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A SURVEY OF PARAMETERIZATION TECHNIQUES FOR THE PLANETARY BOUNDARY LAYER IN ATMOSPHERIC CIRCULATION MODELS

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RAND Corporation

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Atmospheric general circulation models (GCMs) are increasingly used as research tools to test hypotheses and predict climatic variations. A significant phase of that research is the attempt to understand to what extent planetary boundary layer (BL) turbulent processes govern the evolutions of large-scale processes. Parameterization of BL turbulent fluxes in GCMs is one of the most difficult problems confronting atmospheric scientists. The report surveys the parameterization techniques most commonly used in GCMs; these techniques may be based on the so-called K-theory or on similarity theory. The report also discusses special problems in treating the BL in low latitudes as well as over oceans. Realistic determination of BL height is the subject of a separate chapter, which concludes that a rate equation is the most appropriate technique. Existing BL theories are useful for regional or other restricted studies, but are likely to be inadequate for global models. (See also R-1511, R-1654.) (Author)

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PREFACE

Interest in and concern about the problem of climatic variation are growing, because of the realization that earth's climates are undergoing changes that can exert a profound and irreversible influence on human life. It is now considered essential that we explore the mechanisms of climatic variation by developing numerical climate models, with special emphasis on the climatic effects of boundary layer (BL) phenomena.

Parameterization techniques are used to incorporate BL processes in atmospheric general circulation models (GCMs), which serve as tools to study climatic changes. But it appears that owing to preoccupation with the treatment of various other physical and mathematical features of GCMs, BL processes have not received the attention they deserve. It is now realized that more research is necessary in this direction, because, on the time scales relevant to climatic changes, BL turbulent exchanges (in addition to other physical processes) are very important in governing the evolution of large-scale processes. We therefore need to be able to test systematically the elements of our understanding of parameterization hypotheses.

This report surveys existing BL formulations that furnish the basis for parameterizing the BL in atmospheric circulation models. Besides describing BL theories, the report discusses problems encountered in applying the conventional theories to low latitudes and to the atmosphere over oceans. A separate chapter is devoted to the determination of BL height, because recent research has indicated that this is the basic parameter of boundary layer parameterization.

This report is a technical contribution to the broader activities of the Rand Dynamics of Climate Program, sponsored by the Defense Advanced Research Projects Agency and directed to the systematic study of climate variations. Two related Rand publications by the present author are:

- *Numerical Experiments on the Computation of Ground Surface Temperature in an Atmospheric Circulation Model*, R-1511-ARPA, May 1974.
- *Parameterization of the Planetary Boundary Layer in Atmospheric General Circulation Models--A Review*, R-1654-ARPA, March 1975.

SUMMARY

It is generally recognized that the boundary layer (BL) is one of the major energy sources and momentum sinks in the atmosphere. Research on BL dynamics therefore has an important bearing on many atmospheric problems. More emphasis is being placed on the study of the *interaction* between large-scale atmospheric dynamics and BL processes. That interaction is particularly important to researchers attempting to use dynamic methods to study the more comprehensive and pressing problem of climatic variation. The problem involves the most important question of all: the predictability of climatic change, whose determination requires the use of dynamic models.

Atmospheric general circulation models (GCMs) are being used as research tools to test hypotheses and to predict climates. The last few years have seen concerted efforts to improve both simulation modeling and the long-range prediction of climate and its variation. A significant phase of that improvement has resulted from the effort to understand, in addition to other processes such as convection and radiation, how and to what extent BL turbulent exchanges govern the evolution of large-scale processes. In fact, one of the most formidable problems confronting atmospheric scientists is the whole question of determining how smaller-scale conditions (such as BL turbulent fluxes) are excited by large-scale conditions and in turn alter them, i.e., *the question of "parameterization."*

This report surveys the parameterization "theories" used in various atmospheric models. There being no comprehensive fundamental theory of parameterization, various semi-empirical theories have been used. Essentially, they try to relate horizontal stress, heat flux, and moisture flux at the earth's surface and upward through the BL to the external (free atmosphere and underlying surface) parameters.

The so-called K-theory has been widely used for parameterizing BL processes in atmospheric numerical models. The assumptions are related to the form of "internal" quantities, such as eddy coefficient as a function of bulk parameters (stability, for example), and the relevant

equations are solved using appropriate boundary conditions. Though based on more or less arbitrary assumptions, this theory has enabled us to gain considerable insight into the behavior of BL processes in the atmosphere.

The similarity theory is based on the hypothesis that the turbulent regime is unambiguously defined by the values of the parameter U_*, the friction velocity; g/θ, the buoyancy parameter; Q, the vertical heat flux; and f, the coriolis parameter. The last one makes it meaningless to apply this theory near and at the equator. Generally, the results of treatment of the BL by similarity theory show satisfactory agreement with observations. However, sizable discrepancies occur between the results of different authors concerning the values of "universal" constants and "universal" functions of the theory. These discrepancies arise because the theory has been developed for homogeneous barotropic and steady-state conditions, which are rarely found in the real atmosphere. The basic theory has now been made more general by taking into account the nonstationarity and influence of baroclinicity. Another generalization has been made by using, in place of conventional scale height U_*/f, a BL height h that varies in time and space.

On the basis of observations, it has been found desirable to determine h through a rate equation for unstable convection BLs. The use of a rate equation is based on the entrainment hypothesis, which is applied at the top of the BL. So far, however, rate equations have not been considered appropriate for stable BLs. The entrainment hypothesis has been developed basically for cloud-free BLs, though there have been attempts to incorporate effects of radiative fluxes on BLs with clouds within them. To apply the entrainment concept in practice, it is necessary to use a closure assumption that essentially relates the heat flux at the top of the BL to that at the underlying surface.

It is difficult to treat the BL at low latitudes, including the equator, because none of the existing theories are applicable to those regions owing to the presence of f, the coriolis parameter, in almost all basic formulations. There is no alternative but to use the existing schemes in some modified form, however, since no schemes have been designed specifically to handle the low-latitude BL problems.

The treatment of the marine BL, on the basis of existing theories, also presents a peculiar problem related specifically to the determination of "roughness" of the ocean surface. The effects of ocean currents on momentum sources also are to be considered. Inasmuch as the oceanic surface, through its supply of heat and moisture, strongly affects atmospheric processes, concerted research on parameterizing the marine BL is urgently needed.

Existing theories have been useful for regional and other restricted studies, but are likely to be inadequate for global models. It is hoped that this survey will stimulate further application of semi-empirical theories of the BL to global models. This may be achieved by putting together all available parameterization schemes in a GCM and then estimating the advantages and deficiencies of different approaches through systematic numerical experiments.

CONTENTS

SYMBOLS

$A(\mu)$, $B(\mu)$, $C(\mu)$, $D(\mu)$	universal nondimensional functions	
C_D	geostrophic drag coefficient	used in resistance laws, etc.
C_H	heat transfer coefficient	
C_E	moisture transfer coefficient	
C_p	specific heat of air at constant pressure	
E	moisture flux (E_o = surface moisture flux)	
f	coriolis parameter	
G	geostrophic wind	
G_o	surface geostrophic wind	
g	acceleration due to gravity	
H_T	height of tropopause	
h	height of the boundary layer	
K	eddy diffusion coefficient (K_M for momentum, K_H for heat, K_E for moisture)	
k	von Karman constant	
L	Monin-Obukhov length	
ℓ	neutral mixing length	
Q	heat flux (Q_o = surface heat flux)	
q	mixing ratio of water vapor	
q_*	scaling moisture	
R_o	surface Rossby number	
R_C	critical Reynolds number	
Ri_C	critical Richardson number	
Ri_B	bulk Richardson number	

Ri	Richardson number
S_v	virtual static energy
U	horizontal wind speed
U_*	friction velocity
W_h	large-scale vertical velocity at the top of the boundary layer
W_E	entrainment velocity
W_*	mixed layer convective scale
z_o	roughness parameter
z_s	height of the top of constant flux layer
α	angle between surface stress and free atmosphere wind at the top of the boundary layer
γ_{CG}	countergradient heat flux
γ^+	lapse rate above the mixed layer
μ	stability parameter $= \frac{kU_*}{\lvert f \rvert L}$
ϕ	nondimensional wind shear
ρ	density of air
$\vec{\tau}$	stress ($\vec{\tau}_o$ = surface stress)
θ	potential temperature
θ_*	scaling temperature
θ_v	virtual potential temperature

Chapter 1

INTRODUCTION AND BACKGROUND

SIGNIFICANCE OF THE ATMOSPHERIC BOUNDARY LAYER IN ATMOSPHERIC CIRCULATIONS

As our knowledge about the larger-scale dynamics of atmosphere grows, more emphasis is being placed on the study of atmospheric boundary layer (BL) problems, with a consequent emphasis on the *interaction* between these two scales of processes. This is particularly important as the more comprehensive problem of *climate* is studied by dynamic methods. BL researchers must review their understanding in the context of this interaction, while people studying larger-scale processes must think of the BL as an inherent part of the problem. The point deserves emphasis because there has been an apparent trend to treat BL development as a closed or quasi-closed problem, and use research findings only to provide lower boundary conditions for the free atmosphere. Furthermore, many large-scale dynamicists hold the view that a sufficiently detailed model of atmosphere can generate its "own" BL, and that it is not necessary to include the BL (as a part of the troposphere) as an input to the free atmosphere. It therefore will be judicious to apply a brake to these trends and study the role of the BL in the atmospheric general circulation in a composite and comprehensive way.

It is generally accepted that the BL is one of the major energy sources and momentum sinks in the atmosphere. This layer plays a vital role in the exchange of momentum, heat, and moisture between the earth's surface and the free atmosphere. Computations by Kung (1969) suggest that almost one-half of the atmosphere's kinetic energy is lost (dissipated) in the BL, and Wilkins (1963) estimates that, in relation to the entire troposphere, 90 percent of the energy dissipation occurs in the lower km. The atmosphere also receives much of its sensible heat and virtually all of its water vapor through turbulent processes in the BL, which eventually influence the formation of weather disturbances. The role of the BL in cumulus convection associated with fronts, tropical storms, and cloud clusters has also received considerable attention.

It is generally believed that cumulus convection is primarily induced by the frictional veering of wind in the BL. The so-called CISK mechanism (Conditional Instability of Second Kind), which is also induced by frictional convergence in the BL, has been regarded as necessary for the intensification of tropical storms. Over the *oceans* the top of the BL has been observed to be close to the base of cumulus clouds. Oceanic cumuli owe their existence to subcloud layer convergence, believed to be formed by mechanical turbulence within the BL.

Turbulent motion in the BL is a very effective means for the transport of momentum, and this momentum flux from the atmosphere onto the earth has considerable influence on the evolution of weather systems. And since one major goal of meteorological research is to improve long-term (a week or more) weather forecasts and then use the new techniques to study *climatic changes*, it is imperative that we acquire a better understanding of BL dynamics, including energy dissipation and all fluxes. Currently, long-term forecasts are based on subjective (synoptic) methods and *operational* numerical models. The former tend to become unreliable after 2 to 3 days, and the latter leave much to be desired after 5 days or more. These deficiencies are directly related to the relative time scales of energy exchanges within the troposphere vis-à-vis turbulent fluxes through the BL. For example, the reaction time of the atmosphere to *turbulent* fluxes of heat and moisture is about 3 days, to kinetic energy dissipation or to latent heating about 1 day, and to radiative fluxes about one week. In view of the significant influence of non-adiabatic effects, which are related to turbulent fluxes of heat and moisture through the BL, we can expect significant improvements in long-range forecasting for two days or more only if BL processes are well understood.

Even though the most important energy transactions take place at the earth's surface, we cannot simply prescribe boundary conditions of surface characteristics such as roughness, temperature, and moisture, and then compute the input of energy to the atmosphere. We cannot because the atmosphere itself reacts on the surface, and therefore partly controls the boundary conditions and the energy transfers that occur there. By virtue of the interaction between small-scale and large-scale atmospheric processes, the study of turbulence in the atmospheric

BL is an essential factor in studying the physical principles of long-range weather forecasting as well as the theory of climatic change.

PURPOSES OF THE REPORT

The purposes of this report are:

- To present an up-to-date survey of the theories that have been used to parameterize the eddy fluxes of momentum, heat, and moisture, with their limitations,
- To discuss the determination of the height of the BL, considered to be the basic parameter of BL parameterization, and
- To discuss the specific problems encountered in the treatment of the BL in low latitudes and over oceans.

DEFINITIONS AND GENERAL CHARACTERISTICS OF THE BOUNDARY LAYER

The BL has been variously defined in the literature. For example, it has been defined in general terms as:

- That portion of the lower atmosphere in which the wind deviates from gradient or geostrophic flow because of the retarding influence of surface friction.
- The region adjacent to the earth's surface where small-scale turbulence is induced by wind shear and/or thermal convection and *occurs almost continuously* in space and time (Deardorff, 1972a). (Small-scale turbulence is intermittent beyond the BL.)
- The region near the surface in which turbulence, of a scale not much greater than the scale height, carries significant fluxes of heat, momentum, water, etc. (Charnock and Ellison, 1967).
- A region where surface effects remain important but are no longer completely dominant (Kraus, 1972). This definition is wider than generally suggested; it includes regions in which vertical fluxes of momentum, heat, and

moisture determine the vertical distribution of these properties. It is implied thereby that there are as many BLs as there are transported properties. However, the various fluxes are coupled with each other such that the BL usually can be represented by a *single layer* in which surface effects remain significant.

As these definitions indicate, the term "boundary layer" applies to a layer of air above the earth's surface in which significant fluxes of momentum, heat, moisture, and matter are transported by turbulent motions. This definition allows for the inclusion of transport brought about by penetrative (cloudy) convection, which, in the limit, could apply to cumulonimbus clouds. Thus turbulence *does* occur in cumulus clouds and along frontal surfaces that separate air masses. But since these features are considered to be *mesoscale* explosions (through the BL) into the free atmosphere, and are more random and sparsely scattered, the turbulence associated with them is usually excluded from BL, which (unlike the rest of the atmosphere) is continuously in turbulent motion.

In the literature the most commonly used terms describing the resolution of the BL into various layers are

- Interfacial layer,
- Surface (or constant flux) layer, and
- Ekman layer.

The *interfacial layer* is the region at the earth's surface that includes the immediate neighborhood of a land or water surface. The processes within this layer are not well understood, particularly at an air-water interface. However, considering that the fluxes through the BL must be estimated from conditions in the interfacial layer, more research is urgently needed to study it. For example, while the roughness parameter over land has been assumed to be independent of external parameters, we still do not know how to distinguish between aerodynamic roughness and topography. This knowledge, if available, would be useful for global circulation models, which have to consider land surfaces of varying complexities. The problem of specifying the roughness parameter for ocean surfaces is still more complex. In this case the roughness length *cannot* be directly associated with any geometric parameter. It has been suggested that it may be related to the slope of capillary or short capillary gravity waves interacting with the wind (Kraus, 1972). Roughness over the ocean also varies with the fetch and direction of the wind.

The surface (or constant flux) layer extends from immediately above the earth's surface to a depth of some tens of meters. As its name implies, the vertical fluxes of momentum, heat, and moisture are invariant within this layer. Though based on inconclusive observational evidence, "surface layer" has also been defined as the layer in which the vertical integral of the time variation of temperature over the layer is small (<20 percent) compared with the magnitude of the heat flux (Lumley and Panofsky, 1964). The concept of constant flux layer (CFL) is useful because it permits us to estimate the value of this flux from observations of the transported quantity at varying distances from the surface--for example, at the surface and at a level (usually anemometer level) within the CFL. In fact, this layer has been studied much more comprehensively than the BL *as a whole*, and there now exists a reasonably satisfactory description that is partly theoretical and partly observational. The transfer properties of turbulence within the CFL are better defined for near-neutral and unstable conditions than for stable conditions.

The Ekman layer is thicker than the constant flux layer and may fill either a part of or the entire BL. In a *classical* sense, it is the region where the vertical flux of momentum is of the same order of magnitude as the coriolis and the pressure gradient forces; this essentially implies a steady-state, barotropic situation. In the *real* atmosphere, however, the conditions are often infringed by synoptic scale evolution of the pressure field, as well as by the diurnal variation of the radiative heat flux, both of which alter thermal stratification and thereby affect turbulent fluxes and the wind. Nevertheless, though the classical Ekman BL is rarely observed in nature, its concept may be applicable for certain conditions such as the presence of a dense cloud cover, which greatly reduces diurnal variations in the BL.

BASIC EQUATIONS USED IN BL NUMERICAL MODELING

The ensuing description follows that given by Estoque (1973). Numerical models enable us to examine the relative importance of external factors and internal processes in determining the behavior of the planetary BL. The external factors or parameters that must be considered are the large-scale synoptic conditions and the properties of the underlying surface of the earth. The internal processes are the various transport processes, the most important being the eddy transport along the vertical. In general, the large-scale synoptic condition is specified only in terms of the geostrophic wind, $G(z)$ (or the corresponding large-scale horizontal pressure gradient ∇p_L); in some cases it is necessary to specify the large-scale distributions of potential temperature, $\theta_L(z)$, and the mixing ratio, $q_L(z)$. Terrain properties considered are the roughness parameter (z_o), the temperature (T_o), and the mixing ratio (q_o), all of which may vary along the horizontal.

The basic problem in numerical modeling is to determine the space-time variations of the variables (wind, temperature, and moisture) that describe the BL as functions of the different external parameters. So far, generally, the two-dimensional problem (no dependence along one horizontal coordinate) has been treated more extensively than the three-dimensional problem. In mathematical terms, a rather general formulation of the two-dimensional problem is as follows:

GIVEN: $G(z)$ or ∇p_L, $\theta_L(z)$, $q_L(z)$

$z_o(x)$, $T_o(x, t)$, $q_o(x, t)$

TO FIND: $\vec{V}(x, z, t)$, $w(x, z, t)$, $\theta(x, z, t)$

$T(x, z, t)$, $p(x, z, t)$, $q(x, z, t)$

Here, the subscript "L" denotes the value associated with the large-scale synoptic flow patterns, while the subscript "o" refers to the value at the earth surface. Each unknown variable, which may be regarded as the sum of the large-scale value plus a perturbation induced by BL processes, must be calculated in the layer between the surface and a height h. Ideally, the value of h, which we define to be the height of the BL, should be infinitely large. For most practical purposes, however, it is sufficient to assume that h is of the order of one kilometer.

The set of two-dimensional equations that must be solved are:

$$\frac{\partial u}{\partial t} = - u \frac{\partial u}{\partial x} - w \frac{\partial u}{\partial z} + fv - \frac{1}{\rho} \frac{\partial p}{\partial x} + \frac{1}{\rho} \frac{\partial \tau_x}{\partial z} \tag{1.1}$$

$$\frac{\partial v}{\partial t} = - u \frac{\partial v}{\partial x} - w \frac{\partial v}{\partial z} - fu - \frac{1}{\rho} \frac{\partial p}{\partial y} + \frac{1}{\rho} \frac{\partial \tau_y}{\partial z} \tag{1.2}$$

$$\frac{\partial \theta}{\partial t} = - u \frac{\partial \theta}{\partial x} - w \frac{\partial \theta}{\partial z} + \frac{\partial}{\partial z} (F_\theta) + S_\theta \tag{1.3}$$

$$\frac{\partial q}{\partial t} = - u \frac{\partial q}{\partial x} - w \frac{\partial q}{\partial z} + \frac{\partial}{\partial z} (F_q) + S_q \tag{1.4}$$

$$\frac{\partial u}{\partial x} + \frac{\partial w}{\partial z} = 0 \tag{1.5}$$

$$\frac{\partial p}{\partial z} = - \frac{pg}{RT} \tag{1.6}$$

$$\theta \equiv T \left(\frac{p_o}{p} \right)^{R/C_p} \tag{1.7}$$

These equations are based on the assumption that variations of dependent variables along y are zero. Most of the notation is customary. The quantities τ_x, τ_y, F_θ, and F_q represent the eddy fluxes of momentum, heat, and water vapor; S_θ and S_q are the internal sources of heat and water vapor. The density in the pressure gradient term is assumed to be constant. If the eddy fluxes and the internal sources are specified in terms of the unknown variables, then the equations form a closed set.

Boundary conditions are to be specified at the earth surface (assumed to be effectively at $z = z_o$), at the upper boundary, and at the lateral boundaries. The usual lower boundary conditions are as follows:

$$z = z_o: \quad u = v = w = 0$$

$$T = T_o(x, t) \quad \text{or} \quad \theta = \theta_o(x, t)$$

$$q = q_o(x, t)$$

The specification of the lateral and the upper boundary conditions is arbitrary to some extent, and may depend on the particular problem being considered.

Initial conditions involve specifying $\vec{V}(x, z)$, $\theta(x, z)$, and $q(x, z)$ at time $t = 0$.

Chapter 2

APPLICATION OF K-THEORY TO BOUNDARY LAYER PARAMETERIZATION

Atmospheric perturbations depend on spatial and temporal variations of heat and moisture flux from the underlying surface. The relevant transports across the BL are carried by finite-amplitude, three-dimensional perturbations. Since it is not practicable to deal simultaneously with BL perturbations and the large-scale circulations of the atmosphere, the former effects therefore are expressed as functions of *parameters*, which in turn can be determined from a knowledge of large-scale motions. According to K-theory, this can be achieved by replacing molecular diffusivity with a stipulated variable *eddy viscosity*, K, in the classical laminar flow equations.

For parameterizing the BL (as per K-theory concepts), the atmosphere is usually divided into two layers: (a) a surface (or constant flux) layer and (b) the so-called Ekman layer. Parameterization thus involves first modeling the fluxes at the surface, and then the turbulent diffusion of properties (from the top of the surface layer) through the upper Ekman layer.

TREATMENT OF THE SURFACE LAYER

The surface layer usually extends from the top of roughness elements ($z = z_o$) to the top of the constant flux layer ($z = z_s$), which usually represents the *lowest* explicit model level. The quantity z_s can take on values up to a few tens of meters; it is usually made to coincide with anemometer level (~ 10 m) and is kept fixed. Estoque (1963) used a value as large as 50 m for z_s. However, Sasamori (1970) used variable depths for the constant flux layer from the formula:

$$(z_s)^2 \propto T_M(K)_{z_s} \qquad (2.1)$$

where T_M is the response time and K is the diffusion coefficient at $z = z_s$; the formulation shows that depth is large when the intensity of transport is strong.

The fundamental equations within the layer of thickness z_s are:

$$\left.\begin{aligned}\frac{\partial}{\partial z}\left(K_M \frac{\partial U}{\partial z}\right) &= 0 \\ \frac{\partial}{\partial z}\left(K_H \frac{\partial \theta}{\partial z}\right) &= 0 \\ \frac{\partial}{\partial z}\left(K_E \frac{\partial q}{\partial z}\right) &= 0\end{aligned}\right\} \tag{2.2}$$

where K_M, K_H, K_E are, respectively, eddy diffusion coefficients of momentum, heat, and moisture. Businger et al. (1971), Yamamoto and Shimanuki (1966), and Estoque and Bhumralkar (1969) have given formulations that determine the velocity, temperature, and moisture profiles in the surface layer over a wide range of diabatic conditions. These have been used in various studies of the atmospheric BL, both local as well as general circulation. Notable among these are Sasamori (1970) (microscale), Delsol et al. (1971) (GCM), Deardorff (1972a) (GCM), and Pielke (1974) (mesoscale sea breeze). Formulations by Businger et al. (1971) are based on a large number of direct measurements of both heat and momentum fluxes under both stable and unstable conditions. Their formulations are:

$$\left.\begin{aligned}&\frac{kz}{U_*}\frac{\partial \overline{U}}{\partial z} = (1 - 15\xi)^{-\frac{1}{4}} \\ \text{and} \quad &\\ &\frac{kz\ U_*}{(U_*\theta_*)}\frac{\partial \overline{\theta}}{\partial z} = \frac{K_M}{K_H}(1 - 9\xi)^{-\frac{1}{2}}\end{aligned}\right\} \text{unstable case} \tag{2.3a}$$

and

$$\left.\begin{aligned}&\frac{kz}{U_*}\frac{\partial \overline{U}}{\partial z} = (1 + 4.7\xi) \\ &\frac{kz\ U_*}{(U_*\theta_*)}\frac{\partial \overline{\theta}}{\partial z} = \frac{K_M}{K_H} + 4.7\xi\end{aligned}\right\} \begin{array}{l}\text{stable case} \\ \\ U_*\theta_* \text{ may be replaced} \\ \text{by } (-\overline{w'\theta'})\end{array} \tag{2.3b}$$

The ratio K_M/K_H equals 0.74 and is based on observations of Businger et al. (1971). In the above expressions,

$$\xi = \frac{(z + z_o)}{L} \quad \text{where} \quad L = -\frac{U_*^3}{k\,\frac{g}{\theta}\,(U_*\theta_*)} \, .$$

Deardorff (1972a) suggests that Eqs. (2.3a) and (2.3b) are more convenient to integrate in terms of familiar functions than the so-called KEYPS (Lumley and Panofsky, 1964) formulation.

Yamamoto and Shimanuki (1966) developed velocity and temperature profiles in the surface layer under various stability conditions. The momentum exchange coefficient K_M near the ground is given by

$$K_M = \frac{kU_* z}{\phi_i(|\xi|)} \tag{2.4}$$

where ϕ_i is a nondimensional wind shear given by

$$\phi_i = \frac{kz\,\frac{\partial \overline{U}}{\partial z}}{U_*} \quad \left\{ \begin{array}{l} i = 1 \text{ or } 2 \\ \text{in all cases} \end{array} \right\} \quad \left\{ \begin{array}{l} \phi_1 \sim \textit{unstable} \text{ conditions} \\ \phi_2 \sim \textit{stable} \text{ conditions} \end{array} \right. \tag{2.5}$$

ϕ_i is a function of nondimensional height, ξ, given by

$$|\xi| = \frac{z}{|L_*|}$$

where

$$L_* = \frac{-\,U_*^3}{\frac{\sigma k g}{\overline{\theta}}\,(U_*\theta_*)} \equiv \left[\frac{-\,\theta_o U_*^2}{\sigma k g \theta_*}\right] \tag{2.6}$$

$\overline{\theta} \equiv$ mean θ for the layer

σ = empirical constant = (15 ± 3) .

The values of U_* and θ_* are determined from

$$\left.\begin{aligned} U_* &= \frac{kU}{(G_i(|\xi|) - G_i(|\xi_o|))} \\ \text{and} \qquad \theta_* &= \frac{k(\theta - \theta(z_o))}{(G_i(|\xi|) - G_i(|\xi_o|))} \end{aligned}\right\} \begin{array}{l} i = 1,2 \text{ in} \\ \text{all cases} \end{array} \qquad (2.7)$$

where G_1 and G_2 represent profile functions obtained as a function of $|\xi|$: i.e., $U(=\sqrt{u^2 + v^2})$. The nondimensional height $|\xi|$ is given by

$$|\xi| = \frac{z_o}{|L_*|} .$$

The values of ϕ_i and G_i for particular values of $|\xi|$ have been tabulated by Yamamoto and Shimanuki (1966), and relevant values for ϕ_i and G_i can be obtained from interpolation formulas developed by Shimanuki (1969). The relative error, using these approximate relationships, does not exceed 0.4 percent for unstable and 0.6 percent for stable conditions.

The formulations for moisture profile in the surface layer are analogous to those for temperature. The diffusion coefficients for θ and q in surface layer can be obtained from K_M by

$$K_H = K_E = \beta K_M$$

where the variable β (taken as 1.35 by Businger et al. (1971)) may be obtained in terms of dimensionless wind shear from

$$\beta = \frac{1}{\phi_1(|\xi|)} \qquad \text{for neutral and unstable cases}$$

and

$$\beta = 1 \qquad \text{for stable cases} .$$

Estoque and Bhumralkar (1969) computed the wind, temperature, and moisture profiles in the surface layer from the following equations:

$$\left.\begin{aligned} U_z &= \frac{U_*}{k}\left\{\ln\left(\frac{z + z_o}{z_o}\right)(1 + \alpha_c S)\right\} \\ \theta_z &= \theta_o + \frac{\theta_*}{k}\left\{\ln\left(\frac{z + z_o}{z_o}\right)(1 + \alpha_c S)\right\} \\ q_z &= q_o + \frac{q_*}{k}\left\{\ln\left(\frac{z + z_o}{z_o}\right)(1 + \alpha_c S)\right\} \end{aligned}\right\} \qquad (2.8)$$

where S is the stability parameter,

$$\frac{\sqrt{gz}}{\theta}\,\frac{\theta_*}{U_*}/\ln\left(\frac{z + z_o}{z_o}\right),$$

α_c is an empirical constant, and the other variables have usual meaning. These expressions incorporate the effect of thermal stratification somewhat differently than does the so-called KEYPS formulation (Lumley and Panofsky, 1964) and appear to fit empirical data better over a wider range of diabatic conditions.

In all the formulations described above, U_*, θ_*, q_*, the integration constants, are related to the *fluxes through the surface layer*, which are given by

$$K_M \frac{\partial U}{\partial z} = U_*^2$$

$$K_H \frac{\partial \theta}{\partial z} = U_* \theta_*$$

$$K_E \frac{\partial q}{\partial z} = U_* q_*$$

And since the surface layer is characterized by constant fluxes in the vertical, these are also fluxes at $z = z_s$, the top of the surface layer.

In all the preceding formulations, z_o over land surfaces should be presented realistically as a function of essentially aerodynamic roughness. The values of 20 to 70 cm for most land surfaces have been suggested by Fielter et al. (1971). However, *over water*, z_o cannot be associated in any direct way with the mean surface height or some geometric parameter. Charnock (1955) has suggested that z_o is a function of surface stress and thus

$$z_o = \frac{mU_*^2}{g}, \tag{2.9}$$

m being a proportionality constant. Clark (1970) used this formula to obtain z_o by using m = 0.032, and he stipulated that

$$(z_o)_{water} \geq 0.0015 \text{ cm} \tag{2.10}$$

One may use a graph compiled by Deacon and Webb (1962) to determine a functional form of z_o over water.

TREATMENT OF THE EKMAN LAYER

Specific detailed formulation of K for the portion of the BL above the surface layer has been suggested, among others, by Estoque (1963), Yamamoto and Shimanuki (1966), Deardorff (1967), Zilitinkevich et al. (1967), Estoque and Bhumralkar (1969), and O'Brien (1970).

All K formulations involve the assumption that turbulent transports are proportional to the gradient of the transported properties. For example, K for momentum for a *neutral* condition is given by

$$K_M = \ell^2 \left[\left(\frac{\partial u}{\partial z}\right)^2 + \left(\frac{\partial v}{\partial z}\right)^2 \right]^{\frac{1}{2}} \tag{2.11}$$

where ℓ is the so-called mixing length, and u and v are horizontal components of wind. In the following discussions, K_M and K_H (which are not independent quantities) are considered to be related to each other through the following relation suggested by Businger et al. (1971):

$$\frac{K_H}{K_M} = 1.35 .$$

However, because there is considerable controversy about this value, it should be treated as tentative. Various authors have discussed the vertical distribution of K. Whereas well-known and accepted results are available for specifying K(z) in the surface layer, the prescription of K in the Ekman layer is not quite clear, particularly for numerical models in which a large range of stability is expected. It is generally assumed that, within the Ekman layer, K increases to a certain maximum and then decreases in the upper portions. Estoque (1963) used this type of vertical distribution for K in a successful numerical study of the BL. His results compared very favorably with observations of the Great Plains Field Program. At the top of the surface layer (50 m), K was estimated from the results of surface-layer formulations; above this it was assumed to decrease linearly and reach zero at the top of model atmosphere. The main drawback of this profile for K (in the Ekman layer) is that it fails to take into account any variable factors, such as lapse rate and wind shear, which can affect K. Yamamoto and Shimanuki (1966) suggest that K (given by Eq. (2.4)) holds throughout the Ekman layer and thus

$$
\begin{aligned}
K &= k^2z^2\left(\frac{\partial U}{\partial z} + \sqrt{\frac{g}{\theta}\left|\frac{\partial\theta}{\partial z}\right|}\right) && \text{for unstable conditions} \\
&= k^2z^2\left(\frac{\partial U}{\partial z} - \left(\frac{L_*}{z}\right)^p\sqrt{\frac{\sigma g}{\theta}\left|\frac{\partial\theta}{\partial z}\right|}\right) && \text{for stable conditions}
\end{aligned}
\tag{2.12}
$$

(See Eq. 2.6.)

Here σ is an empirical constant (Eq. 2.6) and $p = 1/6$ is another empirical constant introduced by Yamamoto and Shimanuki. Estoque and Bhumralkar (1969) have suggested the following generalized version of an expression devised by Blackadar (1962):

$$
K = \begin{cases} \ell^2 \dfrac{\partial U}{\partial z}(1 - \alpha_c S) & \text{for unstable case} \\ \ell^2 \dfrac{\partial U}{\partial z}(1 + \alpha_c S)^{-1} & \text{for stable case} \end{cases}
\tag{2.13}
$$

where

$$\ell = \frac{k(z + z_o)}{1 + \dfrac{kf(z + z_o)}{27 \times 10^{-5}\, U_{(z_s)}}} \equiv \frac{k(z + z_o)}{1 + \dfrac{k(z + z_o)}{\lambda}} \tag{2.14}$$

and S, the stability parameter,

$$\frac{(g\ell)^{\frac{1}{2}}}{\theta} \frac{\frac{\partial\theta}{\partial z}}{\left|\frac{\partial U}{\partial z}\right|} ;$$

$$\frac{\partial U}{\partial z} = \left[\left(\frac{\partial u}{\partial z}\right)^2 + \left(\frac{\partial v}{\partial z}\right)^2\right]^{\frac{1}{2}} ;$$

α_c is an empirical constant (see Eq. (2.8)). Zilitinkevich et al. (1967), on the basis of dimensional argument, suggest that

$$K = \ell^2 \left[\left(\frac{\partial u}{\partial z}\right)^2 + \left(\frac{\partial v}{\partial z}\right)^2 - 1.3\, \frac{g}{T} \frac{\partial\theta}{\partial z}\right]^{\frac{1}{2}} \tag{2.15}$$

where

$$\ell = \frac{kz}{1 + \frac{kz}{\lambda}} ; \quad \lambda(\text{the asymptotic value of } \ell \text{ at large } z) = \frac{U_g}{f \times 3.7 \times 10^3} \tag{2.16}$$

It may be noted that the expressions for ℓ in Eqs. (2.14) and (2.16) are similar except for some slight variations in the constant. It is also clear that the effect of stratification on K (Eqs. (2.13) and (2.15)) arises primarily from the buoyancy term and not from ℓ, which is for the neutral case.

O'Brien (1970) proposes a functional form of the eddy coefficient K. The values of K and their derivatives, with respect to z_s (height of top of surface layer) and h (height of BL), are used to derive a Hermite-interpolating (third order) polynomial given by

$$K(z) = K(h) + \left(\frac{h - z}{h - z_s}\right)^2 \left[K(z_s) - K(h) + (z - z_s)\left\{\left(\frac{\partial K}{\partial z}\right)_{z_s} + \frac{2(K(z_s) - K(h))}{h - z_s}\right\}\right] \quad (2.17)$$

and analogous formulations for K for θ and q. Here it is assumed that

$$\left(\frac{\partial K}{\partial z}\right)_h = 0 \qquad \text{i.e., the eddy flux divergence is zero.}$$

The values of K_M and K_H at z_s, the top of the surface layer, are determined from

$$K_M(z_s) = \frac{kU_* z_s}{\phi_1(|\xi_{z_s}|)} \qquad \text{(from Eq. (2.4))}$$

and

$$K_H(z_s) = \begin{cases} \dfrac{K_M(z_s)}{\phi_1(|\xi_{z_s}|)} & \text{for unstable case} \\ K_M(z_s) & \text{for stable case.} \end{cases}$$

Here nondimensional height $|\xi_{z_s}|$ is given by

$$|\xi_{z_s}| = \frac{z_s}{|L_*|},$$

where L_* is given by Eq. (2.6).

The value of the diffusion coefficient at the top of the BL (h) may be assumed to be zero (Sasamori, 1970) or be arbitrarily specified (Pielke, 1974). This is done essentially to prevent significant turbulent mixing above h. Deardorff (1971) found that K_M obtained by using Eq. (2.17) gave too large values at intermediate heights compared with the values in the surface layer.

EFFECT OF THERMAL WIND AND STABILITY ON WIND PROFILE IN THE BL

Among other assumptions, the formulations discussed above assume that the large-scale pressure gradient is constant with height. Blackadar (1965), Venkatesh and Csanady (1974), and others have modified the barotropic K-theory to include the more commonly prevailing baroclinic conditions. According to Blackadar (1965), to incorporate thermal wind effects it usually suffices to consider pressure gradient, which varies linearly with height. In terms of geostrophic wind, this implies that

$$\vec{G} = \vec{G}_o + z\vec{T}_H$$

where $\vec{G}_o$ is the surface geostrophic wind and $\vec{T}_H$ is the vector thermal wind.

Blackadar (1965) and Blackadar and Ching (1965) have obtained numerical solutions of BL equations for the following conditions:

(i) neutral and barotropic
(ii) *stratified* and barotropic, and
(iii) neutral and *baroclinic*.

Figures 1 and 2 show comparative results for (i) and (ii). One can see that instability *increases* the surface stress and decreases the cross-isobar inflow angle of the surface wind. Figures 3 and 4 show comparative results for (i) and (iii). The differences can be attributed to distortions in the wind spiral (Fig. 5) produced by the thermal wind. The effects of stratification are more pronounced than those due to thermal wind, e.g., the decrease in angle α (from neutral condition) due to unstable stratification is much greater than that produced by thermal wind (parallel to surface geostrophic wind). Also, the increase in surface stress due to unstable stratification is relatively greater than that due to thermal wind alone.

GENERAL COMMENTS

Questions have been raised about the treatment of the atmospheric BL as consisting of the surface (constant flux) layer and the Ekman

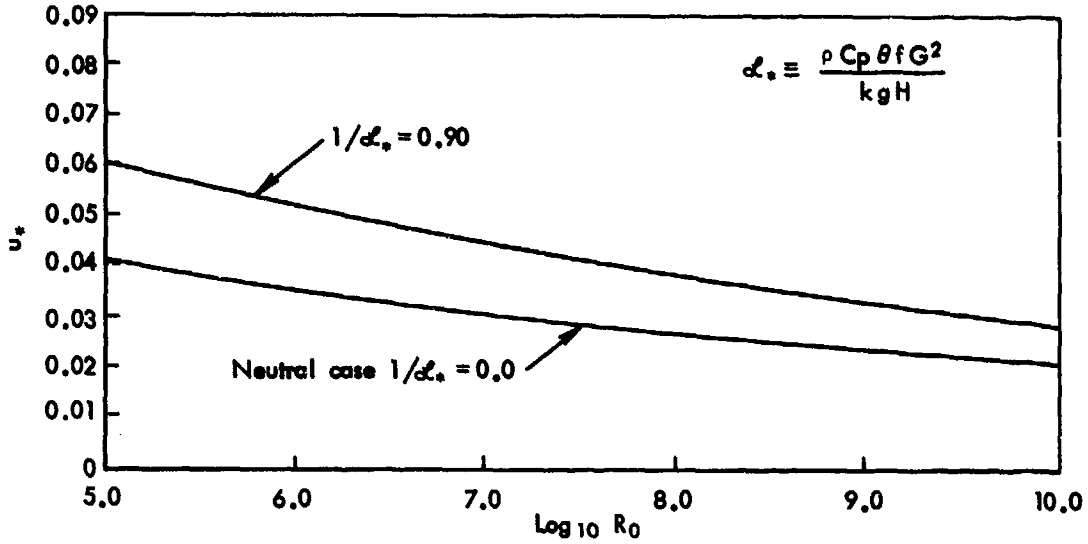


Fig. 1— Graph of u_* versus $\log_{10} R_0$ for the barotropic solution. Effect of instability is to increase the surface stress (Blackadar and Ching, 1965)

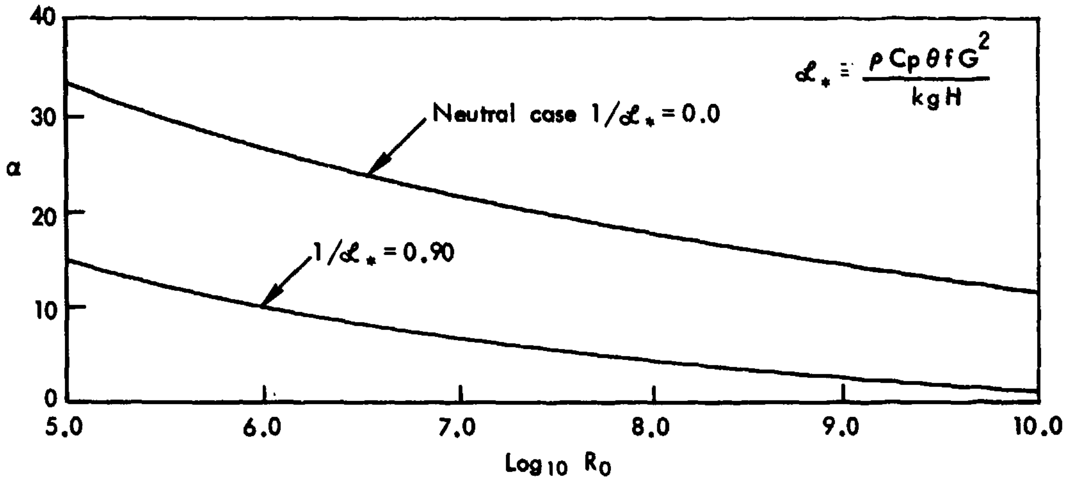


Fig. 2— Graph of α versus $\log_{10} R_0$ for the barotropic solution. Effect of instability is to decrease the cross-isobar angle (Blackadar and Ching, 1965)

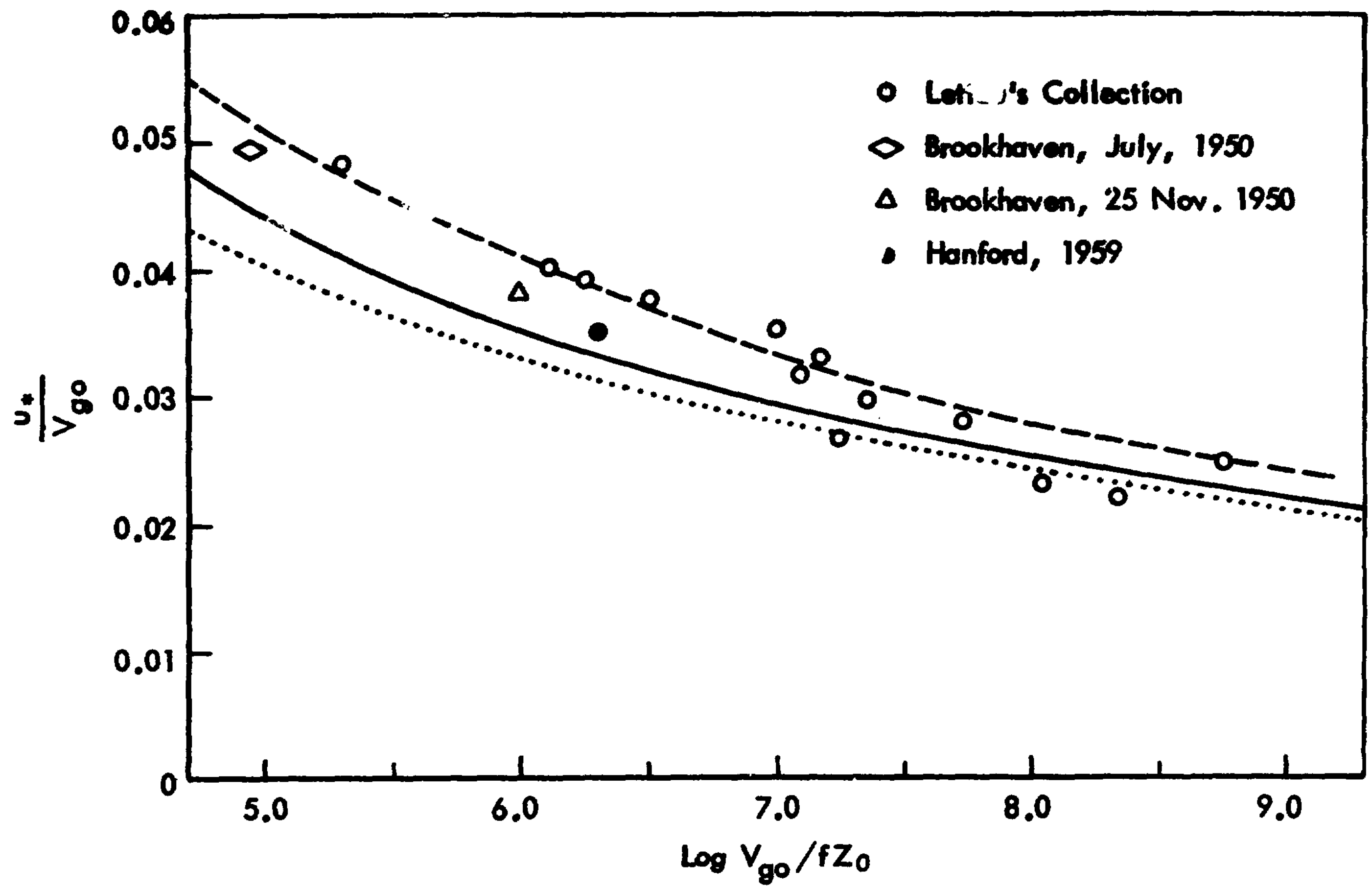
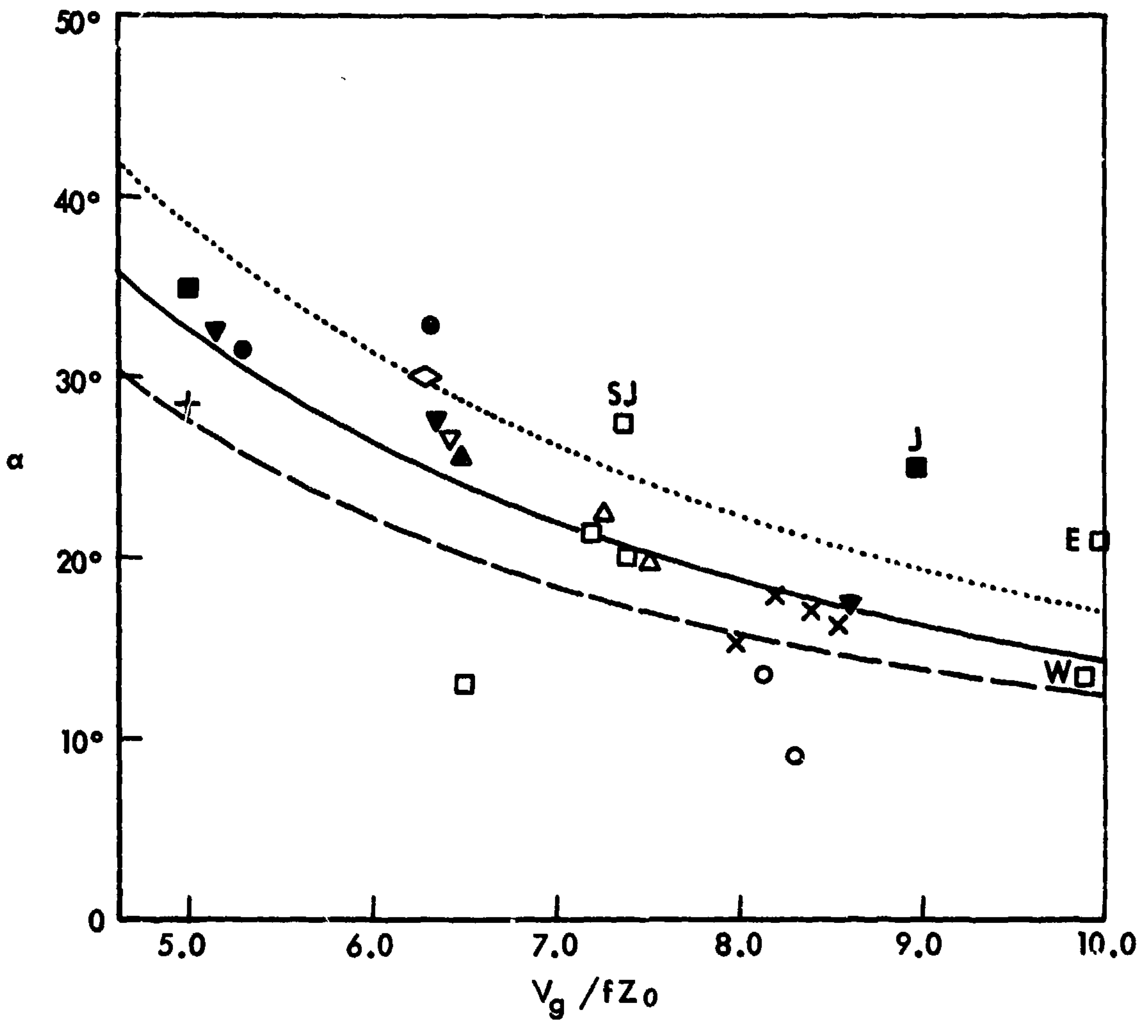


Fig. 3 — Ratio of U_* to surface geostrophic wind speed as function of Surface Rossby Number. Solid curve applies to barotropic neutral boundary layer. Baroclinic curves are also shown for thermal wind parallel (dashed) and opposite (dotted) to the surface geostrophic wind (Blackadar, 1965)



▲ Leipzig (Lettau)
△ O'Neill, neutral
□ Upavon (Dobson)
○ Scilly Is. (Lettau)
■ Sheppard and Omar
E = Eniwetok Is.
W = Wake Is.
J = Johnson Is.
SJ = San Juan

× North Sea (Jeffries)
● Brookhaven, neutral (Bernstein)
⊠ Brookhaven, baratropic (Bernstein)
+ Brookhaven, baroclinic, A = 0.005 (Bernstein)
▽ U.S. Great Plains, neutral (Bernstein)
● U.S. Great Plains, all cases, barotropic
◇ U.S. Great Plains, all cases, A = 0.005 (Bernstein)
▼ Haltiner and Martin, neutral

Fig. 4 — Angle α between the surface wind direction and surface geostrophic wind direction. Solid curve is the computed relation for a barotropic neutral layer. Curves are also shown for thermal wind parallel (dashed) and opposite (dotted) to the surface geostrophic wind (Blackadar, 1965)

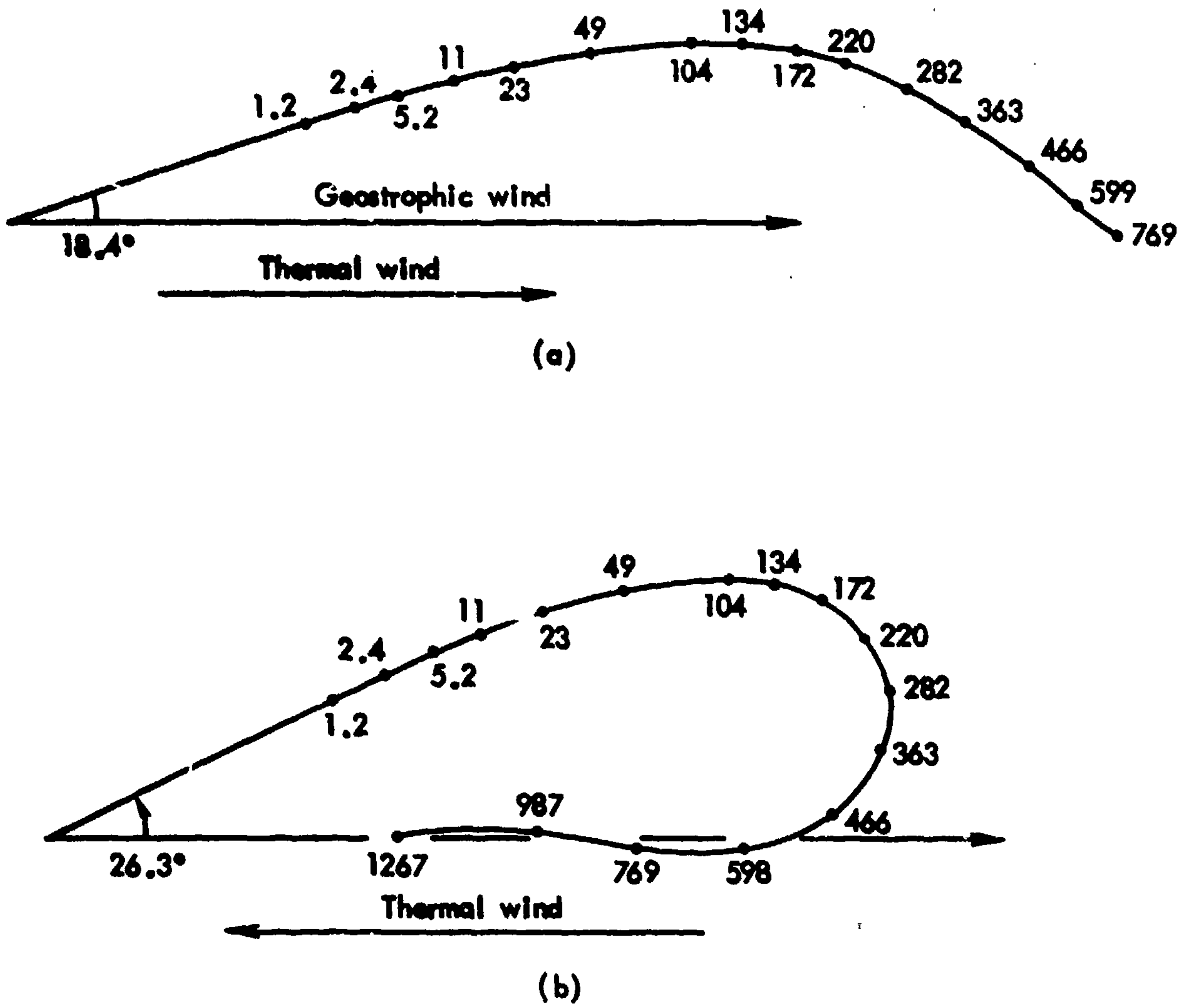


Fig. 5 — Wind hodographs for thermal wind parallel to and
(a) in the same direction as the surface geostrophic wind
(b) opposite to the surface geostrophic wind
(Blackadar, 1965)

layer. Some authors have suggested that the BL be treated as a "single *entity*" (Kraus, 1972; Deardorff, 1973). As a consequence, some recent workers have developed formulations that are applied to the entire depth of the modeled atmospheric BL, instead of using different parameterizations for different regions above the surface. Orlanski et al. (1974) have presented a simple formulation for K that simulates turbulent transfer processes over different layers. Their formulation for eddy coefficient is given by

$$K = \begin{cases} K_o \left[1 + C \left(\dfrac{-g\Delta\theta(\Delta z)^3}{\theta K_o \nu_o} \right) \right]^{1/3} & \text{if } \Delta\theta < 0 \text{ (unstable condition)} \\ K_o & \text{if } \Delta\theta > 0 \text{ (stable condition)} \end{cases}$$

Here, K_o, ν_o are constant values of eddy diffusion coefficient and viscosity, respectively, and $\Delta z, \Delta\theta$ are, respectively, local values of the vertical grid size and potential temperature difference across the grid box. Thus the above formulation has no explicit dependence upon velocity deformation as do those used by Estoque (1963) and others. Here it is assumed that turbulence in stratified fluids is produced only by (i) convective instability if the mean stratification is unstable, or by (ii) wave breaking for mean stable stratification. In other words, it is implied that subgrid turbulent fluxes occur only at those locations in the model where local gravitational instability is produced by "resolved" flow processes of the model. Although this formulation does not involve deformation explicitly, it can still model turbulence generation caused by shear. Also, numerical experiments by Orlanski et al. (1974) have shown that, in general, there is no constancy of heat flux with height for the lower 50 m of the atmosphere, throughout a daytime simulation. They suggest defining "surface layer" as the layer in which vertical integral of the time variation of temperature over the layer is small compared with the magnitude of heat flux. They did find, however, that a constant flux layer can be justified for a few hours (2 to 3) before sunset. They also found that the height of the constant flux layer reaches a minimum after sunrise and sunset and a sharp maximum (~ 500 m) late in the afternoon.

Yamamoto et al. (1968) have also sought a formulation applicable to the entire BL. As described by Yamamoto and Shimanuki (1966), the eddy coefficient K is given by

$$K = \frac{kU_* z}{\phi} \tag{2.18}$$

and ϕ the nondimensional wind shear by

$$\phi^4 - |\xi|^{1-2p}\,\phi^3 - 2\phi^2 + 1 = 0$$

where

$$\xi = \frac{\sigma k \frac{gz}{\theta_o} (U_* \theta_*)}{U_*^3} .$$

It may be noted that both the formulations for K and ξ involve friction velocity U_* and heat flux $U_*\theta_*$, which are independent of height. Since this situation cannot be justified observationally in the upper Ekman layer, Yamamoto et al. (1968) have suggested that if U_* is replaced by $(\tau/\rho)^{\frac{1}{2}}$ in Eq. (2.18) and $U_*\theta_*$ is replaced by a prescribed variable heat flux $Q = -\rho C_p K\, \partial\theta/\partial z$, one can apply the resulting formulations to the entire BL, as in Eqs. (2.19) and (2.20):

$$K = kz \left|\frac{\vec{\tau}}{\rho}\right|^{\frac{1}{2}} \frac{1}{\phi} \tag{2.19}$$

and

$$\xi = \frac{\sigma k \frac{gz}{\theta_o} \frac{Q}{\rho C_p}}{\left|\frac{\vec{\tau}}{\rho}\right|^{3/2}} \tag{2.20}$$

<u>LIMITATIONS OF K-THEORY</u>

Though still widely used, various problems are associated with K-theory. Some of these are:

1. Equations (2.19) and (2.20) imply essentially that the surface layer similarity theory has been extrapolated to cover the entire BL. However, since shear generation of Reynold's stresses can become locally insignificant, it is not always advisable to modify K-theory for the entire BL. For example, observations have indicated that shear generation is important in *stably* stratified low-level flows, but there are also indications (Lenchow, 1970) that it can vanish in a strongly heated BL. Consequently, stresses in such unstable conditions may be almost independent of small local mean wind shear. In such cases, K can become extremely large and/or negative aloft in the BL (Deardorff, 1972b).

2. Another important drawback of K-theory is that it allows only down-gradient transports, which they generally are not when flux is effected by finite amplitude, relatively large-scale eddies. Thus the low-level jet--the velocity maximum within the BL--cannot be explained by any K-type theory.

3. There is ample evidence that under conditions of upward heat flux the lapse rate within the BL is slightly less than adiabatic, i.e., the heat flux is countergradient; and Lenchow (1972) has demonstrated that turbulent momentum fluxes in a heated baroclinic layer may be locally countergradient above a region where mean wind reaches a maximum. Deardorff (1972c) has derived a theoretical expression for counter-potential temperature gradient that can sustain an upward heat flux. He suggests that heat flux Q may be formulated as

$$Q = -\rho C_p K_H \left(\frac{\partial\overline{\theta}}{\partial z} - \gamma_{CG}\right)$$

rather than the usual form,

$$Q = -\rho C_p K_H \frac{\partial\overline{\theta}}{\partial z}$$

Here γ_{CG} is a small positive quantity that allows a countergradient upward heat flow. His findings are mainly based on observations of Bunker (1956), Telford and Warner (1964), Warner (1971), and Lenschow (1970).

Chapter 3

SIMILARITY THEORY OF THE WHOLE BOUNDARY LAYER

GENERAL DISCUSSION

When K theories were found rather wanting in BL studies, attention turned to determining a direct relationship between the geostrophic wind and the surface stress (or friction velocity, U_*) without reference to any hypothetical eddy coefficient. These efforts led to the now well-known "similarity theory," which has been used extensively in BL studies, e.g., Lettau (1959), Kazansky and Monin (1960), Zilitinkevich et al. (1967), Csanady (1972). Gill (1968), Blackadar and Tennekes (1968), and others.

The similarity theory essentially recognizes that the details of the constant stress region near the surface cannot be modeled by the same length scale that characterizes the important features of the planetary BL. In other words, according to similarity theory the turbulent BL has a distinct double structure (Fig. 6), consisting of an "inner" (constant stress) layer and an "outer" (Ekman) layer (Clauser 1956).

The similarity theory is based on the hypothesis that, for horizontally homogeneous and stationary conditions, the turbulent regime

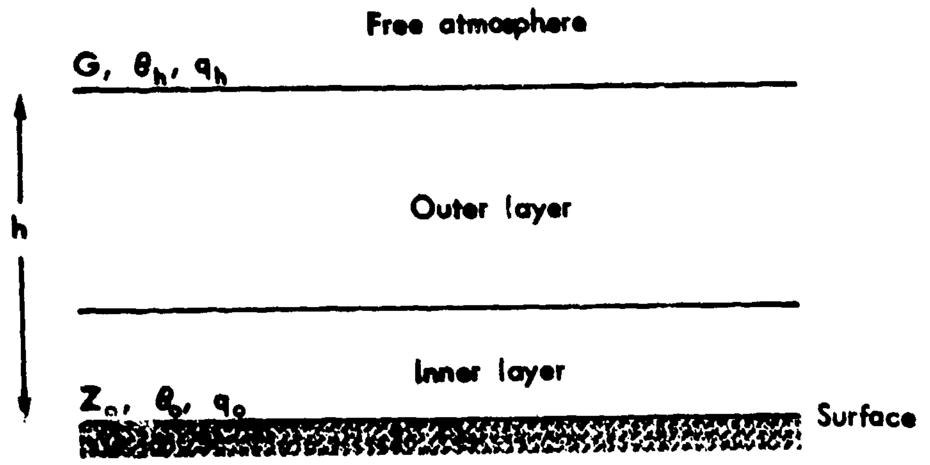


Fig. 6 — Characteristics of the boundary layer

is unambiguously defined by the values of the following "*external*" variables (so called because each is characteristic of the location and is external *relative* to the regime in the boundary layer):

z_o = rougnness parameter of the underlying surface

f = coriolis parameter

G = geostrophic wind

$\frac{g}{\theta}$ = buoyancy parameter

$\Delta\theta \equiv \theta_h - \theta_o$ = difference of potential temperature values at the upper and lower limits of the boundary layer.

The *internal* characteristics of the boundary layer are:

friction velocity: U_*

(or shear stress at surface): τ_o

angle included by τ_o and G_o: α_o

the surface heat flux: Q_o (or θ_*) and

stratification parameter: μ.

Each of the *external* variables is separately a function of all the *internal* variables. These functional relationships are best formulated in terms of the following nondimensional parameters:

angle α_o between surface stress τ_o and G

geostrophic drag coefficient $C_G = \frac{\tau_o}{\rho G^2} = \left(\frac{U_*}{G}\right)^2$

geostrophic heat transfer coefficient $C_H = -\frac{Q_o}{\rho C_p G\Delta\theta}$.

The five external parameters z_o, f, G, g/θ, and $\Delta\theta$ are used to form two nondimensional groups of variables such as

$$\text{Surface Rossby Number} \equiv R_o = \frac{G}{fz_o}$$

and

$$\text{Stability parameter } S = \frac{g}{\theta}\frac{\Delta\theta}{fG}$$

Empirical evidence (Deacon and Webb 1962) indicates that water vapor transfer proceeds by much the same physical mechanism as heat transfer. Thus the geostrophic moisture transfer coefficient may be written as

$$C_E = -\frac{E_o}{\rho G \Delta q}$$

where E_o is the surface moisture flux, and

$\Delta q = (q_h - q_o)$ is the difference of mixing ratio at the upper and lower limits of the BL.

SIMILARITY THEORY AND PARAMETRIC RELATIONS

According to similarity theory, the dimensionless velocity and scalar fields in the BL should be some *universal* functions of

$$\text{a nondimensional height } \xi = \frac{z|f|}{U_*}$$

$$\text{and a nondimensional stability parameter } \mu = \frac{kU_*}{|f|L}$$

where k = von Karman constant

$$L = \text{Obukhov length} = -\frac{U_*^3}{k\left(\frac{g}{\theta}\right)U_*\theta_*}$$

The above similarity parameters are essentially chosen on the assumption that, under all conditions of stability, BL height (h) is proportional to the height scale U_*/f. The constant of proportionality is considered to be uniquely determined by the stability parameter μ.

Let us scale the inner layer by z_o and the outer layer by U_*/f. Let us also assume that the mean velocity, temperature, and moisture, together with their vertical derivatives, can be expressed equally well by the functions appropriate to either the inner or outer region. Also assuming horizontally homogeneous and stationary conditions, and after matching the similarity profiles in a common overlapping region, one can get the following relations:

$$\begin{aligned}
\frac{U_{go}}{U_*} &= \frac{1}{k}\left[\ln\frac{U_*}{|f|z_o} - A(\mu)\right] \\
\frac{V_{go}}{U_*} &= -\frac{B(\mu)}{k}\,\text{sign } f \\
\frac{\Delta\theta}{\theta_*} &= \frac{1}{k\alpha_H}\left[\ln\frac{U_*}{|f|z_o} - C(\mu)\right] \\
\frac{\Delta q}{q_*} &= \frac{1}{k\alpha_H}\left[\ln\frac{U_*}{|f|z_o} - D(\mu)\right]
\end{aligned} \tag{3.1}$$

Here, U_{go}, V_{go} are the x and y components of the surface geostrophic wind,

$\theta_* = \left(-\frac{Q_o}{U_*}\right)$ is the scaling temperature,

$q_* = \left(-\frac{E_o}{U_*}\right)$ is the scaling mixing ratio,

$\alpha_H = \left(\frac{K_H}{K_M}\right)$ is the surface layer value for the ratio of the eddy coefficients for heat and momentum for near-neutral conditions. According to Businger et al. (1971), $\alpha_H = 1.35$ for unstable conditions.

$\left.\begin{array}{l}A(\mu),\ B(\mu), \\ C(\mu) \text{ and } D(\mu)\end{array}\right\}$ are the universal nondimensional functions depending on the internal parameter $\mu = kU_*/|f|L$.

Since Eqs. (3.1) are strictly valid for barotropic conditions, U_{go} and V_{go} may be replaced by U_g and V_g, the geostrophic wind components above the surface.

In the above equations, the unknown internal parameters of the BL to be *determined* are the friction velocity, U_*; angle α_o between surface shear stress and geostrophic wind vectors; θ_*, the temperature scale; and q_*, the mixing ratio scale. And these are to be determined in terms of known external parameters f, z_o, buoyancy parameter g/θ, G_o, $\Delta\theta$, and Δq. The free-atmosphere (large-scale) parameters U, V, θ, and q are considered to be given.

As suggested by Monin and Zilitinkevich (1967) and Clark (1970), Eq. (3.1) can be *inverted* to obtain

$$\frac{\vec{\tau}_o}{\rho G_o^2} = C_D(R_o, S) = k^2 \left\{ \left[\ln\left(\frac{U_*}{|f| z_o}\right) - A(S) \right]^2 + B^2(S) \right\}^{-1}$$

$$\alpha = \tan^{-1}\frac{V_{go}}{U_{go}} = \alpha_o(R_o, S) = \frac{-B(S)}{\ln\left(\frac{U_*}{|f| z_o}\right) - A(S)} \text{ Sign } f$$

$$-\frac{Q_o}{G_o \Delta\theta} = C_H(R_o, S) = \frac{k\alpha_H}{\ln\left(\frac{U_*}{|f| z_o}\right) - C(S)} \tag{3.2}$$

$$-\frac{E_o}{G_o \Delta q} = C_E(R_o, S) = \frac{k\alpha_H}{\ln\left(\frac{U_*}{|f| z_o}\right) - D(S)}$$

Equations (3.2) are in such form that they can be used to compute the internal characteristics U_*, Q_o, and E_o if the external characteristics of the BL, z_o, f, G_o, $\Delta\theta$, and Δq are given. Here C_D, C_H, C_E are, respectively, the geostrophic drag, heat transfer, and moisture transfer coefficients. Note that in Eq. (3.2) a bulk-type stability parameter, $S = (g/\theta)/(\Delta\theta/fG_o)$, is used instead of μ (of Eq. (3.1)) because the latter contains surface fluxes that are actually being sought.

MODIFICATION OF SIMILARITY THEORY EQUATIONS (EQS. (3.1) AND (3.2))

The formulations described above (Eqs. (3.1) and (3.2)) are valid for stationary, horizontally homogeneous, diabatic, and barotropic conditions. These enable determination of surface fluxes in terms of the surface characteristics and the mean variables evaluated *only* at the top of the BL. Two important length-scales are involved: the BL height, h (scaled by U_*/f), and Monin-Obukhov length (L). The universal constants A, B, C, and D are dependent upon the ratio of these two length scales. Various recent studies (Deardorff, 1972a, 1974; Carson, 1973; and others) have demonstrated that when the stability parameter h/L is less than a value of order -1, h is *not* related to U_*/f. As a consequence, Eqs. (3.1) and (3.2) have been modified by replacing U_*/f with h, the actual height of the BL. The value of h itself may be determined from observations or from a rate equation discussed in detail in Chap. 4. Arya (1974) has independently modified these equations for the unstable case only, because of great uncertainty in determining h for stable conditions. This implies that the parameters ought to be switched in going from the unstable case to the stable. One can avoid that, however, by using h instead of U_*/f for *both* the unstable and stable conditions (Zilitinkevich and Deardorff, 1974). Doing so is not likely to introduce any serious errors in computation, particularly considering our ignorance of how h be obtained in stable cases (Melgarejo and Deardorff, 1974).

In view of the above discussion, Eq. (3.1) can be written by replacing U_*/f with h, the height of the BL. Thus we have

$$
\begin{aligned}
\frac{U_{go}}{U_*} &= \frac{1}{k}\left[\ell n\,\frac{h}{z_o} - A_1(\mu_1)\right] \\
\frac{V_{go}}{U_*} &= -\frac{1}{k} B_1(\mu_1)\ \mathrm{Sign}\ f \\
\frac{\Delta\theta}{\theta_*} &= \frac{1}{k\alpha_H}\left[\ell n\,\frac{h}{z_o} - C_1(\mu_1)\right] \\
\frac{\Delta q}{q_*} &= \frac{1}{k\alpha_H}\left[\ell n\,\frac{h}{z_o} - D_1(\mu_1)\right]
\end{aligned}
\qquad (3.3)
$$

where μ_i = kh/L and subscript i has been added to distinguish universal constants A, B, C, D of Eq. (3.1) from those in Eq. (3.3). Note that A_i and B_i in the first two equations of (3.3) are reversed in definition from that of Zilitinkevich and Deardorff (1974).

Similarly, the set (3.2) can be rewritten (by replacing U_*/f with h) as follows:

$$\frac{\vec{\tau}_o}{\rho G_o^2} = C_D\left(\frac{h}{z_o}, S_i\right) = k^2\left\{\left[\ln\frac{h}{z_o} - A_i(S_i)\right]^2 + B_i^2(S_i)\right\}^{-1}$$

$$\tan^{-1}\left(\frac{V_{go}}{U_{go}}\right) = \alpha_o\left(\frac{h}{z_o}, S_i\right) = \frac{-B_i(S_i)}{\ln\frac{h}{z_o} - A_i(S_i)} \text{ Sign } f \qquad (3.4)$$

$$-\frac{Q_o}{G_o\Delta\theta} = C_H\left(\frac{h}{z_o}, S_i\right) = \frac{k\alpha_H}{\ln\frac{h}{z_o} - C_i(S_i)}$$

and

$$-\frac{E_o}{G_o\Delta q} = C_E\left(\frac{h}{z_o}, S_i\right) = \frac{k\alpha_H}{\ln\frac{h}{z_o} - D_i(\mu_i)}$$

Here $S_i = gh\Delta\theta/\theta G^2$ is the stability parameter. Subscript i distinguishes the constants from those in Eq. (3.2). S_i may also be obtained from the formula

$$S_i = \frac{h\left[\ln\frac{h}{z_o} - C_i(S_i)\right]}{L\alpha_H\left\{\left[\ln\frac{h}{z_o} - A_i(S_i)\right]^2 + B_i^2(S_i)\right\}}$$

It may be recognized here that the stability parameter $\mu = kU_*/|f|L$ (of Eq. (3.1)) and μ_i = kh/L (of Eq. (3.3)) are not independent, owing to the presence of L in both. Also, under relatively stationary conditions the variation of L covers a much wider range than that of U_*/f or h.

Experience confirms that Eqs. (3.2) and/or (3.4) are more convenient to use in practice than either (3.1) or (3.3).

EFFECT OF BAROCLINICITY ON WIND PROFILES AND GEOSTROPHIC DRAG LAW

Equations (3.1) to (3.4) were derived for *barotropic* BL. This is perhaps one of the most important factors that cause a large scatter in data points used in empirical determinations of the stability dependent similarity functions A, B, etc. (Arya, 1974; Melgarejo and Deardorff, 1974). (The other factors are uncertainties in measurements and effects of accelerations.) Various observations have shown a large effect of baroclinicity on wind shear and surface cross-isobaric angle α_o. Some theoretical studies also have studied the effect of thermal wind on wind profiles and geostrophic drag laws of the BL, but most are based on K-theory models (Chap. 2), which suffer from the serious limitation that they do not consider effect of geostrophic shear on values of K or mixing length. Also, K becomes meaningless for convective conditions, because of mean gradients being close to zero or of the wrong sign (Deardorff, 1972b and Wyngaard et al., 1974).

Following similarity arguments, Hess (1973) modified similarity functions A_i and B_i (see Eq. (3.3)) to incorporate the effect of thermal wind. He considered the special case of geostrophic shear being constant with height and suggested the use of

$$A_i\left(\mu_i, \frac{T_x}{f}, \frac{T_y}{f}\right)$$

and

$$B_i\left(\mu_i, \frac{T_x}{f}, \frac{T_y}{f}\right)$$

in place of $A_i(\mu_i)$ and $B_i(\mu_i)$, respectively, in the first and second equations of the sets (3.1) to (3.4). Here T_x and T_y are the components of the thermal wind *invariant* with height. Arya and Wyngaard (1974), through a simple physical model of convective BL, have generalized the above concept and expressed the similarity functions A_{io} and B_{io} as sums of a barotropic part (A_i and B_i), dependent only on the stability and BL height parameters, and a baroclinicity dependent part (A_i', B_i'), by using the "more convenient" surface geostrophic wind. Thus

$$A_{io} = A_i + A_i'$$

and

$$B_{io} = B_i + B_i'$$

Here

$$A_i = \ln \frac{h}{z_o} - \frac{kU_g}{U_*}$$

and

$$B_i = -\frac{kV_g}{U_*} \text{ Sign } f$$

from the first two equations of (3.3). According to Arya and Wyngaard (1974) A_i', B_i' due to thermal winds are given by

$$A_i' = aM_o \cos \beta_o$$

$$B_i' = bM_o \sin \beta_o$$

where a and b are coefficients that depend on the variation of geostrophic shear with height, and β_o is the angle between the surface geostrophic shear and the surface wind.

The results of Arya and Wyngaard (1974) show that:

- Actual wind shears in the bulk of the mixed (convective) layer are small compared with the imposed geostrophic shear. It is incorrect to determine geostrophic shear simply from the measured wind profile in the upper part of the BL.
- The flow in the baroclinic mixed layer is far from geostrophic. The deviations from the geostrophic equilibrium are most striking just below the inversion base, and any approach to the former must occur in the stable layer above.

- α_o, the surface cross-isobaric angle, increases toward the equator, a finding that agrees with observations but is opposite to the trend predicted by the drag laws of Zilitinkevich et al. (1967). This variance can be attributed to the use of u_*/f as BL height scale instead of h, the height of the lowest inversion base.
- Since the model does not consider an evolutionary BL, the entrainment effects at the top of mixed layer have been omitted. Consequently, the level where stress vanishes and geostrophic flow is attained must be larger than the estimate for h used in this study. If layer-averaged geostrophic winds are used, however (as they are in general circulation models), then use of A_1 and B_1 is considered adequate for describing both barotropic and baroclinic cases.

DETERMINATION OF THE UNIVERSAL SIMILARITY FUNCTIONS A, B, C, AND D, AS FUNCTIONS OF STABILITY PARAMETER μ

The universal functions are determined empirically from observations, or theoretically.

Evaluations of the universal functions A, B, C and D over wide range of statistics have been provided by Monin and Zilitinkevich (1967), and Zilitinkevich and Chalikov (1968), who used the Great Plain data (Lettau and Davidson, 1957). Clark (1970) has also determined these functions on the basis of observations from south Australia. Examination of these evaluations has shown that there is too much scatter and too many differences to permit reliable estimates of the so-called universal functions. For example, Carson (1972) has compared the plots of $C(\mu)$ obtained by Monin and Zilitinkevich (1967) and by Clark (1970) over a common range of μ (Fig. 7). While Clark's curve is well defined for $\mu < -50$ it shows considerable scatter for $\mu > 50$. On the other hand, the points used to determine "Russian" $C(\mu)$ (μ ranges from -50 to +70) show a wide scatter, thereby emphasizing marked differences between the two curves throughout the common range of μ that is important for estimating heat fluxes. Thus there is an apparent anomalous behavior of $C(\mu)$ for relatively small values of μ; it can be

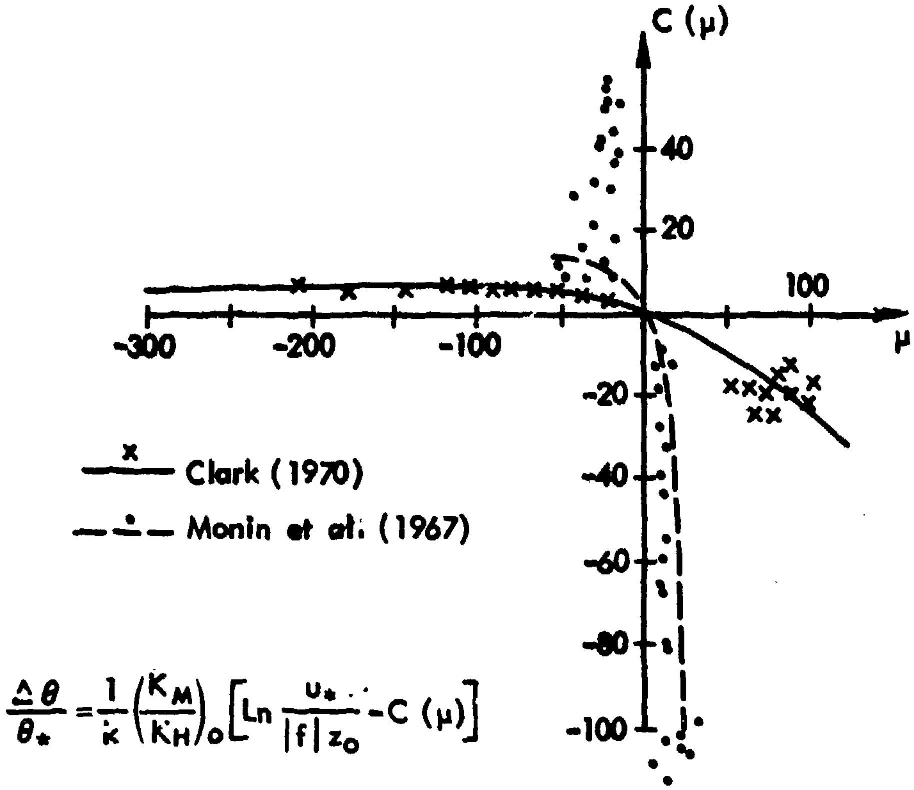


Fig. 7 — Variation of similarity parameter C with stability as suggested by Monin et al. (1967) and Clark (1970) (Carson, 1972)

attributed to the violation of restrictive conditions of similarity theory such as site inhomogeneities, thermal wind, etc. It is also possible that some spurious effects may be caused by the differences in determination of the surface fluxes and external variables, especially the choices of mean flow variables such as θ_h at the top of the BL.

The functions $A(\mu)$, etc. of Eq. (3.1) or (3.3) can be determined if we know U_*, θ_*, z_o, U_{go} and V_{go}, $\Delta\theta$ and Δq. Whereas the empirical determination of $A(\mu)$, $B(\mu)$, $A_i(\mu_i)$, and $B_i(\mu_i)$ is relatively straightforward, that of $C(\mu)/C_i(\mu_i)$ is quite sensitive to the specification of $\Delta\theta(= \theta_h - \theta_o)$. Here θ_o and θ_h are the potential temperatures at $z = z_o$ and $z = h$, the top of the BL (the same is true for $D(\mu)$ or $D_i(\mu_i)$). Zilitinkevich and Chalikov (1968) used a fixed height of 1 km for the top of the BL (irrespective of dynamic and stability considerations), and Clark (1970) assumed a level at the maximum in the U-profile as the top of the BL. It is evident that the former technique is likely to overestimate θ_h (especially if the inversion base lies below the fixed BL top of 1 km), and the latter may underestimate θ_h because the BL depth in this case is about half of the actual BL thickness (Businger and Arya, 1974).

Recently, Arya (1974), after reanalyzing the data from the two sites used by previous investigators, has suggested better estimates of the universal functions. Figure 8 shows a comparison of empirical and theoretical determinations of $C(\mu)$. It may be seen that whereas $C(\mu)$ varies rapidly for the stable ($\mu > 0$) case, it shows no non-monotonic behavior, as is implied by the results of Clark (1970) (Fig. 7). However, there is still considerable scatter in the data as μ increases; this can be attributed to uncertainty in determing U_*, θ_*, U_{go}, V_{go}, etc. under very stable conditions. For the unstable ($\mu < 0$) case and for $-\mu < 50$, $C(\mu)$ shows a continuation of the trend noticed for the stable case. For $-\mu \geq 50$, however, the function shows no trend at all. This implies that either the function approaches a constant value for moderate to strong instability, or that the relevant similarity parameter for the range is $\mu_i = h/L$, not $\mu = kU_*/|f|L$. But as shown by a comparison of $C(\mu)$ for $\mu < 0$ and $C_i(\mu_i)$ (Fig. 9), at least in this case the behavior of C does not seem to differ significantly when μ is replaced by μ_i. This result has also been obtained by Clark and Hess (1973), and thus this study has not established the usefulness of h as scale height for *all* unstable conditions. The very poor comparison of Csanady's (1967) results (Fig. 8) with empirical and other theoretical studies can be attributed to his use of a similarity function that is not dependent on stability. This also caused him to obtain unrealistic results for the surface cross-isobaric angle, which (in his study) increased in unstable conditions and decreased in stable conditions--a result quite opposite to observations.

Melgarejo and Deardorff (1974) have determined the similarity functions $A(\mu)$, $B(\mu)$, $C(\mu)$ by using the modified BL similarity theory (Eqs. (3.3) and (3.4)) in which they have used the observed BL height, h, as

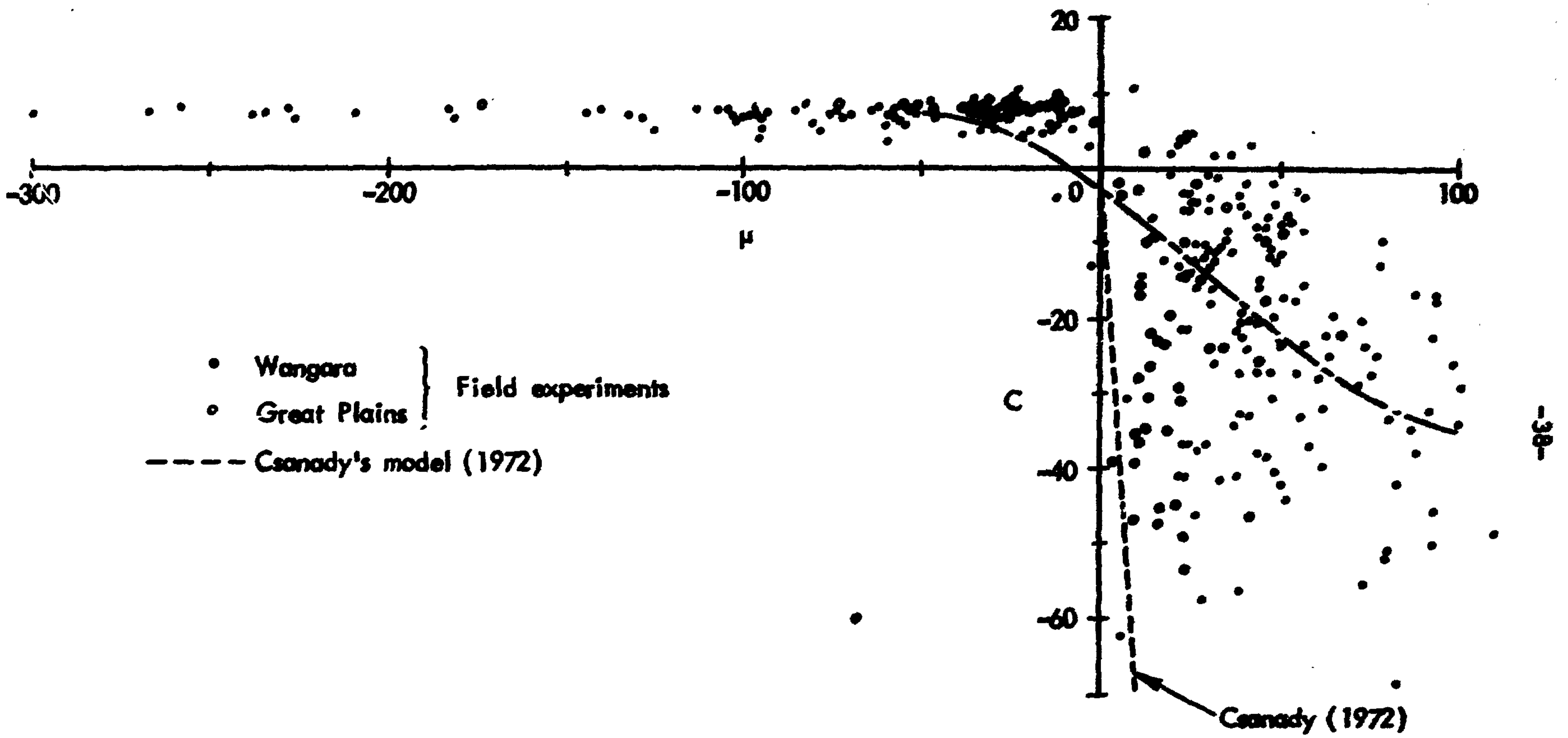


Fig. 8 — Comparison between empirical and theoretical determination of C (μ)
(Arya, 1974)

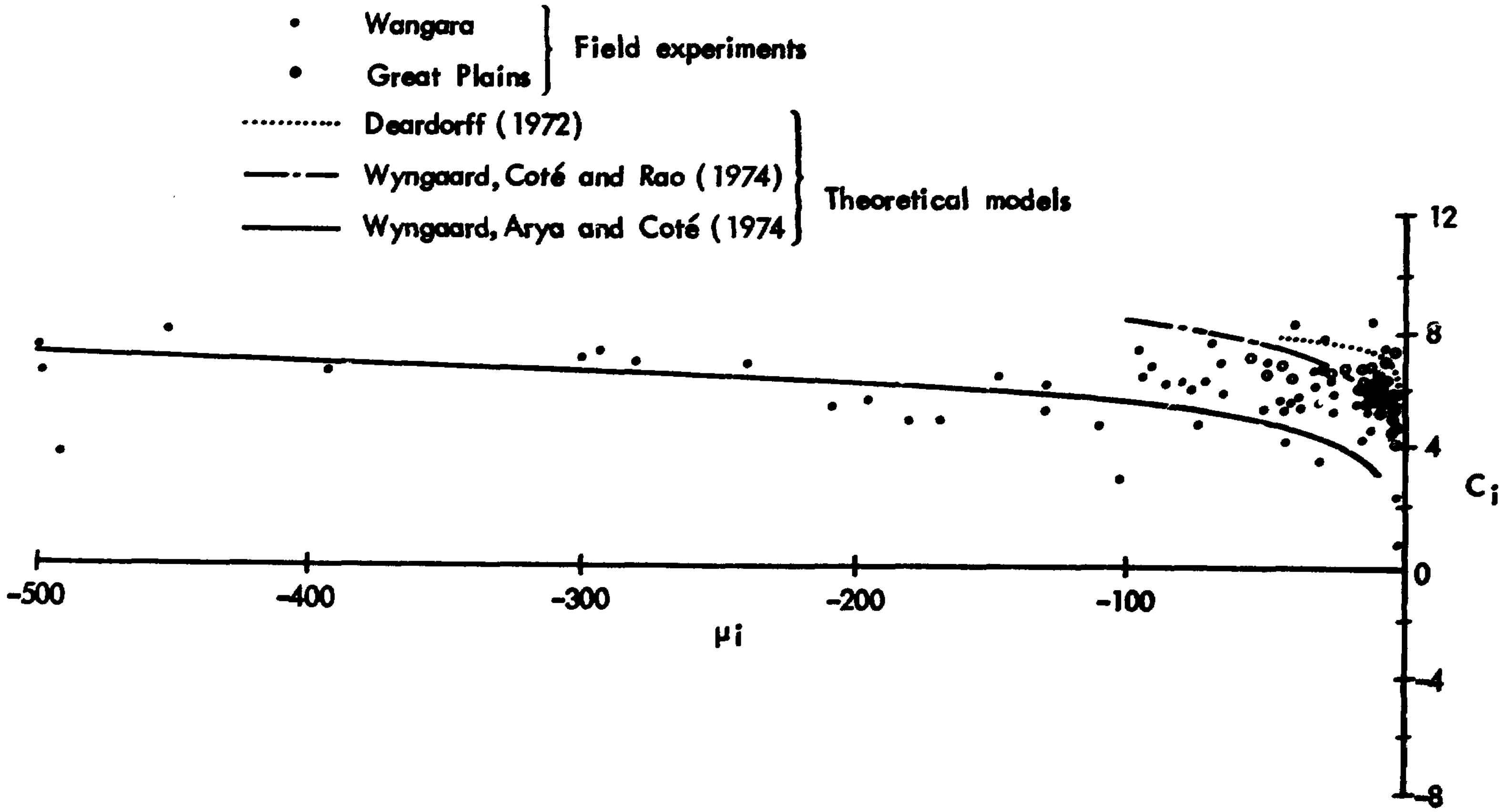


Fig. 9 — Comparison between empirical and theoretical determination of C_i (μ_i) (Arya, 1974)

the physical length scale instead of the conventional U_*/f for *both* stable and unstable cases. Their results, as well as the scatter of data for stability functions, are comparable to those obtained by conventional analyses. There is some uncertainty, however, about the use of h as a relevant parameter for stable cases because of the poor validity of similarity theory on the stable side. In fact, Melgarejo and Deardorff (1974) think that it does not matter whether h or U_*/f is used for stable situations. For unstable conditions, however, h is considered more relevant than U_*/f because then the external variables ($\Delta\theta$, Δq) can be obtained realistically and applied toward the determination of surface fluxes. This is an important aspect, specifically with respect to general circulation models of coarse vertical resolution. It is also suggested that one may use mean actual wind at h rather than geostrophic wind, because

- The results are then applicable closer to the equator, and
- The procedure for obtaining universal functions A and B is more analogous to that for determining C (and D).

DETERMINATION OF GEOSTROPHIC DRAG (C_D), HEAT (C_H), AND MOISTURE (C_E) TRANSFER COEFFICIENTS

An examination of Eqs. (3.1) and (3.3) shows that surface fluxes are implicitly involved in the definitions of the similarity parameters. As a result, these equations are not quite suitable for computing surface fluxes from the known values of external variables. However, Clark (1970) has shown that once $A(\mu)$, $B(\mu)$, $C(\mu)$, etc. have been evaluated empirically, then an *inversion* of Eqs. (3.1) and (3.3) results in relationships (3.2) and (3.4), respectively. And the latter are in a form suitable for determining internal BL characteristics U_*, Q_o, E_o from known external BL characteristics, namely z_o, f, G, $\Delta\theta$, Δq. For this purpose, we need external stability parameter S (or S_i), and surface Rossby number R_o (or h/z_o). Clark (1970) has obtained values of U_*/G, α_o, $\theta_*/\Delta\theta$, $q_*/\Delta q$ as functions of S and R_o and plotted them in the form of *nomograms*. By interpolation on these nomograms, U_* and α_o, and hence τ_o, Q_o and E_o, may be readily computed. Figure 10 is an

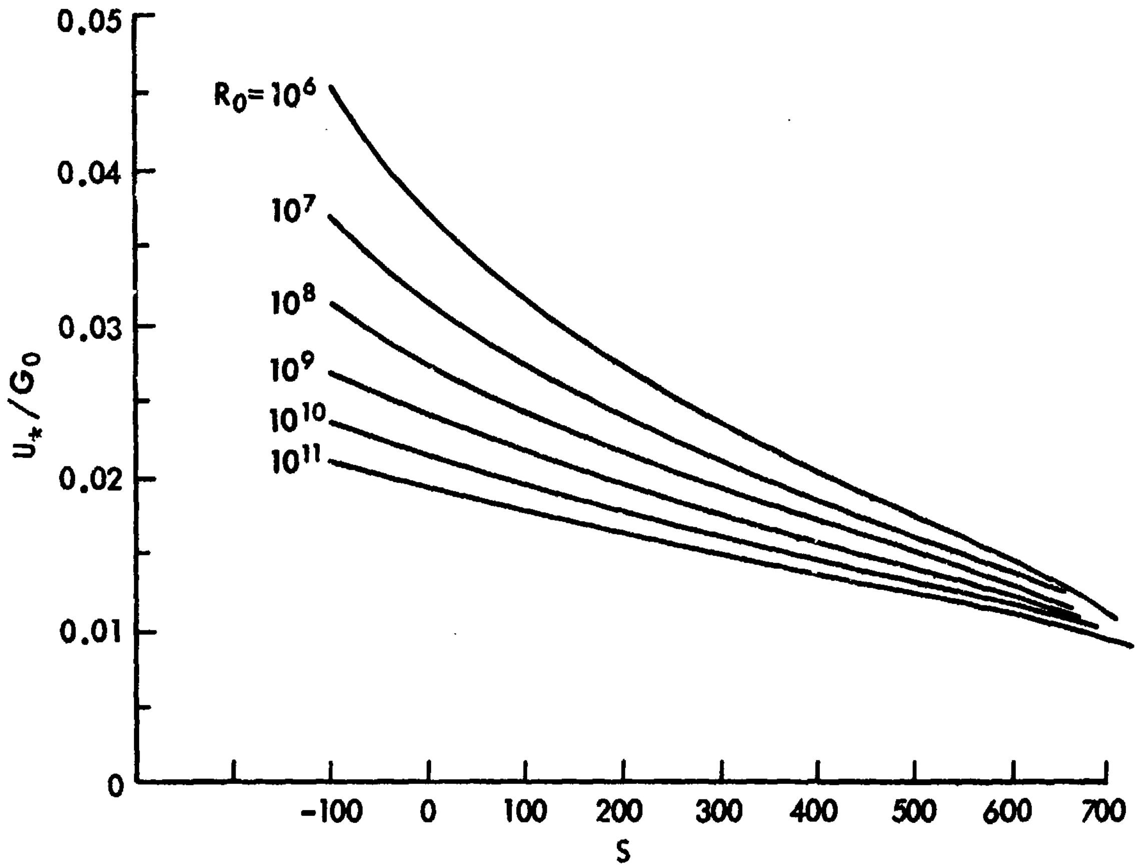


Fig. 10 — Nomogram of $\frac{U_*}{G_0}$ as a function of R_0 and S for near-neutral and stable conditions based on Wangara data (Arya, 1974)

example of a nomogram for the near-neutral and stable case that can be used to obtain surface stress. It is seen that with increasing stability U_*/G decreases, and its dependence on the surface Rossby number weakens. Figure 11 shows a corresponding plot of U_*/G against S_i for unstable (convective conditions). These estimates are not quite reliable, however, because of a large scatter of empirical parameters.

LIMITATIONS OF THE SIMILARITY THEORY

The similarity theory in its original form was based on the following simplifying assumptions. The BL was considered to be:

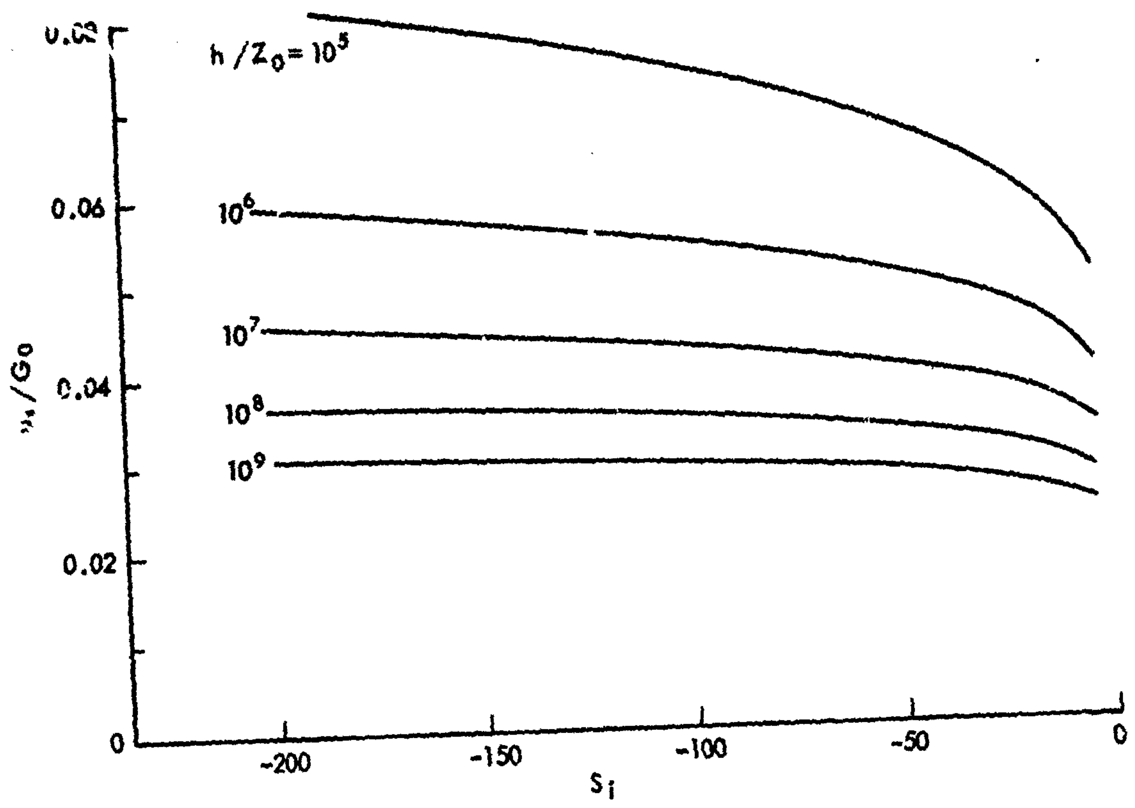


Fig. 11 — Same as Fig. 10 but for u_*/G_0 as a function of h/Z_0 and S_i for convective conditions (Arya, 1974)

- horizontally homogeneous
- barotropic
- neutral
- steady state, and
- with no radiative heat flux.

It is evident that, though the results based on the theory in this form provided reasonable insight into the determination of the variability of BL characteristics, the theory cannot represent the real atmosphere conditions. Consequently, the similarity theory concepts have been modified by adding complexities to the determining parameters. For example, there are now formulations that include the effects of:

- horizontal inhomogeneity
- thermal wind
- diabatic stratification.

Introducing these modifications violates the restrictive conditions of theory, and there is considerable inconsistency in the results based on the similarity theory. This is especially true over land, where there are rapid temporal changes. For example, for a diurnal cycle, $\Delta\theta (= \theta_h - \theta_o)$ is usually large during early morning hours (up to about 0900 hr) because of still-persisting remnants of nocturnal inversion. This results in small values for Q, which cause anomalously large values of C for small negative values of stability parameters μ (Clark, 1972). The theory is also of doubtful validity in the regions of strong horizontal discontinuities, such as coastlines.

From the point of view of "universality" of the theory applications, the most important drawback is the choice of U_*/f for height-scale for all stability conditions. It is obvious that this theory (under this stipulation) has no effective use in general circulation models and is outright meaningless at the equator. As discussed earlier, however, valiant efforts have been made recently to modify the theory to render it usable for more general purposes. For example, while the scale height may be considered to be U_*/f for stable conditions it is redefined to be h (the height of the base of inversion) under unstable conditions. In some cases, doing so perhaps has remedied the large values of C referred to above. Also, the scale height U_*/f has been made a function of stability with a view to studying diurnal variations of BL characteristics (Zilitinkevich, 1972; Carson, 1973). There are also suggestions to use actual wind at h rather than geostrophic wind. However, since the formulations in this case are based on steady-state similarity theory, they cannot adequately simulate the "observed" evolutionary nature of the BL.

Under unstable conditions and especially for marine conditions, the similarity theory has been considered to be of questionable validity. The reason is that marine conditions are usually characterized by inversions (Kraus, 1972), and there is an identical mixed layer for heat and momentum. Consequently, the specification of $\Delta\theta$, the potential temperature difference between the bottom and top of the mixed layer, is ambivalent in these circumstances.

The formulations of similarity theory use a quantity z_o implying a local roughness parameter--on a micrometeorological scale--that varies sharply from point to point over natural surfaces and is determined from towers. This z_o is not, in general, identical with a roughness parameter that should be used in the formula for the large-scale features. This has led Fiedler and Panofsky (1972) to define an effective roughness length z_e for incorporating surface friction into large-scale models of the atmosphere. They suggest values for z_e of 0.42 m (for plains), 0.99 m (for low mountains), and 1.42 m (for high mountains).

Chapter 4

DETERMINATION OF THE HEIGHT (h) OF THE BOUNDARY LAYER

There is increasing evidence that the height h of the BL is the basic parameter to be used in any BL parameterization. This chapter discusses the quantitative determination of h using both diagnostic and prognostic techniques.

DIAGNOSTIC METHODS

In a very detailed survey, Hanna (1969) has discussed various diagnostic methods of estimating h. He has indicated that for many years h was estimated on the basis of the classical theoretical models of Taylor (1915), which predicted that wind velocity approaches geostrophic velocity in an asymptotic manner as heights increase. This implied an estimation of h by means of arbitrary criteria, such as that requiring h to be the level at which the shear of the wind speed first vanishes. While Blackadar (1962), Lettau (1962), and others have solved equations of motion-yielding asymptotic wind spirals whose thicknesses are functions of arbitrary empirical constants, h has also been estimated from observed soundings; for example, Charnock and Ellison (1967) used radiosonde ascents to determine both h and other BL characteristics. The survey also summarizes the various methods for estimating h, together with the tests of their results against the 1953 O'Neill BL observations. Table 1 lists some of the methods he considered for determining h.

More recently, Charney (1969) has determined h on the basis of dynamics of the turbulent motion in the BL. He postulates that h is determined by Ekman layer instability: the eddy viscosity increases until the eddy Reynold number is reduced to a critical value. Thus

$$h = \frac{K_e}{f}$$

where f is coriolis parameter and K_e is the *effective* eddy viscosity determined on the condition that Reynolds number is equal to a critical

Table 1

METHODS FOR ESTIMATING BOUNDARY LAYER HEIGHT h

Definition or Formula	Source	Agreement with Observations
Level at which wind first reaches geostrophic direction	Taylor (1915)	Poor
h is the level of first wind maximum	Clark (1970)	Good at night only
h = 1 km	Zilitinkevich and Chalikov (1968)	Good at neutral and near neutral conditions only
$h = 0.2 \frac{U_*}{f}$	Blackadar (1962)	Poor if applied to diabatic conditions
$h = 0.006 \frac{G}{F}$	Lettau (1962)	Good for neutral and near-neutral conditions
$h = 0.021 \frac{G \sin \alpha}{f}$	Rossby and Montgomery (1935)	
$h = \frac{0.38\, G \sin \alpha}{\left[\frac{g}{\theta} \frac{\partial \theta}{\partial z}\right]^{1/2}}$	Rossby and Montgomery (1935)	Good if constant changed to 1.2
$h = \frac{1.3\, G}{\left[\frac{g}{\theta} \frac{\partial \theta}{\partial z}\right]^{-1/2}}$	Laikhtman (1961)	Good if constant changed to 0.75

SOURCE: Hanna (1969).

value R_c, which has been found from experiments to be 100 for neutral conditions.

The formulations mentioned above are by and large applicable only to neutral (or near-neutral) conditions. Clark (1970) and Carson (1971) studied observational data by using a stability dependent formulation for h given by

$$h = C \frac{U_*}{f} F(\mu)$$

where

$$\mu = \frac{U_*}{|f|L}$$

is a stability parameter. C is a constant of about 0.3. Zilitinkevich (1972), using the relationship between h and "effective" eddy viscosity, K_e, has suggested that with increasing instability h *increases* as $|\mu|^{\frac{1}{2}}$ (where μ is a stability parameter); and with increasing stability h *decreases* as $|\mu|^{\frac{1}{2}}$. Using different reasoning, Businger and Arya (1974) have also shown that for stable conditions, h varies as $|\mu|^{-\frac{1}{2}}$. Figure 12 shows h defined in various ways as a function of stability.

PROGNOSTIC METHODS

In the preceding section we considered the purely diagnostic determination of h. It has been found that, for steady-state and neutral or near-neutral cases (Hanna, 1969), a large scatter is obtained when theoretically derived h is compared with profile-estimated depths of the BL (Fig. 13). Even if h is made stability dependent, the formulations (Carson, 1971; Zilitinkevich, 1972) are totally inadequate for representing the BL's *evolving* nature in real atmosphere. Over land, daytime observations typically have found h to increase from a height on the order of 100 m shortly after sunrise to a height of 1 to 3 km by late afternoon. If $W_h < 0$ (where W_h is the large-scale vertical velocity at the top of boundary layer), then the maximum height reached may be much less. Similarly, over land at night, h typically increases from a height on the order of 100 m shortly after sunset to *only* 200 to 500 m by early morning. In addition to observations, some recent

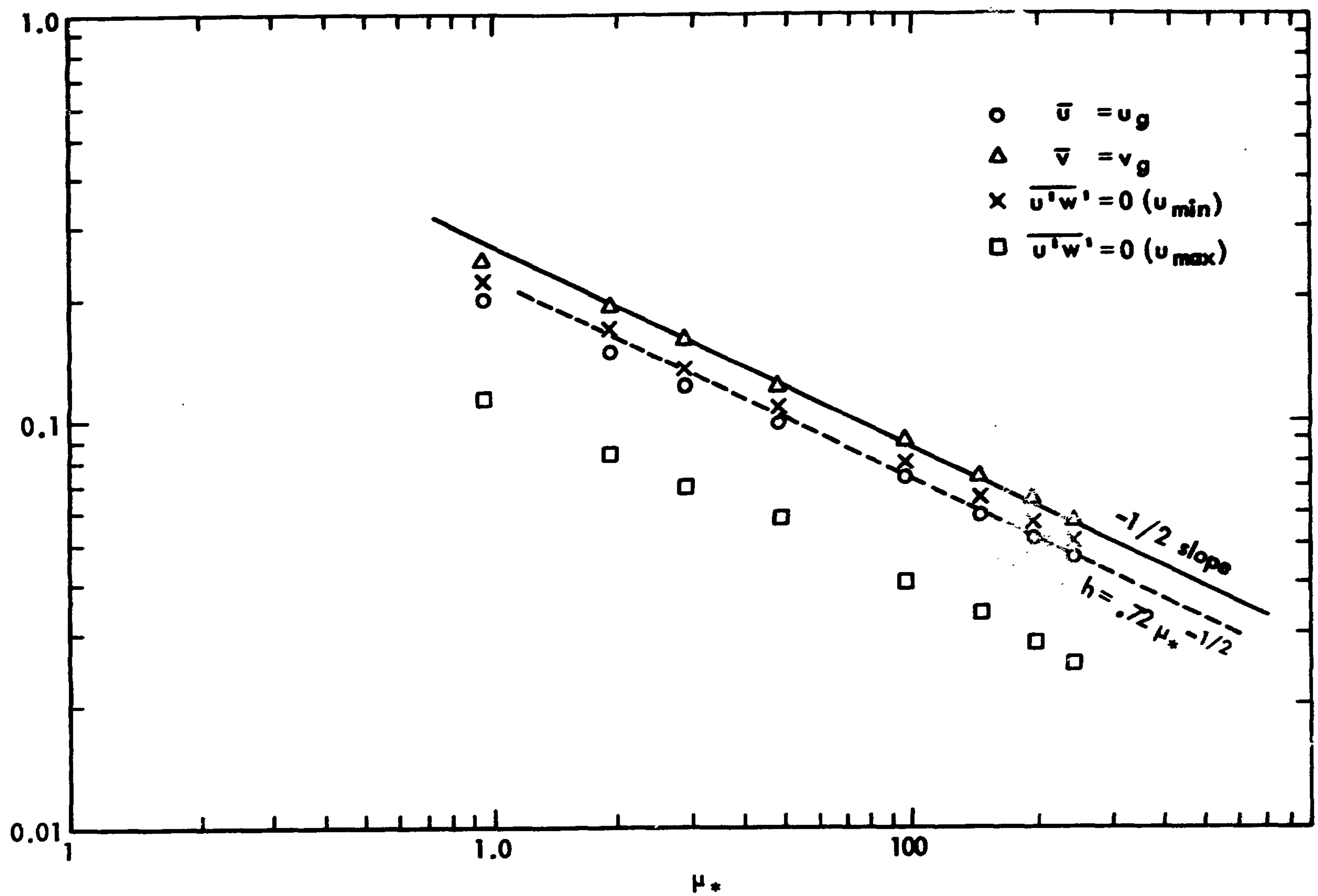


Fig. 12 — The boundary layer height defined in various ways as a function of stability (Businger and Arya, 1974)

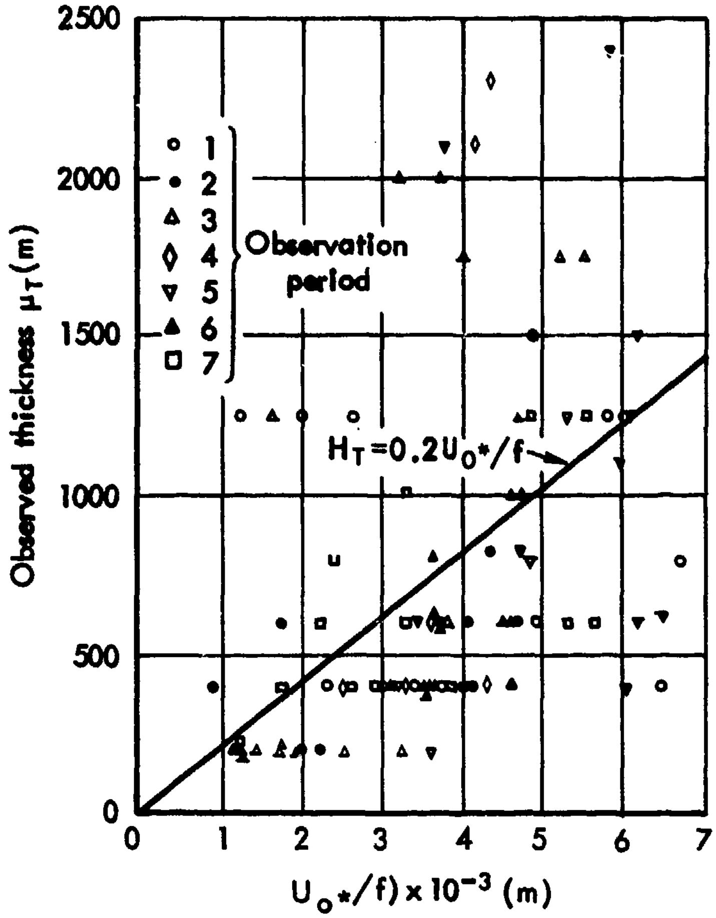


Fig. 13 — The observed boundary layer thickness H_T at O'Neill as a function of the length U_0*/f (Hanna, 1969)

theoretical studies (e.g., Deardorff, 1974) have demonstrated the inadequacy of a diagnostic formula for calculating h. These studies have also pointed out that over land h generally seems determinable from a *rate equation* only. Figure 14 shows that variation of h given by a rate equation compares favorably with the observed h, whereas the diagnostic formula is not at all adequate. It may be remarked here that, while there is general agreement regarding the use of a rate equation for h for *unstable/neutral* conditions, there is some doubt about its use for *stable* conditions. Thus, though Deardorff (1971) suggests that the height of the nocturnal (stable) boundary layer should be predicted by a rate equation, Zilitinkevich and Deardorff (1974)

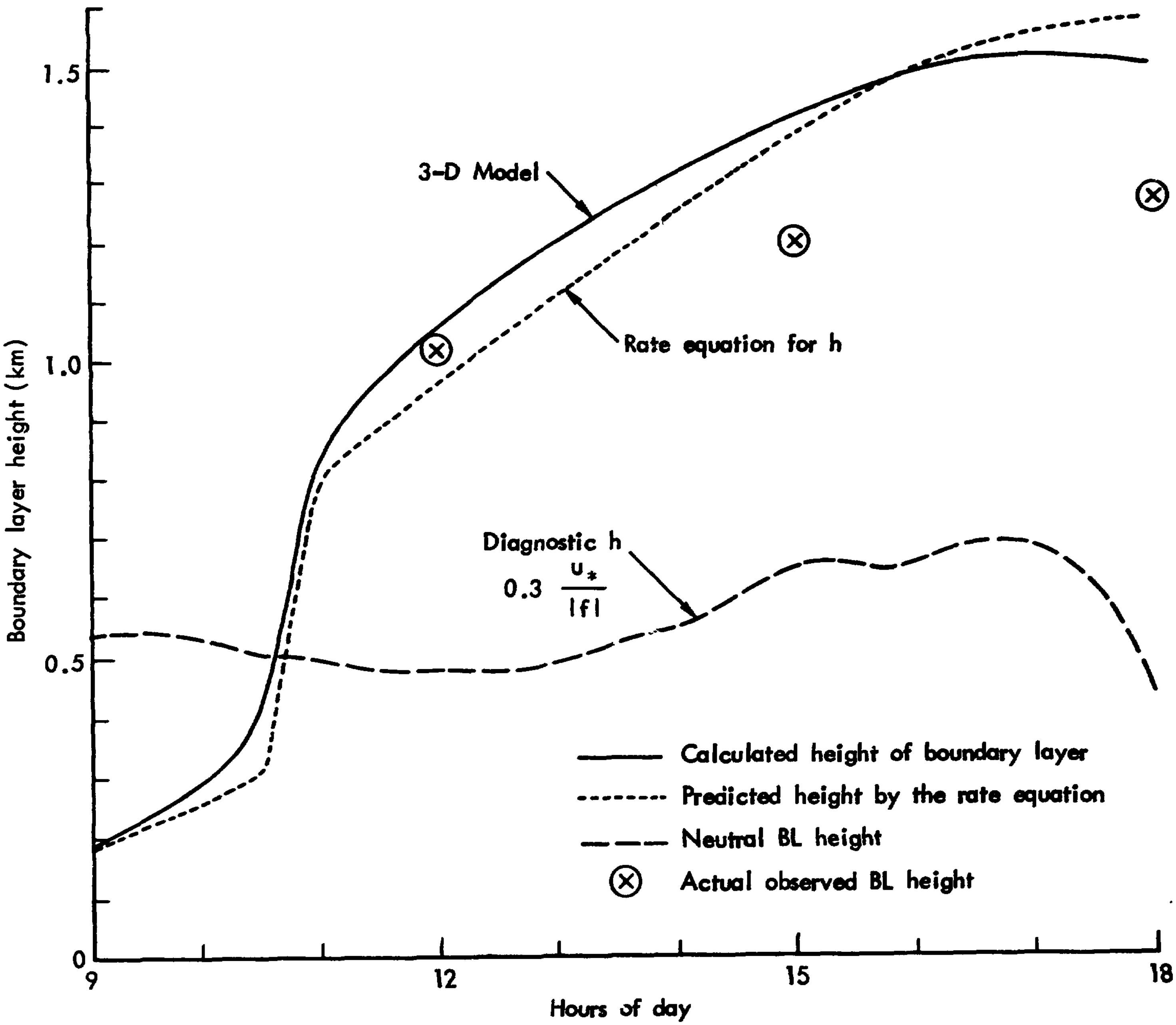


Fig. 14 — Height of the boundary layer (Deardorff, 1974)

believe that a rate equation for the stable case may not be appropriate. We consider below a rate equation for h that is appropriate only for a convectively *unstable* BL.

Physical Descriptions of Convectively Unstable Boundary Layer

The strong heating of ground by solar radiation causes thorough convective mixing in the lower part of the atmosphere, which in turn establishes a mixed layer where potential temperature is virtually independent of height. It is often observed, however, that the upper region of the mixed layer is slightly stable. Next to the ground itself a shallow superadiabatic layer occurs ($\leq$ a few tens of meters), marked by large vertical shears of wind and temperature; here the heat is predominantly transported by mechanically induced turbulent motions. Above the mixed layer a deep *nonturbulent stable* layer occurs. Separating the two, however, is a highly undefined interfacial entrainment layer that results from:

(i) Physical overshooting into the stable layer of convective elements that originate in the surface layer and continually intrude into the stable layer, and

(ii) The entrainment of capping stable air into the mixed layer.

Both factors cause an increase in the depth of the mixed layer (i.e., of h). Figure 15 is a schematic representation of the developing convectively unstable BL. The interface entrainment layers vary widely in depth and character. They are usually represented by step discontinuity in potential temperature $\Delta\theta$ (in the θ profile) at $z = h$, the top of the convectively unstable BL.

The other physical processes that affect the growth rate of h are:

- Horizontal advection at h
- Large-scale vertical velocity at h
- Radiative heat flux.

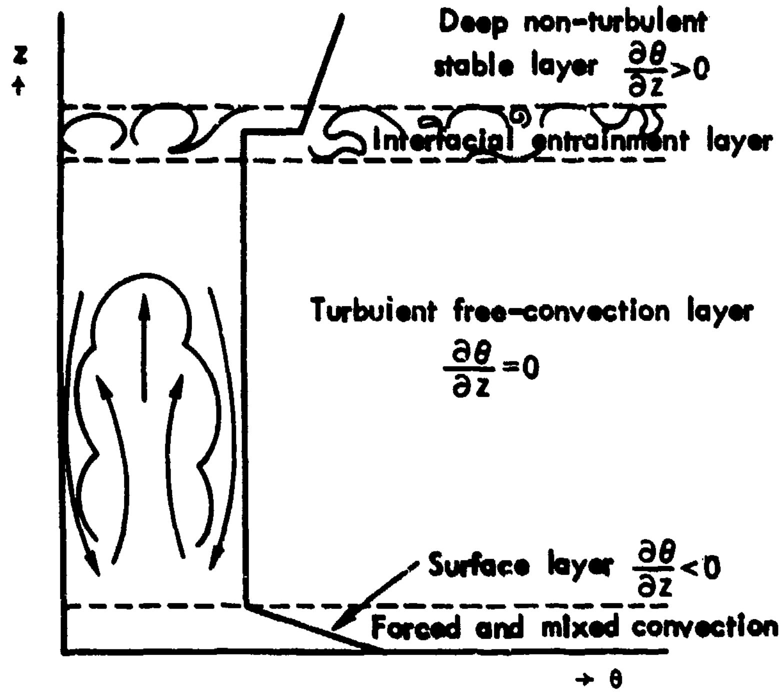


Fig.15 — Schematic representation of the developing convectively unstable boundary layer (Carson, 1973)

Though not much is known about the first item, which is perhaps more predominant over oceans, the second factor (as discussed elsewhere) significantly influences BL evolution. A subsequent section discusses the third factor (radiation), which is especially important when the BL contains fog or stratus or stratocumulus clouds.

Rate Equation for h for the Unstable Case

Many authors have attempted theoretical time-dependent determinations of h for the atmospheric mixed layer. See, for example, Ball (1960), Lilly (1968), Lavoie (1968), Deardorff (1972a, 1973a, 1974), Tennekes (1973), Carson (1973), Stull (1973), and Randall and Arakawa (1974). These studies have pointed out the impossibility of establishing a stationary state except in very special circumstances. The mixed layers have been found to continue to deepen unless counteracted by subsidence or by very special distribution of heating. If an equilibrium is established temporarily, its characteristics depend on previous history as well as instantaneous conditions. Therefore h must be treated as time-dependent in the unstable case. Lavoie (1968) suggests a prognostic equation for h, namely,

$$\frac{\partial h}{\partial t} = W_h - u\frac{\partial h}{\partial x} - v\frac{\partial h}{\partial y} + \left(\frac{1}{\gamma^+}\frac{\partial \theta}{\partial t}\right)_{\theta=\theta_h} \tag{4.1}$$

He assumes that the BL top is a material surface, thereby implying that there is *no entrainment* of mass through the BL top from the overlying atmosphere. However, as shown by observations (both in the atmosphere and by laboratory experiments), entrainment is critically important in determining h, and thus Eq. (4.1) has only limited use. The last term in Eq. (4.1) was added to prevent the development of a superadiabatic layer at the top of the BL whenever the inversion is wiped out by heating from below. It is necessary, of course, to ensure that the physical process represented by this term does not violate the treatment of the thermodynamic energy equation.

We now consider a rate equation for h that incorporates the effects of entrainment but not a radiative heat flux. The equation may be written as

$$\frac{dh}{dt} = W_h + W_E \tag{4.2}$$

where W_h is the large-scale vertical velocity at the top of the BL and W_E is the *entrainment velocity*. As indicated above, entrainment provides an important physical mechanism that leads to deepening of the BL. The determination of entrainment has been treated both theoretically (Lilly (1968), Betts (1973), Tennekes (1973), Deardorff (1972a, 1973a, 1974), Stull (1973) and Randall and Arakawa (1974)), and observationally (Deardorff et al. (1969), Lenschow 1973)). Table 2 shows formulations used in various theoretical studies of the height of the convectively unstable BL; these formulations are primarily based on the so-called "jump" models. It is seen from these expressions that

Table 2

FORMULATIONS USED TO DETERMINE ENTRAINMENT VELOCITY (W_E) FOR UNSTABLE, CLEAR BOUNDARY LAYER

Source	Formulations	Remarks
Lilly (1968) Deardorff (1972a)	$W_E = \begin{cases} \dfrac{-Q_h}{(\Delta\theta)_h} = \dfrac{0.1\ Q_o}{(\Delta\theta)_h} \text{ if } \Delta\theta \geq 0.09\ h\gamma^+ \\ \dfrac{-Q_h}{h\gamma^+} = \dfrac{1.2\ Q_o}{h\gamma^+} \text{ if } \Delta\theta < 0.09\ h\gamma^+ \end{cases}$	• Closure assumption: $Q_h = -kQ_o$ • Factor 0.1 is based on experimental work of Deardorff, et al. (1969) • Virtual potential temperature θ_v may be used instead of θ
Tennekes (1973) Deardorff (1974)	$\begin{cases} \dfrac{dh}{dt} = W_E = -\dfrac{Q_h}{(\Delta\theta)_h} \\ \dfrac{d}{dt}(\Delta\theta)_h = \gamma^+\left(\dfrac{dh}{dt} - W_h\right) - \dfrac{Q_o}{h} + \dfrac{Q_h}{h} \end{cases}$	These formulas are based on 'jump' model and use a prognostic equation to determine $\Delta\theta$, the inversion strength
Randall & Arakawa (1974)	$W_E = \dfrac{kQ_o}{\Delta S_v}$, $k = 0.25$	Virtual static energy S_v is used instead of θ; ΔS_v is the strength of inversion
Carson (1973)	$\begin{cases} W_E = \dfrac{Q_o - 2Q_h}{\rho C_p h\gamma^+} = \dfrac{1.4\ Q_o}{\rho C_p h\gamma^+} \\ (\Delta\theta)_h(t) = \dfrac{Ah\gamma^+}{1+2A} \approx \dfrac{h\gamma^+}{7} \end{cases}$	• Closure assumption $Q_h = -A\ Q_o$; $A = 0.2$ • 'A' measures the degree of interfacial mixing
Stull (1973)	$W_E = 0.57\ E_M\left(\dfrac{Q_o}{\rho C_p}\right)^{2/3}\left(\dfrac{g}{T}\right)^{1/6} h^{-1/3}\left(\gamma^+\right)^{-1/2}$	E_M: empirical parameter. This is a highly parameterized formulation with more than one adjustable constant

Note:
1. All the formulations listed in the table apply to dry, unstable boundary layer only.
2. Randall & Arakawa (1974) have extended their formulation to cloud-topped boundary layers.
3. For stable boundary layer formulations, see the text.

entrainment rate depends upon:

Q_h the downward heat flux at the top of the BL

$(\Delta\theta)_h$ the intensity of the inversion

γ^+ lapse rate just above the jump, and

h height of the BL

Each is discussed below.

Q_h, the Downward Heat Flux at the Top of the BL. This is an immediate consequence of the entrainment of warm air into BL. The physical explanation for the maintenance of downward heat flux at the BL top can also be seen from a consideration of kinetic energy of turbulence near the upper limit of the mixed boundary layer, where entrainment occurs. In general, if the dissipation rate of kinetic energy is negligible, there must be a flux convergence of kinetic energy to maintain a downward heat flux. Tennekes (1973) suggests that, on the basis of dimensions of this flux divergence,

$$-Q_h = \frac{\theta_o}{g}\sigma_w^3 \tag{4.3}$$

where σ_w is the standard deviation of vertical velocity in the BL and g/θ_o is the buoyancy parameter. Also, for a BL in a state of free convection, when the turbulent kinetic energy is maintained by buoyancy only,

$$\sigma_w^3 = AQ_o\left(\frac{g}{\theta_o}\right)h \tag{4.4}$$

(Tennekes, 1970; Deardorff, 1972a and others). Combining Eqs. (4.3) and (4.4) yields:

$$-Q_h = AQ_o \tag{4.5}$$

a result that agrees well with the experimental data of Deardorff et al. (1969) for A = 0. 5. Here, parameter A may be considered the measure of the degree of interfacial entrainment at h. Equation (4.5) constitutes the closure hypothesis for the entrainment theory at the top of the BL.

Ball (1960) assumes a value of unity for A which, though not realistic in the light of the experimental results of Deardorff et al. (1969), provides a maximum possible entrainment criterion for models driven solely by surface heating. Lilly (1968) suggests that the magnitude of actual downward heat flux lies somewhere between 0 and 1. More recently, Deardorff (1974) and Betts (1973) have used a value of 0.25 for A in their models. Carson (1973a) has suggested that A should be time-dependent--partly determined by wind shear across the entrainment layer.

<u>$(\Delta\theta)_h$, the Intensity of the Inversion</u>. This represents a positive jump in average virtual potential temperature at h. Lilly (1968) suggests that the entrainment rate W_E is *inversely* proportional to $(\Delta\theta)_h$. For a realistic determination of W_E, however, it is extremely important to consider the time variation of $(\Delta\theta)_h$ also. Tennekes (1973) uses a rate equation for $(\Delta\theta)_h$ to account for kinematic behavior of $(\Delta\theta)_h$. He postulates that while on the one hand $(\Delta\theta)_h$ tends to decrease as the BL heats up, on the other hand, it tends to increase as the entrainment proceeds at h. The resulting equation is

$$\frac{d}{dt}(\Delta\theta)_h = W_E - \frac{Q_o}{h} + \frac{Q_h}{h} \qquad (4.6)$$

The second and third terms combined represent the situation inside the BL. It can be seen that this use of Eq. (4.6) obviates the problem inherent in using the formulation

$$W_E = -\frac{Q_h}{(\Delta\theta)_h} \qquad (4.7)$$

to determine h, as $(\Delta\theta)_h \to 0$.

<u>γ^+, the Lapse Rate of Potential Temperature Just Above the Jump at h</u>. Deardorff (1972a, 1974) and Tennekes (1973) have suggested that

entrainment rate is inversely proportional to lapse rate in the stable air above inversion. Deardorff (1972a) used this postulate only when $(\Delta\theta)_h$ approached a certain arbitrary small value. In a later paper (1974), he regards $(\Delta\theta)_h$ as small or uncertain, and evaluates entrainment by

$$W_E = \frac{1.2\ Q_o}{h\gamma^+} \tag{4.8}$$

Carson (1973) uses γ^+ to evaluate the intensity of inversion as a function of parameter A (Eq. (4.5)) and of h that varies with time.

Height h of the Convectively Unstable BL. If h becomes very small, most of the entrainment formulations listed in Table 2 break down. Actually, for small h, the dynamics of the surface layer would tend to dominate that of the entire BL. As a result, it has been considered advisable to prescribe a lower limit for h. For example, Deardorff (1972a) has suggested

$$h_{min} = \bar{z}_s + 50\ z_o$$

where $\bar{z}_s$ is the surface height, and z_o is the roughness length. Tennekes (1973) puts h_{min} at 100 m.

"Interpolation" Rate Equations for Determining Unstable Boundary Layer Height

We have discussed above the rate equation of h for a convectively unstable, dry, inversion-capped BL. The physical problems associated with entrainment are not well understood as yet, and hence most of the treatments are exploratory. Recently, Tennekes (1973) and Deardorff (1974) have suggested rate equations for h that are essentially *interpolation* formulations. For example, consider the equation

$$\frac{dh}{dt} - W_h = - \frac{Q_h}{(\Delta\theta)_h} \tag{4.9}$$

Again, if both Q_h and $(\Delta\theta)_h$ tend to zero, $dh/dt - W_h$ becomes indeterminate. However, Deardorff (1974) found from his numerical experiments that when both Q_h and $(\Delta\theta)_h$ become zero,

$$\left.\begin{aligned} &\frac{dh}{dt} - W_h = 0.2\left(\frac{g}{\theta_o}\, hQ_o\right)^{1/3} \\ \text{or}\qquad &\frac{dh}{dt} - W_h = 0.2\, W_* \end{aligned}\right\} \tag{4.10}$$

where the quantity $W_* \equiv \left(\frac{g}{\theta_o}\, hQ_o\right)^{1/3}$ is the mixed layer convective scale. He derives the following interpolation formula by combining Eqs. (4.9) and (4.10):

$$\frac{dh}{dt} - W_h = \frac{1.8\, Q_o\left[1 + 1.1\,\frac{U_*^3}{W_*^3}\left(1 - \frac{3fh}{U_*}\right)\right]}{\left[h\gamma^+ + \frac{gW_*^2}{\left(\frac{g}{\theta_0}\right)^2}\left(1 + 0.8\,\frac{U_*^2}{W_*^2}\right)\right]} \tag{4.11}$$

Equation (4.11) prevents the development of a singularity when

(i) $h \to 0$ or $\gamma^+ \to 0$

(ii) $Q_o \to 0$ and $\gamma^+ \to 0$

(iii) $Q_o \to 0$ *but* $\gamma^+ \neq 0$

Deardorff (1974) tested Eq. (4.11) (with $W_h = 0$) against 3-D model values of h as well as against observed and diagnostic values of h (Fig. 14). The disagreement between observed h and that obtained by rate Eq. (4.11) and the 3-D model can be attributed to his assumption of $W_h = 0$. Empirical adjustment of the numerical coefficients in Eq. (4.11) may be necessary to obtain accurate results.

Tennekes (1973) has also suggested an interpolation formula to treat cases in which BL turbulence is maintained by buoyancy as well as mechanical mixing. Thus,

$$\frac{dh}{dt} = 2.5 \frac{\theta_o}{g} \frac{1}{h} \frac{U_*^3}{(\Delta\theta)_h} + 0.2 \frac{Q_o}{(\Delta\theta)_h} \tag{4.12}$$

This formula cannot be used if h is too small. Also, for $(\Delta\theta)_h = 0$, dh/dt becomes infinite, a physically untenable situation. In that case a limiting value given by

$$\frac{dh}{dt} = 0.2 \ (\sigma_w) \tag{4.13}$$

should be prescribed for the entrainment velocity. Equation (4.13), which is similar to Eq. (4.10) used by Deardorff (1974), gives the rate at which a BL with $(\Delta\theta)_h$ and $\gamma^+ = 0$ entrains aloft (Tennekes and Lumley, 1972).

Rate Equation for a Stable Boundary Layer Height

The BL is almost always stably stratified over land at night, when it is typically much shallower than it was the previous afternoon. As discussed at the beginning of this chapter, h has evolution characteristics for the stable case also, though the variations are not as pronounced as convective conditions. It was also pointed out that there is some doubt about the validity of using a rate equation for h under stable conditions because BL depth then is usually small (in comparison with the similarity scale height $0.3\ U_*/f$) and not likely to be much affected by vertical velocity and turbulence, which are themselves small. Deardorff (1972a) suggests an interpolation formula between the neutral height of BL and a value of h proportional to Monin-Obukhov length L. His formula is

$$h = \left(\frac{1}{30L} + \frac{f}{0.35U_*} + \frac{1}{H_T}\right)^{-1}$$

where H_T is the height of tropopause. The last term ensures that, for a neutral condition at the equator, h does not exceed H_T.

Alternatively, h can be set at 50 m at the start and then allowed to increase at a rate proportional to U_* whenever surface flux (or stability) changes sign from positive to negative.

Despite an uncertainty regarding use of a rate equation for a stable BL, Deardorff (1971) did determine h for a stable BL by means of a prognostic equation:

$$\frac{dh}{dt} - W_h = 0.025\, U_*[1 - h/(0.35\, U_*/f)] \tag{4.14}$$

The factor in brackets has been included on the supposition that the similarity theory for a neutral case provides an upper limit to h in stable atmosphere. Thus

$$\frac{dh}{dt} - W_H - 0.025\, U_*\, \mathrm{Max}\left\{1 - \frac{h}{0.35\,\frac{U_*}{f}},\ 0\right\} \tag{4.15}$$

The factor (0.025) in this equation is only tentative and should be revised on the basis of observations. For example, observations of Clark et al. (1971) suggest a value of 0.04 or 0.05. Randall and Arakawa (1974) have used another version of Eq. (4.15) to predict boundary layer height for stable conditions:

$$\frac{dh}{dt} - W_h = 0.025\, U_*\, \mathrm{Max}\left\{1 - \frac{Ri_B}{Ri_C},\ 0\right\} \tag{4.16}$$

where Ri_B is bulk Richardson number and Ri_C is a critical value of Ri_B. According to Deardorff (1972a),

$$Ri_C = 3.05$$

RADIATIVE FLUX EFFECTS

Previous sections have discussed a rate equation for h that incorporates effects of upward eddy heat flux, large-scale vertical velocity, and entrainment at the interface between the mixed layer and the overlying stable atmosphere. These discussions did not consider the effect of radiative heat flux on the evolution of h in the presence of clouds, though the presence of water vapor can be accounted for by using virtual potential temperature.

Lilly (1968) first put forward a theory that considered radiation off the cloud tops as an essential element. His theory was applied to a marine boundary layer containing fog or stratus or stratocumulus clouds typically observed near the California coast, northern Chile, southern Peru, and southwest Africa most of the year. On the basis of his theory he concluded that the observed inversion (in the trade-wind regime) at a height of 500 to 1000 m and with an intensity of 15 to 20°C cannot be sustained without a radiatively effective cloud cover. This pointed out the importance of incorporating radiative heat flux effects in studying the evolution of a cloud-topped BL. Physically, for such a BL, strong flux divergence gives rise to strong radiative cooling (destabilization) at the BL top, which in turn causes a rise in BL height. Following Lilly (1968), Randall and Arakawa (1974) have described the buoyant instability of a cloud-topped BL in some detail. They have derived an expression for entrainment rate that incorporates effects of radiative heat flux as well as the presence of liquid water within the BL. However, according to Lilly (1968), the choice of entrainment approximations does not appear to strongly affect the general character of steady-state solutions.

Whereas the temporal evolution of a clear BL is fairly well known, that of a cloud-topped BL is still in the speculative stage. In a cloud-free BL, for example, net buoyancy through the BL and kinematic surface heat flux generally have the same direction (Fig. 16). The picture is less certain for a cloud-topped BL. It is possible that in this case net buoyancy and surface heat flux may act oppositely to each other (Fig. 17). Net buoyancy of the BL can be positive (upward), even though surface flux is negative (downward) (Randall and Arakawa, 1974). Lilly (1968) had suggested this argument on the consideration that while the cloud layer is maintained in an active turbulent state because of the release of latent heat there, the sub-cloud layer is characterized by a downward heat flux.

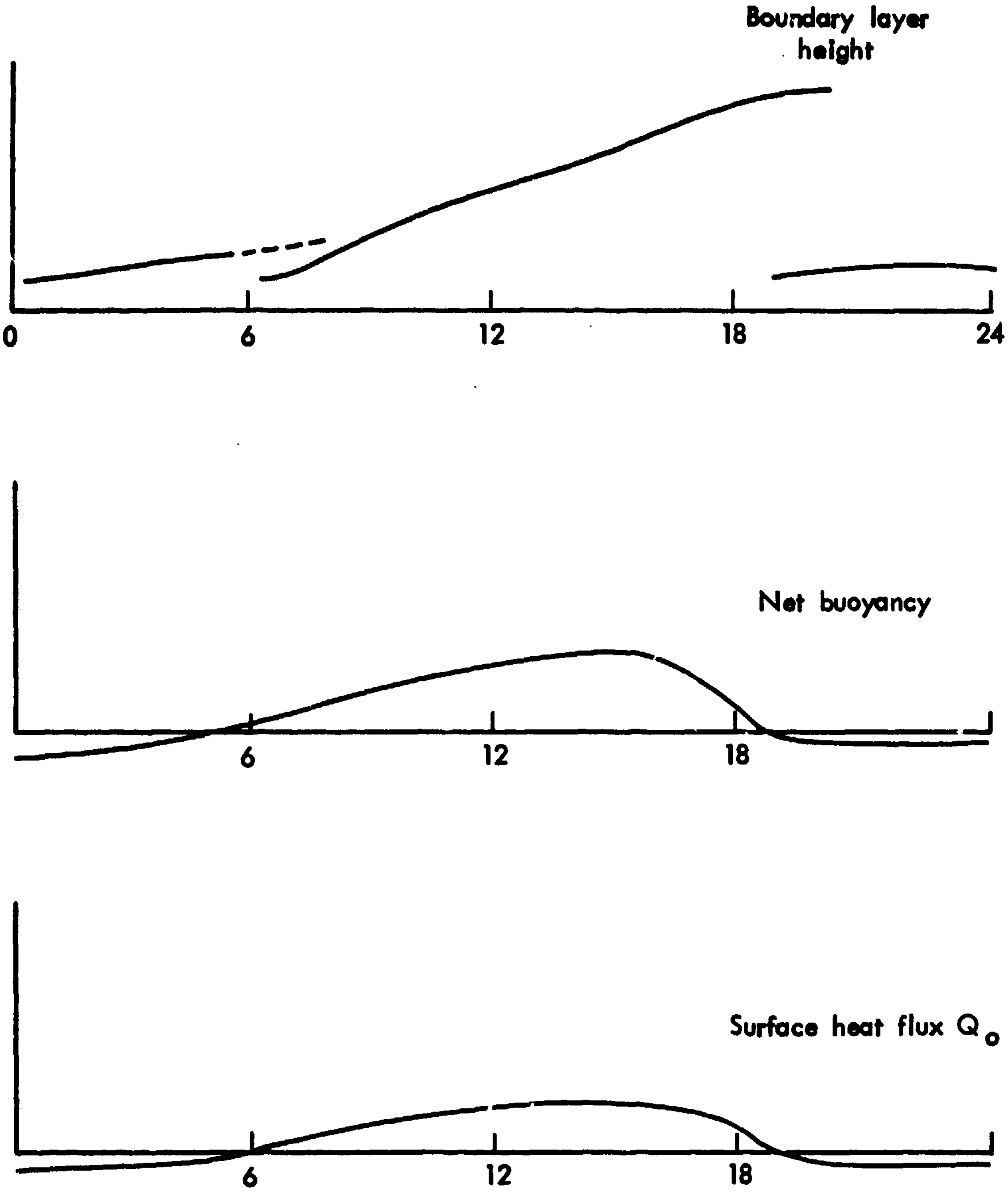


Fig. 16 — Diurnal cycle of clear boundary layer
(Randall and Arakawa, 1974)

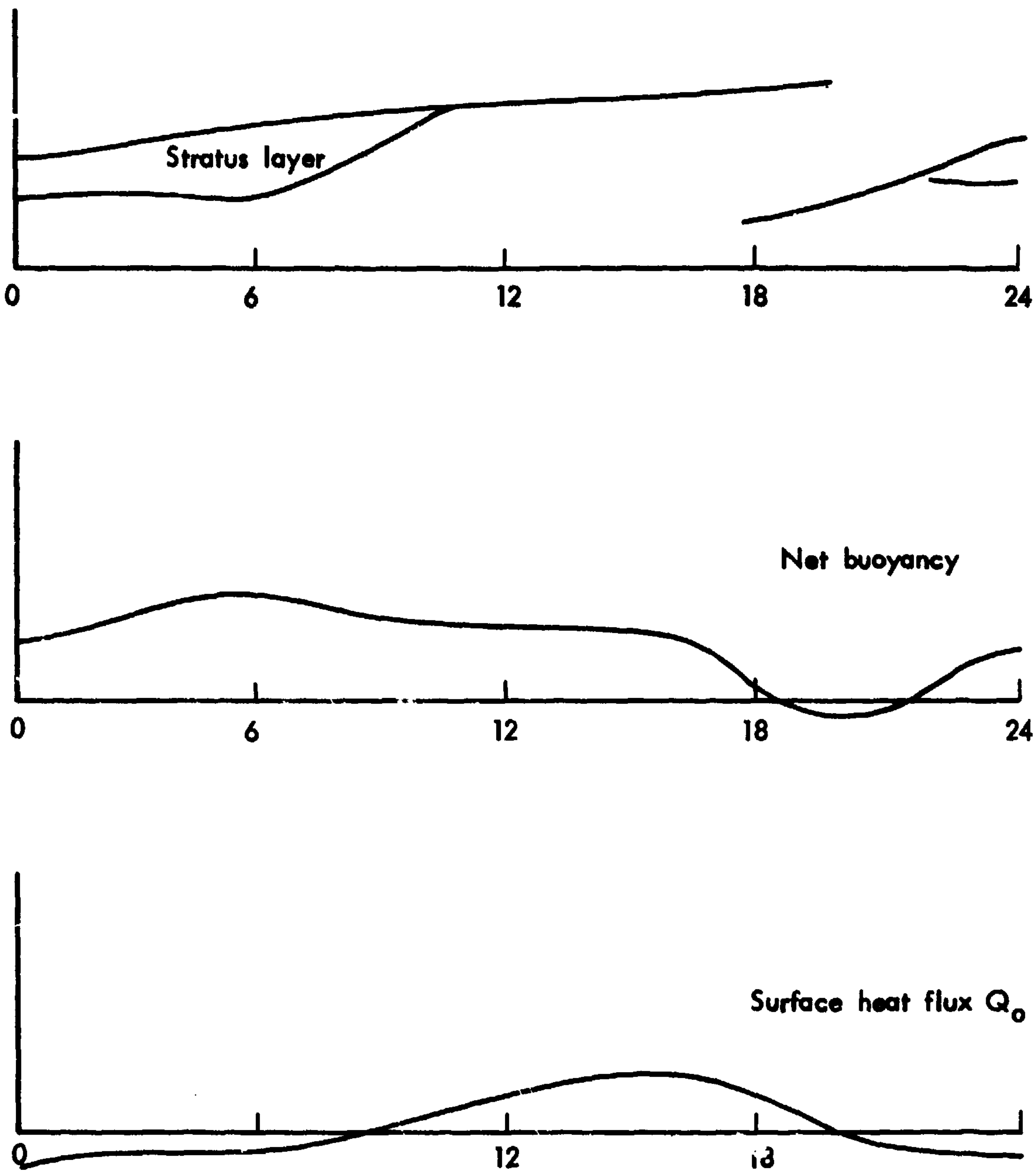


Fig. 17 — Diurnal cycle of cloud topped boundary layer
(Randall and Arakawa, 1974)

Mak (1974) has also studied the role of radiational cooling in a BL. He was motivated by the observation (Lenschow, 1970) that eddy flux decreases markedly with height throughout the BL. He found that for typical circumstances in real atmosphere, no equilibrium BL exists resulting from a balance between eddy heat flux convergence and divergence of radiative flux, because the latter is relatively too weak and other factors predominate. However, he found that if water vapor is mostly concentrated near the surface (in fog, say), an equilibrium BL is possible due to existence of a nearly adiabatic layer overlying a strongly superadiabatic layer. The rapid decrease of sensible heat flux with height is compensated by corresponding radiational cooling. Tennekes (1973) suggests that since radiative heat flux tends to decrease the net heating rate of the BL, it can be parameterized by making a small reduction in the surface heat flux.

GENERAL REMARKS

This chapter has discussed the determination of BL height, h, for both unstable and stable conditions, which are respectively characterized by upward and downward surface heat flux. We have considered both diagnostic and prognostic methods for calculating h. We have also seen that diagnostic methods are inadequate for describing an evolving BL. As regards prognostic equations, we have seen that the large-scale vertical velocity, and the entrainment at the BL, exercise a determining influence on an unstable cloud-free BL. If the BL has cloud layer embedded at its top, the radiative heat flux also significantly affects h. There is some doubt regarding the use of a prognostic equation for a stable BL height.

The foregoing discussions have tacitly assumed that the height of the thermal (mixed) BL is equal to the height of momentum as well as moisture layers. Not many studies have considered prediction of momentum BL height independently of the thermal BL height. However, Deardorff (1973a) has studied the effects of entrainment across a velocity discontinuity at the top of a convectively mixed BL having a wind shear above. He found that the entrainment produces large values of negative stress in the upper two-thirds of a BL. For a positive wind shear above the BL, he obtained large positive values for Reynolds stress in the upper portion of the BL.

Chapter 5

ATMOSPHERIC BOUNDARY LAYER AT LOW LATITUDES

GENERAL FEATURES

The dynamics of BL flow at low latitudes is not well understood because of the simultaneous importance of momentum transport by both turbulent and large-scale motions. Various observational and theoretical studies have revealed the following salient features of the atmospheric BL at low latitudes:

1. Momentum advections are important mainly equatorward from the Intertropical Convergence Zone (ITCZ) (Janota, 1971).

2. The low-level wind vector generally rotates with height, and this is mainly controlled by the height variation of the horizontal pressure gradient (Gray, 1968; Estoque, 1970).

3. Advective accelerations, associated with the rapid latitudinal variation of coriolis parameter, are important in the BL between the equator and a narrow latitudinal transition (Mahrt, 1972a).

4. BL depth is relatively thin and varies slowly with latitude even near the equator; surface cross-isobar angle increases towards the equator and the cross-isobar flow increases with height throughout the BL (Mahrt, 1972a; Kraus, 1972).

5. Moisture, in general, is predominantly confined to the lowest levels near the surface, and thus the BL is essential in the thermodynamics of (low-latitude) circulation systems. This was pointed out by Charney and Eliassen (1964) who, via the CISK theory, postulated that the BL provides moisture convergence for latent heat release in the free atmosphere, which in turn enables the latter to induce upon the boundary layer a pressure field conducive to the low-level flow convergence. However, the application of CISK theory has suffered because of lack of knowledge of BL dynamics in the low latitudes, particularly advective accelerations.

6. The ITCZ, in a climatological sense, is a narrow east-west band of vigorous cumulonimbus convection and heavy precipitation that forms along the equatorward boundary of the trade-wind regimes. (The

picture is more complex when viewed at any particular instant in time.) ITCZ is a unique convective feature directly related to the *larger-scale* circulations in the atmosphere and also to the properties of the oceanic mixed layer (i.e, the sea surface temperature) (Pike, 1970). Also, because of the persistent cloudiness associated with ITCZ, radiation fluxes are important even though the meridional extent of ITCZ is small.

7. A "critical" latitude concept has been put forward to explain, theoretically, the development of ITCZ or a mean convergence zone (Holton et al., 1971; Mahrt, 1972b). This concept assumes that the depth of the Ekman layer depends on the frequency of the wave disturbances in the BL, with a *singularity* at that latitude at which wave frequency is equal to the coriolis frequency. The range of "critical latitude" corresponding to this frequency band is 6° to 7°, which is close to the most frequent ITCZ position.

8. The idea of critical latitude also has been suggested by observations, though on different considerations from those noted above. For example, Janota (1968), on the basis of detailed observational analysis, pointed out that flow characteristics change near 4°N and that the Ekman character of the BL vanishes south of that latitude.

9. There have been various attempts to replace Ekman-type parameterization of BL convergence with the critical latitude concept (Yamasaki, 1971; Hayashi, 1971). They all treat a BL with a top at or around 900 mb. However, observations (Wallace, 1971; Reed and Recker, 1971) have shown that only 10 to 30 percent of the synoptic scale convergence takes place below 900 mb.

10. BL depth in the neighborhood of ITCZ has no clear definition because, with increasing convection, clouds penetrate the stable inversion layer and the fluxes generated at the surface may reach up to the high troposphere. In these regions (low latitude), convective activity generates a vertical circulation that couples sea-surface processes with the upper levels of the atmosphere.

11. Budget studies of cloud clusters embedded in trade winds and ITCZ (Janota, 1971; Yanai et al., 1973, and others) have shown that mean large-scale vertical velocity is directed upward throughout the

entire troposphere. Because there is more mass transport in clouds as compared with the mean large-scale mass flux, there is a compensating sinking motion in clear areas between clouds that reaches its maximum at cloud base (Ogura and Cho, 1973; Yanai et al., 1973). Thus for strongly convective situations (in low latitudes), the stable transitional layer just below cloud-base level acts as an upper barrier for turbulent transports in clear areas between clouds.

PARAMETERIZATION OF THE BOUNDARY LAYER IN LOW-LATITUDE REGIONS

It is now well known that low-latitude regions are characterized by deep convection of subsynoptic scale. Thus, if that convection significantly influences large-scale circulations, and if the processes in the BL have a determining influence on the development or decay of the deep convective systems, then it is obvious that any large-scale model must include parameterization of BL processes in terms of large-scale variables.

Several different types of BL have been devised--for instance, those based on K-theory (Estoque and Bhumralkar, 1969; Yamamoto and Shimanuki, 1966, etc.), on similarity theory (Kazanski and Monin, 1961; Blackadar and Tennekes, 1968; Zilitinkevich, 1969, 1970, and others), and so-called "entity" models (Deardorff, 1973; Tennekes, 1973; Orlanski et al., 1974, and others). But *none* of these parameterization schemes is specifically designed to treat the problem of the low-latitude unstable, baroclinic, inhomogeneous, and transient atmospheric BL commonly found in these regions. Furthermore, simple application of the existing models is highly unlikely to be feasible, at least not until they are improved and refined on the basis of a better physical understanding of BL processes in low latitudes.

With this in view, the Global Atmospheric Research Programme (GARP) has undertaken a Boundary-Layer Subprogramme (BLSP) for the GARP Atlantic Tropical Experiment (GATE), whose objectives have been described in GATE Report No. 5, December 1973. The basic approach of the BLSP is to use existing parameterization schemes as a basis for the experimental programme. One goal is to improve those models by

- Testing the assumptions of each scheme, and
- Determining the importance of physical processes not yet incorporated in the models.

It is evident that any model of the low-latitude BL must necessarily consider the problem of the dynamics at low latitudes, where the coriolis parameter changes significantly, accelerations affect the boundary structure, and cold ocean water generates strong baroclinity (Hoeber, 1974). Also, the most important question relating to the interaction between the mixed layer (usually found in oceanic low latitudes) and the cloud layer has to be considered.

Recently, Deardorff (unpublished) has developed a parameterization scheme specifically for the low-latitude BL. His scheme is based on the general method suggested by Deardorff (1972a), Betts (1973), Tennekes (1973), and Carson (1973) for the clear-skies case and by Lilly (1968) for the case of solid stratocumulus overcast. However, he has extended this method to apply to conditions of *partial* cumulus cloud cover (with bases at the top of the transition layer), which is more usually prevalent in low latitudes. He has expressed the entrainment velocity (W_e in Chap. 4), large-scale vertical velocity at the top of the BL (W_h), and net radiative heat flux from cloud edges and tops in terms of fractional cloud cover. Tests of his modified parameterization scheme have generally given realistic results; e.g.,:

- Cloud-induced subsidence controls the growth of the mixed BL;
- Cloud-base height stays just above the top of the mixed layer;
- Sea surface temperature and large-scale vertical motion variations control the height of the mixed layer, cloudiness, and other properties;
- A vertical velocity (subsidence) of 2 cm/sec at a height of 1 km is sufficient to cause the complete disappearance of convective clouds.

Some of Deardorff's findings, especially the one regarding the role of sinking motion, have also been obtained through much simpler models that treat the mixed layer as a slab (Geisler and Kraus, 1969). The "slab" model computations have shown that the mixed layer continues to deepen unless the atmosphere counteracts entrainment by subsidence. The larger the sinking motion, the smaller the buoyancy forces and the shallower the mixed layer. Large subsidence (typical of low-latitude regions) also results in a large deviation of the BL wind from the geostrophic wind. The model equations of Geisler and Kraus include a drag coefficient whose increased value causes the mixed layer to deepen because of the enhanced generation of turbulent energy. This in turn causes enhanced entrainment of potentially warm air with a geostrophic value of momentum from above. However, because of *increased* air-sea interaction (caused by increased drag coefficient) the model, in substance, yields larger ageostrophic wind and lower temperatures in the mixed layer.

Chapter 6

ATMOSPHERIC BOUNDARY LAYER (ABL) OVER OCEANS

The underlying surface of the atmosphere strongly affects atmospheric processes. Heat transfer and friction, both highly variable in space and time, are especially significant. The atmospheric boundary layer (ABL) is therefore important in studies of general circulation, particularly the ABL over oceans, which cover 71 percent of the earth's surface and predominate in tropical low-latitude regions. Consequently, Chap. 5 is by and large applicable to marine boundary layers also; this chapter, however, deals exclusively with BLs over the sea.

The following are the two essential physical differences between ABL conditions over the ocean and over land.

1. *Roughness*. For continents, whose land surface is almost totally fixed, we can consider roughness as characteristic of the surface itself and independent of the flow. The roughness of oceans is determined by interactions of the turbulent motions of air and water and is therefore variable (see Eqs. (2.9) and (2.10) in Chap. 2).
2. *Storage of heat*. Compared with temperature variations over land, the temperature of water surface does not change appreciably. Also, the dominant daily variation between strong instability and stability over land is nearly absent over sea; the stability or instability of the marine BL is determined predominantly through advection.

It is difficult to describe the turbulent structure in the marine ABL because the sea surface permits only a time-space statistical description, and atmospheric turbulence and the state of the sea interact. The state of the sea cannot be described simply as a local phenomenon due to waves traveling out of the generating area into other regions far from their origin. In fact, some scientists (Hasse, 1970) believe that the above interactions make it hopeless to try to parameterize turbulent transports in a marine ABL.

However, Clark (1970) suggested a simplified approach to parameterize an oceanic BL. He avoided the difficulties of modeling air-sea interaction by assuming a constant sea-surface temperature and a variable roughness parameter, z_o. Otherwise, his model equations were the same ones he used for the ABL over land (see Chap. 3 above). Monin and Zilitinkevich (1970) considered the parameterization of a marine ABL with respect to large-scale atmospheric processes on the assumption that the wind and wave fields are mutually consistent (adapted). They defined an analog of local Reynolds number (R_w) for the water surface by using U_*^2/g as a scale for measuring height; and instead of considering the conventional roughness length, they used an effective roughness length given by

$$z_e = \frac{U_*^2}{g} C_F(R_w)$$

where C_F is a coefficient depending on R_w. Similarly, they expressed differences of temperature ($\Delta\theta$) and moisture (Δq) between the surface value and the mean value (within the BL) as

$$\Delta\theta = \theta_* C_{F\theta}(R_w)$$

$$\Delta a = q_* C_{Fq}(R_w)$$

$C_{F\theta}$, C_{Fq} being coefficients depending on R_w. They regarded the sea-surface water temperature as constant, on the consideration that oceans have extremely large heat inertia. Perhaps they did so because they could not determine sea-surface water temperature by solving a heat balance equation at the sea surface. Recognizing that in reality the knowledge of coefficients C_F, $C_{F\theta}$, C_{Fq} is very poor indeed, they, following Charnock (1955), specified

$$C_F = 0.035$$

and

$$C_{F\theta} = C_{Fq} = 0$$

Pandolfo (1970) has described a nonlinear numerical model of the atmosphere-ocean BL that represents a complex local theory for the study of the vertical structure in the marine ABL. However, it is necessary to prescribe horizontal gradients if the model is to incorporate the effects of horizontal advection. The model uses eddy exchange coefficients (for dependent variables) that are functions of vertical gradients of velocity, temperature (both in atmosphere and ocean), humidity (in atmosphere), and salinity (in ocean). The model equations consider the effect of ocean currents on the momentum source terms.

Radiative flux convergence is incorporated as a temperature source term for computing both the air and water temperatures. The most important deficiency of the model appears to be requirement of some explicit form of stability and wind-wave dependence of eddy coefficients, which is not known at all.

Hasse (1970) has also parameterized stress, heat, and moisture fluxes in the ABL at sea by using a drag coefficient dependent upon stability. However, this parameterization holds only as long as there is no damping of turbulence due to strong stability, which can occur over inland seas and in coastal regions with cold water. Also, drag coefficients are uncertain for higher wind velocities and thus the determination of transports in strong wind regimes is uncertain.

Chapter 7

HIGHER-MOMENT APPROACH TO BOUNDARY-LAYER MODELING

GENERAL DESCRIPTION

A three-dimensional numerical model of Deardorff (1972b) has shown considerable capability of supplying many fine details of BL turbulence structure. The model parameterizes the subgrid Reynolds stresses by means of nonlinear eddy coefficients--a technique proven inadequate in the treatment of a stably stratified layer overlying a well-mixed convective layer. We have seen that the magnitude of the eddy coefficient for heat (in these conditions) proves too large near the top of the BL, and thus excessively smoothes out the temperature jump so evident in natural situations. This problem has been handled by Wilhelmson and Ogura (1972), who use an eddy coefficient term in the equation of temperature deviation from the mean state, but apply no diffusion to the mean state. There are some other limitations and uncertainties in using the eddy coefficient to parameterize subgrid scale fluctuations; above the surface layer, for example, the magnitude of the eddy coefficient and its dependence upon stability are generally not known. (Other limitations were discussed in Chap. 2.)

To overcome the problems associated with lowest-order (K-theory) closure assumptions, a higher-moment model has been suggested by Donaldson (1972). His approach is to use *subgrid transport equations*, and thereby obviate the need to parameterize subgrid (Reynolds) stresses and fluxes. In this technique, mean motion and turbulence are separated in a manner usually employed in treating BL flow but, in addition, the equations for mean velocity and temperature are combined with equations for higher-order statistics, such as variance and covariance, and the resulting equations contain triple products. It may be recognized that for a detailed 3-D numerical study of the BL, the use of higher-moment theory involves equations for 15 independent variables (9 components of velocity gradient tensor, 3 components of the temperature gradient, and 3 components of the moisture gradient). These 15 equations are in addition to a set of three velocity components, temperature, moisture, and pressure. (For typical examples of equations for subgrid Reynolds stresses or fluxes, see Deardorff (1973; 1974), Donaldson (1973).)

Experimentation with higher-moment closure theory is in its very early stages, and most of the studies have used only second-moment equations for generalized subgrid stresses and fluxes. Several problems must be resolved before this technique can be used successfully in BL research--especially for the stable BL. Some of the problems are:

- Ad hoc relationships between third-order terms and gradients of second-order terms are assumed. As a consequence, in some models (Donaldson, 1972), more undetermined quantities have to be introduced with the equations.
- Closure assumption uses a height-dependent mixing length that must be assumed (Donaldson, 1972). This assumption has been avoided in some cases by taking a representative grid scale (Δ) as the relevant length scale in determining the magnitude of subgrid scale quantities. In doing so, however, the constants associated with terms stemming from pressure fluctuation correlations must depend upon the grid aspect ratios $\Delta x/\Delta z$, $\Delta y/\Delta z$, when the latter are not unity (Deardorff, 1973). Presently, not much is known about this dependence.
- The 15 equations for stress and fluxes require computer storage at each grid point and for at least one past time increment. The complexity of these equations in finite difference form increases the burden on computer requirements. Deardorff (1974) indicates that his 3-D model using higher-moment closure assumptions consumes 2-1/2 times as much computer time as it would if eddy coefficients were used instead. To reduce computer requirements, a steady-state nonadvecting

observation of the equations has been suggested, but this version is not self-consistent and tensor-invariant.

APPLICATION OF HIGHER-ORDER MODELING TO THE ATMOSPHERIC BOUNDARY LAYER

Current experimentation in applying the invariant higher-order (second-order) closure model of turbulent shear flow to the ABL is still in its early stages. So far Donaldson (1972, 1973), Deardorff (1973, 1974), and Wyngaard et al. (1974) have published their research in this connection. Deardorff (1974) has studied a heated (well-mixed) BL numerically in a 3-D model using 64,000 grid points (within a volume 5 km × 5 km × 2 km) by using subgrid transport equations in place of eddy coefficient formulations. His results compare very well with Wangara data from southeast Australia (Clark et al., 1971). This model suggests that the momentum BL coincides with the mixed (thermal) BL during the hours of solar heating of the surface; in other words, he finds invalid the concept that, within the mixed layer, the stress vanishes at the lowest height at which the corresponding wind-shear component vanishes. Wyngaard et al. (1974) also use a higher-order-closure model to model the convective BL. They have shown that although coriolis forces cause large production rates of shear, the mechanism associated with mean wind shear prevents the stress level from becoming large. They aver that, in absence of thermal wind, the stress problems are essentially linear regardless of wind direction.

Higher-order closure models have not yet been applied in atmospheric general circulation models, essentially because of prohibitive computer requirements.

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