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A NOTE ON FAVRE AVERAGING IN VARIABLE DENSITY FLOWS

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A NOTE ON FAVRE AVERAGING IN VARIABLE DENSITY FLOWS

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ABSTRACT

The concept of a Favre probability density function is introduced and this formalizes the definition of Favre averages in variable density flows. Current approaches to turbulence modeling suggest that it is the Favre probability density function which models in such flows. This is of particular use in the modeling of conserved scalars since the conservation equations deliver the Favre average and variance.

A Note on Favre Averaging in Variable Density Flows

by

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In many treatments of variable density flows conventional Reynolds averaging of the equations of motion and scalar conservation is used and terms such as $\bar{u}\,\overline{\rho' v'}$ and $\bar{\xi}\,\overline{\rho' v'}$ are neglected (u, v are velocity components, ρ is density and ξ a conserved scalar; primes denote fluctuating part relative to the conventional mean denoted by an overbar, e.g. $\bar{u}$). Stanford and Libby (1974) have measured these quantities in helium-air mixing and find them to be of the same order and sometimes greater than the more familiar momentum and scalar fluxes $\bar{\rho}\,\overline{u'v'}$ and $\bar{\rho}\,\overline{v'\xi'}$. The answer of course lies in Favre averaging (Favre (1969)) which makes the continuity equation exact and eliminates double correlations involving density fluctuations from the turbulent fluxes. In Favre averaging quantities are weighted by the instantaneous density before averaging. The resulting equations are similar in form to the Reynolds equations for uniform density flow except for the terms involving molecular viscosity and diffusivity. Here we confine ourselves to free turbulent flows at low Mach number which obviates the main difficulties with these molecular terms. We concern

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ourselves with modeling the turbulence terms in their Favre form and with handling equation of state information.

We introduce here the concept of the Favre probability density function (p.d.f.) $\tilde{p}_Q(Q)$ of a variable Q:

$$\tilde{p}_Q(Q) \equiv (1/\bar{\rho}) \int_0^\infty \rho \, p_{\rho Q}(\rho, Q) \, d\rho \qquad (1)$$

where $\bar{\rho}$ is the conventional average of the density ρ and $p_{\rho Q}(\rho, Q)$ is the conventional joint p.d.f. of ρ and Q. Favre means, variances, etc. are obtained by weighting by $\tilde{p}_Q(Q)$. Thus

$$\tilde{Q} = \overline{\rho Q}/\bar{\rho} = \int_Q Q \, \tilde{p}_Q(Q) \, dQ \qquad (2)$$

$$\widetilde{Q''^2} = \overline{\rho Q''^2}/\bar{\rho} = \int_Q (Q - \tilde{Q})^2 \, \tilde{p}_Q(Q) \, dQ \qquad (3)$$

$$\widetilde{Q''^3} = \overline{\rho Q''^3}/\bar{\rho} = \int_Q (Q - \tilde{Q})^3 \, \tilde{p}_Q(Q) \, dQ \qquad (4)$$

$$\widetilde{f(Q)} = \int_Q f(Q) \, \tilde{p}_Q(Q) \, dQ \qquad (5)$$

Here $f(Q)$ is any function of Q and $Q'' = Q - \tilde{Q}$ and $\tilde{Q}'' = \overline{\rho Q''}/\bar{\rho} = 0$. Note also that $\tilde{Q} = \bar{Q} + \overline{\rho' Q'}/\bar{\rho}$ and $\overline{Q''} = \overline{\rho' Q'}/\bar{\rho} \neq 0$. It is also possible to define multi-dimensional probability density functions in the above Favre form and so obtain Favre cross correlations.

By way of illustration we consider the axi-symmetric boundary layer equations for low Mach number free turbulent flow with chemical reaction. In Favre form and the usual notation they are:

continuity:

$$\frac{\partial}{\partial x}(\bar{\rho}\,\tilde{u}) + \frac{1}{r}\frac{\partial}{\partial r}(r\,\bar{\rho}\,\tilde{v}) = 0 \tag{6}$$

momentum:

$$\bar{\rho}\,\tilde{u}\,\frac{\partial \tilde{u}}{\partial x} + \bar{\rho}\,\tilde{v}\,\frac{\partial \tilde{v}}{\partial r} = -\frac{\partial p}{\partial x} - \frac{1}{r}\frac{\partial}{\partial r}(r\bar{\rho}\;\widetilde{\overline{u''v''}}) \tag{7}$$

species:

$$\bar{\rho}\,\tilde{u}\,\frac{\partial \tilde{Y}_i}{\partial x} + \bar{\rho}\,\tilde{v}\,\frac{\partial \tilde{Y}_i}{\partial r} = -\frac{1}{r}\frac{\partial}{\partial r}(r\bar{\rho}\,\widetilde{\overline{v''Y_i''}}) + \bar{\rho}\,\widetilde{\overline{(w_i/\rho)}} \tag{8}$$

mixing:

$$\bar{\rho}\,\tilde{u}\,\frac{\partial \tilde{\xi}}{\partial x} + \bar{\rho}\,\tilde{v}\,\frac{\partial \tilde{\xi}}{\partial r} = -\frac{1}{r}\frac{\partial}{\partial r}(r\bar{\rho}\,\widetilde{\overline{v''\xi''}}) \tag{9}$$

standardized enthalpy:

$$\bar{\rho}\,\tilde{u}\,\frac{\partial \tilde{H}}{\partial x} + \bar{\rho}\,\tilde{v}\,\frac{\partial \tilde{H}}{\partial r} = -\frac{1}{r}\frac{\partial}{\partial r}(r\,\bar{\rho}\,\widetilde{\overline{v''H''}}) \tag{10}$$

turbulent "kinetic energy":

$$\bar{\rho}\,\tilde{u}\,\frac{\partial}{\partial x}(\tfrac{1}{2}q^2) + \bar{\rho}\,\tilde{v}\,\frac{\partial}{\partial r}(\tfrac{1}{2}q^2) = -\bar{\rho}\,\widetilde{\overline{u''v''}}\,\frac{\partial \tilde{u}}{\partial r} - \bar{\rho}\,\widetilde{\overline{\frac{v''}{\rho}}}\,\frac{\partial p}{\partial r}$$

$$-\frac{1}{r}\frac{\partial}{\partial r}\{\tfrac{1}{2}\,r\,\bar{\rho}\;\widetilde{\overline{v''(u''^2 + v''^2 + w''^2)}}\} - \bar{\rho}\,\epsilon \tag{11}$$

scalar variance:

$$\bar{\rho}\,\tilde{u}\,\frac{\partial}{\partial x}(\widetilde{\xi''^2}) + \bar{\rho}\,\tilde{v}\,\frac{\partial}{\partial r}(\widetilde{\xi''^2}) = -2\,\bar{\rho}\,\widetilde{\overline{v''\xi''}}\,\frac{\partial \tilde{\xi}}{\partial r}$$

$$-\frac{1}{r}\frac{\partial}{\partial r}\{r\,\bar{\rho}\;\widetilde{\overline{v''\xi''^2}}\} - \bar{\rho}\,X \tag{12}$$

Here $q^2 = \widetilde{\overline{u''^2}} + \widetilde{\overline{v''^2}} + \widetilde{\overline{w''^2}}$ and a tilde over a long overbar denotes a Favre average. The turbulence dissipation ϵ and scalar dissipation χ are given to sufficient accuracy in Cartesian tensor notation by:

$$\epsilon = \frac{1}{2} \nu \widetilde{\overline{\left(\frac{\partial u''_i}{\partial x_j} + \frac{\partial u''_j}{\partial x_i}\right)\left(\frac{\partial u''_i}{\partial x_j} + \frac{\partial u''_j}{\partial x_i}\right)}} \qquad (13)$$

$$\chi = 2 D \widetilde{\overline{\frac{\partial \xi''}{\partial x_k} \frac{\partial \xi''}{\partial x_k}}} \qquad (14)$$

These equations are exactly similar to those normally used for uniform density flows except that Favre averages and Favre fluctuations are used. Favre cross-correlations such as $\widetilde{\overline{u'' v''}}$ are of course formed from the joint probability density function, in this case $\tilde{P}_{uv}(u, v)$. It can be seen that authors that write these equations in Reynolds average form and neglect such terms as $\bar{u}\,\overline{\rho' v'}$ are effectively using these Favre equations.

The system of equations (6) to (12) is not closed, there being more variables than equations. In uniform density flows closure is obtained by modeling some of the turbulent correlations in terms of mean flow variables and/or other turbulent correlations. Launder and Spalding (1972) and Mellor and Herring (1973) survey some of the available methods. In variable density flows the same modeling techniques are used, occasionally with an explicit dependence for density. Without reviewing the literature in detail here the general conclusion is that reasonably good results are obtained with implicit use of the Favre equations (i.e. neglecting

$\bar{u}\,\overline{\rho' v'}$, etc.) and no special dependence on density used in the turbulence model. Libby (1973) studied the two-dimensional mixing layer using the Favre equations explicitly and modeling the Favre turbulence terms in a manner similar to that used in uniform density flows. He found that no special (i.e. extra) allowance was needed for density in low Mach number flows. It appears then that "$\widetilde{u'' v''}$, etc. model like $\overline{u' v'}$, etc. do in uniform density flow" is a viable hypothesis. This almost implies that the Favre joint probability density functions $\tilde{P}_{uv}(u, v)$ model like the conventional ones do in uniform density flow. The "almost" is used since the modeling often only involves the double correlations, i.e. second moments, of the joint p.d.f. although triple correlations and third moments are involved if equations such as (11) and (12) are used.

Closure of the equations (6) to (12) also requires the use of the equation of state. In particular equations (6), (7) and (11) which describe the velocity field are coupled to the rest of the equations by the mean density. $\bar{\rho}$ is required and equations (8) to (10) deliver Favre averages, $\tilde{Y}_i$, $\tilde{H}$ etc. Most of these problems disappear when it is realised that

$$1/\bar{\rho} = \widetilde{(1/\bar{\rho})} = \frac{R}{\bar{p}} \sum_i \widetilde{Y_i T}/W_i \qquad (15)$$

the latter for an ideal gas with R the universal gas constant. For isothermal flows with varying molecular weight or uniform molecular weight flows with varying temperature (15) becomes much simpler involving only $\tilde{Y}_i$ or $\tilde{T}$. For mixing and combustion flows under the assumption of all Lewis numbers equal and fast or frozen chemistry all thermodynamic properties such as Y_i, ρ, and T are functions of a conserved scalar (see Bilger (1975)). The conserved scalar can be chosen to be the mixture fracture ξ with limits of 0 and 1 in the unmixed flows. Assuming that the Favre p. d. f. of ξ, $\tilde{p}_\xi(\xi)$ is available then we have:

$$1/\bar{\rho} = \int_0^1 \{1/\rho(\xi)\} \tilde{p}_\xi(\xi)\, d\xi \tag{16}$$

$$\tilde{Y}_i = \int_0^1 Y_i(\xi)\, \tilde{p}_\xi(\xi)\, d\xi \tag{17}$$

$$\bar{Y}_i = \bar{\rho} \int_0^1 \{Y_i(\xi)/\rho(\xi)\} \tilde{p}_\xi(\xi)\, d\xi \tag{18}$$

with similar results for the other thermodynamic variables. For frozen chemistry and either equal molar specific heats or uniform temperature, specific volume, composition and temperature are linear functions of ξ and so $\bar{\rho} = \rho(\tilde{\xi})$, $\tilde{Y} = Y_i(\tilde{\xi})$, $\tilde{T} = T(\tilde{\xi})$.

This leaves the question of modeling $\tilde{p}_\xi(\xi)$. We recall our discussion above about the modeling of the Favre turbulence correlations and how this was almost equivalent to assuming that the Favre joint p. d. f. 's model as the conventional p. d. f. 's do in uniform density flows. It appears reasonable then to postulate that $\tilde{p}_\xi(\xi)$ will model in variable density flows the same way that $p_\xi(\xi)$ can be modeled in uniform density flows. Some effect of density variations on $p_\xi(\xi)$ can be expected and $\tilde{p}_\xi(\xi)$ is particularly convenient since its mean $\tilde{\xi}$ and variance $\widetilde{\xi''^2}$ and not $\bar{\xi}$ and $\overline{\xi'^2}$ are given by solution of equations (9) and (12). Even if $\tilde{p}_\xi(\xi)$ needs to be modeled with a further dependence on density it will be more convenient to work with $\tilde{p}_\xi(\xi)$ just for this reason.

In summary we have introduced the concept of a Favre probability density function and used it to clarify and elucidate many of the assumptions commonly made in modeling turbulence in variable density flows. In mixing and combustion flows the Favre p.d.f. of the conserved scalar ξ will be particularly useful.

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