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COMPARING INVENTORY DEMAND FORECASTS

Peter W. Zehna, et al

Naval Postgraduate School
Monterey, California

May 1975

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COMPARING INVENTORY DEMAND FORECASTS

by

Peter W. Zehna

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May 1975

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Continued efforts to compare exponential smoothing with other alternatives to demand forecasting are summarized. Using stock-out risk at one extreme and oversupply at the other, the effects of variability in forecasting, even when accurate with respect to the mean, are highlighted. Using a normal model, exponential smoothing is identified as a major source of

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variability. Various forecast methods are compared using simulation relative to mean squared error when mean demand is allowed to vary according to specified patterns. In almost all circumstances, exponential smoothing consistently emerges as a first choice. The same alternatives are compared using real demand data and the results show exponential smoothing and maximum likelihood to be essentially equivalent.

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1. Introduction

Whenever cost parameters in standard inventory models become difficult if not impossible to assess, one is often forced to examine criteria other than the classical minimum cost of the operation as a measure of effectiveness. This consideration is particularly pertinent to military supply systems wherein the traditional "out-of-stock" cost has to be measured in such intangible terms as loss of mission. On the other hand, current budget considerations being what they are, there is a renewed interest in controlling the "other" traditional cost factor of holding too much stock to overprotect the out-of-stock position. Faced with truly random demands on the supply system, however, it is practically certain that one or the other of these two undesirable states will have to be tolerated as time progresses. It naturally follows that the more accurately that random demand can be predicted, the better control one has over the system. Indeed, in a completely deterministic model, where demands are assumed known with certainty, it is possible to structure models (as in Ref [5]) so as to never be out of stock; the system can then be operated optimally with respect to other parameter choices in a simple, and fairly self-regulatory manner.

The problem of forecasting demand thus arises as a very important one in the context of several models presently employed by NAVSUP. Several previous reports ([1], [2] and [3]) have addressed different aspects of some of the difficulties involved in demand forecasting, with special emphasis on statistical considerations. In particular, the method of exponential smoothing, initiated by Brown [4] and presently used extensively by NAVSUP, has been the focus of special attention and analyzed in some depth in those studies. The present report continues that general study and provides some answers to issues that were raised and left unsettled in the previous reports.

One of the problems associated with forecasting has to do with the variability of the forecasts. So much attention is often paid to the accuracy (in the expected value sense) of the forecast that it is easy for variability to become slighted. But in fact, no matter how well the scheme used forecasts mean demand, a large associated variance leaves the decision maker with little in the way of control. This problem is especially highlighted in traditional periodic review models where large buys are made early in the period to protect against what was an accurate but extremely variable forecast that never materializes. The result and unnecessary drain on a constrained budget creates an obvious dilemma. So, while we would like accurate forecasts, we should equally well look for precise ones in the sense of having a small variance.

In previous reports, an issue was made over the fact that exponential smoothing, coupled with MAD as an estimate of variability, seems to produce excessive variability in the reorder levels even under stable assumptions. In the next section, this point is pursued further to determine the extent to which forecasting might be improved upon within the context of smoothing as a technique. Results are given only for the normal distribution with a constant mean and variance. While this is admittedly a special case, it is after all an important one which in fact applies as a model to a number of inventory items. But more importantly, the modifications suggested for comparison were best studied under such controlled assumptions to reveal the effects on variability.

In Section 3, the main purpose of the present study is explored. That purpose is to examine the effects on various forecasting techniques of a changing mean value function. The examination has in turn come about as a result of suggestions from NAVSUP that, while a constant mean model may be a valid assumption in a given period (quarter), that constant value changes from quarter to quarter for many different items in the inventory system. Thus, while exponential smoothing is not an optimal procedure to employ in a constant mean model (as pointed out in [3]) it may be more so in the case of a varying mean. Indeed, such a case would appear to lend itself quite well to some kind of adaptive scheme such as exponential smoothing. Several alternatives, along with smoothing at two levels, are tested against several different models of a changing mean demand. Again, only the normal case is examined in this study. For reasons documented earlier, comparisons are made via computer simulation and the associated program elements are summarized in appendices to this report.

In order to test some of these results with real data, the authors requested and received actual demand data on 10,000 different items for eight quarters. The analysis of these data is presented in Section 4 along with the usual precautions against overgeneralization with such limited information. Finally, some recommendations for further study are presented in a concluding section along with some remarks about processing times.

2. Variability of Smoothing

In previous reports already cited ([2] in particular) the problems associated with forecast variability have been discussed. Special attention was given to the variability associated with exponential smoothing and MAD (Mean Absolute Deviation). This section is a continuation of that study with special attention to the effect in particular on reorder levels.

The theoretical basis for the present discussion is the following: Suppose that random demand in a periodic review inventory model is normally distributed with constant mean μ and standard deviation σ. Since both of these parameters are typically unknown, they must be estimated from data. Such estimators are themselves random variables subject to fluctuation and must, perforce, be inexact. Any application of such estimates are then subject to the same inexactness and, in particular, reorder levels will be so affected.

It is standard in these circumstances to adopt a reorder level of the form $R = \mu + K\sigma$ where K is chosen to satisfy a given risk (stockout) requirement. More specifically, if $0 < \rho < 1$ is a specified stockout risk, then,

$$\rho = \Pr(X>R) = \Pr(X>\mu+K\sigma) \tag{1}$$

defines K, where X is the random demand. Obviously K is then the $100(1-\rho)^{\text{th}}$ percentile of the standard normal distribution and may be found from published tables. Some standard values of K are 1.23, 1.64, 1.96 and 2.57 associated, respectively, with risk values of .10, .05, .025 and .005.

The fact that X is random in the first place is what forces us to even consider a stockout risk. Thus, even if μ and σ were precisely known, there is no way to guarantee that random demand will never exceed a reorder level, wherever we set it. Thus, we adopt a risk value that we are willing to tolerate and choose K to set the reorder level accordingly. The thing that prevents us from making ρ too small, of course, is that the corresponding value of K increases as does R and, if the demand does not materialize, we are left with an oversupply of items for which a holding penalty of some sort must be assessed. There is thus a need for a trade-off between these two opposing penalties.

The situation becomes even more complex when we do not know μ and σ. Indeed, however they might be estimated by, say $\hat{\mu}$ and $\hat{\sigma}$, respectively, the corresponding estimated reorder level $\hat{R} = \hat{\mu} + K\hat{\sigma}$ is very likely different from R. Two awkward situations then arise. First, if $\hat{R} < R$, the theoretical value, the corresponding risk is inflated, which is to say

$$Pr(X>\hat{R}) > Pr(X>R) = \rho.$$

Thus, the probability of running out of stock is greater than ρ, the required stockout risk. On the other hand, if $\hat{R} > R$, there is, of course, a corresponding <u>reduction</u> in risk but only at the expense of overstocking items and paying the holding penalty.

The present study does not address the complex problem of giving a utility value to these two penalties. Thus, it may indeed be more desirable to overstock and pay a holding cost (achieving a stockout risk even smaller than required) than to hold less stock and allow the corresponding stockout risk to inflate. The point of view here is that both situations are to be avoided if possible and our main interest is in the statistical nature of $\hat{R}$ with special attention to its variability.

There are, of course, infinitely many ways to estimate R in the given circumstances. Unfortunately, not all of them produce random variables whose probability distributions are mathematically tractable. In particular, when exponential smoothing is adopted as an averaging technique, and when σ is estimated by means of MAD as well, such tractability is particularly elusive, as documented previously. For this reason, simulation, with its inherent inconclusiveness, was adopted in previous reports to evaluate statistical characteristics. Additional efforts notwithstanding, the intractability remains and simulation is again used in this report as a basic tool for analyzing variability.

When demand is assumed normal with the same parameters from period to period--certainly a very special case--a lot is known about the estimation problem. In particular, if maximum likelihood estimates of μ and σ respectively are substituted (using the invariance principle) into the formula for R, the corresponding estimate of R is itself maximum likelihood with its attendant optimum properties stemming from Gauss-Markov considerations. Consequently, we may use the maximum likelihood estimate as a norm of sorts against which to judge other estimators.

To gain better insight into the source of variability, we will here examine variations of exponential smoothing and MAD and compare the results accordingly.

In particular, we consider smoothed estimates of μ and σ as documented in previous reports. That is, if data $x_1, x_2, \ldots, x_t$ are given,

$$\tilde{\mu} = \alpha \sum_{k=0}^{t-1} \beta^k x_{t-k} \quad \text{and}$$

$$\tilde{\sigma} = \frac{\sqrt{\pi(2-\alpha)}}{2} \tilde{\Delta} \qquad \text{where}$$

$$\tilde{\Delta} = \alpha \sum_{k=0}^{t-1} \beta^k |e_{t-k}|,$$

$$e_{t-k} = x_{t-k} - \hat{x}_{t-k-1},$$

$$\hat{x}_{t-k-1} = \alpha \sum_{j=0}^{t-k-2} \beta^j x_{t-k-1-j}.$$

In words, $\hat{x}_{t-k-1}$ is the forecast of x_{t-k} at time $t-k-1$, based on exponential smoothing of data $x_1, \ldots, x_{t-k-1}$ using a smoothing constant of $\alpha (\beta = 1-\alpha)$; e_{t-k} is the forecast error, being the difference between the actual demand at time $t-k$ and what was forecast one period earlier; $\tilde{\Delta}$ is the exponentially smoothed estimate of error MAD (Δ_e). The formula for $\tilde{\sigma}$ stems from the fact that, under the normal distribution assumptions, the error standard deviation (σ_e) is related to the error MAD by means of $\sigma_e = \sqrt{\frac{\pi}{2}} \Delta_e$ and in turn, σ_e is related to σ by $\sigma = \sqrt{\frac{2-\alpha}{2}} \sigma_e$, at least asymptotically, i.e. for large values of t.

Basically, with these definitions in mind, NAVSUP presently estimates R with the formula

$$\hat{R} = \tilde{\mu} + K\tilde{\sigma}. \tag{3}$$

In the sequel, we will refer to this as Method A.

In our previous studies, we established that a great deal of the variability in $\hat{R}$ stemmed from the excessive variability in $\hat{\sigma}$ and confirmed this by examining the mean squared error of $\hat{\sigma}$ as an estimate of σ in many cases, that is, over a wide range of parameter choices. But, to what extent is that excess variability due to the smoothing operator itself versus the use of MAD to measure variability? To isolate these contributory factors, as well as to test the sensitivity of the results to parameter choices (α in particular), several variations of $\hat{\sigma}$ were chosen.

First, we agree to use a smoothed estimate of μ, that is $\tilde{\mu}$ as defined in (2) to keep that part of the estimate of R constant to each variation of the method. Then, Method B is defined by using a squared version of forecast error in place of absolute value based on smoothing. Thus, smoothed squared forecast error becomes

$$\alpha \sum_{k=0}^{t-1} \beta^k e_{t-k}^2$$

and, to convert back into the unit of scale, the square root is taken and the corresponding estimate of σ becomes,

$$\hat{\sigma} \equiv \hat{\sigma}_B = \sqrt{\frac{2-\alpha}{2}} \cdot \sqrt{\alpha \sum_{k=0}^{t-1} \beta^k e_{t-k}^2} \ .$$

For Method B then, the estimated reorder level is given by

(4) $$\hat{R} = \tilde{\mu} + K\hat{\sigma}_B.$$

When compared to Method A, we should see the effect of absolute value (MAD) on the variability.

For our third variation, we use ordinary averaging on squared forecast error in place of smoothing to test the contribution of the latter. Thus, for Method C we estimate σ by

$$\hat{\sigma} \equiv \hat{\sigma}_C = \sqrt{\frac{2-\alpha}{2}} \ \sqrt{\frac{1}{t} \sum_{k=1}^{t} e_k^2}$$

Accordingly, the estimated reorder level becomes

(5) $$\hat{R} = \tilde{\mu} + K\hat{\sigma}_C.$$

In our fourth method we test the contribution of both smoothing and MAD by dropping them altogether, estimating σ by maximum likelihood but continuing to smooth the data to estimate μ. In Method D then,

$$\hat{\sigma}_D = \sqrt{\frac{1}{t} \sum_{k=1}^{t} (x_k - \bar{x})^2} \quad \text{where} \quad \bar{x} = \frac{1}{t} \sum_{j=1}^{t} x_j$$

and the estimated reorder level becomes,

(6) $$\hat{R} = \tilde{\mu} + K\hat{\sigma}_D.$$

Finally, we label our norm, wherein both μ and σ are estimated by maximum likelihood, as Method E. In that case, the estimated reorder level is maximum likelihood as previously pointed out. That is,

$$(7) \qquad \hat{R} = \bar{X} + K\hat{\sigma}_D .$$

There are a variety of ways that these five methods might be compared and we examined several. For present reporting purposes, however, we were content to examine the effects on reorder levels. For simulation purposes, the following general considerations apply. (More particulars may be found in the program summary in Appendix A.) First, our main concern here is with statistical characteristics. In the absence of analytical results we must have a large sample size (t) and $t = 100$ seemed comfortable. Again, we emphasize that our present concern is not with the real application where a history of 100 periods might be totally unreasonable. Secondly, since we are simulating, we need to replicate the system a large number of times and we chose $NR = 100$ for the number of replications in each case. Finally, it is necessary to restrict ourselves to a selective choice of parameters, notably the smoothing constant α, the mean μ and the standard deviation σ of the underlying demand, the risk level ρ and, as a consequence, what we will refer to as the theoretical reorder level $R = \mu + K\sigma$. The basic parameter choices were made as a result of examining our previous reports for those choices where differences were particularly marked.

For each choice of μ, σ, ρ and R, the scenario was duplicated for $\alpha = .1$ and $\alpha = .2$ which are two choices presently in vogue in applications. The basic results are summarized in Tables 2.1, 2.2 and 2.3. In these tables, we have published, for each choice of parameters under each of methods A, B, C, D and E the following characteristics:

1. Average reorder level, being the average over 100 replications of the reorder level set at the end of the 100$^{\underline{th}}$ period of demand.

2. The standard deviation of the 100 reorder levels (replications) set at the end of period 100.

3. Using three sigma limits as a guide, Oversupply is defined as the number of units over the reorder level R that might be encountered.

4. Again, using three sigma as a range, Max Risk is recorded as the actual value of ρ that might be encountered.

Items 3 and 4, as recorded in the tables, need additional discussion. Imagine the 100 replications as 100 supply managers, each managing the same item for 100 periods. The first case listed in Table 2.1 will do for an illustration. The theoretical reorder level is 116.45 to satisfy a stockout risk of 5%. In Item 1 in the table is recorded, for each method being considered, the average of the 100 reorder levels that each of the supply managers would record after 100 periods of operation. Immediately under that in Item 2 is listed the corresponding standard deviation of those reorder levels. It is easily observed that on the

REORDER LEVEL

PARAMETERS	1. Average 2. Standard Deviation 3. Oversupply 4. Max Risk				
α μ σ ρ R	A	B	C	D	E
.1, 100, 10, .05, 116.45	116.68 3.51 10.76 .27	116.32 3.42 10.13 .27	116.55 2.37 7.21 .17	116.50 2.37 7.16 .17	116.55 1.47 4.51 .11
.2, 100, 10, .05, 116.45	116.78 5.40 16.53 .48	116.22 5.07 14.98 .46	116.57 3.48 10.56 .27	116.53 3.47 10.51 .27	116.55 1.47 4.51 .11
.1, 10, 10, .05, 26.45	26.68 3.49 10.70 .27	26.33 3.39 10.05 .27	26.55 2.34 7.12 .17	26.49 2.34 7.06 .17	26.53 1.41 4.31 .11
.2, 10, 10, .05, 26.45	26.78 5.39 16.50 .48	26.22 5.05 14.92 .46	26.57 3.46 10.50 .27	26.51 3.45 10.41 .27	26.53 1.41 4.31 .11
.1, 100, 25, .05, 141.125	141.69 8.73 26.76 .27	140.81 8.48 25.12 .27	141.38 5.87 17.87 .17	141.22 5.86 17.68 .17	141.34 3.54 10.84 .11
.2, 100, 25, .05, 141.125	141.96 13.47 41.24 .48	140.56 12.62 37.30 .46	141.43 8.65 26.26 .27	141.28 8.62 26.02 .27	141.34 3.54 10.84 .11

Table 2.1. Effects on Reorder Levels.

REORDER LEVEL

PARAMETERS					1. Average 2. Standard Deviation 3. Oversupply 4. Max Risk				
α	μ	σ	ρ	R	A	B	C	D	E
.1,	50,	10,	.05,	66.45	66.68 3.50 10.73 .27	66.32 3.40 10.07 .27	66.55 2.36 7.18 .17	66.49 2.35 7.09 .17	66.54 1.44 4.41 .11
.2,	50,	10,	.05,	66.45	66.78 5.40 16.53 .48	66.22 5.06 14.95 .46	66.57 3.47 10.53 .27	66.51 3.46 10.44 .27	66.54 1.44 4.41 .11
.1,	100,	10,	.10,	112.82	112.89 3.26 9.85 .38	112.75 3.13 9.32 .37	112.96 2.46 7.52 .29	112.94 2.48 7.56 .29	112.76 1.46 4.32 .20
.2,	100,	10,	.10,	112.82	112.71 4.85 14.44 .57	112.46 4.65 13.59 .56	112.92 3.45 10.45 .40	112.90 3.45 10.43 .40	112.76 1.46 4.32 .20
.1,	100,	10,	.20,	108.42	108.55 2.78 8.47 .49	108.45 2.69 8.10 .48	108.60 2.35 7.23 .44	108.58 2.38 7.30 .44	108.40 1.29 3.85 .33
.2,	100,	10,	.20,	108.42	108.42 4.09 12.27 .65	108.25 3.98 11.77 .64	108.56 3.37 10.25 .56	108.55 3.38 10.27 .56	108.40 1.29 3.85 .33

Table 2.2. Continuation of Table 2.1.

REORDER LEVEL

PARAMETERS					1. Average 2. Standard Deviation 3. Oversupply 4. Max Risk				
α	μ	σ	ρ	R	A	B	C	D	E
.1,	100,	10,	.25,	106.74	106.89 2.62 8.01 .54	106.82 2.57 7.79 .54	106.93 2.35 7.24 .50	106.92 2.34 7.20 .50	106.74 1.23 3.69 .38
.2,	100,	10,	.25,	106.74	106.78 3.85 11.59 .68	106.65 3.77 11.22 .68	106.89 3.35 10.20 .63	106.89 3.35 10.20 .63	106.74 1.23 3.69 .38
.1,	10,	1,	.05,	11.645	11.61 .44 1.28 .39	11.57 .41 1.16 .37	11.60 .37 1.06 .31	11.58 .37 1.04 .32	11.58 .35 .98 .30
.2,	10,	1,	.05,	11.645	11.59 .59 1.72 .57	11.54 .55 1.54 .54	11.60 .44 1.28 .39	11.57 .44 1.24 .40	11.58 .35 .98 .30

Table 2.3. Continuation of Table 2.2.

average all methods do well in yielding a reorder level near the theoretical one. But that is an average reorder level. For each method we might ask just how widespread were the actual reorder levels that went in to make up the average? Well, there are two ways here to measure "bad." At one extreme, using three sigma as a guide, it is plausible that some of the 100 supply managers experienced reorder levels as high as the average plus three sigma and others as low as the average minus three sigma. For example, under Method A at least one of the supply managers might have recorded a reorder level of 127.21, thus carrying 10.76 items more than he would have had he known the parameters μ and σ. By the same token, at least one other manager might have estimated R to be as low as 106.15 in which case the actual stockout risk he faces would be 27%, not 5%. This has been recorded, for lack of a better name, as Max Risk in Item 4 of each table.

Now looking at the three tables as a whole, some definite conclusions may be drawn. The actual numbers recorded as Oversupply and Max Risk may or may not be significant. These considerations would depend on the unit cost on the one hand, and on the penalty cost for being out of stock on the other hand. Naturally, these will vary from one item to another and no attempt will be made here to qualify those entries further. What is of concern here is the unmistakable trend toward improvement--either way--as we proceed from Method A to Method E, that is from smoothing with MAD to complete maximum likelihood. Except for Methods C and D, which are always quite close in value and occasionally reverse the order, the improvement trend is overwhelmingly in the stated order. While this does not constitute analytical proof by any means, it certainly lends a great deal of support to the conjecture that, under the assumptions stated, exponential smoothing produces greater variability than maximum likelihood.

But much more can be gleaned out of the results! We set out to test variations of smoothing and the results are ordered in a natural sequence. Starting with smoothed estimates of μ and σ via MAD, some improvement in both Max Risk and Oversupply (however slight) is achieved simply by smoothing squared forecast error rather than absolute forecast error. That is Method B. But the improvement is slight in most cases which supports the conjecture that MAD per se is not the largest contributor to variability.

Method C begins to test the contribution of smoothing since it differs from Method B only in averaging squared forecast error in the traditional sense rather than a smoothed average. In most cases, a reasonable amount of improvement results. We have already remarked that Method D produces results quite similar and occasionally just slightly better than Method C. Since the only difference is averaging squared deviations from the mean rather than squared forecast errors, one may guess that the introduction of the latter is not too significant in contributing to variability.

Finally, in Method E, μ itself is estimated by maximum likelihood along with σ and the results speak for themselves and confirm once again what has been repeatedly observed in previous reports. No need to belabor that point here but we may say in conclusion that it appears to be the method of exponential smoothing that is the real villain in inflating the variability of the reorder level--and that is true after 100 periods of operation it should be pointed out! Additionally, we can see from the tables that the choice of $\alpha = .2$ uniformly produces worse results than the choice $\alpha = .1$, and that improvement is directly proportional to σ. The unmistakable guideline resulting from all this seems to be: If there is any reason to believe that demand is normal with constant parameters over time, do not smooth, and if you do, choose a small value of α. In any case, be aware of the fact that the smoothing operator appears to produce more variability, hence more unnecessary cause for alarm (unreliable reorder levels), than more traditional methods of averaging.

3. Variable Mean Demand

It was, of course, predictable that smoothing would fare worse than maximum likelihood in a constant mean model. The only question is just how inferior in such a case. As suggested in earlier reports, [3] in particular, smoothing is essentially an adaptive scheme and would appear to be more suitable for a situation in which the mean value varies over time. One note of caution should be added, however. The fact that the actual demand itself varies from period to period does not itself indicate a changing mean value. Indeed, it is in the random nature of affairs that, particularly with large variances, an actual demand record may appear to vary a good deal from period to period when in fact the mean is constant. Whether or not the mean is constant, then, is a question of the model.

But it is easy to imagine conditions for which a constant mean model is inappropriate. For example in a military supply system it is clear that the mean demand for certain kinds of parts should shift to a higher, even if constant, level during a sudden global crisis. Or, it might be that even though mean demand is constant, record keeping and reporting is of such a nature that demand records do not reflect such an assumption. The fact that a constant mean is <u>inherent</u> in the process is really of minor concern then to the decision maker who must base his actions on what he actually observes. More about this point in the next section where real demand data have been examined.

For these reasons we decided to compare exponential smoothing to other alternate forecasting schemes including maximum likelihood under various mean value functions, a program suggested in [3] as a continuation of that study. But, how shall the mean value be allowed to vary? Without some regularity, it is almost certain that no general statements would be forthcoming. Moreover, recognizing the lack of analytical results for smoothing even in the constant mean model, simulation would almost surely have to be used as a method of generating statistical properties.

With regard to the mean value function, the authors are aware of no study within NAVSUP to indicate just what assumptions would be realistic. Lacking that, five different patterns were selected as plausible assumptions for various situations reasonably faced by military supply systems and, for purposes of generating data and controlling the parameters, normal demand is universally assumed. Throughout this section, the five demand patterns are identified as follows:

Pattern 1: Mean demand is allowed to increase by 50% in Period 3 and remain at that level thereafter.

Pattern 2: Same as Pattern 1 except the increase in Period 3 is 100%.

Pattern 3: An impulse pattern wherein mean demand is allowed to increase in Period 3 by 100% and then immediately decreases to its previous level where it remains.

Pattern 4: A ramp in which the mean value increases by 10% in each period starting with Period 3.

Pattern 5: Same as Pattern 4 except that the mean value becomes and remains constant at its value in Period 4.

As to the various forecasting schemes to be compared, originally the Bayesian method recommended in [3] was considered along with exponential smoothing, maximum likelihood and a moving average. Specifically, when demand is normal, $N(\mu,\sigma^2)$, <u>with σ^2 known</u>,[†] and when the prior on μ is taken to be also normal, $N(\mu_0,\sigma_0^2)$, with μ_0 and σ_0^2 specified, there is a natural way to generate a posterior normal distribution at the end of each period using the posterior of the last period as the prior for the current one. In this way, one obtains a Bayes estimate, μ_n, of μ (the mean of the posterior in each period) as a forecast of demand for the next period. Reorder levels may then be set at $\mu_n + K\sigma$.[††] As previously reported then, the ratio σ_0^2/σ^2 plays roughly the same role as a smoothing constant and in any case must be specified. But in simulation, σ^2 is deliberately selected and known, so that σ_0^2 can then be selected to reflect the relative degree of imperfect information.

Four different Bayes cases were tested and were found to either be very inferior to the other schemes or at best equivalent to maximum likelihood (at least asymptotically). Consequently, the Bayes procedures were abandoned in the early stages of this investigation and those results are not reported here. This is not to say, however, that Bayesian methods should be ignored in studying a variable mean. They are, after all, adaptive schemes

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† When both μ and σ^2 are assumed unknown, not a lot is available in the way of prior assumptions that lead to tractable results.

††A modification over what appears in [3].

and, broadly speaking are quite amenable to precisely those situations. The particular scheme examined here is a very special one and may not be indicative of the potential of the class as a whole.

Very much related to the abandonment of the Bayes schemes is the question of criteria for comparison. Naturally there are many ways in which forecasting methods might be judged. Clearly the ranking by one method might very well change if a different criterion were to be adopted. With no specific guidelines to dictate a choice, mean squared forecast error was finally selected as the criterion for comparison for several reasons. First, it has a certain amount of universal appeal as a measure of "closeness." And that after all, is our main concern in examining the inherent characteristics of each scheme as regards its forecasting ability. Secondly, mean squared error is functionally related to variance and, with all schemes judged (perhaps prematurely) to be accurate with respect to average, the results would also be a rough indicator of variability, which has been one of our main concerns in this and related studies.

Finally, with regard to the total number of demand periods, we first felt constrained by the fact that only eight quarters of real data are presently maintained in the files by NAVSUP. On the other hand, this places severe limitations on maximum likelihood and moving averages especially. Hence, we decided to examine the status of forecast errors at the end of eight quarters and again at the end of twenty quarters. This allowed us to examine, to some extent short of asymptotic conditions, the effect time might have on the results with obvious implications then for more data storage. Other time periods were examined but are not reported here.

Initially, the number of replications in the simulation was allowed to vary over several values but, with an exception or two, the results reported here are based on 100 replications. We saw no significant changes when that figure was allowed to increase. In any case, an outline of the program used is included in Appendix B so that the interested reader may generate his own data to validate these, as well as any other, observations recorded here.

One disadvantage in adopting mean squared error as a criterion is the difficulty of interpreting the results as meaningful units of measurement error. This is especially true when it comes to comparing two or more methods. What does it mean to say, for example, that the mean squared forecast error of Method A after eight periods was 274.98 while that for Method B was 224.47? We can certainly say that Method B was better than Method A, but it would be very difficult to say how much better. And yet, for reasons already mentioned, we prefer to use this abstract criterion. Consequently, we have summarized our results in terms of relative ranking, ever on the lookout for emerging rank patterns. These results are summarized in Tables 3.1 and 3.2. Some explanatory remarks and summary highlights are in order.

Each set of rankings accompanies a choice of parameter quadruples (μ,σ,N,NF) in which μ and σ are the mean and

PARAMETERS (μ σ N NR)	σ^2/μ	DEMAND PATTERNS #1	#2	#3	#4	#5
(100, .1, 8, 100)	.0001	S_2, MA, S_1, ML	S_2, MA, S_1, ML	ML or S_1, MA, S_2	S_2, MA, S_1, ML	S_2, MA, S_1, ML
(100, .1, 20, 100)		S_2 or MA, S_1, ML	S_2 or MA, S_1, ML	ML, S_1, S_2, MA	S_2 or MA, S_1, ML	S_2 or MA, S_1, ML
(200, 1, 8, 100)	.005	S_2 or MA, S_1, ML	S_2 or MA, S_1, ML	ML, S_1, MA, S_2	S_2, MA, S_1, ML	S_2, MA, S_1, ML
(200, 1, 20, 100)		S_2 or MA, S_1, ML	S_2 or MA, S_1, ML	ML or S_1, S_2 or MA	MA or S_2, S_1, ML	S_2, MA, S_1, ML
(100, 1, 8, 100)	.01	S_2 or MA, S_1, ML	S_2 or MA, S_1, ML	ML or S_1, MA or S_2	S_2, MA, S_1, ML	S_2, MA, S_1, ML
(100, 1, 20, 100)		S_2 or MA, S_1, ML	S_2 or MA, S_1, ML	ML or S_1, S_2 or MA	MA or S_2, S_1, ML	S_2 or MA, S_1, ML
(100, 1, 8, 10)	.1	S_2, MA, S_1, ML	S_2, MA, S_1, ML	ML or S_1, S_2 or MA	S_2 or MA, S_1, ML	S_2, MA or S_1, ML
(100, 1, 8, 100)		S_2, MA, S_1, ML	S_2, MA, S_1, ML	ML or S_1, S_2, MA	S_2, MA, S_1, ML	S_2, MA, S_1, ML
(10, 1, 8, 1000)		S_2 or MA, S_1, ML	S_2 or MA, S_1, ML	ML or S_1, MA or S_2	S_2, MA, S_1, ML	S_2, MA, S_1, ML
(10, 1, 20, 100)		S_2, MA, S_1, ML	S_2, MA, S_1, ML	ML, S_1, S_2, MA	MA or S_2, S_1, ML	S_2, MA, S_1, ML
(10, 1, 100, 100)		S_2 or MA, S_1, ML	S_2, MA, S_1, ML	ML or S_1, S_2 or MA	MA or S_2, S_1, ML	S_2 or MA, S_1, ML
(100, 10, 8, 1000)	1	S_2, MA, S_1, ML	S_2, MA, S_1, ML	ML or S_1, MA or S_2	S_2, MA, S_1, ML	S_2, MA, S_1, ML
(100, 10, 20, 100)		S_2, MA, S_1, ML	S_2, MA, S_1, ML	ML or S_1, S_2 or MA	MA or S_2, S_1, ML	S_2, MA, S_1, ML

Table 3.1. Ranking when $\sigma^2/\mu \geq 1$.

PARAMETERS (μ σ N NR)	σ^2/μ	DEMAND PATTERNS #1	#2	#3	#4	#5
(50, 10, 8, 100)	2	S_2, MA, S_1, ML	S_2, MA, S_1, ML	ML, S_1, S_2, MA	S_2, MA, S_1, ML	S_2, S_1, MA, ML
(50, 10, 20, 100)		S_2, MA, S_1, ML	S_2, MA, S_1, ML	ML, S_1, S_2, MA	MA, S_2, S_1, ML	S_2, MA, S_1, ML
(10, 10, 8, 100)	10	S_1 or ML, S_2, MA	S_2, S_1, MA, ML	ML, S_1, S_2, MA	ML, S_1, S_2, MA	ML, S_1, S_2, MA
(10, 10, 8, 1000)		S_1 or S_2, ML, MA	S_2, S_1 or MA, ML	ML, S_1, S_2, MA	S_1 or ML, S_2, MA	ML, S_1, S_2, MA
(10, 10, 20, 100)		S_1, S_2, ML, MA	S_2, MA, S_1, ML	ML, S_1, S_2, MA	S_2, MA, S_1, ML	ML or S_1, S_2, MA
(10, 10, 100, 100)		ML, S_1, S_2, MA	S_1, S_2, MA, ML	ML, S_1, S_2, MA	MA, S_2, S_1, ML	ML, S_1, S_2, MA
(50, 22, 8, 100)		S_2 or MA, S_1, ML	S_2 or MA, S_1, ML	ML, S_1, S_2, MA	S_2, S_1, MA, ML	ML, S_1, S_2, MA
(50, 22, 20, 100)	—	S_2 or MA, S_1, ML	S_2 or MA, S_1, ML	ML, S_1, S_2, MA	S_2, MA, S_1, ML	S_1, ML or S_2, MA
(100, 70, 8, 100)	50	S_2, S_1, MA or ML	S_2, MA, S_1, ML	ML or S_1, S_2, MA	S_2, S_1, ML, MA	ML or S_1, S_2, MA
(100, 70, 20, 100)	—	S_2, S_1, MA, ML	S_2, MA, S_1, ML	ML, S_1, S_2, MA	S_2, MA, S_1, ML	ML or S_1, S_2, MA
(200, 150, 8, 100)	100	S_2, S_1, ML, MA	S_2, MA, S_1, ML	ML or S_1, S_2, MA	S_2, S_1, ML, MA	ML, S_1, S_2, MA
(200, 150, 20, 100)	—	S_2, S_1, MA, ML	S_2, MA, S_1, ML	ML, S_1, S_2, MA	S_2 or MA, S_1, ML	ML or S_1, S_2, MA
(50, 100, 8, 100)	200	ML, S_1, S_2, MA	S_1 or ML, S_2, MA	ML or S_1, S_2, MA	ML, S_1, S_2, MA	ML, S_1, S_2, MA
(50, 100, 20, 100)	—	ML, S_1, S_2, MA	S_1, S_2, ML, MA	ML or S_1, S_2 or MA	S_1, S_2, MA, ML	ML or S_1, S_2 or MA

Table 3.2. Ranking when $\sigma^2/\mu > 1$.

standard deviation of normal demand, N is the number of periods for which demand is generated and forecast, and NR is the number of replications for simulation purposes. As demand is then generated under each of the demand patterns previously discussed, four different methods of forecasting are used to forecast demand. These are designated S_1 for exponential smoothing with smoothing constant $\alpha = 0.1$ and S_2 for exponential smoothing with $\alpha = 0.2$; ML stands for maximum likelihood and MA for a moving average based on the most recent eight periods of demand. The exact formulas used, along with the initial conditions assumed for each method, are listed in Appendix B with the program summaries.

Under the foregoing conditions then, demand is generated and forecast with the resultant cumulative forecast error recorded after N periods and averaged by dividing by N. The experiment is then replicated NR times and the resulting mean (NR) squared forecast error (MSFE) computed for each method under each demand pattern. For each demand pattern, the forecast schemes are ranked and recorded in the tables with the first one listed corresponding to the <u>smallest</u> mean squared forecast error. Occasionally, two such values are so close as to be (subjectively) considered as ties. In that case, no choice is made in the rankings. For example, for the parameter choice (10,1,20,100) under Demand Pattern 4, the respective mean squared forecast errors for S_1, S_2, ML and MA are 35.38, 15.99, 71.95 and 15.84. It is thus relatively impossible to distinguish between S_2 and MA so that the ranking is listed as MA or S_2, S_1, ML. Finally, a method has been underlined now and then in the ranking list to indicate that its MSFE is of an order of magnitude greater than the rest, thereby singling it out as a particularly poor choice.

As it turned out, the ranking patterns began to emerge according to variance-to-mean ratios and, accordingly, the summary tables are arranged so as to identify groups by values of σ^2/μ. Predictably, results are much more consistent and stable for small values of σ^2/μ in contrast to large values. In particular, for $\sigma^2/\mu \leq 1$, it is almost always the case that the preference, in order of most preferred first, occurs as S_2, MA, S_1, ML for all demand patterns save one. For Pattern 3, the ordering is ML, S_1, S_2, MA and this seems to be the one constant element in all cases. That is, for Pattern 3, maximum likelihood is preferred (if however slightly) to both forms of smoothing and the moving average is the worst choice.

When $\sigma^2/\mu > 1$, results are not so clear cut except as just mentioned for Pattern 3. For Pattern 4, the preference ordering is basically S_2, MA, S_1, ML; for Pattern 5, it is basically ML, S_1, S_2, MA as for Pattern 3. About all that can be said about Pattern 1 when $\sigma^2/\mu > 1$, is that MA seems never to be preferred; otherwise, each of the other techniques occurs at least once on the preferred list.

Finally, it might be remarked, as anticipated, that the standard deviations occur in roughly the same order as the MSFE ranks. This does not show up in the summary tables, however. It might also be observed that, basically, the increase in periods observed from $N = 8$ to $N = 20$ contributes little to the ordering scheme.

4. Sample Characteristics of Real Data

The results of the previous section depend very much on the assumption that demand is normally distributed with a mean value that varies according to specific types of functions. At that, only five such functions were tested. And in each of those patterns the variance was held constant. Naturally, the application of such results to real data is severely restrictive, particularly in view of the fact that little documentation presently exists regarding the true nature of demand for many kinds of items managed by NAVSUP. We therefore set about to run similar comparisons on real data. Even though the underlying model for the data is unknown, one can still ask in retrospect, given the data, what would have happened had a different forecasting scheme been used?

The data used to address this question were supplied from the files of the Ships Parts Control Center (SPCC), Mechanicsburg, Pennsylvania, with the cooperation and assistance of Mssrs. R. Brumbaugh and J. Zerbe of the Fleet Material Support Office (FMSO). Unfortunately, demand history is retained only for the current and seven previous quarters for a total of $N = 8$ quarters or periods. To help counterbalance this very small value, a relatively large sample of 10,000 items was taken. The sample included only secondary, non-reparable items and only recurring maintenance demand was considered. So-called insurance items (those for which demand is less than one per year) were excluded because they obviously generate special forecasting problems of their own, particularly in view of the short history of eight quarters. Even with these items excluded, however, 859 of the 10,000 experienced no demand during the two-year period examined (1973-74).

A histogram displaying the frequency distribution of the data is displayed in Figure 4.1. The frequencies are graphed as proportions (out of 10,000) according to the average of the eight observations. The observed number (in each interval [a,b)) is listed at the end of each bar for further identification. From these data, it was determined that the ensemble mean is 13.545 while the median is only 1.5. As a matter of additional interest, the 90^{th} percentile turned out to be 14.375. Incidentally, if from the 10,000 items, the 859 having zero records were eliminated, the sample mean would only be increased to 14.818.

The fact that only eight quarters of demand are available poses several problems. All of our forecasting techniques require some initial assumptions at the beginning of the recorded demand period. For the two smoothing techniques (S_1 and S_2) that amounts to an initial forecast. For the moving average (MA) technique, each of the preceding eight quarters would be needed, strictly speaking, to implement the technique. In the absence of such information, the assumption made here is that each of these is equal to some initial forecast. Maximum likelihood requires the entire past history so, to put things on an equitable footing, we assume that eight previous quarters is the entire past and that each observation in those quarters is equal to an initial forecast.

This forces the beginning stage of each of our four forecasting techniques to depend on an initial forecast. It was felt

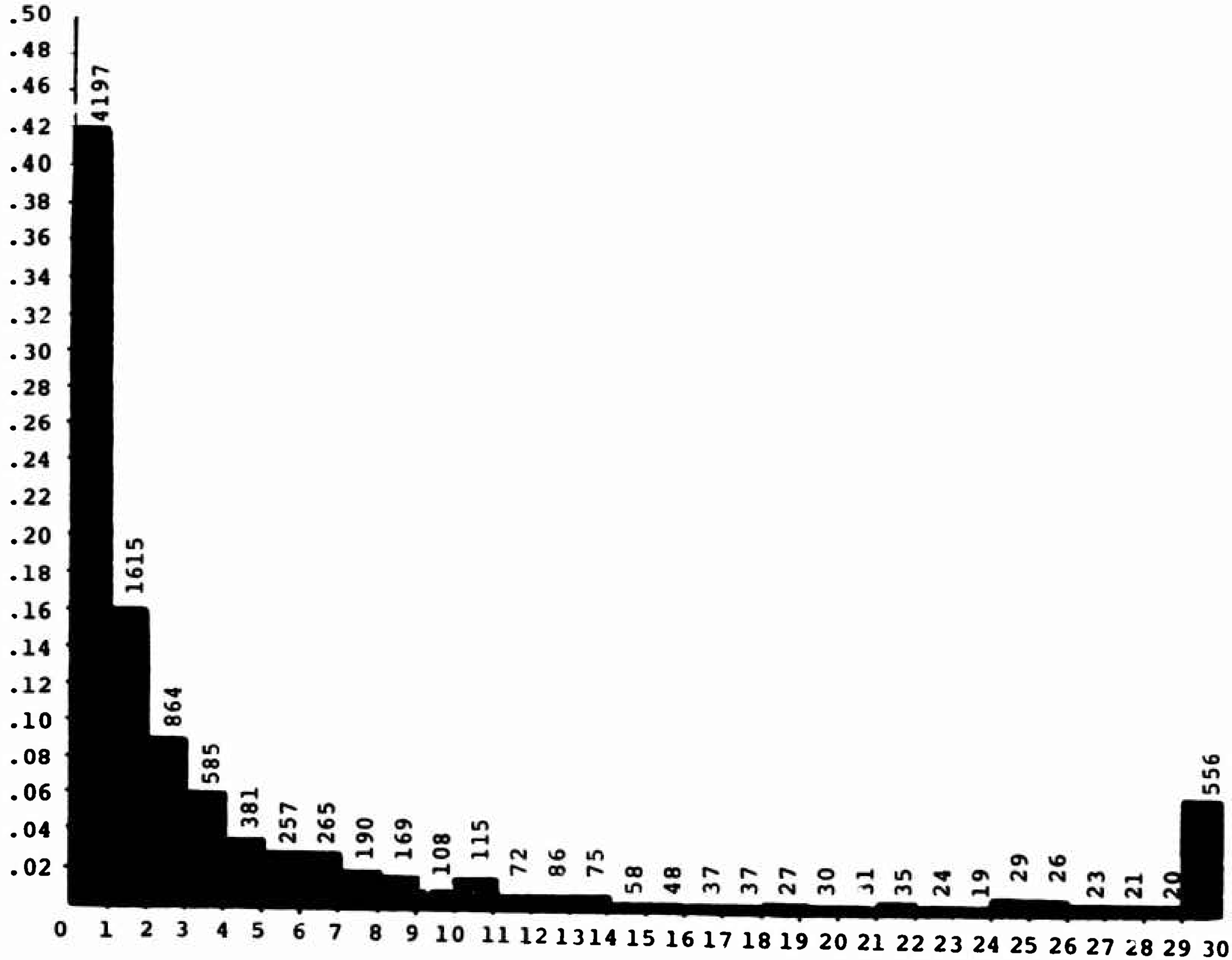

Figure 4.1. Frequency of Sample Means

that any systematic calculation of an initial forecast from the data might introduce extraneous bias in favor of one or the other of the techniques. It was therefore decided to use the ad hoc procedure of taking the midpoint of the range of observations being considered. Thus, for example, for all items whose average falls between 4.0 and 6.0 inclusive, an initial forecast of 5.0 was used; for items whose averages are between 2.0 and 3.0, an initial forecast of 2.5 was used, and so on.

One rather unexpected phenomenon observed in the sample data was the tendency for demands to be clumped together, rather than more evenly distributed throughout the eight observed quarters. A list of just a few such items is displayed in Figure 4.2. Such records, where an observed demand is much larger than expected and occuring rarely at that, make any kind of uniform assumption from quarter to quarter untenable. Certainly it makes the constant mean model, discussed in Section 2, extremely suspect.

	QUARTER							
Item	1	2	3	4	5	6	7	8
A	44	0	0	0	2	1	0	0
B	10	2	0	0	3	1	20	3
C	0	0	0	0	0	12	25	0
D	0	16	10	15	0	0	0	20
E	0	0	10	0	10	20	0	0

Figure 4.2. Distribution by Quarters.

There are many possible explanations for this phenomenon. One possibility is that many of these demands are for storeroom stock at an end-use activity, or at some intermediate echelon. The actual use of the material may indeed be at a relatively uniform rate but the item is ordered in batches at these other levels.[†] Or it may be that the demands themselves satisfy the uniform (identically distributed) assumption well enough but the record keeping is of such a nature that they appear in this form. Nevertheless, a model for such records, if that is in fact what decisions will be based on, will have to take that into account.

In order to compare the four forecasting methods under consideration, again the question of criteria or measures of effectiveness arises. For reasons previously discussed, mean squared forecast error was adopted. Using four different groupings by a range of sample means, the average $(N = 8)$ forecast error for each of the four forecasting methods utilized in Section 3 was computed. The number of items in each group then corresponds to NR in the simulations and permits the computation of an overall mean squared forecast error (MSFE). (Thus, the ad hoc grouping was done to give a respectable value of NR for the group.) This then allows us to order the forecast methods in order of preference as in Section 3. As a matter of added interest, the standard deviation (SD) and the 90^{th} percentile P_{90} are reported along with the mean squared forecast error. These results

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[†]With multiple users, such demands would tend to even out but most items of that type have been transferred to the Defense Supply Agency for management.

are summarized in Figure 4.3 where a group is defined by the indicated interval of means and NR, the total of such observations available is also indicated for each group.

Some obvious degrees of caution should be exercised in interpreting the results of the table. To repeat an earlier point, only $N = 8$ observations were available for computing purposes. This alone makes generalizations difficult. With so few items available in the mid-range values of means, observations were grouped rather arbitrarily to make the value of NR reasonable. At that, only about 2500 of the 10,000 items were finally included in the comparisons. Finally, the initial assumptions were selected on simply a rational basis and certainly affect the results.

With those cautions in mind, however, it may be noted that the results are fairly consistent with the simulation results of the previous section particularly when compared to Demand Pattern 3 of that section. This should not be too surprising since the batching we mentioned earlier has roughly the same effect as Demand Pattern 3. In particular, for these items, ML and S_1 are always preferred to MA and S_2 with MA consistently outperforming S_2, however slightly. We can say, however, that on the average ML would have been as good or better than S_1 had that method been used with the same data. But, considering the prior assumptions made for this study, that preference is certainly not a strong one.

		Forecast Method			
Group	Characteristics	S_1	S_2	MA	ML
I	MSFE	15.69	16.78	16.54	15.65
[2.0,3.0]	SD	13.77	14.86	14.43	13.67
NR = 999	P_{90}	39.02	41.90	40.56	38.25
II	MSFE	51.57	55.11	54.40	51.42
[4.0,6.0]	SD	48.17	51.82	50.67	47.99
NR = 695	P_{90}	111.55	119.31	117.40	111.26
III	MSFE	106.44	113.81	112.43	106.58
[6.0,8.0]	SD	92.01	99.22	96.71	92.36
NR = 488	P_{90}	286.97	308.69	299.04	286.43
IV	MSFE	160.44	171.21	169.75	160.17
[8.0,10.0]	SD	146.68	158.17	153.86	145.98
NR = 300	P_{90}	346.86	376.57	361.13	346.65

Figure 4.3. Characteristics of Forecast Methods.

We should also remark that these results have been carried out as an academic study without regard to modifications in the real applications. But as a matter of fact, SPCC often employs a "trending" technique which is intended to detect any significant trend (up or down) in the mean of the demand distribution. The technique is to compute the value of

$$T = \frac{2(D_0+D_1)}{D_0 + D_1 + D_2 + D_3} \tag{1}$$

where D_i is the demand observed in the $i^{\underline{th}}$ past quarter. If $T \varepsilon [0.9, 1.1]$, a smoothing weight of $\alpha = 0.1$ is used; otherwise α is chosen to be 0.3. Clearly, when two or more consecutive observations are zero, T will fail to be in the test interval.

As we mentioned earlier, the data we have examined are frequently of this very nature so that, for these items, trending can be expected to occur quite often. But, assuming that the patterns shown in these data tend to be repetitive, trending merely adds another harmful effect to the forecast. That is to say, if the data tend to repeatedly peak and then settle back to a lower level (for whatever underlying causes), trending will only add to exponential smoothing in placing undue weight on the misleading peak values. There are obvious implications here for a better understanding of the underlying model for the data since so many items are involved.

5. Conclusions and Recommendations

This is the fourth in a series of reports devoted to the study of demand forecasting (and exponential smoothing in particular) carried out at the Naval Postgraduate School under the sponsorship of the Naval Supply Systems Command. The study is of particular importance to real applications since exponential smoothing has become a basic forecasting tool for supply systems.

When exponential smoothing is coupled with MAD as a method of estimating variability, a great deal of analytical difficulty is encountered even when the most stable assumptions are made with regard to the demand distribution. All of this has been extensively documented in earlier reports and accounts for the continued use of simulation as an investigative tool.

In those earlier reports, it was demonstrated that, at least in a constant mean model, smoothing seems to produce more variable predictions than some other alternatives. We set about in this study to try and isolate the source of that variability. In Section 2 we have shown that exponential smoothing, rather than the use of MAD per se, seems to be the main contributor. While no one case studied can in itself be relied on too heavily due to simulation, the overwhelming consistency in case after case studied of improvement in variability from smoothing with MAD, through several intermediate modifications, to pure maximum likelihood leaves little doubt about the conclusion to be drawn.

But demands in real life seldom have a constant mean it is argued. Surely for a varying mean value, an adaptive procedure like exponential smoothing must be better to use than one designed for more stable conditions. We addressed this problem and the results in Section 3 show that this is sometimes true and sometimes not. In particular, when the mean value has a sudden jump but then settles back to its previous level, maximum likelihood is still a better procedure to use. When we allowed the mean to change in other ways (step and ramp) however, exponential smoothing emerged as a better candidate than either maximum likelihood or a moving average for predicting such changes.

Not content to rely exclusively on simulated results, however, we retrieved real demand data for additional study. Unfortunately, only eight quarters of such demand were available for processing which makes generalization extremely awkward. For this reason, we have attempted to mollify our conclusions in Section 4 to reflect the small sample size. To our surprise, a large number of items revealed demand histories that were more compatible with the one model of Section 3 for which maximum likelihood was a leader. It is not surprising then, that when the data were judged by the same ground rules, maximum likelihood was slightly better. But, with no real model available, certainly any recommendation would be indecisive. The best that can be said is that, had maximum likelihood been used on several of those items observed instead of exponential smoothing, the overall forecast would have improved, if however slightly, in some cases.

The first recommendation growing out of this study then is that more should be known about the underlying model before any decisive conclusions can be drawn. No respectable analysis can be performed on such limited histories but enough is indicated here to suggest the importance of getting closer to the underlying model. The implication for additional data storage is obvious. It should also be pointed out that, even apart from that, only five different demand patterns were studied here. Surely other, perhaps even more appropriate, patterns might occur to users. For this reason, we have included our programs in various appendices for modification and use by the interested reader.

In that regard, we have added an appendix concerned with computational speed. A case for exponential smoothing has been consistently made in the past (particularly by Brown [4]) on the basis of computer storage and speed in computation. But computer technology has made giant strides in recent years. In Appendix D processing times for various forecasting techniques are compared utilizing 1968 state-of-the-art equipment. Processing times are even faster now and hence should no longer be a significant factor in the selection of forecasting techniques, at least of the types discussed in this report.

6. Appendices

APPENDIX A - Comparing Variability

This program simulates normally distributed demands with constant mean and variance. As in the following program (Appendix B) the Learmonth-Lewis random number package LLRANDOM [6] was used. All programs were written in FORTRAN IV and run on the IBM 360/67 computer of the W. R. Church Computer Center at the Naval Post-graduate School.

```
C [illegible] STD DEVIATION OF DEMAND
C [illegible] STATEMENT CHANGED
C [illegible] ALPHA .GE. 1.0
C [illegible]
C [illegible] (NORMAL)
C [illegible] DISTRIBUTION
C [illegible] REPLICATIONS
C [illegible] RIGHT JUSTIFIED ARE PERIODS WHERE
C [illegible]
C [illegible] 55-60 [illegible]
C [illegible] 61-75 SPECIFY .01,.05,.10,.20,.25
C [illegible] USED TO INITIALIZE FORECASTS
C METHOD 1: EXPONENTIALLY SMOOTHED MAD WITH HEAD START
C METHOD 2: EXPONENTIALLY SMOOTHED MAD WITH NO HEAD START
C METHOD 3 SAMPLE STANDARD DEVIATION WITH EXP SMOOTHED FCST OF MEAN
C METHOD 4 EXPONENTIALLY SMOOTHED SQUARED ERROR WITH HEAD START
C METHOD 5 EXPONENTIALLY SMOOTHED SQUARED ERROR WITH NO HEAD START
C METHOD 6 [illegible]
C METHOD 7 MAXIMUM LIKELIHOOD FCST OF MEAN AND SD
C
      DIMENSION X(100),F(100),E(100),SD(7,100),NP(4),SM(7,4),SSD(7,4),
     :          F1(7,4),R2(7,4),S(7,4),SRD(7,4),FML(100)
      [illegible]
      [illegible] (7,4),SD(7,4)
      [illegible]
      [illegible]
 10   [illegible]
      [illegible]
      [illegible]
    1 [illegible] ALPHA,[illegible],(NP(I),I=1,4),ISEED,RHO
      [illegible]
      [illegible]
      [illegible]
      [illegible] = 1.25331 * R1
      [illegible] = 1.0 - ALPHA
      [illegible]
      RK = 2.326
      IF (RHO .EQ. 0.05) RK = 1.645
      IF (RHO .EQ. 0.10) RK = 1.282
      IF (RHO .EQ. [illegible]) RK = [illegible]
      IF (RHO .EQ. [illegible]) RK = [illegible]
      [illegible]
      [illegible] = 1,7
      [illegible] = 1,4
      [illegible](I,J) = 0.
      [illegible](I,J) = 0.
      [illegible](I,J) = 0.0
      [illegible](I,J) = 0.0
      [illegible](I,J) = 0.0
      [illegible](I,J) = 0.0
 11   CONTINUE
 12   CONTINUE
```

```
C     [illegible] INITIALIZE
      [illegible] J = 1,[illegible]
      [illegible] VARIABLES
      CALL NORMAL ([illegible])
      [illegible]
      [illegible]
      [illegible] = [illegible] + SIGMA[illegible]
      [illegible]
C     [illegible] MAX LIKELIHOOD
      [illegible]
      [illegible]
      [illegible]
      [illegible] = [illegible] - MU
      [illegible] SIGMA
      [illegible] = ABS([illegible])
      [illegible]
      [illegible]
      [illegible] = (SIGMA/K1)**2
      [illegible] = ([illegible])
      [illegible]

      [illegible]

C     PROGRAM COMPARES 7 SCHEMES FOR FORECASTING STD DEVIATION OF DEMAND
C     NP(4) MUST BE .LE.100 UNLESS DIMENSION STATEMENT CHANGED
C     PROGRAM TERMINATES WHEN IT READS AN ALPHA .GE. 1.0
C     ALPHA CC 1-10 = SMOOTHING CONSTANT
C     MU    CC 11-20 = MEAN OF DEMAND DISTRIBUTION (NORMAL)
C     SIGMA CC 21-30 = S.D. OF DEMAND DISTRIBUTION
C     NR    CC 31-35 RIGHT JUSTIFIED = NUMBER OF REPLICATIONS
C     NP(I) CC 36-40,41-45,46-50,51-55 RIGHT JUSTIFIED ARE PERIODS WHERE
C        SNAPSHOTS ARE TAKEN
C     ISEED CC 56-60 FOR RANDOM NR GENERATOR
C     RHO   CC 61-70  SPECIFY .01,.05,.10,.20,.25
C     MU AND SIGMA ARE USED TO INITIALIZE FORECASTS
C     METHOD 1:  EXPONENTIALLY SMOOTHED MAD WITH HEAD START
C     METHOD 2:  EXPONENTIALLY SMOOTHED MAD WITH NO HEAD START
C     METHOD 3  SAMPLE STANDARD DEVIATION WITH EXP SMOOTHED FCST OF MEAN
C     METHOD 4   EXPONENTIALLY SMOOTHED SQUARED ERROR WITH HEAD START
C     METHOD 5   EXPONENTIALLY SMOOTHED SQUARED ERROR WITH NO HEAD START
C     METHOD 6   K1*RMSFE
C     METHOD 7   MAXIMUM LIKELIHOOD FCST OF MEAN AND SD

      DIMENSION X(100),F(100),E(100),SD(7,100),NP(4),SM(7,4),SSD(7,4),
     1          R1(7,4),R2(7,4),SR(7,4),SRD(7,4),FML(100)
      REAL*4 K1,K2,MU,MSFE(7,4)
      REAL*8 SSQ(7,4),RL(7,4),RLSD(7,4),SUM(7,4)
      REAL*8 ARG,ORD,RISK,SX3,SSX3
  100 FORMAT (3F10.4,6I5,E10.4)
  190 FORMAT ('0ERROR = ',F10.4)
      CALL OVFLOW
      ITYPE = 4
   10 READ (5,100) ALPHA,MU,SIGMA,NR,(NP(I),I=1,4),ISEED,RHO
C     TEST FOR TERMINATION
      IF(ALPHA.GE.1.0) GO TO 990
      K1 = SQRT(0.5*(2.0 - ALPHA))
      K2 = 1.253314 * K1
      BETA = 1.0 - ALPHA
      N =NP(4)
      RK = 2.326
      IF (RHO .EQ. 0.05) RK = 1.645
      IF (RHO .EQ. 0.10) RK = 1.282
      IF (RHO .EQ. 0.20) RK = 0.842
      IF (RHO .EQ. 0.25) RK = 0.674
      TEL = MU + RK*SIGMA
      DO 12 I = 1,7
      DO 11 J = 1,4
      SUM (I,J) = 0.
      SSQ (I,J) = 0.
      R1(I,J) = 0.0
      R2(I,J) = 0.0
      RL(I,J) = 0.0
      RLSD(I,J) = 0.0
   11 CONTINUE
   12 CONTINUE
```

```
C     SUMS INITIALIZED
      DO 50 J = 1,NR
C     GENERATE N(0,1) VARIATES
      CALL NORMAL (ISEED,X,NP(4))
C     TRANSFORM TO N(MU,SIGMA)
      DO 15 I = 1,N
      X(I) = MU + SIGMA*X(I)
   15 CONTINUE
C     COMPUTE FORECASTS USING SINGLE EXP SMOOTHING   AND MAX LIKELIHOOD
C     INITIALIZE
      F(1) = MU
      FML(1) = MU
      E(1) = X(1) - MU
      SD(1,1) = SIGMA
      SX2 = ABS(E(1))
      SX3 = X(1)
      SSX3 = SX3*SX3
      SX4 = (SIGMA/K1)**2
      SX6 = E(1)*E(1)
      SX5 = SX6
C
C     COMPUTE FORECASTS
C
      DO 20 I = 2,N
      F(I) = ALPHA*X(I-1) + BETA*F(I-1)
      E(I) = X(I) - F(I)
C
C     METHOD 1
      SD(1,I) = BETA*SD(1,I-1) + K2*ALPHA*ABS(E(I-1))
C
C     METHOD 2
      SX2 = BETA*SX2 + ALPHA*ABS(E(I))
      SD(2,I) = K2*SX2
C
C     METHODS 3 AND 7
      FML(I) = (SX3 + MU)/I
      SX3 = SX3 + X(I)
      SSX3 = SSX3 + X(I)*X(I)
C
  160 FORMAT (1H ,4(F20.4,5X))
      ARGU = (SSX3 - SX3*SX3/I)/I
      WRITE (6,160) X(I),SX3,SSX3,ARGU
      SD(3,I) = SQRT (ARGU)
      IF (ARGU .LT. 0.) STOP
C
      SD(7,I) = SD(3,I)
C
C     METHOD 4
      SX4 = BETA*SX4 + ALPHA*E(I-1)*E(I-1)
      SD(4,I) = K1*SQRT(SX4)
C
C     METHOD 5
      SX5 = BETA*SX5 + ALPHA*E(I)*E(I)
      SD(5,I) = K1*SQRT(SX5)
C
C     METHOD 6
      SX6 = SX6 + E(I)*E(I)
      SD(6,I) = K1*SQRT(SX6/I)
   20 CONTINUE
C     COLLECT STATISTICS
      DO 28 K = 1,7
      DO 26 L= 1,4
      SUM (K,L) = SUM(K,L) + SD(K,NP(L))
      SSQ (K,L) = SSQ(K,L) + SD(K,NP(L))*SD(K,NP(L))
      IF (K .EQ. 7) F(NP(L)) = FML(NP(L))
      RP = F(NP(L)) + RK*SD(K,NP(L))
      ARG = (RP - MU)/SIGMA
      CALL NPDA (ARG,ITYPE,PRO,RISK,ERR)
      IF (ERR .NE. 0.0) WRITE (6,150) ERR
      R1(K,L) = R1(K,L) + RISK
      R2(K,L) = R2(K,L) + RISK*RISK
      RL(K,L) = RL(K,L) + RP
      RLSQ(K,L) = RLSQ(K,L) + RP*RP
   26 CONTINUE
   28 CONTINUE
   50 CONTINUE
```

```
C   NOW COMPUTE SAMPLE STATISTICS
       DO 80 J = 1,4
       DO 70 I = 1,7
       SM(I,J) = SUM(I,J)/NR
       VAR = (SSD(I,J) -SUM(I,J)*SUM(I,J)/NR)/NR
       SSD(I,J) = SQRT(VAR)
       MSEE(I,J) = VAR + (SIGMA - SM(I,J))**2
       SR(I,J) = R1(I,J)/NR
       SRD(I,J) = SQRT((R2(I,J) - R1(I,J)*R1(I,J)/NR)/NR)
       RLSD(I,J) =DSQRT((RLSD(I,J) - RL(I,J)*RL(I,J)/NR)/NR)
       RL(I,J) = RL(I,J)/NR
   70 CONTINUE
   80 CONTINUE
C   OUTPUT SECTION
  200 FORMAT ('1THE FOLLOWING PARAMETERS WERE USED FOR THIS RUN:',/)
  201 FORMAT ('0ALPHA',18X,'= ',F10.4,/,' MU',21X,'= ',F10.4,/,' SIGMA',
     1   18X,'= ',F10.4,/,' NUMBER OF REPLICATIONS = ',I10,/,' RISK',
     2   19X,'= ',F10.4,/,' REORDER LEVEL',10X,'= ',F10.4,/,' ISEED',18X,
     3   '= ',I10,/)
  202 FORMAT ('0QUARTER',19X,4(I10,5X))

  210 FORMAT ('0METHOD',I3,5X,'SAMPLE MEAN ',4(F10.4,5X),/,' ',14X,
     1   'SAMPLE S.D. ',4(F10.4,5X),/,' ',14X,'MSE',9X,4(F10.4,5X),/,' ',
     2   14X,'SAMPLE RISK ',4(F10.4,5X),/,' ',14X,'S.D. OF RISK',4(F10.4,
     3   5X),/,' ',14X,'REORDER',5X,4(F10.4,5X),/,' ',14X,'REORDER S.D.',
     4   4(F10.4,5X),/)
       WRITE (6,200)
       WRITE (6,201) ALPHA,MU,SIGMA,NR,RHO,TRL,ISEED
       WRITE (6,202) (NP(I),I=1,4)
       DO 95 I = 1,7
       WRITE (6,210) I,(SM(I,J),J=1,4),(SSD(I,J),J=1,4),
     1 (MSEE(I,J),J=1,4),(SR(I,J),J=1,4),(SRD(I,J),J=1,4)
     2   ,(RL(I,J),J=1,4),(RLSD(I,J),J=1,4)
   95 CONTINUE
       GO TO 10
  999 STOP
       END
```

APPENDIX B - Forecasts of Mean Demand

This program consists of both a demand generator and a forecast analyzer. In the generator, normally distributed demand is first generated and then transformed into the five patterns outlined in Section 3. In the analyzer, each of the four forecast methods outlined in Section 3 is tested against all five demand patterns. For the two smoothing techniques, the usual relationship†

$$f_{i+1} = (1-\alpha)f_i + \alpha x_i$$

was used. For maximum likelihood it was assumed that demands had been recorded for 20 periods prior to the initial testing period and that the 20 period average coincided with the parameter mean used in the generator. For the eight quarter moving average, eight extra quarters of demand were first generated for a set of initial conditions whereupon the relationship

$$f_i = f_{i-1} + \frac{(x_{i-1} - x_{i-9})}{8}$$

was used to forecast. The program follows.

```
      [illegible] FOR FORECASTING DEMAND.
      [illegible] MEAN [illegible] ARE COMPUTED FOR EACH.  VARIOUS NON-STATIONARY
      [illegible] NORMALLY DISTRIBUTED DEMAND PATTERNS ARE CONSIDERED.
   [illegible] INPUTS:   MU                  [illegible] 1-10
                         SIGMA                   11-[illegible]
                         [illegible] PATTERN     25
     [illegible] CARD WILL RUN DEMAND PATTERNS FROM DP TO 5
                         NR REPLICATIONS     31-35
                         NR PERIODS          41-45
                         ISEED               51-55
   PROGRAM TERMINATES WHEN NEGATIVE MU ENCOUNTERED.
   [illegible] USED ARE:
         1.   SS.1 (SINGLE SMOOTHING WITH ALPHA = .1)
         2.   SS.2 (SINGLE SMOOTHING WITH ALPHA = .2)
         3.   MLE20 (MAX LIKELIHOOD ASSUMING 20 WKS PRIOR EXISTENCE)
         4.   MA8 (8 WK MOVING AVERAGE)
   [illegible] IS ASSUMED TO COMMENCE IN PERIOD 3.
```

† f_{i+1} is the forecast for the $(i+1)^{st}$ period using x_i as the most recent observation.

```
C     PATTERN 1:   50% STEP INCREASE
C     PATTERN 2:   100% STEP INCREASE
C     PATTERN 3:   100% SPIKE
C     PATTERN 4:   10% RAMP
C     PATTERN 5:   10% RAMP FOR 2 QTRS, THEN LEVEL

      [illegible]
      INTEGER OP
      DIMENSION X(100),Y(8),F(100),RANK(4),KEY(4)
      REAL MU, SSQ(4),SUM(4),Z(4),SFE
  100 FORMAT (2F10.4,I5,3(5X,I5),E10.4)
      CALL OVFLOW
    1 READ (5,100) MU,SIGMA,OP,NR,N,ISO
      IF (MU.LT.0.) GO TO 999
   11 ISEED = ISO
C     INITIALIZE ARRAYS
      DO 12 I = 1,4
      RANK(I) = 0.
      SUM(I) = 0.
      SSQ(I) = 0.
   12 CONTINUE
C     BEGIN MAIN LOOP
      DO 90 J = 1,NR
C     GENERATE X(J,I) DEMANDS
      CALL NORMAL (ISEED,X,N)
C     TRANSFORM FIRST TWO OBSERVATIONS
      X(1) = MU + SIGMA*X(1)
      X(2) = MU + SIGMA*X(2)
C     GENERATE PREVIOUS 8 QTRS DEMAND FOR MOVING AVERAGE
      CALL NORMAL (ISEED,Y,8)
      DO 16 I = 1,8
      Y(I) = MU + SIGMA*Y(I)
   16 CONTINUE
C     SELECT DEMAND PATTERN AND TRANSFORM REMAINING DEMANDS
      GO TO (21,22,23,24,25),OP
   21 DO 31 I = 3,N
      X(I) = 1.5*MU + SIGMA*X(I)
   31 CONTINUE
      GO TO 50
   22 DO 32 I = 3,N
      X(I) = 2.0*MU + SIGMA*X(I)
   32 CONTINUE
      GO TO 50
   23 X(3) = 2.0*MU + SIGMA*X(3)
      DO 33 I = 4,N
      X(I) = MU + SIGMA*X(I)
   33 CONTINUE
      GO TO 50
   24 DO 34 I = 3,N
      X(I) = (1.0 + (I-2)*0.1)*MU + SIGMA*X(I)
   34 CONTINUE
      GO TO 50
   25 X(3) = 1.1*MU + SIGMA*X(3)
      DO 35 I = 4,N
      X(I) = 1.2*MU + SIGMA*X(I)
   35 CONTINUE
   50 CONTINUE
C     END DEMAND GENERATOR; BEGIN FORECAST ANALYZER
C     TECHNIQUE 1:   SS.1
      F(1) = MU
      SFE1 = (X(1) - MU)*(X(1) - MU)
      SFE = SFE1
      DO 52 I = 2,N
      F(I) = 0.9*F(I-1) + 0.1*X(I-1)
      SFE = SFE + (X(I) - F(I))**2
   52 CONTINUE
      Z(1) = SFE/N
C     TECHNIQUE 2:   SS.2
      SFE = SFE1
      DO 54 I = 2,N
      F(I) = 0.8*F(I-1) + 0.2*X(I-1)
      SFE = SFE + (X(I) - F(I))**2
   54 CONTINUE
      Z(2) = SFE/N
```

```
C     TECHNIQUE 3:  MLE20
      SFE = SFE1
      DO 56 I = 2,N
      F(I) = ((18. + I)*F(I-1) + X(I-1))/(19. + I)
      SFE = SFE + (X(I) - F(I))**2
   56 CONTINUE
      Z(3) = SFE/N
C     TECHNIQUE 4:  MA8
      YSUM = Y(1)
      DO 58 I = 2,8
      YSUM = YSUM + Y(I)
   58 CONTINUE
      F(1) = YSUM/8.0
      SFE = (X(1) - F(1))**2
      DO 60 I = 2,8
      F(I) = F(I-1) + (X(I-1) - Y(I-1))/8.0
      SFE = SFE + (X(I) - F(I))**2
   60 CONTINUE
      IF (N.LE.8) GO TO 64
      F(9) = F(8) + (X(8) -Y(8))/8.0
      SFE = SFE + (X(9) - F(9))**2
      DO 62 I = 10,N
      F(I) = F(I-1) + (X(I-1) - X(I-9))/8.0
      SFE = SFE + (X(I) - F(I))**2
   62 CONTINUE
   64 Z(4) = SFE/N
      DO 86 I = 1,4
      SUM(I) = SUM(I) + Z(I)
      SSQ(I) = SSQ(I) + Z(I)*Z(I)
   86 CONTINUE
C     NOW COMPUTE RANK
      DO 88 I = 1,4
      KEY(I) = I
   88 CONTINUE
      CALL SHSORT (Z,KEY,4)
      DO 89 I = 1,4
      RANK(KEY(I)) = RANK(KEY(I)) + I
   89 CONTINUE
   90 CONTINUE
C     END MAIN LOOP
C     NOW COMPILE STATISTICS
      DO 92 I = 1,4
C     MEAN RANK
      RANK(I) = RANK(I)/NR
C     SAMPLE MEAN OF MSFE'S
      Y(I) = SUM(I)/NR
C     SAMPLE STANDARD DEVIATION OF MSFE'S
      VAR = (SSQ(I) - SUM(I)*SUM(I)/NR)/NR
      IF (VAR .LT. 0.0) GO TO 600
   91 Z(I) = SQRT (VAR)
   92 CONTINUE
C     OUTPUT SECTION
  200 FORMAT ('0',///////,' PARAMETERS USED FOR THIS RUN:',//,' MU',18X,
     1 F10.4,/,' SIGMA',15X,F10.4,/,' DEMAND PATTERN',6X,I10,/,
     2 ' REPLICATIONS',8X,I10,/,' PERIODS',13X,I10,//)
  201 FORMAT ('0METHOD',15X,'MEAN',11X,'MSFE',8X,'SAMPLE',/,16X,' RANK',
     1 23X,'STD DEV',/)
  202 FORMAT ('0',I6,4X,3F15.4)
      WRITE (6,200) MU,SIGMA,DP,NR,N
      WRITE (6,201)
      DO 210 I = 1,4
      WRITE (6,202) I,RANK(I),Y(I),Z(I)
  210 CONTINUE
      IF (DP .EQ. 5) GO TO 10
      DP = DP + 1
      GO TO 11
  250 FORMAT ('1I = ',I10,/,'0SUM(I) = ',F15.6,/,'0SSQ(I) = ',E15.6,/,
     1 '0DEMAND PATTERN = ',I10,/,'0VAR = ',E15.6,//)
  600 WRITE (6,250) I,SUM(I),SSQ(I),DP,VAR
      VAR = 0.0
      GO TO 91
  999 STOP
      END
```

APPENDIX C - Comparing SPCC Data

The assumptions leading to an initial forecast for each of the forecasting schemes examined were discussed in Section 4. Once these are given, forecasting then proceeds as in Appendix B. In connection with both maximum likelihood and moving average, it was assumed further that eight previous demands, each equal to the initial forecast, was part of the recorded history. Some assumption of this nature is essential for the computational forms required. These matters are obvious in the program which follows.

```
C     COMPARES 4 FORECASTING TECHNIQUES USING SPCC DATA
C     CONSIDERS ITEMS WHOSE AVG DEMAND IS BETWEEN DL AND DH
C     CSFE1=SS.1,CSFE2=SS.2,CSFE3=MA8,CSFE4=MLE8
C
C
      REAL*4 SS1 (1000),SS2 (1000),MA8(1000),MLE8(1000),X(10),Y(10)
      INTEGER*4 A(10),B(10),D(10,8)
  100 FORMAT (10(2X,2A4,20X,8I10,F8.2,F7.2))
  200 FORMAT (1X,I5,2X,2A4,8I4,2X,2F9.2,4F15.4)
  300 FORMAT (1H1,4X,'N',6X,'NIIN',2X,'D0',2X,'D1',2X,'D2',2X,'D3',2X,
     1  'D4',2X,'D5',2X,'D6',2X,'D7',7X,'MEAN',6X,'VMR',3X,'SMOOTHING .1
     2',3X,'SMOOTHING .2',' MOVING AVERAGE',12X,'MLE')
      DATA NR,FO,DL,DH/0,2.5,2.0,3.0/
      CALL ERRSET (215,300,-1,1)
      WRITE (6,300)
      REWIND 2
    5 READ (2,100,END=20) ((A(I),B(I),(D(I,J),J=1,8),X(I),Y(I)),I=1,10)
      DO 10 I = 1,10
      IF ((X(I).LT.DL ).OR.(X(I).GT.DH )) GO TO 10
      NR = NR + 1
      CALL FCST(D,SS1 ,SS2 ,MA8,MLE8,NR,I,FO)
      WRITE (6,200) NR,A(I),B(I),(D(I,J),J=1,8),X(I),Y(I),SS1 (NR),
     1  SS2 (NR),MA8(NR),MLE8(NR)
      IF (NR.GT.1000) GO TO 20
   10 CONTINUE
      GO TO 5
   20 CALL HISTG(SS1 ,NR,0)
      CALL HISTG(SS2 ,NR,0)
      CALL HISTG(MA8,NR,0)
      CALL HISTG(MLE8,NR,0)
      STOP
      END
```

```
      SUBROUTINE FCST(D,SS1,SS2,MA8,MLE8,NR,I,F0)
      INTEGER*4 D(10,8)
      REAL*4 SS1 (1000),SS2 (1000),MA8(1000),MLE8(1000)
      F = F0
      SFE1 = (F - D(I,8))**2
C     SS.1 FORECASTS
      SFE = SFE1
      DO 50 K = 1,7
      J = 8 - K
      F = 0.9*F + 0.1*D(I,J+1)
   50 SFE = SFE + (F - D(I,J))**2
      SS1(NR) = SFE
C     SS.2 FORECASTS
      F = F0
      SFE = SFE1
      DO 52 K = 1,7
      J = 8 - K
      F = 0.8*F + 0.2*D(I,J+1)
   52 SFE = SFE + (F - D(I,J))**2
      SS2(NR) = SFE
C     MA8 FORECASTS
      F = F0
      SFE = SFE1
      DO 54 K = 1,7
      J = 8 - K
      F = F + 0.125*(D(I,J+1) - F0)
   54 SFE = SFE + (F - D(I,J))**2
      MA8(NR) = SFE
C     MLE8 FORECASTS
      F = F0
      SFE = SFE1
      DO 56 K = 1,7
      J = 8-K
      F = ((K+7.0)*F + D(I,J+1))/(K+8.0)
   56 SFE = SFE + (F - D(I,J))**2
      MLE8(NR) = SFE
      RETURN
      END
```

APPENDIX D - Computation Times

The following comparisons are based on processing times for an IBM 360/65 computer using 1968 technology. For newer large computers, such as the Burroughs 3500 presently installed at Naval Supply Centers, times will be faster. This is particularly true for division and square root operations. All times reported here are in microseconds (10^{-6} seconds).

The summary table below has the following legend.

A. Exponential Smoothing.

B. Maximum Likelihood (Mean).

C. Moving Average.

D. Exponentially Smoothed MAD.

E. Maximum Likelihood (Standard Deviation).

F. Exponentially Smoothed Squared Error.

G. Root Mean Squared Forecast Error.

TIME PER OPERATION		NUMBER OF OPERATIONS						
		A	B	C	D	E	F	G
Additions	(0.65)	1	2	1	1	2	1	1
Subtractions	(0.65)	1	0	1	1	1	1	1
Multiplications	(4.05)	2	0	1	3	2	4	2
Divisions	(6.55)	0	1	0	0	2	0	1
Absolute Value	(0.95)	1	0	0	1	0	0	0
Square Root	(59.1)	0	0	0	0	1	1	1
Total		10.35	7.85	5.35	14.40	82.25	76.60	75.05

Table D.1. Times Versus Operations (Per Iteration).

From the table we see that the difference in processing time for the various methods of forecasting mean demand (A, B, C) is less than one second per 100,000 iterations. For estimators of variability, the difference is less than seven seconds per 100,000 iterations. For the kinds of estimators that we have been discussing, then, processing time certainly ought not to be an overriding factor in choosing one method over another.

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