*PIE*DIA -PSIMN*AREA*DEPLDT/778.
      DSDT=DSDT/WORK
      IF(T6.GE.PARIN1) GO TO 201
  128 CONTINUE
C CALCULATE THE CHANGE IN P AND Q
      UG36=(UG3+UG6)/2.
      RHO36=(RHO3+RHO6)/2.
      PSI36=(PSI3+PSI6)/2.
      AREA=(AREA3+AREA6)/2.
      A36=(A3+A6)/2.
      EPL36=(EPL3+EPL6)/2.
      TRY=A36*A36*778.*(S6-S3)/(X6-X3)/GAMMA/RGAS
      DELPDT=-A36*UG36*(AREA3-AREA6)/(X3-X6)/AREA
      DELPDT=DELPDT+A36*778.*DSDT/RGAS
      WORK=TAUW*PIE*DIA/AREA/(1.-EPL36)/RHO36
      WORK=WORK+FD56/RHO36/(1.-EPL36)
      DELPDT=DELPDT-WORK
      DELPDT=DELPDT+A36*DEPLDT/(1.-EPL36)+((A36**3)/PSI36/GAMMA-UG36/RHO
     >36)*AMG56/AREA/(1.-EPL36)
      DELPDT=DELPDT+TRY
      PSI46=(PSI4+PSI6)/2.
      EPL46=(EPL4+EPL6)/2.
      A46=(A4+A6)/2.
      UG46=(UG4+UG6)/2.
      RHO46=(RHO4+RHO6)/2.
      AREA=(AREA4+AREA6)/2.
      TRY=A46*A46*778.*(S6-S4)/(X6-X4)/GAMMA/RGAS
      DELQDT=-A46*UG46*(AREA4-AREA6)/(X4-X6)/AREA
      DELQDT=DELQDT+A46*778.*DSDT/RGAS
      WORK=(TAUW*PIE*DIA/AREA+FD56)/(1.-EPL46)/RHO46
      DELQDT=DELQDT+WORK+A46*DEPLDT/(1.-EPL46)
      DELQDT=DELQDT+((A46**3)/PSI46/GAMMA+UG46/RHO46)*AMG56/AREA/(1.-EPL
     #46)
      DELQDT=DELQDT-TRY
      P6=P3+DELPDT*(T6-T3)
      Q6=Q4+DELQDT*(T6-T4)
      ANEW6=(GAMMA-1.)*(P6+Q6)/4.
      UGNEW6=(P6-Q6)/2.
      S6=S7+DSDT*(T6-T7)*778./RGAS/GAMMA
      EPL6=EPL5+DEPLDT*(T6-T5)
      IF(EPL6.LE.0.00)EPL6=0.0
      UPN6=UP5+DUPDT*(T6-T5)
       IF(EPL6.LE.0.00)UPN6=UGNEW6
      TG6=ANEW6*ANEW6/GAMMA/RGAS
      PSI6=PSI7*((ANEW6/A7)**WCRK2)
      PSI6=PSI6*EXP(-GAMMA*(S6-S7))
      RHO6=PSI6/RGAS/TG6
      IF(T6.GE.TMLZE) GO TO 260
 5003 FORMAT(1X,5E11.3)
      CHECK=ABS((ANEW6-A6)/A6)
      IF(CHECK.GT.TOL) GO TO 180
      CHECK=ABS((UGNEW6-UG6)/UG6)
      IF(UG6.LT.300.) CHECK=0.0001
       TOLL=10.*TOL
      IF(CHECK.GT.TOLL) GO  TO  180
```

```
      UOLD4=UG4
      AOLD4=A4
      XOLD4=X4
      SOLD4=S4
      PSIOL4=PSI4
      UPOLD4=UP4
      XOLD4=X4
      RETURN
  180 ITRY=ITRY+1
      IF(ITRY.LT.21) GO TO 200
      LOC=1
      CALL DIAGO(LOC,UGNEW6,ANEW6)
  200 UG6=(UGNEW6+UG6)/2.
      A6=(ANEW6+A6)/2.
      UP6=(UPN6+UP6)/2.
      GO TO 30
  201 WRITE(6,5004)VOL5,VOL6,SURF5,SURF6,ETA56,AMG56,EPL56,DUPDT,DEPLDT
      WRITE(6,5004)L5,L6,X5,X6,BR56
      WRITE(6,5004)P6,Q6,A6,UG6,UP6
      DVOL=2.*((3.*VOL5/(4.*PIE))**0.33)
      WRITE(6,5004)FD56,TAUW,EGAS,RS1,F1,UGMN,FOR56,UP3,UP4,UP5,DUPDX,UP
     #56,UG5,DBAR,DVOL,UPN6
 5004 FORMAT(1X,5E20.5)
      GO TO 128
  250 T6=TMUZE
      X6=X3+SLOPEP*(T6-T3)
      AQ=(UG3+UOLC4-A3-AOLD4)/2.
      AP=(UOLD4+UG4+A4+AOLD4)/2.
      TI=(X4-X6+AQ*T6-AP*T4)/(AQ-AP)
      XI=X4+AP*(TI-T4)
      UG4=(UG4-UOLD4)*(XI-X4)/(XCLD4-X4)+UG4
       A4=(A4-AOLC4)*(XI-X4)/(XOLD4-X4)+A4
      TG4=A4*A4/GAMMA/RGAS
      PSI4=(PSI4-PSIOL4)*(XI-X4)/(XOLD4-X4)+PSI4
      RHO4=PSI4/RGAS/TG4
      UP4=(UP4-UPCLD4)*(XI-X4)/(XCLD4-X4)+UP4
      S4=(S4-SOLD4)*(XI-X4)/(XCLC4-X4)+S4
      X4=XI
      T4=TI
      GO TO 31
  260 UG6=UGNEW6
      A6=ANEW6
      UP6=UPN6
      RETURN
      END
```

APPENDIX B

Listing of Flame Front Program

```
// JOB MCASSEY 6044999919                                        DATE 04/01
// EXEC WATFIV

      /PROGRAM MCASSEY
      C      PROGRAM CALCULATES THE PROPAGATION OF A FLAMEFRONT THRU A
      C      POROUS PROPELLANT BED
      C      F=PROPELLANT IMPETUS (FT-LBS/LB)
      C      C= CHARGE WT (LBS)
      C      RHOP= PROPELLANT DENSITY (LBS/IN**3)
      C      ALFA=FLAMEFRONT VELOCITY PRESSURE COEFFICIENT (IN/SEC-PSI**BETA)
      C      BETA=FLAMEFRONT VELOCITY PRESSURE EXPONENT (DIMEN)
      C      B=PROPELLANT BURNING RATE COEFFICIENT (IN/(SEC-PSI**EN))
      C      EN=PROPELLANT BU  NG RATE EXPONENT (DIMEN)
      C      RO=EQUIV. MEAN R  IUS OF PROP. GRAIN (IN)
      C      RI=1/2 WEB  IN)
      C      XLO=LENGTH OF COMPACTION REGION (IN)
      C      ETA=PROPELLANT COVOLUME (IN**3/LB)
    1       DIMENSION P(30),PBAR(30),VF(30),DL(30),XL(30,30),RL(30,33),V(30,30
     $),XMG(30,30),ZL(30),CL(30),E(30,30)
      C      SUBSCRIPT J DENOTES THE NUMBER OF TIME INTERVALS (PT) DURING WHICH
      C      THE FLAMEFRONT PENETRATES THE PROPELLANT,J=1 CORRESPONDE TO IGNITIO
    2       DATA RHOP,RO,RI,XLO,ETA/0.0560,0.0115,0.0075,0.23,32.2/
    3     2 READ(5,5000)F,ALFA,BETA,B,EN,FACT
    4       READ(5,5000)RGAS,GAMMA,CV,TZERO
    5  5000 FORMAT(2E20.5)
    6        WRITE(6,5000)F,ALFA,BETA,B,EN,FACT
    7       WRITE(6,5000)RGAS,GAMMA,CV,TZERO
    8       VEL=0.0
    9       AZERO=SQRT(GAMMA*RGAS*TZERO)
   10       C=4.014E-03
   11       ICOUNT=0
   12        DO 10 I=1,30
   13       DO 10 J=1,30
   14       E(I,J)=0.0
   15    10 XL(I,J)=0.0
   16       PIE=3.1415
   17       T=0.0001
   18       XTG=0.0
   19       RR=RO-RI
   20       VOI=2.*PIE*RR*RR*RI+PIE*PIE*RI*RI*(RR+1.333*RI/PIE)
      C     XN=THE NUMBER OF GRAINS PER UNIT VOLUME
      C     A=CROSS SECTIONAL AREA OF CARTRIDGE
      C     D=DIAMETER OF CARTRIDGE CASE 0.335
   21       A=PIE*0.335*0.335/4.
      C     VC=VOLUME OCCUPIED BY COMPRESSED PROPELLANT
   22       VC=A*(1.5-XLO)
   23       XN=C/(RHOP*VOI*VC)
   24       WRITE(6,1234)
   25       P(1)=2750.
   26       DO 1000 J=2,29
   27       P(J)=P(J-1)
   28       DT=0.25E-04
   29       T=T+DT
   30  801  ICOUNT=ICOUNT+1
      C     SUBSCRIPT J DENOTES THE NUMBER OF TIME INTERVAL(DT) DURING WHICH
      C     THE FLAMEFRONT PENETRATES THE PROPELLANT BED(IGNIT TIME=0.0001S)
      C
      C     CALCULATE THE DISTANCE (D(J)) THE FLAMEFRONT HAS TRAVELED
   31       PBAR(J)=(P(J-1)+P(J))/2.
   32       VF(J)=ALFA*PBAR(J)**BETA
   33       DL(J)=VF(J)*DT
   34       ZL(1)=XLO
            CL(1)=XLO
```

```
      IF(ZL(J).GE.1.5) ZL(J)=1.5
      IF(ZL(J).GE.1.5)DL(J)=ZL(J)-ZL(J-1)
      IF(ZL(J).GE.1.5)T=T-DT
      IF(ZL(J).GE.1.5)DT=DL(J)/VF(J)
      IF(ZL(J).GE.1.5)T=T+DT
C
C     CALCULATE THE DISTANCE(XL(I,J)) BURNED NORMAL TO PROP. SURFACE(IN)
C     CELL I DURING TIME DT,I=1 IS INITIAL COMPACTION REGION(NO PROP.)
      PXTG=0.0
      VOLS=0.0
      DO 500 I=2,J
      DXL=B*PBAR(J)**EN*DT
      DXL=DXL*FACT
      XL(I,J)=XL(I,J-1)+DXL
      RL(I,J)=RI-XL(I,J)
      V(I,J)=2.*PIE*(RR**2*RL(I,J))+PIE*PIE*RL(I,J)**2*(RR+1.333/PIE*RL(
     #I,J))
C
C     XMG(I,J)IS THE AMOUNT OF GAS GENERATED FROM CELL I DURING TIME T
      XMG(I,J)=DL(I)*XN*A*(VOI-V(I,J))*RHOP
C     XTG=THE TOTAL GAS GENERATED IN TIME T
      DXMG=XMG(I,J)
      E(I,J)=(XN*A*DL(I)*VOI-XMG(I,J)/RHOP)/A/DL(I)
      VOLS=VOLS+A*DL(I)*E(I,J)
      PXTG=PXTG+DXMG
  500 CONTINUE
      XNUM=12.*PXTG*F+P(1)*A*ZL(1)
      DENOM=A*ZL(J)-VOLS-PXTG*ETA
      PPBAR=XNUM/DENOM
      IMN=J
      IF(ABS((PPBAR-P(J))/PPBAR)-0.05)55,55,800
  800 IF(ICOUNT.GT.25) GO TO 1001
      P(J)=PPBAR
      GO TO 801
   55 CONTINUE
      XTG=PXTG
      P(J)=PPBAR
      ICOUNT=0
      DO 900 I=2,J
      WRITE(6,2468)T,P(J),ZL(J),XTG,E(I,J),XL(I,J),I,J
      TIME=T
  900 CONTINUE
      IF(ZL(J).GE.1.5) GO TO 1001
      K=J
       CL(J)=ZL(J)
 1234 FORMAT(1H1,9X,'TIME',15X,'HP(J)',15X,'ZL(J)',15X,'XTG',15X,'E(I,J)
     #')
 2468 FORMAT(6(4X,E15.6),2I3)
 1000 CONTINUE
      DO 1002 J=2,9
       DO 1002 I=2,9
      WRITE(6,2468)XL(I,J),DXL,V(I,J),XMG(I,J),RL(I,J),DL(I),I,J
 1002 CONTINUE
      GO TO 1005
 1001 CONTINUE
      J=IMN
      P(J)=P(J)*144.
      DENS=P(J)/RGAS/TZERO
      ENTR=(RGAS/778.+CV)*ALOG(TZERO/530.)-RGAS/778.*ALOG(P(J)/2075.)
      ENTR=ENTR*778./RGAS/GAMMA
      DO 1004 I=1,IMN
      DIST=ZL(I)/12.
      XL(I,J)=XL(I,J)/12.
```

```
 92            TIM=T+(0.125-DIST)/AZERO
 93           WRITE(7,4445)I,J
 94           WRITE(7,4444)DIST,TIM,E(I,J),AZERO,VEL,VEL,P(J),DENS,ENTR,XL(I,J)
             $,TZERO
 95           WRITE(6,4444)DIST,TIM,E(I,J),AZERO,VEL,VEL,P(J),DENS,ENTR,XL(I,J)
             $,TZERO
 96           WRITE(7,4445)I,J
 97      1004 CONTINUE
 98      4445 FORMAT(5X,'********',5X,I4,5X,I4,5X,'*********'5X)
 99      4444 FORMAT(3E20.5)
100      1005 CONTINUE
101           GO TO 2
102           END

 /GO
0.32060E 06          0.31500E 02
0.48000E 00          0.26300E-02
0.70000E 00          0.10000E 01
0.25643E 04          0.12506E 01
0.11470E 02          0.46150E 04
```