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ROTOR INDUCED POWER

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St. Louis, Missouri

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Methodology is presented for rapid estimation of the induced power requirements of a helicopter rotor. Operating ranges treated are hover in or out of ground effect, vertical climb, and level flight. Earlier work on this subject is modified by more recent flight test information on ground effect; enlarged in scope by the consideration of a generalized trapezoidal inflow velocity distribution at the rotor; and refined to account for the influence of level flight on the effect of a non-uniform induced velocity distribution at the rotor.

DISCLAIMER STATEMENT

The findings in this report are not to be construed as an official Department of the Army position.

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## INTRODUCTION

It is possible to evolve a method for rapidly estimating helicopter rotor induced power by means of the momentum theory. This has been done and is well documented in the literature on rotary-wing aircraft. However, the theory requires the assumption of ideal conditions that are not fully realizable in the physical world. For practical application, the theoretical results are generally modified by appropriate factors that lead to more realistic results. The usual nature and magnitude of those modifying factors is a product of logic, intuition, and empiricism.

The present report unifies the most generally accepted modifying factors and enlarges on some of them to provide broader applicability over a helicopter's flight regime. The results are presented in a form that enables rapid estimation of helicopter rotor induced power. The following elements of the method, as presented in this report, are believed to differ from earlier published approaches.

a. The application of correlated flight test data on the thrust increase in ground effect, reference 1, to provide an indication of ground effect on induced power requirements.

b. The effect of a trapezoidal inflow velocity distribution on the induced power requirements in hover.

c. The effect of a non-uniform inflow velocity distribution on the induced power requirements in a vertical climb.

d. The variation with forward flight velocity of the effect of a non-uniform induced velocity distribution on the induced power requirements.

## DISCUSSION

From momentum theory, with the assumption of uniform inflow, the induced power requirement of a rotor, out of ground effect, can be expressed as the product of the rotor thrust and the velocity of the flow induced through the rotor,

$$P_{i_\square} = T\, u_\square \tag{1}$$

When the induced flow is the only flow through the rotor, as in hover, the induced velocity can be expressed explicitly as a function of the rotor thrust,

$$u_\square = u_H = \sqrt{\frac{T}{2\rho A}} \tag{2}$$

so that

$$P_{i_\square} = \frac{(T)^{3/2}}{\sqrt{2\rho A}} \tag{3}$$

With additional velocity through the rotor, as in vertical climb or level flight, the induced velocity is not explicitly related to the rotor thrust alone. However, the induced velocity in those cases can be referred to an equivalent hover value, $u = K\, u_H$, so that equation (3) can be used with the modifying factor, K. Values of this factor, to be used for the cases of vertical climb and level flight, are derived in the following sections.

Experience has indicated that rotor induced velocity distributions, rather than being uniform, are more nearly triangular in shape, with maximum values near the rotor tips. An expression is derived herein which relates the induced power requirement in hover for the general case of a trapezoidal inflow distribution to that of a uniform inflow, $P_i = \eta P_{i_{\square}}$. Equation (3) can then be applied with the appropriate value of this factor.

When a rotor is close to the ground, its flow pattern no longer corresponds to that assumed for the momentum theory. Although other theoretical approaches can provide reasonably valid indications of ground effect, the complexities of the flow pattern, and viscous effects, limit the validity of theory. As an alternative, experimental data can be used to relate the induced power required in ground effect to that required out of ground effect, $P_{i_{IGE}} = \Lambda P_{i_{OGE}}$. In a following section, the results of a recent study of flight test data, Reference 1, are used to determine appropriate values of $\Lambda$.

The above three factors, combined with the usual rotor tip loss factor, B, and with an allowance for the vertical drag of the body surfaces in the rotor's wake, $D_V$, in the form

$$T = F_{vd} W_G = \left(1 + \frac{D_V}{W_G}\right) W_G \qquad (4)$$

can be applied to equation (3) to derive a general expression for the induced power requirement of a helicopter,

$$P = \frac{\Lambda \eta K}{B} \frac{\left(F_{vd} W_G\right)^{3/2}}{\sqrt{2 \rho A}} \qquad (5)$$

Expressed in coefficient form, equation (5) becomes

$$C_{P_i} = \frac{\Lambda \eta K}{B\sqrt{2}} (C_T)^{3/2} \tag{6}$$

GROUND EFFECT, $\Lambda$:

Numerous ground effect studies have been made with both model scale and full scale flight articles. There are disagreements among the results as presented in the literature. A recent thorough analysis of flight test data is presented in Reference 1, and its results are used in the present report. They are expressed in terms of a thrust increase for the rotor in ground effect, rather than an induced power decrease. An error analysis was used in Reference 1 to account for data scatter, and led to the relation

$$\lambda = \frac{(C_T)_{IGE}}{(C_T)_{OGE}} = \frac{Z/D}{\left(1.099 - .289\frac{(C_T)_{OGE}}{\nabla}\right)\frac{Z}{D} + .391\frac{(C_T)_{OGE}}{\nabla} - .104} \tag{7}$$

Since $(C_T)_{OGE} = \frac{(C_T)_{IGE}}{\lambda}$, equation (7) can be rearranged to express $\lambda$ as an explicit function of $\frac{(C_T)_{IGE}}{\nabla}$ and Z/D,

$$\lambda = \frac{Z/D + \frac{(C_T)_{IGE}}{\nabla}\left(.289\frac{Z}{D} - .391\right)}{1.099\frac{Z}{D} - .104} \tag{8}$$

A comparison of equation (8) with the earlier empirical work of Reference 2 shows good agreement between the two for values of Z/D greater than about 0.3 and values of $C_T/\nabla$ greater than about 0.06. In general, equation (8) is more conservative than Reference 2 at lower values of $C_T/\nabla$ and more optimistic than Reference 2 at lower values of Z/D.

Equation (8) can be applied to the induced power relation by noting that, out of ground effect, $C_{P_i} \sim C_{T_{OGE}}^{3/2}$. The same cannot be said for the rotor in ground effect; however, for the same induced power coefficient,

$$C_{P_{i_{IGE}}} = C_{P_{i_{OGE}}} \sim \left(\frac{C_{T_{IGE}}}{\lambda}\right)^{3/2}$$

Therefore, the power ratio for the case of equal rotor thrusts is

$$\Lambda = \frac{P_{i_{IGE}}}{P_{i_{OGE}}} = \left(\frac{1}{\lambda}\right)^{3/2} \tag{9}$$

Equation (9) is plotted in Figure 1 using values of $\lambda$ computed by means of equation (8) for a range of Z/D and $C_T/\sigma$. The flight test data on which the preceding is based are limited to values of Z/D greater than 0.28 and values of $C_T/\sigma$ in the range of 0.05 to 0.13. Consequently, values of $\Lambda$ outside of those limits in Figure 1 are extrapolations from the data and should be used with caution.

## NON-HOVERING INFLOW VELOCITY FACTOR, K:

### Vertical Climb:

For a rotor rising vertically at a constant velocity, $P_c = T_c\,(V_c + u_c)$, where $T_c = 2\,\rho\,A\,u_c\,(V_c + u_c)$. The latter equation can be solved for $u_c$, to yield.

$$u_c = -\frac{V_c}{2} + \sqrt{\left(\frac{V_c}{2}\right)^2 + \frac{T_c}{2\rho A}} \tag{10}$$

Expressed in coefficient form, the induced power portion of the total climb power required is

$$C_{P_{i_c}} = \frac{C_{T_c} u_c}{\Omega R} = \frac{C_{T_c}}{2}\sqrt{\left(\frac{V_c}{\Omega R}\right)^2 + 2C_{T_c}} - \frac{C_{T_c}}{2}\left(\frac{V_c}{\Omega R}\right) \qquad (11)$$

This can be put into the form

$$C_{P_{i_c}} = \frac{K_c}{B} \frac{(C_{T_c})^{3/2}}{\sqrt{2}} \qquad (12)$$

where

$$K_c = \frac{u_c}{u_H} = \sqrt{\left(\frac{1}{2}\frac{V_c}{u_H}\right)^2 + 1} - \frac{1}{2}\frac{V_c}{u_H} \qquad (13)$$

$K_c$ is plotted as a function of $\frac{V_c}{u_H}$ in Figure 2.

Level Flight

For a rotor in forward flight the resultant velocity at the rotor (V+u), is generally expressed as $V'$, where

$$V' = \sqrt{(V_o \cos\alpha)^2 + (V_o \sin\alpha - u_o)^2} \qquad (14)$$

so that

$$T_o = 2\rho A V' u_o \qquad (15)$$

in contrast with the hovering case, where

$$T_H = 2\rho A u_H^2 \qquad (16)$$

The corresponding induced power expressions

$$P_{i_o} = T_o u_c \quad \text{and} \quad P_{i_H} = T_H u_H$$

can be combined, so that

$$\frac{P_{i_o}}{P_{i_H}} = \frac{T_o}{T_H}\left(\frac{u_c}{u_H}\right)$$

or
$$P_{i_c} = P_{i_H}\left(\frac{T_o}{T_H}\right)\left(\frac{u_o}{u_H}\right) \tag{17}$$

From equations (15) and (16),

$$\frac{T_c}{T_H} = \left(\frac{V'}{u_H}\right)\left(\frac{u_o}{u_H}\right)$$

If we assume that $T_o = T_H$, then

$$\frac{u_o}{u_H} = \frac{u_H}{V'}$$

or, calling

$$\frac{u_o}{u_H} = K_u \text{ ,}$$

$$K_u = \frac{u_H}{\sqrt{(V_o \cos\alpha)^2 + (V_o \sin\alpha - K_u u_H)^2}} \tag{18}$$

Equation (18) can be rearranged, see Appendix I, as

$$\frac{V_o}{u_H} = K_u \sin\alpha + \frac{1}{K_u}\sqrt{1 - K_u^4 \cos^2\alpha} \tag{19}$$

Equation (19) has been used to prepare a graph of $K_u$ as a function of $V_o/u_H$, Figure 3.

The effect of rotor angles of attack within the normal range is seen to be very slight. Accordingly, equation (19) could be reduced to

$$\frac{V_o}{u_H} = \sqrt{\frac{1}{K_u^2} - K_u^2} \tag{20}$$

with no significant loss of accuracy.

We can express equation (17) as $P_{i_o} = P_{i_H} K_u$ for the case of $T_o = T_H$ . In forward flight, $T_o \neq T_H$ in general, however, if $P_{i_H}$ is defined as the induced power that would be required in a hover at a thrust level of $T_o$ , then $P_{i_H} = T_o u$, where

$$u = \frac{1}{B}\sqrt{\frac{T_o}{2\rho A}}$$

so that

$$P_{i_H} = \frac{T_o}{B}\sqrt{\frac{T_o}{2\rho A}}$$

and

$$P_{i_o} = K_u\left(\frac{1}{B}\right)\frac{(T_o)^{3/2}}{\sqrt{2\rho A}} \qquad (21)$$

or

$$C_{P_{i_o}} = \frac{K_u}{B}\frac{(C_{T_o})^{3/2}}{\sqrt{2}} \qquad (22)$$

for the general case, for the hovering case, $K_u = 1.0$, and $T_o = T_h = F_{vd} W_G$.

<u>NON-UNIFORM INFLOW FACTOR, $\eta$:</u>

<u>Hover:</u>

Reference 3 includes a derivation of a factor which relates the induced velocity for a triangular inflow distribution to that for a uniform distribution. In general, such a distribution is more realistic than the uniform. The result of the analysis in Reference 3, in terms of induced power required relative to that required with a uniform inflow, in hover, is

$$\eta = \frac{(C_{P_i})_\Delta}{(C_{P_i})_\square} = 1.131$$

It is possible, with appropriate combinations of rotor blade taper and twist, to induce rotor inflows in distributions that are less severe than the triangular. To allow for a rotor inflow distribution of any trapezoidal form, Appendix II presents a derivation of the value of $\eta$ applicable to any assumed trapezoidal shape of inflow distribution. That analysis includes the uniform and triangular distributions as two of its possible cases. Figure 4 is a plot of $\eta$, for a hovering rotor, as a function of the trapezoidal shape factor, $\xi$, where

$$\xi = {}^{u_A}/_{u_T}$$

Vertical Climb:

The non-uniform inflow factor, $\eta$, can be computed for a rotor climbing vertically, in the same manner as was done in Appendix II, for the hovering case. The result of such an analysis, for an assumed triangular inflow distribution, is shown in Appendix III, where $\eta$ is expressed as a function of ${}^{V_c}/_{u_H}$, the ratio of the vertical climb velocity to the ideal induced velocity for hover at the same thrust. The result is also plotted in Figure III-1. The same can be done for a climbing rotor with an assumed generalized trapezoidal inflow distribution. However, as shown by Figure III-1, the effect of vertical climb on $\eta$ for a triangular inflow distribution is small, and since that is the trapezoidal shape that will experience the greatest changes in $\eta$ due to the superposition of a climbing velocity, the more involved algebra of the generalized trapezoidal inflow assumption is not warranted. It is assumed, as a consequence of this study, that the effect of vertical climb on $\eta$ is of second order and does not merit consideration in the rapid estimation method contemplated here.

Level Flight:

In forward flight, the induced velocity becomes a decreasingly significant portion of the flow through the rotor as the forward velocity increases. The induced power, expressed as an induced drag, has been shown to approach that of a circular wing, Ref. 4; i.e.,

$$\left(\frac{D}{L}\right)_i = \frac{C_L}{4}\left[\frac{\mu}{\cos^3\alpha\,(\mu^2+\lambda^2)^{1/2}}\right] \tag{23}$$

at low speeds, and

$$\left(\frac{D}{L}\right)_i \approx \frac{C_L}{4} \tag{24}$$

at high speeds.

This behavior can be used to postulate a variation of the non-uniform inflow velocity factor, $\eta$ , with forward speed.

In Appendix IV, using Glauert's method, Reference 5, it is shown that, for a circular wing, where

$$\left(\frac{D}{L}\right)_i = \frac{C_L}{4}(1+\delta) \tag{25}$$

$(1 + \delta) = 1.038$. This, then, is a measure of the non-uniformity of the downwash at a circular wing. For the case of a rotor, $\eta$ can be assumed to vary from its value in hover; $\epsilon$ ~., 1.131, to a value of 1.038 at high speed. A transition between these two values can be derived as follows: Since $\alpha$ is generally small, $\cos\alpha \approx 1.0$, and equation (23) can be expressed as

$$\left(\frac{D}{L}\right)_i \approx \frac{C_L}{4}\left(\frac{V_o}{V'}\right)$$

As shown in the derivation of $K_u$, for $T_o = T_H$, $u_H^2 = V' u_o$; therefore,

$$V' = \frac{u_H^2}{u_o}$$

or

$$\left(\frac{D}{L}\right)_i = \frac{C_L}{4}\left(\frac{V_o u_o}{u_H^2}\right) = \frac{C_L}{4}\left(K_u \frac{V_o}{u_H}\right)$$

Then, for $\left(K_u \frac{V_o}{u_H}\right)$ approaching 1.0, $(D/L)_i$ approaches $\frac{C_L}{4}$.

Now, using the previous derived expression for $\frac{V_o}{u_H}$,

$$\frac{V_o}{u_H} = \sqrt{\frac{1}{K_u^2} - K_u^2} \qquad (20)$$

so that $\left(K_u \frac{V_o}{u_H}\right) = \sqrt{1 - K_u^4}$, and Figure 3, we can obtain Figure 5. $\left(K_u \frac{V_o}{u_H}\right)$ approaches 1.0 asymptotically, but for practical purposes can be assumed to reach that value when $\frac{V_o}{u_H} = 2.0$. The variation of $\eta$ with $\frac{V_o}{u_H}$ can be assumed to follow the same pattern as Figure 5, Examples, using various possible values of $\eta_H$, are shown in Figure 6.

RECOMMENDED ESTIMATION PROCEDURE:

The preceding discussion is summarized here in a set of procedures and equations recommended for estimation purposes.

Given: Aircraft gross weight, rotor geometry, and rotor RPM.

Hover:

(1) Determine $C_T/\sigma$ from

$$\frac{C_T}{\sigma} = \frac{F_{vd}\, W_G}{b\, c\, \rho (\Omega R)^2 R}\ , \quad \text{where} \quad \sigma = \frac{b\, c}{\pi R}$$

(2) For desired rotor height, Z/D, and $C_T/\sigma$, as computed in Step (1), determine from Figure 1 whether rotor is in or out of ground effect.

(3) For hover out of ground effect

$$C_{P_i} = \frac{\eta_H}{\sqrt{2}\, B} (C_T)^{3/2}$$

or

$$HP_i = \frac{\eta_H}{\sqrt{2}} \frac{(F_{vd})^{3/2} (W_G)^{3/2}}{550\, B \sqrt{\rho A}}$$

where $\eta_H$ may be selected from Figure 4.

(4) For hover in ground effect,

$$C_{P_i} = \Lambda \left[ \frac{\eta_H}{\sqrt{2}\,B} (C_T)^{3/2} \right]$$

or

$$HP_i = \Lambda \left[ \frac{\eta_H}{\sqrt{2}} \frac{(F_{vd})^{3/2} (W_G)^{3/2}}{550\ B \sqrt{\rho A}} \right]$$

where $\eta_H$ may be selected from Figure 4,

and $\Lambda$ is taken from Figure 1.

Vertical Climb:

For vertical climb, at a given climb velocity, $V_c$, and thrust, $T_c$,

$$\frac{V_c}{u_H} = \frac{V_c}{\sqrt{\frac{T_c}{2\rho A}}}$$

Obtain corresponding value of $K_c$ from Figure 2.

$$C_{P_i} = \frac{K_c \eta_c}{\sqrt{2}\,B} (C_T)^{3/2}$$

or

$$HP_i = \frac{K_c \eta_c}{\sqrt{2}} \frac{(F_{vd})^{3/2} (W_G)^{3/2}}{550\ B \sqrt{\rho A}}$$

where the value of $\eta_H$ selected from Figure 4 can be used as $\eta_c$. If desired, the methods of Appendices II and III can be followed to derive a more precise value of $\eta_c$.

**Forward Flight:**

For forward flight, at a given velocity, $V_0$ , and thrust, $T_0$ ,

$$\frac{V_0}{u_H} = \frac{V_0}{\sqrt{\frac{T_0}{2\rho A}}}$$

Obtain corresponding values of $K_u$ from Figure 3. and $\eta_0$ from Figure 6.

$$C_{P_i} = \frac{K_u \eta_0}{\sqrt{2}\,B} (C_T)^{3/2}$$

or

$$HP_i = \frac{K_u \eta_0}{\sqrt{2}} \frac{(T_0)^{3/2}}{550\,B\sqrt{\rho A}}$$

The above equations require values of B and $F_{vd}$ for their solution. For rapid approximation purposes, and the case of a conventional single rotor helicopter, the values recommended here are

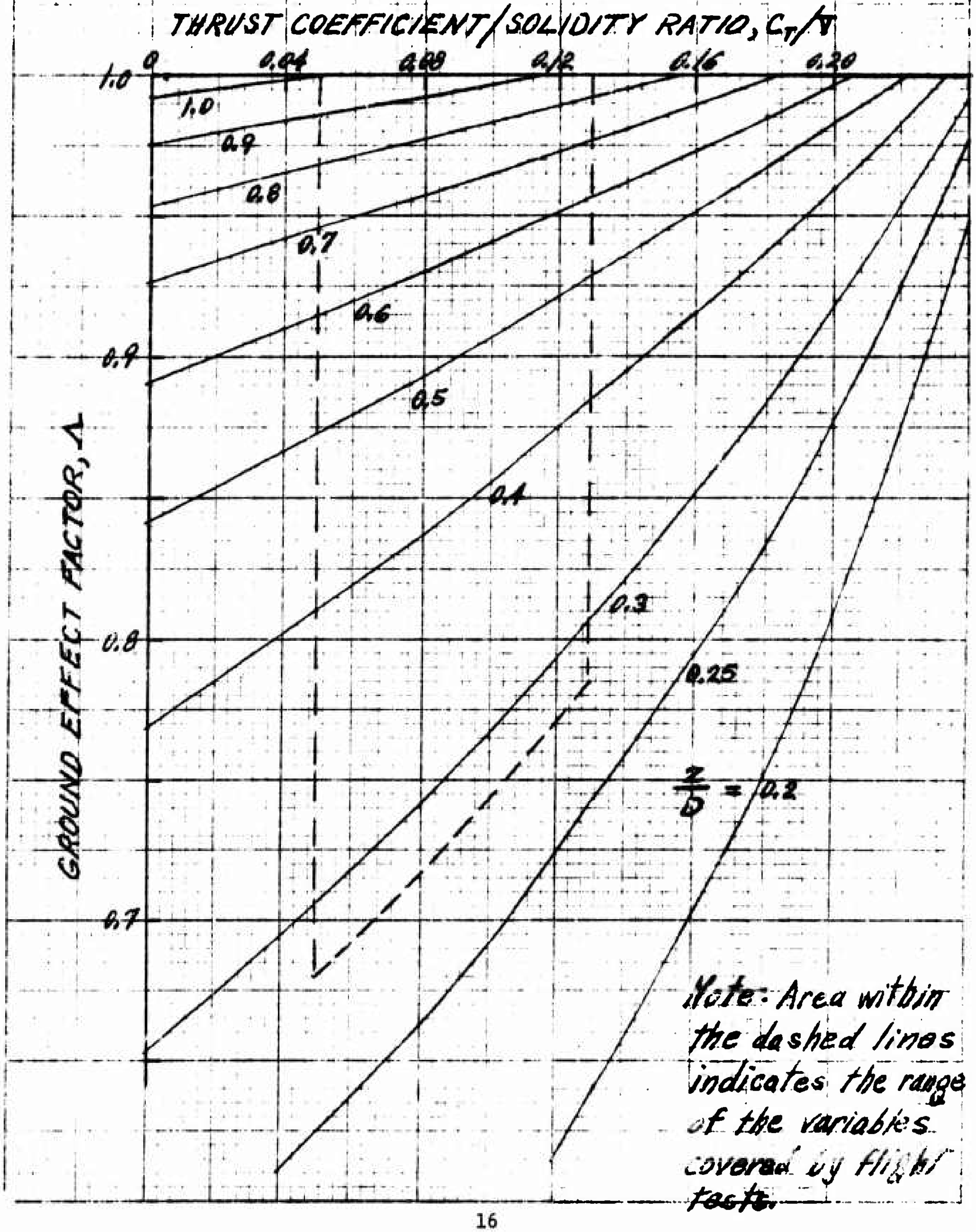
$$B = 0.97 \qquad\qquad F_{vd} = 1.05$$

**Notes:**

1. The same vertical drag factor is suggested for vertical climb as for hover since vertical velocity will normally be small compared to u.

2. The assumption that $\eta$ will remain unchanged in the presence of the ground, implied in the recommendation of $\eta_H$ from Figure 4 for hover in ground effect, admittedly still requires justification.

FIG. 1. GROUND EFFECT ON INDUCED POWER



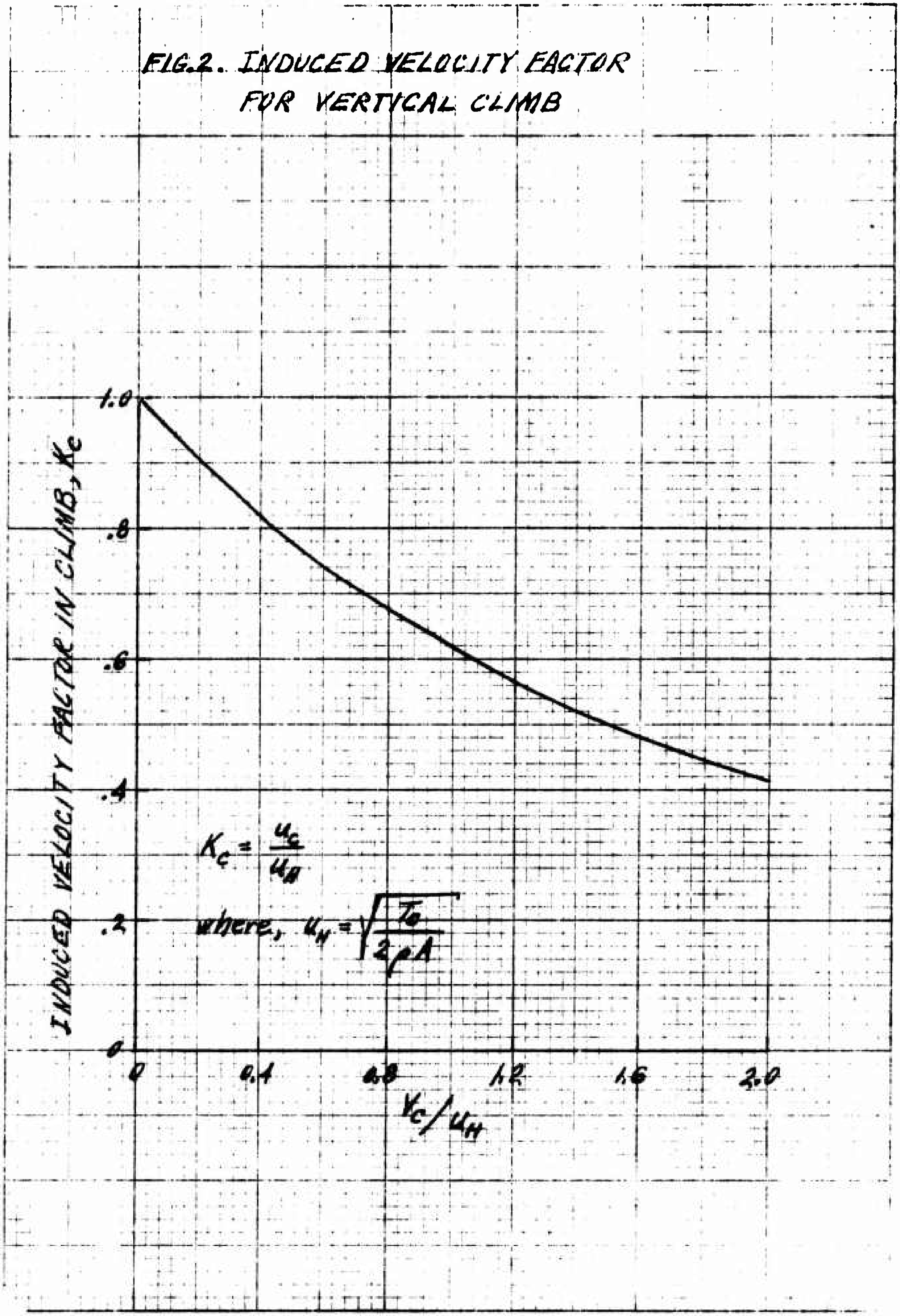


FIG. 2. INDUCED VELOCITY FACTOR FOR VERTICAL CLIMB
INDUCED VELOCITY FACTOR IN CLIMB, $K_c$
1.0
.8
.6
.4
.2
0
0
0.4
0.8
1.2
1.6
2.0
$V_c/u_H$
$K_c = \frac{u_c}{u_H}$
where, $u_H = \sqrt{\frac{T_0}{2\rho A}}$

FIG. 3. INDUCED VELOCITY FACTOR FOR LEVEL FLIGHT

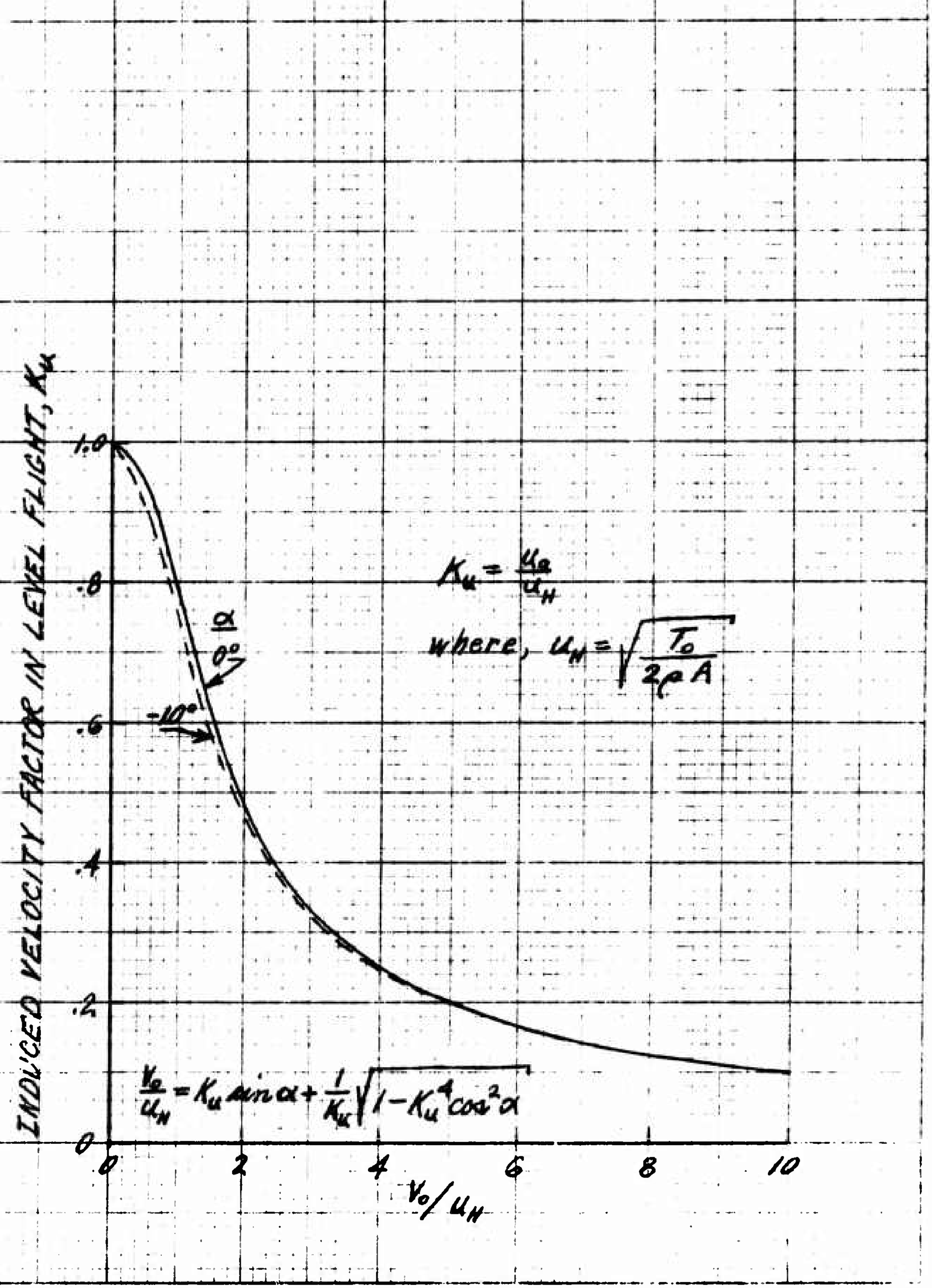


FIG. 4. VARIATION OF INFLOW FACTOR FOR HOVER WITH SHAPE OF INFLOW DISTRIBUTION

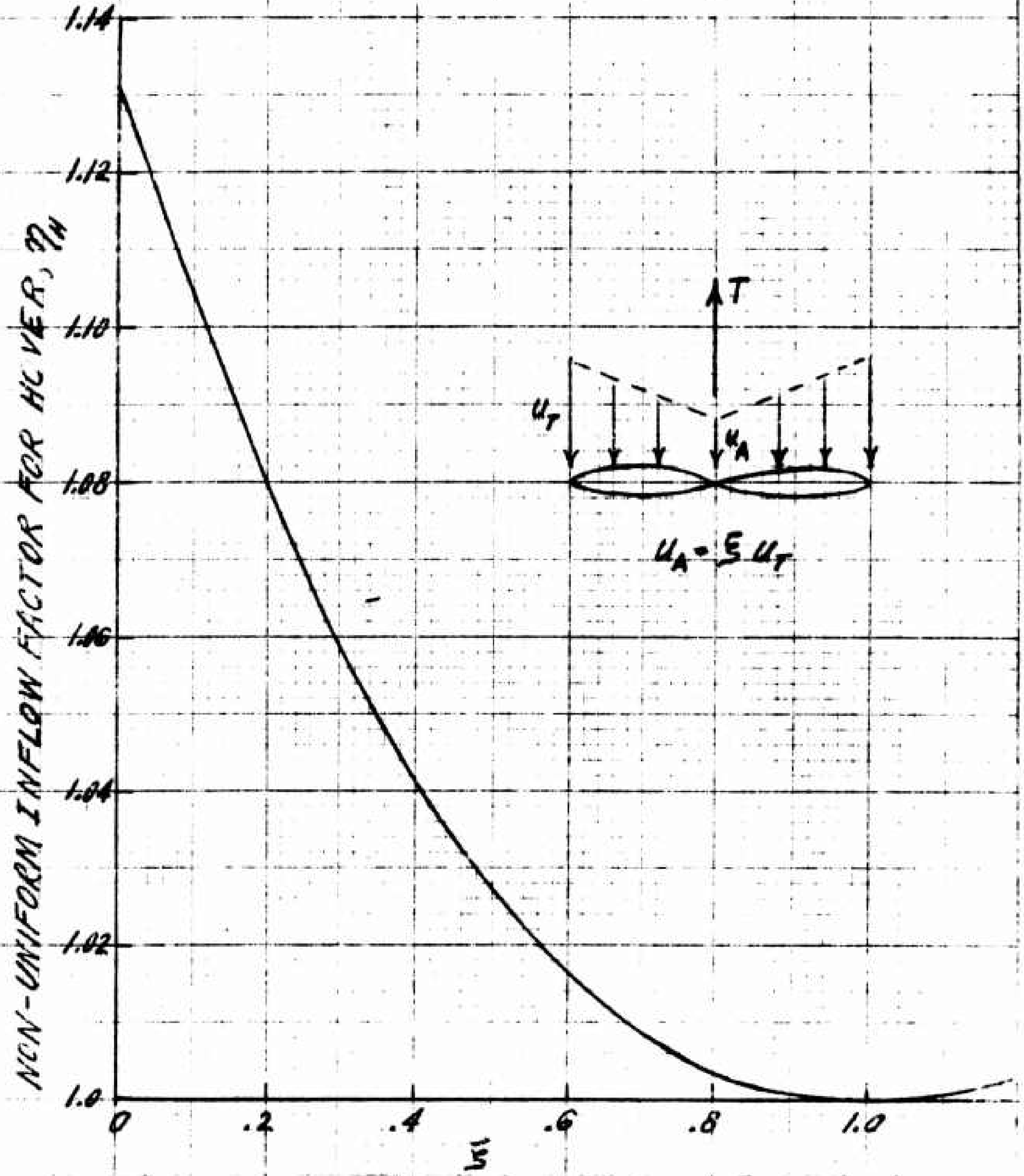


FIG. 5. VELOCITY RATIOS IN LEVEL FLIGHT

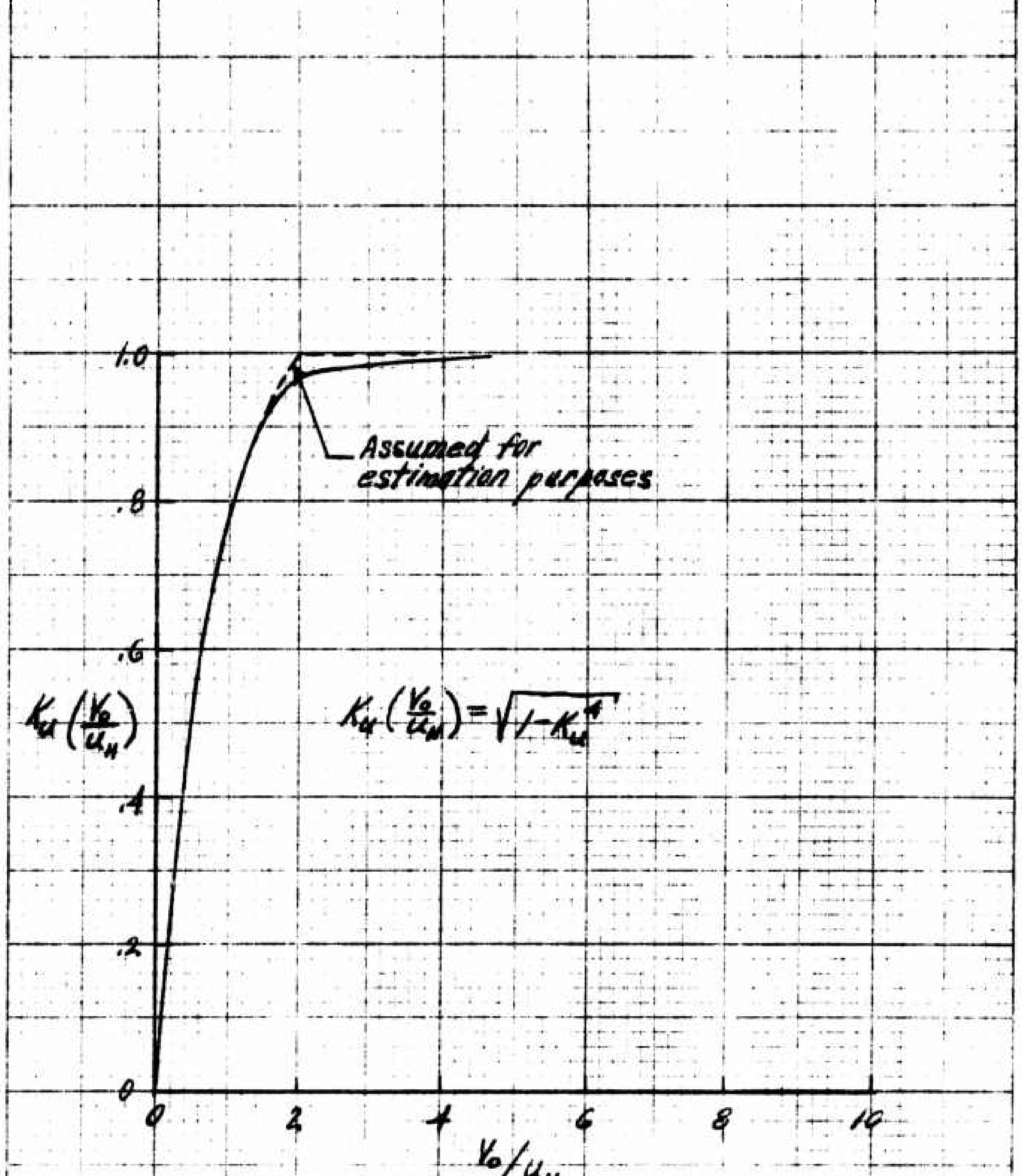
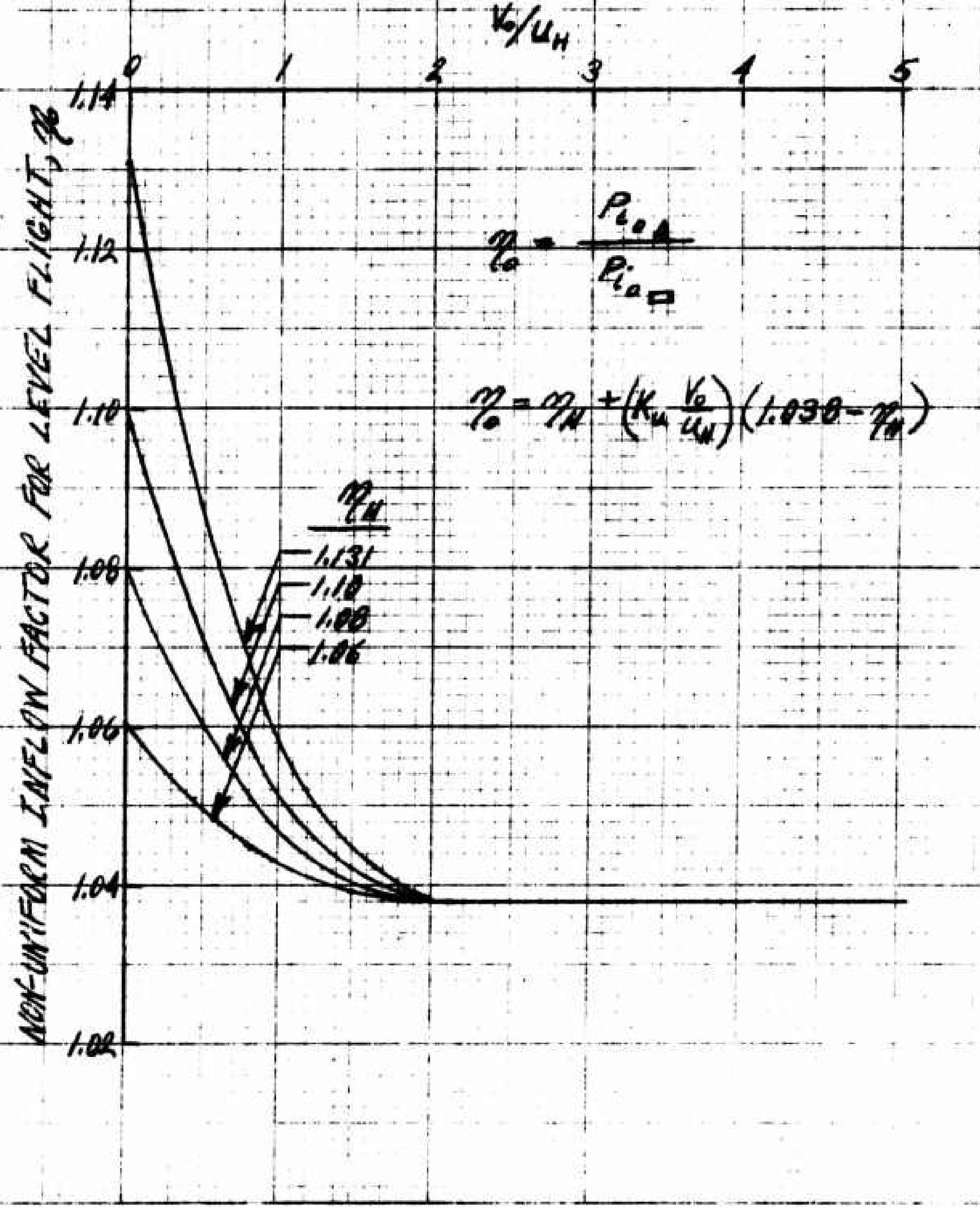


FIG. 6. NON-UNIFORM INFLOW FACTOR FOR LEVEL FLIGHT



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## APPENDIX A

### Induced Velocity Factor For Forward Flight

$$K_u = \frac{u_H}{\sqrt{(V_0 \cos\alpha)^2 + (V_0 \sin\alpha - K_u u_H)^2}}$$

or

$$K_u = \frac{1}{\sqrt{\left(\frac{V_0}{u_H}\right)^2 \cos^2\alpha + \left(\frac{V_0}{u_H}\right)^2 \sin^2\alpha - 2\sin\alpha\, K_u \left(\frac{V_0}{u_H}\right) + K_u^2}}$$

$$\therefore\; K_u^2 = \frac{1}{(V_0/u_H)^2 - (V_0/u_H)\, 2\sin\alpha\, K_u + K_u^2}$$

so

$$\left(\frac{V_0}{u_H}\right)^2 K_u^2 - \left(\frac{V_0}{u_H}\right) 2\sin\alpha\; K_u^3 + K_u^4 - 1 = 0$$

or

$$\left(\frac{V_0}{u_H}\right)^2 - \left(\frac{V_0}{u_H}\right) 2\sin\alpha\, K_u + K_u^2 - \frac{1}{K_u^2} = 0$$

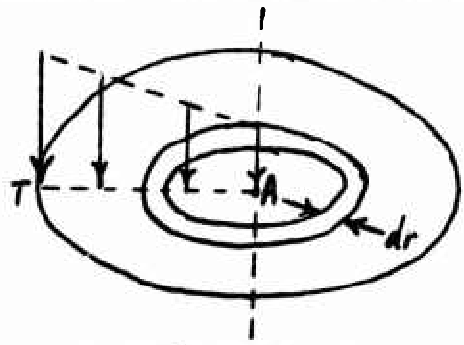
$$\therefore\; \frac{V_0}{u_H} = K_u \sin\alpha \pm \sqrt{K_u^2 \sin^2\alpha - K_u^2 + \frac{1}{K_u^2}}$$

and, for forward flight,

$$\frac{V_0}{u_H} = K_u \sin\alpha + \frac{1}{K_u}\sqrt{1 - K_u^4 \cos^2\alpha}$$

## APPENDIX B

## NON-UNIFORM INFLOW FACTOR IN HOVER AS A FUNCTION OF TRAPEZOIDAL INFLOW DISTRIBUTION




Inflow velocity at any radial station is

$$u_r = u_A + \frac{r}{R}(u_T - u_A)$$

Let: $u_A = \xi u_T$

Then

$$u_r = u_T\left[\xi + \frac{r}{R}(1-\xi)\right]$$

Now

$$dT = \rho(2\pi r\,dr)\,u_r(2u_r) = 4\pi\rho r u_r^2\,dr$$

$$\therefore\quad T = 4\pi\rho u_T^2\left\{\xi^2\int_0^R r\,dr + \frac{2\xi}{R}\int_0^R r^2 dr - \frac{2\xi^2}{R}\int_0^R r^2 dr + \frac{1}{R^2}\int_0^R r^3 dr - \frac{2\xi}{R^2}\int_0^R r^3 dr + \frac{\xi^2}{R^2}\int_0^R r^3 dr\right\}$$

$$\therefore\quad T = 4\pi R^2\rho u_T^2\left[\xi^2/12 + \xi/6 + 1/4\right]$$

or

$$u_T = \sqrt{\frac{T}{\pi R^2\rho}}\sqrt{\frac{3}{\xi^2+2\xi+3}}$$

Now

$$u_{H_\square} = \sqrt{\frac{T}{2\pi R^2\rho}}$$

$$\therefore\quad \frac{u_T}{u_{H_\square}} = \sqrt{2}\sqrt{\frac{3}{\xi^2+2\xi+3}}$$

Now

$$dP = dT(u_r) = 4\pi \rho\, r\, u_r^3\, dr$$

$$\therefore \quad P = 4\pi\rho\, u_T^3 \Big\{ \xi^3 \int_0^R r\,dr + \frac{3\xi^2}{R}\int_0^R r^2 dr - \frac{3\xi^3}{R}\int_0^R r^2 dr$$

$$+ \frac{3\xi}{R^2}\int_0^R r^3 dr - \frac{6\xi^2}{R^2}\int_0^R r^3 dr + \frac{3\xi^3}{R^2}\int_0^R r^3 dr$$

$$+ \frac{1}{R^3}\int_0^R r^4 dr - \frac{3\xi}{R^3}\int_0^R r^4 dr + \frac{3\xi^2}{R^3}\int_0^R r^4 dr - \frac{\xi^3}{R^3}\int_0^R r^4 dr \Big\}$$

$$\therefore \quad P = 4\pi R^2 \rho\, u_T^3 \left[ \frac{\xi^3}{20} + \frac{\xi^2}{10} + \frac{3\xi}{20} + \frac{1}{5} \right]$$

Now $\quad P_{\square} = 2\pi R^2 \rho\, u_{H_\square}^3$

$$\therefore \quad \frac{P}{P_{\square}} = 2\left[ \frac{\xi^3}{20} + \frac{\xi^2}{10} + \frac{3\xi}{20} + \frac{1}{5} \right] \left( \frac{u_T}{u_{H_\square}} \right)^3$$

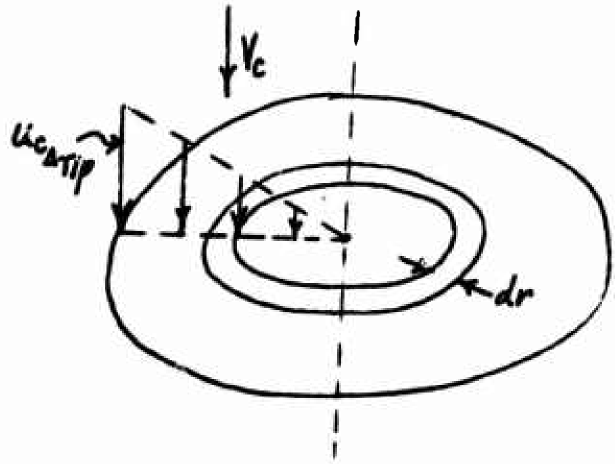
and, since $\quad \left( \dfrac{u_T}{u_{H_\square}} \right)^3 = \dfrac{6\sqrt{6}}{\left[\xi^2 + 2\xi + 3\right]^{3/2}}$

then

$$\frac{P}{P_{\square}} = \frac{3\sqrt{6}}{5}\left[ \frac{\xi}{(\xi^2 + 2\xi + 3)^{1/2}} + \frac{4}{(\xi^2 + 2\xi + 3)^{3/2}} \right]$$

## APPENDIX C

### NON-UNIFORM INFLOW FACTOR IN VERTICAL CLIMB, ASSUMING TRIANGULAR INFLOW DISTRIBUTION




$$dT = \rho(2\pi r\, dr)\left(V_c + \frac{r}{R} u_{c_{\Delta Tip}}\right)\left(2\frac{r}{R} u_{c_{\Delta Tip}}\right) \qquad \text{(III-1)}$$

$$\therefore\ dT = 4\pi\rho\, u_{c_{\Delta Tip}}\left[\frac{r^2}{R}V_c + \frac{r^3}{R^2}u_{c_{\Delta Tip}}\right]dr$$

$$\therefore\ T = 4\pi\rho\, u_{c_{\Delta Tip}}\left\{\frac{1}{R}\int_0^R r^2 V_c\, dr + \frac{1}{R^2}\int_0^R r^3 u_{c_{\Delta Tip}}\, dr\right\}$$

$$\therefore\ T = 4\pi R^2\rho\left\{\frac{V_c}{3} + \frac{u_{c_{\Delta Tip}}}{4}\right\}u_{c_{\Delta Tip}} \qquad \text{(III-2)}$$

Then $$u_{c_{\Delta Tip}}^2 + \frac{4}{3}V_c\, u_{c_{\Delta Tip}} - \frac{T}{\pi R^2\rho} = 0$$

$$\therefore\ u_{c_{\Delta Tip}} = -\frac{2}{3}V_c + \sqrt{\left(\frac{2}{3}V_c\right)^2 + \frac{T}{\pi R^2\rho}} \qquad \text{(III-3)}$$

Now $$dP = dT\left(V_c + \frac{r}{R} u_{c_{\Delta Tip}}\right)$$

$$\therefore\ dP = 4\pi\rho\, u_{c_{\Delta Tip}} \left[\frac{r^2}{R} V_c^2 + 2 V_c \frac{r^3}{R^2} u_{c_{\Delta Tip}} + \frac{r^4}{R^3} u_{c_{\Delta Tip}}^2\right] dr \qquad \text{(III-4)}$$

$$\therefore\ P = 4\pi\rho\, \frac{u_{c_{\Delta Tip}}}{R} \left\{ V_c^2 \int_0^R r^2 dr + \frac{2 V_c u_{c_{\Delta Tip}}}{R} \int_0^R r^3 dr + \frac{u_{c_{\Delta Tip}}^2}{R^2} \int_0^R r^4 dr \right\}$$

$$\therefore\ P = 4\pi R^2 \rho\, u_{c_{\Delta Tip}} \left( \frac{V_c^2}{3} + \frac{V_c u_{c_{\Delta Tip}}}{2} + \frac{u_{c_{\Delta Tip}}^2}{5} \right) \qquad \text{(III-5)}$$

$$\text{or}\quad P = 4\pi R^2 \rho\, u_{c_{\Delta Tip}} \left\{ V_c \left( \frac{V_c}{3} + \frac{u_{c_{\Delta Tip}}}{4} \right) \right\} + 4\pi R^2 \rho\, u_{c_{\Delta Tip}}^2 \left( \frac{V_c}{4} + \frac{u_{c_{\Delta Tip}}}{5} \right)$$

so that, from eq. (III-2),

$$P_\Delta = T V_c + 4\pi R^2 \rho\, u_{c_{\Delta Tip}}^2 \left( \frac{V_c}{4} + \frac{u_{c_{\Delta Tip}}}{5} \right)$$

Now, for uniform inflow at the rotor,

$$P_{\square} = T V_c + 2\pi R^2 \rho\, u_{c_{\square}}^2 \left( V_c + u_{c_{\square}} \right)$$

$$\therefore\ \left( \frac{P_\Delta}{P_{\square}} \right)_i = 2 \left( \frac{u_{c_{\Delta Tip}}}{u_{c_{\square}}} \right)^2 \left( \frac{\frac{V_c}{4} + \frac{u_{c_{\Delta Tip}}}{5}}{V_c + u_{c_{\square}}} \right)$$

From eqs. (III-3) and (10),

$$\frac{u_{c_{\Delta Tip}}}{u_H} = -\frac{2}{3}\frac{V_c}{u_H} + \sqrt{\left(\frac{2}{3}\frac{V_c}{u_H}\right)^2 + 2}$$

$$\frac{u_{c_{\square}}}{u_H} = -\frac{1}{2}\frac{V_c}{u_H} + \sqrt{\left(\frac{1}{2}\frac{V_c}{u_H}\right)^2 + 1}$$

so that

$$\eta_c = \left(\frac{P_\Delta}{P_\square}\right)_i = 2\left\{\frac{-\frac{2}{3}\frac{V_c}{u_H} + \sqrt{\left(\frac{2}{3}\frac{V_c}{u_H}\right)^2 + 2}}{-\frac{1}{2}\frac{V_c}{u_H} + \sqrt{\left(\frac{1}{2}\frac{V_c}{u_H}\right)^2 + 1}}\right\}^2$$

$$\times \left\{\frac{\frac{7}{60}\frac{V_c}{u_H} + \frac{1}{5}\sqrt{\left(\frac{2}{3}\frac{V_c}{u_H}\right)^2 + 2}}{\frac{1}{2}\frac{V_c}{u_H} + \sqrt{\left(\frac{1}{2}\frac{V_c}{u_H}\right)^2 + 1}}\right\}$$

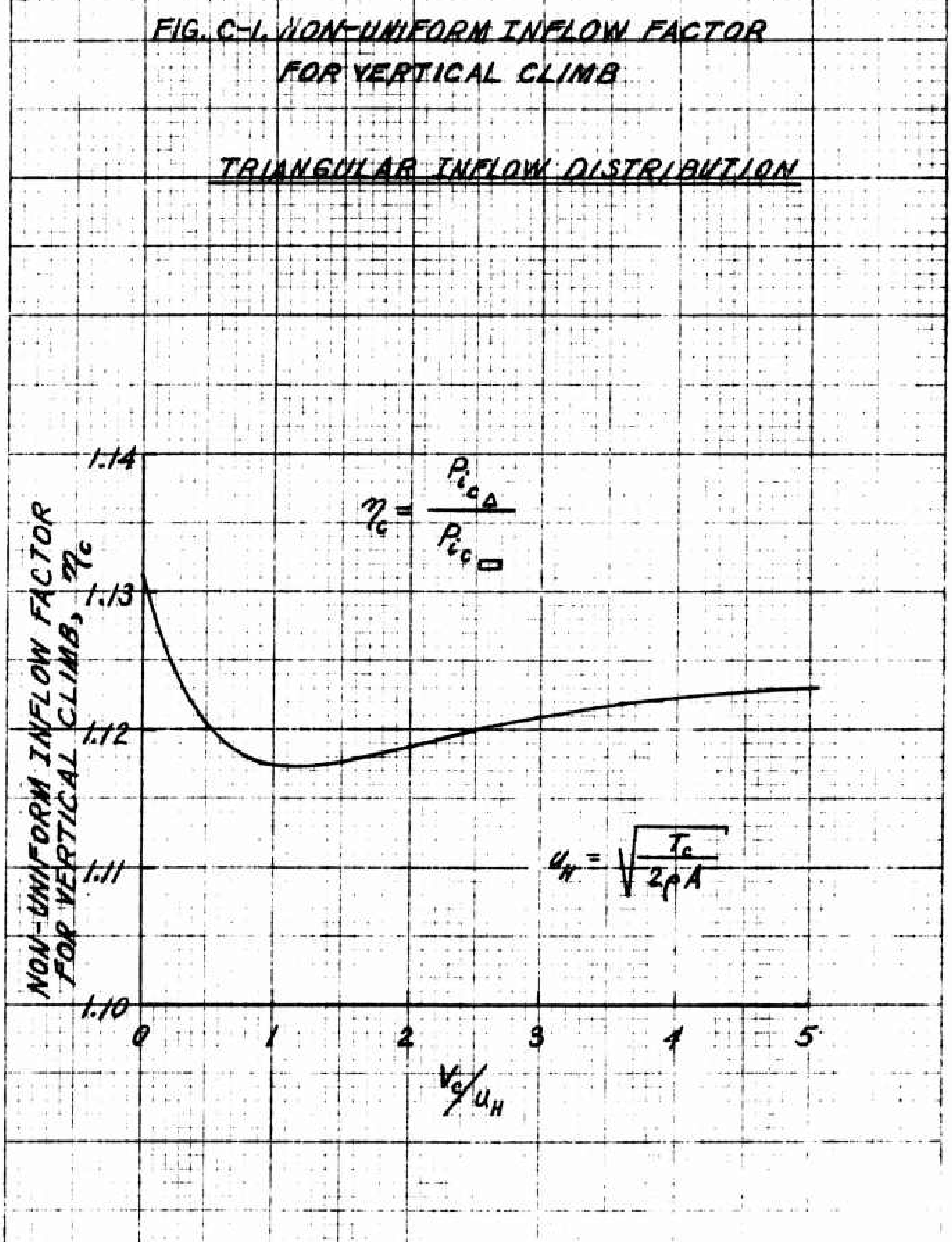


FIG. C-1. NON-UNIFORM INFLOW FACTOR
FOR VERTICAL CLIMB
TRIANGULAR INFLOW DISTRIBUTION
$\eta_c = \frac{P_{i_{c_\Delta}}}{P_{i_{c_\square}}}$
$u_H = \sqrt{\frac{T_c}{2\rho A}}$
NON-UNIFORM INFLOW FACTOR FOR VERTICAL CLIMB, $\eta_c$
1.14
1.13
1.12
1.11
1.10
0
1
2
3
4
5
$V_c/u_H$

## APPENDIX D

## INDUCED DRAG OF A CIRCULAR WING

The general expression for the induced drag coefficient of a wing, from Reference 5,

$$C_{D_i} = \frac{C_L^2}{\pi AR}(1+\delta)$$

or

$$\frac{D_i}{L} = \frac{C_{D_i}}{C_L} = \frac{C_L}{\pi AR}(1+\delta)$$

reduces to

$$\frac{D_i}{L} = \frac{C_L}{4}(1+\delta)$$

for a circular wing, where $AR = 4/\pi$.

In Ref. 5 a solution is developed in the form

$$1+\delta = \frac{\sum n A_n^2}{A_1^2}$$

where, in general,

$$\sum A_n \sin n\theta \,(n\mu + \sin\theta) = \mu\alpha \sin\theta \qquad \text{(IV-1)}$$

and, for a circular wing,

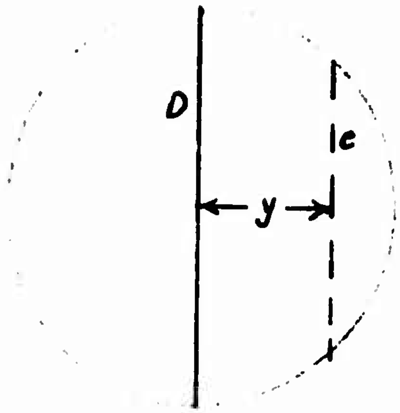
$$\mu = \frac{a_0}{4}\left(\frac{c}{D}\right)$$

If we express Eq. (IV-1) as,

$$\sum \frac{A_n}{\alpha} \sin n\theta \,(n\mu + \sin\theta) = \mu \sin\theta \qquad \text{(IV-2)}$$

and retain the first four coefficients (odd integral values of $n$ only) we can satisfy the equation at the four points,,

$\theta$ = 22 1/2°, 45°, 67 1/2°, 90°.




For a circular wing

$$\frac{c}{D} = \sqrt{1-\left(\frac{y}{R}\right)^2}$$

Then

| $\theta$ | $y/R$ | $c/D$ |
|---|---|---|
| 22 1/2° | .924 | .383 |
| 45° | .707 | .707 |
| 67 1/2° | .383 | .924 |
| 90° | 0 | 1.0 |

Solution of Eq. (IV-2) for the four selected points leads to

$$\frac{A_1}{\alpha} = \frac{a_0/4}{a_0/4 + 1} \qquad \frac{A_3}{\alpha} = \frac{1}{4}\,\frac{a_0/4}{3a_0/4 + 1}$$

$$\frac{A_5}{\alpha} = \frac{A_7}{\alpha} = 0$$

$$\therefore\ 1+\delta = \frac{\left(\frac{a_0}{a_0+4}\right)^2 + 3\left(\frac{a_0}{12a_0+16}\right)^2}{\left(\frac{a_0}{a_0+4}\right)^2} = 1 + \frac{3}{16}\left(\frac{a_0+4}{3a_0+4}\right)^2$$

For $a_0 = 2\pi$

$$1+\delta = 1.038$$

## SYMBOLS

| | |
|---|---|
| A | Rotor disk area, $\frac{\pi D^2}{4}$, square feet |
| B | Rotor tip-loss factor |
| b | Number of blades per rotor |
| $C_L$ | Lift coefficient, $L/\frac{1}{2}\rho V_o^2 A$ |
| $C_P$ | Power coefficient, $P/\rho A(\Omega R)^3$ |
| $C_T$ | Thrust coefficient, $T/\rho A(\Omega R)^2$ |
| c | Blade-section chord, feet |
| D | Rotor diameter, feet |
| $D_V$ | Vertical drag, pounds |
| $F_{vd}$ | Hovering vertical drag factor, $(W_G+D_V)/W_G$ |
| HP | Horsepower |
| K | Non-hovering inflow-velocity factor, $u/u_H$ |
| $K_u$ | Induced velocity factor for forward flight, $u_o/u_H$ |
| $K_c$ | Induced velocity factor for vertical climb, $u_c/u_H$ |
| L | Lift, pounds |
| P | Power, foot-pounds/second |
| R | Rotor radius, feet |
| T | Thrust, pounds |
| u | Induced rotor inflow velocity, feet/second |
| V | Velocity, feet/second |
| V' | Resultant velocity at rotor, feet/second |
| $W_G$ | Gross weight, pounds |

| | |
|---|---|
| Z | Rotor height above ground, feet |
| $\alpha$ | Rotor angle of attack, radians |
| $\eta$ | Induced power coefficient ratio, $\dfrac{(C_{P_i})_{Non\text{-}Uniform\ Inflow}}{(C_{P_i})_{\square}}$ |
| $\Lambda$ | Induced horsepower ratio, $(HP_i)_{IGE}/(HP_i)_{OGE}$ |
| $\lambda$ | Thrust coefficient ratio, $(C_T)_{IGE}/(C_T)_{OGE}$; |
| | also, Rotor inflow-ratio, $(V_o \sin\alpha - u)/\Omega R$ |
| $\mu$ | Rotor tip-speed ratio, $V_o \cos\alpha/\Omega R$ |
| $\xi$ | Trapezoidal inflow velocity distribution parameter, $u_A/u_T$ |
| $\rho$ | Mass density of air, slugs/cubic foot |
| $\sigma$ | Rotor solidity, $\dfrac{bc}{\pi R}$ |
| $\Omega$ | Rotor angular velocity, radians/second |

SUBSCRIPTS:

| | |
|---|---|
| A | At rotor hub |
| c | Vertical climb |
| H | Hover |
| i | Induced |
| IGE | In ground effect |
| o | Forward flight |
| OGE | Out of ground effect |
| T | At rotor tip |
| $\Delta$ | Triangular inflow distribution |
| $\square$ | Uniform inflow distribution |