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TECHNICAL REPORT 4775

**THE SABRE METHOD: DESIGNING A RELIABILITY
TEST PROGRAM FOR AN ARTILLERY FIRED
ATOMIC PROJECTILE (AFAP)**



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20. ABSTRACT continued

Carlo experiments are performed, each utilizing different sample sizes and assumed failures. Each Monte Carlo produces a Bayesian system posterior distribution as in the Tri-Service approach. All posteriors are compared against design requirements, and the sample size which produced the best posterior was selected. This technique had resulted in a suggested reduction in sample size of 75% at a total saving of over \$15,000,000.

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INTRODUCTION

In October 1967, a Tri-Service Group was formed to investigate state-of-the-art approaches to reliability assessments. Following the publication of the Tri-Service Report (Ref 1), Picatinny Arsenal began to explore areas of use for Bayesian analysis in the reliability testing of systems. It became obvious that for any Bayesian approach to be truly useful, it must be designed directly into the data collection scheme. Clearly, it would be of little advantage to utilize an analysis which could have been effective for five items if 50 were actually tested. The savings could only be realized if the test program itself was designed so as to only test five items. In order to achieve these savings, the SABRE method was developed. SABRE provides a method of determining a reasonable sample size requirement for reliability testing of atomic projectiles.

CRITERIA FOR DATA COLLECTION AND ANALYSIS

Assumptions

It has been assumed that the data from each test may be combined. If, during actual testing, it is found that the data is not combinable, new analyses will have to be developed to reflect the reduced equivalent sample size. Reliability analyses performed assume both component and system reliabilities behave as random variables, distributed approximately as the Beta distribution.

$$R(p/\alpha, \beta) \approx \left\{ \frac{\Gamma(\alpha+\beta+2)}{\Gamma(\alpha+1)\Gamma(\beta+1)} \right\} p^\alpha (1-p)^\beta$$

for $0 \leq p \leq 1$ and $\alpha, \beta > -1$

where $R(p/\alpha, \beta)$ is reliability as a function of the random variable p given α and β

α is the equivalent number of successes

β is the equivalent number of failures

Γ is the gamma function*

$$*\Gamma(x+1) = \int_0^{\infty} y^x e^{-y} dy, \quad \Gamma(n+1) = n! \quad n \in \mathbb{N}$$

Analyses on these variables are performed using the Tri-Service Method (Ref 1). The general idea of a Bayesian approach has been supported (Ref 2). Finally, it is assumed that the component and system reliabilities are monotonic increasing functions of time at least up until the time of production. Reliability growth occurs both as a function of detecting and correcting deficiencies and of increased confidence in reliability estimates as more applicable data is collected.

Design of the Test Plan

The purpose of this study was to recommend a sample size for the reliability test portion of a test plan, with the goal towards keeping the overall sample size to a minimum. Upon examination of a test plan, it was determined, for that item, both the system safety test rounds (fuzed) and full function test rounds would be of proper configuration to provide suitable data for evaluating reliability. A further examination utilizing the SABRE technique, as presented, indicated that an acceptable reliability estimate could be made well within the bounds of the system safety test sample size. As a result, no additional rounds were proposed solely to evaluate reliability. A still further investigation into the system safety test indicated that, at best, only a "gut feeling" of the system safety could be achieved within the limits of an economically feasible sample size. As a result, an alternative plan was proposed to incorporate the reliability test within the system safety test, and reduce the system safety test sample size to only that level necessary to establish a reliability estimate. The feeling being that if the reliability is sufficiently demonstrated to give creditability to our "paper studies" of reliability then we have justification in assuming that the system does indeed perform as anticipated; thus, in effect, lending support to our "paper studies" of safety. While not entirely scientific, i.e. "statistically valid", this procedure for evaluating safety is no worse than the present method of firing a totally arbitrary number of rounds and postulating extremely low system premature rates from the data. We can indeed subject the test rounds to all environments of interest by using a suitably designed experiment and still determine the effects of each individual condition.

DESIGN PROCEDURES

Assumptions

- (a) Components and system reliabilities are Beta distributed.

- (b) Each component has the same failure rate (attribute).
- (c) Excessive component failures will lead to redesign.
- (d) In each test plan, fewer failures than those stated will be seen for each component.
- (e) Model of system exists.

Discussion

The subsequent section on Simulation Procedure provides the mathematics used in obtaining a sample size for estimating reliability. That procedure consists of a rather simple and straightforward analytical approach. Simply put, it was assumed a certain experiment was performed (i.e., a given test plan adopted) and that certain data was collected. Then an analysis using that data was performed (i.e., a system posterior was calculated). The results of this analysis were examined and, if suitable, then different data was assumed for the same experiment sample size. If the posterior was unsuitable, then a different experiment was postulated. The end result is an "optimal experiment", i.e., one producing the best posterior over a range of possible experimental outcomes. A graph is included to indicate some of the experiments and outcomes which were examined. A brief explanation of the various assumptions may now be of help. The assumptions, needless to say, form the basis for the procedure and, as such, deserve individual examination. Item (a) assumes a Beta fit can be found for each set of data. This is a rather general assumption about the property of components, and with the exception of all but the most wildly misbehaving data a good approximation to reality for those systems considered. Also assumed by item (a) is that the system reliability is Beta-distributed. This again is a reasonable assumption, barring any evidence to the contrary (i.e., bi- or tri-modal distribution caused by multiple manufacturers, etc). Assumptions (b), and (c), and (d) form a limiting case of the considerations and, as such, pose a condition for rejecting a design/system as unsuitable. Assumption (c) is an outcome of the necessity for using component level information to make system level statements. Without a model of the system, based upon the components, this procedure fails. As a result, the system estimates can only be as accurate as the model being used. This area will, in fact, be one for close future surveillance to assure that the model is always representative of the system as fielded. The simulation procedure shown is a very simple Monte Carlo approach. The only areas worth noting are:

(a) The conservative prior of $\alpha = 0, \beta = 0$ is used on the component level pending the outcome of the engineering design (ED) phase testing when a less conservative prior can be determined. The effect is to possibly require a slightly higher sample size at that time than may ultimately be required.

(b) The number of simulations stated, 200, is purely arbitrary at this time, decided on a computer cost basis. Before each test phase, a finalized test program will be determined as indicated in above paragraph. Then, a sufficiently large number (N) of Monte Carlo simulations will be used to make the results insensitive to such N.

Simulation Procedure

Using engineering judgment, pick the highest failure rate/component. Suppose, in a sample of 40 components, we will have one component failure. Our Beta parameters for the component reliability distribution are:

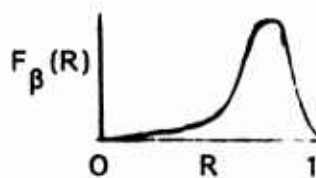
$$\alpha = \# \text{ successes} = 39$$

$$\beta = \# \text{ failures} = 1$$

These parameters are sufficient to describe a Beta curve

$$F_{\beta}(R) = \frac{\Gamma(\alpha+\beta+2)}{\Gamma(\alpha+1)\Gamma(\beta+1)} R^{\alpha} (1-R)^{\beta}$$

The Beta curve looks like:

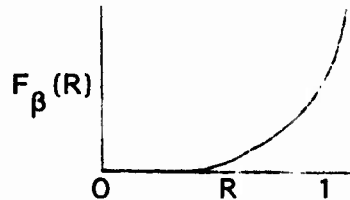


$$\text{Mean} = (\alpha+1) / (\alpha+\beta+2)$$

$$\text{Mode} = \alpha / (\alpha+\beta)$$

For each block in the system Model generate a random variate from $F_{\beta}(R/\alpha, \beta)$.

Having a numerical value in each block of the system model, calculate a point estimate for system success. This can be done with a system success equation, a system reliability computer program, or a failure equation. Storing this point estimate, repeat the process. Generate random variates for components and calculate a system estimate. Continue this procedure until 200 system point estimates have been compiled. Calculate the mean and standard deviation of these 200 values and calculate the equivalent system Beta parameters α and β . The mode is the best estimate of system reliability. In certain cases, where the equivalent number of system failures is less than zero, the Beta curve looks like the following:



The curve is asymptotic to $R = 1.00$. In this case, the mode is invalid and the mean must be used as a best estimate for the system reliability. After the equivalent system parameters and best estimate are computed, the 90% confidence value can be calculated. This is done by finding the point on the axis which 90% of the area under the curve falls to the right of. We now have the following information:

- (a) Component sample size and failure rate.
- (b) Best estimate of system reliability.
- (c) 90% confidence value for system reliability.

It is now necessary to review these values and see if they meet the requirements. If these values are too low, then it may be necessary to increase our sample size tested and/or reduce the number of component failures which are acceptable. Either of these actions will shift the curve to the right, increasing the best estimate and 90% confidence value. Figure 1 gives an example of various possible results for a range of test plans.

EXAMPLE OF APPROACH

This section provides an example of how data may be analyzed. For simplicity, the conservative $\alpha=0$, $\beta=0$ prior was again used; however, a

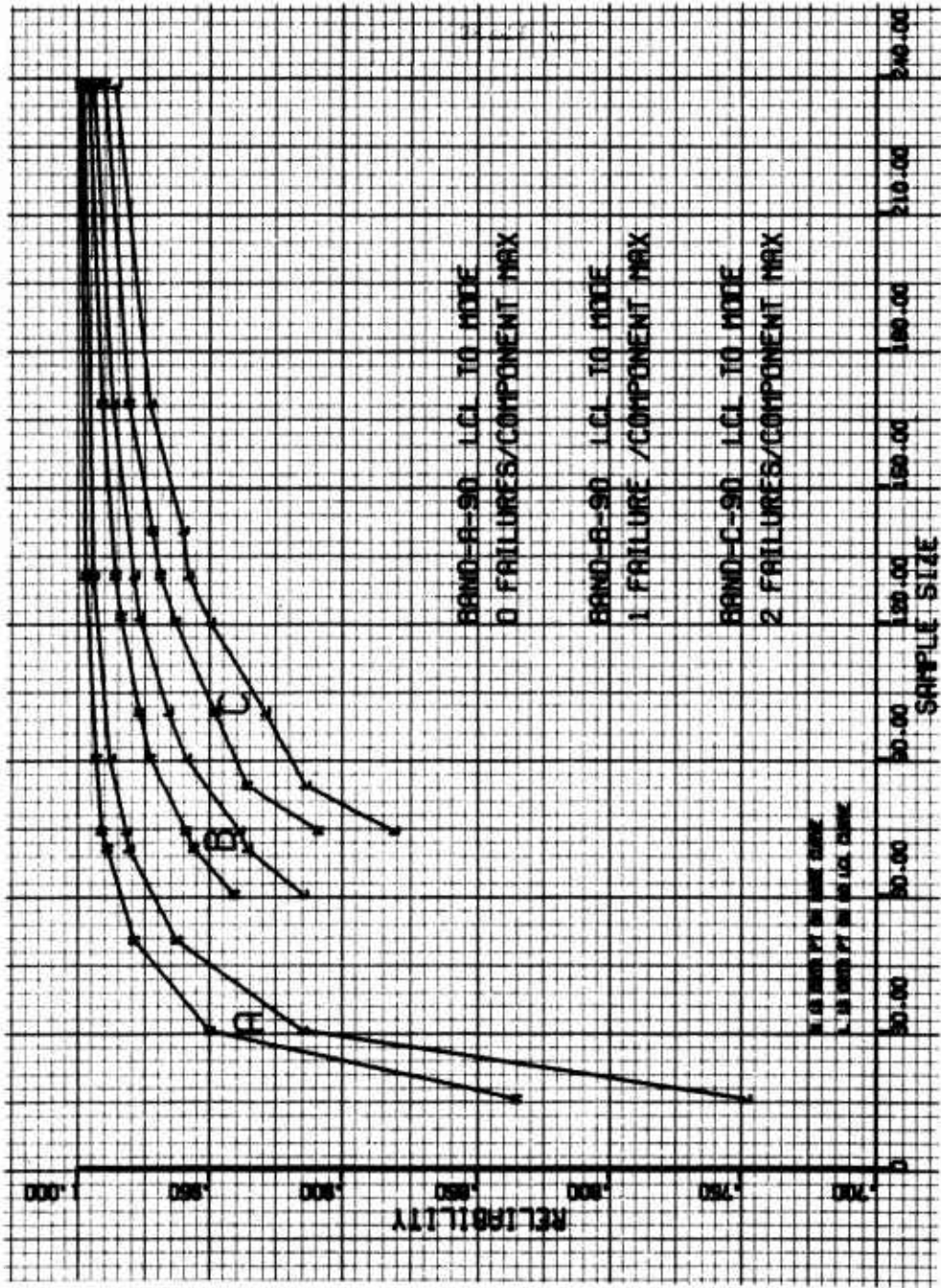
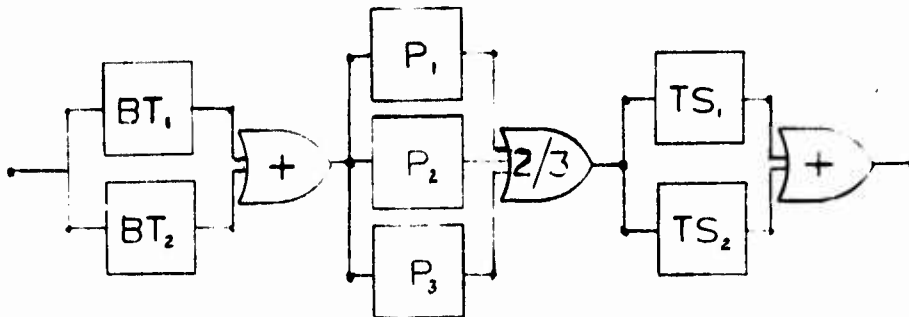


Fig 1 System reliability versus sample size for various assumed failure rates

less conservative prior could be applied if applicable prior test data exists. In this example, a total of 994 components are tested as parts of systems, with six component failures occurring. In all likelihood, corrective action would be taken to preclude the recurrence of failures similar to those six which occurred. However, barring a total major system redesign, all six failures should be included as indicative of other failures which potentially exist and are at present uncorrected. At no time are these statistical procedures meant to outweigh or replace sound engineering judgment, and each failure which occurs should be examined, not only for its impact on the system but, more importantly, how strongly this failure contradicts what were previously held "truths" about the manner in which the system operates. Bearing this in mind, the statistical procedure is performed once the engineers are convinced that they can continue to accent their "prior beliefs".

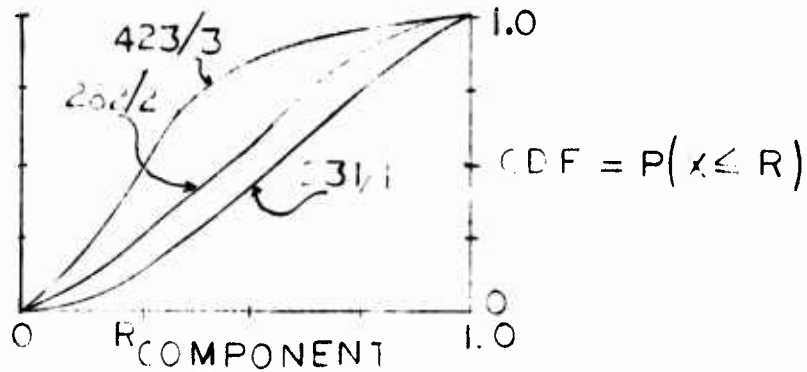
Let us say we have actually tested 45 safety test rounds, 18 full function rounds, and 90 firing table rounds. This gives us a total sample size of 153 rounds. This should give 306 tests of double-redundant components and 459 tests of triple-redundant items. Say, for example, we use a system as below:



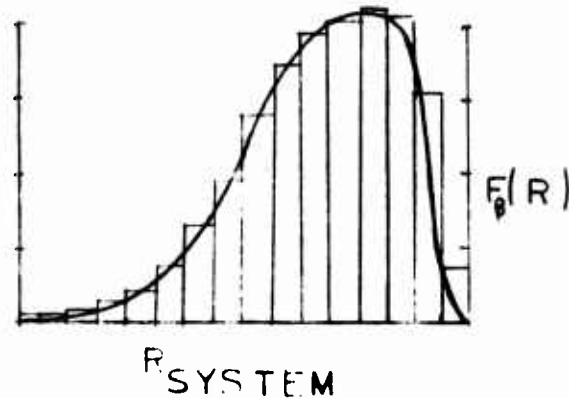
And say our 153 rounds result in three failures for p, one failure for BT, and two failures for TS, with 11 system no tests (no TM at all). We have test results below:

<u>COMPONENT</u>	<u>SAMPLE SIZE</u>	<u>SUCCESS</u>	<u>FAILURE (b)</u>
BT	284	283	1
P	426	423	3
TS	284	282	2

We construct cumulative density functions for the Beta distributions of each component using the α and β parameters as above (roughly pictured below).



We will then randomly sample along the CDF axis to get a corresponding value of R for each of the blocks in the system diagram (1) and calculate one point estimate for system reliability. Repeating the sampling procedure many times, we will calculate a large number of system point estimates (like 200) and plot a histogram of these points. We can then take a Beta fit to this histogram and calculate any desired statistics for this posterior system Beta distribution as required.



SUMMARY

The result of one study was the recommendation for a substantial reduction in the size of the system safety portion of the test program. This reduction was brought about through the judicious use of component level reliability test data for determining system reliability. While a numerical safety estimate was not made based upon test data, an engineering analysis of system safety would be made from all available data. This engineering analysis would depend in large part on the belief that the system operates as expected; and that belief is substantiated to a degree by the reliability data taken.

The SABRE method is simply an analytical approach to reliability test program design. Various experiments are assumed, worst case data is then assumed, and experimental results calculated by the techniques described. The outcomes are screened to determine the optimum experiment yielding desired outcomes. Key points in the analytical procedure are formulation of priors, Monte Carlo analysis to yield system posteriors, and a risk analysis using the parameters of the posterior distribution. Imbedded in the procedures is a reliance on a mathematical model for the system. A computer routine "SABRE" soon to be available from Picatinny Arsenal will utilize the most advanced modeling techniques yet available; but that we will save for future discussion.

REFERENCES

1. Mardo, J. et al, *A Tri-Service Bayesian Approach to Nuclear Weapons Reliability Assessment*, Picatinny Arsenal Technical Report 4359, April 1972
2. Atzinger, E. and Brooks, W., *Comparison of Bayesian and Classical Analysis for a Class of Decision Problems*, Army Materiel Systems Analysis Agency (AMSAA), Technical Report 59, April 1972

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