

AD-A013 326

INFLUENCE OF HIGH HYDROSTATIC PRESSURE EXTRUSION ON
MECHANICAL BEHAVIOR OF MATERIALS

Oleg D. Sherby, et al

Stanford University

Prepared for:

Office of Naval Research
Advanced Research Projects Agency

July 1975

DISTRIBUTED BY:

NTIS

National Technical Information Service
U. S. DEPARTMENT OF COMMERCE

ADA013326

230102

FIRST ANNUAL TECHNICAL REPORT

June 1, 1974 - May 30, 1975

INFLUENCE OF HIGH HYDROSTATIC PRESSURE EXTRUSION

ON

MECHANICAL BEHAVIOR OF MATERIALS

Sponsored by
Advanced Research Projects Agency
ARPA Order No. 2783

Program Code Number: N63375

Contractor: Stanford University

Effective Date of Contract: June 1, 1974

Contract Expiration Date: May 31, 1975

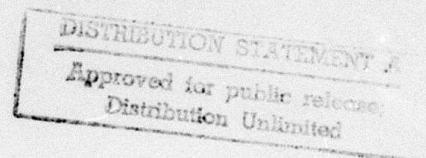
Amount of Grant: \$80,000

Contract Number: N0014-67-A-0112-0083

Principal Investigators: Oleg D. Sherby & John C. Shyne
Phone: (415) 497-2636 or 497-2534

Co-Principal Investigator: Erastus H. Lee
Phone: (415) 497-4067

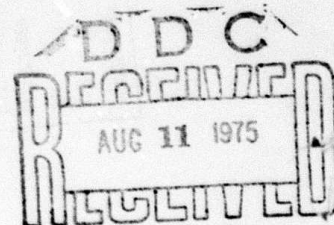
Scientific Officer: Director, Metallurgy Program
Materials Science Division
Department of the Navy
Office of Naval Research
800 North Quincy Street
Arlington, Virginia 22217



Short Title: Hydrostatic Extrusion

The views and conclusions contained in this document are those of the authors and should not be interpreted as necessarily representing the official policies, either expressed or implied, of the Advanced Research Projects Agency or the U. S. Government.

Department of Materials Science and Engineering
Stanford University
Stanford, California 94305
(415) 497-2534
SU-DMS-75-T-64
July 1975



A

Reproduced by
NATIONAL TECHNICAL
INFORMATION SERVICE
US Department of Commerce
Springfield, VA. 22151

Unclassified

Security Classification

DOCUMENT CONTROL DATA - R & D

(Security classification of title, body of abstract and indexing annotation must be entered when the overall report is classified)

1. ORIGINATING ACTIVITY (Corporate author) Department of Materials Science and Engineering Stanford University Stanford, California 94305		2a. REPORT SECURITY CLASSIFICATION Unclassified	
3. REPORT TITLE INFLUENCE OF HIGH HYDROSTATIC PRESSURE EXTRUSION ON MECHANICAL BEHAVIOR OF MATERIALS		2b. GROUP	
4. DESCRIPTIVE NOTES (Type of report and inclusive dates) First Annual Technical Report - June 1, 1974 - May 30, 1975			
5. AUTHOR(S) (First name, middle initial, last name) Oleg D. Sherby, John C. Shyne and Erastus H. Lee			
6. REPORT DATE May 1975		7a. TOTAL NO. OF PAGES 25 43	7b. NO. OF REFS 17
8a. CONTRACT OR GRANT NO. N00014-67-A-0112-0083		9a. ORIGINATOR'S REPORT NUMBER(S)	
b. PROJECT NO. 471		9b. OTHER REPORT NO(S) (Any other numbers that may be assigned this report)	
c.			
d.			
10. DISTRIBUTION STATEMENT Unlimited			
11. SUPPLEMENTARY NOTES		12. SPONSORING MILITARY ACTIVITY Procuring Contracting Officer Department of the Navy Arlington, Virginia 22217	
13. ABSTRACT Hydrostatic extrusion is a promising new metal working process. The intent of this research program is to relate the process variables of hydrostatic extrusion to the microstructure and properties of a wide variety of extruded materials and to analyze this process of metal forming by theoretical mechanics. A hydrostatic extrusion press has been designed and built for these experiments. This device is capable of subjecting a 3/8 inch diameter billet to 500,000 psi of hydrostatic pressure. Special tooling has been built for the mechanical testing of the experimentally extruded materials. Certain specialized specimen materials preparation equipment has been set up, for example a mechanical attritor to facilitate the preparation of particulate composite alloys. The theoretical mechanics analysis has been directed toward the development of a practical finite element analysis of the mechanics of the hydrostatic extrusion process. Progress has been made toward a physically and mathematically rigorous formulation of the problem amenable to numerical solution by means of finite element computer methods.			

Unclassified

Security Classification

14

KEY WORDS

hydrostatic extrusion
mechanical properties
stress analysis
extrusion pressure
extrusion rate
extrusion ratio

LINK A

LINK B

LINK C

ROLE

WT

ROLE

WT

ROLE

WT

;;
//

Unclassified

Security Classification

I. Introduction

Hydrostatic extrusion is the metal working operation that consists in forcing metal through a die as in conventional extrusion but with the addition of a superimposed hydrostatic pressure. The effect of the superimposed hydrostatic pressure is to permit the extrusion of materials not practically extrudable with the conventional process, it allows greater extrusion ratios to be used, and it seems to impart improved mechanical properties. This promising metal forming technique has already had some technological applications, and its potential for industrial utilization is very attractive.

Despite the considerable technological promise of hydrostatic extrusion there is a lack of understanding about the basic engineering mechanics of the process, and there is little information about how the variables of the process relate to the resulting mechanical properties of hydrostatically extruded materials. It is the intent of the present program to make inroads on both these deficiencies.

Our work on hydrostatic extrusion follows three main lines of investigation:

1. The systematic investigation of the influence of the processing variables (extrusion ratio, extrusion rate, superimposed hydrostatic pressure, extrusion temperature) and the resulting mechanical properties and microstructure of several fairly simple materials such as pure Fe, Ni, Mg and Ti; and Cu-30Zn.
2. Exploratory attempts to improve the properties of more complex materials chosen for their technological utility such as Mg-Li-B for high specific stiffness, Fe-C for high strength, T-D nichrome for high temperature strength, Ti-6Al-4V for its high strength to weight ratio, and NbTi a metallic superconductor.
3. Analysis of the mechanics of hydrostatic extrusion. This will involve both theoretical mechanics and complementary experiments to test the theoretical predictions. The thrust of the theoretical mechanics program is toward the

development of finite element analysis methods capable of yielding an accurate numerical description of the hydrostatic extrusion process taking into account the deformation characteristics of the extruded materials, die geometry, strain rates, etcetera.

During this first year of work considerable progress has been made toward developing experimental apparatus and techniques, and in overcoming some formidable analytical and computational problems. These are detailed in this report.

II. Experimental Equipment and Techniques

1. Hydrostatic Extrusion Press

The key piece of apparatus for this work is, of course, the hydrostatic extrusion press. Funds for the purchase of the hydrostatic extrusion press were provided by an equipment grant from the National Science Foundation and by Stanford University. This device has been designed and built by Revere Copper and Brass Company. The time required for the construction of the extrusion press considerably exceeded the manufacturers estimate. Originally planned for installation at Stanford University in January of 1975; it was delivered early in June 1975 and as yet no experimental hydrostatic extrusions have been made with it.

The hydrostatic extrusion press was given trial runs at the Rome, New York plant of the Revere Copper and Brass Company where Mr. Robert Whalen of Stanford observed and discussed all details of the design and operation of the device with Revere engineers. Much of our consideration in the initial operation of the apparatus will be with safety, both with respect to the operating personnel and for the equipment itself. Safety is obviously a matter of great importance in the successful operation of ultra high pressure equipment.

In most respects the design and fabrication of the Revere Copper and Brass Company hydrostatic machine, Model 0310, has followed very closely the normal pattern of prototype equipment development. Major and minor design alterations

and modifications have resulted in numerous unpredicted delays. It is true that one other prototype of this basic design had been built, but significant improvements have clearly established the uniqueness and superiority of the Stanford model. The original machine should be considered a prototype for industrial applications with lesser emphasis on its value as a basic research tool. Our machine has been altered somewhat to provide more precise control of those processing variables (e.g. temperature, extrusion rate, back pressure) involved with the empirical and theoretical prediction of material and processing properties. Thus its utility is essentially for research with indirect applications to industrial process development. It is with great enthusiasm that we await the installation of the hydrostatic extrusion machine at Stanford scheduled for June 1975.

The hydrostatic extrusion system consists of three separate parts (Fig. 1) the extruder, the hydraulic control system, and the control panel which electronically links the two together. In addition to the control panel, the extruder and hydraulic system are interconnected by a system of microswitches mounted on the extruder. These switches, which will be discussed in more detail later, establish the position of the piston for extrusion and behave as limit switches as a precaution against over-extruding.

The hydraulic system was custom manufactured by Airline Hydraulic Corporation to provide the necessary oil flow and pressure as regulated by the specific function controls to be set by the operator. Since the extruder and hydraulic system work together, it is best to combine the discussion of their operation.

From the standpoint of design, the extruder is the more interesting. The extrusion cylinder and mounting stand occupy a volume of approximately sixty-two inches long by sixty-four inches high by forty-eight inches wide. The extrusion cylinder which is about fifty-two inches long is divided into two parts with

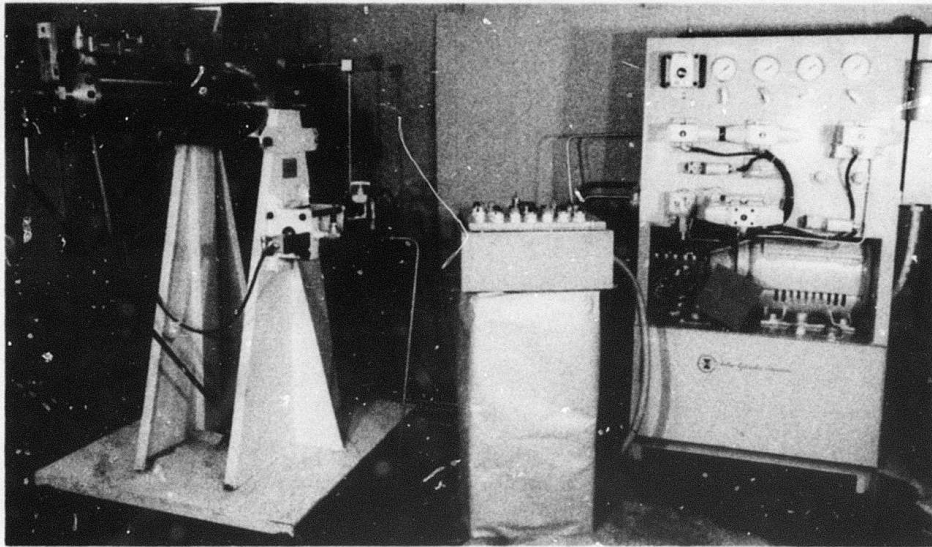


Figure 1. Hydrostatic Extrusion system.

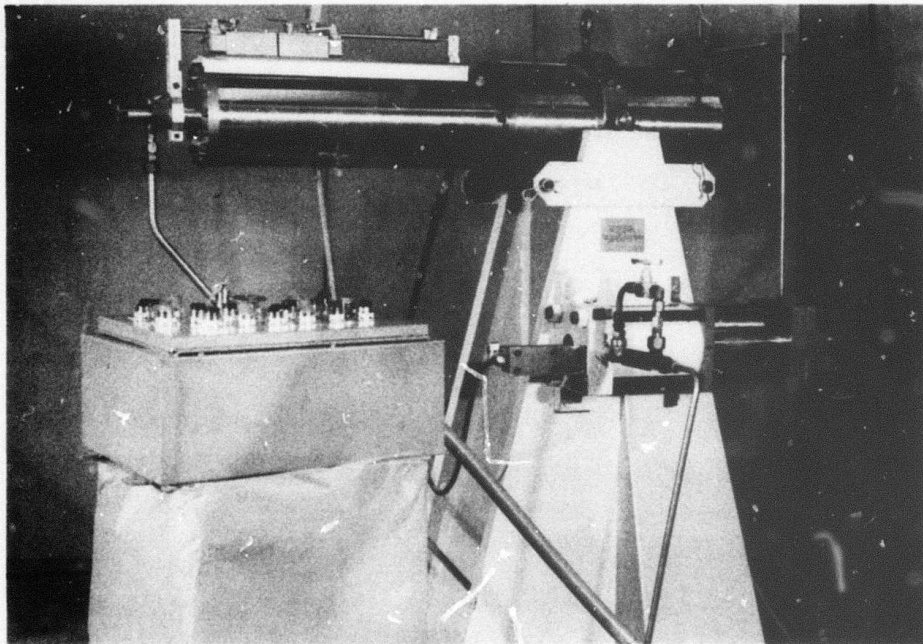


Figure 2. Hydrostatic Extrusion and control panel.

distinct diameters. The smaller diameter section of the cylinder contains the extrusion piston and die stem guides. On top of the cylinder are located the previously mentioned microswitches (see Fig. 2 and drawing 3a). Also visible in Fig. 2 are the hydraulic ports mounted on the face and wall of the small diameter cylinder. The attached larger diameter port of the cylinder contains the extrusion load chamber and all the accompanying equipment necessary for the containment of the high pressures which develop during extrusion (see Fig. 2 and drawing 3b).

The floating die jacket design for the containment of the high hydrostatic pressures is quite interesting. It was chosen over other designs for several reasons. For one, the cylinder fatigue life (estimated to be 10^6 cycles) is far superior than with other systems designed to hold comparable pressures. Secondly, this design is more tolerant of dimensional changes which must naturally occur under such loading conditions. When changes do occur the machine will either continue to operate normally or require only minor adjustments of the die size. Finally, loading and reloading for either direct or indirect extrusion is rapid, permitting many extrusions in a day.

Although some of the ideas and principles involved are ingenious the operation of the extrusion press is quite simple. To extrude in the indirect mode the following steps are taken. The reverse button is pushed on the control panel to fully retract the piston, die stem, and die stem guides. The microswitches are adjusted for the proper extrusion stroke length. Next the die, billet, oil and high pressure seal placed into the loading tube shown in Fig. 4. The loading tube is used to transfer the assembled die, billet, oil, and seal intact into the liner of the die jacket in the high pressure extrusion chamber. Once the transfer has taken place, a tungsten carbide anvil is secured against the seal to prevent backward motion due to the hydrostatic pressure, (see drawing

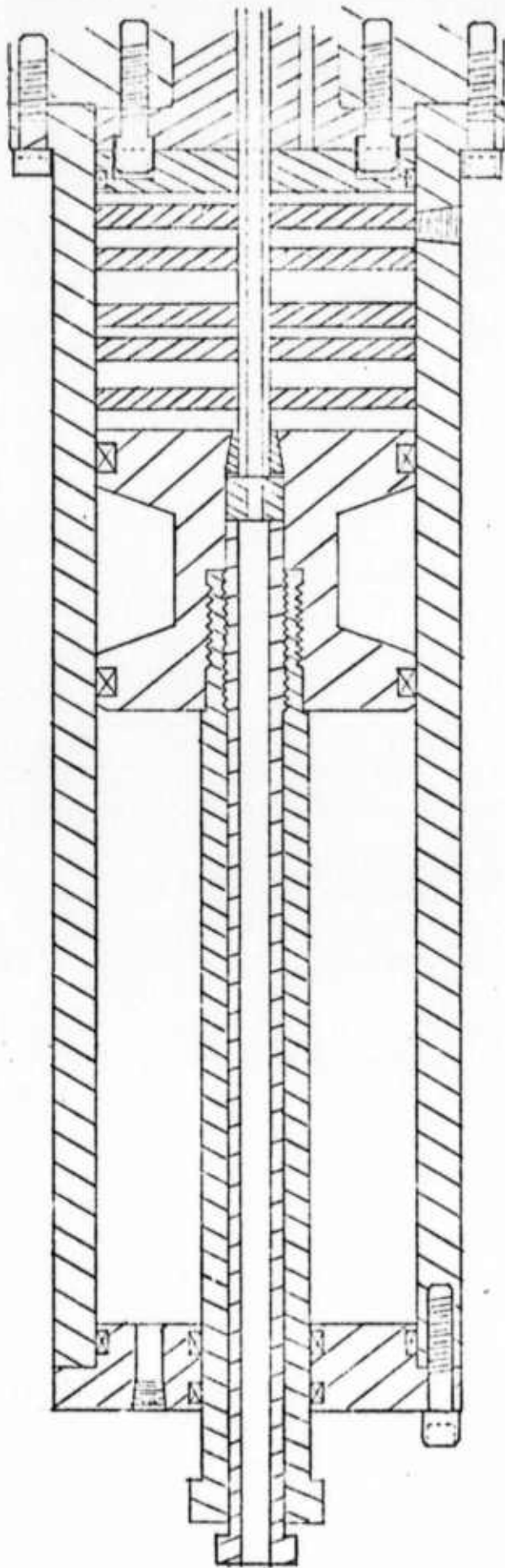


Figure 3a. Cross sectional drawing of extrusion piston, die stem and die stem guides.

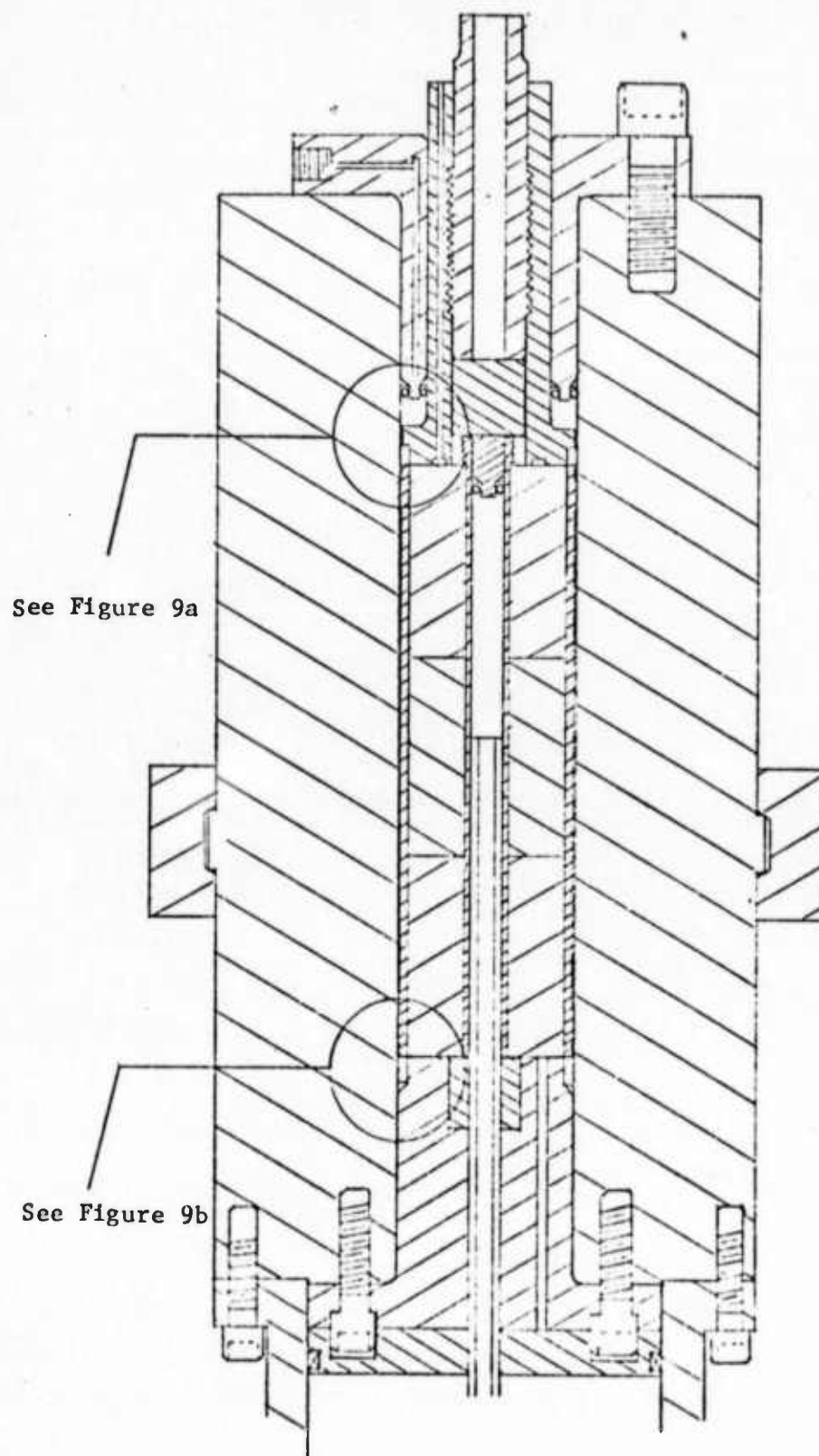


Figure 3b. Cross sectional drawing of high pressure extrusion cylinder.

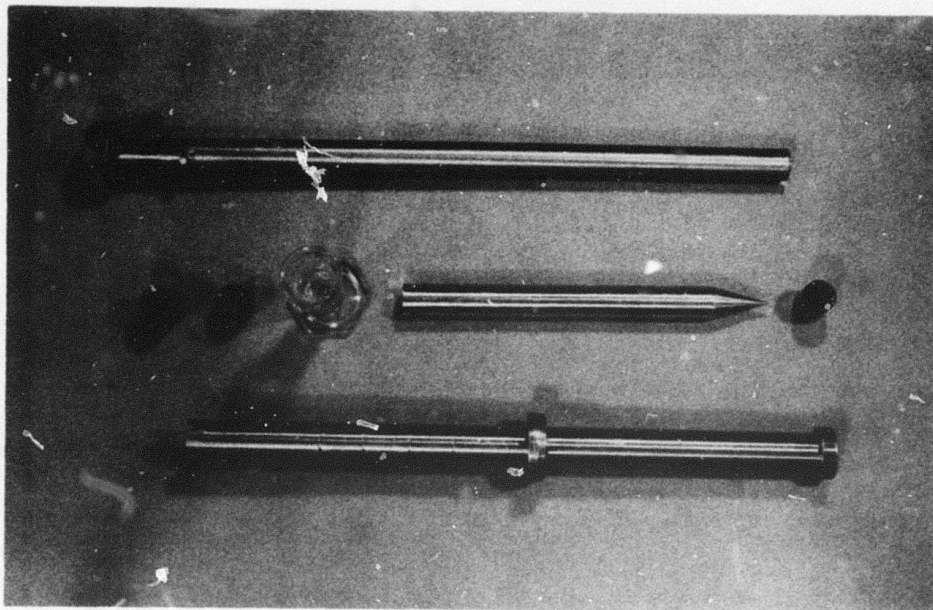


Figure 4. Proper loading tube assembly for indirect extrusion. In the order shown, die, billet, oil, and high pressure seal are loaded into the tube at the bottom of the photo.

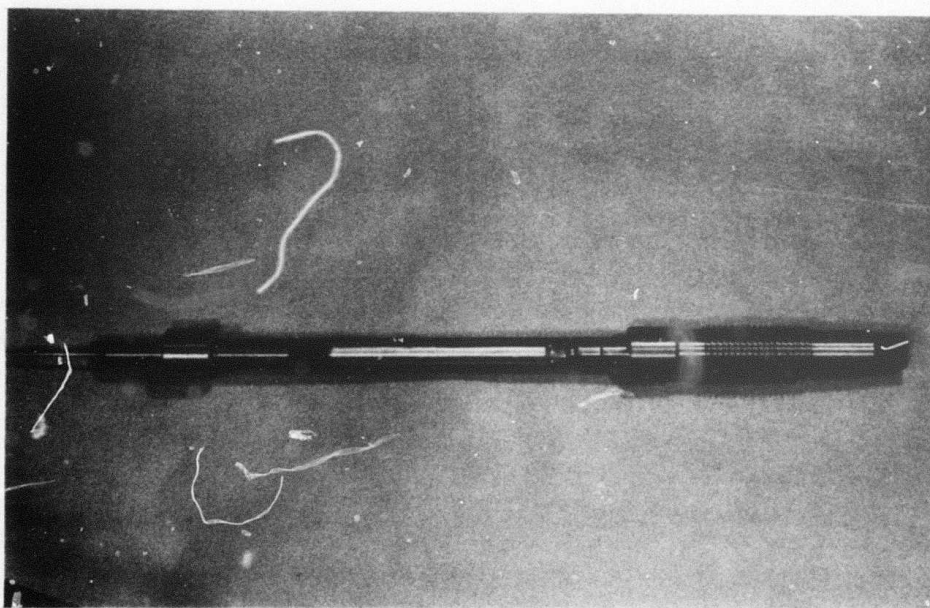


Figure 5. Cut-away view of loaded hydrostatic extrusion chamber - indirect extrusion mode. The material has been transferred intact from the loading tube (Figure 3).

3b, and Figs. 5 and 6. Note the direction in which the material extrudes.) The machine is now ready for the extrusion cycle. A push of the "pressure intensifier" button directs a system hydraulic pressure of 3,000 psi to the intensifier piston (located approximately half way up the load frame, see Fig. 2). The pressure intensifier boosts the pressure to 60,000 psi and delivers the high pressure through stainless steel high pressure tubing to the load chamber cylinder, pressurizing the external surface of the die jacket. The uniform compressive force on the die jacket outer surface performs two functions. First, it provides the necessary counterbalancing compression force on the liner to contain the high internal pressures developed during extrusion. Secondly, it contracts the liner sufficiently to form a metal to metal seal with the die, a condition necessary for the prevention of oil leakage from the load chamber. By activating the extrusion function at the control panel, the piston and die stem begin to move at a pre-selected rate toward the load chamber until the hollow die stem contacts the die. At this point the extrusion pressure will build until the material begins to extrude. Provided the necessary pressure to extrude is not greater than the maximum system pressure, the piston will always move at the selected rate. If the pressure to extrude is too great, a static condition will prevail, and no extrusion will take place. Other values for the processing variables must then be chosen. Since all the load is carried by the die stem, the die stem guides are a necessary precaution against buckling of the die stem. When the piston has travelled a certain pre-set length, a microswitch is activated which automatically stops the extrusion. This is a precaution against over-extruding, a potentially costly error which will be discussed later. Removing the materials from the load chamber after extrusion is a simple procedure. First, remove the breach plug securing the anvil against the load chamber seal. Then remove the anvil. Now, by using an over-ride button on the control panel move the piston in the forward direction

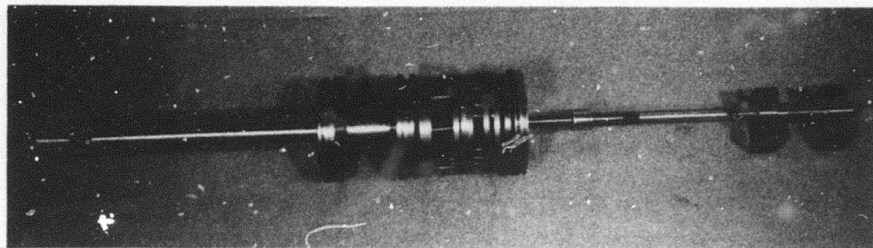


Figure 6. Internal assembly (incomplete) of the hydrostatic extruder. (Compare with Figures 2a and 2b).

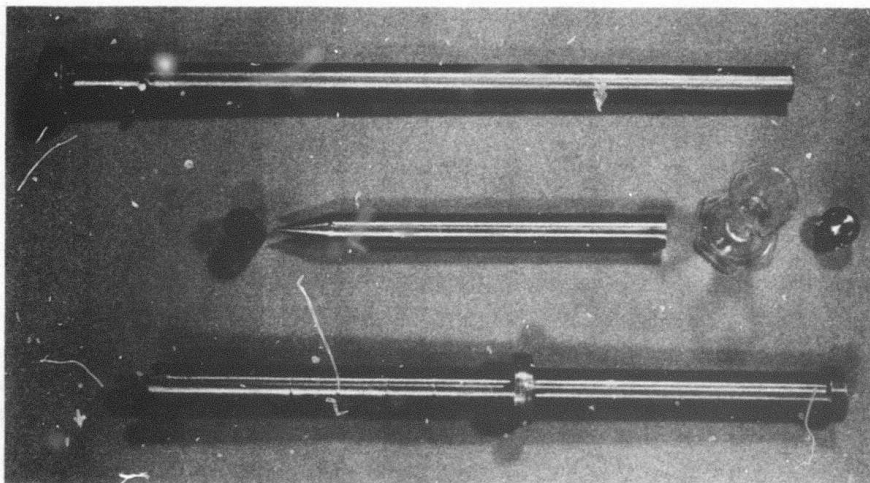


Figure 7. Proper loading tube assembly for direct extrusion.



Figure 8. Cut-away view of loaded hydrostatic extrusion chamber-direct extrusion mode. Note the extrusion direction for the direct mode.

until the seal, remaining billet material, and die are pushed out of the load chamber by the die stem. Protection from moving the piston too far forward and jamming it against the high pressure cylinder is provided by another microswitch which shuts the system off when tripped.

The same procedure is followed for extrusion in the direct mode, except that the order and direction in which the material (seal, material, die) are loaded is reversed. See Figs. 7 and 8 for the positioning in the loading tube and die jacket liner, respectively. Note also that the extrusion direction is opposite to indirect extrusion.

The machine is interchangeable for either three-eighths inch or three-quarter inch billets. To change the system, the die jacket assembly, die stem and die stem guides must be removed and replaced by the other set. Such an exchange consumes about three-quarters of a day. The machine is designed to operate at maximum system pressure (3,000 psi) with safety when rigged for a three-quarter inch billet. A hydrostatic pressure of 250,000 psi is developed in the load chamber when the piston cylinder pressure is 3,000 psi. One must exercise caution when using the three-eighths inch billet system since hydrostatic pressures of 1,000,000 psi could be developed with a piston cylinder pressure of 3,000 psi, while both the die stem and die jacket were designed to contain pressures only up to 500,000 psi. For this reason, the maximum permissible extrusion pressure developed by the hydraulic system is adjustable; and it is only a matter of a few calculations to determine the appropriate maximum allowable extrusion pressure before critical stresses build up in the die stem and the die jacket.

The hydrostatic machine is a precision engineered design with most tolerances $\pm .001$ inches. With proper care and respect no significant wear should occur. Fortunately, most moving parts are well lubricated. The fit of die size to die jacket liner size must be constantly checked to ensure metal to metal contact

during extrusion. Since the die and liner do not make well-lubricated contact, some wear is expected. The liner should periodically be checked. However, it is constructed of polished tool steel and usually will be harder than the chosen die material. Therefore, its lifetime should be somewhere around 250 to 1,000 cycles. Its replacement is of minor consequence. If a die is found to be in good condition but undersized, the diameter may be built up easily by electroplating with copper.

As with most high pressure equipment potentially hazardous and costly situations may arise. All precautions will be taken from the start to ensure that the operator is well protected from any possible machine failure. All high pressure cylinders and tubing will be covered by heavy metal shielding. The main pressure cylinder should not present a major problem since the material was chosen for its ability to absorb energy through slow crack propagation. The volume of oil compressed at 60,000 psi around the die jacket is too small to impart much energy to the system during a failure. In any case, these areas will be protected by shields. More importantly, the machine will be oriented so that both ends of the extrusion cylinders face barriers. It is quite possible under certain conditions for high pressure oil, die stems, or material projectiles to be ejected at dangerous velocities from either end.

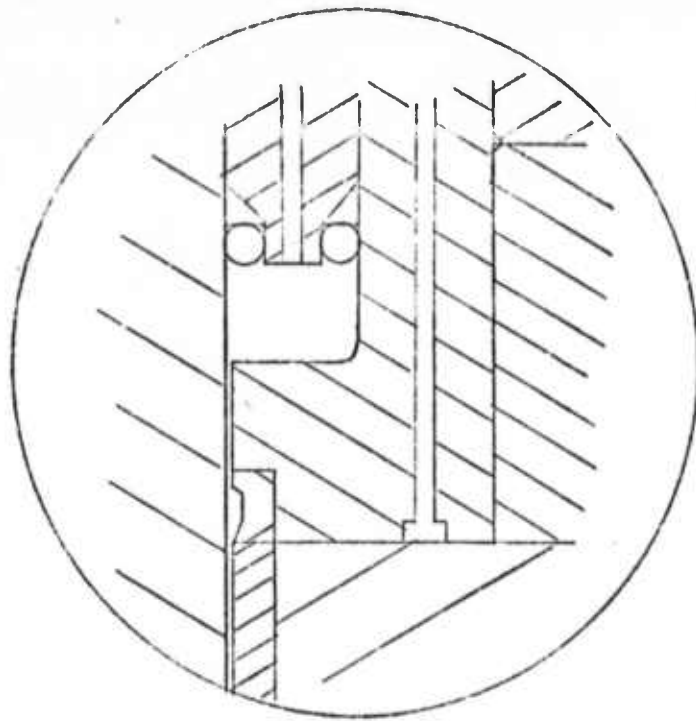
Care must be exercised to prevent two things that could go wrong which would cause no physical harm but would require costly replacement or repair. (1) The intensifier pressure could drop below 60,000 psi resulting in a split liner and expanded die jacket assembly, and (2) the die stem could break.

The die stem could be broken by simply miscalculating and exceeding the compressive yield stress of the die stem during extrusion with the three-eighths inch system. Failure of the die stem might also result from a sudden loss of hydrostatic pressure in the die jacket load chamber. This might occur from

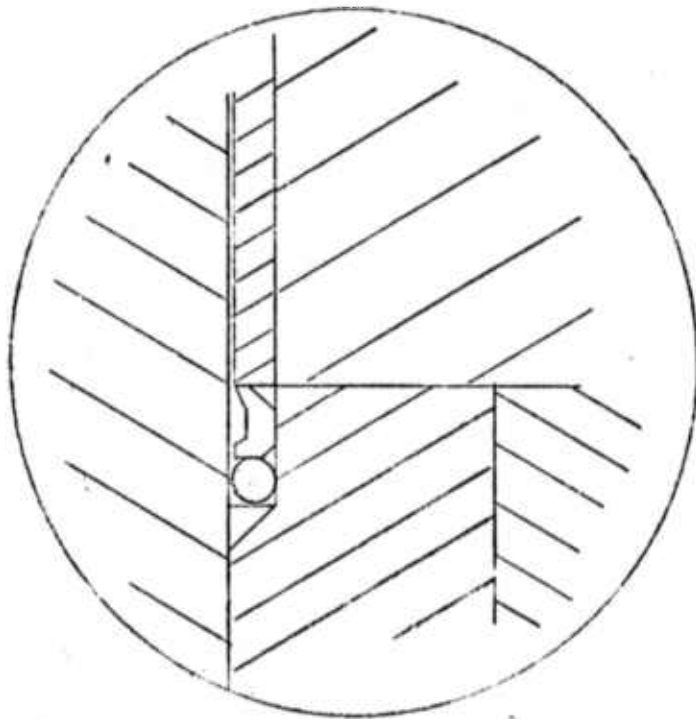
overestimating the extrusion stroke. At the beginning of the stroke, the pressure builds until the extrusion is initiated; steady state extrusion follows. But instead of triggering a microswitch to halt the forward motion of the piston before the billet is completely extruded, the piston continues forward and extrudes the entire billet creating a pressure drop to atmospheric pressure in the chamber. Such almost instantaneous unloading on the die stem could cause critical tensile stresses to develop and the die stem to fracture. This condition may also arise if oil loss occurs in the load chamber. Extrusion of oil instead of material due to improper billet to die seal, or cracked dies can result in such a failure. However, in most situations, loss of oil can only stall the machine at maximum extrusion piston pressure with no damage resulting.

Failure or loss of intensifier pressure is potentially a very serious problem. Machine damage (although reparable) is almost certain to occur if the pressure on the outer surface of the die jacket drops below some critical value during extrusion. This critical pressure varies according to the applied extrusion pressure and whether the three-eighths inch or three-quarter inch system is in use. As a safety measure, the die jacket is always pressurized to the maximum allowable, i.e. 60,000 psi. Failure could result in several ways. Perhaps the most serious failure mode would be when the pressure seals actually prevent the high pressure oil from entering the clearance between the die jacket and outer large diameter cylinder (see Fig. 9a,b for seal design). The pressure in the line and on the gage would read the equivalent 60,000 psi, but the actual pressure within the area of the die jacket would be near atmospheric pressure (see Fig. 10a,b). The other potential mode of failure is through leaks in the seals or in the check valves in the hydraulic line.

Because of the seriousness of the problem, we will mount strain gages externally on the large cylinder over the area of the die jacket. The strain gages will be connected by a relay network to the control panel. The machine will be programmed not to operate if either pressure does not develop around the die jacket or if



9a



9b

Figure 9. Detailed drawings of die jacket seal design.

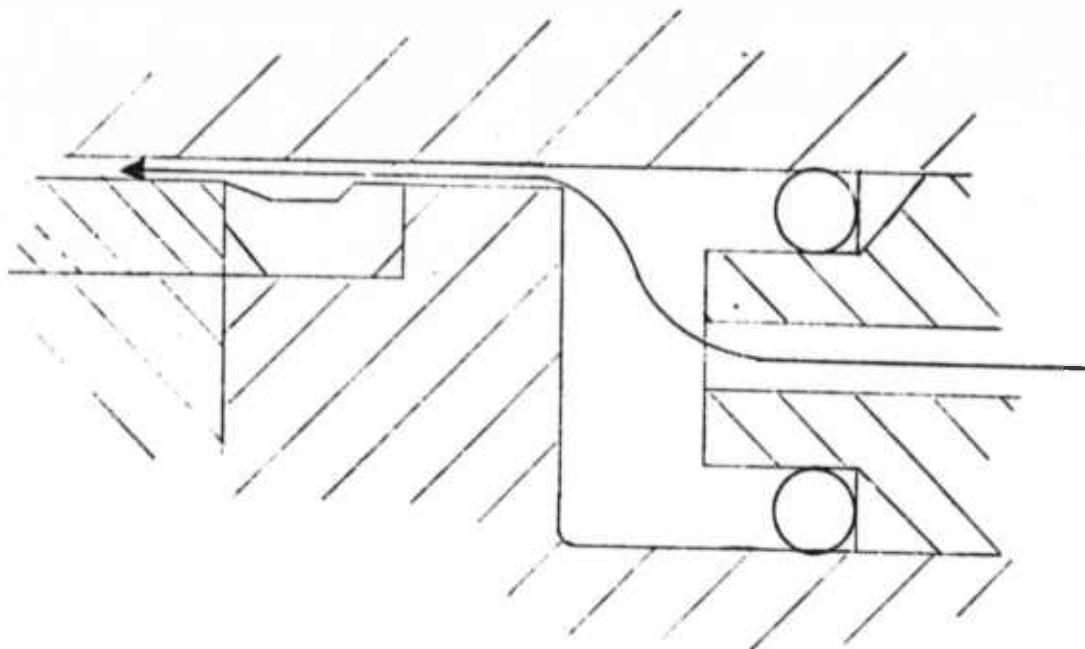


Figure 10a. Schematic diagram of proper high pressure hydraulic fluid flow for normal operation. The space between die jacket liner and high pressure cylinder must be pressurized to 60,000 psi in order to contain the high pressures which develop in the load chamber during extrusion.

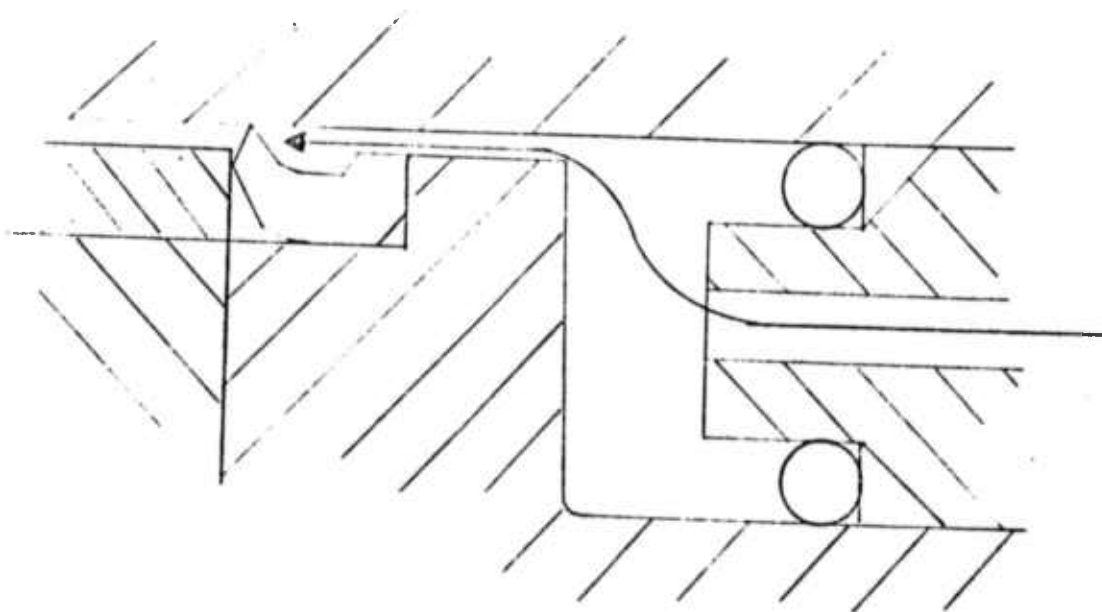


Figure 10b. Possible seal failure. Due to the absence of a high compensating compressive stress, tensile stresses now develop and likely lead to load chamber liner failure.

pressure suddenly drops around the die jacket.

Much of the initial work done with the machine will involve calibration of the strain gages, flow meters and determination of die friction and oil compressibility. Presently, the applied load to the die is measured as strain on a strain gage resulting from the load transmitted to it from the die stem. This is calibrated with the pressure in the extrusion piston cylinder to give a numerical value of the applied load and therefore hydrostatic pressure. Although this technique accurately measures the load on the die stem, one must be careful in relating it to the hydrostatic pressure in the load chamber or the pressure at the extrusion die. In the indirect mode of extrusion the most accurate results of hydrostatic pressure at the die are possible. Since the die stem and the die are in contact, only the wall friction between die and liner contributes to the difference between internal hydrostatic pressure and load on the die stem. Through experiments it will be possible to gain an approximate value for die friction under various condition. The direct mode of extrusion presents considerable problems. The die stem is no longer close to the extrusion die and under high pressures the oil tends to solidify, thus gaining the capacity to carry a load. This result is a pressure drop from the point of application of the load (i.e. the die stem), to the actual point of extrusion. Such readings at the die stem obviously involve unpredictable errors. It may be possible to mount strain gages on the die for the direct extrusion mode to get more reliable results.

Oil compressibility in the load chamber is something which should be known in order to accurately position the microswitches for proper extrusion stroke length. Knowledge of the compressibility will become especially important when using the back pressure system. Here the available length of the outlet chamber into which the material will be extruded will be related to fluid compressibility. This length will, of course, vary with pre-set back pressure reading.

While it is expected that the back pressure system will be an extremely useful tool in the future, it is arriving at Stanford as untested equipment. Such a system has never been used before with this machine. Even though the back pressure system is a simple design it may require considerable trial and error to make it function properly.

2. Testing Equipment

Mechanical property measurements will be an important part of this program. It is planned that both tension compression and fatigue tests will be performed on hydrostatically extruded experimental materials. For that purpose a fully instrumented, 20,000 pound capacity, hydraulic mechanical testing system (MTS model No. 901.29, Research Inc.) has been set up adjacent to the location of the hydrostatic extrusion press. This mechanical test equipment is fully operational.

In addition to room temperature properties, we are also interested in mechanical properties at elevated temperatures. Furthermore, we are restricted to very small compression samples due to the limited size of the extruded materials. To meet these needs, a furnace-cooling water system has been fitted onto the MTS frame in such a way that we can compress very small samples (as small as 0.15 inches in diameter by 0.200 inches long) in air up to 1000°C. This furnace attachment was not simple to set-up because of two design problems. The first problem was the non uniform deformation of the small compression samples caused by even the slightest misalignment of the compression columns. This problem was eventually solved (or at least minimized) by decreasing the length of the compression columns to 6 inches. This in turn required that the furnace be placed on rollers and allowed to slide along a vertical track. The second problem was the conduction of heat out of the furnace and into the load cell and hydraulic fluid on the MTS frame by the compression columns. This problem was eliminated by installing alumina columns. The furnace MTS system is now fully operational. A photograph of the

system is included as Figure 11.

3. Materials Preparation Equipment

Included in our planned hydrostatic extrusion program is the study of hydrostatically extruded particulate composites. Such composites are oftentimes difficult to extrude by ordinary methods and should be ideally suited for testing the feasibility of hydrostatic extrusion. Earlier work by us in the preparation of particulate composites involved the use of a special powder metallurgy-mechanical comminution process^(1,2) developed at Stanford. This work has been successful in obtaining unusual mechanical properties in large volume fraction particulate composites based on model systems (zinc and cadmium base metals containing alumina, tungsten and boron)⁽³⁻⁶⁾. Recently we have concentrated on magnesium and copper base particulate composites. As discussed in the first semi-annual report⁽⁷⁾ these systems have posed some new experimental difficulties.

1) The compaction press previously used could only deliver a maximum of 30,000 psi pressure on a 1 1/2 inch bore die, which is inadequate for hot pressing of these newer materials. 2) A large size difference between the matrix powders (30-40 microns) and the second phase powders (about 1 micron) prevented uniform distribution of the hard particles in the matrix material. 3) Copper and magnesium readily oxidize at the elevated temperatures of hot pressing which inhibits interparticle diffusion and results in a poorly bonded final product (hence cracks during extrusion).

The first problem has been solved by the use of a 120,000 pound capacity pneumatic compaction press provided by the Civil Engineering Department at Stanford. This virtually doubles our maximum compaction pressure.

The second problem has been solved by the acquisition of a mechanical attritor from Union Process Co., of Akron, Ohio. The attritor not only mixes powders very well but also can grind powders in a protective atmosphere to submicron size.

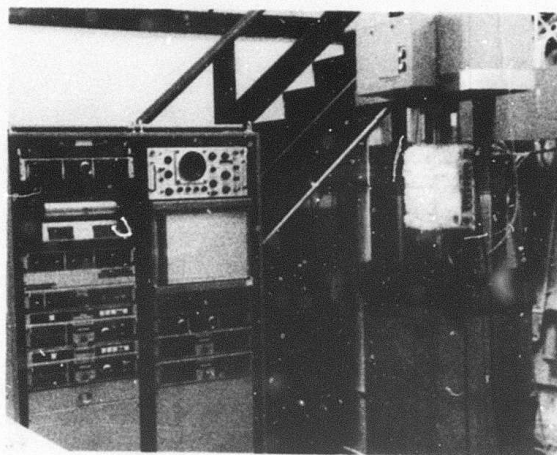


Figure 11. Photograph of the MTS model No. 901.29, Research Inc. with the corresponding high temperature mechanical testing system attached to the load frame. Not shown is the furnace temperature controller and the water cooling system for the load cell.

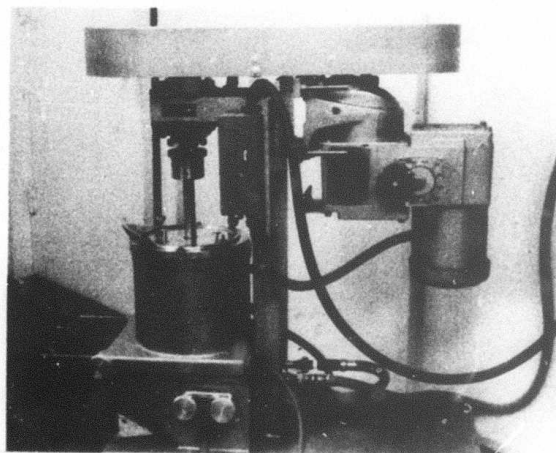


Figure 12. Photograph of the attritor acquired from Union Process Co. as installed in our Powder Metallurgy Laboratory.

Large particle matrix powders can thus be reduced to a size comensurate to the second phase particles. Our attritor, only recently delivered (May 9, 1975), has already been installed and used to grind cast iron powders. The first test runs have only been partially successful - the powders are still coarse even after many hours of grinding. This can be attributed to the large size of the charged cast iron particles (0.010 inches thick and 0.25 inches long). Better results are expected as the influence of the process conditions on the effectiveness of the mechanical comminution are learned through experience. A photograph of the installed attritor is included as Figure 12.

The third problem has been solved by the design of an inert atmosphere hot-pressing furnace, which is shown schematically in Figure 13. This furnace allows for the hot pressing of powders at high stresses (up to 60,000 psi) and temperatures (up to 900°C) in a protective (or reducing) atmosphere. The compaction chamber is floating, pressure being applied simultaneously to both the top and bottom of the powder charge, thereby minimizing density gradients along the length of the compacts. The device is presently under construction.

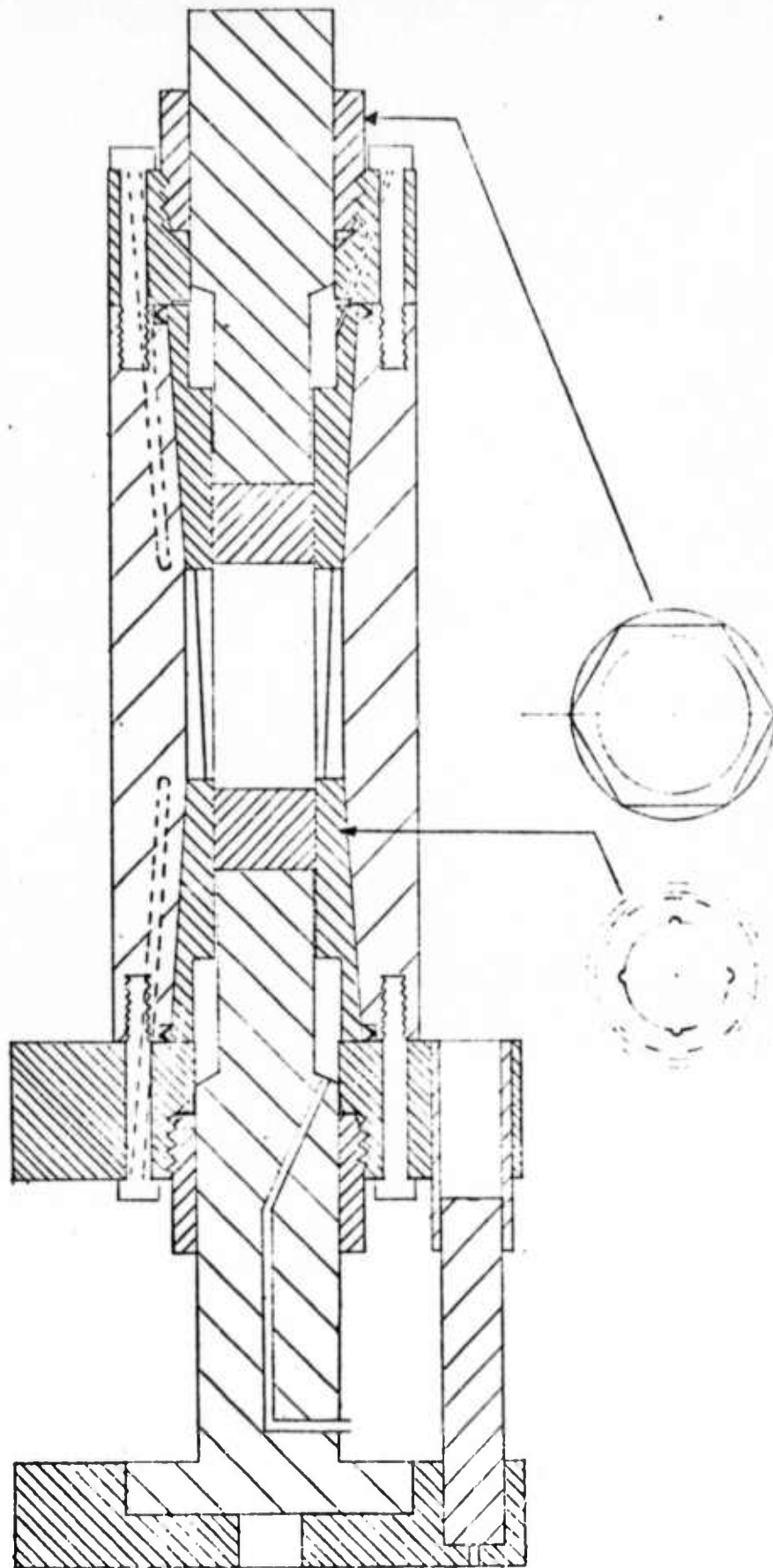


Figure 13. Schematic diagram of the inert atmosphere hot-pressing furnace currently under construction. This furnace is designed to handle up to 900°C environments and compaction stresses up to 60,000 psi.

III. Development of a Finite-Element Program for Metal Forming

It was pointed out in the First Semi-Annual Technical Report,⁽⁷⁾ that in spite of the large strains which occur in metal forming processes, it is necessary to carry out an elastic-plastic analysis in order to be able to anticipate the limits on satisfactory operation. Even though the plastic strains are very large compared with elastic strains, the latter play the major role in determining the stress distribution, and this governs the development of defects, through, for example, the initiation of internal fractures and instabilities. Elastic strains are of the order (yield stress)/(elastic modulus), which is about 10^{-3} for most technologically important metals. Thus computational programs must be accurate to this order of strain if stresses are to be accurately predicted throughout the work-piece. In regions where large plastic flow is currently taking place, the stress is effectively determined by the plasticity laws, but in the final product, for example, plastic flow has ceased and elastic strains determine the distribution of residual stresses which are important from a technological standpoint. Thus, while it might appear from a purely formal computational standpoint that since strains of the order unity occur, strains of magnitude 10^{-3} could be neglected, this is not the case for metal forming analysis aimed at assessing satisfactory operational limits for process variables.

The need to adopt elastic-plastic theory has a major influence on the mathematical structure of the analysis. This aspect of the problem

has received concentrated attention of the theoretical group during the first year of the project. Some of the considerations which arise in this connection are discussed in subsequent sections of this report.

1. Rigid-Plastic and Elastic-Plastic Theories

For simplicity of the initial discussion, consider ideal plasticity obeying the Mises yield condition. The stress at a material particle is given by the Cauchy or "true" stress tensor σ_{ij} , with six independent components because it is symmetric: the direct stresses σ_{11} , σ_{22} and σ_{33} and the shear stresses σ_{12} , σ_{23} and σ_{31} . It is convenient to write the plasticity laws in terms of the stress deviator, δ_{ij} , which is the stress tensor less the average hydrostatic component:

$$\delta_{ij} = \sigma_{ij} - \frac{1}{3} \sigma_{kk} \delta_{ij} \quad (1)$$

where δ_{ij} is the Kronecker delta:

$$\begin{aligned} \delta_{ij} &= 1, \quad i = j \\ &= 0, \quad i \neq j \end{aligned} \quad (2)$$

and the summation convention for the repeated suffix k is utilized. The deviator expresses plastic flow characteristics since it in effect represents the shear type stress components. Plasticity occurs when the yield stress is reached at the Mises limit:

$$J_2 = \frac{1}{2} \delta_{ij} \delta_{ij} = \frac{1}{3} Y^2 \quad (3)$$

where Y is the yield stress in tension. The equations determining the plastic deformation are flow or incremental in nature, giving the rate of plastic strain $\dot{\epsilon}_{ij}^P$, or equivalently the increment in plastic strain in a time increment dt

$$d\epsilon_{ij}^P = \dot{\epsilon}_{ij}^P dt \quad (4)$$

For ideal or perfect plasticity, flow occurs with the stress remaining at the yield limit (3), and the plastic strain-rate is given by

$$\dot{\epsilon}_{ij}^P = \lambda \delta_{ij} \quad (5)$$

where λ is a free parameter, since indefinite flow can occur at constant stress with ideal plasticity and thus no work-hardening. Since δ_{ij} is a deviator in (5), so also must $\dot{\epsilon}_{ij}^P$ be, hence the rate of volume strain is zero:

$$\dot{\epsilon}_{ii}^P = 0 \quad (6)$$

These relations govern the plastic components in elastic-plastic theory, to which must be added the elastic components of strain-rate:

$$\dot{\epsilon}_{ij} = \dot{\epsilon}_{ij}^e + \dot{\epsilon}_{ij}^p \quad (7)$$

so that the total deviatoric strain-rate is given by

$$\dot{\epsilon}_{ij} = \dot{s}_{ij}/2G + \lambda s_{ij} \quad (8)$$

since Hooke's law for stress and strain deviators takes the form

$$\dot{\epsilon}_{ij}^e = \dot{s}_{ij}/2G \quad (9)$$

where G is the shear modulus.

Equations (5) and (7) illustrate the different structures of rigid-plastic and elastic-plastic theories. For rigid-plastic theory, $\dot{\epsilon}_{ij}^p$ is equal to the total strain-rate $\dot{\epsilon}_{ij}$. For a velocity field $\underline{v}(\underline{x}, t)$ - the subscript curls denote vectors - where $\underline{x}$ are position coordinates in a body in the current (or deformed) configuration, the strain-rate is given by

$$\dot{\epsilon}_{ij} = \frac{1}{2} \left(\frac{\partial v_i}{\partial x_j} + \frac{\partial v_j}{\partial x_i} \right) \quad (10)$$

For a given velocity field, (5) in conjunction with (3) determines the stress deviators, s_{ij} , for rigid-plastic response to stress, whereas for elastic-plastic analysis, (8) yields only a differential equation for the determination of $\dot{s}_{ij}$, or equivalently it yields $\dot{s}_{ij}$ if s_{ij} is known. This difference is well illustrated by the graphical construction for the case of principal directions of stress and strain fixed in the

body, known as the π diagram,⁽⁸⁾ Figures 14 and 15.

The π diagram is a projection of the principal stress space onto the plane

$$\sigma_1 + \sigma_2 + \sigma_3 = 0 \quad (11)$$

that is a projection parallel to the line subtending equal angles with the three stress axes. Strain can be plotted on the same diagram, and the scale factor $2G$, where G is the elastic shear modulus, produces coincidence of the stress and strain points for purely elastic deformation. Fig. 1a shows the rigid-plastic case for which the plastic strain-rate $\dot{\epsilon}^p$ is identical with the total strain rate. The strain-rate vector DE is parallel to the stress vector OA . Thus given the strain-rate DE , the stress point A on the yield circle is uniquely determined.

In the elastic-plastic case, Fig. 1b, DE again represents the total strain-rate vector, now broken down into a plastic component FE and an elastic component DF . The latter, including the factor $2G$, is equal to the stress-rate vector AB . In this case, the total strain-rate vector DE does not determine the stress OA , but given OA , it then determines the plastic component of the strain-rate FE parallel to OA , and hence the elastic component DF .

Prandtl⁽⁹⁾ gives a geometrical or mechanical analogy for determining the stress variation associated with a prescribed strain path. For ideal plasticity, if a bead is dragged behind the strain point on a thread of

length Y , the thread, as the bead follows a curve of pursuit, gives the stress vector according to (8). The curve of pursuit in effect solves the differential equation (8).

The two types of response to prescribed strain-rate just illustrated both analytically and graphically, and corresponding to rigid-plastic and elastic-plastic behavior, represent quite different ways in which the history of straining influences the current stress. For rigid-plastic material, the rate of strain, or more precisely the velocity strain (10), determines the stress deviator uniquely. The stress tensor itself is determined to within an arbitrary hydrostatic pressure, which is associated with the incompressible characteristic of plastic flow. Thus no influence of the prior history of strain appears in the equations which determine the stress. In contrast, for the elastic-plastic case, the current stress is given by the solution of the differential equation (8) integrated over the history of deformation from prescribed initial conditions. This circumstance determines the structure of the solution process for stress and deformation distributions in metal forming. Consider the solution to have been obtained up to the time t . The stress and deformation distributions are then known at that instant. The equilibrium equations, constitutive relations and velocity strain-rate relations can be most conveniently written in terms of stress rates, for as in the case of Equation (8) these are determined uniquely. Equivalently, for a sufficiently small time step dt , stress, strain and displacement increments are determined. The resulting new stress and deformation distributions constitute the "initial conditions" for the next time step.

An analogous situation arises in the case of more realistic work-hardening laws of plasticity, which again require integration through time for the determination of stress distributions. A generalized π diagram construction has also been developed for this case (8,10).

It is interesting to observe that the extent to which strain history affects the current stress has a bearing on the convection of stress magnitudes through motion of the material elements. In the rigid-plastic case this would be zero since the current velocity strain distribution determines stress quite uninfluenced by its prior values. In the actual evaluation for elastic-plastic materials, this aspect appears in the appropriate choice of convected derivative for substitution in the constitutive relation.

The need to evaluate the stress and deformation distributions in terms of stress rates and velocities in each time increment raises the question of the appropriate rates for expressing the equilibrium equations and the constitutive relation. The former can clearly be most simply treated in terms of the partial time derivative, for this commutes with the space derivative in the equilibrium equation:

$$\frac{\partial \sigma_{ij}}{\partial x_i} = 0 \quad (12)$$

according to the relation:

$$\frac{\partial}{\partial t} \left(\frac{\partial \sigma_{ij}}{\partial x_i} \right) = \frac{\partial}{\partial x_i} \left(\frac{\partial \sigma_{ij}}{\partial t} \right) = 0 \quad (13)$$

However the stress-rate term in the constitutive equation, $\dot{\sigma}_{ij}$, must follow the material particle, and Rice⁽¹¹⁾ has pointed out that however

small the time increments, substitution of $\dot{\sigma}_{ij}$ for the partial derivative with respect to t in (13) would usually involve major error.

If, for illustration, $\dot{\sigma}_{ij}$ is taken to be the material derivative, then (13) takes the form

$$\frac{\partial}{\partial x_i} \left(\frac{\partial \sigma_{ij}}{\partial t} \right) = \frac{\partial}{\partial x_i} \left(\dot{\sigma}_{ij} - v_k \frac{\partial \sigma_{ij}}{\partial x_k} \right) = 0 \quad (14)$$

For a time increment dt , we write

$$d\sigma_{ij} = \dot{\sigma}_{ij} dt, \quad d\epsilon_{ij} = \dot{\epsilon}_{ij} dt, \quad du_k = v_k dt \quad (15)$$

and (14), multiplied by dt , takes the form

$$\frac{\partial}{\partial x_i} (d\sigma_{ij}) - \frac{\partial}{\partial x_i} (du_k) \frac{\partial \sigma_{ij}}{\partial x_k} = 0 \quad (16)$$

In order of magnitude, $d\sigma_{ij} \sim E^t d\epsilon_{ij}$, where E^t is the appropriate tangent modulus, $\frac{\partial}{\partial x_i} (du_k) \sim d\epsilon$, so that the first term dominates if

$$E^t \gg \sigma \quad (17)$$

In this case it is permissible to replace $\frac{\partial \sigma_{ij}}{\partial t}$ by $\dot{\sigma}_{ij}$ and the influence of convection on the material stress rate is negligible. For loading in the plastic range of many metals, the tangent modulus is of the order of the stress, so that the terms in (16) are of the same order. Thus the circumstance which applies in classical elasticity, for which the elastic modulus is large compared with the stress, that the difference in using deformed or undeformed coordinates is negligible, does not usually apply

for plastic loading. It is clear from (15) and (16) that this circumstance is independent of the size of the time increment dt . It is perhaps worth pointing out that the additional terms commonly added to generate objective stress-rates are of the same order as the second term in (16) so that the same conclusion results. Moreover, when steady state operation of a metal forming process has been achieved, $\partial\sigma_{ij}/\partial t = 0$, and the two terms in the second expression in (14) are exactly equal in magnitude, whatever the characteristics of the material, and must clearly both be retained.

2. Formulation of Finite-Element Relations

A discussion of the generation of finite element relations for metal forming problems has been given by Lee and Kobayashi⁽¹²⁾ and forms the basis for the present discussion. Some of the pitfalls which must be avoided in applying this approach to elastic-plastic analysis are emphasized.

In accordance with the structure of elastic-plastic theory discussed in the previous section, which showed that stress and strain rates (or equivalently increments of stress and strain in the time Δt) are sought on the basis of knowledge of the current stress and deformation, a variational principle in the stress and strain rates is utilized⁽¹³⁾:

$$\delta\phi = 0 \quad (18)$$

where

$$\dot{\Phi} = \frac{1}{2} \int_V \dot{\underline{\sigma}}^T \dot{\underline{\epsilon}} dV - \int_S \underline{\dot{v}}^T \underline{\dot{T}} dS \quad (19)$$

Superscript T denotes the transpose of a matrix or a vector. The stress tensor σ_{ij} is written as a vector $\underline{\sigma}$ with nine components, as also is the strain tensor, so that the first integral gives the rate of work carried out by the stress-rates. The second integral expresses the rate of external work expended by the traction rate vector $\underline{\dot{T}}$ acting over the surface S .

The body being analysed is divided into M finite elements considered to be interconnected at a finite number of nodal points P . The functional (19) evaluated for the m^{th} element is denoted by $\dot{\Phi}^{(m)}$, so that

$$\dot{\Phi} = \sum_{m=1}^M \dot{\Phi}^{(m)} \quad (20)$$

The discretization of the variational problem for each element is accomplished by approximating the functional $\dot{\Phi}^{(m)}$ by a function $\varphi^{(m)}$ of the m^{th} element nodal point values. This is achieved by replacing the $\underline{v}$ distribution with an approximating velocity distribution in each element, continuity between elements being maintained. The functional $\dot{\Phi}$, (19), is thus approximated by a function of the P nodal-point velocity variables $\underline{v}_P$

$$\dot{\Phi} \approx \varphi(\underline{v}_1, \underline{v}_2, \dots, \underline{v}_P) = \sum_{m=1}^M \varphi^{(m)}(\underline{v}_k^{(m)}) \quad (21)$$

Thus the initial variational problem is reduced to the determination of the vector $\underline{v}_p$ in the P -dimensional vector space of vectors which minimize $\varphi(\underline{v}_p)$. This vector combined with the elemental velocity distributions provide the approximation to the solution for the actual velocity distribution $\underline{v}(x,t)$.

The approximate velocity variation in the m^{th} element is given by

$$\underline{v}^{(m)}(x,t) = [N]^{(m)}(x) \underline{v}^{(m)}(t) \quad (22)$$

where $\underline{v}^{(m)}$ and $[N]^{(m)}$ are of the m^{th} element nodal point velocity components and the m^{th} distribution matrix respectively. From (22) the approximate strain-rate vector can be expressed in terms of the nodal velocities by the matrix operator $[B]^{(m)}$:

$$\dot{\underline{\epsilon}}^{(m)} = [B]^{(m)} \underline{v}^{(m)} \quad (23)$$

Both in the elastic and plastic regions, the vectors of rates of stress and strain are related by

$$\dot{\underline{\sigma}}^{(m)} = [D]^{(m)} \dot{\underline{\epsilon}}^{(m)} \quad (24)$$

where, for elastic response the matrix $[D]^{(m)}$ has constant elements, and for plastic behavior it depends on the current stress and on the loading history. Substituting (22), (23) and (24) in (19) for the m^{th} element gives

$$\begin{aligned} \varphi^{(m)} = & \frac{1}{2} \int_V ([D]^{(m)} [B]^{(m)} \underline{v}^{(m)})^T [B]^{(m)} \underline{v}^{(m)} dV \\ & - \int_{S^{(m)}} ([N]^{(m)} \underline{v}^{(m)})^T \underline{\dot{T}}^{(m)} dS \end{aligned} \quad (25)$$

Evaluation of the volume and surface integrals for a single element results in the discrete representation of the functional for an element

$$\varphi^{(m)} = \frac{1}{2} \underline{v}^{(m)T} [K]^{(m)} \underline{v}^{(m)} - \underline{v}^{(m)T} \underline{\dot{R}}^{(m)} \quad (26)$$

where $[K]^{(m)}$ is the element stiffness matrix and $\underline{\dot{R}}^{(m)}$ is the nodal force-rate vector equivalent to the external traction rate $\underline{\dot{T}}^{(m)}$ distributed over the surface $S^{(m)}$. The global stiffness matrix $[K]$ and the equivalent nodal force-rate vector $\underline{\dot{R}}$ are the sums of the corresponding element values as called for in (21). The condition for φ to assume a stationary value yields a system of simultaneous linear algebraic equations which determine the node velocities $\underline{v}_p$. From these, stress-rates can be obtained from (24) and (23).

When mixed boundary conditions are considered for which velocity vectors over part of the surface S_U are prescribed, the unknown nodal velocity vectors are given by

$$\underline{\dot{R}}_a = [K]_{aa} \underline{v}_a + [K]_{ab} \underline{v}_b \quad (27)$$

where $\underline{\dot{R}}_a$ is the known nodal force-rate vector and $\underline{v}_a$ and $\underline{v}_b$ are respectively the unknown and prescribed nodal velocity vectors. The

matrices $[K]_{aa}$ and $[K]_{ab}$ are submatrices of the global stiffness matrix $[K]$, according to

$$[K] \begin{Bmatrix} v_a \\ v_b \end{Bmatrix} = \begin{bmatrix} [K]_{aa} & [K]_{ab} \\ [K]_{ba} & [K]_{bb} \end{bmatrix} \begin{Bmatrix} v_a \\ v_b \end{Bmatrix} \quad (28)$$

Note that in this finite-element formulation the stress rate appearing in the functional (19) was obtained from the material constitutive relation (24) through the relation (23) based on nodal point velocities. Thus rates of stress and strain following specific particles are evaluated, called material derivatives (convected derivatives in the fluid mechanics literature). But (19) was established⁽¹³⁾ from a virtual work hypothesis which assumed that $\dot{\sigma}_{ij}$ satisfied equilibrium equations of the form (12). We saw in the last section that this is not valid for plastic flow since the partial time derivative $\partial \sigma_{ij} / \partial t$ is not approximated by the material derivative, and it is the former which satisfies the equilibrium equation. The situation becomes particularly anomalous in the case of steady-state problems, since then the partial derivative of the stress is zero, and substitution of the material derivative for it is totally suspect.

As discussed by McMeeking and Rice⁽¹⁴⁾, Hill⁽¹⁵⁾ has attempted to avoid this difficulty by working with stress as force per unit reference area (Piola-Kirchhoff stress) instead of the "true" or Cauchy stress utilized so far in this report, which is defined per unit current area. McMeeking and Rice consider an increment of deformation from a current state which for the time interval Δt is taken to be the reference state. For plasticity laws it is not appropriate to use a permanent reference state to define the stress and strain tensors, since plastic flow is an incremental or flow type phenomenon which can most naturally be expressed

in the current geometry as for a fluid⁽¹⁶⁾. However, taking the current states sequentially as a series of reference states for a sequence of deformation increments, flow type response can be accommodated with the advantage of a referential theory rather than a spacial theory (Euler coordinates). In a referential type theory (Lagrange coordinates) the equations of motion are written in terms of the reference coordinates, so that partial differentiation with respect to time corresponds to a fixed reference or material point, and so is simultaneously a material rate.

Even when an appropriate reference formulation is adopted, care must be exercised in selecting appropriate tensor rate definitions, for constitutive relations must be objective, and this is most easily achieved if all variables occurring in them are objective. Objectivity is the requirement that a constitutive relation for a material must be such that if a time dependent rigid body rotation is superposed on the body, the stress determined by the constitutive equation for the new motion must simply be the original current stress rotated by the current value of the superposed rotation. In this definition it is assumed that any change in inertia forces is equilibrated by body forces so that they do not complicate the situation.

Some variables which would appear to be plausible candidates for constitutive relations are found not to be objective and are therefore unsuitable. For example the material time derivative of stress

$$\dot{\sigma}_{ij} = \frac{\partial \sigma_{ij}}{\partial t} + v_k \frac{\partial \sigma_{ij}}{\partial x_k} \quad (29)$$

is not objective (see, for example, Malvern⁽¹⁷⁾ p. 402). But the co-rotational stress-rate or Jaumann rate, which is equal to the material

rate less the stress rate associated with the rotation, is objective [10]. Thus an appropriate rate is needed for substitution in the elastic-plastic relation (24), and the influence of this choice on the form of the variational principle (19) must be investigated.

We are looking into these questions to develop the most effective form of finite-element analysis on which to base our attack on the steady-state extrusion problem. We have already formulated possible approaches to the satisfaction of steady-state boundary conditions at fixed spacial positions as the work material moves through them. We expect to run corresponding numerical programs during the summer.

References

1. W. H. McCarthy, J. C. Shyne and O. E. Sherby, Trans. ASM, 62, 117, 1969.
2. G. R. Edwards and O. D. Sherby, Int. Jnl. of Powder Metall., 7, 21, 1971.
3. T. R. McNelley, G. R. Edwards, D. Francois, W. H. McCarthy, J. C. Shyne and O. D. Sherby, Met. Trans., 3, 1316, 1972.
4. G. R. Edwards, "The Mechanical Behavior of Powder Metallurgy, Zinc and Zinc-Tungsten Particulate Composites", Ph.D. Dissertation, Stanford University, August 1971.
5. Shu-en Hsu, "The Mechanical Behavior of Cadmium Base Particulate Composites", Ph.D. Dissertation, Stanford University, June 1972.
6. T. R. McNelley, "Mechanical Behavior of Powder Metallurgy Zinc-Alumina Particulate Composite Materials", Ph.D. Dissertation, Stanford University, March 1973.
7. O. D. Sherby, J. C. Shyne, and E. H. Lee, "Influence of High Hydrostatic Pressure Extrusion on Mechanical Behavior of Materials - First Semi-Annual Technical Report", Report No. SU-DMS-75-T-32, Department of Materials Science and Engineering, Stanford University, Stanford, Calif., 1975.
8. R. Hill, The Mathematical Theory of Plasticity, p. 42, Clarendon Press Oxford, 1950.
9. L. Prandtl, Proc. 1st Int. Cong. Appl. Mech., p. 43, Delft, 1924.
10. E. H. Lee, A Geometrical Construction of Plastic Flow with Work Hardening, Proceedings of the First Midwestern Conference on Solid Mechanics. pp. 136-140, 1954.
11. J. R. Rice, A Note on the "Small Strain" Formulation for Elastic-plastic Problems. Technical Report N00013-67-A-0191-0003/8, Div. of Engg., Brown University, 1970.
12. C. H. Lee and S. Kobayashi, Elastoplastic Analysis of Plane-Strain and Axisymmetric Flat Punch Indentation by the Finite-Element Method, Int. J. Mech. Sc., 12, 349-370, 1970.
13. D. C. Drucker, Variational Principles in the Mathematical Theory of Plasticity, Proc. Symp. Appl. Math., 8, 7-22, 1958.
14. R. M. McMeeking and J. R. Rice, Finite-Element Formulations For Problems of Large Elastic-Plastic Deformation, Tech. Rep. NASA NGL 40-002-080/15, Div. of Engg., Brown University, 1974.
15. R. Hill, Some Basic Principles in the Mechanics of Solids Without a Natural Time, J. Mech. Phys. Solids, 7, 209-225, 1959.
16. E. H. Lee, Elastic-Plastic Theory at Finite Strain, Problems of Plasticity, Ed. A. Sawczuk, 117-133, Noordhoff Int. Publ., London, 1974.

17. L. E. Malvery, Introduction to the Mechanics of the Continuous Medium,
Prentice Hall, New Jersey, 1969.

Reproduced from
best available copy.

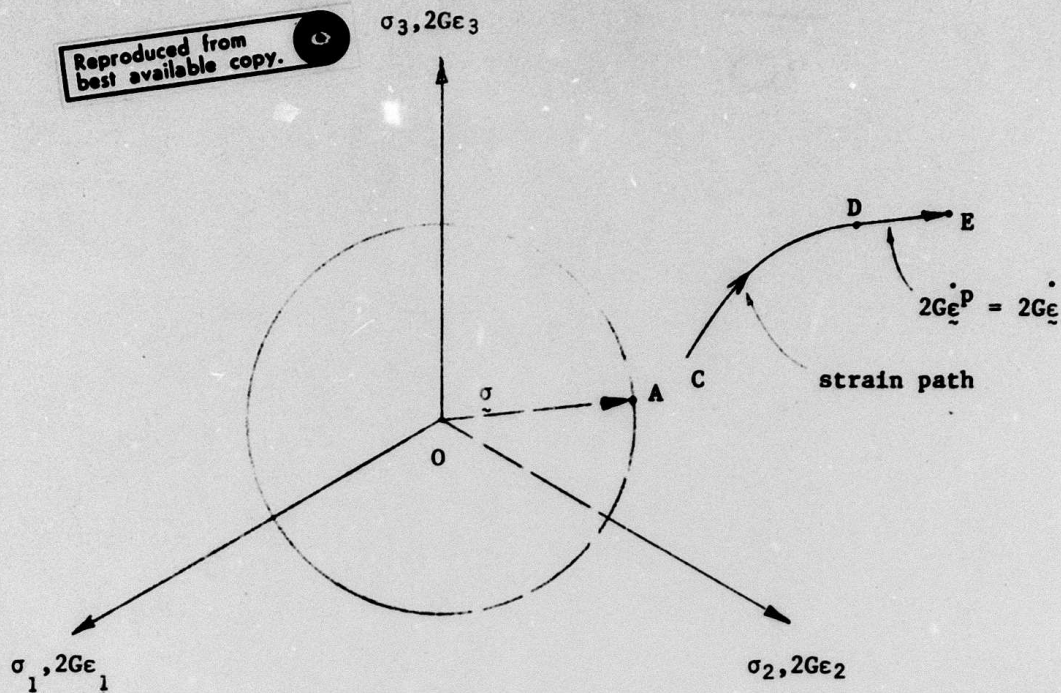


Fig. 1a. Rigid-plastic response

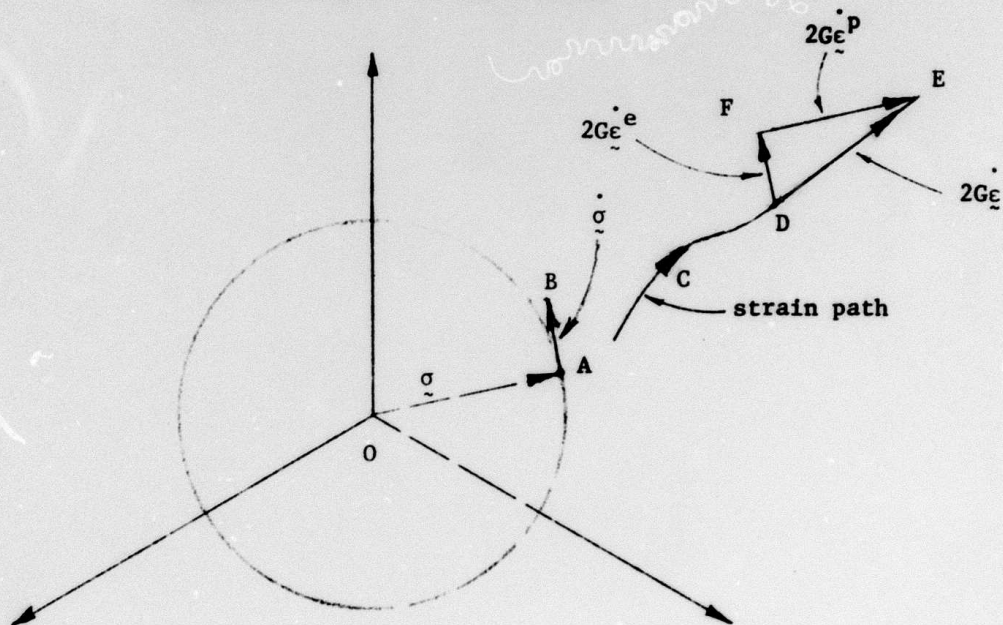


Fig. 1b. Elastic-plastic response