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FLUERICS 37: A GENERAL PLANAR NOZZLE DISCHARGE COEFFICIENT REPRESENTATION

Tadeusz M. Drzewiecki

Harry Diamond Laboratories
Adelphi, Maryland

August 1974

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This report presents an improved theory for the calculation of incompressible, laminar-flow, nozzle discharge coefficients in planar nozzles. The theory includes the effects of: acceleration of the free stream in a constant area section due to boundary layer growth; three dimensionality, due to corner boundary layer interaction; and, fully developed flow at very low Reynolds numbers due to opposing boundary layer interaction. The discharge coefficient, data and theory, regardless of aspect ratio σ and

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effective nozzle length, ℓ, (e.g., nozzle shape) is a unique function of a modified Reynolds number N_R', defined in the following manner:

$$N_R' = \frac{N_R}{\ell/b\ (1 + 1/\sigma)^2}$$

where the usual definition of Reynolds number N_R is: $N_R = \frac{b}{\nu} \sqrt{2P_s/\rho}$, b = nozzle width, ν = density, P_s = plenum stagnation pressure. Discharge data are presented for ten nozzles of different planviews, with throat lengths varying from zero to four nozzle widths and with five aspect ratios varying from 0.5 to 3.0. It is found that the theory agrees to within 3 percent of the data, and that the unique representation of c_d is quite general, independent of most smooth nozzle shapes.

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1. INTRODUCTION

The determination of discharge coefficients in nozzles has been of interest to engineers for a considerable time. The emergence of the technology of fluidics and its attendant requirement of the *a priori* knowledge of nozzle power consumption created an additional interest in the solution of three-dimensional nozzle flows. Simmons,[1] Williams and Smetana,[2] McRee and Moses,[3] and Drzewiecki[4], provided solutions for discharge coefficients. The first two studies considered axisymmetric and two-dimensional nozzles respectively, while the last two considered planar three-dimensional nozzles. Drzewiecki's[4] discharge computation was a very simple but general treatment, using two-dimensional momentum integral equations. The boundary layer solutions for the mutually-perpendicular sides were used directly with no interactions and the acceleration of the free stream flow in constant area sections due to boundary layer growth was ignored.

Many characteristics of fluidic devices depend directly on the discharge coefficient of the supply nozzle. In beam deflection flueric devices, Manion and Mon[5] showed that gain and dynamic response are related to the discharge coefficient, c_d. In digital devices, Drzewiecki and Goto[6] showed a dependence of the switching time and the input characteristics on c_d. In both analog and digital devices, for example, the flow in a control nozzle may vary from zero or very low values to a final nonzero value. Since c_d varies with flow, the characteristics depending on c_d will also vary. For example the operation of fluidic devices depends on nozzle impedances. Since the discharge coefficient is a measure of the impedance, the operation is thus dependent on the discharge characteristics.

In analog devices, for example, pressure gain usually increases with Reynolds number for $N_R < 1500$ and has a slope that depends on c_d. In digital devices the switch time is dependent on the resistance of the control nozzle which modulates the control flow. The discharge coefficient varies from zero upward as a function of the Reynolds number of the control flow. It is therefore of considerable interest to determine discharge coefficient at low Reynolds numbers. Drzewiecki[4] pointed out that his discharge coefficient solution became less accurate as the Reynolds number decreased.

[1]Simmons, F., "Analytic Determination of the Discharge Coefficients of Flow Nozzles," NACA TN 3447, Lewis Flight Propulsion Labs, Cleveland OH, April 1955.

[2]Williams, J. and Smetana, F., "Theoretical Study of a Convergent Nozzle and Free Jet Flow," Proc. Fluid Amplification Symposium, Harry Diamond Laboratories, Washington DC, Vol. 1, October 1965.

[3]McRee, D., and Moses, H., "The Effect of Aspect Ratio and Offset on Nozzle Flow and Jet Reattachment," Advances in Fluidics, ASME Fluidics Symposium, Chicago, IL, May 1967.

[4]Drzewiecki, T. M., "Planar Nozzle Discharge Coefficients," Developments in Mechanics, Vol. 7, Proc. 13th Midwestern Mechanics Conference, August 1973.

[5]Manion, F. M. and Mon, G., "Fluerics 33: Design and Staging of Laminar Proportional Amplifiers," HDL-TR-1608, September 1972, AD-751182.

[6]Drzewiecki, T. M. and Goto, J. M., "Analytical Model for the Response of Flueric Wall Attachment Amplifiers," Fluidics Quarterly, Vol. 5, No. 1, January 1973.

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It is the purpose of this paper, then, to present suitable improvements to the simple two-dimensional theory to include corner boundary layer interaction, boundary layer displacement effects in constant area ducts resulting in axial flow acceleration, and effects of total boundary layer interaction resulting in a quasi-fully-developed flow. The effects of c_d on various flueric, jet-interaction devices will also be discussed.

2. ANALYSIS

The discharge coefficient of a planar nozzle may be thought of as the effective, reduced exit area, for the passage of ideal flow, due to exit displacement thickness. The discharge coefficient is defined as the ratio of actual flow to ideal flow passed through a nozzle. The actual flow is simply the ideal flow through the effective area A_{eff}, hence:

$$c_d = \frac{U_\infty A_{eff}}{U_\infty A_a} = \frac{A_{eff}}{A_a}$$

The effective area previously chosen by Drzewiecki[4] was determined by assuming that there was no interaction between the mutually perpendicular boundary layers. Hence, the height and width are decreased by twice the displacement thickness or:

$$A_{eff} = (b - 2\delta_1^*)\ (h - 2\delta_2^*)$$

$$A_a = bh$$

and then

$$c_d = (1 - 2\delta_1^*/b)\ (1 - 2\delta_2^*/h)$$

However, to fully describe the discharge coefficient, it is necessary to consider all the effects of the boundary layers, including interaction and displacement.

2.1 <u>The Displacement Effect in a Constant Area Section</u>

When flow enters a rectangular duct, the boundary layers get thicker in the axial direction until they merge; whereupon, the flow becomes fully developed. In the entrance region the free stream is accelerated axially since the effective area is decreasing. The same effect happens in the straight section of a nozzle. In order to determine the effect of this acceleration, the nonconstant axial velocity distribution must be known. Drzewiecki[4] assumed it constant.

Consider then the formulation for discharge coefficient, c_d, given by Drzewiecki[4] for the nozzle shape shown in figure 1. Equation (1) results from the application of the solution of the momentum integral equations to the definition of c_d above.

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[4] Drzewiecki, T. M., "Planar Nozzle Discharge Coefficients," Developments in Mechanics, Vol. 7, Proc. 13th Midwestern Mechanics Conference, August 1973.

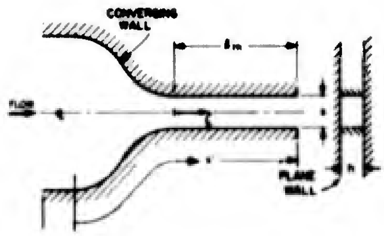

Figure 1. Schematic of a planar nozzle.

$$c_d = \left|1 - 2D_1/\sqrt{N_R}\right| \left|1 - 2D_2/(\sigma\sqrt{N_R})\right| \qquad (1)$$

where

$$D_1/\sqrt{N_R} = \delta_1^*/b,\ D_2/(\sigma\sqrt{N_R}) = \delta_2^*/h \qquad (1a)$$

and from reference 4

$$D_i = 2.554\sqrt{0.47 \int_0^{x'} (U_i(\chi)/U_\infty)^5 \, d\chi} \qquad (1b)$$

for

i = $_{1,2}$, where $_1$ refers to the converging walls and $_2$ refers to plane walls (fig. 1)

where

N_R = $\frac{U_\infty b}{\nu}$ - Reynolds number based on nozzle width, b

D_i = constant

$U_i(x)$ = free-stream velocity in the local direction of flow computed as a function of cross-section. For smooth nozzles in contraction section,[4] $U_1/U(\zeta=0)=1/[(Y(x)/b)\cdot\cos\gamma(x)]$, $U_2/U(\zeta=0)=1/(Y(x)/b)$

U_∞ = $\sqrt{2P_s/\rho}$ - nozzle-exit velocity

γ = angle converging wall makes with nozzle centerline

b = nozzle width

$Y(x)$ = local width of nozzle section

h = nozzle height

P_s = nozzle stagnation pressure

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[4]Drzewiecki, T. M., "Planar Nozzle Discharge Coefficients," Developments in Mechanics, Vol. 7, Proc. 13th Midwestern Mechanics Conference, August 1973.

ν = kinematic viscosity

ρ = fluid density

x' = total distance along a wall normalized by b

χ = distance variable along a wall normalized by b

σ = h/b - nozzle aspect ratio

This formulation as it stands would be adequate in describing the flow in a constant area section of a nozzle (neglecting corner effects) if the axial free-stream-velocity distribution were known and if the Reynolds number were very high. However, only the exit free-stream velocity is known provided the opposing boundary layers have not coalesced resulting in a quasi-fully-developed flow.

The calculation of the axial velocity distribution, and hence the discharge coefficient, lends itself quite readily to an iterative procedure. The final velocity distribution, and hence c_d, may be obtained by initially assuming an axial free-stream-velocity distribution and then solving for the resulting axial displacement-thickness distribution. By applying the continuity equation, one may, then calculate a new axial-velocity distribution, and hence a new displacement-thickness distribution, then another velocity distribution, and continue until the calculated velocity distribution does not differ from that of the previous interation. The easiest initial guess, of course, is constant axial velocity.[4] One can then make the usual assumption[7] that the displacement thickness, δ_i^*, grows as the square root of downstream distance, ζ, such that:

$$\delta_i^* (\zeta) = \delta_i^* (0) + \sqrt{\zeta/\ell_{th}}\, [\delta_i^* (\ell_{th}) - \delta_i^* (0)] \quad (2)$$

where

$\delta_i^*(\zeta)$ = displacement thickness on one of the walls at down-distance, ζ, in the throat and

ℓ_{th} = throat length

In the constant area section, the free-stream velocity is virtually the same for all the boundary layers; hence the subscript i is dropped from the velocity. The initial guess for (U/U_∞) = constant in eq (1) provides values for δ_i^* (0) and δ_i^* (ℓ_{th}) by solving eq (1a) and (1b) for values of x' at the start and end of the straight section. Applying the continuity equation, one determines a new value for the velocity, U(0), at the start of the straight section, so that

$$U_\infty [b - 2\delta_1^*(\ell_{th})]\ [h - 2\delta_2^*(\ell_{th})] = U(0) [b - 2\delta_1^*(0)]\ [h - 2\delta_2^*(0)]$$

or

$$U(0)/U_\infty = \frac{[1 - 2\delta_1^*(\ell_{th})/b]\ [1 - 2\delta_2^*(\ell_{th})/h]}{[1 - 2\delta_1^*(0)/b]\ [1 - 2\delta_2^*(0)/h]} \quad (3)$$

[4]Drzewiecki, T. M., "Planar Nozzle Discharge Coefficients," Developments in Mechanics, Vol. 7, Proc. 13th Midwestern Mechanics Conference, August 1973.
[7]Schlichting, H., Boundary Layer Theory, 6th ed., McGraw-Hill, New York, 1960, p. 123.

and similarly one notes that for any point ζ in the throat the free-stream velocity, $U(\zeta)$, is simply:

$$U(\zeta)/U_\infty = \frac{[1 - 2\delta_1^*(\ell_{th})/b]\ [1 - 2\delta_2^*(\ell_{th})/h]}{[1 - 2\delta_1^*(\zeta)/b]\ [1 - 2\delta_2^*(\zeta)/h]} \quad (4)$$

Combining eq (2) and (4) results in the next estimate for the velocity distribution to be used in eq (1).

The new value of free-stream velocity at the start of the straight sections, eq (3), is used as the terminal velocity for computing the velocity distribution in the converging section. When a new approximation for the overall free-stream velocity distribution has been deterined, a new discharge coefficient is calculated along with the new displacement thicknesses. The procedure is repeated until convergence is obtained when a further approximation for the velocity does not appreciably change the new value for c_d from the old value. In general, for low Reynolds numbers, where the displacement effect is large, as many as 20 iterations may be necessary to insure a convergence of c_d to within 0.05 percent. At high Reynolds numbers the iteration procedure converges rapidly and approaches the value obtained from the uncorrected solution, eq (1) with $U/U_\infty = 1$. This procedure is not difficult but it is extremely cumbersome if carried out by hand; hence, it lends itself readily to a computer or even a programmable calculator.

The net effect of including the accelerated flow is to decrease the exit displacement thickness, and to increase the discharge coefficient over what was previously predicted in reference 4. This is in substantial agreement with what would be expected since eq (1) underestimates c_d at low Reynolds number due to its mathematical form (both roots $c_d = 0$ occur for $N_R > 0$ but in reality $c_d > 0$ for all $N_R > 0$).

The relationships derived for the displacement thickness, δ^* are based on a solution for the momentum thickness, θ. Schlichting[7] shows a relation between these two dimensions that depends on the axial, static-pressure gradient in the flow. If the gradient is zero, then $\delta^*/\theta = 2.554$.

The static-pressure gradient generated by the accelerating flow is small enough so that one can assume that the relationships between the thickness, θ, and δ^* are unchanged from the zero gradient case.

2.2 The Corner Effect

In addition to the displacement effect, discussed above, which increased c_d, there is the effect of the coalescence of the mutually-perpendicular boundary layers (side wall and top or bottom wall) into what could be described as a fillet, i.e., a concave portion connecting the perpendicular boundary layers. Also, there are secondary flows such as described by Trask and Drzewiecki[8] and Owczarek and Rockwell[9] that tend to deform the boundary layers. Figure 2 depicts how the boundary layers might appear. There is a helical flow generated in the axial direction by the contraction (fig. 2) that produces

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[4]Drzewiecki, T. M., "Planar Nozzle Discharge Coefficients," Developments in Mechanics, Vol. 7, Proc. 13th Midwestern Mechanics Conference, August 1973.

[7]Schlichting, H., Boundary Layer Theory, 6th ed., McGraw-Hill, New York, NY, 1960, p. 247.

[8]Trask, R. P. and Drzewiecki, T. M., "Secondary Flows in Jets and Their Effect on Fluidic Components," HDL-TM-70-23, 1970, AD-719232.

[9]Owczarek, J. and Rockwell, D. O., "An Experimental Study of Flows in Planar Nozzles," ASME Publication No. 72-Flcs-2, San Francisco, CA, March 1972.

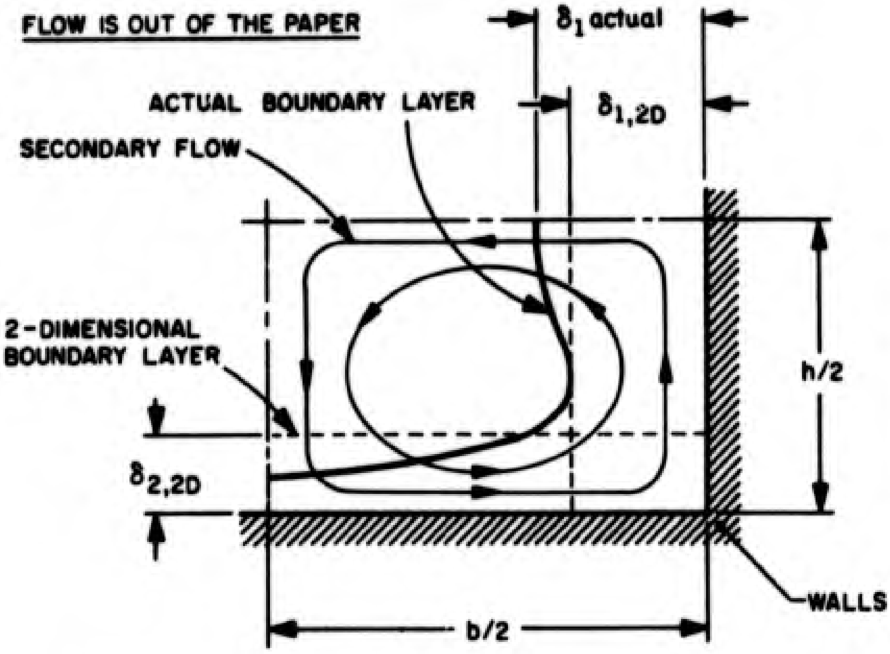

Figure 2. Actual and two-dimensional boundary layer configuration in nozzle throat.

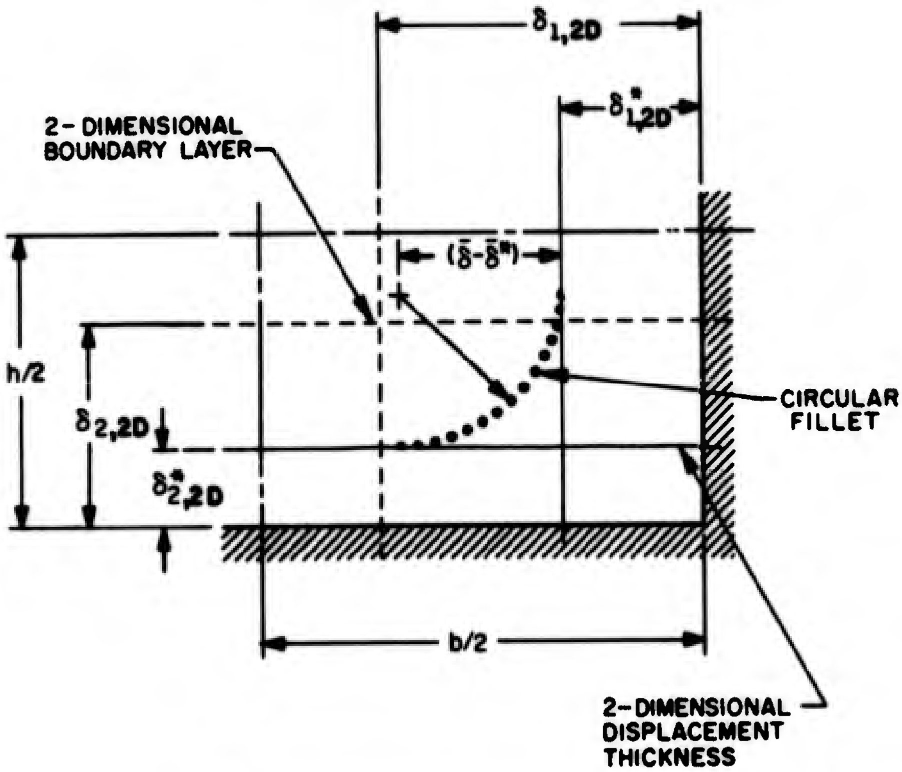

Figure 3. Assumed three-dimensionality for nozzle flow.

velocity components normal to the boundary layers. These velocity components deform the boundary layers somewhat. An impinging flow compresses a boundary layer, and an outward flow distends it. The heavy line in figure 2 demonstrates what the boundary layer shape might actually be, and the dashed lines show what it would be without secondary flow or coalescence. The engineering approximation of introducing a fillet between the ideal displacement boundary layers is shown in figure 3. Consider the ideal boundary layer and displacement thickness envelopes shown in this figure. Now, assume that the ideal boundary layer thickness $\delta_{1,2D}$ is a good representation at midplane. The bottom displacement thickness $\delta^*_{2,2D}$ will not be affected until the boundary layer $\delta_{1,2D}$ is above it. Similarly, the side displacement thickness $\delta^*_{1,2D}$ is not affected until boundary layer $\delta^*_{1,2D}$ is above it. Similarly, the side displacement thickness $\delta^*_{1,2D}$ is not affected until boundary layer $\delta^*_{1,2D}$ is above it. Using this reasoning, a circular fillet is assumed between the points where the displacement thickness is affected. The radius of the fillet on the displacement thickness is the distance between the average boundary layer thickness,

$$\bar{\delta} = (\delta_{1,2D} - \delta_{2,2D})/2,$$

and the average displacement thickness, $\delta^* = (\delta^*_{1,2D})/2$. Figure 3 shows this assumption where the boundary layer thicknesses, $\delta_{1,2D}$ and displacement thickness, $\delta^*_{1,2D}$, determine the fillet radius size. For incompressible flow, it has been shown in section 2 and in references 2 and 4 that the discharge coefficient is merely the ratio of the effective flow area (exit cross section reduced by the displacement thickness) to the nozzle exit area. The fillet area, A_f, further reduces the theoretically calculated effective area, A_{eff}, such that the discharge coefficient relation becomes:

$$c_d = \frac{A_{eff} - A_f}{bh} = c_d\big|_{2D} - \frac{A_f}{bh} \qquad (5)$$

where

$c_d\big|_{2D}$ = discharge coefficient calculated without fillets by the iterative procedure.

and the fillet area is

$$A_f = 4(\bar{\delta} - \bar{\delta}^*)^2 - \pi(\bar{\delta} - \bar{\delta}^*)^2$$

[2]Williams, J. and Smetana, F., "Theoretical Study of a Convergent Nozzle and Free Jet Flow," Proc. Fluid Amplification Symposium, Harry Diamond Laboratories, Washington DC, Vol. 1, October 1965.

[4]Drzewiecki, T. M., "Planar Nozzle Discharge Coefficients," Developments in Mechanics, Vol. 7, Proc. 13th Midwestern Mechanics Conference, August 1973.

and simplifying

$$A_f = (f - \pi)\ (\bar{\delta} - \bar{\delta}^*)^2 = (4 - \pi)\bar{\delta}^2\ (1 - \frac{\bar{\delta}}{\bar{\delta}}) \qquad (6)$$

The ratio of displacement thickness to boundary layer thickness for zero, or negligible pressure gradient, is given by Schlichting[7] to be $\delta^*/\delta = 0.3$ so that:

$$A_f = 0.42\bar{\delta}^2 = 4.67\bar{\delta}^{*2} \qquad (7)$$

including the corner effect, thus gives a final expression for the discharge coefficient

$$c_d = c_d\Big|_{2D} - \frac{4.67\bar{\delta}^*}{bh} \qquad (8)$$

2.3 Quasi-Fully-Developed Flow——Duct Solution

At very low Reynolds numbers, the nozzle exit velocity is no longer the Bernoulli velocity since the flow may be fully developed. In this case, the value of the nozzle exit velocity, U_∞, is no longer known. Fortunately, the nozzle flow can be considered as an entrance flow into a rectangular duct. The effective length of the duct, ℓ, is made up of two straight sections, the nozzle throat length, ℓ_{th}, and an effective length due to the contracting section, ℓ_{eff}.

$$\ell = \ell_{th} + \ell_{eff} \qquad (9)$$

To determine ℓ_{eff} consider the integral in eq (1b) over the entire length of a nozzle

$$\int_0^{x'} [U(\chi)/U_\infty]^5\ d\chi$$

where $x' = 0$ is defined as the start of the contraction.

Breaking the integral into two parts, one for the contraction and one for the throat, the integral yields

$$\int_0^{x'} = \int_0^{x'(\zeta=0)} [U(\chi)/U_\infty]^5\ d\chi + \int_{x(\zeta=0)}^{x'(\zeta=\ell_{th})} [U(\chi)/U_\infty]^5\ d\chi$$

[7]Schlichting, H., Boundary Layer Theory, 6th ed., McGraw-Hill, New York, NY, 1960.

In the simple formulation it was assumed that the velocity in the throat was constant, $U(\chi)/U_\infty = 1.0$. Therefore,

$$\int_0^{x'} = \int_0^{x'(\xi=0)} [U(\chi)/U_\infty]^5 \, d\chi + \int_0^{x'(\xi=\ell_{th})} d\chi = \int_0^{x'(\xi=0)} [U(\chi)/U_\infty]^5 \, d\chi + \ell_{th}$$

The integral over the contraction must be an effective length ℓ_{eff} added to the throat length ℓ_{th}

$$\ell_{eff} = \int_0^{x'(\zeta=0)} [U(\chi)U_\infty]^5 \, d\chi \tag{10}$$

The overall integral, therefore, must be an effective length of constant area duct in which a boundary layer grows.

It is now possible to compute the discharge coefficient. It can be shown from the data of Sparrow, et al.[10] that the incremental, additional pressure drop, ΔP_{add}, due to the entrance region of a rectangular duct of moderate aspect ratios, $0.2 \leq \sigma < 5.$ is 0.95 of the average dynamic head within 5 percent or

$$\Delta P_{add}/\tfrac{1}{2}\rho U^2 = 0.95 \pm .05 \tag{11}$$

Therefore, for a given length of duct the pressure drop is the drop if all the flow were fully developed, ΔP_{FD}, plus the additional drop, ΔP_{add}, or:

$$\Delta P = \Delta P_{FD} + \Delta P_{add} \tag{12}$$

Dividing by the volumetric flow through the duct, Q, yields the equation for the resistance of the rectangular duct including the losses in the developing entrance flow.

$$\frac{\Delta P}{Q} = R = \frac{\Delta P_{FD}}{Q} + \frac{\Delta P_{add}}{Q} \tag{13}$$

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[10] Sparrow, E. M., Hixon, C. W. and Shavitt, G., "Experiments on Laminar Flow Development in Rectangular Ducts," J. Basic Engineering, Trans. ASME, March 1967.

The first term, $\Delta P_{FD}/Q$ is simply the fully developed resistance, R_{FD}, which given by Drzewiecki[11] as:

$$R_{FD} = \frac{12\mu \ell}{(bh)^2} \left[\sigma\left(1 + \frac{1}{\sigma^2}\right) + C\right] \quad (14)$$

where μ is the dynamic viscosity and C is an empirical constant given as $0.35 \leq C \leq 0.5$ for $1 \leq \sigma \leq 2$, and $C = 0.5$ for $\sigma \geq 2$. Substituting eq (11) into eq (13) results in

$$R = R_{FD} + 0.475\ N_R \frac{\mu}{b^2 h} \quad (15)$$

To find the discharge coefficient we note:

$$Q = \frac{\Delta P}{R} = \frac{P_s}{R}$$

and

$$c_d = \frac{Q}{Q_{id}} = Q/(bh\sqrt{2P_s/\rho} \quad (16)$$

where

$Q_{id} = bh\sqrt{2P_s/\rho_b}$ - ideal, potential volumetric flow.

Substituting $N_R = \frac{\rho b\sqrt{\frac{2P_s}{\rho}}}{\mu}$, and $Q = \frac{P_s}{R}$ into eq (16) gives

$$c_d = \frac{\mu N_R}{2b^2 hR} \quad (17)$$

Substituting eq (14) and (15) into eq (17) results in an expression for the discharge coefficient for fully developed flow through a nozzle:

$$c_d = N_R/[24\ell/h)\ (\sigma(1+/\sigma^2) + C + 0.95\ N_R] \quad (18)$$

2.4 The Modified Reynolds Number

The value of discharge coefficient, as can be seen from the above discussions, depends on three parameters: the aspect ratio, $\sigma = b/h$, the effective nozzle length, ℓ, and the Reynolds number based on width, N_R. If there exists a unique combination of these three parameters on which c_d depends, then one general curve would describe all discharge coefficients.

[11] Drzewiecki, T. M., "The Fluidic Load Cell," HDL-TR-1622, Harry Diamond Laboratories, Washington D. C., 1973

F. M. Manion, of HDL, has suggested that the simple form of discharge coefficient from reference 4 be examined for a dominant group. Equation (1) is the expression for discharge coefficient[4]

$$c_d = [1 - 2D_1/\sqrt{N_R}]\ [1 - 2D_2/(\sigma\sqrt{N_R})]$$

It is noted from eq (1b) and (10) that the integral in the equation for D_i contributes an effective length for the nozzle so that:

$$D_i = K\sqrt{\frac{\ell_{eff} + \ell_{th}}{b}}$$

where

$K = 1.75$

Then the equation for discharge coefficient becomes

$$c_d = \left[1 - 2K\sqrt{\frac{(\ell_{eff} + \ell_{th})/b}{N_R}}\right]\left[1 - 2\frac{K}{\sigma}\sqrt{\frac{(\ell_{eff} + \ell_{th})/b}{N_R}}\right] \quad (19)$$

Multiplying yields the following equation:

$$c_d = 1 - 2K\sqrt{\frac{(\ell_{eff} + \ell_{th})/b}{N_R}}\ (1 + \frac{1}{\sigma}) + \frac{4K^2}{\sigma}\ \frac{(\ell_{eff} + \ell_{th})/b}{N_R} \quad (20)$$

For large Reynolds numbers

$$4K^2\ \frac{(\ell_{eff} + \ell_{th})/b}{N_R} << 1, \ell = \ell_{eff} + \ell_{th}$$

$$c_d \cong 1 - 2K\sqrt{\frac{\ell/b(1 + 1/\sigma)^2}{N_R}} \quad (21)$$

From eq (21) it can be seen that the discharge coefficient is a function of a single parameter, which is called here the modified Reynolds number

$$N_R' = \frac{N_R}{\ell/b(1+1/\sigma)^2} \quad (22)$$

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[4]Drzewiecki, T. M., "Planar Nozzle Discharge Coefficients," Developments in Mechanics, Vol. 7, Proc. 13th Midwestern Mechanics Conference, August 1973.

A similar analysis of eq (18) for the fully-developed flow case indicates that at very low Reynolds numbers the appropriate dimensionless group would be:

$$\frac{N_R}{(\ell_{th}/h + 1)}$$

Since the above fully developed dimensionless group is different than the modified Reynolds number N_R' it means that c_d is not a function of only N_R' at very low Reynolds numbers. One therefore would not expect a unique curve for c_d versus N_R' at low N_R'.

2.5 Analytical Results

Eq (8) and (18) form a description of the incompressible laminar flow discharge coefficient. The curves of c_d vs N_R' are shown in figure 4. Note here that there is indeed some spread at low N_R' as was indicated in section 2.3.

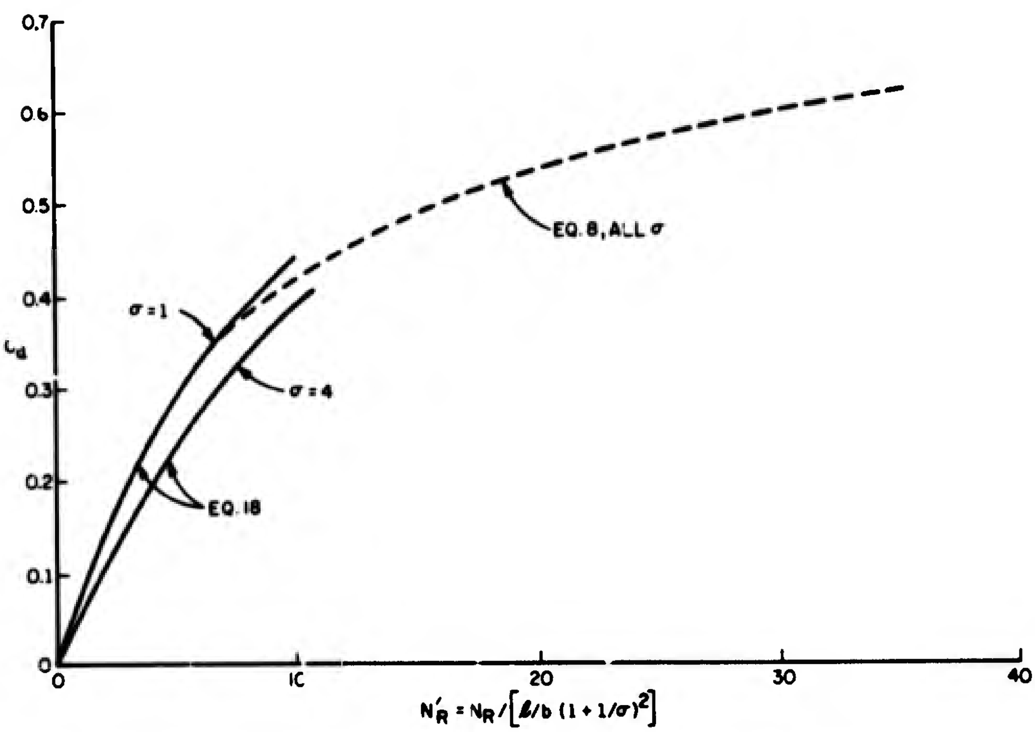

Figure 4. Graphic representation of discharge coefficient.

3. EXPERIMENTAL VERIFICATION

Four nozzles of the planview shown in figure 5, with $b \doteq 1.0$ mm and $h = 0.5$, 1.0, 1.5, 2.0 mm, and a moderately long throat, $\ell_{th} = 4.0$ mm were used in the experimental measurement of the discharge coefficient. The plenum chambers were made long in order to allow the flow, making a right-angled turn from a fitting, to become uniform. The static pressure in the plenum was measured with a static wall tap located at the start of the contraction. Electronic manometers were used to measure both static pressure, P_{st}, in the plenum and the pressure differential across a calibrated laminar flow meter that monitors the supply flow, Q_a. The data were recorded on an X-Y plotter as actual volumetric flow rate, Q_a, versus plenum static pressure, P_{st}. Figure 6 presents the data. The supply stagnation pressure P_s, is then calculated from the plenum dimensions and the flow, as:

$$P_s = P_{st} + \frac{\rho}{2}\left[\frac{Q_a}{Bh}\right]^2 \tag{23}$$

where

B = width of the plenum chamber.

The experimental value for discharge coefficient is the ratio of actual flow through the nozzle to the ideal flow as in eq (16).

$$c_d = \frac{Q_a}{Q_{id}} = Q_a/(bh\sqrt{2P_s/\rho})$$

The resultant experimentally determined values of discharge coefficients are compared with the analysis in figure 7. As can be seen, the agreement is good except in the case of the low-aspect ratio nozzle, $\sigma = 0.5$. Close examination of the actual nozzle used for $\sigma = 0.5$, showed that bonding glue had seeped into the nozzle channel. The reduced cross-section resulted in increased impedance to flow and hence yielded a low value of c_d, based on the ideal nozzle exit area.

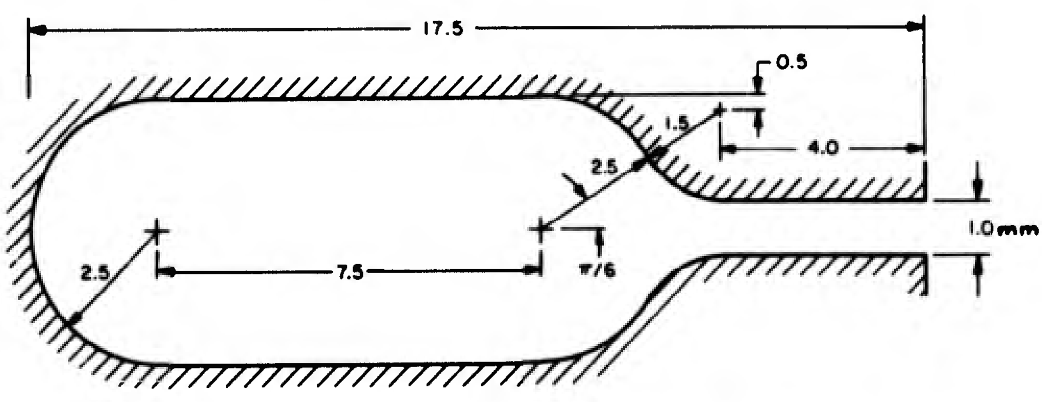

Figure 5. Planview of the planar nozzle used in the experiments.

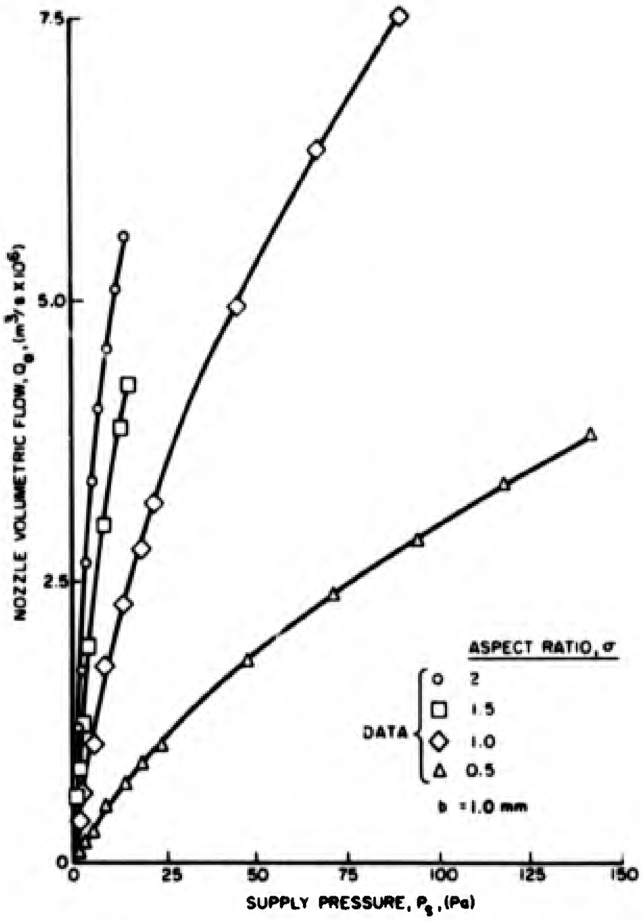

Figure 6. Raw flow-pressure data for the experimental nozzle.

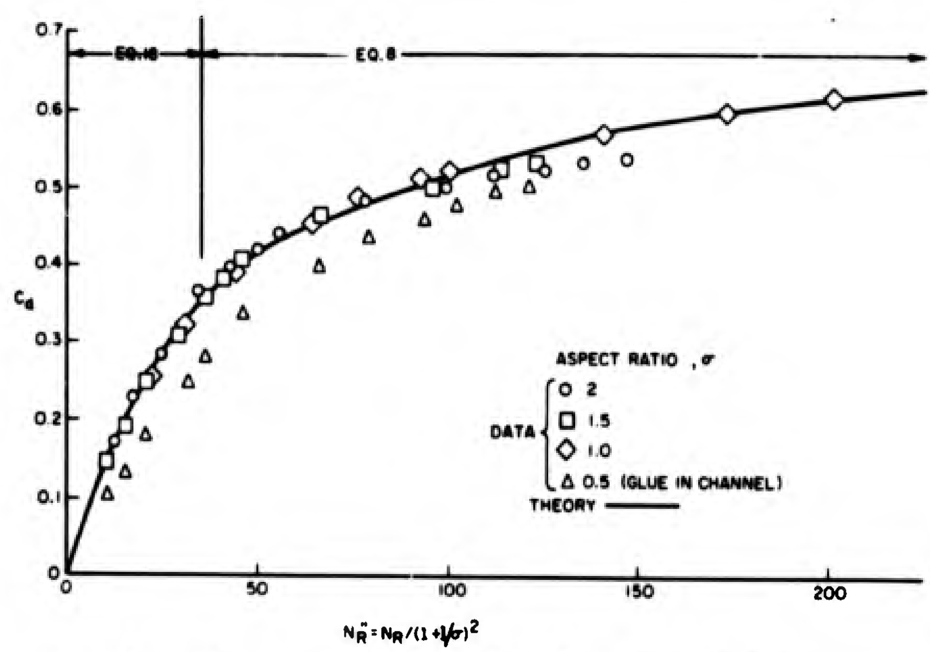

Figure 7. Comparison of theory and data for discharge coefficient.

In addition all the discharge coefficient data from reference 4 as well as the data presented above, have been replotted as a function of the modified Reynolds number, in figures 8 and 9. Figure 8 is plotted for low N_R' and figure 9 for high N_R'. Not only does all the data taken on differently shaped nozzles collapse to virtually one curve, as predicted by the theory, but also theory describes it remarkably well to within 3 percent. In all, the data for c_d in figures 8 and 9 are obtained from ten different nozzles, seven different planviews, with throat lengths varying from 0 to 4.0 nozzle widths and with five aspect ratios, $0.5 \leq \sigma \leq 3.0$. Hence, figures 8 and 9 can be considered as the general curves for all smooth planar nozzles.

4. THE EFFECTS OF c_d IN FLUID AMPLIFIERS

McRee and Moses[3] in their paper on the effects of aspect ratio on the attachment distance of a bounded jet to an adjacent wall, show that decreasing c_d, by lowering the aspect ratio, increases the distance to the attachment point. Discharge coefficient, however, is not the only important parameter. Viscous shear at the bounding plates probably has an effect of equal magnitude on the attachment point as c_d. Attachment distance, therefore, should correlate not only with c_d but also with the size of the bounding plates, or the distance to attachment. Since the effect of the bounding plates is to add boundary layer thickness, a square root dependence in the downstream direction would be expected. An empirical examination of the data[3] shows that the quantity $c_d \sqrt{x/b}$ is constant ($\cong 3.5$) where c_d is defined by eq (21), and x is the distance to the attachment point.

In another example where the downstream location is important, consider the location of the jet laminar-to-turbulent transition.[12] For a given bounding plate size with different nozzle shapes the dimensionless group

$$N_R \left(\frac{x_t}{b}\right)^{1/2} c_d^2 = \text{constant} \tag{24}$$

where x_t, is the centerline axial location of laminar-to-turbulent transition from the nozzle exit. Different dimensionless groups can probably be found for different bounding plates such that

$$N_R \left(\frac{x_t}{b}\right)^{1/2} c_d^2 \, f\left(\frac{L}{b}, \frac{W}{b}\right) = \text{constant}$$

where L is the length of a bounding plate and W is the half width of a bounding plate

The work on the modeling of bistable switching mechanisms[6] prompted this author to start investigating the discharge coefficient.

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[3]McRee, D. and Moses, H., "The Effect of Aspect Ratio and Offset on Nozzle Flow and Jet Reattachment," Advances in Fluidics, ASME Fluidics Symposium, Chicago, IL, May 1967.

[4]Drzewiecki, T. M., "Planar Nozzle Discharge Coefficients," Developments in Mechanics, Vol. 7, Proc. 13th Midwestern Mechanics Conference, August 1973.

[6]Drzewiecki, T. M. and Goto, J. M., "An Analytical Model for the Response of Flueric Wall Attachment Amplifiers," Fluidics Quarterly, Vo. 5, No. 1, January 1973.

[12]Drzewiecki, T. M., "The Interpretation of Surface Static Pressure Distributions in Fluid Amplifier Applications," HDL-TR-1627, Harry Diamond Laboratories, Washington D.C. 1973

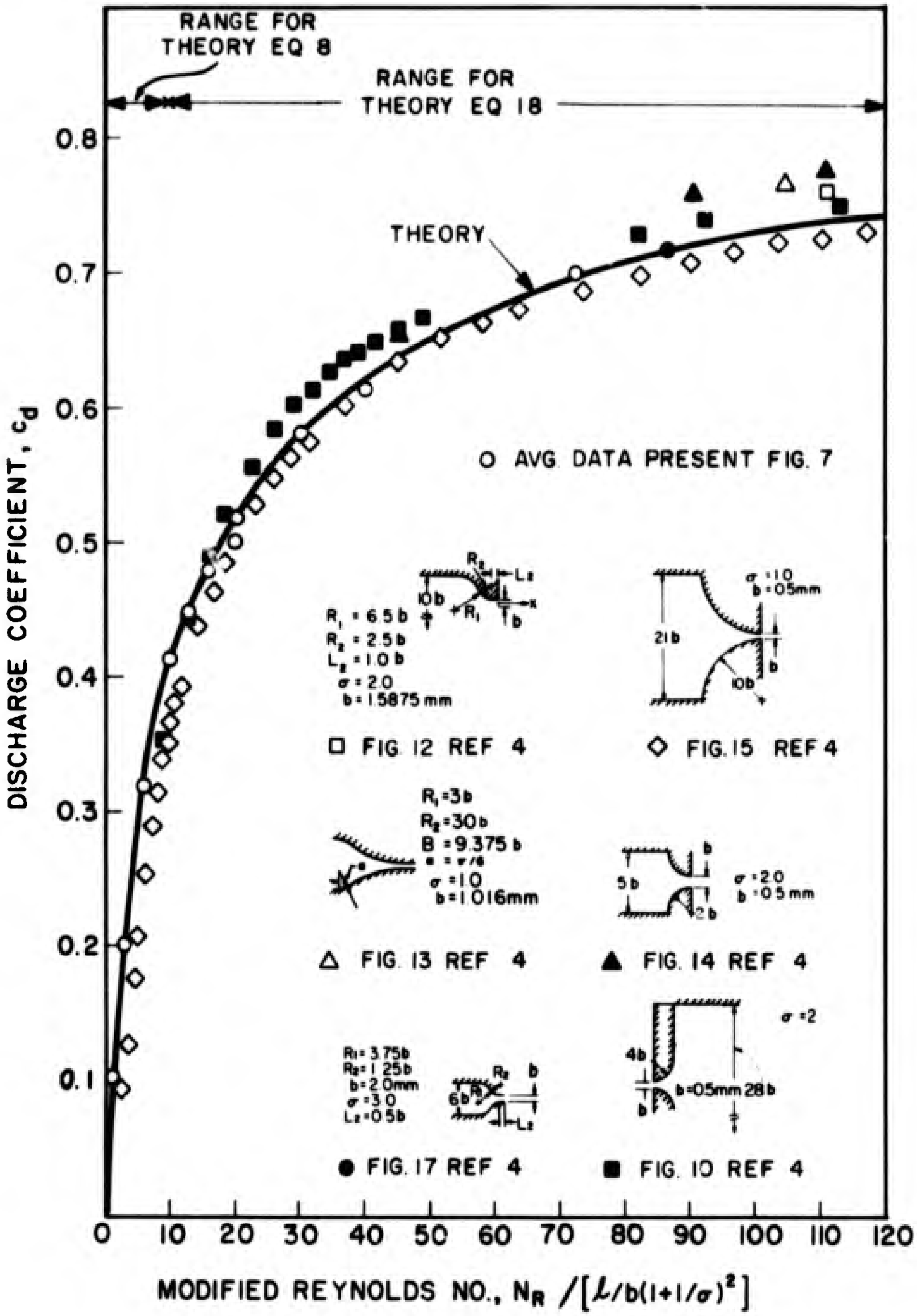

Figure 8. Comparison of theory and data for c_d for nozzles of widely varying planview - low N_R'.

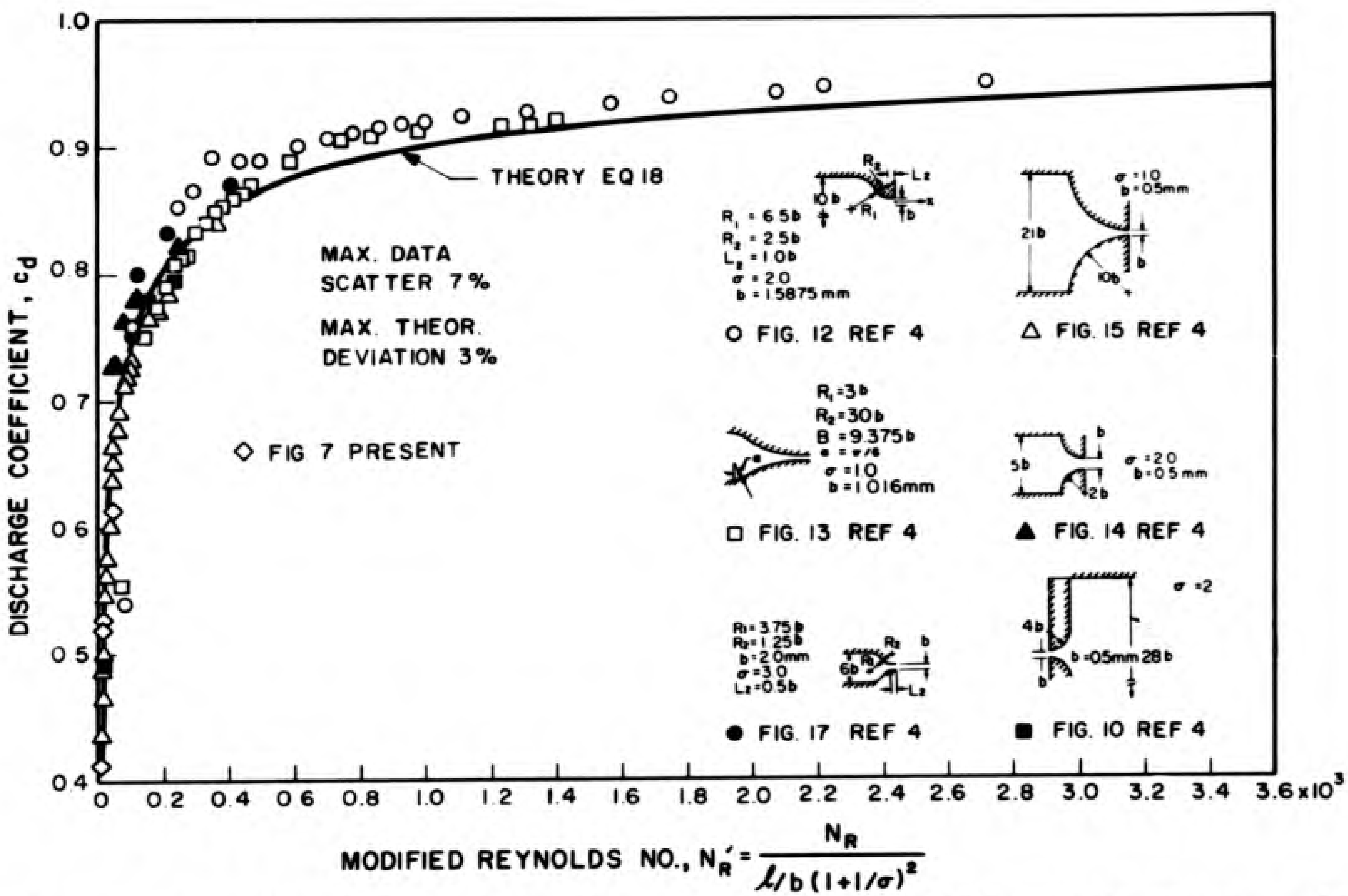

Figure 9. Comparison of theory and data for c_d for nozzles of widely varying planview - high N_R'.

The switch time of a flip-flop is directly related to the impedance of the control nozzle. Initially if an orifice impedance with c_d = constant was used the result was switch times that were too short. It was then realized that $c_d \neq$ constant because the control flow varied and consequently, the Reynolds number varied. Then the control impedance was modeled as a quasi-static function of time following the experimental static c_d versus N_R' curve, the switch time was brought into agreement with data. In order to eliminate empiricism from the switching analysis, a general discharge coefficient theory was necessary.

The work of Manion and Mon[5] on the design of laminar-proportional amplifiers has shown the effects of supply nozzle c_d on gain and frequency response. They converted the flow discharge coefficient to a momentum discharge coefficient and showed that pressure gain, G_p, changes with discharge coefficient and that the normalized corner frequency, Ω_c, is a constant for the given geometry:

$$\Omega_c = f_c \, b/(c_d \sqrt{2P_s/\rho}) = 0.01 \qquad (25)$$

where

f_c - actual corner frequency in Hz

It can thus be seen that the actual corner frequency response is low for a highly resistive (low c_d) supply nozzle.

The work of Drzewiecki and Goto[6] and Manion and Mon[5] considers lumped parameter, passive circuitry and parts of models for fluidic devices; nozzles are parts of such circuitry. If meaningful results are to be obtained for passive circuitry containing nozzles, the non-constant nature of discharge coefficient must be accounted for.

Supply and input characteristics (flows versus pressure), for fluid amplifiers are strongly dependent on the discharge coefficients. A knowledge of c_d versus N_R is necessary if an accurate analytical representation of any fluid amplifier is to be made.

5. SUMMARY AND CONCLUSIONS

The simple nozzle, discharge coefficient, analysis[4] has been extended to include: flow acceleration effects in constant area section; three-dimensional effects; and fully-developed flow effects. Very good correlation is obtained between theory and experiment, and a unique representation of c_d versus N_R' is established for most smooth nozzle designs.

The analysis is simple enough to be programmed on a desk-top calculator, without the necessity of sophisticated hardware.

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[4] Drzewiecki, T. M., "Planar Nozzle Discharge Coefficients," Developments in Mechanics, Vol. 7, Proc. 13th Midwestern Mechanics Conference, August 1973.

[5] Manion, F. M. and Mon, G., "Fluerics 33: Design and Staging of Laminar Proportional Amplifiers," HDL-TR-1608, September 1972, AD-751182.

[6] Drzewiecki, T. M. and Goto, J. M., "An Analytical Model for the Response of Flueric Wall Attachment Amplifiers," Fluidics Quarterly, Vol. 5, No. 1, January 1973.

The analysis indicates that the discharge coefficient is a function of only one parameter, a modified Reynolds number, N_R'. All the discharge coefficient data, regardless of nozzle shape or aspect ratio, collapse to a single curve when plotted against N_R'. Experimental results verify the analysis.

It has been pointed out that the operation of both digital and analog fluidic devices and other phenomena such as jet attachment, transition-to-turbulence, gain and frequency response all depend strongly on c_d.

In conclusion, then, a relatively simple, but accurate analysis for the determination of nozzle discharge coefficients, as found in fluid amplifiers, has been presented. This analysis can lead to the design of improved nozzles for use in fluid amplifiers and to the better understanding of fluid amplifier operation.

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ACKNOWLEDGEMENT

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NOMENCLATURE

A = cross-sectional area, (m^2)

b = nozzle width, (m)

B = plenum chamber width, (m)

c_d = nozzle discharge coefficient, (dimensionless)

C = empirical constant in resistance, (dimensionless)

D = constant, (dimensionless)

f_c = corner frequency, (Hz)

G_p = pressure gain, (dimensionless)

h = nozzle height, (m)

K = constant, (dimensionless)

ℓ = length of a duct, (m)

L = length of a bounding plate, (m)

N_R = Reynolds number based on width, $\frac{b}{\nu}\sqrt{\frac{P_s}{\rho}}$, (dimensionless)

N_R' = Modified Reynolds number, $N_R/[(\ell/b)\ (1 + 1/\sigma)^2]$, (dimensionless)

P = pressure, (Pa)

Q = volumetric flow, (m^3/s)

R = fluid resistance, $(kg/(m^4 \cdot s))$

U = free stream fluid velocity, (m/s)

u = fluid velocity in the boundary layer, (m/s)

W = half width of a bounding plate, (m)

x = coordinate in local direction of flow, (m)

x' = coordinate in local direction of flow, (dimensionless)

y = coordinate normal to x, (m)

Y = local width of nozzle section (m)

GREEK SYMBOLS

γ = angle converging wall makes with nozzle centerline, (rad)

δ = boundary layer thickness, (m)

δ^* = displacement thickness, (m) $\int_0^{Y/2} (1 - u/U_\infty)\, dy$

θ = momentum thickness, (m), $\int_0^{Y/2} \frac{u}{U_\infty}(- u/U_\infty)\, dy$

μ = dynamic viscosity, (kg/(m · s)

ν = kinematic viscosity, (m^2/s)

ζ = axial coordinate in nozzle throat, (m)

ρ = fluid density, (kg/m^3)

σ = aspect ratio, h/b, (dimensionless)

χ = dummy variable of integration, (dimensionless)

Ω_c = normalized corner frequency response of a proportional amplifier, (dimensionless)

SUBSCRIPTS

a = actual

add = additional

c = corner

eff = effective

f = fillet

FD = fully developed

i = integer count, 1 or 2

id = ideal

s = supply, reference or stagnation condition

st = static

t = transition

th = throat

2D = refers to a solution neglecting boundary layer coalescence

$_1$ = refers to condition on converging walls

$_2$ = refers to condition on plane walls

$_\infty$ = nozzle exit condition