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GEAR TOOTH SCORING INVESTIGATION

P. M. Ku, et al

Southwest Research Institute

Prepared for:

Army Air Mobility Research and
Development Laboratory

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GEAR TOOTH SCORING INVESTIGATION

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The predictive scheme comprises basically two steps. The first step involves the prediction of the ideal scoring-limited power-transmitting capacity, assuming perfect tooth alignment and no dynamic tooth load. The probable, actual scoring-limited power-transmitting capacity is then deduced from the ideal scoring-limited power-transmitting capacity by applying corrections for the misalignment and dynamic effects.

In order to evaluate the quality of the predictions, scoring tests have been performed on typical aircraft-quality gears, including spur gears of the same design but of two different surface characteristics, and spiral bevel gears of one design. The experimentally determined scoring-limited power-transmitting capacities have all been found to be within ten percent of the predictions.

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EUSTIS DIRECTORATE POSITION STATEMENT

The work reported herein is the result of controlled disk tests, analytical computations, and verification gear tests. The resulting computer programs predict the scoring potential and scoring-limited, power-transmitting capacity of spur, helical, and spiral bevel gears. The verification gear test data have all been found to be within 10 percent of the predictions.

E. Rousee Givens of the Technology Applications Division served as Project Engineer for this effort.

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PREFACE

This report presents the results of a program conducted by Southwest Research Institute for the Eustis Directorate, U.S. Army Air Mobility Research and Development Laboratory, under Contract DAAJ02-70-C-0071. The USAAMRDL technical direction was provided by Mr. R. Givens.

The work described herein was performed with Southwest Research Institute as the prime contractor and Bell Helicopter Company as the subcontractor. SwRI had overall responsibility in the administration of the program, in addition to the development of the gear scoring predictive technique, the performance of sliding-rolling disk tests, and the analyses of all disk-test and gear-test results. BHC was responsible for the manufacture of all test disks and test gears, as well as gear testing.

In addition to the above gear tests, valuable spur gear test results were made available to this program by Mr. P. Lynwander, Chairman, Tribology Division, Aerospace Gearing Committee, American Gear Manufacturers Association. These test results were utilized in the formulation of the gear scoring predictive methodology.

Active participants of this program included P. M. Ku (principal investigator), H. E. Staph, H. J. Carper, D. M. Deffenbaugh, and H. Haufler of SwRI; C. E. Braddock, R. Battles, and R. T. Jenkins of BHC; as well as other supporting personnel of the two organizations.

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CHAPTER I

INTRODUCTION

The investigation reported herein had as its basic objective the development of an improved methodology for predicting the power-transmitting capacity of gears as limited by gear-tooth scoring, with special reference to aircraft power gears. The gear types of interest included spur gears, straight helical gears, and spiral bevel gears. The gear material selected for study was carburized vacuum-melt AISI 9310 steel. The lubricants were MIL-L-7808 and MIL-L-23699 synthetic lubricants, with emphasis on a MIL-L-7808G lubricant.

In order to accomplish the above objective, pertinent information in the literature was reviewed and made use of where deemed applicable; controlled sliding-rolling disk tests were performed and their results analyzed and generalized to provide the basic inputs to the predictive scheme; computer programs were written for predicting the scoring-limited power-transmitting capacity of spur, helical, and spiral bevel gears; and full-scale gear tests were conducted to test the validity of the computer predictions.

Aircraft power gears should ideally be designed to give maximum power-transmitting capacity per unit size and weight, and at the same time possess a high degree of operating reliability. Obviously, it is not possible to assess operating reliability and hence the maximum permissible power-transmitting capacity of gears as limited by all possible modes of failure without considering the nature of these failure modes and their respective impact on gear performance. A comprehensive gear failure analysis of this character is not only beyond the scope of this investigation, but also impossible to accomplish in many respects. However, in order to provide some perspective to the overall problem, a brief qualitative discussion of the major gear-tooth failure modes and their effects will be given in the next chapter. It suffices to state at this juncture that gear teeth may fail basically by either strength-related or lubrication-related causes. However, many strength-related failures are influenced by lubrication or can be induced by lubrication-related failures. The importance of lubrication to gear performance is thus abundantly clear.

The current investigation is concerned with gear-tooth scoring, which is one particular mode of lubrication-related gear failure. In contrast to the other modes of lubrication-related gear failure which generally take time to develop or reach destructive magnitude, scoring

occurs quite precipitously and is therefore the most urgent one confronting the designer. Obviously, as long as scoring cannot be overcome, all time-dependent modes of gear failure are essentially academic. In other words, with gears that are adequately designed and manufactured strengthwise, scoring is the first performance barrier that must be crossed. It is only after scoring can assuredly be controlled that the other failure modes become truly relevant.

As will be seen later, the existing technology of gear design is such that the risk of strength-related failures can be quantitatively assessed with some confidence, especially for gears of relatively simple geometry operating under conditions such that misalignment and dynamic effects are not large. However, the same cannot be said of all lubrication-related failures due to the enormous complexity of the phenomena involved. This investigation emphasizes the scoring problem. Assuming that a scoring criterion can be established and quantitatively related to gear performance, then the next logical step will be the development of criteria for quantitative assessment of the effects of the other lubrication-related failures, as well as further refinements in handling the strength-related failures particularly for the complex gear types and the subtle influence of gear mechanics on lubrication. Unfortunately, all such information is not yet at hand; hence meaningful optimization of gear design is now not possible.

In view of the overwhelming importance of scoring, it is understandable that the gear designer's concern, other than the strength considerations, has so far been directed primarily toward the avoidance of scoring almost at any cost. In this connection, it is well to emphasize the difference between the avoidance of scoring and the quantitative prediction of scoring in the design stage. Thanks to decades of efforts on the part of many workers, the existing gear design technology is such that although the onset of scoring cannot yet be predicted accurately, nevertheless enough is known in a general way to avoid scoring by design without regard to the price to be paid. The advancement that is being sought in this program is to be able to predict scoring in the design stage and estimate the scoring-limited power-transmitting capacity. It is a far more difficult task than mere avoidance of scoring without inquiring as to what penalties are thereby entailed.

The program was essentially in the nature of engineering application, and as such no serious effort has been made to delve into the basic mechanism of scoring. Nevertheless, tangible progress is believed to have been made in the phenomenological sense to provide a

methodology for predicting the scoring-limiting power-transmitting capacity of gears. As outlined in Chapter VII, the basic procedure involves first the prediction of the ideal scoring-limited power-transmitting capacity of a gear set, assuming perfect tooth alignment and no dynamic tooth load. The probable, actual scoring-limited power-transmitting capacity is then deduced from the ideal scoring-limited power-transmitting capacity by applying corrections for the misalignment and dynamic effects.

As will be shown in Chapter VII, the prediction of the ideal scoring-limited power-transmitting capacity requires the use of certain numerical coefficients, and the prediction of the actual scoring-limited power-transmitting capacity further requires quantitative estimates of the misalignment and dynamic correction factors. Tentative values for these coefficients and correction factors were derived from the results of the sliding-rolling disk tests performed under this program, supplemented by several sets of full-scale spur gear test results made available by the Aerospace Gearing Committee of the American Gear Manufacturers Association. It is realized that these coefficients and correction factors are subject to refinements as additional disk and gear test results become available. Nevertheless, using the tentative values thus far developed, predictions were made for the full-scale gear tests performed under this program. As will be seen in Chapter VIII, the predicted scoring-limited power-transmitting capacities were within 10 percent of the statistically deduced test results from two series of tests on spur gears and one series of tests on spiral bevel gears. Helical gears were not tested in this program, due to difficulties encountered by the subcontractor in the scheduling of gear manufacturing and testing.

The investigation has brought out certain fundamental issues related to gear lubrication and gear mechanics, the resolution of which is believed to be essential before gear design, performance prediction, and performance optimization can be put on a truly rational basis. These problems are discussed in Chapter IX.

CHAPTER II

GEAR-TOOTH FAILURE AND LUBRICATION

A. Major Modes of Gear-Tooth Failure

Since this investigation is concerned with gear design and performance analysis, the nomenclature and symbols employed herein follow generally those adopted by the American Gear Manufacturers Association.¹⁻⁶ However, departures from the AGMA practice are made in several instances for the sake of clarity, as evident from the List of Symbols presented at the end of the report.

AGMA cites 21 modes of gear-tooth failure,² divided into four broad categories of wear, surface fatigue, plastic flow, and breakage. For the purpose of the present discussion, it is convenient to classify the major gear-tooth failure modes as shown in Table 1.

There are two broad classes of gear-tooth failures, namely, lubrication-related failures and strength-related failures. Major modes of lubrication-related failure are rubbing wear, scoring, and pitting. Major modes of strength-related failure are plastic flow and breakage.

Rubbing wear is a loss of metal by the rubbing action between two relatively moving surfaces, when there is a lack of an intact oil film of sufficient thickness to separate the surfaces.^{7, 8} One form of rubbing wear is adhesive wear, caused by metal transfer due to localized adhesion or a solid-phase welding process, and subsequent detachment of particles from one or both surfaces. The other form of rubbing wear is abrasive wear, caused by abrasive action between the relatively moving surfaces, or by the presence of abrasive particles between them. These particles may be dirt or other solid contaminants, or particles detached from the surfaces themselves due to severe pitting or wear.

Rubbing wear takes time to reach damaging proportion. It is of course harmful if severe and continued at an undiminishing rate. However, rubbing wear which diminishes with time, such as that associated with a break-in process, is not damaging but in fact beneficial.

Scoring (or scuffing) is a severe form of adhesive wear, which results in rapid damage to one or both surfaces in relative motion.^{9, 10} In contrast to the other modes of lubrication-related tooth failure which generally take time to develop or reach destructive magnitude, scoring occurs quite precipitously and is therefore the most urgent one

TABLE 1. MAJOR MODES OF GEAR-TOOTH FAILURE

<u>Mode</u>	<u>Basic cause</u>
<u>Lubrication-related failure</u>	
1. Rubbing wear	Lack of an intact oil film of sufficient thickness.
a. Adhesive wear	Metal transfer by localized adhesion, and subsequent detachment of particles.
b. Abrasive wear	Abrasive action.
2. Scoring	Lack of intact oil film coupled with intense localized frictional heating.
3. Pitting	Repeated surface stress cycling.
<u>Strength-related failure</u>	
1. Plastic flow	Surface deformation under heavy load, often aggravated by inadequate lubrication.
2. Breakage	Bending fatigue, severe pitting or abrasive wear.

confronting the designer. Obviously, as long as scoring cannot be avoided, all time-dependent modes of failure are essentially academic. In other words, with gears that are adequately designed and manufactured strengthwise, scoring is the first performance barrier that must be crossed. It is only after scoring can assuredly be controlled that the other failure modes become truly relevant. This accounts for the enormous emphasis to date on gear scoring by researchers and designers alike.

Since scoring is a form of adhesive wear, it cannot occur if an oil film of sufficient thickness separates the surfaces. However, mere lack of an intact oil film, while it inevitably leads to adhesive wear, may not cause scoring. In order for adhesive wear to advance to scoring, another necessary condition must be satisfied. Although the precise mechanism of scoring is at present not yet understood, the consensus is that it is the result of intense, localized frictional heating at the rubbing contact and is thus thermal in character.

Pitting (or surface fatigue) is the consequence of repeated stress cycling of the surfaces beyond the metal's endurance limit, which leads to surface or subsurface cracks and eventually the detachment of fragments from and the formation of pits on one or both surfaces.⁹⁻¹¹ Being a fatigue phenomenon, pitting takes time to develop. However, while rubbing wear and scoring cannot take place if an intact oil film of adequate thickness separates the two surfaces, pitting can occur even though it takes more time. This is because the presence of such an oil film merely modulates the intensity of the repeated surface stressing, but does not eliminate it altogether.

Plastic flow is the surface deformation resulting from plastic yielding of one or both surfaces in relative motion, usually associated with heavy loads or high temperatures. Although basically a strength-related phenomenon, it can nevertheless be influenced by lubrication. For example, high temperature which results in a reduction of the metal's yield strength may be due to inadequate lubrication. Moreover, rippling (a form of plastic flow) is apparently related to a complex interaction between the oil film and surfaces.¹²

Breakage of a gear tooth is caused by the bending stress imposed on it by the transmitted torque.⁴⁻⁶ Outright breakage due to excessive bending stress beyond the fracture strength of the tooth is rather rare. A more common form of tooth breakage is that due to bending fatigue. Breakage is basically a strength-related failure. However, severe pitting or wear may so weaken the tooth (otherwise

adequate for the service) as to cause breakage. In this connection, a lubrication-related failure may lead to a breakage failure.

Consideration of the strength-related modes of gear-tooth failure is beyond the scope of this report. However, the subject is well covered by the standard treatises.¹³⁻¹⁵ Moreover, the AGMA Standards for rating the strength of several gear types⁴⁻⁶ provide tangible, straightforward, and quantitative guides to design, provided the many correction factors contained therein are selected with care.

In Chapters III, IV, and V which follow, the mechanics of spur, helical, and spiral bevel gears will be briefly discussed, with the objective of providing a background for assessing the risks of gear-tooth scoring as well as the other modes of lubrication-related failure.

It is only necessary to emphasize here that although gear-tooth failures may be due to either strength-related causes or lubrication-related causes, many strength-related failures are directly or indirectly influenced by lubrication.

B. Nature of Gear-Tooth Scoring

Although the basic mechanism of the scoring phenomenon is still largely not understood, there is good agreement that the lack of an intact oil film between the relatively moving surfaces is only a necessary but insufficient condition for scoring.^{9, 10, 16-31} In other words, in order for scoring to occur, the operation not only must move into the boundary lubrication regime,* but also must meet an additional requirement. However, largely because the mechanism of scoring is basically unsettled, what form this additional scoring criterion must take is still very much an open question. All available evidence appears to suggest that how deeply the operation may safely extend into the boundary lubrication regime without resulting in scoring depends upon the physical and chemical nature of the oil, the metal surface, and the surrounding atmosphere, as well as the operating conditions. Moreover, if there is a generalized scoring criterion, the consensus is that it is thermal in character, i. e., it is the consequence of the intense frictional heat generation at the potential scoring site.

* The term "boundary lubrication" is used herein to designate collectively the rather ill-defined modes of classical boundary lubrication, mixed lubrication, partial-elastohydrodynamic lubrication, and micro-elastohydrodynamic lubrication.

Of the various thermal scoring criteria that have been proposed, the most famous is no doubt the critical temperature criterion.^{16, 17} Other principal criteria include the critical power intensity criterion²¹ and the critical power criterion.²⁸ These criteria have been compared,²⁷⁻³⁰ but the comparisons are basically inconclusive. In short, there is no lack of data or arguments either to support or to refute any of these criteria in some way. However, from the point of view of generality, i. e., the potential of the criterion to account for the greatest number of design, material, lubricant, surface, and operating variables, it appears at this time that Blok's critical temperature criterion is the most promising, provided suitable refinements are added to Blok's original hypothesis. Such refinements will be discussed later in Chapter VI as applied to sliding-rolling disks and in Chapter VII as applied to gears.

Critical Temperature Hypothesis. In a sliding-rolling system, the friction due to the relative motion results in a localized, instantaneous rise in the temperatures of the surfaces in the conjunction which, under steady-operating conditions, establishes the quasi-steady temperatures of the two surfaces and the bulk temperatures of the two bodies. In most practical cases, the partition of the frictional heat and the general heat transfer conditions at the conjunction are such that the two surface temperatures are approximately equal.³² The very unusual case when this is not so requires a more elaborate treatment.²³ However, such a refinement is hardly warranted at this time, inasmuch as the basic mechanism of scoring is, as previously stated, still not understood.

Taking the two surface temperatures as being equal, then one may write for the sliding-rolling system

$$T_c = T_s + \Delta T \quad (1)$$

where T_c = maximum instantaneous surface temperature in the conjunction, °F

T_s = quasi-steady surface temperature, °F (often erroneously taken as the "bulk temperature")

ΔT = maximum rise of instantaneous surface temperature in the conjunction above the quasi-steady surface temperature, °F (also called the "flash temperature" by Blok)

Note that the transient temperature, T_C , is made up of two components: a quasi-steady component, T_s , and a transient component, ΔT . Both of these components are basically caused by the frictional heat generation at the conjunction. Their principal difference is that the ΔT component arises almost instantaneously, so practically no heat loss can take place in the process. On the other hand, the T_s component can be influenced by the heat loss from the conjunction, because it is taken over a period of time.²⁸ This problem will be considered further in Chapters VI and VII.

The basic premise of Blok's critical temperature hypothesis is that scoring will occur when T_C reaches a critical value.^{16,17} In other words, the criterion for scoring is

$$T_{Cr} = T_s + \Delta T \quad (2)$$

where T_{Cr} = critical temperature, °F.

In Blok's postulate, T_{Cr} is considered as a constant for a given metal-oil combination, regardless of the surface and operating conditions. However, this has not been found to be quite true by other investigators.^{24-26, 28-30} The matter will be considered in some detail in Chapter VI.

If the critical temperature hypothesis is accepted, then scoring prediction for a given gear set becomes basically a problem of establishing the proper value of T_{Cr} and estimating the value of T_s for various operating conditions, from which the ΔT required to reach scoring may be determined. Once this scoring-limited ΔT is known, the scoring-limited power-transmitting capacity may readily be computed.

The basic expression for computing the scoring-limited power-transmitting capacity is Blok's equation for a steady-operating sliding-rolling system with a rectangular conjunction (the so-called "line contact"), such as a system comprising a pair of perfectly aligned, straight cylinders.¹⁹ This equation relates ΔT to the unit normal load as follows:

$$\Delta T = \frac{1.11 fw \left| \sqrt{V_1} - \sqrt{V_2} \right|}{\beta \sqrt{B}} \quad (3)$$

where f = coefficient of friction
 w = unit normal load, ppi
 V_1, V_2 = surface velocities (absolute) of the two bodies, ips
 β = Blok's thermal coefficient, $\text{lb}/^\circ\text{F-in.-sec}^{\frac{1}{2}}$
 B = width of Hertzian band in the direction of motion, in.

The various quantities involved in Equation (3) will be discussed in Chapter VI for steady-operating sliding-rolling disks. Application to gears, which undergo transient operation, must consider the effect of gear mechanics as given in Chapter VII.

Note that Equation (3) applies strictly to a sliding-rolling system with a rectangular conjunction. In case the conjunction is elliptic in shape, B must be replaced by the width of the Hertzian ellipse and w be replaced by an "equivalent unit normal load" such as that proposed by Kelley.¹⁸

C. Elastohydrodynamic Lubrication

It follows from Section A of this chapter that the presence of an intact oil film between the surfaces of mating gear teeth appears to be a desirable design goal, as it precludes rubbing wear and scoring, and minimizes pitting danger. However, whether or not such a design goal is practical or even achievable requires critical examination.

When counterformal bodies are loaded against each other, their surfaces experience significant localized elastic deformations. Elastohydrodynamic lubrication deals with the interaction between the hydrodynamic action of the lubricant and the localized elastic deformations of the surfaces. It basically explains why an intact oil film may exist under certain conditions between highly-loaded counterformal systems.

It has been found, both analytically and experimentally,^{33, 34} that the oil film thickness in an EHD conjunction is not uniform. Accordingly, the oil film thickness of particular interest is the minimum oil film thickness, because if rubbing contact were to occur, it would be apt to occur where the oil film thickness is the least.

The basic equation for the minimum oil film thickness in a

rectangular EHD conjunction of perfectly smooth surfaces, in a steady-state, flooded, and isothermal flow, has been given in dimensionless form by Dowson.³⁴ This equation may be written in conventional engineering units as follows:

$$h_m = 26.5 \frac{\alpha_o^{0.54} (\mu_o V_t)^{0.70} R^{0.43}}{w^{0.13} E^* 0.03} \quad (4)$$

where h_m = minimum oil film thickness, $\mu\text{in.}$

α_o = pressure-viscosity coefficient of oil at conjunction-inlet temperature and near-atmospheric pressure, psi^{-1}

μ_o = absolute viscosity of oil at conjunction-inlet temperature and near-atmospheric pressure, cp

V_t = sum velocity, ips

R = equivalent radius of curvature at the conjunction, in.

w = unit normal load, ppi

E^* = equivalent Young's modulus, psi

The above equation applies strictly to a sliding-rolling system with a rectangular conjunction, provided the numerous assumptions stated above are met. In practical applications, the conjunction shape may not be rectangular, but may be elliptic in shape. If the aspect ratio of the ellipse normal to the motion is large (> 5), little error results from the rectangular assumption. However, if the aspect ratio is small, then the problem becomes more complex. In that event, an approximate correction for "side flow effect," due to Cheng,³⁵ may be used, provided all the other assumptions are not substantially violated.

Additionally, the assumption of an isothermal flow process may not be approached in practice, due to heating caused by the viscous shear of the oil in the inlet region. This effect can be very significant at high sum velocities, particularly when the oil viscosity is high. In that event, another approximate correction for the "inlet-shear thermal effect," also due to Cheng,³⁶ may be applied, provided again all the other assumptions are not substantially violated.

When Cheng's side flow and inlet-shear thermal corrections are applied to Equation (4), the minimum oil film thickness for a flooded, elliptic EHD conjunction of perfectly smooth surfaces is obtained:

$$h_m' = 26.5 \frac{\alpha_o^{0.54} (\mu_o V_t)^{0.70} R^{0.43} \phi_s \phi_t}{0.13 \frac{*}{E} 0.03} \quad (5)$$

where h_m' = minimum oil film thickness, $\mu\text{in.}$

ϕ_s = side flow correction factor

ϕ_t = inlet-shear thermal correction factor

The procedure for estimating the values of ϕ_s and ϕ_t is conveniently summarized in a recent paper by Cheng.³⁷ The other quantities involved in Equation (5) will be further dealt with in Appendix C, primarily as a matter of general interest. However, the calculation of the EHD film thickness will not be emphasized in this report. The reasons are many;^{9,10} but the crucial ones are as follows:

1. The inlet-shear thermal correction factor does not account for the nonuniform temperature distribution across the oil film, yet this effect can be quite significant in practice.³⁸ Reliable assessment of the temperature gradient across the film, particularly considering sliding³⁹ and the complex participating flow and heat transfer involved, is currently not available.

2. Similarly, an improved method for estimating the side flow correction factor is required.

3. Actual surfaces are never perfectly smooth. Surface roughness and surface texture affect the EHD film formation in a complex manner;^{25,40-42} but there is as yet no confident way to assess these effects. Indeed, if the composite surface roughness involved is about the same order of magnitude as the nominal film thickness, EHD lubrication in the classical sense no longer prevails. In that event, the meaning of classical EHD lubrication becomes quite obscure, and the computed EHD film thickness resulting therefrom is apt to be very misleading.

4. Due to the action of the gear teeth and the conventional manner of oil supply, the state of gear-tooth lubrication is probably always starved, or far from the flooded assumption. Although the effect of starvation on film thickness behavior is quite well understood by assuming an arbitrary inlet boundary location and shape with a uniform temperature distribution across the film,⁴³ these assumptions are, as stated above, not realistic for gears. In any case, there is presently no reliable way to relate the extent of starvation (i. e., the inlet boundary location) to lubricant, design, and operating parameters, even under these idealized conditions.

5. Gear-tooth action also introduces dynamic tooth loading, augments oil film development due to normal approach of the tooth surfaces, and causes flow acceleration and deceleration which affect film formation. These effects are difficult to account for quantitatively.

6. As stated previously, it is now well established that the lack of an intact oil film between the relatively moving surfaces is only a necessary but insufficient condition for scoring. In other words, scoring always occurs in the boundary lubrication regime, and the scoring risk is more realistically assessed by another criterion, such as the critical temperature. Accordingly, gear design based on obtaining full EHD lubrication is not only unnecessary, but far too conservative from the standpoint of size and weight.

7. Even if one chooses to employ full EHD lubrication as a design goal, he has no assurance that it will be achieved in practice, since the current film thickness predictive technique is inadequate for this purpose, for reasons stated above. It can be readily seen from Equation (5) that, everything else being equal, h_m^l is inversely proportional to $w^{0.13}$, or w is inversely proportional to $(h_m^l)^{7.7}$. Thus, any errors in predicting the minimum oil film thickness will be greatly magnified in the prediction of the scoring-limited power-transmitting capacity.

D. Boundary Lubrication

The above remarks are not intended to minimize the important contributions of the EHD theory to the current understanding of the lubrication of counterformal surfaces. It is only that as knowledge on the details of EHD lubrication expands, complications begin to emerge and further refinements appear necessary. In particular, once the operation leaves the full EHD lubrication regime, a continuous and undisturbed oil film no longer exists between the mating surfaces. The operation then enters the boundary lubrication regime, and the

complex chemical interactions involved cannot be ignored.

It has been argued previously that of the three major lubrication-related gear-tooth failure modes, EHD lubrication is not a necessary condition for pitting and not a sufficient condition for scoring. Therefore, in assessing the effect of lubrication-related failure modes on gear performance, the crucial question is not when and how full EHD film ceases to prevail; but rather when and how the boundary film formed by the oil-metal-atmosphere interaction ceases to inhibit or minimize surface failures.

Boundary lubrication is of course the most investigated but perplexed subject in lubrication. Literally thousands of references exist that pertain to various aspects of boundary lubrication; but a few broad treatises should suffice to illustrate its scope and tremendous complexity.⁴⁴⁻⁴⁷ It is not possible to deal with the subject of boundary lubrication in simple terms. What appears particularly important as pertains to the lubrication-related failure modes of gear teeth will be commented on later, mainly in Chapter VI. In any case, as the operation moves into the boundary lubrication regime, i. e., when contact between the gear-tooth surfaces takes place, rubbing wear becomes inevitable, scoring becomes a possibility, and pitting becomes more likely. The manifestation of rubbing wear and pitting damages is time-dependent, and their rates of damage depend upon the physical and chemical oil-metal-atmosphere interactions. The occurrence of scoring is quite precipitous, and is also controlled by boundary lubrication considerations in some way.

E. Lubrication-Limited Gear Performance

It is very difficult to describe the lubrication-related modes of gear-tooth failure, or to evaluate their impact on gear performance with much confidence, mainly because the mechanisms of the failure modes are still not well understood and the effect of gear mechanics on these failure modes—while known to be large—cannot be accurately assessed. However, regardless of the approximations involved, an analysis made by Blok¹⁹ is instructive.

In his analysis, Blok derived general expressions for the maximum power transmittable by a set of homologous gears, or gears having similar design and materials, using a straight mineral oil. He assumed that rubbing wear would not take place if the operation is in the full EHD regime. He considered that scoring was governed by his critical temperature hypothesis, with the critical temperature and

instantaneous coefficient of tooth friction assumed constant. He regarded pitting as strictly a mechanical consideration by ignoring the effect of lubrication. Using these assumptions, Blok concluded that in the absence of strength-related failures, the maximum power transmittable through a set of homologous gears is primarily limited at low speeds by rubbing wear, at intermediate speeds by pitting or scoring, and at high speeds by scoring.

As has been discussed in the preceding sections of this chapter, Blok's assumptions are rather drastic oversimplifications of very complex phenomena. Quite apart from the difficulty of defining the mechanisms of the failure phenomena, which will hopefully come about in due time, it is believed that the time-dependent nature of the damages due to rubbing wear and pitting can be introduced in the scheme of analysis. While such refinements will alter Blok's predicted trends substantially in some regimes of gear operation, it is not believed that his major conclusions will be greatly altered. The Blok analysis gives probably the most convincing reason for the practical importance of scoring, especially for aircraft power gears.

F. Impact of Gear Mechanics

Lubrication is concerned with the behavior of interacting surfaces in relative motion. Accordingly, the study of the lubrication of any machine element must, by necessity, include a consideration of the total effect of the following participating factors:

1. Motions without regard to the forces acting, or a study of kinematics.
2. Forces, displacements, and motions, which are the concern of what may be termed statics (where a state of rest is assumed) and dynamics (the general case).
3. Material and surface characteristics, both physical and chemical.
4. Lubricant characteristics, both physical and chemical.
5. Characteristics of the surrounding atmosphere, both physical and chemical.

In other words, lubrication deals with the total interaction including all physical and chemical causes and effects, which are in most respects time-dependent in character. In an effort to gain an insight into some specific aspects of the total problem, one customarily begins by isolating the problem into neat, individual packages that can

be more readily attacked. This is a logical and necessary learning process; but one must not lose sight of the fact that these prescribed packages may or may not neatly simulate gear operation. As explained previously, although much has been learned about the lubrication of idealized sliding-rolling systems in steady-state operation, certain basic issues still remain. Additionally, in order to translate such idealized knowledge into practice, the mechanical behavior of gears must be brought into focus; but this is an area that has not engaged the needed attention of lubrication engineers.

Gears employ counterformal surfaces and are thus subject to high normal stresses. As they go through a mesh cycle, the tooth load, sum velocity, and sliding velocity all vary in manners dependent on the gear type and design.

Gear kinematics can be precisely defined by assuming completely rigid gears.^{13-15, 48} Even so, the matter acquires much complexity with such gear types as the hypoids and spiral bevels. In reality, gears are never completely rigid, hence one must deal with the interactions between forces and displacements or motions. One then encounters the problems of statics and dynamics of gears, which will now be highlighted.

Surface Deformation — Since gears are not completely rigid, one must consider the consequences of this fact. One important consequence is the local elastic deformation of the counterformal surfaces under load, which gives rise to elastohydrodynamic lubrication, the application of which to gear lubrication has been discussed.

Tooth Deflection — The elastic deflection of the gear teeth affects tooth profile modification and the manner of load sharing among the teeth. Although the subject of load sharing will be covered later, some general remarks appear in order here. Consider, for example, a set of involute spur gears (assuming no manufacturing errors) with a contact ratio of less than 2, for which the load is carried by two pairs of teeth at the beginning and end of the mesh cycle, and by only one pair of teeth during the remaining portion of the mesh cycle. In this simple case, the relation between the load sharing pattern and tooth profile modification for a particular design load can be established by statics with relative ease, but still with some measure of empiricism. If the contact ratio is, say, between 2 and 3, the load is carried by three pairs of teeth at the beginning, middle, and end of the mesh cycle, and by two pairs of teeth during the remaining portions of the mesh cycle. The load sharing and profile modification problem of

high-contact-ratio gears is considerably more difficult to solve. Design optimization is far more complex, because the propensity of both strength-related and lubrication-related failures depends markedly on how the high contact ratio is achieved.⁴⁹ Nevertheless, high contact ratio normally exists in such gears as the helicals and spiral bevels; and it is gaining in popularity for aircraft spur gears.

Other Deflections — The gear bodies, shafts, support bearings, and housing also deflect under load. These deflections may modify load sharing among the teeth, or cause tooth misalignment. Analysis of these deflections is even more difficult than that of tooth deflection; a rational approach is currently lacking.

Tooth Misalignment — Tooth misalignment may be due to the numerous bulk deflections mentioned above, manufacturing errors, stackup of tolerances in the assembly process, or differential thermal expansion. Whatever the causes, misalignment can greatly affect both strength-related failures⁴⁻⁶ and lubrication-related failures.²² Misalignment is one of the most nasty problems to handle, because it is difficult to measure and control in practice, and reliable prediction of its effects is still not available.

Dynamics — The dynamics of gear-tooth behavior, due to the transient nature of tooth engagement, operation away from the profile-modified design point, manufacturing errors, and externally imposed dynamic conditions, is an exceedingly complex subject. Clearly, if the actual tooth load is much higher than that derived for the static case, then estimates for both strength-related and lubrication-related failures based on the static load can be overly optimistic. As will be seen later, the dynamics of gear teeth of simple geometry, under idealized conditions, has been a subject of much study, mainly with regard to strength-related failures. Even so, the dynamics of a complete gear system, and also the dynamics of lubricant flow to and over the gear teeth, are quite different matters. The effects of gear and lubricant flow dynamics on lubrication-related failures, as well as the time-dependent chemical interactions involved in the failure processes, are by and large not well understood at present.

CHAPTER III

SPUR GEAR MECHANICS

A. Spur Gear Kinematics

Spur gears are the most common form of gears in use. They operate on parallel shafts and all elements of a gear tooth are parallel to those shafts. The shape of a gear tooth may take one of many forms, provided the contact between mating pairs of teeth results in a conjugate motion; i. e., the transmission of motion at constant angular velocity. The most commonly used tooth form is derived from the involute of a circle.

This chapter will discuss several important aspects of involute gear mechanics as applicable to the problem of gear scoring. More detailed treatment of involute gear mechanics may be found in standard texts. 13-15

Figure 1 shows a transverse view of two involute gears in mesh. The smaller one of the pair, usually called the pinion, is the driving member in this illustration. The larger one, usually called the gear, is the driven member. Contact initiates at point A between a pair of teeth and continues along the line AD until the pair of teeth disengage at point D. The line AD is the path of contact. To avoid interference between the tooth profiles, contact must lie between points V and W, the interference points.

The line VW is tangent to the base circles of the pinion and gear, which are unique to any given pair of involute gears. This line, of which AD is a part, makes an angle ϕ with the normal to the line of centers. This angle is the pressure angle, as shown in the lower portion of Figure 1.

Figure 1 shows two adjacent pairs of teeth in contact, one pair at A and the second pair at B. If the line VW, assumed to be a flexible but inextensible cord, were loosened at point W and wrapped around the pinion base circle, and then loosened at point V and wrapped around the gear base circle, scribes attached at points A and B would alternately trace involutes on the pinion and gear blanks which would form part of the active portions of the mating tooth surfaces. The length AB is equal to the arc length A'B', the normal base pitch.

The distance from any point of tooth contact to the interference

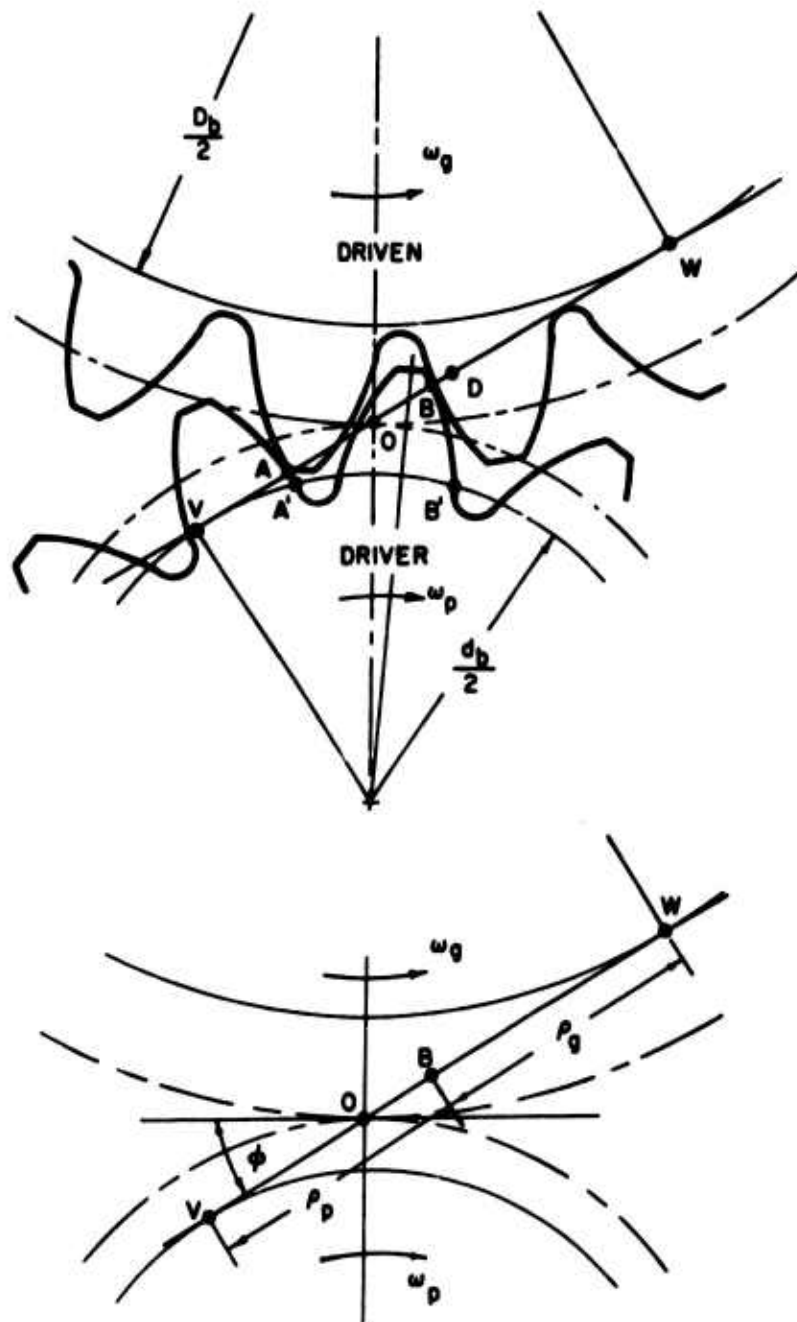


Figure 1. Spur gear geometry and kinematics

point is the instantaneous radius of curvature of the tooth profile at the point of contact. Thus in Figure 1, the radii of curvature at A are AV for the pinion tooth and AW for the gear tooth. Similarly at B, the radii are BV and BW.

The instantaneous sliding and sum velocities at the contact point may be determined from the instantaneous radii of curvature of the two teeth at that point and the angular velocities of the pinion and gear. Referring to Figure 1, they are

$$V_s = \rho_p \omega_p - \rho_g \omega_g \quad (6)$$

$$V_t = \rho_p \omega_p + \rho_g \omega_g \quad (7)$$

where V_s = instantaneous sliding velocity, ips

V_t = instantaneous sum velocity, ips

ρ_p = instantaneous radius of curvature of pinion tooth, in.

ρ_g = instantaneous radius of curvature of gear tooth, in.

ω_p = angular velocity of pinion, rad/sec

ω_g = angular velocity of gear, rad/sec

The gear ratio, or the ratio of the larger to the smaller number of teeth in the mating gears, is

$$G = \frac{N_g}{N_p} = \frac{D}{d} = \frac{\omega_p}{\omega_g} \quad (8)$$

where G = gear ratio

d = pitch diameter of pinion, in.

D = pitch diameter of gear, in.

N_p = number of pinion teeth

N_g = number of gear teeth

It can be readily shown that as a pair of teeth go through the mesh, the sum velocity remains positive throughout the mesh cycle. On the other hand, the sliding velocity starts at a maximum negative value at point A, rises to zero at the pitch point O, then it becomes positive and increases to a maximum positive value at point D. For the case of $G = 1$, the sum velocity is constant throughout the mesh cycle, while the absolute value of the sliding velocity variation is symmetrical with respect to the pitch point. If the pinion is the driver, as shown in Figure 1, the sum velocity will increase as the mesh progresses from A to D, and the absolute value of the sliding velocity will be greater at A than at D. With the gear as the driver, the sum velocity will decrease as the mesh progresses from A to D, and the absolute value of the sliding velocity will be less at A than at D.

B. Spur Gear Statics

It has been pointed out that a pair of spur gear teeth first engage at point A (Fig. 1) and finally disengage at point D. Figure 2 shows two involute spur gears at the instant that a pair of teeth engage at A. A second pair of teeth are already in mesh at B. As the gears rotate, the pair of teeth previously in mesh at B will move to point D, where they are ready to disengage. The pair of teeth previously at A are engaged at point C. Any further motion will result in the teeth at D separating and the teeth at C being the only pair of teeth in contact. It is obvious that as the pair of teeth at A move to C, and the pair of teeth at B move to D, the load is shared between these two tooth pairs. However, as the pair of teeth at C move to B, the load is carried entirely by this single pair of teeth. A measure of the portion of the total contact time during which two pairs of teeth share the load is given by the contact ratio, defined as

$$m_c = \frac{AD}{P_b} \quad (9)$$

where m_c = contact ratio

AD = length of path of contact, in.

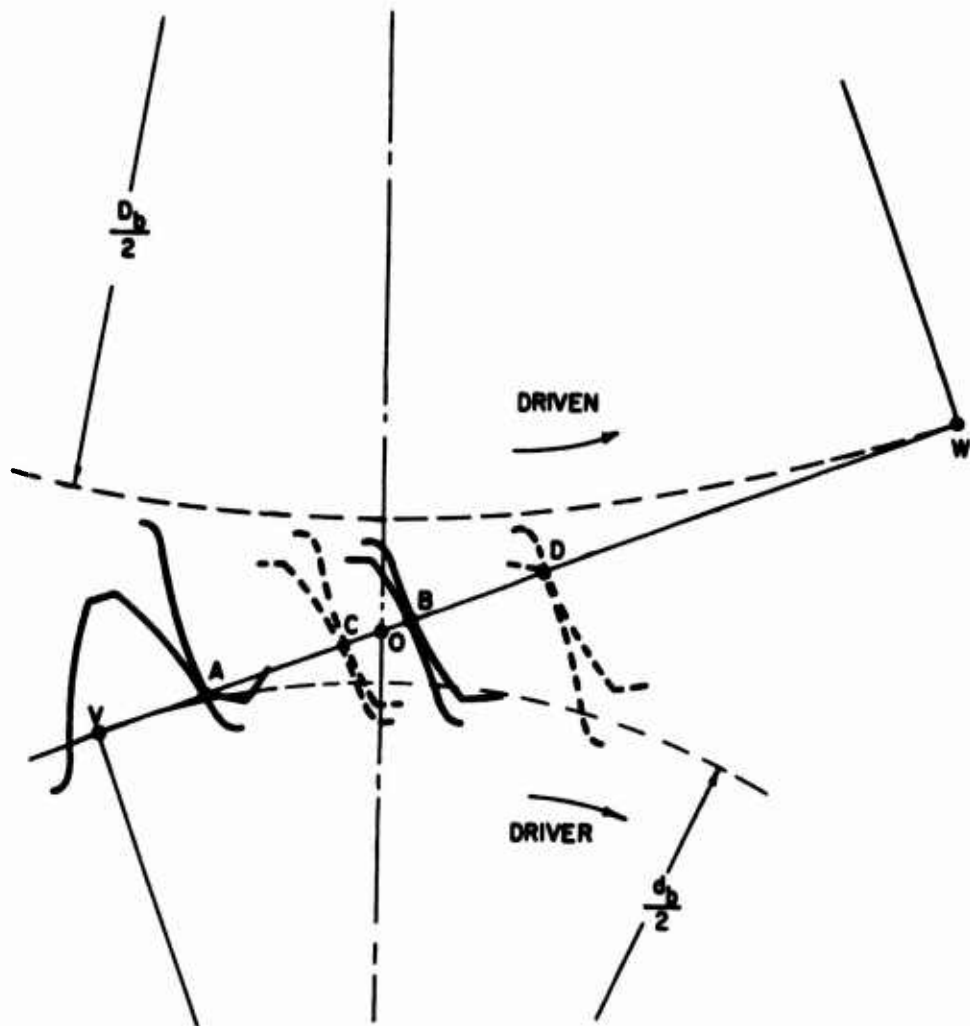


Figure 2. Spur gear contact condition

P_b = base pitch, in.

Contact ratios of properly designed spur gears are normally in the range of 1.3 to 1.7; and the greater the contact ratio, the longer two pairs of teeth share the load during the mesh cycle.

The load distribution between the two pairs of teeth simultaneously in contact between AC and BD is a function of the tooth deflection under load and the tooth profile modification.

Walker⁵⁰⁻⁵² has shown to an engineering approximation that the normal deflection at an arbitrary point M of a full-depth involute spur gear tooth under a normal load at M is given by the expression

$$\delta_m = \frac{14 w_m h}{E t} \cos \theta \quad (10)$$

where δ_m = normal deflection of tooth, in.

w_m = unit normal load on a single tooth at point M, lb/in.

E = Young's modulus, psi

h = height of inscribed parabola, in.

t = width of inscribed parabola, in.

θ = load angle, deg.

The parabola height and width are those associated with the tooth-equivalent beam of uniform bending stress inscribed in the tooth profile^{4, 52} and illustrated in Figure 3. The load angle is the angle between the load line and a line normal to the center line of the tooth. It is also shown in Figure 3.

The total deflection between a pair of teeth in contact at M is the sum of the individual deflections of each tooth, or

$$\delta_m = \delta_{mp} + \delta_{mg} \quad (11)$$

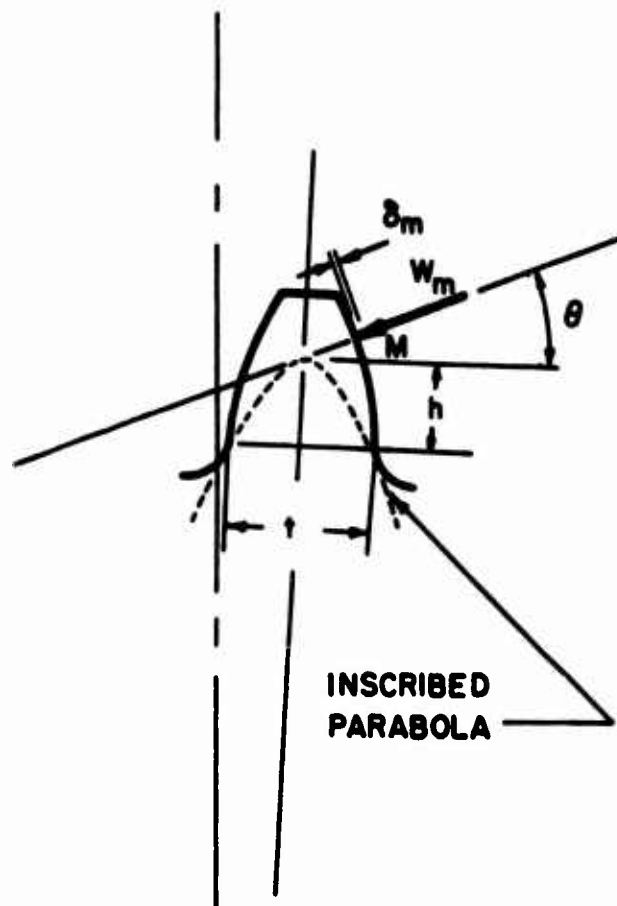


Figure 3. Load on a single tooth

When the gear teeth are loaded, they deflect; and the deflection manifests itself as a relative angular shift between the gears. The result is that the teeth interfere with each other at the point of load engagement. This is illustrated in Figure 4. The interference at point A may be eliminated if the tooth on the driven gear were to have the "overlapping" material at its tip removed. Then the tip of this gear tooth would once again make initial contact with the driver gear tooth at the point A. To ensure smooth engagement of the teeth as the mesh continues, material is removed along the entire tip of the working face of the driven gear tooth from A to C. To avoid a similar interference during the mesh cycle from B to D, the driver gear tooth profile is relieved from B to D. It is seen from Figure 4 that the maximum profile modification to be applied to the tip of the driven gear tooth is equal to the total tooth deflection at point B. Similarly, the maximum profile modification to be applied to the tip of the driver gear tooth is the total tooth deflection at point C. The amount of modification between points A and C, and between points D and B, usually reduces linearly along the profile.*

It should be apparent that if the modification is determined by the deflection of the teeth at some given load, then the modification will not be ideal at other loads. If the load is greater than that for which modification was selected, some interference will occur, but certainly not to the extent had there been no modification at all. If, on the other hand, the load is less than that for which modification was selected, the teeth will be slightly late in engaging and early in disengaging, but no interference will occur. The usual practice is to choose the modification to give a smooth load transfer at the load level most often encountered during operation.

If it is assumed that during contact between two pairs of teeth (double tooth contact), the total deflections at each contact pair are equal, and that the sum of the loads at each pair are equal to the total transmitted load, the individual normal tooth loads may be determined.

For contact between A and C (Fig. 2), the normal load at an arbitrary point i is

$$W_i = W \frac{\delta_j + \Delta_j - \Delta_i}{\delta_i + \delta_j} \quad (12)$$

* In practice, the calculated modification is applied fully at the first point of contact, but only partially at the last point of contact, in an effort to allow the gear set to run smoother at less than the design load.

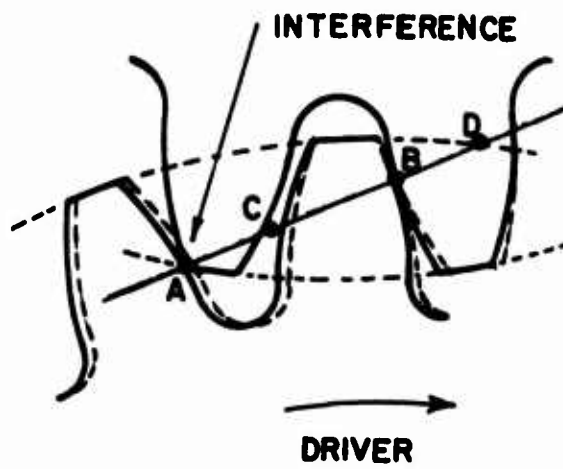


Figure 4. Tooth interference due to bending

where W_i = normal contact load at any arbitrary point i of double tooth contact, lb

W = total normal load, lb

δ_i = total tooth deflection at i , in.

δ_j = total tooth deflection at j , one base pitch from i , in.

Δ_i = modification of driven tooth at i , in.

Δ_j = modification of driver tooth at j , in.

For contact between B and D, at a point j one base pitch along the line of contact AD from point i , the normal load is

$$W_j = W - W_i \quad (13)$$

where W_j = normal contact load at j of double tooth contact, one base pitch from i , lb

For single tooth contact between C and B, the normal load is constant, as for example at an arbitrary point k ,

$$W_k = W \quad (14)$$

where W_k = normal contact load at any arbitrary point k of single tooth contact, lb.

Equations (12), (13), and (14), when applied to points along the path of contact, define the load sharing pattern through the mesh cycle of spur gears with a contact ratio between 1 and 2. This fairly laborious calculation can easily be performed by means of a computer program, as is done in the current work (App. H). If such a computer program is not available, an approximate load sharing pattern, such as that used in the AGMA gear scoring design guide,³ is often used in practice. The AGMA load sharing pattern is fixed, and is independent of the load level. However, in view of its approximate nature, this objection is mainly academic.

Spur gears with contact ratios greater than 2 are fairly rare; but are gaining in popularity for aircraft power gears. The impact of high contact ratio on spur gear performance has been discussed recently by Staph.⁴⁹ If the contact ratio is between 2 and 3, then 3 pairs of teeth share the load at the beginning, middle, and end of the mesh cycle; while only 2 pairs of teeth share the load during the remaining portions of the mesh cycle. The computation of the load sharing pattern for such gears can be done in a similar manner,⁴⁹ by using the Walker procedure. The AGMA load sharing pattern mentioned above does not apply to this case.

Tooth loads found by the above procedures are static loads, or those for very low-speed operation. Rigorous analysis of load sharing under dynamic conditions is currently not available. Gear-tooth dynamics can thus only be treated in an approximate manner, which will be discussed in the section which follows.

C. Spur Gear Dynamics

In the previous section, equations were given for finding the tooth loads between mating gear teeth for static or very low-speed operation. If the tooth profiles were perfectly designed, if the gears were perfectly manufactured and assembled, if the gears were operated at that unique load level for which the tooth profile modifications were designed, and if no other dynamic stimuli were present in the system, then these equations would also be valid for high-speed operation. Unfortunately, these conditions are never achievable in practice. Because of these deviations from the ideal, the actual tooth loads are higher than those calculated from the static load equations.

Dynamic loads result when, for one reason or another, the gear teeth undergo an angular speed change in the meshing process. For example, in a gear set in which the pinion is the driver, if the gear tooth just ready to engage the pinion tooth is too thick (a manufacturing tolerance problem), contact between the teeth will not occur on the line of action; but somewhere ahead of and off the line. The action will not be conjugate. The teeth will deflect to some small extent under load, lessening the shock of the sudden loading; but the main result will be an acceleration of the gear or a deceleration of the pinion, each in an effort to bring the point of contact, now off the line of contact, back onto the line. This acceleration or deceleration gives rise to an overload, or a dynamic load increment. The static load plus the dynamic increment is the dynamic load.

Although the previous example illustrates the dynamic load as being produced by the velocity change occurring at the first point of contact, in fact no such limitation exists. At any time during the nominal engagement of a pair of teeth whose profiles are such that contact moves off the theoretical line of action, a velocity change will occur to the pinion and the gear, producing a dynamic load. Generally speaking, however, the velocity change at the initial point of contact is the greatest and hence produces the largest dynamic loading in the cycle.

All gear pairs experience dynamic loads to some degree since gear perfection is not a reality. For high-precision gears operating at moderate speeds and loads, the dynamic loads are not very high and they do not cause serious problems.⁴⁻⁶ On the other hand, highly-loaded gears and very high-speed gears, even lightly-loaded, may experience dynamic loads sufficiently high to cause concern. This is particularly true where a gear pair is designed for the ultimate in power-to-weight ratio such as aircraft gearing.

Of particular concern are gear systems which operate at speeds near the natural frequency of the gear mass/tooth spring system. At resonance the dynamic increment can equal the load due to the input power.⁵³⁻⁵⁵ Some evidence exists which shows even higher dynamic increments if the damping in the system is less than about 7 percent of critical.⁵³⁻⁵⁵ Fortunately, most combinations of material, lubricant, and gear blank design will provide this value of damping.

Dynamic loads may result from manufacturing tolerances in the pitch, pressure angle, tooth thickness, tooth profile, and lead, or from misalignment or tooth deflection, or anything which causes the gears to deviate dimensionally or operationally from perfection. Likewise, dynamic loads may arise from the operation of modified profile gears at loads other than that for which the modification was based, since the effect is the same as tooth mesh errors due to manufacturing tolerances.

From a practical standpoint, dynamic loads may be related to tooth deflection, δ , pitch error, e , and profile modification, Δ . Table 2 shows the relationships involved for several combinations of pitch error, deflection, and profile modification. The pitch error is the sum of the allowable pitch tolerances of each gear. Pitch tolerances are found in the AGMA Gear Handbook⁵⁶ for the class of gear under consideration. The total tooth deflection δ is the deflection at the point B (Fig. 2), since it is this deflection that affects the mesh at A.

TABLE 2. EFFECTIVE TOOTH ERRORS

<u>Case</u>	<u>Pitch error, e</u>	<u>Total tooth deflection, δ</u>	<u>Effective pitch error, e_f</u>	<u>Profile modification, Δ</u>	<u>Effective error, e_e</u>
a	0	0	0	0	0
b	e	0	e	0	e
c	e	δ	$e + \delta$	0	$e + \delta$
d	e	δ	$e + \delta$	$-\delta$	e
e	e	δ	$e + \delta$	$-\Delta$	$e + \delta - \Delta$

The effective pitch error, e_f , is the algebraic sum of e and δ .

The profile modification Δ is the sum of the tip modification of the driven gear and the root modification (if any) of the mating driving gear. Normally, profile modification tolerances are positive (i. e. , removal of material), reducing the effective error and are thus not considered. Note in Table 2 for case d, the modification is equal to the total tooth deflection, thus negating the effect of the deflection at engagement.

The effective error, that error which the teeth "see" as they engage, is the algebraic sum of e_f and Δ . It is this error, regardless of its source or makeup, that will cause the dynamic load.

Gear-tooth dynamics has been a subject of considerable study from both theoretical and experimental standpoints; and the complexity of the problem is well illustrated by some of the references cited herein. 53-55, 57-65 Among the works that are fairly typical of the state of the art, the Seireg and Houser method^{61, 62} represents a combined theoretical and experimental approach but is rather difficult to apply, while the Tuplin method⁶³ is based on a simple theoretical approach but is easier to use in practice.

It should be emphasized that the subject of gear-tooth dynamics is exceedingly complex. It involves by necessity not only the dynamic behavior of the gear teeth themselves, but also of the other components in the system which participate in governing the dynamic behavior of the gear teeth. The displacement, elastic, damping, and inertia characteristics of all these participating components are difficult to define and account for; and how well the currently available approaches actually work out in practice remains intriguing. In view of this situation, only the relatively simple Tuplin's method⁶³ will be discussed herein and employed in the computer program in this report (App. H). In addition to the Tuplin method, the empirical and even simpler AGMA approach⁴⁻⁶ will also be mentioned.

A convenient way to account for the dynamic effect is to introduce a dynamic factor, K_v , defined as

$$\begin{aligned} K_v &= \frac{\text{static load}}{\text{dynamic load}} \\ &= \frac{\text{static load}}{\text{static load} + \text{dynamic increment}} \end{aligned} \quad (15)$$

The static load is computed by the procedure outlined in the preceding section. The dynamic load is then the static load divided by the dynamic factor.

Tuplin's Method. In order to apply the Tuplin method,⁶³ it is necessary to calculate the period of the natural frequency of the gear mass/tooth spring system. It may be calculated by any convenient method, as for example, the Holzer method, or by the following approximation:

$$T_n = 2\pi \sqrt{\frac{m_e}{k_l}} \quad (16)$$

where T_n = period of natural frequency of gear mass/tooth spring system, sec

m_e = equivalent mass of pinion and gear, lb-sec²/in.

k_l = spring rate of a pair of mating spur gear teeth, lb/in.

The spring rate of a pair of mating teeth is approximately constant through the mesh; thus a fair approximation to the spring rate may be found by considering a cantilever beam loaded with a uniform load across the tip. This gives

$$k_l = \frac{FE_p^3}{32 (D_o - D)^3} \quad (17)$$

where F = face width, in.

E = Young's modulus, psi

p = circular pitch, in.

D_o = gear outside diameter, in.

D = gear pitch diameter, in.

If the gear drives, the equivalent pinion diameters should be used for D_o and D .

The time for the tooth error to be applied is assumed to be the time for the gears to turn through one circular pitch, or

$$t_e = \frac{p}{V_t} \quad (18)$$

where t_e = time for tooth error to be applied, sec

V_t = pitchline velocity, ips.

The ratio t_e/T_n is used in Figure 5 to determine the value of e_a/e_e . With e_e obtained from Table 2, the apparent tooth error, e_a , is calculated; and the dynamic increment is then

$$F_i = k_l e_a \quad (19)$$

where F_i = dynamic increment, lb

e_a = apparent error, in. (from Fig. 5)

The dynamic factor, K_v , may then be calculated from Equation (15).

If the gear train operates at speeds near the resonant frequency of the gear mass/tooth spring system, it may be shown that the critical value of t_e/T_n is given by the reciprocal of the number of pinion teeth.⁶³ Assuming a minimum number of pinion teeth as being 12, the critical value of t_e/T_n is 0.083, and $e_a/e_e = 0.96$ from Figure 5. From Equation (19),

$$F_i = k_l \left(\frac{e_a}{e_e} \right) e_e = 0.96 k_l e_e \sim k_l e_e$$

In other words, the dynamic increment is nearly equal to the static load, or K_v is nearly 0.5, a result suggested earlier.

AGMA Method. The AGMA method⁴ is extremely simple to apply in practice; but its use requires much experience and judgment.

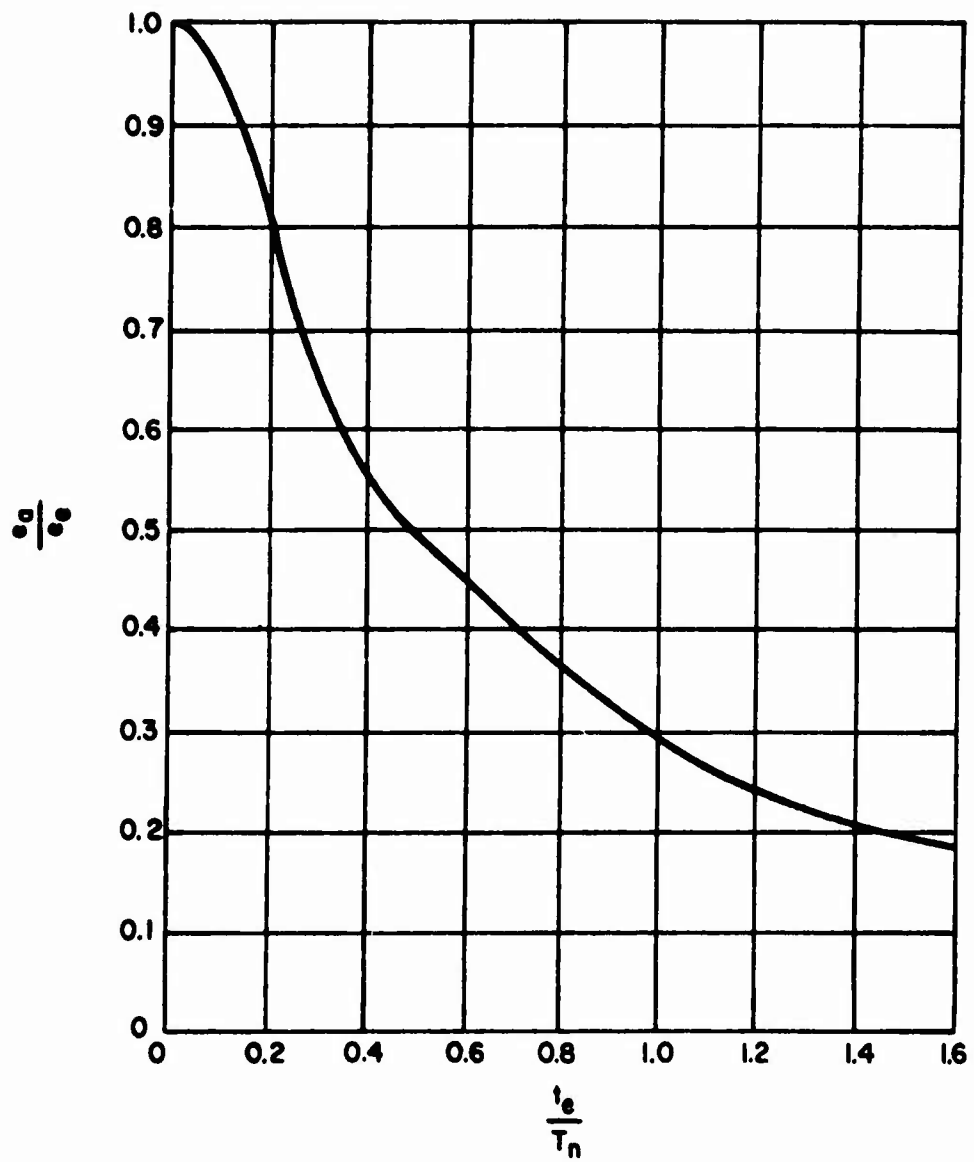


Figure 5. e_a/e_e vs. t_e/T_n for Tuplin's method

This method states that the dynamic factor, K_v , depends on: "(a) effect of tooth spacing and profile errors, (b) effect of pitchline and rotational speeds, (c) inertia and stiffness of all rotating elements, (d) transmitted load per inch of face, and (e) tooth stiffness." It then furnishes three simple equations for the dynamic factor, herein designated as K_{v1} , K_{v2} , and K_{v3} for convenience, for the following three situations:

For "high-precision" gears when the effect of the items listed above are such that "no appreciable dynamic load is developed,"

$$K_{v1} = 1 \quad (20)$$

For "high-precision" gears when the items listed above "can develop a dynamic load,"

$$K_{v2} = \sqrt{\frac{78}{78 + \sqrt{V_t}}} \quad (21)$$

For less precise gears,

$$K_{v3} = \frac{50}{50 + \sqrt{V_t}} \quad (22)$$

In Equations (21) and (22), V_t is the pitchline velocity, fpm.

The AGMA equations can of course be faulted for their lack of sophistication. However, in view of the complexity of the total gear-tooth dynamics problem and the difficulty of handling the problem in a rigorous but realistic manner, they do provide some easy and practical means of accounting for the dynamic effect if caution is exercised.

Further discussion of the dynamic factor as applied to practical prediction of the scoring-limited performance of gears will be deferred until Chapter VII.

CHAPTER IV

HELICAL GEAR MECHANICS

A. Helical Gear Kinematics

If a spur gear were sliced transversely into a number of thin plates, and each plate were displaced through a small positive angle with respect to the preceding one, the result would be a stepped gear. To carry the process further, the plates could become infinitesimal in thickness and infinite in number, then the result would be a helical gear. Consequently, the tooth profiles in any transverse plane of a helical gear are identical, but are shifted through an angle proportional to the axial displacement. The intersection of the tooth surface with the pitch cylinder is a helix.

The contact in a pair of uncrowned helical gears on parallel axes is approximately a rectangle. In actual practice, the contact may not be like this if misalignment is present, but may be more nearly a distorted ellipse.

Two helical gears viewed in the transverse plane (normal to the axes of rotation) are shown in Figure 6. The plane of action is projected above the gears. The slant lines in the plane of action are the lines of contact of the contacting tooth pairs. There are, in this case, two pairs of teeth simultaneously in contact, spaced one normal base pitch apart. Contact between a pair of teeth starts at point A and sweeps across the plane of action to end at point D'.

The geometry at an arbitrary point H on a line of contact may be found by determining the point's projection H' in the transverse plane. Then methods of analysis similar to those for spur gears may be used to determine tooth geometry at H'. The base helix angle, ψ_b , may be taken into account to translate the results back into the normal plane.

The projected radii of curvature at H' in the transverse plane are

$$\rho_p = VH'$$

$$\rho_g = WH'$$

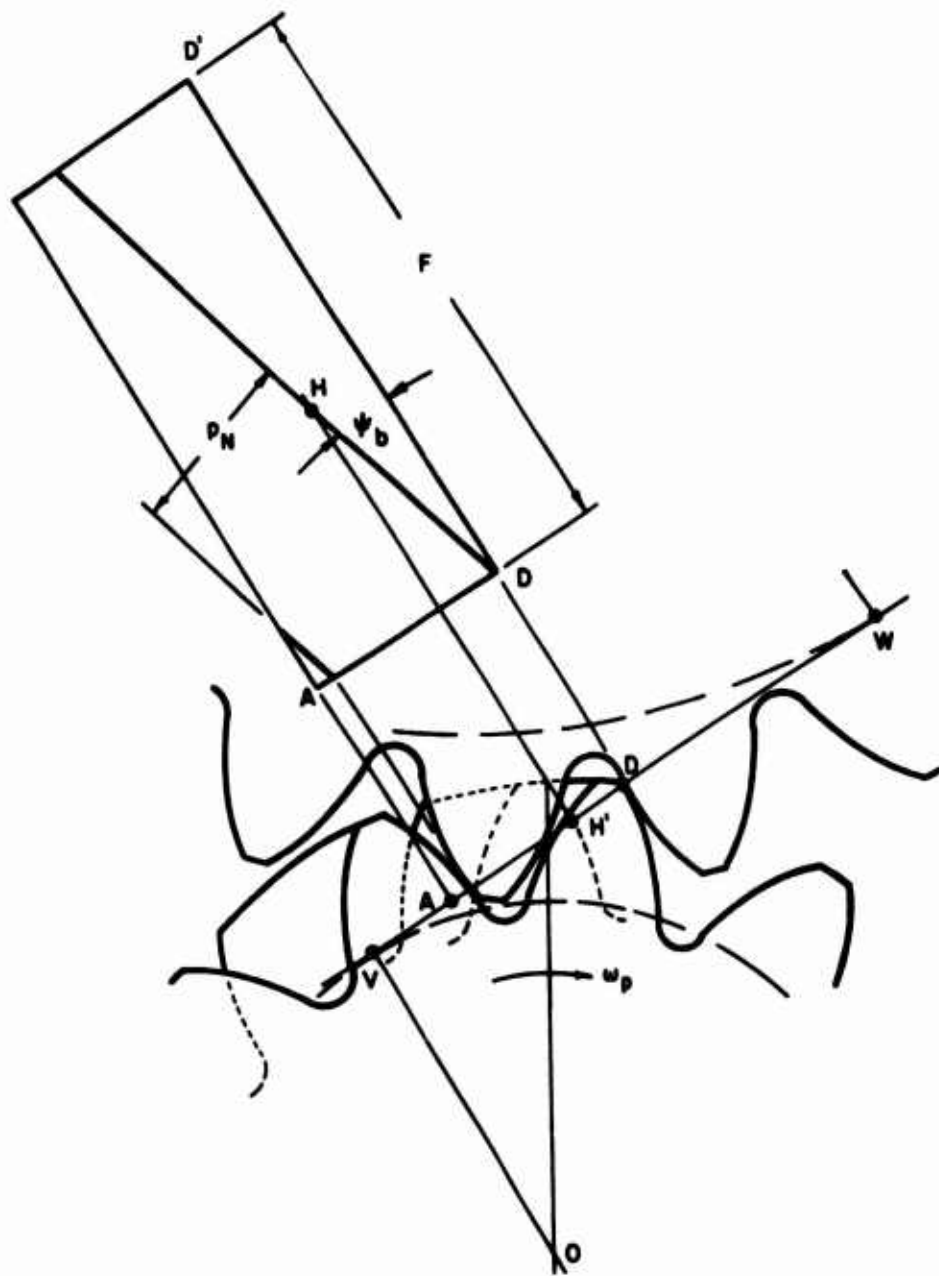


Figure 6. Helical gear geometry and kinematics

These radii may be translated back to the point H in the normal plane by taking into account the base helix angle, ψ_b , thus

$$\rho_{pn} = VH' \sec \psi_b$$

$$\rho_{gn} = WH' \sec \psi_b$$

Using these radii in the normal plane, the instantaneous sliding and sum velocities at the point H are

$$V_s = (\rho_{pn}\omega_p - \rho_{gn}\omega_g) \cos \psi_b \quad (23)$$

$$V_t = (\rho_{pn}\omega_p + \rho_{gn}\omega_g) \cos \psi_b \quad (24)$$

where ρ_{pn} = radius of curvature of pinion in normal plane, in.

ρ_{gn} = radius of curvature of gear in normal plane, in.

ω_p = angular velocity of pinion, rad/sec

ω_g = angular velocity of gear, rad/sec

ψ_b = base helical angle, deg.

B. Helical Gear Statics

As illustrated in Figure 6, two pairs of teeth are simultaneously in contact at the instant shown. This set of gears will always have two pair of teeth in contact at any one time, since as the leading line of contact nears point D', a new line enters at point A. Properly designed helical gears will always have at least two and often more pairs of teeth in contact at any one time. Because several pairs of teeth are in contact at one time, helical gears operate more quietly than spur gears, and with less shock at tooth engagement.

Like spur gears, a measure of the load sharing between pairs of teeth simultaneously in contact is the contact ratio. However, since contact in the plane of action of helical gears is in two dimensions, there are two contact ratios. The face contact ratio is that due to the

helical nature of the tooth elements that cause overlap between pairs of teeth. In terms of gear parameters, it is

$$m_f = \frac{F \tan \psi}{p} \quad (25)$$

where m_f = face contact ratio

F = face width, in.

ψ = helix angle, deg.

p = transverse circular pitch, in.

The transverse contact ratio is that due to load sharing in the transverse plane and is identical in nature to the contact ratio in spur gears, or

$$m_c = \frac{AD}{p_b} \quad (26)$$

where p_b is the transverse base pitch, in.

The total contact ratio is the sum of the face and transverse contact ratios, or

$$m_t = m_c + m_f \quad (27)$$

Because the load extends from the tip, diagonally across the gear face, to the root of a tooth, the tooth deflection is a complex function of tooth form and helix angle. No rational method is available for determining the tooth deflection. However, the flexibility of the tooth loaded near the tip, and the rigidity of the same tooth simultaneously loaded near the root, together with the several teeth in contact at one time, tend to distribute the load more evenly over the instantaneous lines of contact for uncrowned helical gears. This approximation is implied in the AGMA standard for rating the strength of helical and herringbone gear teeth⁵ in the calculation of the load sharing ratio

$$m_N = \frac{F}{L} \quad (28)$$

where m_N = load sharing ratio

L = total length of lines of contact for all tooth pairs simultaneously in contact, in.

This equation implies that the fraction of the total normal load carried at any instant by any tooth is proportional to the ratio of the length of the instantaneous line of contact on that tooth to the total length of all instantaneous lines of contact on all teeth.

Figure 7 is a view of the plane of action of a pair of helical gears. Three pairs of teeth are in simultaneous contact in this illustration. The position of a line of contact from the time it starts at point A until it exits at point D' is determined by the parameter f , measured normally to the lines of contact. There may be, as shown in Figure 7, lines of contact ahead of and behind this line at a distance of p_N , the normal base pitch, apart.

The line of contact at distance f may be divided into a number of equal divisions and each such division may be treated for purposes of analysis as an elemental spur gear tooth having a width in the normal plane equal to the length of the division. The load is assumed constant over the length of the division, so that

$$W_i = Wl/L \quad (29)$$

where W_i = normal load on the elemental gear, lb

W = total normal load, lb

l = length of a division, in.

The instantaneous sliding and sum velocities in the normal plane at the point of contact in the middle of the division, as for example at point M, are used as being representative of the conditions on the elemental gear.

C. Helical Gear Dynamics

Dynamic loads in helical gears arise from the same causes as they do in spur gears; namely, manufacturing tolerances, tooth

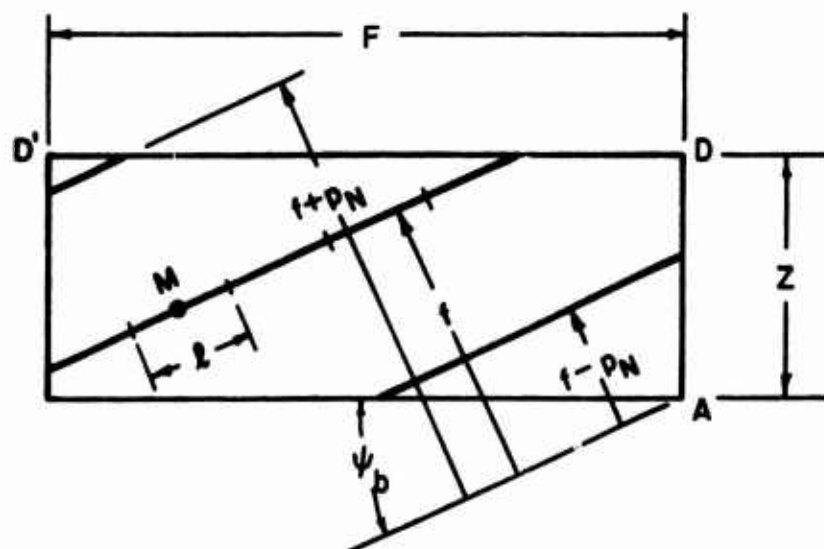


Figure 7. Helical gear contact condition

deflections, system dynamics, or anything that results in a change in the angular speeds of the teeth in action. However, since the load on helical gear teeth is applied diagonally across the tooth face, and because at least two and usually more teeth are simultaneously in contact, the tooth deflections are not as severe as they are in spur gears. Consequently, dynamic loads are, on the whole, somewhat less for helical gears than they are for spur gears. With only minor changes, the methods for determining the dynamic factors described previously for spur gears may be applied to helical gears.

Tuplin's Method. The dynamic factor for helical gears may be computed by the same equations presented in the preceding section for spur gears, with one exception. This exception relates to the tooth spring rate, k_t . Equation (17) previously given applies to spur gears. For helical gears, the approximate equation is

$$k_t = \frac{\pi E (p \cos \psi)^3}{96 (D_o - D)^2} \quad (30)$$

where k_t = tooth spring rate for helical gears, lb/in.

E = Young's modulus, psi

p = transverse circular pitch, in.

ψ = helix angle, deg

D_o = gear outside diameter, in.

D = gear pitch diameter, in.

Note that the quantity p in Equation (18) should be the transverse circular pitch, since it and not the normal circular pitch governs the distance moved by points on the pitchline of the rotating gear.

AGMA Method. The AGMA method for estimating the dynamic factor of helical gears is also basically identical to that for spur gears. For helical gears, the AGMA procedure specifies two dynamic factors, K_{v1} and K_{v2} , exactly as in Equations (20) and (21), respectively. The third dynamic factor, K_{v3} , is not used for helical gears.

CHAPTER V

SPIRAL BEVEL GEAR MECHANICS

A. Spiral Bevel Gear Kinematics

Spiral bevel gears are related to straight bevel gears in much the same manner as helical gears are related to spur gears. That is to say, the element of a spiral bevel gear tooth forms a spiral helix about the pitch cone, whereas the element of a helical gear tooth forms a cylindrical helix about the pitch cylinder. Many of the advantages and disadvantages of helical gears, such as multiple tooth contact and thrust loading, are present in spiral bevel gears.

The bases of spiral bevel gears are pitch cones which intersect at a common point. The axial plane contains the gear axes. Figure 8, looking normal to the axial plane, shows some of the parts of a pair of spiral bevel gears. The pitch plane is seen as the line PO, and the pitch point is point P.

Figure 9 looks at the pitch cone normal to the pitch plane. Elements of a tooth make an angle ψ with the cone element midway of the face width. This is the spiral angle, and it is equivalent to the helix angle in helical gears with the exception that, due to the taper of the pitch cone, the value of the spiral angle is not constant but depends upon where along the cone element it is measured.

A section of the tooth on the normal plane is also shown in Figure 9. The pressure angle, ϕ_n , is specified in this plane. The most common spiral angle is 35° ; the usual pressure angle is 20° . The tangent plane is also shown in Figure 9.

Since the plane of action shows the contact on all teeth in action simultaneously, it is more convenient to use the plane of action for study than to use the tooth surface. Accordingly, Figure 10 shows the plane of action bounded by the pairs of curved and tapering lines. To simplify the analysis, the plane of action is assumed to be rectangular with width F and length Z as shown in Figure 10.

The three diagonal lines in the plane of action represent the contact between three pairs of teeth which at the instant are sharing the load. These lines are actually the major axes of contact ellipses. The ellipse passing through the ends of the instantaneous lines of contact represents the limits of contact. If it were not for mismatch of

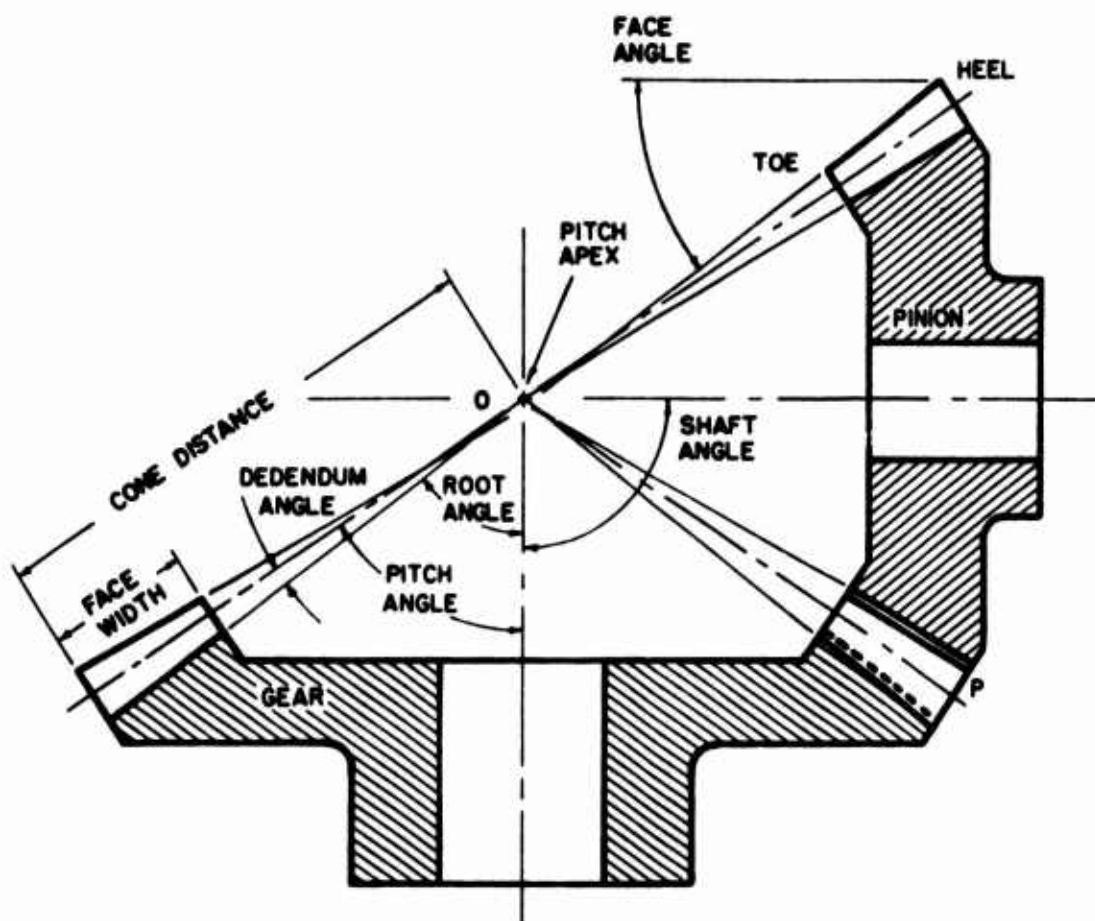


Figure 8. Spiral bevel gear geometry

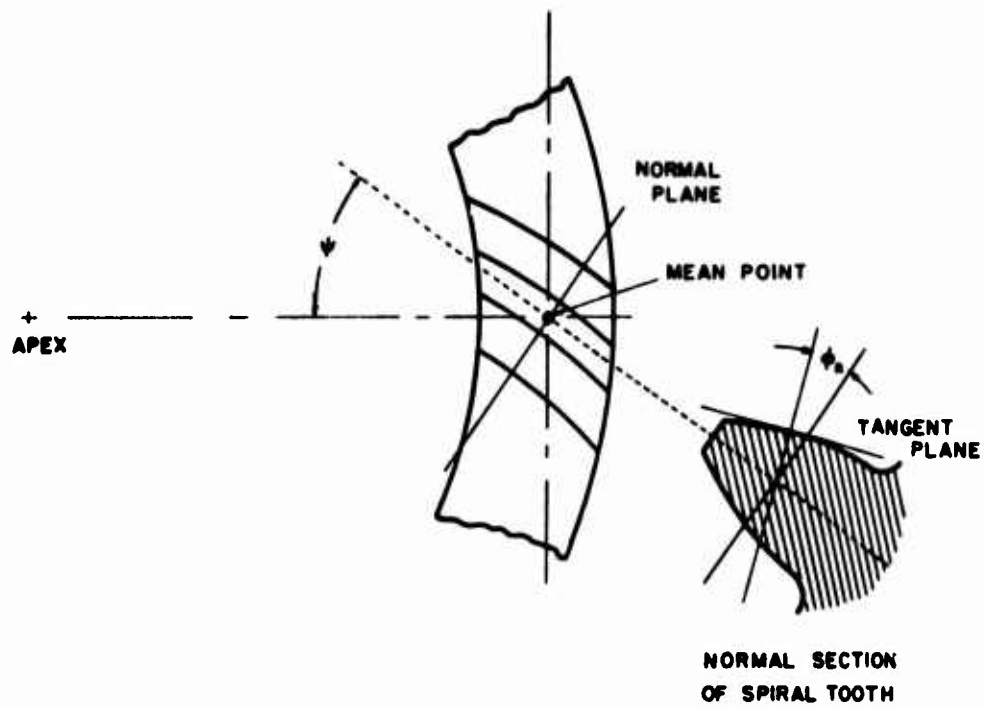


Figure 9. View normal to pitch plane

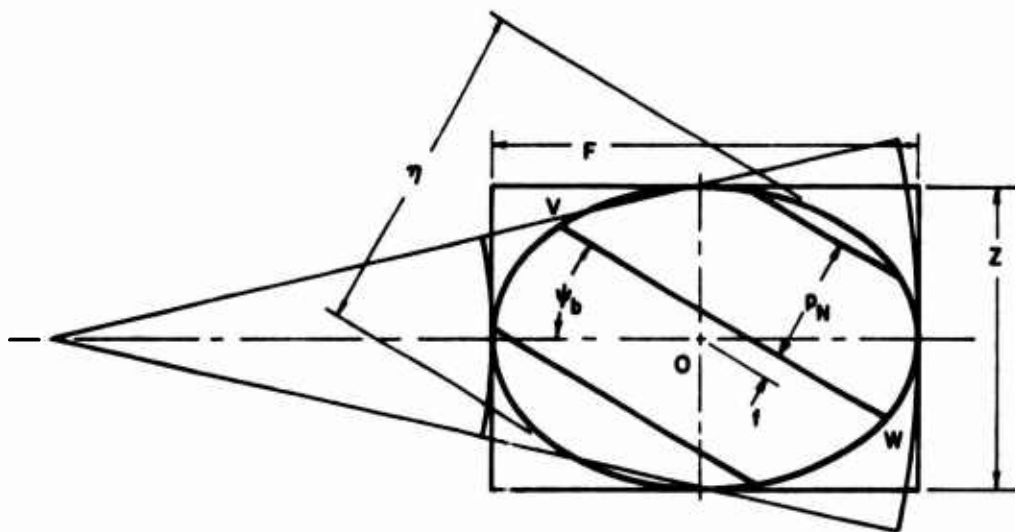


Figure 10. Spiral bevel gear contact condition

the teeth (to be discussed more fully later), the lines of contact would extend theoretically to the edges of the rectangle. Because the plane of action is tangent to the base cone, the lines of contact are inclined to the cone element by the angle ψ_b , instead of ψ as in Figure 9.

Because contact in the plane of action of spiral bevel gear teeth is in two dimensions, there are, as in helical gears, two contact ratios. These ratios are calculated from Equations (25) and (26) for helical gears, but they are combined differently to give a modified contact ratio as

$$m_o = \sqrt{m_f^2 + m_c^2} \quad (31)$$

where m_o = modified contact ratio

m_f = face contact ratio

m_c = transverse contact ratio

Sliding and sum velocities between tooth contact points midway of the line of contact VW are found by determining the components of the absolute velocities of the contact points on the pinion and gear in the tangent plane.

It may be shown⁶⁶ that the component of the velocity of the point of contact of the pinion tooth in the direction along the tooth element is

$$V_{Fp} = V_n \left[\tan \psi + |Z_o| \sin \phi_n \frac{\tan \psi}{A \tan \gamma} - |Z_o| \cos \phi_n \frac{1}{A \cos \psi} \right] \quad (32)$$

where V_{Fp} = component of velocity in the tangent plane of contact point on pinion along tooth element, ips

V_n = normal component of pitch point velocity in the pitch plane, ips

ψ = mean spiral angle (i. e., spiral angle at mean point, Fig. 9), deg.

Z_o = distance in the plane of action from center of contact to projection of contact point in mean normal section, in.

ϕ_n = pressure angle in normal plane, deg.

A = mean cone distance, in.

γ = pinion pitch angle, deg.

The component of the velocity of the point of contact of the pinion tooth in the profile direction is

$$V_{Pp} = V_n \left[\sin \phi_n + Z_o \frac{1}{A \tan \gamma} \right] \quad (33)$$

where V_{Pp} = component of velocity in the tangent plane of contact point on pinion in the profile direction, ips

Similar equations may be written for the corresponding contact point on the gear, in the tangent plane, as

$$V_{Fg} = V_n \left[\tan \psi - |Z_o| \sin \phi_n \frac{\tan \psi}{A \tan \Gamma} - |Z_o| \cos \phi_n \frac{1}{A \cos \Gamma} \right] \quad (34)$$

$$V_{Pg} = V_n \left[\sin \phi_n - Z_o \frac{1}{A \tan \Gamma} \right] \quad (35)$$

where V_{Fg} = component of velocity in tangent plane of contact point on gear along tooth element, ips

Γ = gear pitch angle, deg.

V_{Pg} = component of velocity in tangent plane of contact point on gear in the profile direction, ips

The difference and the sum of the components of the velocities in the two directions give the sliding and sum velocity components in

the direction along a tooth element and in the profile direction. Since these directions are orthogonal, these resulting components may be combined to give the sliding and sum velocities as

$$V_s = \sqrt{(V_{Fp} - V_{Fg})^2 + (V_{Pp} - V_{Pg})^2} \quad (36)$$

$$V_t = \sqrt{(V_{Fp} + V_{Fg})^2 + (V_{Pp} + V_{Pg})^2} \quad (37)$$

In order to apply Blok's conjunction temperature rise equation, it is necessary to determine the time for a contact point on each of a pair of sliding surfaces to cross the heat zone, the area of contact between the two surfaces. This requires a knowledge of the distance across the contact area. In spur and helical gears, this distance is the same for the point of contact of the pinion and gear. Because the axes of spiral bevel gears are not parallel, the corresponding contact points on the pinion and on the gear move across the area of contact in different directions.

Figure 11 shows a typical instantaneous contact area in the tangent plane of a pair of spiral bevel gears. V_p and V_g are the vector sums of the velocity components V_{Fp} and V_{Pp} for the pinion, and V_{Fg} V_{Pg} for the gear, respectively. The distance that the contact point on the pinion travels as it sweeps across the contact area is d_p , in the direction of V_p . Similarly, the distance that the contact point on the gear travels as it sweeps across the contact area is d_g , in the direction of V_g . These distances are

$$d_p = 2 \sqrt{\frac{a^2 b^2}{a^2 \sin^2 (\alpha_p + \omega) + b^2 \cos^2 (\alpha_p + \omega)}} \quad (38)$$

$$d_g = 2 \sqrt{\frac{a^2 b^2}{a^2 \sin^2 (\alpha_g + \omega) + b^2 \cos^2 (\alpha_g + \omega)}} \quad (39)$$

where d_p = instantaneous sliding distance of the pinion across the contact area, in.

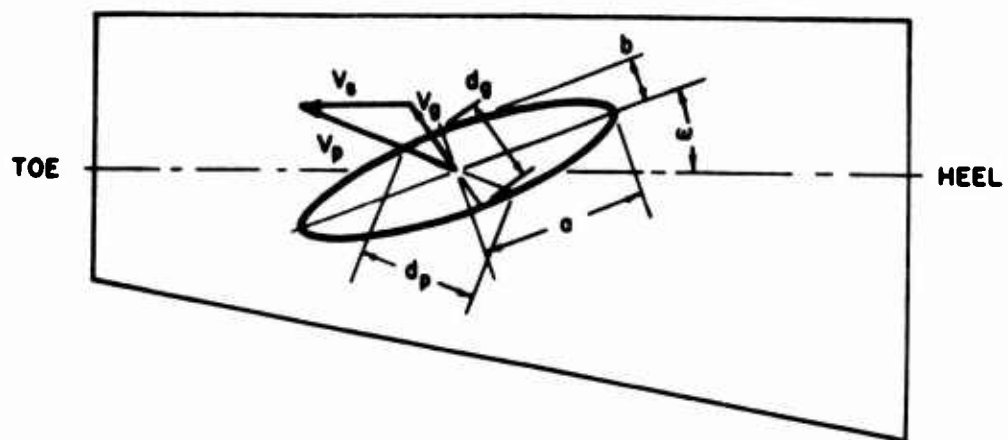


Figure 11. Spiral bevel gear kinematics

- d_g = instantaneous sliding distance of the gear across the contact area, in.
- a = major semiwidth of the contact ellipse, in.
- b = minor semiwidth of the contact ellipse, in.
- α_p = angle the resultant pinion velocity vector makes with pitchline, deg.
- α_g = angle the resultant gear velocity vector makes with the pitchline, deg.
- ω = angle of inclination of the line of contact with the pitchline, deg.

B. Spiral Bevel Gear Statics

In this section, an equation for the tooth contact stress will be developed. The method follows that of Reference 66 to which the reader is referred for more details. Other helpful references are References 67 and 68.

Theoretically, spiral bevel gear teeth should operate with conjugate motion. Contact would be along a line extending diagonally across the tooth surface, and moving generally from the heel to the toe (or reverse). However, because of their sensitivity to the effect of manufacturing and assembly tolerances, and deflection under loading, spiral bevel gear teeth do not operate with conjugate motion. Instead, contact shifts to the edge and load concentrations occur. To counteract this shifting of load, the tooth profiles are modified to produce "mismatch." This mismatch causes the point of contact to move back onto the tooth face and, although the mating surfaces are no longer conjugate, the resulting action is smoother and far better than that produced by the theoretical conjugate motion. The contact, with mismatch, is theoretically a point, but local yielding of the surface results in an elliptical contact zone (Fig. 11). Reference 69 gives an excellent review of mismatch techniques used in industry.

Since the mating surfaces of spiral bevel gear teeth are no longer conjugate, the classical Hertz contact stresses between cylinders do not apply. Reference 66 uses a combination of experimental results and approximations to the Hertz theory to obtain the tooth contact stress.

Referring to Figure 10, the variable f measures the displacement of the line of contact VW from the center of the surface of action. In the load analysis to be presented the line of contact VW is swept across the surface of action by varying f until the point is reached where the load on VW is a maximum.

From Reference 67, the load sharing ratio, the ratio of the load carried on the line VW to the total load is

$$m_N = \frac{\eta_1^3}{\eta_1^3 + A + B} \quad (40)$$

where m_N = load sharing ratio

η_1 = a function of variable dimension f

and

$$A = \sum_{k=1}^n \sqrt{[\eta_1^2 - 4kp_N(kp_N + 2f)]^3} \quad (41)$$

$$B = \sum_{k=1}^n \sqrt{[\eta_1^2 - 4kp_N(kp_N - 2f)]^3} \quad (42)$$

and p_N = mean normal base pitch, in.

f = a variable dimension locating the line of contact VW with respect to the center of the surface of action, in.

k = a positive integer which takes on successive values from 1 to n , generating all real terms in the series

The dimension f in Figure 10 is varied from $-\eta/2$ to $+\eta/2$ until maximum m_N is obtained. It is at this point that the scoring potential will be the greatest.

The length of the contact line VW is given by⁶⁷

$$S_G = \frac{FZ\eta_1}{\eta^2} \quad (43)$$

where S_G = length of the contact line VW, in.

F = face width, in.

Z = mean length of path of contact in transverse plane, in.

η = length of contact normal to lines of contact, in.

The normal load on the line of contact VW is

$$W_j = W m_N \quad (44)$$

where W_j = normal load on line of contact VW, lb

W = total normal load, lb

consequently,

$$W_j = \frac{W_t m_N}{\cos \phi \cos \psi} \quad (45)$$

where W_t = tangential load at pitch point, lb

ϕ = pressure angle, deg.

ψ = mean spiral angle, deg.

C. Spiral Bevel Gear Dynamics

The actual tooth load or dynamic load is greater than the static load for exactly the same reasons as for spur and helical gears. The dynamic load is caused by manufacturing inaccuracies, tooth deflections, and system dynamics.

The Tuplin method for determining the dynamic factor for spur and helical gears is not applicable to spiral bevel gears. The dynamic load is, instead, evaluated by the use of factors relating to the type of dynamic load-producing function.⁶⁶

The dynamic load on spiral bevel gears is obtained from

$$W_d = W \frac{K_i}{K_v} \quad (46)$$

where W_d = dynamic load, lb

K_i = inertia factor

K_v = dynamic factor

The inertia factor is related to the modified contact ratio, m_o . Because there are normally several pairs of teeth in contact simultaneously, load transfer is smooth and the inertia factor is customarily taken as unity.⁶⁶⁻⁶⁸ However, if m_o is less than 2, load transfer is no longer smooth, the rotating velocities are variable, and gear inertia becomes a factor in the dynamic load. For a modified contact ratio of less than 2, the inertia factor is taken as $K_i = 2/m_o$.

The dynamic factor⁶ is defined by the same AGMA formulas for K_{v1} and K_{v2} , i. e., Equations (20) and (21), respectively. As in the case of helical gears, the factor K_{v3} is not used for spiral bevel gears.

CHAPTER VI

BASIC SCORING PREDICTIVE DATA

A. Disk Test Program

As mentioned in Chapter I, the prediction of the scoring-limited performance of gears at the design stage is a far more difficult task than the mere avoidance of gear scoring by design without regard to the performance penalty to be paid. In order to make such a prediction, it is necessary to devise a suitable predictive scheme and to develop certain quantitative data that are required in the predictive process.

The formulation of the predictive scheme can be approached essentially in three ways. One way is to lay out a scheme that is as completely rational as possible, regardless of how complex it is. However, for reasons enumerated in Chapters II through V, the current state of the art does not permit this level of sophistication without a great deal of further work on the basic mechanism of scoring, the thermal behavior involved, as well as the influence of gear mechanics. The second way is to approach the problem in a primarily empirical manner, such as the current AGMA gear scoring design guide,³ which involves assumptions that are basically arbitrary. The third alternative is to devise an interim scheme which recognizes the importance of the above-mentioned basic problems, but accepts approximations without waiting for definitive answers to these basic problems. One of the requirements of this program is that the predictive scheme to be developed should be simple enough for the practical engineers to use without having to resort to elaborate computer programs; but yet represent a tangible advance beyond the current state of the art. The third alternative is believed to satisfy this requirement best, and is therefore the one adopted herein.

The proposed predictive procedure entails two basic steps. The first step is to estimate the ideal scoring-limited power-transmitting capacity for a gear set assuming no tooth misalignment and dynamic load. The second step is to apply corrections for the misalignment and dynamic effects, thus enabling an estimate to be made of the actual scoring-limited power-transmitting capacity when misalignment and dynamic load are inevitably present. This chapter of the report will be concerned with the development of the basic data for predicting the ideal scoring-limited power-transmitting capacity. The prediction of the actual scoring-limited power-transmitting capacity will be taken up in the next chapter.

The data to be presented in this chapter were generated from controlled sliding-rolling disk tests. The bulk of the basic information was deduced from 187 disk tests performed under this program, using a modified Caterpillar disk tester herein designated as SwRI disk tester A for convenience, carburized AISI 9310 steel test disks of 10 different surface characteristics, a MIL-L-7808G synthetic oil (Oil F) and a MIL-L-23699 synthetic oil (Oil E), under a variety of test conditions.

The 10 disk types of different surface characteristics are herein referred to, for the sake of brevity, as follows:

Type 1 — Soft circumferentially-ground, plain
Type 1A — Soft circumferentially-ground, oxidized

Type 3 — Rough circumferentially-ground, plain
Type 3A — Rough circumferentially-ground, oxidized

Type 5 — Honed, plain
Type 5A — Honed, oxidized

Type 7 — Rough cross-ground, plain
Type 7A — Rough cross-ground, oxidized

Type 9 — Smooth circumferentially-ground, plain
Type 9A — Smooth circumferentially-ground, oxidized

The "plain" disks are those which were not surface-treated after the grinding or honing process. The "oxidized" disks were treated with a black oxide by a proprietary process after grinding or honing (App. D). The average surface characteristics of each type of test disk pairs are given in Table D-1. The properties of the test steel and test oils are given in Appendixes A and B.

Some of the test results from the above disk test program, as well as some other results obtained from SwRI disk tester A have been reported earlier in the literature.^{25, 28} For the purpose of this report, supplementary information was also extracted from tests performed on an AFAPL disk tester^{26, 32} herein designated as SwRI disk tester B for convenience; published results from a Thornton disk tester;^{24, 29} as well as other unpublished results of disk tests conducted in the authors' laboratory.

A brief description of SwRI disk tester A, test conditions and

procedure, and a summary of the test results from this program are presented in Appendix D. These results, together with the results from other sources as mentioned above, will be examined at some length in the subsequent sections of this chapter. However, some cursory remarks of a general character appear pertinent at this juncture.

Of the 187 tests reported in Appendix D, 133 tests were performed with Oil F (MIL-L-7808G) and 54 tests with Oil E (MIL-L-23699). Viewed in another way, 98 tests were performed with the plain disks, while 89 tests were performed with the oxidized disks. It is of interest to inquire how the test oil and the surface treatment influence the scoring-limited performance under otherwise comparable conditions.

In attempting to answer the above question, it should be recognized that scoring is a highly scattered phenomenon, and the scoring loads observed in even the best controlled, replicate disk tests may vary in the range of 3 to 1 or more under ostensibly identical test conditions.²⁰⁻³² Consequently, in order to answer the above question with real confidence, a large number of replicate tests must be performed for each disk-oil combination and each set of test conditions; and the results must be analyzed statistically. The performance of a large number of replicate tests was not feasible in a program of limited size when so many variables must be varied. It was therefore necessary to conduct the disk tests based partially on prior experience as to the relative importance of the many variables involved, and partially on the major emphasis of the overall program. The number of tests conducted for each disk-oil combination, covering all sets of test conditions, is presented in Table D-2. For each disk-oil combination, the number of tests conducted for each set of test conditions may be deduced from Tables D-3 to D-23, which also present the results at scoring or at test termination for each test.

In Tables D-3 to D-23, a quantity of special interest, i. e., the critical temperature, T_{cr} , for each set of replicate tests is derived by the Weibull analysis.⁷⁰ However, for the sake of convenience, the normal load reached at scoring, W , is reported only in terms of an algebraic average.

Effect of Test Oil. Figure 12 compares the average scoring load, W_f (i. e., the average W in Tables D-3 to D-23), of Oil E with that of Oil F, for those disk-oil-test variable combinations where

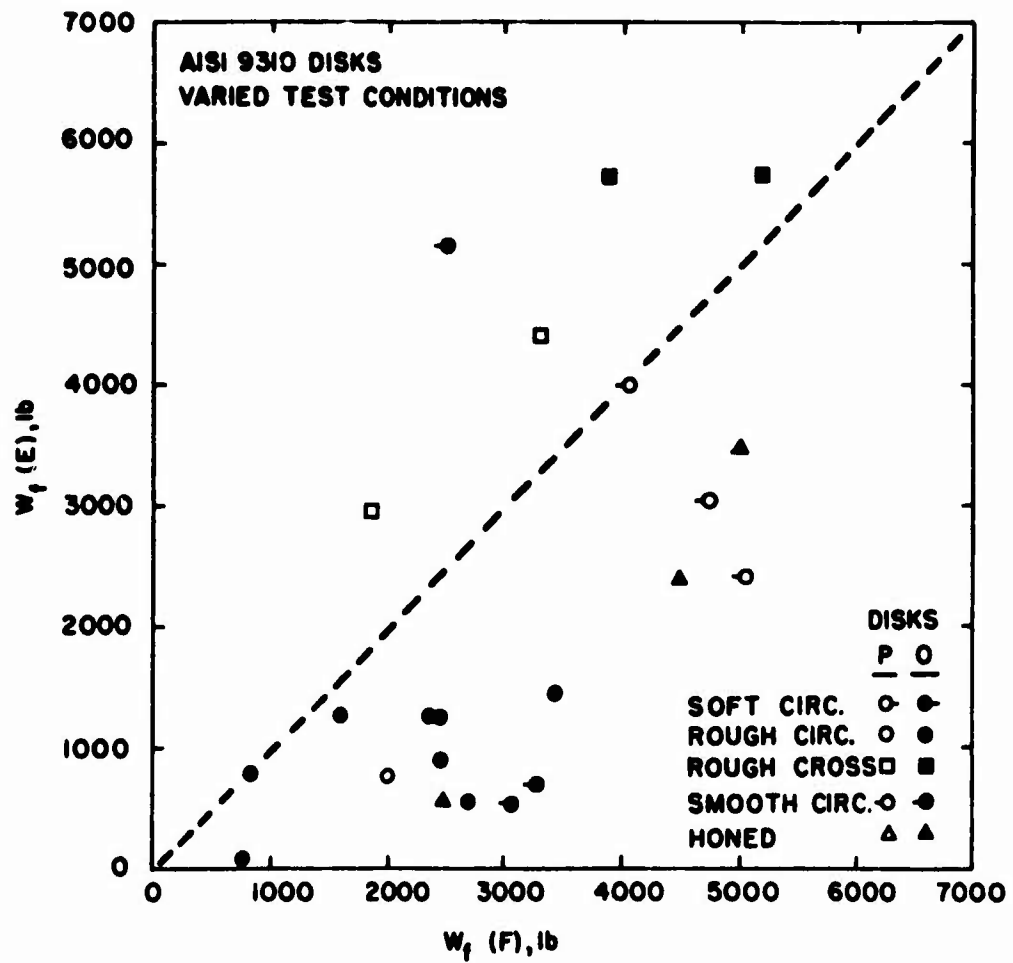


Figure 12. Effect of test oil on average scoring load

comparative results are available. In this figure, the results from the different disk types are represented by different symbols, using the hollow symbols for the plain disks and the solid symbols for the oxidized disks. Although the test conditions are not shown in the figure, nor is this information necessary for the present comparison, each plotted point gives the W_f of Oil E as ordinate vs. the corresponding W_f of Oil F as abscissa. It is obvious that if the two oils should give equal performance, then regardless of the disk type and test conditions, all plotted points would lie on the diagonal line shown.

The fact that these points show a great deal of scatter is largely a matter of statistics. As mentioned earlier, only relatively few tests could be run for each disk-oil-test variable combination. As a matter of fact, of the 5 points that lie above the diagonal line, 4 had only one test conducted on Oil E while the remaining point had only 2 replicate tests. Since the range of scatter of the scoring load can be as large as 3 to 1 or more as mentioned before, not much confidence should be attached to these points. Thus, taken as a whole, the figure suggests that the scoring load of Oil E is lower than that of Oil F, despite the fact that Oil E has a higher viscosity than Oil F. A similar result was also observed in tests conducted with SwRI disk tester B using different-size test disks, with Oils E and F and another straight mineral oil of still higher viscosity.²⁶ It was found that the scoring load was actually highest with Oil F, intermediate with Oil E, and lowest with the straight mineral oil, under otherwise identical test conditions. The fact that oil viscosity as such affects the scoring load the "wrong" way is clear indication that elastohydrodynamic lubrication is not meaningful in controlling scoring as previously stated in Chapter II. Indeed, examination of Tables D-3 to D-23 will show that the computed EHD film thickness ratio, Λ , for all except a very few tests was substantially less than unity. The ratio Λ reported in these tables was based on the computed h_m by Equation (4) without applying side flow and inlet shear thermal corrections, and the composite surface roughness of the disk pair at scoring as defined by Equation (B-2) in Appendix B. The few instances with $\Lambda > 1$ all occurred at $V_t \geq 1080$ ips, when the inlet shear thermal correction would be expected to be more significant. Moreover, the effect of inlet starvation, even if small in the disk tests, was not, nor could it be, accounted for. Besides, it was very difficult to measure accurately the surface roughness of the disks after they had scored. Finally, the definition of the composite surface roughness of the disk pair is, after all, quite arbitrary in this or any other work, so that Λ is normally expected to be greater than unity when full, classical elastohydrodynamic lubrication ceases to exist. Considering all these factors, as well as the preponderance of the $\Lambda < 1$ values

obtained in all other tests, there can be little doubt that scoring occurred in the boundary lubrication regime as emphasized from the theoretical standpoint in Chapter II.

The fact that Oil E gave less scoring protection than Oil F, despite its higher viscosity, emphasizes the importance of surface chemistry in controlling scoring—a boundary lubrication phenomenon. No attempt has been made to draw a "weighted" curve in Figure 12 for the $W_f(E)$ vs. $W_f(F)$ results, because their relationship is not simple, but depends on the operating conditions. This fact will be obvious from the subsequent sections of this chapter.

Effect of Surface Treatment. The effect of surface treatment is presented in Figure 13, in a similar manner. As in the preceding case, there is a great deal of scatter in the results, and the statistics are generally weak. Nevertheless, if more weight is given to those points with greater number of tests for both the plain and oxidized cases, one must conclude that the W_f with the oxidized disks is generally lower than the W_f with the plain disks. Again, the reason will be obvious later.

The use of black oxide surface treatment appears to be detrimental from the scoring standpoint, except with cross-ground disks to be discussed later. However, this detrimental effect is, in all likelihood, not significant in actual gears, because the practical limit of scoring for actual gears is generally more advanced than in the disk tests where the "true" incipient scoring can more readily be detected and identified. At a more severe level of scoring, the thin black oxide layer is apt to be worn off, so that the actual gears, whether oxidized or not, are apt to behave as if there were no black oxide present so far as scoring is concerned. The practical advantages of the black oxide treatment are apparently that it serves as a rust preservative in storage, and that it is a very useful aid in checking the accuracy of gear manufacture or assembly. If the gear alignment is poor, or if the surface contour is not correct, the wearing off of the black oxide layer provides a convenient visual indication.

Effect of Surface Texture. Before leaving Figures 12 and 13, it is of interest to note that, despite their statistical weakness, there appears to be a tendency for the cross-ground disks to behave better when a black oxide layer is present. It is speculated that if the black oxide should offer any advantage, it would be more apt to show up with cross-ground disks because the grinding grooves, which are normal to the sliding motion, tend to retain the black oxide better. Apart

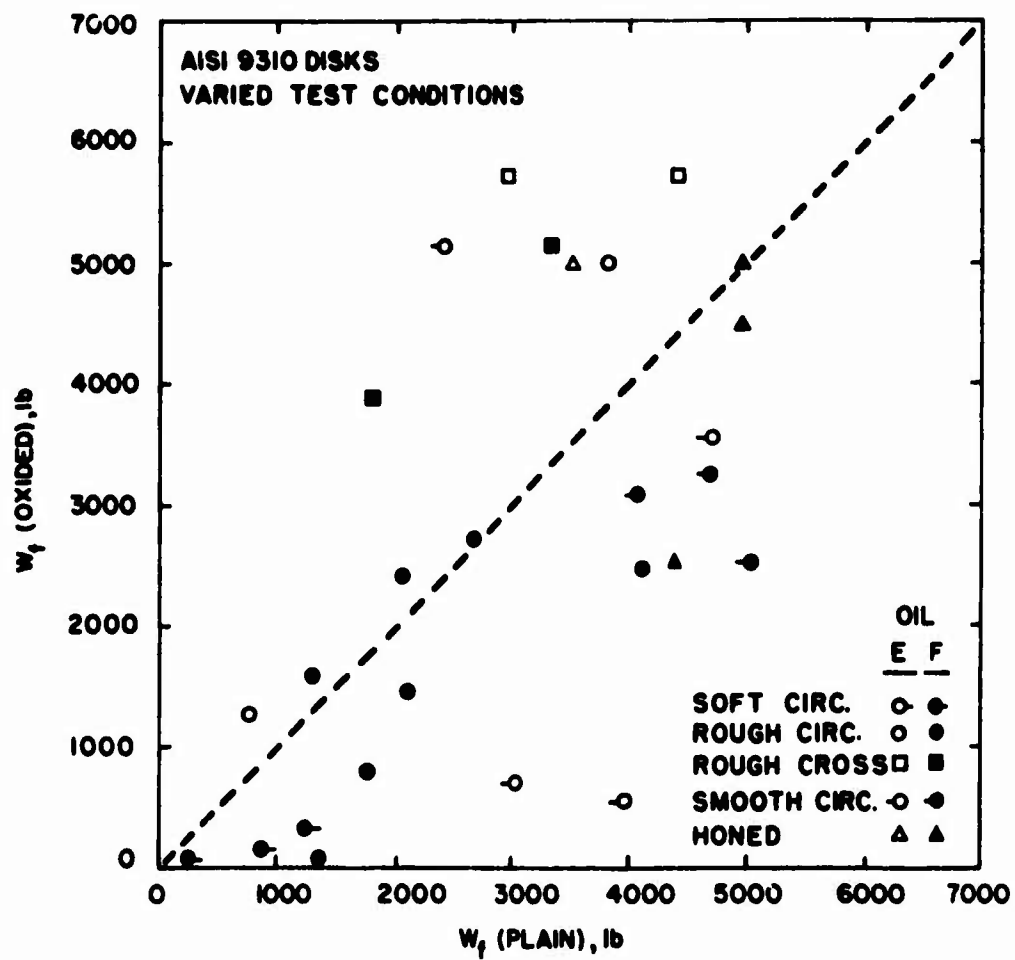


Figure 13. Effect of black oxide surface treatment on average scoring load

from this, it will be seen later that cross-ground disks give a lower coefficient of friction than circumferentially-ground disks of equal composite surface roughness even without a black oxide treatment, possibly due to micro-EHD action.⁴⁰⁻⁴² This is of course beneficial from the scoring viewpoint,²⁵ and by inference from the viewpoint of rubbing wear also. There is also evidence that the presence of the micro-EHD film is beneficial from the viewpoint of pitting as well.⁷¹

Now, since sliding in gears usually takes place normal to or nearly normal to the grinding grooves, the possible advantage suggested by the cross-ground disks, particularly in the presence of a black oxide treatment, is certainly worth investigating. Unfortunately, due to cost and disk delivery considerations, only very few tests were run with cross-ground disks in this program.

B. Critical Temperature

As originally postulated by Blok,^{16, 17} and subsequently defended by him,^{23, 27} the critical temperature, i. e., the maximum instantaneous surface temperature in a sliding-rolling conjunction for scoring to occur, is a function of the metal-oil combination but independent of the surface characteristics and operating conditions. On the other hand, there is a large volume of experimental disk test data^{9, 10, 20-22, 24-26, 28-30} to show that Blok's constant critical temperature hypothesis is not strictly true. This controversy cannot be resolved on theoretical ground. However, as a practical design index, much depends upon how much accuracy or statistical confidence one wishes to attach to the scoring prediction. It is the authors' current thinking that elaborate refinements on the critical temperature is justified in the scoring prediction of sliding-rolling disks; but not for gears mainly because of the pronounced effects of tooth misalignment and dynamic load, the magnitudes of which, as will be seen in the next chapter, can only be reasonably well inferred, but not accurately established, at this time.

The authors have shown^{25, 26, 28} that the effects of surface characteristics and operating variables on the critical temperature of a given metal-oil combination can be satisfactorily expressed by

$$T_{cr} = F(\zeta, M) \quad (47)$$

where F = an experimentally determined function, and

$$\zeta = \frac{\mu_o^2 V_s V_t}{R^2 \sigma_m^2} \quad (48)$$

$$M = \frac{V_s}{V_t} \quad (49)$$

where V_s = sliding velocity, ips

V_t = sum velocity, ips

ζ = a dimensionless parameter

M = sliding-to-sum velocity ratio

R = equivalent radius of curvature of the conjunction, in.

σ_m = maximum Hertz stress in the conjunction, psi

μ_o = oil viscosity at the conjunction inlet, lb-sec/in.²

Equation (47) is extremely difficult to apply to practical gear design. This is partly because μ_o is very sensitive to the oil temperature at the conjunction inlet, T_o , and thus difficult to estimate accurately. Also, σ_m is difficult to estimate because misalignment and dynamic effects cannot be accurately quantified. Therefore, for the present purpose, a simpler approach which requires no estimates of these quantities, is being proposed.

Oil F. The critical temperature data for Oil F (MIL-L-7808G) in combination with AISI 9310 steel are presented in Tables D-3 to D-13 for the different disk types and operating conditions. Consider the plain, rough circumferentially-ground disks (Type 3) for the time being, the effects of V_s and V_t , at three constant M values, are presented in Figures 14 and 15, respectively.

In Tables D-3 to D-13, data for the scoring temperature, T_{cr} , for all individual sets of replicate tests, at 10-percent probability by Weibull analysis, are given, together with their 90-percent confidence limits. These are shown in the two figures by appropriate symbols and vertical bars, except that they are now designated as T_f as a matter of clarification. The symbol T_{cr} is now reserved to represent

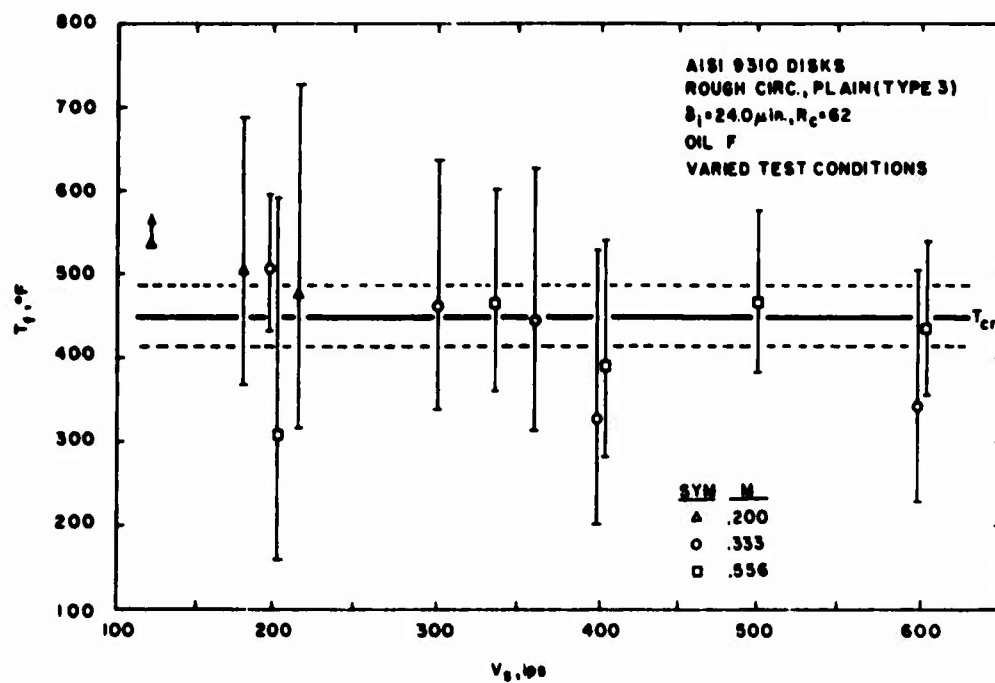


Figure 14. Effect of sliding velocity on critical temperature for Oil F and AISI 9310 steel

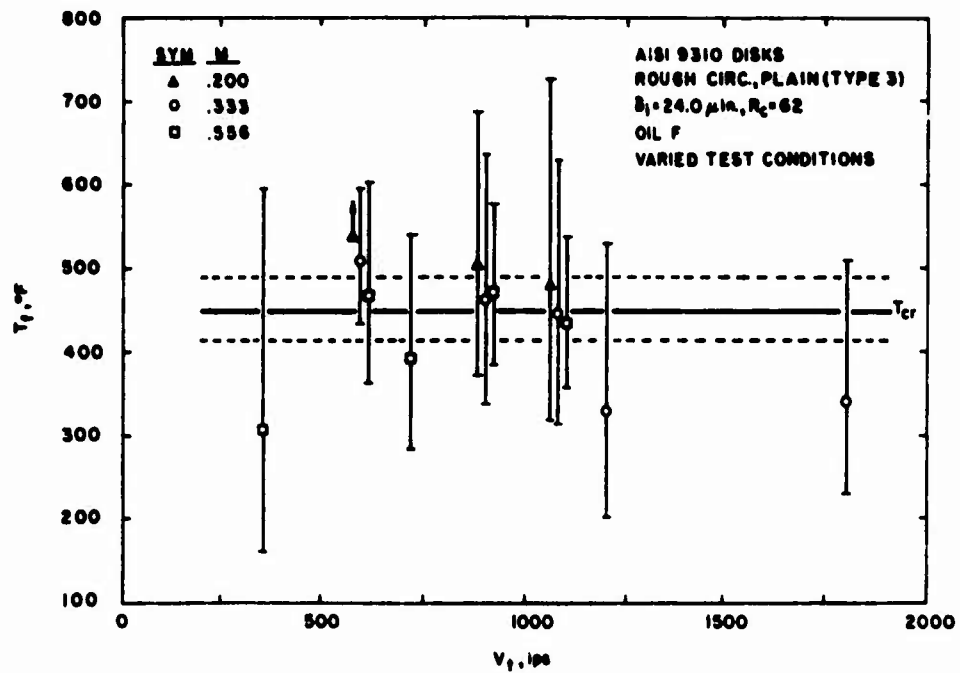


Figure 15. Effect of sum velocity on critical temperature for Oil F and AISI 9310 steel

a constant critical temperature by pooling all of the data on Oil F-Disk Type 3 combination together by a similar Weibull analysis. The point for $V_s = 120$ ips and $M = 0.2$ signifies that its T_f is higher than shown, since no scoring was obtained (Tests F130, F186, F187, Table D-5).

Close examination of the data presented in the two figures will show that T_f does indeed vary with V_s at constant V_t , and also with V_t at constant V_s , as previously reported.^{25,26,29} However, it is evident that these variations are generally not significant when compared with the confidence limits involved. In view of this fact, the pooled result of all 61 tests for this particular metal-oil combination at 10-percent scoring probability can be regarded as constant and independent of operating conditions, defined herein as its critical temperature, T_{cr} . In this case, $T_{cr} = 450^\circ\text{F}$ and the lower and upper 90-percent confidence limits are 415°F and 488°F , respectively.

While the above analysis supports Blok's hypothesis on a statistical basis so far as the effect of operating variables are concerned, it has been found that the effect of surface roughness appears more significant and can, at any rate, be conveniently accounted for. This situation is summarized in Table 3, which includes data not only from this program; but also from other tests on similar metal-oil combinations performed in the authors' laboratory. Among these other tests, those on Type X disks (circumferentially-ground) were conducted also with SwRI disk tester A, while those on Type Y disks (honed) were conducted with SwRI disk tester B. Note that the number of tests included in the pooled Weibull analysis does not necessarily correspond with the number of scored tests given in Table D-2. This is because some of the unscored tests at high loads were treated in the Weibull analysis as suspended tests.

Figure 16 presents the above results in graphic form. Note that the critical temperature, T_{cr} , quite consistently decreases with increasing the initial composite surface roughness of the disk pair, δ_i ; and is generally lower for the oxidized disks than for the plain disks. Moreover, within the 90-percent confidence limits, neither the surface texture (i. e., whether the surfaces are circumferentially-ground, cross-ground, or honed) nor the small variation of surface hardness makes a significant difference. Accordingly, since more tests are available for the plain disks, its T_{cr} vs. δ_i line has been established by linear regression. The line for the oxidized disks is then drawn parallel to that for the plain disks, giving some weight on the statistical distribution. The equations for the two straight lines are

TABLE 3. CRITICAL TEMPERATURE FOR OIL F
AND AISI 9310 STEEL DISKS

Disk type	δ_i , <u>μin.</u>	<u>R_c</u>	<u>M</u>	<u>No. of tests in analysis</u>	<u>T_{cr}, °F</u>	<u>Confidence limits, °F</u>	
						<u>Lower</u>	<u>Upper</u>
<u>Plain disks</u>							
1	26.0	58	0.556	5	425	386	468
3	24.0	62	Mixed	61	450	415	488
7	23.5	62	0.556	6	480	402	574
X	15.2	62	0.333	5	408	345	483
9	9.5	62	0.556	3	530	362	775
Y	6.0	62	Mixed	86	490	458	524
5	5.5	62	0.556	3	560	532	590
<u>Oxided disks</u>							
1A	26.0	58	0.556	4	330	290	372
3A	24.0	62	Mixed	27	355	292	432
7A	23.5	62	0.556	4	445	393	504
9A	9.5	62	0.556	6	440	304	636
5A	5.5	62	0.556	5	500	403	620

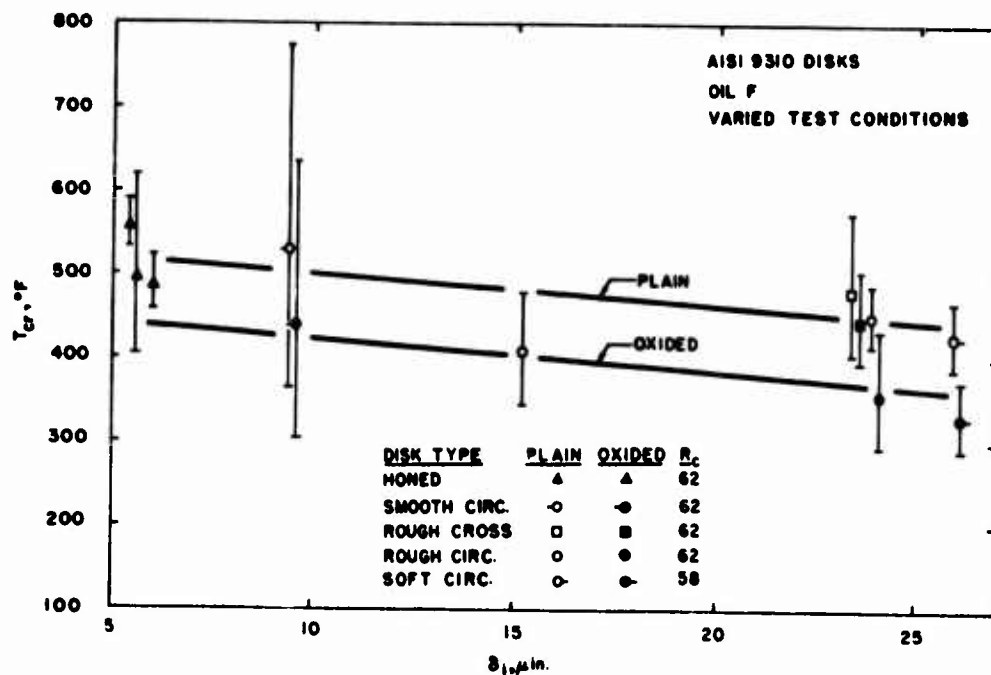


Figure 16. Effect of initial composite surface roughness on critical temperature for Oil F and AISI 9310 steel

$$\text{Plain surfaces: } T_{cr} = 540 - 3.80 \delta_i \quad (50)$$

$$\text{Oxided surfaces: } T_{cr} = 460 - 3.80 \delta_i \quad (51)$$

where T_{cr} = critical temperature for Oil F and AISI 9310 steel combination, °F

δ_i = initial composite surface roughness of the mating surfaces, μ in. AA

The above equations can of course be criticized for not using the prevalent composite surface roughness at the moment of scoring. The reasons for using here the initial value, δ_i , are several. First, the prevalent surface roughness, though theoretically correct, is difficult to measure even on disks; and the after-scoring measurement is not necessarily the correct one since tests cannot be stopped instantaneously and some or severe surface deterioration inevitably occurs in the stopping process. Second, in practice, the initial surface roughness of gears can be measured; but the surface roughness in service depends on the length of service and operating conditions, and is thus a nebulous quantity. Third, if the gears are properly broken in and put into service, the surfaces generally become smoother. Thus, scoring is most prone to occur when the gear set is first run under the design conditions; and once this hurdle is crossed, scoring is not likely to occur unless the operating conditions are drastically made more severe. Considering these factors, it is felt that δ_i is more within the control of the designer and is usually the critical quantity to watch. Of course, if a prescribed break-in procedure is followed, the surface roughness after the break-in would be a better figure to use if it is measured, or it can be estimated if not measured such as by the relationship reported in Reference 25. In those instances where severe operating conditions are anticipated after extended service, it is certainly wise to measure the surface roughness before that occurs, and use what amounts to the δ_i for that new set of operating conditions.

Oil E. The critical temperature data for Oil E (MIL-L-23699) in combination with AISI 9310 steel, given in Tables D-14 to D-23 in Appendix D, may be treated similarly. Table 4 presents a summary of the data based on these results, plus data from Type Z disks (circumferentially-ground) of a similar metal-oil combination obtained also with SwRI disk tester A.

It should be noted that the data for Oil E are statistically much

TABLE 4. CRITICAL TEMPERATURE FOR OIL E
AND AISI 9310 STEEL DISKS

Disk type	δ_i , <u>μin.</u>	<u>R_c</u>	<u>M</u>	No. of tests in <u>analysis</u>	<u>T_{cr}</u> , <u>°F</u>	Confidence limits, °F	
						<u>Lower</u>	<u>Upper</u>
<u>Plain disks</u>							
3	24.0	62	0.333	1	406	—	—
7	23.5	62	Mixed	6	573	479	685
Z	16.7	62	0.333	5	480	426	540
9	9.5	62	Mixed	5	572	516	634
5	5.5	62	0.333	3	440	413	468
<u>Oxided disks</u>							
3A	24.0	62	Mixed	18	340	276	419
9A	9.5	62	Mixed	7	380	309	468
5A	5.5	62	Mixed	8	345	252	473

weaker than for Oil F presented in Table 3. Consequently, some liberty must be taken in treating these data. The two straight lines drawn in Figure 17 are based on the assumption that the slope of the two lines is the same as that in Figure 16, while giving some weight to statistics. With this assumption, the equations for the two straight lines are

$$\text{Plain surfaces: } T_{cr} = 515 - 3.80 \delta_i \quad (52)$$

$$\text{Oxided surfaces: } T_{cr} = 415 - 3.80 \delta_i \quad (53)$$

where T_{cr} = critical temperature for Oil E and AISI 9310 steel combination, °F

δ_i = initial composite surface roughness of the mating surfaces, $\mu\text{in. AA}$

C. Coefficient of Friction

Apart from the critical temperature, the coefficient of friction is another primary parameter in controlling scoring. For general performance analysis, the friction behavior through the entire range of EHD to boundary lubrication is of interest. However, for scoring analysis, emphasis should clearly be on the boundary friction regime.

Attempts to generalize the friction behavior of sliding-rolling systems have not been fruitful.^{9,10} This is because friction is not only exceedingly difficult to determine accurately by experiment, but also equally difficult to understand theoretically. Consequently, empirical approach has been necessary, and considerable uncertainties must be expected.

The authors have attempted to correlate the coefficient of friction of sliding-rolling disks with the dimensionless parameters ζ and M mentioned in the preceding section of this chapter. But due essentially to the same reasons, this method has been found to be most difficult to employ in practical gear design. Accordingly, a simpler correlation was proposed recently,²⁸ which will be used herein.

Oil F. Figures 18 and 19 for Oil F and plain, circumferentially-ground and cross-ground, AISI 9310 steel disks are taken from Reference 28 to illustrate the nature of the correlation. In these

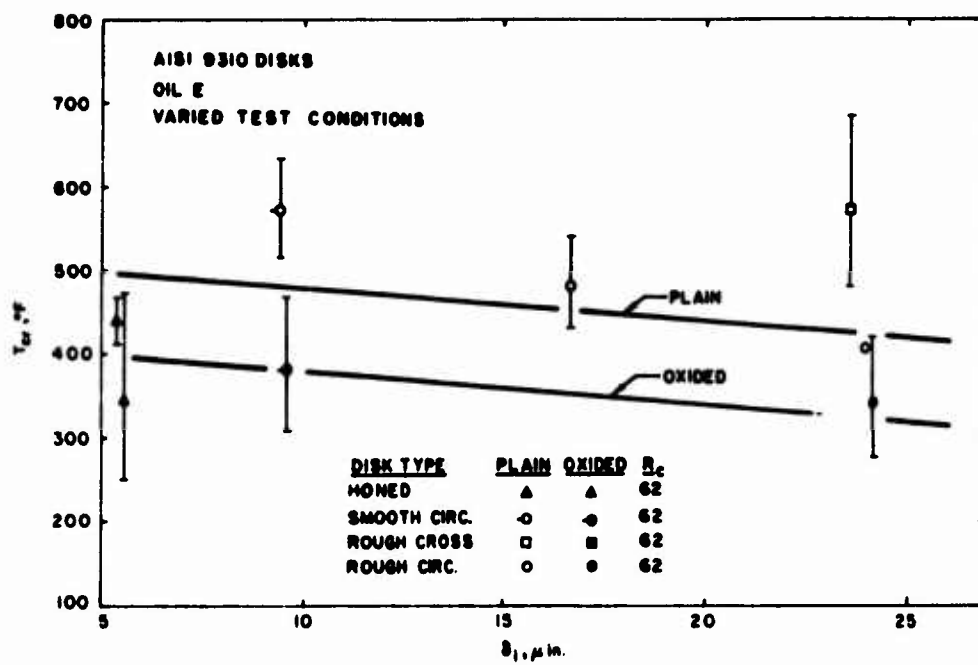


Figure 17. Effect of initial composite surface roughness on critical temperature for Oil E and AISI 9310 steel

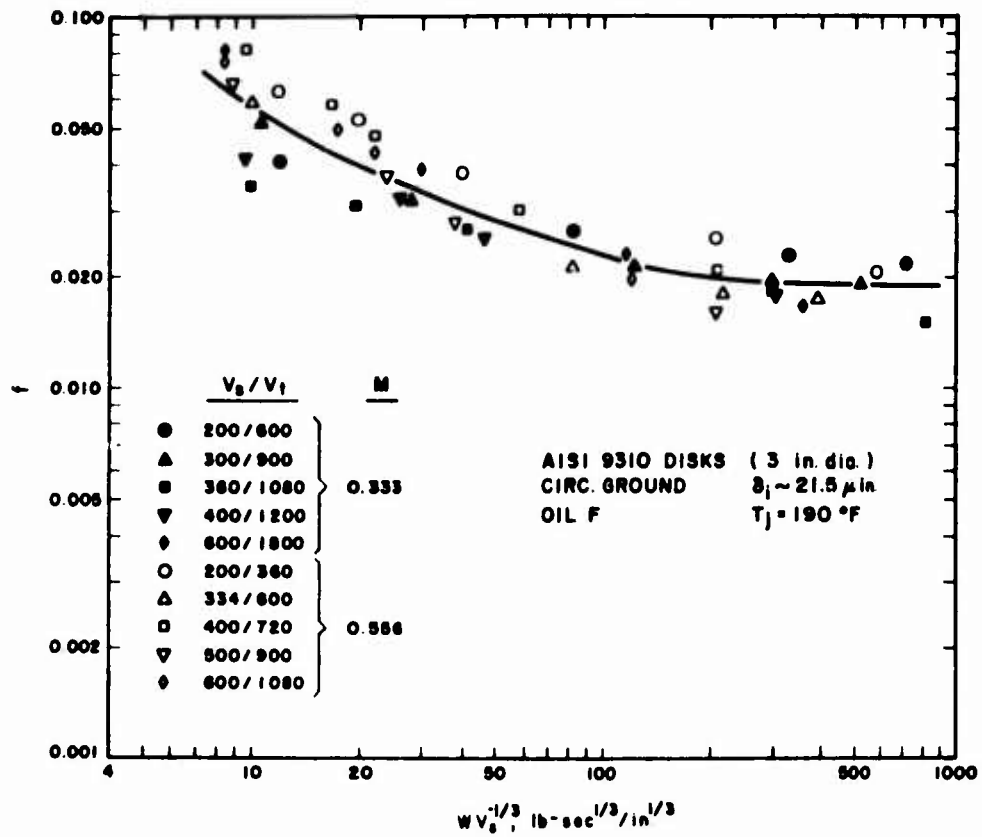


Figure 18. Friction behavior of plain circumferentially-ground AISI 9310 steel disks with Oil F

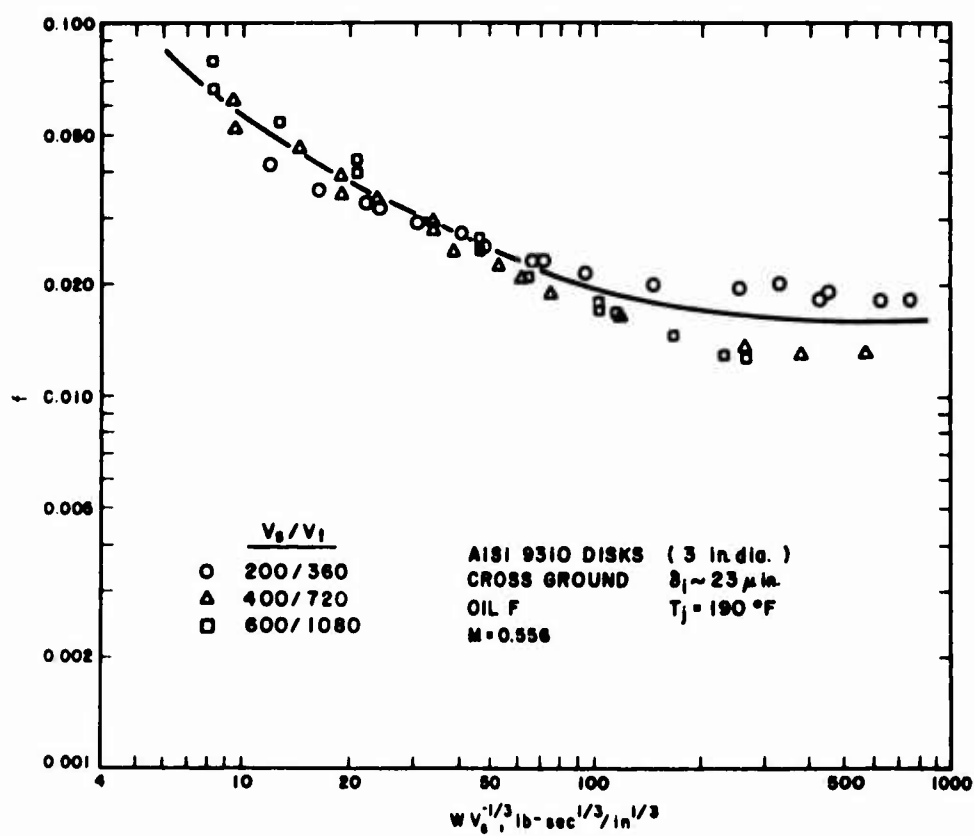


Figure 19. Friction behavior of plain cross-ground AISI 9310 steel disks with Oil F

figures, the coefficient of friction, f , is related to the quantity $WV_s^{-\frac{1}{3}}$, where W is the normal load, lb, and V_s is the sliding velocity, ips. It should be remarked that literally hundreds of data points are available not only from the disk scoring tests, but also from a great deal of additional friction measurements. However, for the sake of clarity, only very few "extreme values" are shown in the figures to indicate the scatter involved, which is about ± 20 percent from the average curves at high values of $WV_s^{-\frac{1}{3}}$ (a range of greater interest in scoring prediction) and about ± 40 percent at low values of $WV_s^{-\frac{1}{3}}$. Within this scatter range, it is seen that f bears an approximate relationship to $WV_s^{-\frac{1}{3}}$.

By comparing the two figures, it will be noted that at the same value of $WV_s^{-\frac{1}{3}}$, f is somewhat lower with the cross-ground disks than with the circumferentially-ground disks, even though the composite surface roughness of the cross-ground disks was slightly greater. This could be due to a micro-EHD effect.²⁵

Figures 18 and 19 were based on partial data available in 1973. The more complete, updated results are now presented in Figure 20, for all 10 disk types investigated. For the sake of clarity, these updated results are presented herein merely as average curves, by omitting the data points. Note that with more data on the rough, circumferentially-ground disks (Type 3), the average δ_i is raised, as is the average f . Otherwise, the general trends and maximum scatter are about the same as in Figures 18 and 19.

Figure 20 reveals two items of major interest. First, with the exception of the cross-ground disks, the coefficient of friction is not measurably influenced by the black oxide surface treatment. The substantially lower f observed for the Type 7A (oxidized, cross-ground) disks is based on data from only 4 scoring tests with Oil F (Table D-11). However, 2 other tests run on Type 7A disks with Oil E (Table D-21) gave similar results. These data are admittedly skimpy; but the enormous effect observed is certainly intriguing. As suggested in the preceding section, this effect could be due to the fact that the grinding grooves normal to the sliding direction tend to help retain the black oxide in the grooves. A similar mechanism is not present for all other surface textures.

The other item of interest is that the value of f on the flat portion of each curve, where f could be determined more accurately, generally increases with increasing the composite surface roughness of the disk pair. However, one wonders what effects the surface

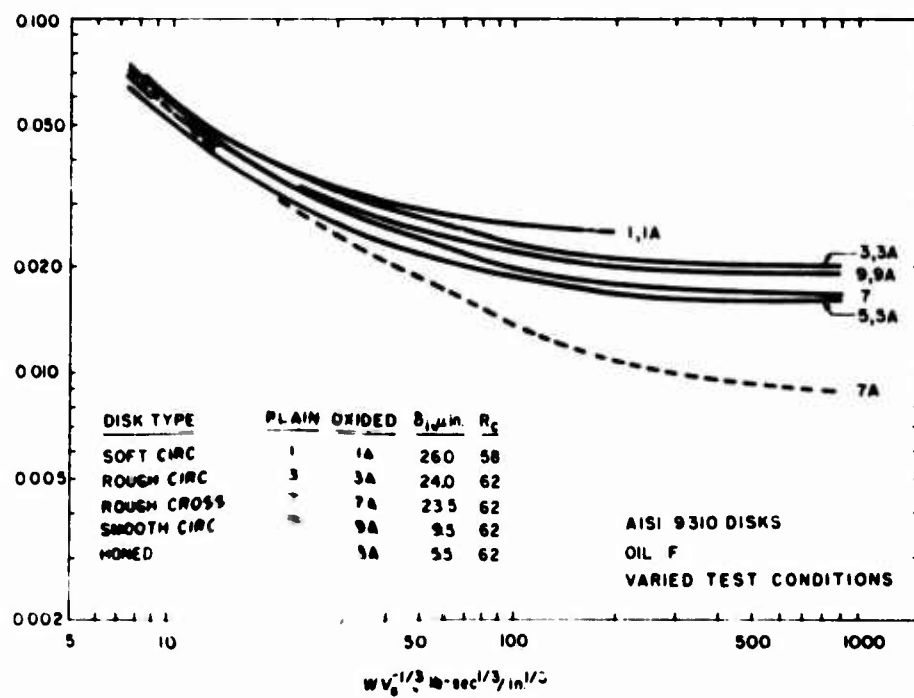


Figure 20. Friction behavior of ten types of AISI 9310 steel disks with Oil F

texture and hardness might have on the friction behavior. These latter effects will now be examined.

Figure 21 portrays the relationship between the flat-value f and δ_i , without including the data for Type 7A disks. In the absence of more data, it is believed logical to infer that the effect of δ_i on f is best represented by a straight line through the points for the smooth circumferentially-ground (Types 9 and 9A) and rough circumferentially-ground (Types 3 and 3A) disks. If this inference is correct, then the higher f for the soft circumferentially-ground disks (Types 1 and 1A) appears to be largely due to their lower case hardness. The lower f for the cross-ground disks (Type 7) then appears to be due to a micro-EHD effect suggested previously. Finally, the lower f for the honed disks (Types 5 and 5A) appears to be due to their "neutral texture," a surface texture which does not provide leakage paths for the oil in the conjunction as does the circumferentially-ground grooves.

It should be remarked that the general level of f shown here is much lower than that assumed in the AGMA gear scoring design guide,³ and the effect of surface roughness on f is also less. A detailed comparison of the AGMA friction behavior with that observed here is presented in Appendix E for the sake of convenience. It is only necessary to state that the level and trend observed in this work are generally consistent with those obtained for the same and other oil-disk combinations on both SwRI disk testers A and B,^{25, 26, 32} as well as those reported on mineral oils and different disks using the Thornton disk tester.^{24, 29, 72}

In order to adapt the information presented in Figures 20 and 21 to computer programs, one way is to use first the data from Figure 20 recognizing that they apply only to the particular δ_i values specified, and then apply a correction for the effect of δ_i by assuming the same f vs. δ_i slope for all surface textures as that for the circumferentially-ground case shown in Figure 21. Another way is to approximate the curves in Figures 20 and 21 by straight lines, so that the variations can be written as equations for convenience.

For this approximation, it is assumed that each curve in Figure 20 may be represented by a horizontal line (i. e., constant f) for $WV_s^{-\frac{1}{3}} \geq 200$, and by an inclined straight line (i. e., an exponential variation of f with $WV_s^{-\frac{1}{3}}$) for $WV_s^{-\frac{1}{3}} < 200$ lb-sec ^{$\frac{1}{3}$} /in. ^{$\frac{1}{3}$} . The slope of the f vs. δ_i line in Figure 21 is 0.00007 per μ in. AA. Using these values it can be readily shown that the friction equations for Oil F and

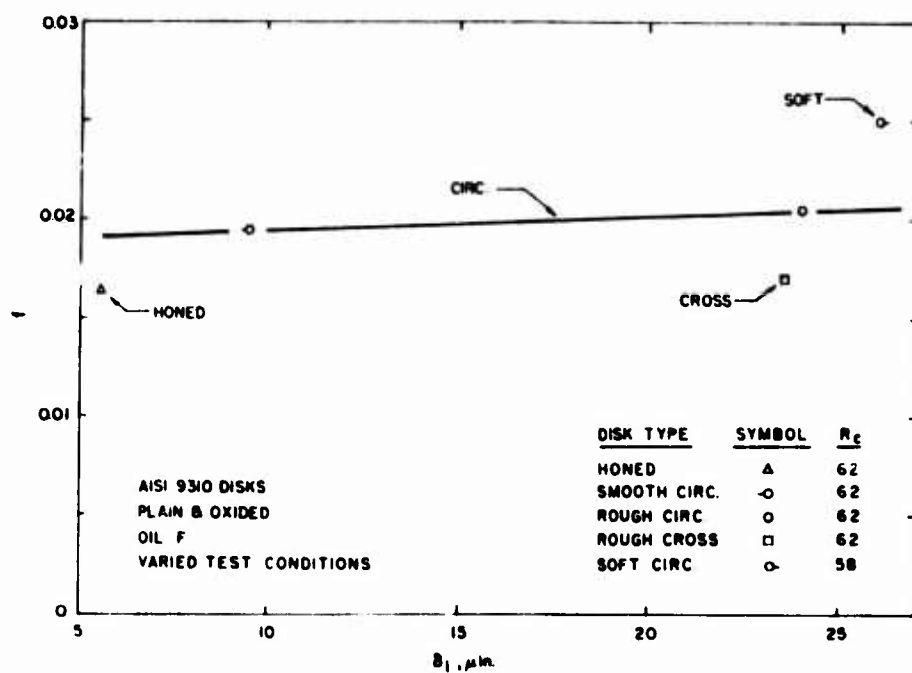


Figure 21. Effect of initial composite surface roughness on friction for Oil F and AISI 9310 steel

AISI 9310 steel combination are as follows.

For plain and oxidized circumferentially-ground surfaces:

$$f = (0.0920 + 0.00034 \delta_i) (WV_s^{-\frac{1}{3}})^{-0.3}$$
$$\text{at } WV_s^{-\frac{1}{3}} < 200 \quad (54)$$

$$f = 0.0188 + 0.00007 \delta_i$$
$$\text{at } WV_s^{-\frac{1}{3}} \geq 200 \quad (55)$$

For plain cross-ground surfaces:

$$f = (0.0755 + 0.00034 \delta_i) (WV_s^{-\frac{1}{3}})^{-0.3}$$
$$\text{at } WV_s^{-\frac{1}{3}} < 200 \quad (56)$$

$$f = 0.0154 + 0.00007 \delta_i$$
$$\text{at } WV_s^{-\frac{1}{3}} \geq 200 \quad (57)$$

For oxidized cross-ground surfaces:

$$f = (0.0407 + 0.00034 \delta_i) (WV_s^{-\frac{1}{3}})^{-0.3}$$
$$\text{at } WV_s^{-\frac{1}{3}} < 200 \quad (58)$$

$$f = 0.0083 + 0.00007 \delta_i$$
$$\text{at } WV_s^{-\frac{1}{3}} \geq 200 \quad (59)$$

For plain and oxidized honed surfaces:

$$f = (0.0789 + 0.00034 \delta_i) (WV_s^{-\frac{1}{3}})^{-0.3}$$
$$\text{at } WV_s^{-\frac{1}{3}} < 200 \quad (60)$$

$$f = 0.0161 + 0.00007 \delta_1$$

$$\text{at } W V_s^{-\frac{1}{3}} \geq 200 \quad (61)$$

In Equations (54) to (61), W = normal load, lb; V_s = sliding velocity, ips; and δ_1 = initial composite surface roughness of the mating surfaces, μ in. AA. The reasons for using the initial composite surface roughness were given in the preceding section.

Oil E. Within the precision of the friction measurements, the friction behavior of AISI 9310 steel disks is substantially the same with Oil E as with Oil F. Equations (54) to (61) are therefore also recommended for use with Oil E and AISI 9310 steel combination.

D. Surface Temperature

According to the critical temperature hypothesis (Chap. II, Sect. B), the scoring-limited power-transmitting capacity is controlled by the maximum rise of the instantaneous surface temperature in the conjunction, ΔT . Assuming that the critical temperature, T_{cr} , is known, it is then necessary to estimate the quasi-steady surface temperature, T_s , in order to arrive at an estimate for ΔT .

The value of T_s is determined by the frictional power loss in the conjunction and the heat loss to the environment by various heat transfer processes. The frictional power loss is defined as

$$\phi = f W V_s / 9336 \quad (62)$$

where ϕ = frictional power loss, Btu/sec

f = coefficient of friction

W = normal load, lb

V_s = sliding velocity, ips

The heat transfer processes vary, of course, with the system configuration and operating conditions.

The authors showed in a recent publication²⁸ that for a specific system configuration and operating conditions, in steady operation, the following approximate relationship holds:

$$T_s - T_j = C \phi^n \quad (63)$$

where T_j = oil jet temperature, °F

C, n = fitting constants

Figure 22 illustrates such a relationship²⁸ for SwRI disk tester A, using a "horn" to supply the test oil to the exit side of the conjunction and halfway around both disks (Fig. D-1, App. D). Plain, rough circumferentially-ground (Type 3) and cross-ground (Type 7) disks were used. The test oil was Oil F, the total oil flow rate was 20 gpm, and T_j was 140°F and 190°F. The operating conditions (i. e., V_s , V_t , and W) were varied over a wide range. For the sake of clarity, the plotted points do not include all available data; but they do portray the maximum scatter of the data, which are due largely to errors in the friction measurement as explained in the preceding section. Note that within the experimental scatter, the relationship is as shown by Equation (63); and this relationship is not systematically influenced by the disk type, the oil jet temperature, or the operating conditions.

Figure 23 presents the more complete data for SwRI disk tester A, using both Oil E and Oil F, supplied by the horn at 20 gpm total flow rate, at 140°F and 190°F jet temperatures, all 10 disk types investigated in this program; and with test conditions widely varied. Again, the plotted points indicate the maximum scatter observed. It is seen that, within the scatter range, the data exhibit an almost identical exponential relationship as that shown in Figure 22—a relationship essentially unaffected by the disk type or surface characteristics, the oil or the oil jet temperature, or the operating conditions.

Figure 24 shows the trends computed from data reported by Bell and Dyson,^{24, 29} using the Thornton disk tester. The test disks were straight cylindrical disks made of EN 34 steel, at two levels of surface finish, F. A straight mineral oil and the same oil with an EP additive were used. The oil was supplied by means of two jets, located on the inlet and exit sides of the conjunction. The tests covered two T_j and total oil flow rate combinations; and the test conditions were widely varied. It is seen that an exponential relationship

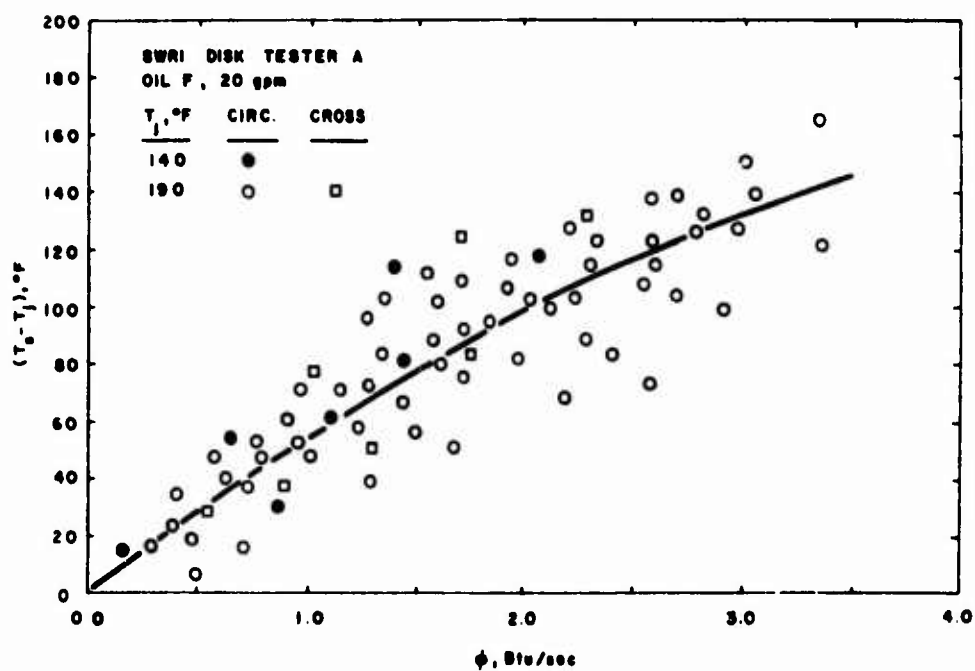


Figure 22. Variation of $(T_s - T_j)$ with ϕ for Oil F and two disk types

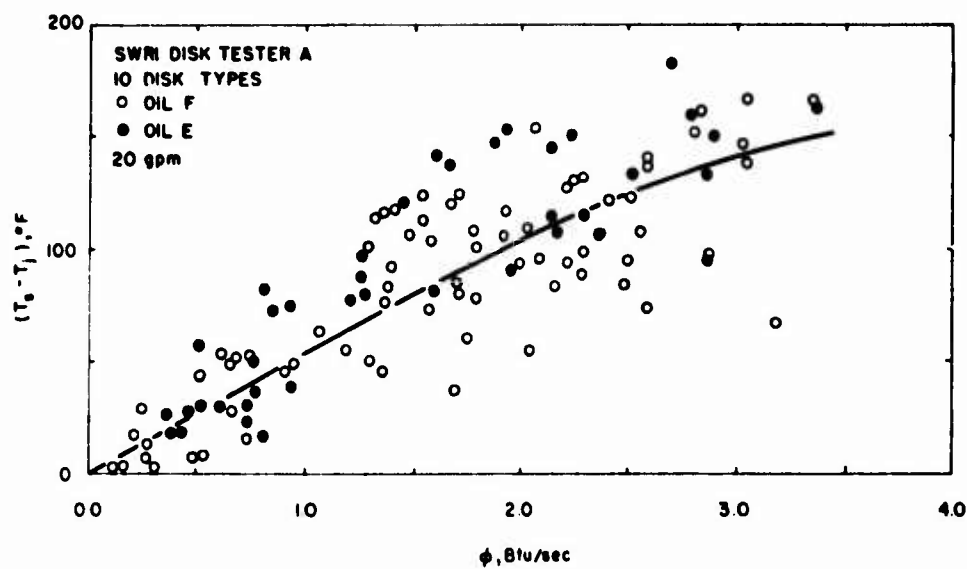


Figure 23. Variation of $(T_s - T_j)$ with ϕ for Oils E and F and ten disk types

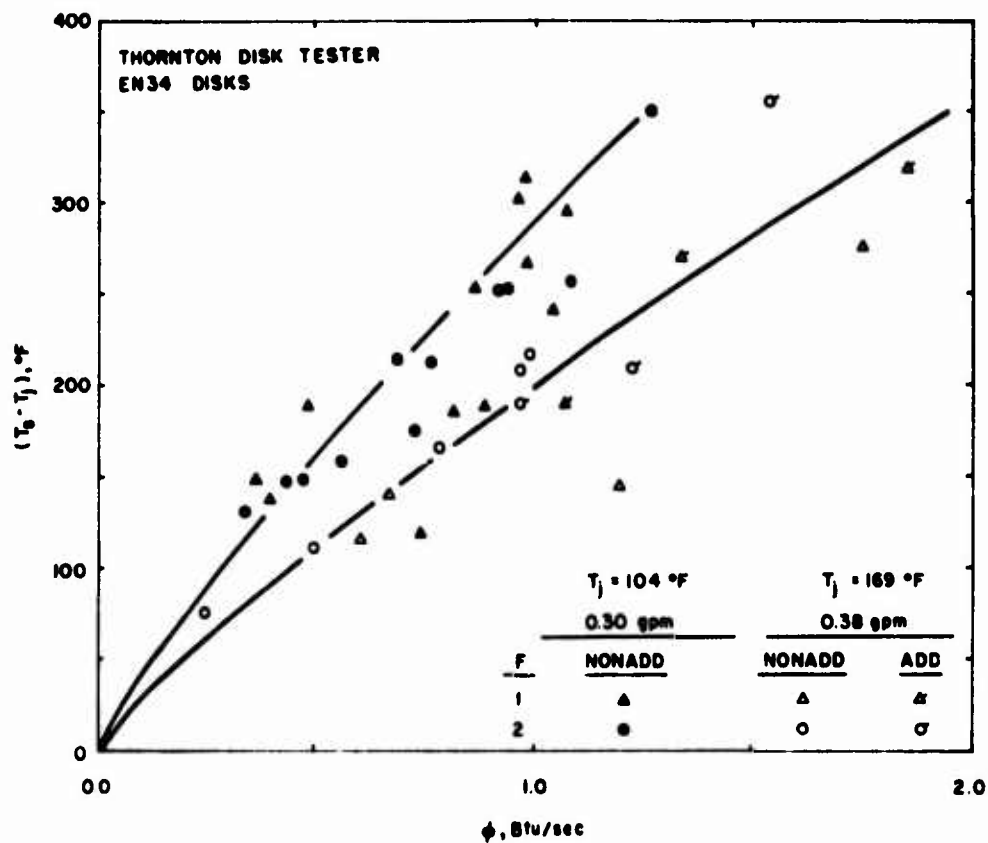


Figure 24. Variation of $(T_s - T_j)$ with ϕ based on the results of Bell and Dyson

also holds, one for each oil flow rate; and each such relationship is not significantly affected by the oil or the oil jet temperature, the surface finish of the disks, or the operating conditions.

The preceding figures serve to show that the $(T_s - T_j)$ vs. ϕ relationship is essentially exponential and not affected by the disk material, disk surface characteristics, and oil type; and, within the range of the investigation, not affected also by the oil jet temperature and operating conditions. In other words, while these factors are expected to affect the magnitude of friction, they do not appreciably affect the relationship between $(T_s - T_j)$ and ϕ since friction enters the makeup of both $(T_s - T_j)$ and ϕ almost similarly. On the other hand, the quantitative behavior is seen to be significantly influenced by the oil flow rate and, by implication, the disposition of the oil jet and the system design, i. e., by those considerations which control the overall heat transfer from the disks. This latter situation is well illustrated by Table 5.

In compiling the information presented in Table 5, it was found that the data shown in Figures 22, 23, and 24, as well as those to be discussed, all yield a value of n of about 0.80 in Equation (63); but the value of C depends on those factors which influence the overall heat transfer. In other words, Equation (63) may now be written as

$$T_s - T_j = C \phi^{0.80} \quad (64)$$

and the constant C is the sole parameter which reflects the system's thermal characteristics. For example, the data for SwRI disk tester A with the horn oil jet at a total oil flow rate of 20 gpm (i. e., Figs. 22 and 23) represent Case 10 in Table 5, and the corresponding value of C is 55. The data for the Thornton disk tester with inlet-exit oil jets (Fig. 24) yield $C = 285$ at a total oil flow rate of 0.30 gpm (Case 6), and $C = 200$ at a total oil flow rate of 0.38 gpm (Case 7).

The variation of C shown in Table 5 with the oil flow rate is presented in Figure 25. Note that the value of C is generally highest with the Thornton disk tester, intermediate with SwRI disk tester B, and lowest with SwRI disk tester A—reflecting the influence of design on system heat transfer. Note further that with the same SwRI disk tester B at two constant oil flow rates, the exit jet location gives a lower value of C , or more effective cooling, than the inlet jet location. While this latter effect is believed to be real and is apparently also

TABLE 5. VALUE OF C FROM THREE DISK TESTERS

<u>Case</u>	<u>Disk tester</u>	<u>Oil jet type</u>	<u>Disk material</u>	<u>Disk type</u>	<u>Oil type</u>	<u>T_j, °F</u>	<u>Oil flow, gpm</u>	<u>C</u>
1	SwRI B	Inlet	AISI 9310	Honed	Oil F	190	0.033	360
2	SwRI B	Inlet	AISI 9310	Honed	Oil F	190	0.25	180
3	SwRI B	Exit	AISI 9310	Honed	Oil F	190	0.033	260
4	SwRI B	Exit	AISI 9310	Honed	Oil F	190	0.25	130
5	SwRI A	Exit	SAE 8620	Circ.	Mineral	Varied	0.50	125
6	Thornton	In-exit	EN 34	Circ.	Mineral	104	0.30	285
7	Thornton	In-exit	EN 34	Circ.	Mineral	169	0.38	200
8	SwRI B	Horn	AISI 9310	Honed	Oil F	190	2.5	100
9	SwRI B	Horn	AISI 9310	Honed	Oil F	190	10	100
10	SwRI A	Horn	AISI 9310	Varied	E & F	Varied	20	55

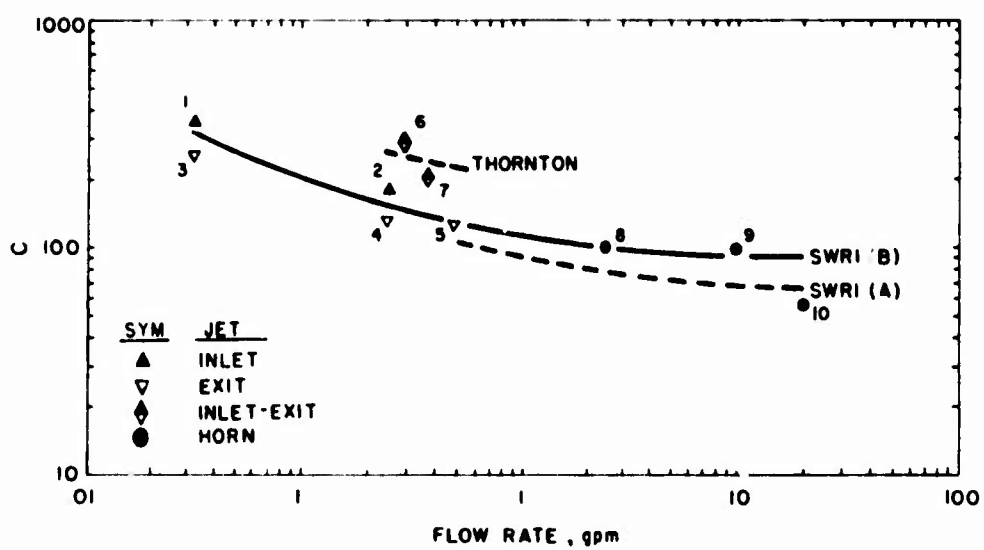


Figure 25. Variation of C with total oil flow rate and system design

experienced in gear operation, a single curve is herein drawn for SwRI disk tester B to emphasize the major trends, namely, the oil flow rate and the tester design. Finally, the effect of oil flow rate on C is quite marked at low flow rates, and generally tends to level off at high flow rates.

It is believed that Equation (64), with the value of C judiciously selected (such as from Fig. 25 or further refined data), gives a tangible basis for estimating the value of T_s . In the AGMA gear scoring design guide,³ an implied assumption is that T_s is equal to T_j . It is obvious from the data presented herein that this assumption is far from being true.

E. Conjunction-Inlet Oil Temperature

The temperature of the oil at the conjunction inlet, T_o , is of interest mainly in elastohydrodynamic film thickness calculations (App. C). Although EHD film thickness will not receive emphasis in this report for reasons given in Chapter II, Section C, information for estimating T_o is presented for the sake of completeness.

Figure 26 shows the variation of $(T_o - T_j)$ with ϕ previously reported²⁸ for SwRI disk tester A, Types 3 and 7 disks, Oil F supplied by the horn at 20 gpm flow rate, at $T_j = 140^\circ\text{F}$ and 190°F . Figure 27 presents the more complete results for the same tester and horn, same T_j and oil flow rate, Oil E and Oil F, and the 10 disk types. Note that substantially the same exponential relationship is shown in these figures. This relationship may be represented by

$$T_o - T_j = C_o \phi^{0.80} \quad (65)$$

where T_o = conjunction-inlet oil temperature, $^\circ\text{F}$

T_j = oil jet temperature, $^\circ\text{F}$

ϕ = frictional power loss, Btu/sec

C_o = a fitting constant

By comparing Figures 26 and 27 for $(T_o - T_j)$ and Figures 22 and 23 for $(T_s - T_j)$, it will be found that

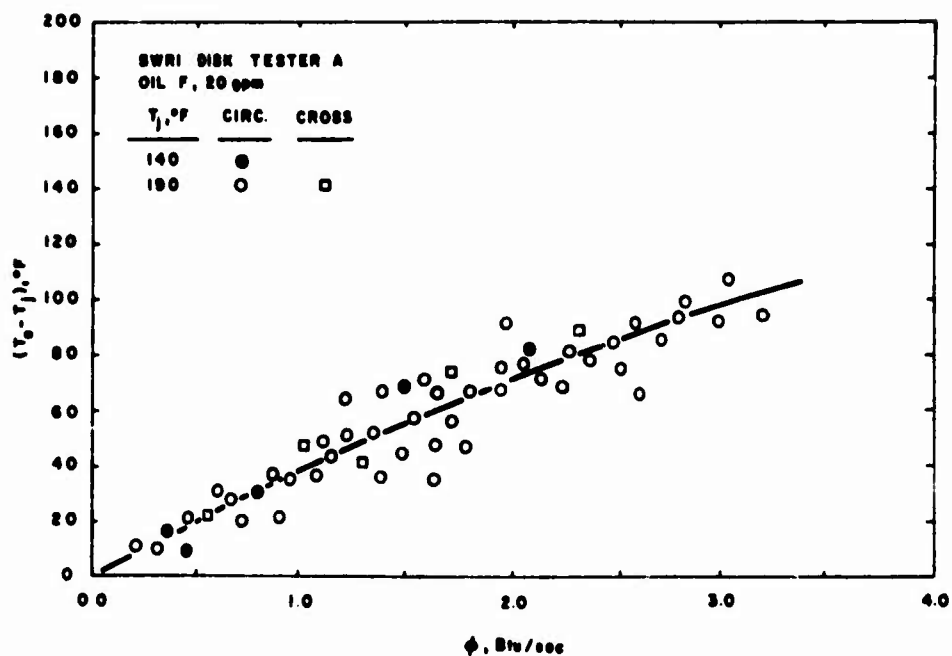


Figure 26. Variation of $(T_o - T_j)$ with ϕ for Oil F and two disk types

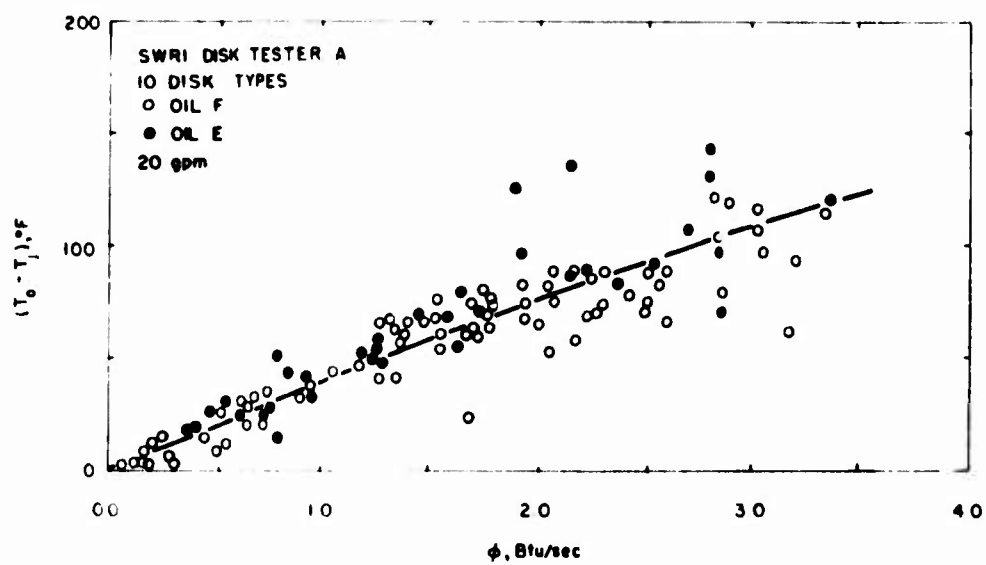


Figure 27. Variation of $(T_o - T_j)$ with ϕ for Oils E and F and ten disk types

$$C_o = 0.70 C \quad (66)$$

is a very good approximation over the range of variables investigated.

Equation (65) gives a tangible basis for estimating the value of T_o . It is seen from Equation (66) that the assumption of either $T_o = T_j$ or $T_o = T_s$, as commonly used, is not satisfactory.

CHAPTER VII

GEAR SCORING PREDICTION

A. Basic Procedure

It was noted in Chapter II, Section B, that the mechanism of scoring is still basically not understood. However, phenomenological observations to date suggest that it is probably triggered by a thermal interaction between the relatively moving surfaces; and that among the several thermal scoring models that have been proposed, the critical temperature model appears to be the most plausible and promising. It was also noted that apart from the need for establishing a meaningful critical temperature criterion, the prediction of the scoring-limited power-transmitting capacity of gears further requires a consideration of the important influences of gear mechanics—a subject whose complexity was broadly indicated in Chapter II, Section F, and further discussed in Chapters III, IV, and V.

In view of the aforementioned problems, it was concluded in Chapter VI, Section A, that a completely rational procedure for predicting the scoring-limited power-transmitting capacity of gears is not now possible. On the other hand, an essentially empirical procedure, such as the AGMA gear scoring design guide,³ certainly leaves much to be desired. Accordingly, an interim predictive scheme, which recognizes the importance of the above-mentioned problems but accepts certain necessary approximations, appears to be the only viable approach.

The predictive scheme to be discussed herein can readily be applied to practical gear design. It is assumed that the scoring behavior of gear teeth follows the same phenomenological behavior of sliding-rolling disks, except for the effects of gear mechanics. This assumption implies that the basic data derived from steady-operating sliding-rolling disks (Chap. VI) may be applied to the transient process of the gear-tooth action, provided the effect of gear-tooth dynamics can be isolated and taken into account. Additionally, it is implied that the effect of gear-tooth misalignment can also be isolated and accounted for. These assumptions are difficult to defend from a rigorous theoretical standpoint, and as such they are "approximations" being forced upon the problem by the current state of the art. Refinements or revisions of the technique are naturally possible; nevertheless, the results thus far obtained appear quite plausible. The analysis also shows that thermal behavior, gear-tooth misalignment, and gear-tooth

dynamics exert major impacts on the scoring-limited power-transmitting capacity of gears.

The basic premise of the proposed gear scoring predictive method may be stated by the equation

$$P_A = P_I S_m S_d \quad (67)$$

where P_A = actual scoring-limited power-transmitting capacity, hp

P_I = ideal scoring-limited power-transmitting capacity, hp

S_m = misalignment factor

S_d = dynamic factor

In other words, the proposed predictive procedure entails, in effect, two basic steps. The first step is to estimate the ideal scoring-limited power-transmitting capacity of a gear set, by considering the thermal effect and assuming perfect tooth alignment and no dynamic load. The second step is to estimate the actual scoring-limited power-transmitting capacity, by applying corrections for the misalignment and dynamic effects. For the sake of convenience, the general features of this procedure will first be considered. Some specific details related to gear types will be discussed in the final sections of this chapter.

B. Ideal Scoring-Limited Power-Transmitting Capacity

The prediction of the ideal scoring-limited power-transmitting capacity, P_I , of a given gear set operating under specified conditions requires, in principle, successive comparisons of the ideal maximum instantaneous conjunction surface temperature, T_c , somewhere in the gear mesh, as the transmitted power is being progressively increased at the specified gear speed and operating conditions, with the critical temperature, T_{cr} , of the metal-surface-oil combination. The word "ideal" signifies that these comparisons are being made for the assumed case of perfect tooth alignment and static tooth load. From Equation (1) in Chapter II,

$$T_c = T_s + \Delta T \quad (68)$$

where T_C = maximum instantaneous surface temperature at any point in the gear mesh, °F

T_s = quasi-steady gear surface temperature, °F

ΔT = maximum instantaneous surface temperature rise at any point in the gear mesh, °F

Note that as the power level is increased at otherwise constant operating conditions, ΔT increases, and its magnitude varies through the gear mesh. At the same time, T_s also increases; but its magnitude remains constant with respect to time. Consequently, T_C also increases, and its magnitude varies through the gear mesh. When the maximum value of T_C somewhere in the gear mesh equals the critical temperature, T_{cr} , scoring occurs and the corresponding power transmitted is, by definition, the ideal scoring-limited power-transmitting capacity of the gear set under the specified operating conditions.

Outline of Predictive Process. The prediction of P_I then comprises the following steps:

1. An estimate of the critical temperature, T_{cr} .
2. At constant gear speed and operating conditions, calculate the quasi-steady surface temperature, T_s , by progressively increasing the power level, while assuming perfect tooth alignment and static tooth load.
3. Similarly, calculate the maximum instantaneous surface temperature rise, ΔT , through the gear mesh.
4. From Steps 2 and 3, calculate the maximum instantaneous conjunction surface temperature, T_C , through the gear mesh.
5. When the maximum value of T_C at some point in the gear mesh (from Step 4) equals the critical temperature, T_{cr} (from Step 1), the scoring criterion is met; hence the corresponding power transmitted is the ideal scoring-limited power-transmitting capacity of the gear set under the specified operating conditions.

Input Data. The prediction process as described can readily be carried out by means of a computer. The required equations and numerical constants or coefficients for each step will now be outlined.

1. As noted in Chapter VI, Section B, the statistically

defined critical temperature, T_{CR} , of a given metal-oil combination is a function of the composite surface roughness of the mating surfaces, but independent of the surface texture and operating conditions. Moreover, although the black oxide surface treatment was found to have a substantial effect on T_{CR} in the disk tests, the practical scoring level of gears, which is generally more severe, tends to permit removal of the thin black oxide layer so that its effect is apt to be essentially absent in practical gears. Therefore, if the gears are made of carburized AISI 9310 steel, whether or not surface-treated with black oxide, and if the oil is Oil F, then the critical temperature is given by Equation (50). For the same metal and Oil E, the critical temperature is given by Equation (52).

2. The quasi-steady surface temperature, T_s , is a time-averaged quantity, whose magnitude depends upon the frictional heat generated at the meshing surfaces and the heat removal from these surfaces by various means—principally by conduction and convection. In other words, T_s is highly dependent upon gear design, system design, and operating conditions; and its prediction requires a quantitative knowledge of the effects of these factors on both the frictional heat generation and the heat removal.

For the simple case of steady-operating sliding-rolling disk systems, it was shown in Chapter VI, Section D, that the frictional heat generated per unit time, or the frictional power loss, is

$$\phi = f W V_s / 9336$$

where ϕ = frictional power loss, Btu/sec

f = coefficient of friction

W = normal load, lb

V_s = sliding velocity, ips

and the relation between T_s and ϕ may be approximated by the equation

$$T_s - T_j = C \phi^{0.80}$$

where T_j = oil jet temperature, °F

C = a fitting constant for sliding-rolling disk systems

The constant C in the above equation was found to depend quite markedly on the rate at which the oil supplied by the jet (or jets, or horn) impinges on the sliding-rolling disk surfaces and also the disk system design.

When applied to gears, the quantities V_s , W , and f generally all vary through the gear mesh, or with respect to time. Accordingly, ϕ also varies with respect to time; and it is the time-averaged ϕ that controls the quasi-steady surface temperature. In other words, the quasi-steady surface temperature of gears is expected to be governed by the equation

$$T_s - T_j = C' \phi_{av}^{0.80} \quad (69)$$

where ϕ_{av} = average frictional power loss, Btu/sec

C' = a fitting constant for gear systems

The solution to Equation (69) requires the assignment of the magnitude of the constant C' and the evaluation of the quantity ϕ_{av} . As will be seen presently, the selection of the value of C' is very difficult basically because little is known about the heat transfer processes involved in gear operation. However, the calculation of ϕ_{av} can readily be accomplished by an integration process which accounts for the variations of V_s , W , and f in the gear mesh. Depending upon the gear type, the instantaneous sliding velocity, V_s , may be deduced from kinematic analysis (Sect. A of Chaps. III, IV, V). The instantaneous static normal load, W , may be obtained from static load analysis (Sect. B of Chaps. III, IV, V). The instantaneous coefficient of friction, f , is a function of $WV_s^{-\frac{1}{3}}$ and the oil and surface characteristics (Chap. VI, Sect. C).

In Chapter VI, Section C, it was found that with AISI 9310 steel, Oils E and F gave substantially the same friction behavior. For practical gear scoring predictions, the effect of black oxide surface treatment on friction will be ignored, as suggested in Step 1. Accordingly, the relationship between f and $WV_s^{-\frac{1}{3}}$ and composite surface roughness, for carburized AISI 9310 steel with either Oil E or Oil F, may be approximated by Equations (54) and (55), or Equations (56) and (57), or Equations (60) and (61), as applicable. With honed surfaces, the use of Equations (60) and (61) presents no complication. With ground surfaces, the rather significant effect of the sliding

direction with respect to the grinding grooves should be noted. For example, with ground spur gears, the sliding motion is usually normal to the grinding grooves; thus Equations (56) and (57) for cross-ground surfaces should be used. On the other hand, with helical and spiral bevel gears, the sliding motion is usually at an angle to the grinding grooves; thus an interpolation between Equations (56) and (57) for cross-ground surfaces and Equations (54) and (55) for circumferentially ground surfaces is required. The interpolation procedure will be explained in Sections C and D of Chapter VIII, by reference to specific examples of helical and spiral bevel gears.

3. The calculation of the maximum instantaneous surface temperature rise, ΔT , through the gear mesh utilizes basically Equation (3), repeated herein for convenience:

$$\Delta T = \frac{1.11 f w \left| \sqrt{V_1} - \sqrt{V_2} \right|}{\beta \sqrt{B}} \quad (70)$$

In applying this equation, Blok's thermal coefficient, β , is a property of the gear steel (App. A). The instantaneous surface velocities, V_1 and V_2 , may be deduced from kinematic analysis. The equation applies directly to spur gears with perfect alignment, which give an instantaneous rectangular contact so that w is simply the instantaneous unit static normal load. With helical gears, the equation applies to an elemental instantaneous contact area, which may be treated as a rectangle. With spiral bevel gears, the instantaneous contact area is an ellipse; thus application of Equation (70) will require an approximation. This is customarily done, as first suggested by Kelley,¹⁸ by replacing the ellipse with an equivalent rectangle of the same major and minor widths as the ellipse and the same maximum Hertz stress at the center. In that case, the quantity w will be replaced by an "equivalent unit load," which is equal to $3w/4$.

Calculation of ϕ_{av} of Gears. In calculating the average frictional power loss, ϕ_{av} , in Equation (69), it should be borne in mind that both the sliding velocity, V_s , and the normal load, W , change cyclically through the mesh; and, by Equations (54) to (61), the coefficient of friction, f , also changes cyclically through the mesh. Consider, for example, a set of gears with a contact ratio less than 2, as illustrated in Figure 2. In the course of a mesh cycle, as any one tooth experiences its double-tooth contact in approach from point A to point C, its preceding tooth simultaneously experiences its double-tooth

contact in recess from point B to point D. Following this, the tooth which has reached point C experiences its single-tooth contact from point C to point B. After this, the next mesh cycle begins, with the tooth which is now at point B completing its double-tooth contact in recess from point B to point D, while at the same time a new tooth goes through its double-tooth contact in approach from point A to point C; and the process repeats.

The total frictional heat generated in each mesh cycle is thus

$$\begin{aligned} & \int_A^C \phi(t) dt + \int_B^D \phi(t) dt + \int_C^B \phi(t) dt \\ &= \left[\int_A^C \phi'(\epsilon) d\epsilon + \int_B^D \phi'(\epsilon) d\epsilon + \int_C^B \phi'(\epsilon) d\epsilon \right] \frac{dt}{d\epsilon} \\ &= \left[\int_A^C \phi'(\epsilon) d\epsilon + \int_B^D \phi'(\epsilon) d\epsilon + \int_C^B \phi'(\epsilon) d\epsilon \right] \frac{1}{6n_p} \end{aligned}$$

where $\phi(t)$ = instantaneous frictional power loss expressed as a function of time, Btu/sec

$\phi'(\epsilon)$ = instantaneous frictional power loss expressed as a function of pinion roll angle, Btu/sec

n_p = rotative speed of pinion, rpm

and the degrees of pinion roll angle per second is

$$\frac{d\epsilon}{dt} = \frac{360 n_p}{60} = 6n_p$$

Now, in each pinion revolution, as many mesh cycles take place as there are the number of pinion teeth. Moreover, the number of pinion revolutions per second is $n_p/60$. Therefore, the number of mesh cycles per second is equal to $N_p (n_p/60)$. Accordingly, the average frictional power loss for the gear set is

$$\phi_{av} = \left[\int_A^C \phi'(\epsilon) d\epsilon + \int_C^B \phi'(\epsilon) d\epsilon + \int_B^D \phi'(\epsilon) d\epsilon \right] \left(\frac{N_p}{6n_p} \right) \left(\frac{n_p}{60} \right)$$

or

$$\phi_{av} = \left[\int_A^C \phi'(\epsilon) d\epsilon + \int_C^B \phi'(\epsilon) d\epsilon + \int_B^D \phi'(\epsilon) d\epsilon \right] \frac{N_p}{360} \quad (71)$$

where ϕ_{av} = average frictional power loss of the gear set, Btu/sec

N_p = number of pinion teeth

Equation (71) was derived for a gear set with a contact ratio between 1 and 2. If the contact ratio is greater than 2, a similar reasoning yields

$$\phi_{av} = \left[\sum \int \phi'(\epsilon) d\epsilon \right] \frac{N_p}{360} \quad (72)$$

For a contact ratio between 2 and 3, the expression in the brackets contains 5 terms, to be integrated over the first triple-tooth contact, the first double-tooth contact, the second triple-tooth contact, the second double-tooth contact, and the third triple-tooth contact.⁴⁹ Similarly, for still higher contact ratios, the number of terms in the brackets must be correspondingly increased.

Equation (72) is generally applicable to any gear type at any contact ratio, provided it is noted that the expression for $\phi'(\epsilon)$ must consider the effect of gear type on f , W , and V_s , and provided the number of terms in the brackets is set commensurate with the contact ratio. The procedure for solving Equation (72) is therefore dependent on the gear design, as will be explained in Sections D, E, and F of this chapter, and also by reference to specific examples in Sections B, C, and D of Chapter VIII.

Selection of C' of Gears. The way Equation (69) is set up, the constant C' is the sole parameter which defines the heat removal characteristics of the gear system. The value of C' is dependent on the heat transfer processes involved, particularly conduction and convection from the meshing gear surfaces; and these processes are

expected to be influenced by gear design, system design, and operating conditions. Confident assessment of the value of C' requires a detailed analysis of the heat transfer processes; and in view of the direct dependence of $(T_s - T_j)$ on C' in Equation (69), it is clear that each case should be examined individually and "rules of thumb" are difficult to apply. Unfortunately, no rational analysis of the heat transfer behavior of gears, or of any rotating lubricated machine elements, is known to have been made to date.¹⁰ Therefore, for the purpose at hand, a tentative guideline, however crude, is required.

The basis of this tentative guideline is the variation of the constant C for sliding-rolling disk systems presented in Figure 25, to which a "correction" is applied to obtain the corresponding value of C' for gear systems. As seen in Figure 25, the value of C for a disk system is markedly influenced by the rate at which the oil supplied by the jet (or jets, or horn) impinges on the meshing surfaces, as well as the system design as it affects heat convection and conduction from the meshing surfaces. For the present purpose, the curve for SwRI disk tester B in Figure 25 will be taken as the basis for estimating purposes. It will be assumed that the value of C' for any gear system is related to the value of C for SwRI disk tester B, at the same oil flow rate, as follows:

$$C' = KC \quad (73)$$

where the factor K accounts for the difference in heat transfer behaviors between the gear system and the reference disk system. The value of K is expected to depend on the gear design, system design, and operating conditions. In general, the oil which is supplied by the jet (or jets, or horn) impinges directly on the meshing surfaces of the disks where frictional heat is generated, and thus performs the best job of removing the frictional heat by convection. This is not so with gears, because the oil jetted toward the gears usually cannot penetrate deep into the gear mesh and tends to break up or atomize due to gear rotation. Moreover, the gear type would be expected to exert a considerable influence on this behavior, with the influence being smaller for spur gears due to their "open" configuration, and greater for helical and spiral bevel gears due to their less "open" configuration.

In addition to heat convection, the effect of heat conduction along the gear shafts is also important. In the disk systems referred to in Figure 25, the disks were straddle-mounted so that there were two

paths to remove the heat by conduction from the disks. With gears that are straddle-mounted, it is clear that there are likewise two heat conduction paths. On the other hand, with overhung gears, heat conduction can take place only along one end of the shafts, so that the heat removal rate is expected to be greatly reduced.

While the general heat transfer behavior described above is believed to be qualitatively correct, the assignment of the quantitative value of K in Equation (73) for different gear types and gear mounting arrangements is of course very difficult. As said before, the authors are not aware of any available information of this kind. In the absence of such information, the following tentative values of K are assumed:

$$K = 1.5 \text{ for straddle-mounted spur gears} \quad (74)$$

$$K = 3.0 \text{ for overhung spur gears} \quad (75)$$

$$K = 2.5 \text{ for straddle-mounted helical and spiral bevel gears} \quad (76)$$

$$K = 5.0 \text{ for overhung helical and spiral bevel gears} \quad (77)$$

The K values given above are admittedly very arbitrary; but no viable and more precise alternative appears possible at this time. They serve at least as a tentative element in the overall predictive framework, until more refined solution to the problem becomes available.

C. Actual Scoring-Limited Power-Transmitting Capacity

It is clear from Equation (67) that in order to predict the actual scoring-limited power-transmitting capacity of the gear set, it is necessary to estimate the misalignment factor, S_m , and the dynamic factor, S_d . Confident assessment of S_m and S_d is, as indicated in Chapters II to V, exceedingly difficult. Empirical correlations are likewise difficult, because even if P_A for a gear set is known by test and the corresponding P_I is obtained from the preceding analysis, one only knows the magnitude of the product $S_m S_d$, but not the individual magnitudes of S_m and S_d . An attempt will be made presently to deduce the probable magnitudes of these two factors, based upon some spur gear scoring test results furnished by AGMA. However, before doing so, an overview of the problem appears in order.

Misalignment Factor. The effect of angular misalignment on

the strength-related failures of spur gear teeth was first examined by Van Zandt,⁷³ then applied to helical gears by Wellauer,⁷⁴ and later adopted for use in the AGMA strength design standards for spur⁴ and helical gears.⁵ The effect of tooth misalignment on the scoring-limited power-transmitting capacity of spur gears was emphasized by Kelley and Lemanski.²² The procedure of calculation has been outlined in References 4 and 5.

Using the above-mentioned procedure, the effect of angular misalignment on the value of S_m of spur and helical gears has been calculated and is presented in Figure 28. In this figure, w_t denotes a fictitious unit tangential tooth load, which gives a measure of the so-called load-carrying capacity; e is the angular misalignment; and F is the effective face width. Note that at any value of w_t and F , S_m reduces markedly with increasing e . Note further that, due to elastic deformation across the tooth face, the misalignment effect is substantially reduced with narrow gears and with increased torque or w_t . These latter effects are illustrated in Figure 29 for an angular misalignment of 0.001 rad.

The misalignment problem is considerably more difficult to handle for spiral bevel gears, mainly because the gear surfaces are curved three-dimensionally and therefore misalignment will produce a very complex tooth contact condition. The problem was discussed by Coleman,⁷⁵ who stated that aircraft spiral bevel gears frequently require a load distribution factor of 1.4 or more. In the AGMA strength design standard for spiral bevel gears,⁶ the load distribution factor is taken as 1.10 to 1.40 for straddle-mounted aircraft-type spiral bevel gears, and 1.25 to 1.50 for overhung-mounted aircraft-type spiral bevel gears. The misalignment factor, S_m , as employed herein, is the reciprocal of the AGMA load distribution factor. Therefore, accepting the AGMA values as being representative, the misalignment factor, S_m , for aircraft-type spiral bevel gears is then in the range of 0.71 to 0.91 if straddle-mounted, and in the range of 0.67 to 0.80 if overhung-mounted.

It is seen that the effect of tooth misalignment in reducing the scoring-limited power-transmitting capacity from the ideal case of perfect alignment is very powerful. A particularly disturbing aspect of the problem is that the misalignment in actual gears can be caused by manufacturing errors, tolerance stackup in the assembly process, support bearing misalignment, shaft deflections under load, and differential thermal expansions of the housing and related components. The situation is so involved, particularly with the more complex gear

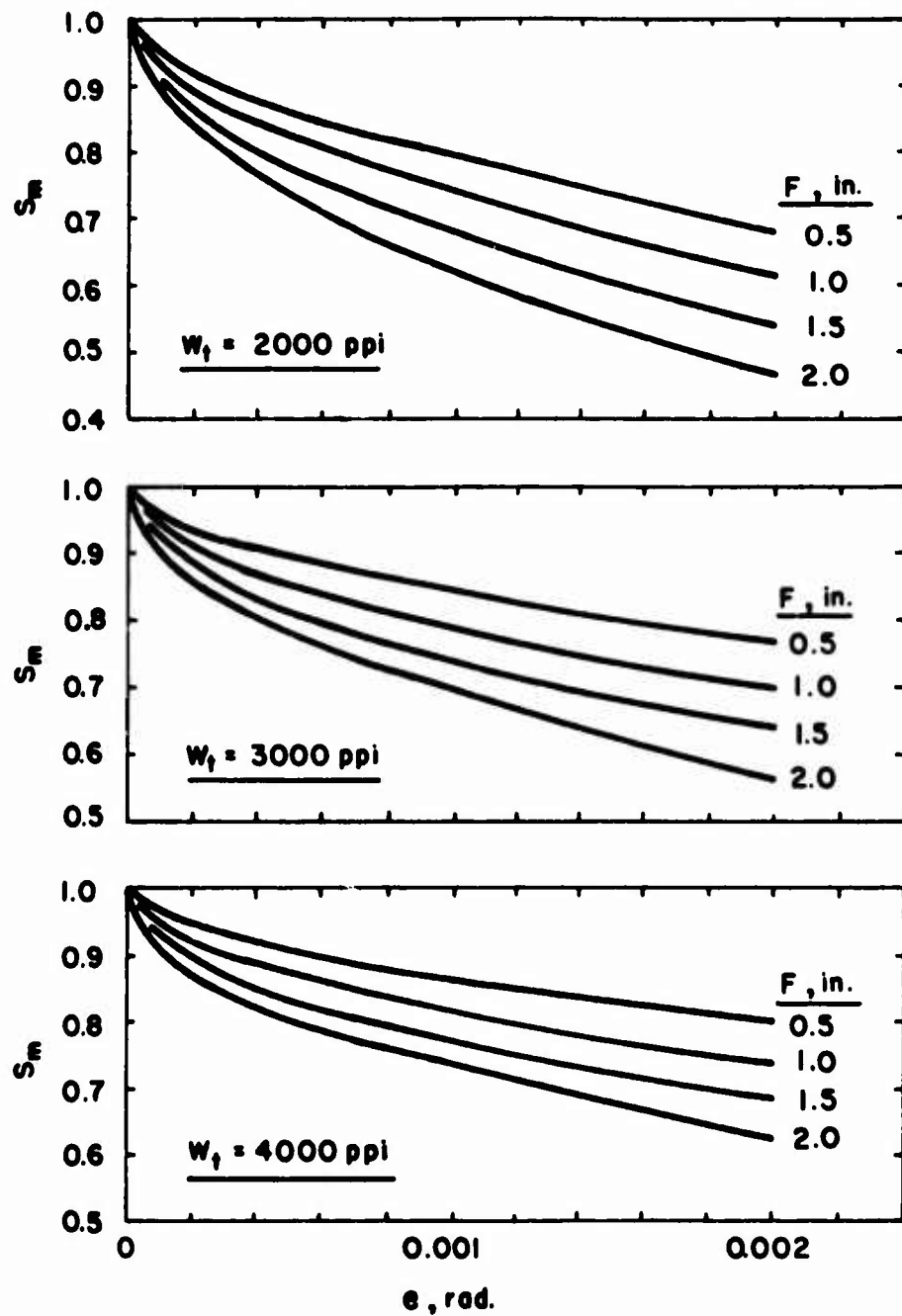


Figure 28. Misalignment factor for spur and helical gears vs. angular misalignment

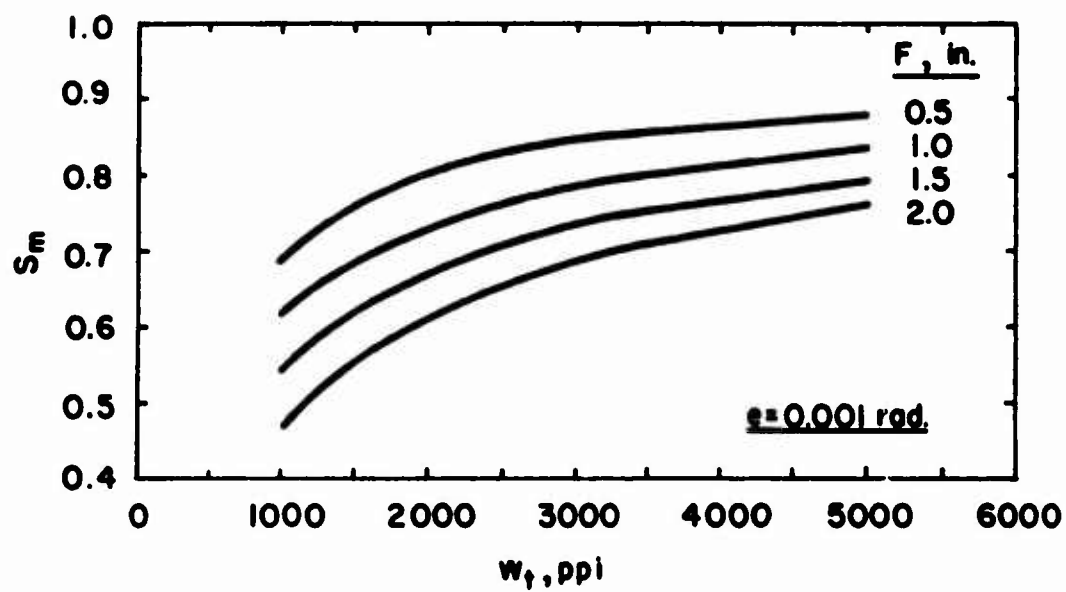


Figure 29. Misalignment factor for spur and helical gears at constant angular misalignment

types, that the net value of e cannot be realistically predicted or controlled in the design process. Measurement of e in an assembled gear set requires extreme accuracy, because S_m is so sensitive to e , especially in the range of low e which is of major interest. Finally, examination of the "contact pattern" across the tooth face can be quite misleading, because "full contact" can be expected unless gross misalignment is present. For example, with spur and helical gears, "full contact" would prevail for any value of S_m of 0.5 or greater, regardless of the combination of F and w_t . In other words, "full contact" would be observed for all values of F up to 1.5 in. shown in Figure 28; but yet the misalignment may be very much in excess of 0.002 rad.

For aircraft power gears, which are designed, manufactured, and assembled with care, it is felt that an angular misalignment of 0.001 rad. may very likely be a realistic optimal limit—an amount significant enough in reducing the power-transmitting capacity from perfect alignment; but small enough so that it could easily escape notice in practice. Thus, in the absence of specific information to the contrary, it is believed that $e = 0.001$ rad. is probably a fairly reasonable value to use in aircraft-type spur and helical gear design, and Figure 29 can be used to estimate the value of S_m . For aircraft-type spiral bevel gears, an assumed value of S_m of 0.75 for straddle-mounted gears, or 0.70 for overhung gears, is believed to be realistically optimal. If the actual value of S_m should be found to be substantially lower than these target values, more stringent control in design, manufacture, and assembly would appear warranted.

Dynamic Factor. It should be clear from Chapter II, Section F, and Chapters III to V that the dynamic factor is another extremely difficult quantity to handle, both theoretically and practically. However, its pronounced effect in reducing the actual gear performance from ideal can readily be seen from Figure 30.

Figure 30 plots the dynamic factor, S_d , as a function of the pitchline velocity, V_t . The horizontal line designated as K_v1 and the curves designated as K_v2 and K_v3 are taken from the AGMA strength design standard for spur gears.⁴ The AGMA strength design standards for helical gears⁵ and spiral bevel gears⁶ use the same K_v1 and K_v2 ; but not K_v3 due essentially to the greater degree of load sharing and hence smoother load transfer in these gear types.

The equations for these AGMA K_v values were given in Chapter III, Section C, but are repeated below for convenience:

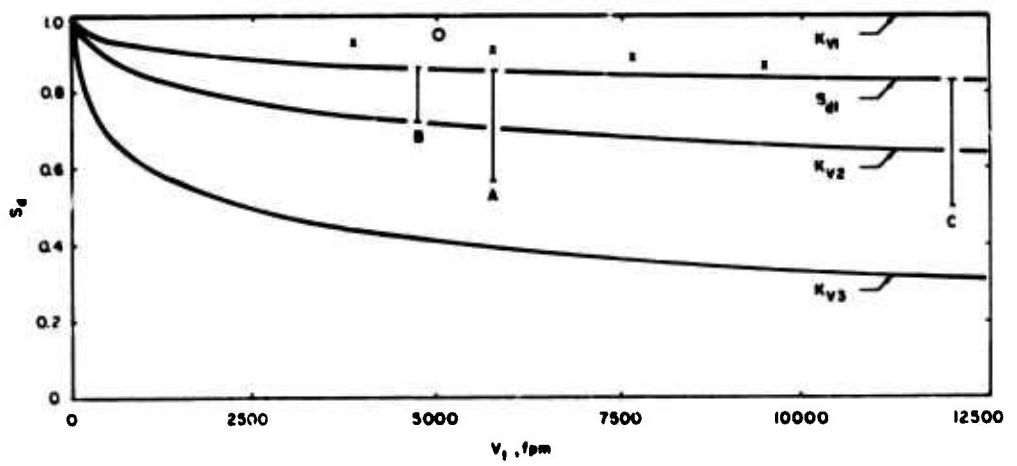


Figure 30. Dynamic factor vs. pitchline velocity

$$K_{v1} = 1 \quad (78)$$

$$K_{v2} = \sqrt{\frac{78}{78 + \sqrt{V_t}}} \quad (79)$$

$$K_{v3} = \frac{50}{50 + \sqrt{V_t}} \quad (80)$$

These equations are empirical and do not explicitly account for the effects of gear and system design, manufacturing errors, and operating variables as discussed, for example, in Chapter III, Section C. It is clear that K_{v1} is in reality a definition of the static load, which is not strictly achievable even for "high precision" gears in theory or practice. At the other extreme, K_{v3} would appear to be rather pessimistic for aircraft-quality spur gears and too pessimistic for aircraft-quality helical and spur gears. In order to provide some measure of flexibility for the purposes of design and performance analysis when no reliable estimate of the system dynamic behavior is available, another empirical dynamic factor, defined as

$$S_{d1} = \frac{K_{v1} + K_{v2}}{2} \quad (81)$$

has been found to be quite helpful.

Figure 30 compares the four curves with the dynamic factors calculated by Tuplin's method (Sect. C, Chaps. III and IV) for an aircraft-type spur gear set at four pitchline velocities (shown by the crosses) and for an aircraft-type helical gear set at one pitchline velocity (shown by a circle), as well as the dynamic factors deduced from the test results for several aircraft-type spur gear sets furnished by AGMA (shown by vertical bars A, B, and C). The deduction of the AGMA data will be discussed presently. It is seen that, if these data are indicative of what may happen in practice, then K_{v2} appears to be a fairly typical average for aircraft-type spur gears, with an uncertainty range as high as S_{d1} and as low as halfway between K_{v2} and K_{v3} .

In view of the enormous complexity of the problem and a lack of other specific information at present, it is suggested that K_{v2} be taken as a reasonable dynamic factor for aircraft spur gears, and S_{d1} as a reasonable dynamic factor for aircraft helical and spiral bevel gears, provided no unusual system dynamic stimulus is present. If substantially lower dynamic factors are suspected in practice, a review of the details of design, manufacture, and system dynamics would seem to be indicated.

AGMA Spur Gear Test Results. As an aid to evaluating the scoring-limited performance of typical aircraft power gears, 13 sets of full-scale spur gear scoring test results were supplied to this program by the Tribology Division, AGMA Aerospace Gearing Committee. Only 5 sets of such data, from tests employing AISI 9310 steel gears and MIL-L-7808 or MIL-L-23699 oils that went far enough to reach scoring, were analyzed and made use of herein in an attempt to deduce the probable values of the misalignment and dynamic factors.

A description of the five AGMA test series, their experimentally determined P_A , and their estimated P_I by the procedure outlined in the preceding section, are detailed in Appendix E. It will be noted that, in each cited case, only one test was available for scoring analysis. Since there is no statistical basis for a single test, it was not thought necessary to estimate the P_I for each case by a full computer program. Rather, making use of the AGMA computer printouts, their results were converted by simple mathematical manipulations, which are identical to the procedure outlined in the preceding section in all respects, except by assuming a different but constant f in each case. The assumption of a constant f for each test is not a drastic one, because scoring generally occurs in the fairly flat region of the f vs. $WV_s^{-\frac{1}{3}}$ curve (Fig. 20). But this assumption greatly simplifies the computations, so that they could easily be handled manually. These rather simple manipulations are explained in detail in Appendix E.

Table E-7 compares the actual and ideal performance of the cases examined. The quantity of special interest here is the ratio of P_A to P_I , which, from Equation (67), is the product of S_m and S_d in each case.

In an effort to estimate the individual values of S_m and S_d for each case when their product is known, it is necessary to assume some value for S_m or S_d and examine whether the resulting S_d or S_m would appear plausible. The process is then repeated until plausible answers are obtained. Table 6 summarizes the results of this type of analysis.

**TABLE 6. ESTIMATES FOR MISALIGNMENT AND DYNAMIC
FACTORS FOR AGMA SPUR GEAR TESTS**

	<u>Series B</u> <u>(Test 272)</u>	<u>Series B</u> <u>(Test 273)</u>	<u>Series A1</u> <u>(Test 87)</u>	<u>Series A2</u> <u>(Test 118)</u>	<u>Series A3</u> <u>(Test 110)</u>	<u>Series C</u> <u>(Test 10)</u>
F, in.	1.55	1.55	0.50	0.50	0.50	0.25
δ_i , μ in. AA	15.0	15.0	12.8	30.0	39.0	18.0
V _t , fpm	4760	4760	5749	5749	5749	11968
w _t , ppi	2308	3078	4160	2398	1956	2800
S _m S _d	0.71	0.76	0.37	0.32	0.34	0.31
<u>Assume S_d = S_{d1}</u>						
S _d	0.87	0.87	0.86	0.86	0.86	0.83
S _m	0.82	0.87	0.43	0.37	0.40	0.37
e, rad.	0.0003	0.0002	0.0104	0.0073	0.0054	0.0120
<u>Assume S_d = K_{v2}</u>						
S _d	0.72	(0.76)*	0.71	0.71	0.71	0.64
S _m	0.99	1.00	0.52	0.45	0.48	0.48
e, rad.	~0	0	0.0077	0.0057	0.0042	0.0075
<u>Assume e = 0.005 rad.</u>						
S _m	-	-	0.65	0.50	0.45	0.62
S _d	-	-	0.57	0.64	0.76	0.50

* In this case, a maximum value of S_m = 1.00 is assumed. Thus S_d = 0.76, which is greater than K_{v2} = 0.72.

Table 6 lists the effective face width, F ; initial composite surface roughness, δ_i ; pitchline velocity, V_t ; unit tangential load at scoring, w_t ; and $S_m S_d$ product for the six tests in which scoring was obtained. The values of the $S_m S_d$ product are taken directly from Table E-7 for Series A1, A2, A3, and C. For each of the two tests in Series B, the two $S_m S_d$ products deduced for two assumed oil jet temperatures in Table E-7 are averaged to give a single value listed in Table 6.

Let it first be assumed that the dynamic factor, S_d , in each test is as high as S_{d1} at that particular V_t . Figure 30 or Equation (81) then gives the tabulated value of S_d for the test. Since $S_m = (S_m S_d / S_d)$, then the corresponding value of S_m for the test can be calculated. Knowing this S_m and given F and w_t , the appropriate value of e for the test can be read off Figure 28 by interpolation, or calculated by the AGMA method⁴ if e is greater than 0.002 rad. Note from Table 6 that the values of e for the two Series B tests are very small, indicating good alignment in these tests. On the other hand, the values of e for the other four tests appear to be excessive. This is because at values of S_m less than 0.5, less than "full contact" across the tooth face would be expected, and this condition would probably have been noticed by an alert test operator.

Let it now be assumed that the dynamic factor, S_d , in these tests is equal to K_v2 ; then similar calculations will yield the values of S_m and e shown next in Table 6. It is seen that Test B272 would indicate almost perfect tooth alignment. As to Test B273, even the assumption of a perfect tooth alignment would yield $S_d = 0.76/1.00 = 0.76$, which is greater than the assumed $S_d = 0.72$ for Test B272. If it is then assumed that Test B272 also has $S_d = 0.76$, the corresponding misalignment factor would be $S_m = 0.71/0.76 = 0.93$, which gives $e = 0.0001$ rad., or still an extremely good tooth alignment.

The assumption of $S_d = K_v2$ gives significantly more plausible results for the Series A and Series C tests. Note from Table 6 that the values of e are considerably reduced as compared with the case of $S_d = S_{d1}$. Further, the values of S_m are now all close enough to 0.5 so that "full contact" across the tooth face might have resulted, and the presence of the misalignment could have escaped notice.

Finally, if it is assumed that $e = 0.005$ rad. in the Series A and Series C tests, then the resulting values of S_m , and the corresponding values of S_d , would be as shown at the bottom of Table 6. These S_m values appear, on the whole, to be even more plausible than those given

by the assumption of $S_d = K_v2$. However, with the assumed $e = 0.005$ rad., the S_d values would all be less than K_v2 .

In reviewing the various values of S_m and S_d presented above, it should be kept in mind that only single tests are involved, so the results should be interpreted with caution. It would appear that perfect tooth alignment would be more likely accidental than realistically achievable; but a misalignment in excess of, say, 0.005 rad. should have been detected by alert test personnel. Thus, selecting only those values of S_d in Table 6 that lie within these misalignment limits, then the probable range of S_d for these tests would be as shown by the vertical bars in Figure 30.

In other words, depending upon what sort of tooth misalignment was assumed for these AGMA spur gear tests, the dynamic factor, S_d , could be as portrayed by the vertical bars shown in Figure 30, giving a rather good average corresponding to K_v2 but with a large range of uncertainty. This exercise shows clearly that more definitive data on both misalignment and dynamic effects are urgently needed. It also shows that while both factors are important, poor misalignment can easily mask the probable effect of the dynamic load.

In the prediction of the actual scoring-limited power-transmitting capacity of gears, careful assessment of the probable misalignment and dynamic effects is desirable but obviously not easy. In the absence of specific information, performance predictions based on an assumed angular misalignment of 0.001 rad., and an assumed dynamic factor equal to K_v2 for spur gears or S_d1 for helical and spiral bevel gears, would appear to be reasonable, though perhaps somewhat optimistic, for aircraft power gear practice.

D. Spur Gear Scoring Prediction

The preceding sections have dealt with the prediction of the ideal and actual scoring-limited power-transmitting capacities of gears in detail, except for those items related to specific gear types. This section will be concerned with those aspects of the predictive procedure dealing specifically with spur gears. The next two sections will deal with helical and spiral bevel gears.

Instantaneous Coefficient of Friction. The quantity f is required in calculating T_s and ΔT , and thus in the prediction of the ideal scoring-limited power-transmitting capacity. As noted in Section B of this chapter, the selection of the proper equations for f for spur gears is

straightforward. Specific examples will be given in Section B of the next chapter.

Quasi-Steady Surface Temperature. The quantity T_s is calculated by Equation (69), and the procedures for calculating ϕ_{av} and assigning the value of the constant C' have been explained in Section B of this chapter.

In calculating ϕ_{av} by Equation (72), note that the instantaneous frictional power loss, ϕ' , expressed as a function of the pinion roll angle, ϵ , is given in the computer printout (App. H). With this information, Equation (72) may be solved graphically by plotting ϕ' vs. ϵ and measuring the areas under the curve. However, numerical integration by the computer is by far the easier, as will be explained in Section B of the next chapter. A specific example will be given in Appendix K.

Maximum Instantaneous Surface Temperature Rise. The quantity ΔT is also required in the prediction of the ideal scoring-limited power-transmitting capacity. The basic equation for ΔT is given by Equation (70). For spur gears,

$$B = (32 w R / \pi E^*)^{\frac{1}{2}}$$

$$V_1 = 2\pi \rho_p n_p / 60$$

$$V_2 = 2\pi \rho_g n_g / 60$$

Substituting the above expressions into Equation (70), and taking $E^* = 33 \times 10^6$ psi, one obtains

$$\Delta T = \frac{15.23 f w^{\frac{3}{4}} \left| \sqrt{\rho_p n_p} - \sqrt{\rho_g n_g} \right|}{\beta R^{\frac{1}{4}}} \quad (82)$$

where β is given in Appendix A for AISI 9310 steel.

Critical Temperature. The quantity T_{cr} is required to establish the scoring condition and obtain the ideal scoring-limited power-transmitting capacity. This quantity was dealt with in Section B of this chapter.

Misalignment and Dynamic Factors. The misalignment factor, S_m , and dynamic factor, S_d , are required in the prediction of the actual scoring-limited power-transmitting capacity by Equation (67). If no other specific information is available at the design stage, it is recommended that S_m be based on an assumed angular misalignment of 0.001 rad., and S_d be assumed to be equal to K_v2 .

E. Helical Gear Scoring Prediction

The prediction of the scoring-limited power-transmitting capacity of helical gears is basically similar to that of spur gears. However, on account of the high contact ratios normally used in helical gears, the load sharing problem is far more complex (Chap. IV, Sect. B). The customary, approximate way to handle the problem has been illustrated in Figure 7. At any instant in the mesh cycle, several pairs of teeth are sharing the total normal tooth load; and the fraction of this load carried by any tooth pair is assumed to be proportional to the ratio of the length of the line of contact on that tooth pair to the total length of lines of contact on all contacting tooth pairs. This is tantamount to assuming that the instantaneous unit static normal load at any point on all lines of contact of all simultaneously contacting teeth at that instant is constant and equal to

$$w = W/L \quad (83)$$

and the instantaneous static normal load carried by any elemental gear tooth with midpoint at M (Fig. 7) is

$$W_i = wl = Wl/L \quad (84)$$

where w = instantaneous unit static load, ppi

W = total static normal load, lb

W_i = instantaneous static normal load on an elemental tooth, lb

L = total length of instantaneous lines of contact on all simultaneously contacting tooth pairs, in.

l = length of line of contact on an elemental tooth, in.

Note that although w is constant at any given instant in the mesh, it is not constant throughout the mesh cycle because L is not constant throughout the mesh cycle. Also, since l is chosen to be an integral divider of the length of the particular instantaneous line of contact on which it lies, it is generally not the same on all simultaneous lines of contact. Accordingly, W_i is generally not constant at any instant spatially and not constant through the mesh cycle.

Instantaneous Coefficient of Friction. As noted in Section B of this chapter, the sliding motion in the helical gear mesh is inclined at an angle to the orientation of the grinding grooves. This orientation effect is taken as a function of the helix angle, and should be accounted for in writing the equations for f . A specific example will be given in Section C of the next chapter.

Quasi-Steady Surface Temperature. The quantity T_s is calculated by Equation (69), and the procedure for assigning the value of the constant C' needs no further comment. The quantity ϕ_{av} is calculated basically by Equation (72) which is general for all gear types. However, some additional manipulations are required due to the manner in which the helical gear analysis is made.

In calculating ϕ_{av} for helical gears, it is first necessary to calculate the elemental contribution $\Delta\phi = \Delta(f W V_s)$ at any arbitrary point M on an instantaneous line of contact (Fig. 7), and to obtain the total instantaneous ϕ along this line by summing all of the elemental contributions over this line. This summation is comparable to ϕ' in Equation (72). If this summation were expressed as a function of the pinion roll angle at the line of contact under consideration, then the successive values of $\phi'(\epsilon)$ obtained as contact progressed over the plane of action could be applied directly to Equation (72).

However, for the sake of convenience, the computer program (App. I) is written with successive lines of contact spaced equal distances apart as a function of their distance f from the initial contact point A , as shown in Figure 7. Since ϕ is thus a function of the linear distance f and not the pinion roll angle ϵ , Equation (72) must be transformed to read

$$\phi_{av} = \left[\frac{\int \phi'(f) df}{f_{D'}} \right] \Delta\epsilon \left(\frac{N_p}{360} \right) \quad (85)$$

where $\phi''(f)$ = instantaneous frictional power loss expressed as a function of parameter f , Btu/sec

$f_{D'}$ = normal distance from the first point of contact A to the final point of contact D' (Fig. 7)

$\Delta\epsilon$ = angle the pinion turns through from the first point of contact A to the final point of contact D', deg

N_p = number of pinion teeth

Note that the integration is performed over the distance f , normal to the instantaneous lines of contact. The bracketed term in Equation (85) represents the average ϕ'' over the plane of action.

The value of $\Delta\epsilon$ is found from the length of the path of contact in the transverse plane as

$$\Delta\epsilon = \frac{2Z}{d_b} \cdot \frac{180}{\pi} \quad (86)$$

where Z = length of path of contact in transverse plane, in.

d_b = base diameter of pinion, in.

The ratio $180/\pi$ converts the expression to angular degrees.

Maximum Instantaneous Surface Temperature Rise. The quantity ΔT at any point M in Figure 7 is obtained by substituting W_i/l for w in Equation (82), thus:

$$\Delta T = \frac{15.23 f (W_i/l)^{\frac{3}{4}} \left| \sqrt{\rho_p n_p} - \sqrt{\rho_g n_g} \right|}{\beta R^{\frac{1}{4}}} \quad (87)$$

where all quantities except β , n_p and n_g are the instantaneous values at point M.

Critical Temperature. The quantity T_{cr} was dealt with in Section B of this chapter.

Misalignment and Dynamic Factors. If no other specific information is available at the design stage, it is recommended that the misalignment factor, S_m , be based on an assumed angular misalignment of 0.001 rad.; and the dynamic factor, S_d , be assumed to be equal to S_{d1} .

F. Spiral Bevel Gear Scoring Predictions

The prediction of the scoring-limited power-transmitting capacity of spiral bevel gears is similar in principle to that of helical gears, but with the added complications of the cross-axes arrangement and the varying spiral angle. For the present purpose, the approximate kinematic and static load analyses, due largely to Coleman^{66, 67} and briefly covered in Chapter V, will be adapted for use.

Instantaneous Coefficient of Friction. As in the case of helical gears, the sliding motion in the spiral bevel gear mesh is also inclined at an angle to the orientation of the grinding grooves. This orientation effect is taken as a function of the spiral angle, and should be accounted for in writing the equations for f . A specific example will be given in Section D of the next chapter.

Quasi-Steady Surface Temperature. The calculation of T_s for spiral bevel gears is similar to that for helical gears, except that the quantity ϕ'' for any instantaneous line of contact is directly calculated (App. J), and a summation along the line is thus not required.

The contact condition in a pair of spiral bevel gears is shown in Figure 10. Motion sweeps across the plane of action in the axial as well as the transverse directions, much as in the helical gear. However, unlike the uniform contact assumed along a contact line in helical gears, the contact in spiral bevel gears is assumed concentrated at the middle of the contact line. Thus to estimate T_s , it is only necessary to integrate over the diagonal length of action, η , the instantaneous values of ϕ'' for a large number of contact lines uniformly spaced over the contact ellipse in the plane of action.

Using an expression similar to Equation (85), ϕ_{av} is then

$$\phi_{av} = \left[\frac{\int \phi''(f) df}{\eta} \right] \Delta \epsilon \left(\frac{N_p}{360} \right) \quad (88)$$

where $\phi''(f)$ = instantaneous frictional power loss expressed as a function of parameter f , Btu/sec

f = distance from the center of contact area to a line of contact (Fig. 10), in.

η = length of contact normal to lines of contact, in.

$\Delta\epsilon$ = angle through which the pinion turns from initial contact to final contact, deg.

N_p = number of pinion teeth

The value of $\Delta\epsilon$ in this case is

$$\Delta\epsilon = \frac{Z}{A \sin \gamma} \cdot \frac{180}{\pi} \quad (89)$$

where Z = length of contact in transverse plane, in.

A = mean cone distance, in.

γ = pitch angle of pinion, deg.

Maximum Instantaneous Surface Temperature Rise. In Coleman's analysis,⁶⁶ the quantity ΔT at the midpoint on any instantaneous line of contact (Fig. 10) was expressed in terms of the maximum Hertz stress, q_0 , at that point. He used basically Kelley's approximation¹⁸ to enter Equation (70). Coleman's expression for ΔT may then be shown to be

$$\Delta T = \frac{\sqrt{\pi} f q_0 V_s}{2 \beta \left(\sqrt{V_p/d_p} + \sqrt{V_g/d_g} \right)} \quad (90)$$

with

$$q_0 = C_p \sqrt{\frac{W_i}{SGR}} \quad (91)$$

and

$$C_p = \sqrt{\frac{3 E^*}{4 \pi}} \quad (92)$$

Substituting Equations (91) and (92) into Equation (90), and taking $E^* = 33 \times 10^6$ psi, one obtains

$$\Delta T = \frac{2487 f \sqrt{W_j} V_s}{\beta \sqrt{SGR} \left(\sqrt{V_p/d_p} + \sqrt{V_g/d_g} \right)} \quad (93)$$

In Equations (90) and (93), the kinematic relationships have been dealt with in Chapter V, Section A, while the static load relationships have been dealt with in Chapter V, Section B. Thus, V_s is given by Equation (36), and d_p and d_g by Equations (38) and (39), respectively. V_p and V_g , not previously given, are

$$V_p = \sqrt{V_{Fp}^2 + V_{Pp}^2} \quad (94)$$

$$V_g = \sqrt{V_{Fg}^2 + V_{Pg}^2} \quad (95)$$

where V_{Fp} , V_{Pp} , V_{Fg} , and V_{Pg} are given by Equations (32), (33), (34), and (35), respectively. SG is given by Equation (43). W_j is given by Equation (45). Note that except for β and E^* , which are properties of the gear steel, all other quantities entering Equation (90) or Equation (93) are not constant either spatially or through the mesh cycle.

Critical Temperature. The quantity T_{cr} was dealt with in Section B of this chapter.

Misalignment and Dynamic Factors. If no other specific information is available at the design stage, it is recommended that the misalignment factor, S_m , be taken as 0.75 for straddle-mounted spiral bevel gears and 0.70 for overhung spiral bevel gears, and that the dynamic factor, S_d , be assumed to be equal to S_{d1} .

CHAPTER VIII

GEAR SCORING TEST PROGRAM

A. General

In order to evaluate the validity of the gear scoring prediction method outlined in the preceding chapter, the plan was to select or design typical aircraft-type spur, helical, and spiral bevel gears, and to predict their probable scoring-limited power-transmitting capacities under specified operating conditions. Concurrently, these selected or designed gears were to be procured or manufactured, and then tested under the specified operating conditions to determine their actual scoring-limited power-transmitting capacities. The predicted and the experimentally-determined values were then to be compared.

Recommendations on the specific gear designs and test plans were made with the aid of Bell Helicopter Company (subcontractor) and submitted to the Eustis Directorate, U.S. Army Air Mobility Research and Development Laboratory for prior approval. After USAAMRDL approval was received, the procurement and manufacture of the gears were handled by BHC. Testing of these gears was subsequently performed by BHC under SwRI supervision.

The gear test program originally called for a total of 30 tests, comprising 10 tests each on spur, helical, and spiral bevel gears. The 10 tests on each gear type were further divided into two sets of 5 tests each, with some design feature or test condition varied. All gears were to be made of AISI 9310 CEVM steel, carburized to give an effective case thickness of 0.030-0.040 in., a case hardness of 60-63 R_C, a core hardness of 33-41 R_C, and surface-treated with black oxide as per BHC Specification BPSFW 4084—essentially the same as the test disks used in the disk tests (Chap. VI and App. D). The test oil was to be Oil F (App. A), the same MIL-L-7808G oil used in the disk tests.

The other gear and test details will be given later in Sections B, C, and D of this chapter and in Appendixes F and G. However, the gear test program had undergone certain changes as it developed, and these changes will now be reviewed.

Spur Gears. All spur gears were to be 31 x 76 teeth, with a diametral pitch of 8.5 in.⁻¹ Five sets of these gears were to be ground to about 17 μ in. AA surface finish and 5 sets honed to about 7 μ in. AA surface finish. All tests were to be run at a pinion speed of 8,000 rpm, at an oil jet temperature of 190°F.

As it turned out, the gears received from the vendor were found to be much smoother than specified; but it was decided to proceed with the testing in order to expedite the program. It was also found that the torque capacity of the test rig available at BHC was not enough to score the gears at 190°F oil jet temperature and 8,000 rpm pinion speed. It was felt that increasing the test speed at maximum rig torque capacity was risky. Accordingly, the test plan was modified to test all spur gears at an oil jet temperature of 250°F.

Helical Gears. The helical gears were to be 31 x 138 teeth, with a diametral pitch of 8.5 in.⁻¹ and a helix angle of 18.3°. Five sets of these gears were to be tested at 5,000 rpm, and 5 sets at 25,000 rpm. The oil jet temperature was to be 190°F.

As it happened, manufacturing problems were encountered by the vendor, and delivery of the helical gears was repeatedly delayed. It became necessary to modify the program to delete testing of the helical gears entirely.

Spiral Bevel Gears. It was planned to test 5 sets each of two spiral bevel gear designs. One design was to be 22 x 23 teeth, with a diametral pitch of 6.11 in.⁻¹, and a spiral angle of 35°. These 5 sets were duly tested at the planned pinion speed of 4,500 rpm and the planned oil jet temperature of 190°F.

The other design was to be 19 x 62 teeth, with a diametral pitch of 6.33 in.⁻¹, and a spiral angle of 35°. These 5 sets were to be tested at a pinion speed of 6,000 rpm and an oil jet temperature of 190°F. Due to scheduling problems of the available test rig at BHC, it was not possible to conduct these tests expeditiously. Accordingly, the program was modified to delete the 5 tests on the 19 x 62 spiral bevel gears.

In summary, due to various difficulties encountered, the originally scheduled 30 gear tests were not all run. Rather, only 15 tests were performed, 5 each on the ground spur gears, the honed spur gears, and the 22 x 23 spiral bevel gears.

The subsequent sections of this chapter will first present the predicted scoring-limited power-transmitting capacities of the gear sets, and then compare these with the experimentally-determined values. For the sake of convenience, sample runs of the computer programs are presented in Appendixes H, I, and J for the three gear types. Likewise, summaries of the spur gear and spiral bevel gear

test data are presented in Appendixes F and G, respectively.

B. Spur Gear Test Program

The spur gear test program consisted of replicate tests on 5 sets of ground spur gears and 5 sets of honed spur gears of same design and material, tested for scoring at the same speed and oil jet temperature. Details of the test equipment, test procedure, and results are summarized in Appendix F.

The test gears were made of AISI 9310 CEVM steel, carburized to a case thickness of 0.030-0.040 in., a case hardness of 60-63 Rc, a core hardness of 33-41 Rc, and surface-treated with a black oxide. The dimensions of the pinions and gears are given in Table 7. The measured surface roughnesses in the profile and lead directions of the pinions and gears are presented in Appendix F, where it is estimated that the average initial composite surface, δ_i , was 13.7 μ in. AA for the 5 sets of ground gears, and 15.8 μ in. AA for the 5 sets of honed gears.

The test oil was Oil F, a MIL-L-7808G synthetic oil.

The tests were conducted at a pinion speed of 8,000 rpm, with the pinion as the driver. The corresponding pitchline velocity was 7,638 fpm. The gears were mounted on vertical shafts, and were lubricated by cascading oil and by an oil jet directed into the mesh. Flow through the oil jet was 0.28 gpm and the oil jet temperature was 250°F. The cascading oil flow rate and temperature were not measured.

Dimensional inspection of the test section housing and bearings revealed that the test rig had an assembled misalignment of 0.0007 rad., with the lower end of the teeth (i.e., the S/N end in Fig. F-2) being more heavily loaded than the upper end. Apart from this, tooth misalignment also resulted from the lead errors on the pinion and gear teeth. An attempt was made to balance the lead errors on the mating pairs of pinions and gears. As shown in Appendix F, the best estimate for the average resultant tooth misalignment was 0.00076 rad. for the ground gear sets, and 0.00084 rad. for the honed gear sets.

Ideal Scoring Power of Ground Spur Gears. The ideal scoring limited power-transmitting capacity of the ground spur gears was estimated by the procedure outlined in Chapter VII. For reasons given in Chapter VII, the critical temperature, T_{cr} , and the coefficient of

TABLE 7. SPUR GEAR DESIGN DATA

	<u>Pinion</u> <u>(Driver)</u>	<u>Gear</u> <u>(Driven)</u>
Number of teeth	31	76
Diametral pitch, in. ⁻¹	8.5	8.5
Pitch diameter, in.	3.6471	8.9412
Face width, in.	1.375	1.250
Pressure angle, deg	22.0	22.0
Outside diameter, in.	3.907	9.184
Root diameter, in.	3.354	8.632
Mean circular tooth thickness, in.	0.1848	0.1768
Start of tip modification, deg roll	27.49	25.00
End of tip modification, deg roll	32.15	26.89
Nominal slope of tip modification line, in.	-0.00035	-0.00045
Nominal slope of profile slope line, in.	0	0
Nominal slope of tooth slope line, in.	0	0
Maximum allowable errors:		
Tooth to tooth spacing, in.	0.0002	0.0002
Accumulated spacing, in.	0.0006	0.0006
Slope of tip modification line, in.	±0.00015	±0.00015
Slope of profile line, in.	±0.0001	±0.0001
Slope of tooth slope line, in.	±0.00025	±0.00025
Material and surface:		
Material, AISI	9310	9310
Case thickness, in.	0.030-0.040	0.030-0.040
Case hardness, R _C	60-63	60-63
Core hardness, R _C	33-41	33-41
Surface finish, μin. AA	See text	See text

friction, f , are assumed to be those of the plain surfaces, i.e., surfaces as if there were no black oxide surface treatment. In other words, noting that $\delta_i = 13.7 \mu\text{in. AA}$ for the ground spur gears, then the critical temperature, from Equation (50), is

$$T_{cr} = 540 - 3.80 \delta_i = 488^\circ\text{F} \quad (96)$$

and the coefficient of friction, from Equations (56) and (57), is

$$\begin{aligned} f &= (0.0755 + 0.00034 \delta_i)(WV_s^{-\frac{1}{3}})^{-0.3} \\ &= 0.0802 (WV_s^{-\frac{1}{3}})^{-0.3} \\ &\quad \text{at } WV_s^{-\frac{1}{3}} < 200 \end{aligned} \quad (97)$$

$$\begin{aligned} f &= 0.0154 + 0.00007 \delta_i = 0.0164 \\ &\quad \text{at } WV_s^{-\frac{1}{3}} \geq 200 \end{aligned} \quad (98)$$

The quasi-steady surface temperature, T_s , is given by Equation (69), which at $T_j = 250^\circ\text{F}$ reads

$$T_s = T_j + C' \phi_{av}^{0.80} = 250 + 225 \phi_{av}^{0.80} \quad (99)$$

where ϕ_{av} is defined by Equation (72). In estimating the value of C' in Equation (99), the cascading oil is assumed to contribute negligibly toward the cooling of the gear meshing surfaces, since this oil from the upper support bearings is immediately flung off as it falls on the top sides of the gears, and thus has little chance of entering the gear mesh. Accordingly, at an oil jet flow rate of 0.28 gpm, Figure 25 gives $C = 150$; and $C' = 1.5 \times 150 = 225$ by Equation (74).

The calculation of ϕ_{av} by Equation (72) is straightforward since $\phi'(\epsilon)$ is known from the computer program (App. H) from the

kinematic and static load analyses. The numerical integration process to obtain ϕ_{av} is also written into the computer program. However, to illustrate how this is done, a numerical example will be given in Appendix K.

The maximum instantaneous surface temperature rise, ΔT , is given by Equation (82), with β taken as $42.15 \text{ lb/}^\circ\text{F-in. sec}^{\frac{1}{2}}$ (App. A). In other words,

$$\Delta T = \frac{0.361 f w^{\frac{3}{4}} \left| \sqrt{\rho_p n_p} - \sqrt{\rho_g n_g} \right|}{R^{\frac{1}{4}}} \quad (100)$$

The maximum instantaneous surface temperature, T_c , is then

$$T_c = T_s + \Delta T \quad (101)$$

Equations (97), (98), (99), (72), (100), and (101) are then employed to compute the instantaneous values of T_c in the gear mesh at different power levels. The computer program is presented in Appendix H, along with a sample run at 600 hp. The major computer results at 600 hp and several additional power levels are summarized in Table 8. In this table, P is the power transmitted; W is the normal tooth load, which depends only on P ; T_s is the quasi-steady surface temperature, which depends only on P ; ΔT is the maximum instantaneous surface temperature rise somewhere in the gear mesh, which depends on both P and mesh position; T_c is the maximum instantaneous surface temperature, which also depends on both P and mesh position; and the critical temperature is constant for the problem.

It will be recalled from Equation (96) that $T_{cr} = 488^\circ\text{F}$ in this case. Thus, from Table 8, the ideal scoring-limited power-transmitting capacity, P_I , is expected to be between 900 and 1000 hp. A graphical way to determine the value of P_I is shown in Figure 31, by plotting the tabulated values of T_c vs. P . The intersection of this curve with $T_{cr} = 488^\circ\text{F}$ yields P_I equal to 957 hp for the ground spur gears.

Ideal Scoring Power of Honed Spur Gears. With the honed spur gears, the initial composite surface roughness, δ_i , is $15.8 \mu\text{in}$.

TABLE 8. IDEAL SPUR GEAR PERFORMANCE SUMMARY

<u>P, hp</u>	<u>W, lb</u>	<u>ϕ_{av}, Btu/sec</u>	<u>T_s, °F</u>	<u>ΔT, °F</u>	<u>T_c, °F</u>	<u>T_{cr}, °F</u>
<u>Ground gears ($\delta_i = 13.7 \mu\text{in. AA}$)</u>						
600	2795.7	0.5763	394.7	18.6	413.4	488
700	3261.7	0.6688	413.1	21.5	434.6	488
800	3727.6	0.7631	431.2	24.2	455.5	488
900	4193.6	0.8595	449.3	26.7	476.0	488
1000	4659.6	0.9583	467.5	28.8	496.3	488
<u>Honed gears ($\delta_i = 15.8 \mu\text{in. AA}$)</u>						
600	2795.7	0.6048	400.5	19.6	420.1	480
700	3261.7	0.7018	419.5	22.6	442.1	480
800	3727.6	0.8005	438.3	25.5	463.8	480
900	4193.6	0.9015	457.1	28.0	485.1	480
1000	4659.6	1.0052	475.9	30.3	506.2	480

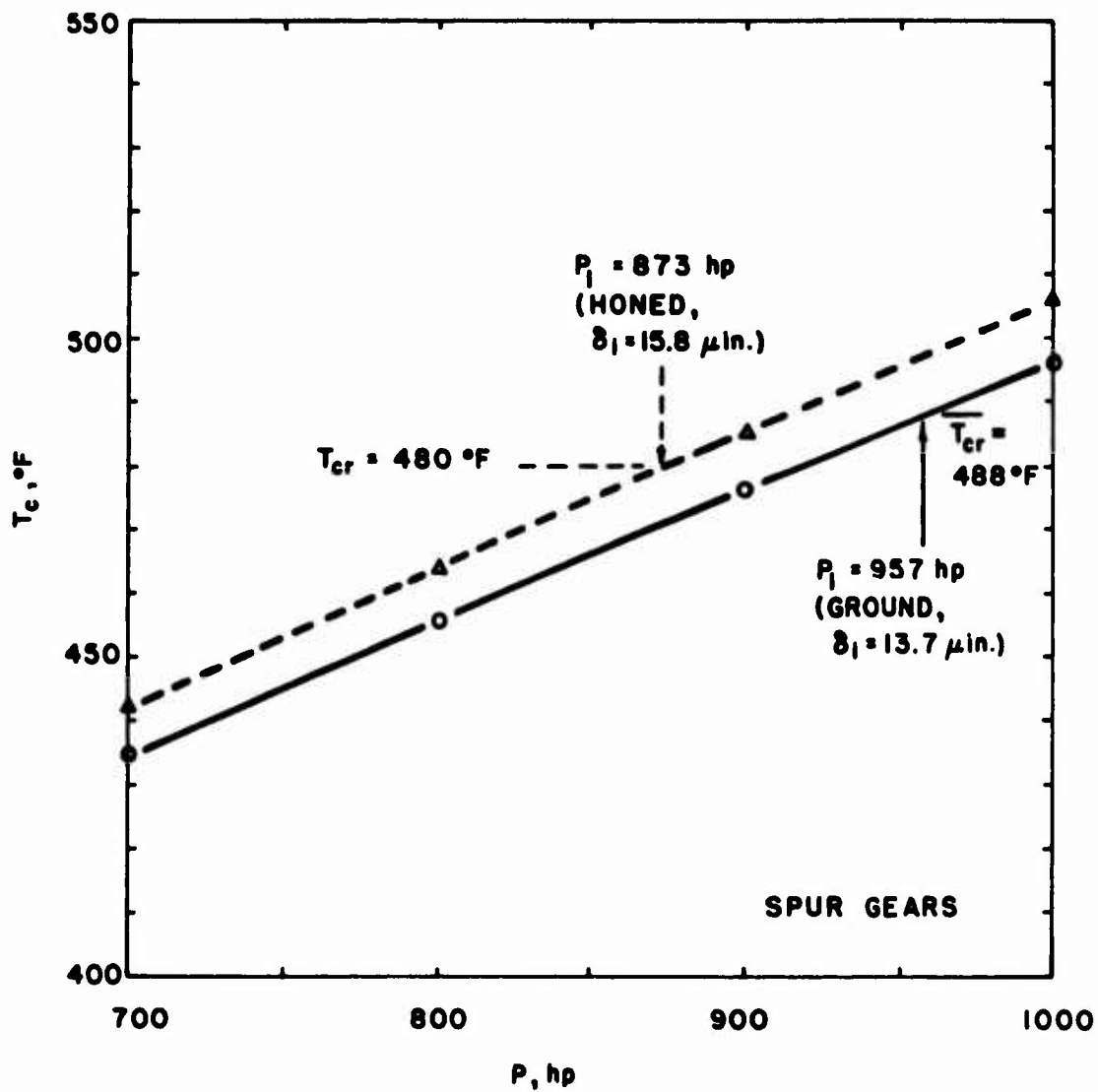


Figure 31. Determination of ideal scoring power of ground and honed spur gears

AA. Accordingly, Equation (50) gives

$$T_{cr} = 540 - 3.80 \delta_i = 480^\circ F \quad (102)$$

and Equations (60) and (61) give

$$\begin{aligned} f &= (0.0789 + 0.00034 \delta_i)(WV_s^{-\frac{1}{3}})^{-0.30} \\ &= 0.0842 (WV_s^{-\frac{1}{3}})^{-0.30} \\ &\quad \text{at } WV_s^{-\frac{1}{3}} < 200 \end{aligned} \quad (103)$$

$$\begin{aligned} f &= 0.0161 + 0.00007 \delta_i = 0.0172 \\ &\quad \text{at } WV_s^{-\frac{1}{3}} \geq 200 \end{aligned} \quad (104)$$

It is clear that Equations (99), (100), and (101) apply equally to this case, by noting that f will now be defined by Equations (103) and (104).

Equations (99), (72), (100), (101), (103), and (104) are now employed in the computer program in the same manner as in the preceding case. A summary of the major computer results is presented in Table 8. A plot of T_c vs. P for this case is shown dashed in Figure 31. The ideal scoring-limited power-transmitting capacity, P_I , is then equal to 873 hp for the honed spur gears.

Actual Scoring Power of Ground Spur Gears. In order to predict the actual scoring-limited power-transmitting capacity, it is necessary to estimate the misalignment factor, S_m , and the dynamic factor, S_d , and then apply Equation (67).

In order to estimate S_m , both the angular misalignment, e , and the unit tangential tooth load, w_t , are required. The unit tangential tooth load for any gear set is

$$w_t = \frac{126050 P}{n_p d F} \quad (105)$$

where w_t = unit tangential tooth load, ppi

P = power transmitted, hp

n_p = pinion speed, rpm

d = pitch diameter of pinion, in.

F = effective face width, in.

For the ground gear sets, the average tooth misalignment, e , was, as stated previously, estimated to be 0.00076 rad. At $P_I = 957$ hp, the unit tangential tooth load, w_t , is $(126050 \times 957)/(8000 \times 3.6471 \times 1.25) = 3308$ ppi. Accordingly, with $e = 0.00076$ rad. and $w_t = 3308$ ppi, Figure 28 gives $S_m = 0.80$ by interpolation.

To estimate the dynamic factor, it is assumed that $S_d = K_v2$ (Chap. VII, Sect. D). The pitchline velocity is $V_t = 7638$ fpm. At this V_t , Figure 30 gives $S_d = K_v2 = 0.68$.

Having thus estimated the values of S_m and S_d , Equation (67) then gives a predicted actual scoring-limited power-transmitting capacity of

$$\begin{aligned} P_A &= P_I S_m S_d \\ &= 957 \times 0.80 \times 0.68 = 521 \text{ hp} \end{aligned}$$

This predicted P_A may be compared with the experimentally determined P_A given in Appendix F. If the comparison is made with the average experimentally determined P_A of >679 hp, then the prediction is at least 23 percent too low. Such a comparison is, however, not correct because the average P_A has no statistical meaning. The proper basis of comparison should be the statistically deduced, experimentally determined P_A of 507 hp. On this basis, the prediction is seen to be only 3 percent too high.

Actual Scoring Power of Honed Spur Gears. As stated earlier, the average tooth misalignment for the honed spur gears was 0.00084

rad. At $P_I = 873$ hp, $w_t = (126050 \times 873)/(8000 \times 3.6471 \times 1.25) = 3017$ ppi. At $e = 0.00084$ and $w_t = 3017$ ppi, Figure 28 gives $S_m = 0.78$.

Let it be assumed again that $S_d = K_{v2} = 0.68$ at $V_t = 7638$ fpm. Then the predicted P_A is

$$P_A = 873 \times 0.78 \times 0.68 = 463 \text{ hp}$$

Appendix F shows that the average experimentally determined P_A is 606 hp, and the statistically deduced, experimentally determined P_A is 425 hp. As argued earlier, the proper basis of comparison is the statistically deduced P_A of 425 hp. On this basis, it is seen that the predicted P_A is 9 percent too high.

C. Helical Gear Test Program

As mentioned earlier, the helical gear test program was not run. However, a computer program for helical gears has been written (App. I), the basis for which will be explained.

The test gears intended for the test program were to be made of AISI 9310 CEVM steel, carburized and surface-treated with a black oxide as shown in Table 9, which also presents the design data. The surface finish on both pinion and gear was specified to be $22 \mu\text{in. AA}$ maximum. If the gears were produced to this maximum surface finish, then, by Equation (B-4) in Appendix B, the initial composite surface roughness of a gear set would be $\delta_i = 3(22 + 22)/4 = 33 \mu\text{in. AA}$.

The test oil was to be Oil F, a MIL-L-7808G synthetic oil.

The tests were to be run at a pinion speed of 5,000 rpm, with the pinion as the driver. The corresponding pitchline velocity is 5,028 fpm. The oil jet temperature was to be 190°F .

Ideal Scoring Power of Helical Gears. For reasons given in Chapter VII, the critical temperature, T_{cr} , and the coefficient of friction, f , are herein assumed to be those of the plain surfaces, i.e., surfaces as if no black oxide were present. Therefore, at the assumed value of $\delta_i = 33 \mu\text{in. AA}$, Equation (50) gives

$$T_{cr} = 540 - 3.80 \delta_i = 415^\circ\text{F} \quad (106)$$

TABLE 9. HELICAL GEAR DESIGN DATA

	<u>Pinion (Driver)</u>	<u>Gear (Driven)</u>
Number of teeth	31	138
Diametral pitch (normal plane), in. ⁻¹	8.5	8.5
Pitch diameter, in.	3.8413	17.6998
Face width, in.	2.500	2.380
Pressure angle (normal plane), deg	22.0	22.0
Outside diameter, in.	4.032	17.290
Root diameter, in.	3.528	16.786
Mean circular tooth thickness (normal plane), in.	0.1782	0.1782
Helix angle, deg	18.2966	18.2966
Lead of helix, in.	36.5000	162.4681
Hand of helix	LH	RH
Start of tip modification, deg roll	26.88	25.18
End of tip modification, deg roll	30.74	25.95
Nominal slope of tip modification line, in.	0.00008	0.00008
Nominal slope of profile slope line, in.	0	0
Nominal slope of tooth slope line, in.	0	0
Maximum allowable errors:		
Tooth to tooth spacing, in.	0.0002	0.0003
Accumulated spacing, in.	0.0006	0.0010
Slope of tip modification line, in.	±0.00008	±0.00008
Slope of profile slope line, in.	±0.0002	±0.0002
Slope of tooth slope line, in.	±0.0015	±0.0002
Material and surface:		
Material, AISI	9310	9310
Case thickness, in.	0.030-0.040	0.030-0.040
Case hardness, R _C	60-63	60-63
Core hardness, R _C	33-41	33-41
Surface finish, μin. AA max.	22	22

To calculate the coefficient of friction, note that the direction of sliding in helical gears is not normal, but inclined, to the orientation of the grinding grooves. In this case, the helix angle of the gear set is $\psi = 18.2966^\circ$, so the direction of sliding makes an angle of $90 - 18.2966 = 71.7034^\circ$ to the grinding grooves; and $\cot(71.7034^\circ) = 0.33065$. An approximate way to account for this orientation effect is to interpolate between Equations (56) and (57) for the cross-ground situation and Equations (54) and (55) for the circumferentially-ground situation. The equations for f are then approximately

$$\begin{aligned} f &= \left[0.0755 + 0.33065 (0.0920 - 0.0755) \right. \\ &\quad \left. + 0.00034 \delta_i \right] (WV_s^{-\frac{1}{3}})^{-0.30} \\ &= (0.0810 + 0.00034 \delta_i) (WV_s^{-\frac{1}{3}})^{-0.30} \\ &= 0.0922 (WV_s^{-\frac{1}{3}})^{-0.30} \\ &\quad \text{at } WV_s^{-\frac{1}{3}} < 200 \end{aligned} \quad (107)$$

$$\begin{aligned} f &= 0.0154 + 0.33065 (0.0188 - 0.0154) + 0.00007 \delta_i \\ &= 0.0165 + 0.00007 \delta_i = 0.0188 \\ &\quad \text{at } WV_s^{-\frac{1}{3}} \geq 200 \end{aligned} \quad (108)$$

The quasi-steady surface temperature, T_s , is given by Equation (69), thus:

$$T_s = T_j + C' \phi_{av}^{0.80} = 190 + 312.5 \phi_{av}^{0.80} \quad (109)$$

where ϕ_{av} is now defined by Equation (85). In assigning the value of

C' , it is assumed that the oil jet flow rate is 0.60 gpm. Then Figure 25 gives $C = 125$, and Equation (76) yields $C' = 2.5 \times 125 = 312.5$.

The computation of ϕ_{av} by Equation (85) is accomplished by the computer program (App. I) as explained in Chapter VII, Section E. A numerical example for this computation will be given in Appendix K.

The maximum instantaneous surface temperature rise, ΔT , is given by Equation (87), with $\beta = 42.15 \text{ lb/}^\circ\text{F-in.}^{\frac{1}{2}}\text{sec}^{\frac{1}{2}}$.

The maximum instantaneous surface temperature, T_c , is thus given by Equation (101).

Equations (107), (108), (109), (85), (87), and (101) are then employed in the computer program to compute the instantaneous values of T_c at various points on various instantaneous lines of contact in the gear mesh, at different power levels. The computer program is presented in Appendix I, along with a sample run at 600 hp. The major computer results at 600 hp and several additional power levels are summarized in Table 10. The symbols in Table 10 are defined the same way as in Table 9, except that ΔT and thus T_c are the maximas somewhere on one of the instantaneous lines of contact somewhere in the mesh.

Figure 32 presents a plot of T_c vs. P given in Table 10. At the assumed $T_j = 190^\circ\text{F}$, the figure yields a predicted ideal scoring-limited power-transmitting capacity, P_I , of 1210 hp.

As a matter of interest, Figure 32 also shows the ideal performance for an assumed $T_j = 250^\circ\text{F}$, the same oil jet temperature at which the spur gears were tested. For this case, Equation (109) should be changed to read $T_s = 250 + 312.5 \phi_{av}^{0.80}$, with all other equations remaining the same. The corresponding performance is shown dashed in Figure 32, and the corresponding predicted P_I is 796 hp.

Actual Scoring Power of Helical Gears. In the absence of other specific information, let it be assumed that the gear misalignment is $e = 0.001$ rad. The procedure for estimating the misalignment factor, S_m , of spur gears will be employed for helical gears as well. The procedure will yield $S_m = 0.68$ for the case of $T_j = 190^\circ\text{F}$, and $S_m = 0.61$ for the case of $T_j = 250^\circ\text{F}$.

In estimating the dynamic factor, S_d , it will be assumed, in

TABLE 10. IDEAL HELICAL GEAR PERFORMANCE SUMMARY

<u>P, hp</u>	<u>W, lb</u>	<u>ϕ_{av}, Btu/sec</u>	<u>T_s, °F</u>	<u>ΔT, °F</u>	<u>T_c, °F</u>	<u>T_{cr}, °F</u>
<u>$T_j = 190^\circ\text{F}$</u>						
600	4471.4	0.2690	299.3	26.1	325.4	415
700	5216.6	0.3115	312.9	27.9	340.8	415
800	5961.9	0.3541	326.2	29.7	355.9	415
900	6707.1	0.3973	339.3	31.3	370.6	415
1000	7452.3	0.4404	352.2	32.8	385.0	415
1100	8197.4	0.4835	364.7	34.3	399.0	415
1200	8942.8	0.5265	377.1	36.6	413.6	415
1300	9688.0	0.5695	389.2	38.8	428.0	415
1400	10433.2	0.6124	401.1	41.1	442.2	415
<u>$T_j = 250^\circ\text{F}$</u>						
600	4471.4	0.2690	359.3	26.1	385.4	415
700	5216.6	0.3115	372.9	27.9	400.8	415
800	5961.9	0.3541	386.2	29.7	415.9	415
900	6707.1	0.3973	399.3	31.3	430.6	415
1000	7452.3	0.4404	412.2	32.8	445.0	415
1100	8197.4	0.4835	424.7	34.3	459.0	415
1200	8942.8	0.5265	437.1	36.6	473.6	415
1300	9688.0	0.5695	449.2	38.8	488.0	415
1400	10433.2	0.6124	461.1	41.1	500.2	415

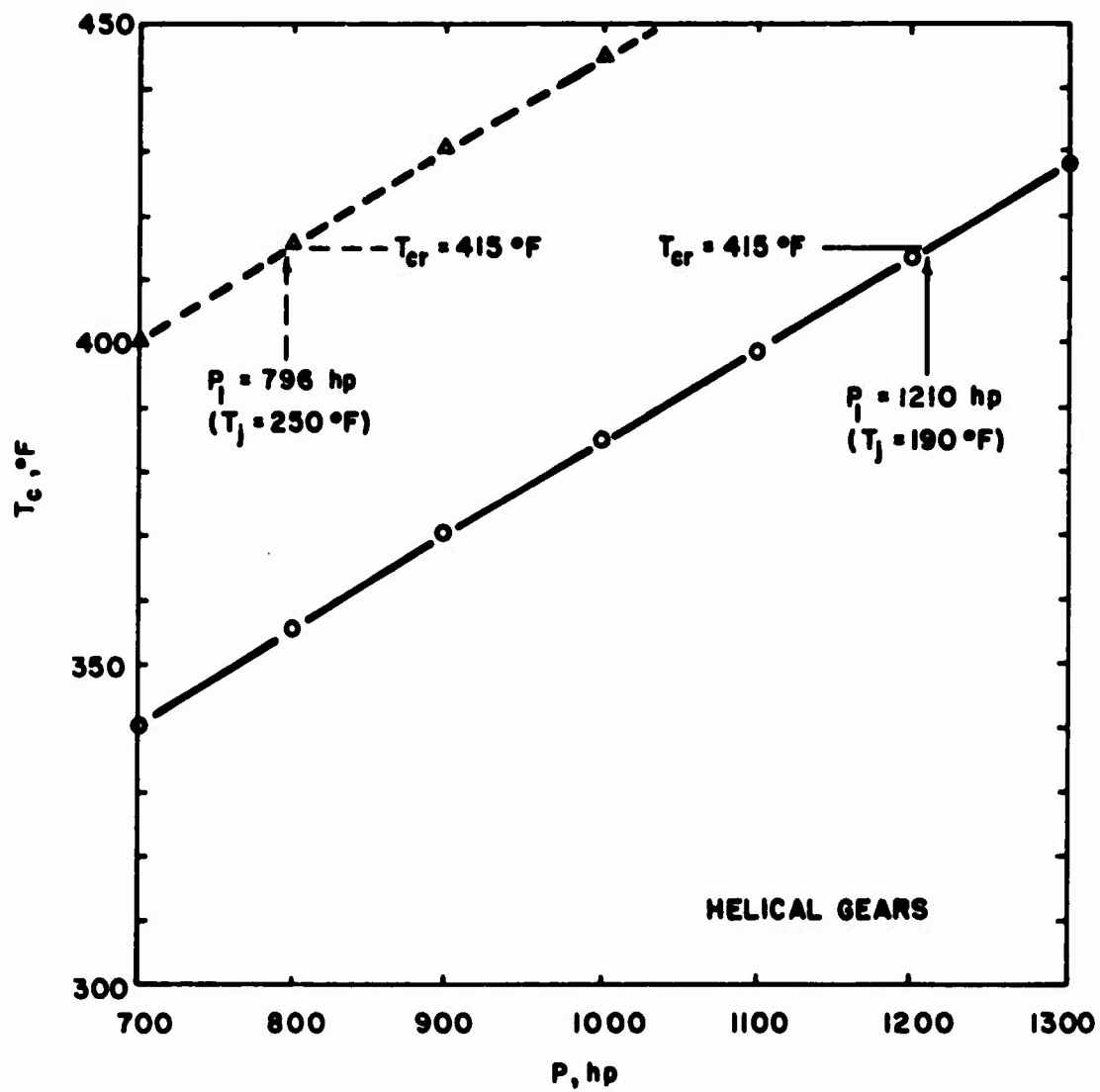


Figure 32. Determination of ideal scoring power of helical gears

the absence of other specific information, that $S_d = S_{d1}$ for helical gears. At $V_t = 5028$ fpm, Figure 30 gives $S_d = S_{d1} = 0.86$.

From Equation (67), the predicted actual scoring-limited power-transmitting capacity is thus $P_A = 708$ hp for the case of $T_j = 190^\circ\text{F}$, and $P = 418$ hp for the case of $T_j = 250^\circ\text{F}$. The powerful influence of the oil jet temperature on scoring-limited power-transmitting capacity is clearly indicated.

The quality of the above predictions is not known, since no tests were actually conducted.

D. Spiral Bevel Gear Test Program

As mentioned earlier, only one design of spiral bevel gears was tested. The predicted performance and experimentally determined performance of these gears will now be compared.

The spiral bevel gears in question were made of AISI 9310 CEVM steel, carburized and surface-treated with a black oxide as shown in Table 11, which also presents the design data. As in the case of the helical gears, the surface finish of the spiral bevel gears was specified to be $22\text{ }\mu\text{in. AA}$ maximum. Attempts were made to measure the actual surface roughness of these gears, but without success (App. G). Accordingly, for the present estimating purposes, a surface finish of $22\text{ }\mu\text{in. AA}$ on both pinion and gear is assumed. The corresponding initial composite surface roughness of the gear set is then $\delta_i = 3(22 + 22)/4 = 33\text{ }\mu\text{in. AA}$ by Equation (B-4) in Appendix B.

The test oil was Oil F, a MIL-L-7808G synthetic oil.

The tests were conducted at a pinion speed of 4,500 rpm, with the pinion as the driver. The corresponding pitchline velocity is 4,241 fpm. The oil jet temperature was 190°F . The oil jet flow rate was 0.45 gpm.

Ideal Scoring Power of Spiral Bevel Gears. For reasons given in Chapter VII, the critical temperature, T_{cr} , and the coefficient of friction, f , are assumed to be those of the plain surfaces, i. e., surfaces as if no black oxide were present. Consequently, at the assumed value of $\delta_i = 33\text{ }\mu\text{in. AA}$, Equation (50) gives

$$T_{cr} = 540 - 3.80 \delta_i = 415^\circ\text{F} \quad (110)$$

TABLE 11. SPIRAL BEVEL GEAR DESIGN DATA

	<u>Pinion (Driver)</u>	<u>Gear (Driven)</u>
Number of teeth	22	23
Diametral pitch, in. ⁻¹	6.111	6.111
Pitch diameter, in.	3.6000	3.7637
Face width, in.	0.871	0.871
Pressure angle, deg	22.5	22.5
Outside diameter, in.	3.794	3.935
Mean circular tooth thickness, in.	0.213	0.208
Outer cone distance, in.	2.604	2.604
Mean cone distance, in.	2.171	2.171
Working depth, in.	0.258	0.258
Whole depth, in.	0.289	0.289
Addendum, in.	0.134	0.124
Dedendum, in.	0.155	0.165
Pitch apex to crown, in.	1.789	1.710
Outer normal top land, in.	0.073	0.058
Mean normal top land, in.	0.073	0.077
Inner normal top land, in.	0.074	0.062
Outer normal backlash, in.	0.004	0.006
Pitch angle, deg	43.727	46.273
Face angle to flank, deg	46.244	48.556
Root angle, deg	41.444	43.756
Dedendum angle, deg	2.283	2.517
Outer spiral angle, deg	44.835	44.835
Mean spiral angle, deg	35.000	35.000
Inner spiral angle, deg	25.990	25.990
Hand of spiral	LH	RH
Material, AISI	9310	9310
Case thickness, in.	0.030-0.040	0.030-0.040
Case hardness, R _C	60-63	60-63
Core hardness, R _C	33-41	33-41
Surface finish, μ in. AA max.	22	22

In calculating the coefficient of friction, f , it is noted that the mean spiral angle of the gear set is $\psi = 35^\circ$. Thus, the sliding motion makes an angle of $90 - 35 = 65^\circ$ to the grinding grooves. Thus, using the same procedure as for the helical gears, and noting that $\cot 65^\circ = 0.70021$, the approximate equations for f are

$$\begin{aligned}
 f &= \left[0.0755 + 0.70021 (0.0920 - 0.0755) \right. \\
 &\quad \left. + 0.00034 \delta_i \right] (WV_s^{-\frac{1}{3}})^{-0.30} \\
 &= (0.0871 + 0.00034 \delta_i) (WV_s^{-\frac{1}{3}})^{-0.30} \\
 &= 0.0983 (WV_s^{-\frac{1}{3}})^{-0.30} \\
 &\quad \text{at } WV_s^{-\frac{1}{3}} < 200 \quad (111)
 \end{aligned}$$

$$\begin{aligned}
 f &= 0.0154 + 0.70021 (0.0188 - 0.0154) + 0.00007 \delta_i \\
 &= 0.0178 + 0.00007 \delta_i = 0.0201 \quad (112)
 \end{aligned}$$

The quasi-steady surface temperature, T_s , is again given by Equation (69), so

$$T_s = T_j + C' \phi_{av}^{0.80} = 190 + 675 \phi_{av}^{0.80} \quad (113)$$

where ϕ_{av} is defined by Equation (88). In this case, at an oil jet flow rate of 0.45 gpm, $C = 135$ from Figure 25, and $C' = 5 \times 135 = 675$ from Equation (77).

The computation of ϕ_{av} by Equation (88) is accomplished by the computer program (App. J), as explained in Chapter VII, Section F. A numerical example of this computation will be given in Appendix K.

The maximum instantaneous surface temperature rise, ΔT , is

given by Equation (90), with $\beta = 42.15 \text{ lb/}^\circ\text{F-in.-sec}^{\frac{1}{2}}$.

The maximum instantaneous surface temperature, T_c , is given by Equation (101).

Equations (111), (112), (113), (88), (90), and (101) are then employed in the computer program to compute the maximum instantaneous values of T_c through the mesh at different power levels. The computer program is presented in Appendix J, along with a sample run at 600 hp. The major computer results at 600 hp and several additional power levels are summarized in Table 12. In this table, W_t is the tangential tooth load, and ΔT and T_c are the maximas on an instantaneous line of contact somewhere in the mesh.

Figure 33 presents a plot of T_c vs. P given in Table 12. At $T_j = 190^\circ\text{F}$, the predicted ideal scoring-limited power-transmitting capacity is seen to be $P_I = 627 \text{ hp}$.

Figure 33 also presents a plot of T_c vs. P for $T_j = 250^\circ\text{F}$. By increasing T_j to 250°F , it is seen that P_I is reduced to 420 hp.

Actual Scoring Power of Spiral Bevel Gears. In the absence of other specific information, the misalignment factor for the overhung spiral bevel gear set is taken as $S_m = 0.70$, in accordance with the recommendation made in Chapter VII, Section F. The pitchline velocity is $V_t = 4241 \text{ fpm}$. Thus, from Figure 30, $S_d = S_{d1} = 0.87$.

Applying Equation (67), the predicted actual scoring-limited power-transmitting capacity is then

$$P_A = 627 \times 0.70 \times 0.87 = 382 \text{ hp}$$

for the case of $T_j = 190^\circ\text{F}$. From Appendix G, the average experimentally-determined P_A at $T_j = 190^\circ\text{F}$ is 367 hp, and the statistically deduced, experimentally determined P_A is 346 hp. The latter value, which is statistically more meaningful, should be used as the basis for comparison. It is seen that the predicted P_A is 10 percent too high.

The predicted actual scoring-limited power-transmitting capacity of the same gears, if operated at an oil jet temperature of 250°F , is

$$P_A = 420 \times 0.70 \times 0.87 = 256 \text{ hp}$$

**TABLE 12. IDEAL SPIRAL BEVEL GEAR
PERFORMANCE SUMMARY**

<u>P, hp</u>	<u>W_t, lb</u>	<u>φ_{av}, Btu/sec</u>	<u>T_s, °F</u>	<u>ΔT, °F</u>	<u>T_c, °F</u>	<u>T_{cr}, °F</u>
<u>T_j = 190° F</u>						
300	2333.3	0.1013	298.1	20.5	318.7	415
400	3111.1	0.1333	324.7	24.2	348.9	415
500	3888.8	0.1651	349.8	28.6	378.4	415
600	4666.6	0.1975	374.4	32.8	407.3	415
700	5444.3	0.2299	398.3	36.9	435.1	415
800	6222.1	0.2623	421.4	40.7	462.1	415
<u>T_j = 250° F</u>						
300	2333.3	0.1029	358.1	20.5	378.7	415
400	3111.1	0.1353	384.7	24.2	408.9	415
500	3888.8	0.1676	409.8	28.6	438.4	415
600	4666.6	0.2005	434.4	32.8	467.3	415
700	5444.3	0.2334	458.3	36.9	495.1	415
800	6222.1	0.2662	481.4	40.7	522.1	415

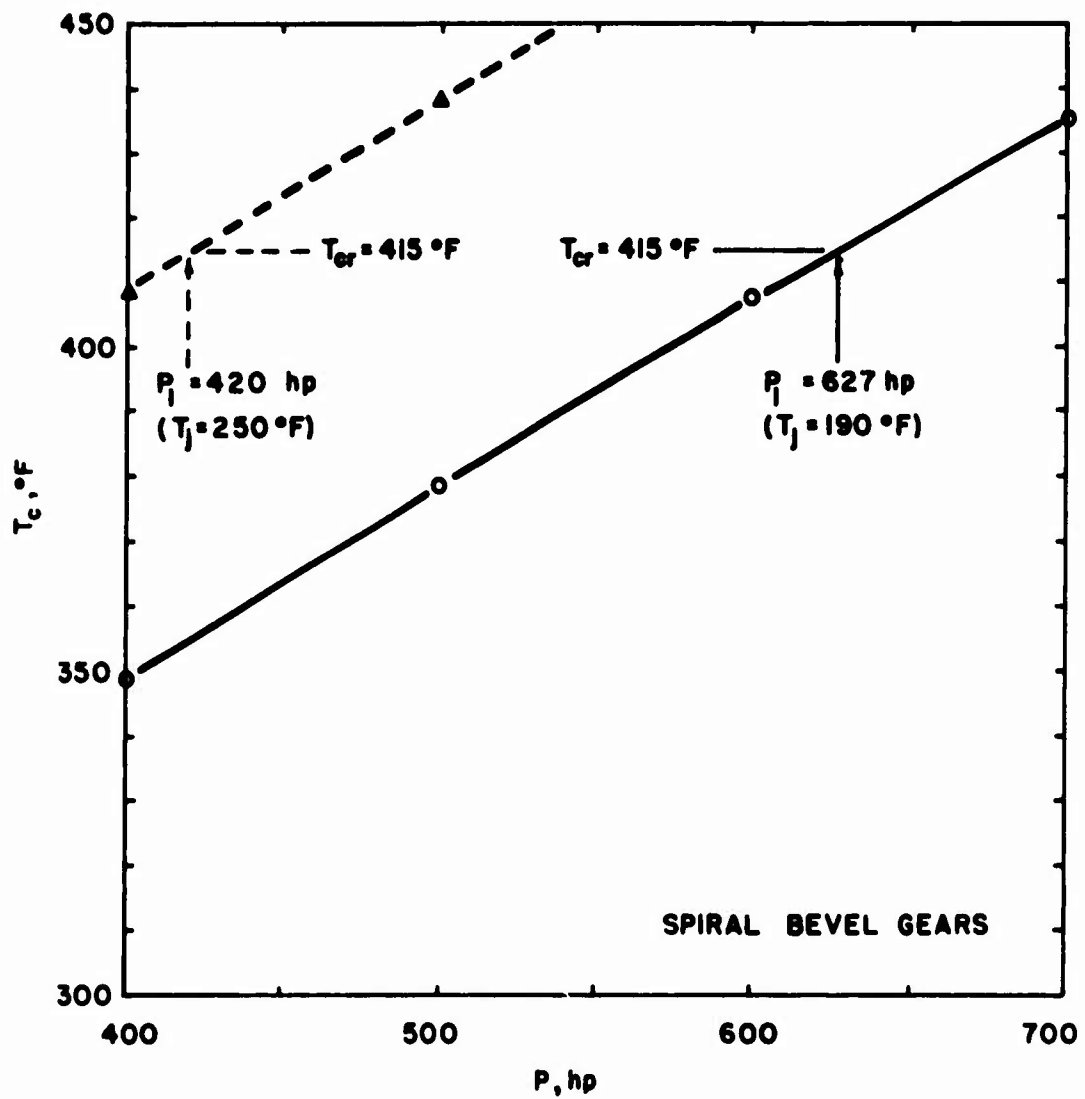


Figure 33. Determination of ideal scoring power of spiral bevel gears

which is much lower than that predicted for $T_j = 190^\circ\text{F}$. However, no test data are available for comparison in this instance.

CHAPTER IX

CONCLUSIONS AND RECOMMENDATIONS

A. Conclusions

A method has been devised for predicting the scoring potential and scoring-limited power-transmitting capacity of spur, helical, and spiral bevel gears. Computer programs for making such predictions for the three gear types have been written and are presented herein.

In order to evaluate the quality of the predictions made, full-scale scoring tests have been performed on typical aircraft-quality gears by Bell Helicopter Company, the subcontractor. The predicted scoring-limited power-transmitting capacities have been found to be within 10 percent of the statistically deduced test results from two series of tests on spur gears and one series of tests on spiral bevel gears. Helical gears were not tested in this program, due to difficulties encountered by the subcontractor in the scheduling of gear manufacturing and testing.

The predictive scheme comprises basically two steps. The first step involves the prediction of the ideal scoring-limited power-transmitting capacity, assuming perfect tooth alignment and no dynamic tooth load. The probable, actual scoring-limited power-transmitting capacity is then deduced from the ideal scoring-limited power-transmitting capacity by applying corrections for the misalignment and dynamic effects.

Due to the current lack of a fundamental understanding of the mechanism of scoring, the thermal behavior involved, and the detailed effects of gear mechanics, a completely rational approach in gear scoring prediction is deemed impossible at this time. On the other hand, a basically empirical procedure, such as the current AGMA gear scoring design guide,³ leaves much to be desired. The proposed scheme is accordingly in the nature of an engineering compromise, which recognizes the importance of the above-mentioned basic problems but accepts certain approximations imposed by the current state of the art.

The key assumption involved in the methodology presented herein is that the effects of tooth misalignment and dynamic tooth load can be isolated in the prediction of the ideal scoring-limited power-transmitting capacity, and later separately accounted for in the

assessment of the actual scoring-limited power-transmitting capacity. Once this assumption is accepted, the entire predictive scheme is relatively straightforward; and the only tasks that remain are those of establishing the various functional relationships and the magnitudes of the several constants and coefficients involved.

In the prediction of the ideal scoring-limited power-transmitting capacity, Blok's well-known critical temperature hypothesis^{16, 19, 23} has been modified in several respects, including quantitative descriptions of the critical temperature and the coefficient of tooth friction, based on data derived from steady-operating sliding-rolling disk tests. Perhaps the most important advance that has been made here is a technique for estimating the quasi-steady surface temperature of gears, which has been a totally neglected subject so far. However, due to a lack of information on the heat transfer behavior of gear systems, the quantitative magnitude of the constant C' in Equation (69) had to be assigned rather arbitrarily.

In the prediction of the actual scoring-limited power-transmitting capacity, the major tasks have been assigning the quantitative magnitudes of the misalignment factor and dynamic factor. The approach employed herein follows basically the AGMA procedure for rating the strength of gear teeth.⁴⁻⁶ The numerical constants have been deduced from five sets of spur gear scoring test results made available to this program by AGMA.

It goes without saying that the basic equations and particularly the numerical constants and coefficients used herein are tentative, since they were deduced from a rather limited data base. Refinements or improvements are to be expected as additional disk and gear test results become available.

B. Recommendations

The mechanism of scoring has been a subject of serious research for almost 40 years, since Blok published his first paper in 1937.¹⁶ Any refinements on this hypothesis that have been introduced since that time have been relatively minor. The general concept of a critical temperature for scoring can neither be defended nor be refuted on strictly theoretical ground. It appears that further understanding of the mechanism of scoring will require a fundamental approach backed up by detailed, combined theoretical analysis and sophisticated experimental observations.

Granting the tentative nature of the predictive scheme presented herein, the results clearly emphasize the importance of the thermal behavior of the gear system in affecting the quasi-steady gear surface temperature, and of the effects of misalignment and dynamic load on transient tooth action and hence on scoring. A definitive understanding of these three facets of gear performance is sorely needed, either to improve the predictive methodology as proposed herein, or hopefully to enable the development of a completely rational scheme of gear scoring prediction.

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APPENDIX A PROPERTIES OF TEST STEEL AND TEST OILS

Test Steel

All test disks and test gears employed in this program were made of AISI 9310 CEVM steel, carburized to give a specified case thickness, case hardness, and core hardness.

The bulk properties of the AISI 9310 steel are taken as follows:

<u>Quantity</u>	<u>Symbol</u>	<u>Unit</u>	<u>Value</u>	<u>Remarks</u>
Young's modulus	E	psi	30×10^6	
Poisson's ratio	ν	-	0.30	
Equivalent Young's modulus	E^*	psi	33×10^6	$E^* = E / (1 - \nu^2)$
Density	ρ	lb/in. ³	0.283	
Specific heat	c	in. /°F	1075	
Thermal conductivity	k	lb/°F-sec	5.84	
Blok's thermal coefficient	β	lb/°F-in.sec ^{$\frac{1}{2}$}	42.15	$\beta = (\rho c k)^{\frac{1}{2}}$

Test Oils

Two synthetic aviation gas turbine lubricants, a MIL-L-7808G lubricant herein designated as Oil F and a MIL-L-23699 lubricant herein designated as Oil E, were employed in the program. Adequate quantities of these oils, each from a single production batch, were supplied for use in both the disk tests and the gear tests by the USAF Aero Propulsion Laboratory, under the code designations of O-67-23 for Oil F and O-64-2 for Oil E.

The measured properties of these two oils at atmospheric pressure are as follows:

<u>Quantity</u>	<u>Symbol</u>	<u>Unit</u>	<u>Oil F</u>	<u>Oil E</u>
Specification MIL-L-	-	-	7808G	23699
Density at 60°F	ρ_{60}	g/ml	0.953	1.007
Kin. viscosity at 100°F	ν_o	cs	13.4	27.5
Kin. viscosity at 210°F	ν_o	cs	3.23	5.07
Sp. ht. at 300°F	c	Btu/lb-°F	0.541	0.541
Th. cond. at 300°F	k	Btu/ft-°F-sec	0.0841	0.0703
Neutralization no.	-	mg KOH/g	0.2	0.2

The oil properties at any temperature and pressure may be calculated by the following expressions:

$$\rho = \rho_{60} - G(T - 60)$$

$$\log \log (\nu_o + 0.60) = A - B \log (T + 460)$$

$$\mu_o = \nu_o \rho_o$$

$$\mu = \mu_o e^{\alpha p}$$

$$\alpha_o = \frac{K}{T\beta}$$

where T = temperature, °F

p = pressure, psig

ρ_o = density at atmospheric pressure and temperature T, g/ml

ρ_{60} = density at atmospheric pressure and 60°F, g/ml

ν_0 = kinematic viscosity at atmospheric pressure and temperature T, cs

μ_0 = absolute viscosity at atmospheric pressure and temperature T, cp

μ = absolute viscosity at pressure p and temperature T, cp

α = pressure-viscosity coefficient at pressure p and temperature T, psi^{-1}

α_0 = pressure-viscosity coefficient at atmospheric pressure and temperature T, psi^{-1}

B, A, B, K, β = fitting constants which are functions of the oil in question

The various fitting constants to be used in the preceding equations are as follows:

	<u>$G \times 10^4$</u>	<u>A</u>	<u>B</u>	<u>$K \times 10^4$</u>	<u>β</u>
Oil F	3.94	11.75543	4.24477	12.717	0.566
Oil E	4.14	11.16683	3.99787	9.496	0.492

APPENDIX B

COMPOSITE SURFACE ROUGHNESS

Surface roughness has been found to have a significant effect on the lubrication-related failures.^{9, 10, 24-26} In order to relate the lubrication and failure behaviors to surface roughness, some way of quantitatively describing the "composite surface roughness" of two interacting surfaces is required. The method employed herein is given below.

Composite Surface Roughness of a Single Surface

The composite surface roughness of a single surface is herein defined as

$$\delta = (\delta_x + \delta_y)/2 \quad (B-1)$$

where δ = composite surface roughness of a single surface,
 $\mu\text{in. AA}$

δ_x = surface roughness in one direction (usually the direction of sliding), $\mu\text{in. AA}$

δ_y = surface roughness in the perpendicular direction,
 $\mu\text{in. AA}$

Composite Surface Roughness of a Pair of Surfaces

The composite surface roughness of a pair of interacting surfaces is herein defined as

$$\delta_c = \delta_1 + \delta_2 \quad (B-2)$$

where δ_c = composite surface roughness of a pair of surfaces,
 $\mu\text{in. AA}$

δ_1 = composite surface roughness of surface 1, $\mu\text{in. AA}$

δ_2 = composite surface roughness of surface 2, $\mu\text{in. AA}$

Approximate Procedure

In general, it is good practice to measure the surface roughnesses of both surfaces in two directions, and then calculate the composite surface roughness of each surface by Equation (B-1) and that of the pair by Equation (B-2). However, in many practical cases, the surface roughness is usually measured in only one direction. In such cases, the surface roughness in the normal direction has to be assumed.

With ground surfaces, it has been found from measurements on both the test disks and test gears, as well as a large volume of additional data on hand in the authors' laboratory, that the ratio of the surface roughness across the grinding marks to that in the direction of grinding is generally quite close to 2. In other words, if δ_x is the surface roughness across the grinding marks, then $\delta_y \sim \delta_x/2$ with good approximation. It then follows from Equation (B-1) that

$$\delta \sim 3\delta_x/4 \quad (B-3)$$

and from Equation (B-2) that

$$\delta_c \sim 3(\delta_{x1} + \delta_{x2})/4 \quad (B-4)$$

With honed surfaces, δ_x and δ_y are usually nearly equal. Thus

$$\delta \sim \delta_x \quad (B-5)$$

and

$$\delta_c \sim \delta_x + \delta_y \quad (B-6)$$

APPENDIX C

ELASTOHYDRODYNAMIC FILM THICKNESS

As explained in Chapter II, Section C, EHD film thickness calculations will not be given emphasis in this report. However, as a matter of general interest, the procedure for such calculations will be given below.

Sliding-Rolling Disks

The minimum oil film thickness developed by EHD action in a flooded, elliptic conjunction of perfectly smooth surfaces, in steady operation, is given by Equation (5) and repeated below:

$$h'_m = 26.5 \frac{\alpha_o^{0.54} (\mu_o V_t)^{0.70} R^{0.43} \phi_s \phi_t}{w^{0.13} E^{*0.03}} \quad (C-1)$$

where h'_m = minimum oil film thickness, μ in.

α_o = pressure-viscosity coefficient of oil at conjunction-inlet temperature and near-atmospheric pressure, psi^{-1}

μ_o = absolute viscosity of oil at conjunction-inlet temperature and near-atmospheric pressure, cp

E^* = equivalent Young's modulus of material, psi

R = equivalent radius of curvature at the conjunction, in.

w = unit normal load, ppi

V_t = sum velocity, ips

ϕ_s = side flow correction factor

ϕ_t = inlet-shear thermal correction factor

In EHD oil film thickness analysis, the controlling oil properties are those prevalent at the conjunction inlet, i.e., at the

conjunction-inlet oil temperature, T_o , and near-atmospheric pressure. If T_o is known, the appropriate values of α_o and μ_o may be calculated, as recommended in Appendix A.

In general, T_o is higher than the oil jet temperature, T_j , and lower than the quasi-steady surface temperature, T_s . For a sliding-rolling disk system in steady operation, T_o can easily be measured by means of a thermocouple. In case T_o is not known, then it may be estimated, as shown in Chapter VI, Section E, as follows:

$$T_o - T_j = C_o \phi^{0.80} \quad (C-2)$$

where C_o = a fitting constant, and ϕ = frictional power loss, Btu/sec.

For sliding-rolling disks, C_o may be estimated as given in Chapter VI, Section E; and ϕ is given by

$$\phi = f W V_s / 9336 \quad (C-3)$$

where ϕ = frictional power loss, Btu/sec

f = coefficient of friction

W = normal load, lb

V_s = sliding velocity, ips

The values of R , w , ϕ_s , and ϕ_t may be obtained by the procedure outlined by Cheng.³⁷

Gears

To compute h_m^i at any point in a gear mesh, the effect of gear mechanics on quantities entering Equation (C-1) must be taken into account. This matter is considered in Chapters III, IV, and V for spur, helical, and spiral bevel gears, respectively.

In order to estimate T_o for gears, it should be recognized that ϕ in Equation (C-2) varies cyclically through a mesh cycle. Thus

Equation (C-2) should be modified to read

$$T_o - T_j = C_o' \phi_{av}^{0.80} \quad (C-4)$$

where ϕ_{av} = average frictional power loss, Btu/sec, and $C_o' = 0.70 C'$ by applying Equation (66). The estimation of C' and the calculation of ϕ_{av} for gears are given in Chapter VII, Section B.

EHD Film Thickness Ratio

As explained in Chapter II, Section C, there is at present no viable way to account for the effect of surface roughness and surface texture on the EHD film thickness. However, an empirical parameter is often used in practice to indicate, in a very approximate way, whether or not the operation is in the EHD regime, or how deeply the operation is in the boundary lubrication regime. This parameter is defined as

$$\Lambda = \frac{h_m'}{\delta_c} \quad (C-5)$$

where Λ = EHD film thickness ratio

h_m' = minimum oil film thickness, $\mu\text{in.}$, as given by Equation (C-1)

δ_c = composite surface roughness of a pair of surfaces, as given in Appendix B.

APPENDIX D

SUMMARY OF DISK TEST DATA

Test Equipment

The basic equipment employed in the work reported herein is the Caterpillar disk tester,²⁰ which has, however, been rather substantially modified in the authors' laboratory.²⁵ This tester employs two identical test disks of 3 in. diameter and 14 in. crown radius, whose material and surface characteristics may be varied. Crowned disks are used to minimize misalignment problems.

Figure D-1 shows the general arrangement of the test disks and some major instrumentation used. The two test disks are mounted on parallel shafts and a normal load, W , is applied between them. The lower shaft is driven by a variable-speed hydraulic motor through pulleys and timing belt. The upper shaft is driven off the lower shaft by means of one of several sets of phase gears of different speed ratios. The surface velocities of the two disks are V_1 and V_2 , thus the sliding velocity is $V_s = V_1 - V_2$, the sum velocity is $V_t = V_1 + V_2$, and the sliding-to-sum velocity ratio is $M = V_s/V_t$.

The test oil, at temperature T_j , is jetted toward the center of the conjunction in the center plane of the disks. The disks are cooled largely by impinging jets of the test oil supplied by a horn which envelopes halves of the disks. The oil pressure is maintained at 40 psig, and the total oil flow rate is approximately 20 gpm. The oil temperature at the conjunction inlet, T_o , is measured by a thermocouple probe placed in the center plane of the disks, 0.250 in. ahead of the conjunction center, and riding with a slight pressure on one of the disks. The quasi-steady surface temperature of the disks, T_s , is estimated from T_o by means of a relationship established by a separate calibration employing thermocouples embedded in the disks and operating the disks with V_1 , V_2 , W , T_j and the oil systematically varied.

The upper shaft of the Caterpillar tester is so instrumented that the reaction torque on this shaft can be measured. The disk friction torque, T_f , is then the difference between the reaction torque and the machine-loss torque, the latter being due principally to losses in the upper-shaft support bearings. The machine-loss torque is derived from a separate calibration involving operating the disks in both normal and reverse directions of rotation with the same

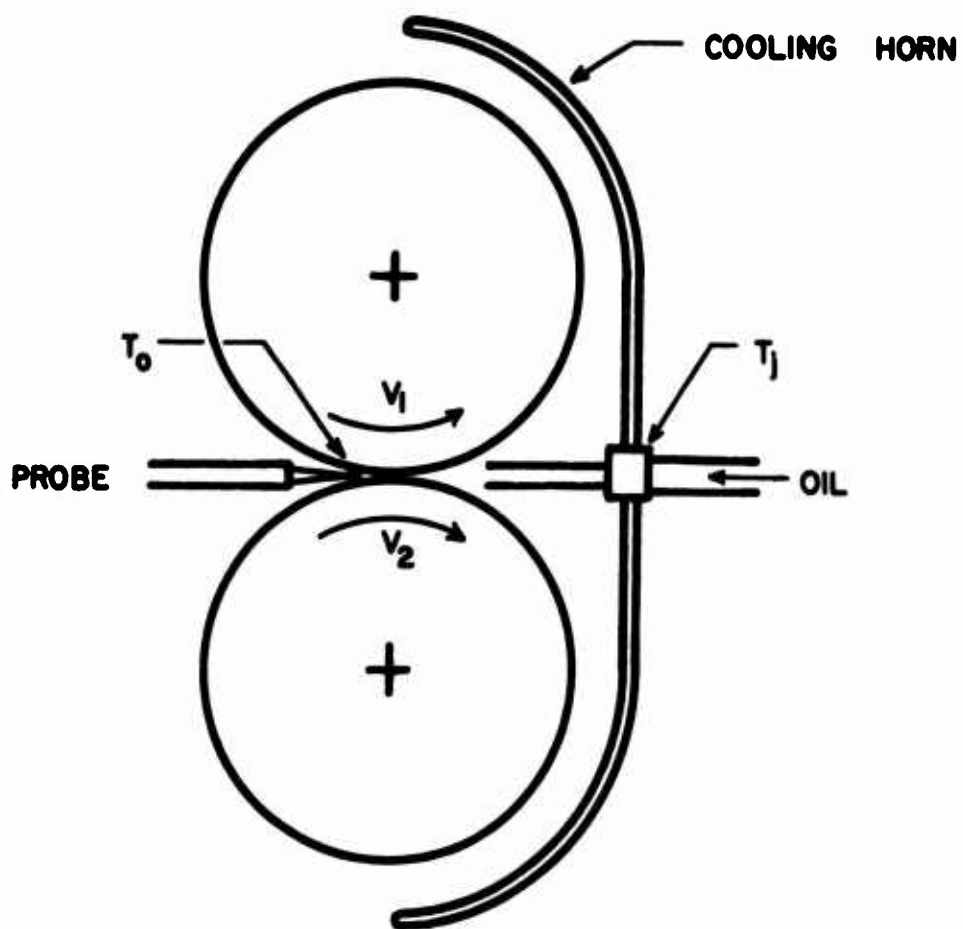


Figure D-1. Arrangement of test disks and instrumentation for SwRI disk tester A

V_1 , V_2 , W , T_f and oil; and with these variables varied systematically. The disk coefficient of friction is simply $f = T_f/rW$, where $r = 1.5$ in. is the disk radius.

The occurrence of metallic contact between the disks is observed by the instantaneous contact resistance technique. With straight mineral oils, reduction of the contact resistance to near zero is a good indication of metallic contact and usually of scoring when the test conditions are such that scoring occurs. With reactive oils, the contact resistance often collapses only partially due to the presence of a reactive surface film, so that the technique provides no reliable indication of asperity contact. A slight but abrupt increase in the reaction torque is usually the best indication of scoring under these circumstances. In all cases, actual scoring is always verified by visual examination.

Test Disks

The test disks employed in this program were all fabricated by the Bell Helicopter Company. They were all 3 in. in diameter and had a crown radius of 14 in. They were made of AISI 9310 CEVM steel, carburized to give a case thickness of 0.045-0.053 in., a case hardness as given in Table D-1, and a core hardness of 36-41 R_C .

As shown in Table D-1, 10 different types of test disks were used, consisting of 5 types of surface finishes and 2 types of surface treatments. The "plain" disks were not surface-treated, i.e., they were as ground or honed. The "oxidized" disks were surface-treated with a black oxide after grinding or honing. The black oxide was applied in accordance with BHC Specification BPSFW 4084, to a nominal thickness of "less than 100 μ in." However, measurements made at SwRI showed virtually no effect on the surface roughness by the black oxide treatment. The symbol δ_i denotes the initial composite surface roughness of the disk pairs.

Disk Test Program

The 10 types of disks were tested with 2 test oils, over a range of sliding and sum velocities to be detailed later. Most of the tests were conducted at an oil jet temperature of 190°F, with some at 140°F. As shown in Table D-2, 187 tests were performed, 160 of which resulted in scoring failure.

TABLE D-1. AVERAGE PROPERTIES OF TEST DISK PAIRS

<u>Disk type</u>	<u>Surface finish</u>	<u>Surface treatment</u>	<u>Case hardness, R_c</u>	<u>δ_i, μin. AA</u>
1	Soft circ. ground	Plain	58 $\pm$ 1	26 $\pm$ 2
1A	Soft circ. ground	Oxided	58 $\pm$ 1	26 $\pm$ 2
3	Rough circ. ground	Plain	62 $\pm$ 1	24 $\pm$ 2
3A	Rough circ. ground	Oxided	62 $\pm$ 1	24 $\pm$ 2
5	Honed	Plain	62 $\pm$ 1	5.5 $\pm$ 1
5A	Honed	Oxided	62 $\pm$ 1	5.5 $\pm$ 1
7	Rough cross ground	Plain	62 $\pm$ 1	23.5 $\pm$ 1
7A	Rough cross ground	Oxided	62 $\pm$ 1	23.5 $\pm$ 1
9	Smooth circ. ground	Plain	62 $\pm$ 1	9.5 $\pm$ 1
9A	Smooth circ. ground	Oxided	62 $\pm$ 1	9.5 $\pm$ 1

TABLE D-2. NUMBER OF DISK TESTS PERFORMED
AND SCORED

Oil code	T _j , °F	Disk type	No. of tests		Disk type	No. of tests	
			Total	Scored		Total	Scored
F	190	1	5	5	1A	5	5
F	190	3	65	58	3A	24	20
	140	3	-	-	3A	7	7
F	190	5	3	3	5A	5	4
F	190	7	6	6	7A	4	3
F	190	9	<u>3</u>	<u>2</u>	9A	<u>6</u>	<u>5</u>
Total for Oil F			<u>82</u>	<u>74</u>		<u>51</u>	<u>44</u>
E	190	3	1	1	3A	12	12
	140	3	-	-	3A	6	6
E	190	5	3	3	5A	8	5
E	190	7	4	3	7A	2	0
	140	7	2	2	7A	-	-
E	190	9	<u>6</u>	<u>4</u>	9A	<u>10</u>	<u>6</u>
Total for Oil E			<u>16</u>	<u>13</u>		<u>38</u>	<u>29</u>
Grand total			<u>98</u>	<u>87</u>		<u>89</u>	<u>73</u>

Note further from Table D-2 that the number of tests performed with Oil F was 82 with plain disks and 51 with oxidized disks, or a total of 133. The number of tests performed with Oil E was 16 with plain disks and 38 with oxidized disks, or a total of 54. Viewed in another way, 98 tests were performed with the plain disks, while 89 tests were performed with the oxidized disks.

Further, most of the tests were performed with Types 3 and 3A disks (a total of 109 tests for both oils), mainly due to the customary use of circumferentially-ground disks in disk testing and also cost considerations. As it turned out, it was found that, at about the same initial composite surface roughness (Table D-1), Types 7 and 7A disks gave substantially different results from Types 3 and 3A disks (Chap. VI). Since spur gears generally slide normal to the grinding marks, and the helical and spiral bevel gears approximate this condition, it would have been more desirable to run more tests with the cross-ground disks. Unfortunately, this could not be done in the program, as the disks had to be ordered and fabricated in advance, and the program could not be modified by the time this effect was noted.

Disk Test Procedure

The test disks were examined for nicks, scratches, rust spots, etc., and wiped dry; after which they were inspected for case hardness and transverse and circumferential surface roughnesses. Then they were covered with a straight mineral oil and placed in storage. Just prior to testing, they were cleaned with trisolvant, wiped dry with a clean cloth, and assembled on the shafts to check the runout.

The break-in of the test disks was performed using the desired test oil at $T_j = 90^\circ\text{F}$, $V_s = 23.6$ ips, and $V_t = 70.8$ ips. The load schedule comprised 4 equal increments of 500 lb of 15 min duration each, which gave a maximum break-in load of 2000 lb. This break-in procedure was used regardless of the test conditions to be employed later. Following the break-in, the disks were removed from the shafts for inspection and surface roughness measurement, and then reinstalled for subsequent testing.

The testing was performed using the desired T_j at an initial load of 70 lb, with V_s and V_t brought up to the desired values. Subsequent to this, the load was increased in equal increments of 35 lb

of 3 min duration each, until scoring failure occurred or until the machine capacity (about 6000 lb) was reached. Upon conclusion of the test, the disks were removed for inspection and surface roughness measurement.

When changing from an oil to one of another type, the following procedure was used: The used test oil was first drained from the system. The system was flushed completely with a clean tri-solvent (equal parts of reagent grade acetone, benzene, and isopropyl alcohol), which was drained and discarded. This was followed by two more flushings with the clean solvent. The system was then charged with new test oil. This oil was circulated for 30 min, then drained and discarded. A second charge of test oil was handled likewise. After this oil was drained, the sump was filled with its normal charge of test oil. When testing for extended periods with the same oil, samples were periodically drawn and checked for viscosity and neutralization number changes, and the oil was replaced as necessary by draining and one flushing with new oil. The criteria for oil change were a viscosity increase of 5 percent, or a neutralization increase of 0.2 mg KOH/g. These conditions were, however, never reached in the tests reported herein.

The surface roughnesses of the disks were measured at the end of each break-in and also at the end of each test, from which the corresponding composite surface roughnesses of the disk pairs were calculated by the procedure given in Appendix B. For the break-in and test procedures used herein, it was found that the composite surface roughness of the disk pairs was generally reduced about 20 percent after break-in and about 30 percent after test.^{25, 26} These values did not vary much when all tests were taken as a whole. Therefore, for the sake of brevity, these composite surface roughness values are not included in the data tabulation in this report.

Summary of Disk Test Results

The summaries of all disk test results are given in Tables D-3 to D-23. Each table presents the conditions reached at scoring in each test for a given test disk and test oil combination, at a given test oil jet temperature. Replicate tests were run in most cases, sometimes to as many as 5 to 6 tests.

The nomenclature and symbols used in these tables have mostly been introduced earlier, and also given in the List of Symbols at the end of this report. The only necessary explanations are given

below.

The "mode" of scoring failure is identified as follows: The symbol "CR" represents that scoring was detected by a collapse of the instantaneous contact resistance and subsequently verified by visual inspection. The symbol "TO" represents that scoring was detected by a torque increase and subsequently verified by visual inspection. The symbol "PS" represents premature scoring, i. e., scoring occurring during the first load step ($W = 70$ lb) and in most instances before the conditions stabilized. The symbol "No" represents that no scoring was obtained at the reported highest load, at which time the test was terminated.

Except as noted below, the quantities W , f , T_o , and T_s are measured quantities. The quantity ΔT was computed by Equation (3) in Chapter II, by using Kelley's equivalent unit load¹⁸ since the test disks are crowned. T_c is as defined by Equation (1) in Chapter II.

The minimum oil film thickness, h_m , was computed by Equation (C-1) in Appendix C, by taking $\phi_s \phi_t = 1$. The EHD film thickness ratio, Λ , was calculated by Equation (C-5) in Appendix C, based on h_m as computed above and the composite surface roughness of each disk pair at the end of the test.

For each set of replicate tests, the scoring load or the highest test load, W , for each test is tabulated. The average scoring load, Avg. W , for the replicate tests is given immediately below the tabulated values of W .

For each set of replicate tests, the conjunction temperature, T_c , for each test at scoring is tabulated. The critical temperature, T_{cr} , is computed by Weibull analysis from the individual values of T_c for the replicate tests, at 10-percent probability. This T_{cr} value is given immediately below the tabulated T_c values. The values given in the parenthesis immediately after the T_{cr} value are the lower and upper limits of T_{cr} at 90-percent confidence.

In performing the Weibull analysis, all T_c values for the prematurely scored tests were treated as if these tests actually scored at the initial load of 70 lb, and the coefficient of friction at scoring was estimated from the f vs. W history of the other tests in the same set. From these f and W values, the approximate values of T_o , T_s , ΔT , and T_c were computed. For tests with no scoring, the tabulated values of T_c were treated as suspensions, and included in

the Weibull analysis.

These results are discussed in Chapter VI, along with other available data.

TABLE D-3. SCORING RESULTS FOR TYPE 1 DISKS AND OIL F ($T_j = 190^\circ\text{F}$)

Test no.	V_s, ips	V_t, ips	M	Mode	W, lb	f	$T_o, ^\circ\text{F}$	$T_s, ^\circ\text{F}$	$\Delta T, ^\circ\text{F}$	$T_c, ^\circ\text{F}$	$h_m, \mu\text{in.}$	Λ
F122	200	360	0.556	CR	977	0.0248	217	233	202	435	2.8	0.19
F125				CR	1077	0.0282	220	240	241	481	2.8	0.21
				Avg. W = 1027						$T_{cr} = 390 (300/508)$		
F105	400	720	0.556	TO	1073	0.0199	223	235	240	475	4.4	0.36
F126				CR	657	0.0229	211	218	216	435	5.0	0.38
				Avg. W = 925						$T_{cr} = 395 (322/485)$		
F106	600	1080	0.556	CR	273	0.0304	201	199	226	426	7.8	0.56

TABLE D-4. SCORING RESULTS FOR TYPE 1A DISKS AND OIL F ($T_j = 190^\circ\text{F}$)

Test no.	V_s, ips	V_t, ips	M	Mode	W, lb	f	$T_o, ^\circ\text{F}$	$T_s, ^\circ\text{F}$	$\Delta T, ^\circ\text{F}$	$T_c, ^\circ\text{F}$	$h_m, \mu\text{in.}$	Λ
F121	200	360	0.556	CR	355	0.0296	202	208	145	353	3.5	0.26
F124				CR	312	0.0	199	202	139	341	3.6	0.30
				Avg. W = 334						$T_{cr} = 330 (302/360)$		
F107	400	720	0.556	CR	212	0.0321	197	197	172	369	6.2	0.36
F126				PS	70	-0.0596	-191	-191	-184	-374	-	-
				Avg. W = 141						$T_{cr} = 362 (341/384)$		
F108	600	1080	0.556	PS	70	-	-	-	-	-	-	-

TABLE D-5. SCORING RESULTS FOR TYPE 3 DISKS AND OIL F ($T_j = 190^\circ\text{F}$)

Test no.	V_s, ips	V_t, ips	M	Mode	W, lb	f	$T_o, ^\circ\text{F}$	$T_s, ^\circ\text{F}$	$\Delta T, ^\circ\text{F}$	$T_c, ^\circ\text{F}$	$h_m, \mu\text{in.}$	A
F82	200	360	0.556	CR	2816	0.0211	246	286	297	578	2.1	0.15
F104				CR	3076	0.0214	255	298	309	607	2.0	0.14
F163				CR	3400	0.0209	257	204	317	621	1.9	0.15
F168				TO	3240	0.0199	258	306	295	601	1.9	0.13
F176				PS	-70	-0.0518	-193	-193	-113	-306	-	-
F194				TO	3446	0.0209	259	303	319	622	1.9	0.15
				Avg. W = 2675						$T_{cr} = 307 (160/592)$		
F130	120	600	0.200	No	>6373	<0.0272	-258	-284	>253	>538	-2.6	-0.19
F186				No	>5353	<0.0281	-258	-297	>239	>536	-2.6	-0.19
F187				No	>5924	<0.0258	-281	-307	>231	>538	-2.3	-0.16
				Avg. W = 5883						$T_{cr} > 537$		
F83	200	600	0.333	CR	3406	0.0236	242	265	269	534	3.0	0.18
F133				TO	4144	0.0219	258	289	276	565	2.7	0.14
F145				TO	5306	0.0228	278	331	325	656	2.3	0.14
F155				CR	3316	0.0220	251	294	248	542	2.9	0.23
F171				CR	4475	0.0207	264	302	271	573	2.6	0.19
				Avg. W = 4129						$T_{cr} = 508 (434/596)$		
F142	334	600	0.556	CR	2705	0.0173	265	310	302	612	2.7	0.20
F160				CR	2470	0.0198	254	278	331	610	2.9	0.23
F165				CR	1893	0.0199	242	274	290	565	3.2	0.25
F172				CR	905	0.0211	223	242	213	455	3.9	0.27
F180				CR	2504	0.0188	257	293	316	609	2.8	0.23
				Avg. W = 2095						$T_{cr} = 467 (361/604)$		
F80	400	720	0.556	CR	2855	0.0159	266	307	313	620	3.0	0.24
F102				CR	1848	0.0176	239	258	280	437	3.7	0.26
F164				CR	1509	0.0207	238	260	295	556	3.8	0.27
F169				TO	3328	0.0144	267	298	305	604	3.0	0.24
F177				PS	-70	-0.0670	-193	-193	-207	-399	-	-
F193				CR	1031	0.0199	227	242	235	478	4.3	0.32
				Avg. W = 1774						$T_{cr} = 392 (284/541)$		

TABLE D-5. SCORING RESULTS FOR TYPE 3 DISKS AND OIL F ($T_j = 190^\circ\text{F}$) (Cont'd)

Test no.	V _a , ips	V _t , ips	M	Mode	W, lb	f	T _o , °F	T _s , °F	ΔT, °F	T _c , °F	h, μin.	A
F131	180	900	0.200	No	>6392	<0.0243	-282	-316	>277	>593	-3.0	-0.22
F188				No	>5950	<0.0245	-284	-316	>269	>586	-3.0	-0.18
F189				TO	5906	0.0266	298	335	292	627	2.8	0.19
F195				TO	4900	0.0243	262	379	243	522	3.4	0.24
				Avg. W = 5787						T _{cr} = 505 (370/688)		
F134	300	900	0.333	No	>3468	<0.0190	-264	-295	>269	>564	-3.5	-0.24
F146				TO	4167	0.0194	281	327	300	626	3.1	0.22
F156				CR	1596	0.0207	235	253	198	451	4.5	0.32
F173				CR	3897	0.0188	275	313	282	595	3.2	0.21
F181				CR	4099	0.0184	261	274	282	556	3.5	0.27
				Avg. W = 3445						T _{cr} = 465 (339/637)		
F143	500	900	0.556	TO	1646	0.0164	242	257	274	531	4.3	0.35
F161				CR	2709	0.0153	277	317	329	646	3.3	0.25
F166				CR	1198	0.0192	231	248	274	523	4.8	0.34
F174				CR	666	0.0244	224	237	259	496	5.3	0.34
F182				PS	-70	-0.0852	-192	-192	-294	-486	-	-
F192				CR	1164	0.0222	234	246	312	557	4.7	0.34
				Avg. W = 1242						T _{cr} = 470 (384/577)		
F132	216	1080	0.200	No	>6031	<0.0243	-283	-311	>294	>605	-3.4	-0.23
F190				TO	5654	0.0234	287	329	274	603	3.3	0.22
F191				TO	5776	0.0192	272	297	228	525	3.6	0.29
				Avg. W = 5820						T _{cr} = 480 (318/726)		
F141	360	1080	0.333	TO	5758	0.0151	305	355	301	656	3.0	0.25
F147				CR	3689	0.0186	286	326	296	622	3.5	0.26
F157				CR	1152	0.0213	228	239	190	429	5.6	0.44
F175				CR	3706	0.0182	278	307	290	597	3.6	0.28
F183				CR	4244	0.0173	287	323	296	619	3.4	0.33
				Avg. W = 3710						T _{cr} = 446 (316/629)		

TABLE D-5. SCORING RESULTS FOR TYPE 3 DESKS AND OIL F ($T_j = 190^\circ\text{F}$ (Comp'd))

Test no.	V_s , ips	V_t , ips	M	Mode	W, lb	f	T_o , °F	T_s , °F	ΔT , °F	T_c , °F	h _m , μin.	A
F81	600	1080	0.556	CR	1030	0.0174	235	253	252	505	5.3	0.45
F103				CR	202	0.0388	200	197	248	445	8.1	0.56
F129				CR	981	0.0184	229	243	260	503	5.6	0.43
F144				CR	1019	0.0199	234	247	287	534	5.4	0.37
F162				CR	2196	0.0161	258	268	340	608	4.3	0.34
F167				CR	2861	0.0141	255	263	341	604	4.3	0.33
				Avg. W = 1382						$T_{cr} = 438 (356/539)$		
F84	400	1200	0.333	CR	817	0.0180	214	217	142	359	6.9	0.44
F128				CR	710	0.0249	214	218	184	402	7.0	0.49
F158				CR	2219	0.0177	241	242	231	473	5.2	0.43
F178				TO	3327	0.0176	278	312	282	594	3.9	0.35
F184				CR	2956	0.0187	267	288	281	569	4.2	0.32
				Avg. W = 2006						$T_{cr} = 329 (204/530)$		
F127	600	1800	0.333	CR	547	0.0207	210	206	164	371	9.7	0.63
F159				CR	1023	0.0228	231	240	247	487	7.8	0.53
F170				CR	818	0.0183	226	230	177	407	8.3	0.60
F179				CR	1302	0.0195	226	244	238	482	8.0	0.67
F185				CR	2993	0.0167	284	298	310	608	5.1	0.39
				Avg. W = 1337						$T_{cr} = 340 (229/506)$		

TABLE D-6. SCORING RESULTS FOR TYPE 3A DISKS AND OIL F (T_j = 190°F)

Test no.	V _s , ips	V _t , ips	M	Mode	W, lb	f	T _o , °F	T _s , °F	ΔT, °F	T _c , °F	hm, μin.	Λ
F74	200	360	0.556	CR	2798	0.0236	256	308	325	634	2.0	0.13
F78				CR	2627	0.0240	254	306	320	627	2.0	0.15
				Avg. W = 2712						T _{cr} = 615 (583/648)		
F135	120	600	0.200	No	>6384	<0.0295	-269	-312	>274	>586	-2.4	-0.16
F136				No	>6392	<0.0258	-264	-307	>240	>547	-2.5	-0.18
F152				No	>6419	<0.0255	-268	-310	>238	>548	-2.4	-0.19
F154				No	>6434	<0.0273	-275	-320	>255	>575	-2.3	-0.16
				Avg. W = 6407						T _{cr} = 564		
F62	200	600	0.556	CR	2649	0.0245	241	277	247	524	3.1	0.18
F63				CR	945	0.0276	214	231	166	397	4.2	0.22
F64				CR	127	0.0472	193	192	104	296	5.9	0.32
F137				TO	5188	0.0199	272	321	280	601	2.4	0.17
F138				CR	3567	0.0233	258	299	273	572	2.7	0.17
F148				CR	2394	0.0211	231	244	202	446	3.4	0.21
				Avg. W = 2478						T _{cr} = 290 (165/510)		
F139	334	600	0.556	CR	1315	0.0263	244	277	322	599	3.3	0.27
F140				CR	2652	0.0188	268	315	325	640	2.6	0.20
F149				CR	932	0.0226	226	244	232	475	3.8	0.28
F150				CR	1084	0.0218	242	272	242	513	3.4	0.24
F151				CR	1247	0.0212	235	264	252	516	3.5	0.26
				Avg. W = 1446						T _{cr} = 435 (322/574)		
F75	400	720	0.556	CR	1511	0.0206	251	291	295	585	3.5	0.31
F77				PS	-70	-0.0574	-194	-194	-177	-373	-	-
				Avg. W = 791						T _{cr} = 245 (74/806)		
F76	600	1080	0.556	PS	-70	-0.660	-202	-209	-248	-457	-	-
F61	400	1200	0.333	CR	2720	0.0172	256	283	249	532	4.6	0.30
F65				CR	2087	0.0175	243	264	222	486	5.1	0.39
				Avg. W = 2404						T _{cr} = 450 (358/565)		
F66	600	1800	0.333	CR	1190	0.0178	231	236	208	443	7.7	0.58
F67				CR	2009	0.0162	265	285	246	530	5.9	0.40
				Avg. W = 1600						T = 375 (234/601)		

TABLE D-7. SCORING RESULTS FOR TYPE 3A DISKS AND OIL F ($T_j = 140^\circ\text{F}$)

Test no.	V_s, ips	V_t, ips	M	Mode	W, lb	f	$T_o, ^\circ\text{F}$	$T_s, ^\circ\text{F}$	$\Delta T, ^\circ\text{F}$	$T_c, ^\circ\text{F}$	$h_{\text{min}}, \mu\text{in.}$	A
F68	200	360	0.556	CR	561	0.0210	156	170	130	299	6.4	-
F79				CR	1056	0.0271	172	194	229	423	5.1	0.35
				Avg. W = 809						$T_{\text{cr}} = 215 (87/531)$		
F72	200	600	0.333	CR	3443	0.0181	206	254	208	462	4.4	0.28
F69	400	720	0.556	CR	2464	0.0146	216	264	268	531	4.6	0.32
F70	600	1080	0.556	CR	1390	0.0144	206	240	242	482	7.0	0.46
F71	400	1200	0.333	CR	2984	0.0151	223	258	228	486	6.4	0.37
F73	600	1800	0.333	CR	1159	0.0160	186	195	184	380	13.1	0.68

TABLE D-8. SCORING RESULTS FOR TYPE 5 DISKS AND OIL F ($T_j = 190^\circ\text{F}$)

Test no.	V_s, ips	V_t, ips	M	Mode	W, lb	f	$T_o, ^\circ\text{F}$	$T_s, ^\circ\text{F}$	$\Delta T, ^\circ\text{F}$	$T_c, ^\circ\text{F}$	$h_{\text{min}}, \mu\text{in.}$	A
F96	200	360	0.556	CR	4975	0.0166	259	292	306	597	1.8	0.39
F94	400	720	0.556	CR	4972	0.0117	260	274	304	578	3.0	0.63
F95	600	1080	0.556	CR	4402	0.0102	269	285	304	589	3.8	1.05

TABLE D-9. SCORING RESULTS FOR TYPE 5A DISKS AND OIL F ($T_j = 190^\circ\text{F}$)

Test no.	V_s, ips	V_t, ips	M	Mode	W, lb	f	$T_{o, }^\circ\text{F}$	$T_{s, }^\circ\text{F}$	$\Delta T, ^\circ\text{F}$	$T_{c, }^\circ\text{F}$	$\text{hrr}, \mu\text{in.}$	Λ
F99	200	360	0.556	No	>4979	<0.0169	-264	-303	>310	>613	-1.8	-0.24
F97	400	720	0.556	CR	4733	0.0123	265	286	313	598	2.9	0.39
F100				CR	4260	0.0127	265	290	305	594	2.9	0.41
				Avg. W = 4496						$T_{cr} = 594$		
F98	600	1080	0.556	CR	2499	0.0128	242	246	289	534	4.7	0.76
F101				CR	2507	0.0130	247	274	239	513	3.4	0.44
				Avg. W = 2503						$T_{cr} = 500 (403/620)$		

TABLE D-10. SCORING RESULTS FOR TYPE 7 DISKS AND OIL F ($T_j = 190^\circ\text{F}$)

Test no.	V_s, ips	V_t, ips	M	Mode	W, lb	f	$T_{o, }^\circ\text{F}$	$T_{s, }^\circ\text{F}$	$\Delta T, ^\circ\text{F}$	$T_{c, }^\circ\text{F}$	$\text{hrr}, \mu\text{in.}$	Λ
F111	200	360	0.556	CR	4000	0.0174	256	298	287	585	1.9	0.12
F114				CR	4417	0.0180	264	315	312	626	1.8	0.10
				Avg. W = 4208						$T_{cr} = 560 (490/641)$		
F109	400	720	0.556	CR	2507	0.0130	247	274	239	513	3.4	0.20
F112				CR	4161	0.0129	278	322	307	629	2.7	0.12
				Avg. W = 3334						$T_{cr} = 430 (261/709)$		
F110	600	1080	0.556	CR	2222	0.0122	251	274	259	534	4.5	0.25
F113				CR	1404	0.0143	231	241	242	483	5.3	0.36
				Avg. W = 1813						$T_{cr} = 450 (360/562)$		

TABLE D-11. SCORING RESULTS FOR TYPE 7A DISKS AND OIL F ($T_j = 190^\circ\text{F}$)

Test no.	V_s, ips	V_t, ips	M	Mode	W, lb	f	$T_o, ^\circ\text{F}$	$T_s, ^\circ\text{F}$	$\Delta T, ^\circ\text{F}$	$T_c, ^\circ\text{F}$	hm, $\mu\text{in.}$	A
F115	400	720	0.556	No	>5068	<0.0084	-260	-291	>219	>510	-3.6	-0.26
F118				CR	5260	0.0078	251	270	208	478	3.8	0.32
				Avg. W =	5164					$T_{cr} = 440$ (358/540)		
F116	600	1080	0.556	CR	3424	0.0078	254	275	205	480	5.1	0.30
F117				CR	4356	0.0073	273	299	218	517	4.5	0.25
				Avg. W =	3890					$T_{cr} = 450$ (366/553)		

TABLE D-12. SCORING RESULTS FOR TYPE 9 DISKS AND OIL F ($T_j = 190^\circ\text{F}$)

Test no.	V_s, ips	V_t, ips	M	Mode	W, lb	f	$T_o, ^\circ\text{F}$	$T_s, ^\circ\text{F}$	$\Delta T, ^\circ\text{F}$	$T_c, ^\circ\text{F}$	hm, $\mu\text{in.}$	A
F93	200	360	0.556	No	>5050	<0.0201	-267	-302	>372	>674	-1.8	-0.26
F91	400	720	0.556	CR	4705	0.0141	294	351	356	707	2.4	0.35
F92	600	1080	0.556	CR	4072	0.0122	25	257	350	607	4.2	0.58

TABLE D-13. SCORING RESULTS FOR TYPE 9A DISKS AND OIL F ($T_j = 190^\circ\text{F}$)

Test no.	V_s, ips	V_t, ips	M	Mode	W, lb	f	$T_o, ^\circ\text{F}$	$T_s, ^\circ\text{F}$	$\Delta T, ^\circ\text{F}$	$T_c, ^\circ\text{F}$	$h_m, \mu\text{in.}$	Λ
F87	200	360	0.556	No	>4996	<0.0194	-278	-343	>357	>700	-1.6	-0.17
F90				PS	-70	-0.0490	-192	-192	-107	-298	-	-
				Avg. W = 2533								
F85	400	720	0.556	CR	4755	0.0138	292	341	351	692	2.5	0.24
F88				CR	1742	0.0188	250	282	290	572	3.5	0.27
				Avg. W = 3249						$T_{cr} = 485 (301/782)$		
F86	600	1080	0.556	CR	4477	0.0105	306	356	518	674	3.1	0.34
F89				CR	1671	0.0160	251	271	294	565	3.6	0.39
				Avg. W = 3074						$T_{cr} = 465 (288/750)$		

TABLE D-14. SCORING RESULTS FOR TYPE 3 DISKS AND OIL E ($T_j = 190^\circ\text{F}$)

Test no.	V_s, ips	V_t, ips	M	Mode	W, lb	f	$T_o, ^\circ\text{F}$	$T_s, ^\circ\text{F}$	$\Delta T, ^\circ\text{F}$	$T_c, ^\circ\text{F}$	$h_m, \mu\text{in.}$	Λ
E19	400	1200	0.333	CR	781	0.0231	220	228	179	406	9.3	0.53

TABLE D-15. SCORING RESULTS FOR TYPE 3A DISKS AND OIL E ($T_j = 190^\circ\text{F}$)

Test no.	V_s, ips	V_t, ips	M	Mode	W, lb	f	$T_o, ^\circ\text{F}$	$T_s, ^\circ\text{F}$	$\Delta T, ^\circ\text{F}$	$T_c, ^\circ\text{F}$	hm, $\mu\text{in.}$	Λ
E24	200	360	0.556	PS	-70	-0.1154	-195	-195	-252	-447	-	-
E25				CR	2080	0.0281	240	278	334	612	3.1	0.20
E29				PS	-70	-0.1154	-195	-195	-252	-447	-	-
E30				PS	-70	-0.1154	-195	-195	-252	-447	-	-
				Avg. W = 572						$T_{cr} = 400 (295/542)$		
E14	200	600	0.333	CR	764	0.0227	207	216	123	340	6.4	0.43
E16				CR	1030	0.0179	208	217	113	330	6.2	0.41
				Avg. W = 897						$T_{cr} = 320 (301/343)$		
E39	400	720	0.556	PS	-70	-0.0574	-194	-194	-177	-373	-	-
E15	400	1200	0.333	CR	2595	0.0123	249	266	173	439	6.6	0.46
E17				CR	550	0.0812	212	218	527	725	10.2	0.78
E18				CR	657	0.0258	214	220	183	404	9.9	0.63
				Avg. W = 1267						$T_{cr} = 235 (67/693)$		
E21	600	1800	0.333	TO	710	0.0207	225	230	187	417	11.9	0.80
E22				CR	1831	0.0185	279	297	269	566	7.4	0.48
				Avg. W = 1270						$T_{cr} = 325 (154/687)$		

TABLE D-16. SCORING RESULTS FOR TYPE 3A DISKS AND OIL E ($T_j = 140^\circ\text{F}$)

Test no.	V_s, ips	V_t, ips	M	Mode	W, lb	f	$T_o, ^\circ\text{F}$	$T_s, ^\circ\text{F}$	$\Delta T, ^\circ\text{F}$	$T_c, ^\circ\text{F}$	hm, $\mu\text{in.}$	Λ
E26	200	360	0.556	CR	781	0.0280	166	188	204	392	6.9	0.44
E37	200	600	0.333	CR	1457	0.0273	185	213	204	417	7.6	0.49
E27	400	720	0.556	CR	1279	0.0222	192	218	292	510	8.0	0.61

TABLE D-16. SCORING RESULTS FOR TYPE 3A DISKS AND OIL E ($T_j = 140^\circ\text{F}$) (Cont'd)

Test no.	V_s, ips	V_t, ips	M	Mode	W, lb	f	$T_{O_2}, ^\circ\text{F}$	$T_{S_2}, ^\circ\text{F}$	$\Delta T, ^\circ\text{F}$	$T_{C_2}, ^\circ\text{F}$	$h_m, \mu\text{in.}$	Λ
E31	600	1080	0.556	CR	234	0.0536	105	157	369	526	19.0	1.26
E36	400	1200	0.333	CR	1333	0.0225	200	220	227	448	10.6	0.63
E38	600	1800	0.333	CR	2311	0.0190	261	285	310	595	8.1	0.56

TABLE D-17. SCORING RESULTS FOR TYPE 5 DISKS AND OIL E ($T_j = 190^\circ\text{F}$)

Test no.	V_s, ips	V_t, ips	M	Mode	W, lb	f	$T_{O_2}, ^\circ\text{F}$	$T_{S_2}, ^\circ\text{F}$	$\Delta T, ^\circ\text{F}$	$T_{C_2}, ^\circ\text{F}$	$h_m, \mu\text{in.}$	Λ
E3	200	600	0.333	CR	4054	0.0149	289	268	186	455	4.2	0.85
E1	400	1200	0.333	CR	3549	0.0105	253	272	173	446	6.3	1.14
E2	600	1800	0.333	CR	3289	0.0092	268	281	180	461	7.5	1.20

TABLE D-18. SCORING RESULTS FOR TYPE 5A DISKS AND OIL E ($T_j = 190^\circ\text{F}$)

Test no.	V_s, ips	V_t, ips	M	Mode	W, lb	f	$T_{O_2}, ^\circ\text{F}$	$T_{S_2}, ^\circ\text{F}$	$\Delta T, ^\circ\text{F}$	$T_{C_2}, ^\circ\text{F}$	$h_m, \mu\text{in.}$	Λ
E20-2	200	360	0.556	No	>4000	<0.0170	-258	-310	>280	>590	-2.6	-0.27
E23				No	>4972	<0.0156	-268	-327	>287	>614	-2.3	-0.37
E28				CR	1564	0.0156	222	248	161	409	3.7	0.53
Avg. W > 3512												
										$T_{Cr} = 340 (162/713)$		

TABLE D-18. SCORING RESULTS FOR TYPE 5A DISKS AND OIL E ($T_j = 190^\circ\text{F}$) (Cont'd)

Test no.	V_s, ips	V_t, ips	M	Mode	W, lb	f	$T_o, ^\circ\text{F}$	$T_s, ^\circ\text{F}$	$\Delta T, ^\circ\text{F}$	$T_c, ^\circ\text{F}$	hm, $\mu\text{in.}$	A
E47	400	720	0.556	CR	3243	0.0115	279	331	242	573	3.7	0.69
E50				CR	1528	0.0124	241	272	179	451	5.2	0.83
				Avg. W = 2386						$T_{cr} = 360 (191/677)$		
E48	600	1080	0.556	CR	764	0.0124	216	221	155	375	9.0	1.40
E49				CR	422	0.0151	206	208	140	348	10.3	1.67
				Avg. W = 593						$T_{cr} = 317 (254/395)$		
E20-1	400	1200	0.333	No	>4986	<0.0112	274	296	218	515	5.2	0.61

TABLE D-19. SCORING RESULTS FOR TYPE 7 DISKS AND OIL E ($T_j = 190^\circ\text{F}$)

Test no.	V_s, ips	V_t, ips	M	Mode	W, lb	f	$T_o, ^\circ\text{F}$	$T_s, ^\circ\text{F}$	$\Delta T, ^\circ\text{F}$	$T_c, ^\circ\text{F}$	hm, $\mu\text{in.}$	A
E33	200	360	0.556	CR	7035	0.0149	279	341	325	666	2.1	0.15
E35	400	720	0.556	CR	4417	0.0134	283	322	328	650	3.5	0.20
E34	600	1080	0.556	CR	2951	0.0114	275	297	278	575	5.0	0.28
E32	400	1200	0.333	No	>3997	<0.0126	-280	-302	>221	>524	-5.1	-0.25

TABLE D-20. SCORING RESULTS FOR TYPE 7 DISKS AND OIL E ($T_j = 140^\circ\text{F}$)

Test no.	V_s, ips	V_t, ips	M	Mode	W, lb	f	$T_o, ^\circ\text{F}$	$T_s, ^\circ\text{F}$	$\Delta T, ^\circ\text{F}$	$T_c, ^\circ\text{F}$	$h_m, \mu\text{in.}$	Λ
E52	400	720	0.556	TO	6241	0.0112	322	356	325	681	9.9	0.64
E53	600	1080	0.556	CR	4296	0.0123	335	351	365	715	12.6	0.51

TABLE D-21. SCORING RESULTS FOR TYPE 7A DISKS AND OIL E ($T_j = 190^\circ\text{F}$)

Test no.	V_s, ips	V_t, ips	M	Mode	W, lb	f	$T_o, ^\circ\text{F}$	$T_s, ^\circ\text{F}$	$\Delta T, ^\circ\text{F}$	$T_c, ^\circ\text{F}$	$h_m, \mu\text{in.}$	Λ
E55	400	720	0.556	No	5768	0.0077	315	337	214	551	10.4	0.76
E56	600	1080	0.556	No	5754	0.0058	326	335	199	534	13.0	0.50

TABLE D-22. SCORING RESULTS FOR TYPE 9 DISKS AND OIL L ($T_j = 190^\circ\text{F}$)

Test no.	V_s, ips	V_t, ips	M	Mode	W, lb	f	$T_o, ^\circ\text{F}$	$T_s, ^\circ\text{F}$	$\Delta T, ^\circ\text{F}$	$T_c, ^\circ\text{F}$	$h_m, \mu\text{in.}$	Λ
E12	200	360	0.556	CR	2418	0.0244	243	288	313	600	3.0	0.39
E45	400	720	0.556	CR	3047	0.0148	287	342	300	643	3.5	0.38
E46	600	1080	0.556	No	>3993	<0.0109	-332	-405	>312	>717	-3.5	-0.52
E6	400	1200	0.333	CR	4410	0.0152	288	323	279	603	4.8	0.60
E9				No	>4996	<0.0158	-310	-354	>309	>662	4.2	0.49
				Avg. W = 4703						$T_{cr} = 565(458/697)$		
E7	600	1800	0.333	TO	3840	0.0118	310	341	248	589	5.7	0.50

TABLE D-23. SCORING RESULTS FOR TYPE 9A DISKS AND OIL E ($T_j = 190^\circ\text{F}$)

Test no.	V_s , ips	V_t , ips	M	Mode	W, lb	f	T_o , °F	T_s , °F	ΔT , °F	T_c , °F	hm, $\mu\text{in.}$	Λ
E13	200	360	0.556	No	>7967	<0.0159	-298	-372	>369	>742	-1.9	-0.29
E10-2				CR	2346	0.0187	465	265	236	502	3.3	0.31
					Avg. W = 5156					$T_{cr} = 360 (131/989)$		
E4	200	600	0.333	No	>3965	<0.0193	-245	-278	>239	>517	-4.0	-0.43
E10-1				No	>5043	<0.0214	-263	-305	>298	>603	-3.5	-0.30
					Avg. W = 4504					$T_{cr} > 560$		
E41	400	720	0.556	CR	774	0.0232	223	242	238	480	6.3	0.82
E42				CR	636	0.0192	215	220	178	392	7.0	0.93
					Avg. W = 705					$T_{cr} = 330 (197/554)$		
E43	600	1080	0.556	CR	347	0.0196	206	208	165	374	10.5	1.35
E44				CR	749	0.0155	224	234	191	425	8.4	1.00
					Avg. W = 548					$T_{cr} = 330 (229/475)$		
E5	400	1200	0.333	CR	3556	0.0115	260	284	190	474	5.9	0.55
E11	600	1800	0.333	No	>4979	<0.0088	-643	-349	>210	>559	-5.2	-0.57

APPENDIX E

ANALYSIS OF AGMA SPUR GEAR TEST DATA

As an aid to evaluating the scoring-limited performance of typical aerospace power gears, 13 sets of full-scale spur gear scoring test results, collected by the Tribology Division, AGMA Aerospace Gearing Committee, were supplied to this program. Only five sets of such data, from tests employing AISI 9310 steel gears and MIL-L-7808 or MIL-L-23699 oils that went far enough to reach scoring, were analyzed and made use of herein.

A brief description of the five test series selected for analysis is given in Table E-1. Note that Series A1, A2, and A3 were identical except for the surface roughness of the test gears. Other than Series B, all tests were performed at an oil jet temperature of 200°F at an unreported oil flow rate, and the gear surface temperature was not measured. Series B was performed at an unreported oil jet temperature with the oil flow rate varied but not reported; however, the gear surface temperature was measured and reported.

The tests were generally performed under the stated conditions by progressively increasing applied load (Series A1, A2, A3, and C) or by progressively decreasing the oil flow rate (Series B), until scoring was obtained.

Basis of AGMA Reported Data

The AGMA test results are reported in form of computer printouts for each test, listing, among other items, the values of the maximum conjunction-surface temperature rise, ΔT , vs. the roll angle, based on the AGMA gear scoring design guide.³ This ΔT herein designated as $\Delta T(\text{AGMA})$ for clarity, is given for 21 values of roll angle through the mesh, and also for the pitch point and for the lowest and highest points of single tooth contact. The equation for $\Delta T(\text{AGMA})$ is, by simple algebraic manipulation of the expression given in the AGMA design guide,

$$\Delta T(\text{AGMA}) = 0.0175 \left(\frac{50}{50-S} \right) \frac{w^{\frac{3}{4}} \left| \sqrt{\rho_p n_p} - \sqrt{\rho_g n_g} \right|}{R^{\frac{1}{4}}} \quad (\text{E-1})$$

TABLE E-1. DESCRIPTION OF AGMA SPUR GEAR TESTS

	Series A1	Series A2	Series A3	Series B	Series C
Test gear material, AISI	9310	9310	9310	9310	9310
Driver (pinion):					
N_p , no. of teeth	30	30	30	28	32
d , pitch diameter, in.	6.000	6.000	6.000	5.600	2.286
S' , surface finish, μ in. AA	8	20	25	10	12
Driven (gears):					
N_g , no. of teeth	30	30	30	39	64
D , pitch diameter, in.	6.000	6.000	6.000	7.800	4.572
S' , surface finish, μ in. AA	9	20	27	10	12
Other gear characteristics:					
ϕ , pressure angle, deg	25	25	25	25	20
F , effective face width, in.	0.500	0.500	0.500	1.550	0.250
m_c , contact ratio	1.47	1.47	1.47	1.48	1.78
Test oil, MIL-L-	7808D	7808D	7808D	7808G	23699
Test conditions:					
n_p , driver speed, rpm	3660	3660	3660	3247	20000
n_g , driven speed, rpm	3660	3660	3660	2311	10000
V_t , pitchline velocity, fpm	5749	5749	5749	4760	11948
T_j , oil jet temperature, °F	200	200	200	—	200
Oil flow rate, gpm	—	—	—	—	—

where the various quantities are defined in the List of Symbols. Note that S = surface roughness in the profile direction (after break-in), μ in. rms.

The critical temperature hypothesis (Chap. II, Sect. B) states that

$$T_c = T_s + \Delta T \quad (E-2)$$

and that scoring occurs when T_c reaches the critical temperature, T_{cr} , for the metal-oil combination concerned. Thus, the AGMA design guide gives

$$T_{cr}(AGMA) = T_s(AGMA) + \Delta T(AGMA) \quad (E-3)$$

where $T_{cr}(AGMA) = T_{cr}$ by the AGMA procedure, °F

$T_s(AGMA)$ = "initial temperature, " °F ("may be oil inlet")

$\Delta T(AGMA)$ = as defined by Equation (E-1)

Note that in the AGMA design guide, $\Delta T(AGMA)$ is calculated from the actual scoring-limited power, and as such it includes the effects of gear-tooth misalignment and dynamic load. Since these effects vary for different gear sets and for different operating conditions, the $\Delta T(AGMA)$ thus computed as well as the resulting $T_{cr}(AGMA)$ are not basic quantities. This is one of the reasons for introducing in Chapter VII the concept of the ideal scoring-limited power-transmitting capacity, to which corrections are applied for misalignment and dynamic load in order to arrive at the actual scoring-limited power-transmitting capacity.

Another source of error in the AGMA design guide is the assumption that $T_s(AGMA)$ may be taken as the oil inlet temperature, T_j . This has not been found to be true as discussed in Chapters VI and VII.

Finally, the $\Delta T(AGMA)$ equation, i. e., Equation (E-1), entails certain assumptions principally related to the coefficient of friction. This matter will now be examined.

Comparison of AGMA and True ΔT

As derived by Blok (Chap. II, Sect. B), the basic equation for ΔT is

$$\Delta T = \frac{1.11 f w}{\beta \sqrt{B}} \left| \sqrt{V_1} - \sqrt{V_2} \right| \quad (\text{E-4})$$

For spur gears, it can be shown that

$$B = (32 w R / \pi E^*)^{\frac{1}{2}}$$

$$V_1 = 2\pi \rho_p n_p / 60$$

$$V_2 = 2\pi \rho_g n_g / 60$$

where all the quantities are defined in the List of Symbols.

Substituting the above expressions into Equation (E-4), and taking $E^* = 33 \times 10^6$ psi, one obtains

$$\Delta T = \frac{15.23 f w^{\frac{3}{4}}}{\beta R^{\frac{1}{4}}} \left| \sqrt{\rho_p n_p} - \sqrt{\rho_g n_g} \right| \quad (\text{E-5})$$

Accordingly, if Equation (E-1) were to be equal to Equation (E-5), then the coefficient of friction implicit in Equation (E-1) must conform to

$$0.0175 \left(\frac{50}{50-S} \right) = 15.23 \frac{f(\text{AGMA})}{\beta}$$

Taking $\beta = 42.15 \text{ lb/}^\circ\text{F-in.-sec}^{\frac{1}{2}}$, then

$$f(\text{AGMA}) = 0.04843 \left(\frac{50}{50-S} \right) \quad (\text{E-6})$$

which is equivalent to assuming that $f(\text{AGMA}) = 0.060$ at $S \sim 10 \mu\text{in. rms.}$

The value of $f(\text{AGMA})$ as given by Equation (E-6) is, in general, much higher than the value of f derived from the sliding-rolling disk tests (Chap. VI, Sect. C). Their ratio is, from Equation (E-6),

$$\frac{f}{f(\text{AGMA})} = 20.65 f (1 - 0.02 S) \quad (\text{E-7})$$

and consequently

$$\frac{\Delta T}{\Delta T(\text{AGMA})} = 20.65 f (1 - 0.02 S) \quad (\text{E-8})$$

where f is, in general, a variable depending upon the metal-oil combination and operating conditions. Since f is usually lower than $f(\text{AGMA})$, it follows from Equation (E-8) that the true ΔT must be smaller than $\Delta T(\text{AGMA})$.

Calculation of T_s

As mentioned earlier, the AGMA design procedure does not provide a specific guideline for assigning the value of T_s . In practical gear scoring analysis, either T_s is taken as T_j as implied by the AGMA design guide, or else some sort of estimate must be made.

In calculating T_s , use will be made of the method outlined in Chapter VII, Section B, i. e., by the relation

$$T_s - T_j = C' \phi_{av}^{0.80} \quad (\text{E-9})$$

where ϕ_{av} = average frictional power loss, Btu/sec

C' = a fitting constant for gear systems

The selection of C' and the calculation of ϕ_{av} were discussed in Chapter VII, Section B.

Conversion of AGMA Test Data

In order to obtain the values of T_s , ΔT , and T_c for the AGMA tests, one alternative is to assign appropriate values of f and C' and to calculate the values of ϕ_{av} by a computer program. However, inasmuch as the AGMA computer printouts have already furnished key numerical results on the basis of the AGMA procedure, these results can readily be converted without using a computer. This latter alternative is explained below for the Series A1 tests.

Composite Surface Roughness. From Table E-1, the surface roughnesses of the pinion and gear in the profile direction for Series A1 are 8 and 9 $\mu\text{in. AA}$, respectively. Thus, by Equation (B-4) in Appendix B, the initial composite surface roughness of the pinion and gear surfaces is

$$\delta_i = 3(8 + 9)/4 = 12.8 \mu\text{in. AA}$$

Coefficient of Friction. It was shown in Chapter VI, Section C, that f is a function of $WV_s^{-\frac{1}{3}}$, but that f is nearly constant when $WV_s^{-\frac{1}{3}}$ is greater than 200. For the sake of convenience, this constant value of f is used in the present data conversion. For AISI 9310 steel gears and MIL-L-7808 oil (Oil F), this is, by Equation (57)

$$f = 0.0154 + 0.00007 \delta_i = 0.0163$$

Average Friction Power Loss. The average frictional power loss is given by Equation (72) as follows:

$$\phi_{av} = \left[\sum \int \phi'(\epsilon) d\epsilon \right] \frac{N_p}{360} \quad (\text{E-10})$$

where ϕ_{av} = average frictional power loss, Btu/sec

$$\phi' = f W V_s / 9336, \text{ Btu/sec}$$

$$N_p = \text{number of pinion teeth}$$

In this conversion process, f is taken as constant; thus ϕ_{av} becomes a function of $\int d(WV_s)$ only. Accordingly, the values of W and V_s can be readily deduced from the AGMA printouts and plotted vs. the roll angle. The areas under the curve for the single and double tooth contact regions are then measured by means of a planimeter.

Applying the above equation and with proper units, then for Series A1

$$\phi_{av} = 0.001324 P$$

where P = power transmitted, hp.

Quasi-Steady Surface Temperature. The quantity $(T_s - T_j)$ is then computed from Equation (E-9) by taking $C' = 173$. This is equivalent to assuming that the oil flow rate jetted toward the gear mesh is 1.0 gpm. At this flow rate, $C = 115$ from Figure 25; and applying Equation (74), $C' = 1.5 \times 115 = 173$. In other words,

$$T_s - T_j = 173 \phi_{av}^{0.80}$$

Since $T_j = 200^\circ\text{F}$ for Series A1, then

$$T_s = 200 + 173 \phi_{av}^{0.80}$$

Conjunction Surface Temperature Rise. The quantity ΔT is given by Equation (E-8), where S is expressed in $\mu\text{in. rms}$ by the AGMA procedure. In these tests, S' in $\mu\text{in. AA}$ was given. By taking $S = 0.9 S'$, then Equation (E-8) becomes

$$\frac{\Delta T}{\Delta T(\text{AGMA})} = 20.65 f (1 - 0.018 S')$$

For Series A1, $S' = (8 + 9)/2 = 8.5 \mu\text{in. AA}$, and $f = 0.0163$ as given earlier. Thus

$$\Delta T = 0.285 \Delta T(\text{AGMA})$$

Critical Scoring Point. The ΔT of special interest in gear scoring analysis is that which has the highest value in the mesh cycle. The AGMA computer printouts tabulate all $\Delta T(\text{AGMA})$ values vs. the roll angle, and the highest $\Delta T(\text{AGMA})$ values are attained approximately midway on the recess portion of double tooth contact. These values are therefore extracted from the AGMA computer printouts, and the revised ΔT is calculated by the preceding equation.

Revised Data for AGMA Tests

Series A1. Using the above procedure, the revised data for Series A1 at critical scoring point are calculated, and are tabulated in Table E-2. Note that scoring first occurred in Test 87 at 362 hp. At this power level, $T_s = 296^\circ\text{F}$, $\Delta T = 38^\circ\text{F}$, and $T_c = 334^\circ\text{F}$. By the AGMA rationale, the critical temperature would have to be 334°F . Yet, from Equation (50), the true critical temperature in this case is

$$T_{cr} = 540 - 3.80 \delta_i = 491^\circ\text{F}$$

Thus assuming no tooth misalignment and dynamic tooth load, the ideal scoring-limited power-transmitting capacity must be considerably greater than 362 hp. An estimate for the ideal power-transmitting capacity for this case will be given later. It is only necessary to emphasize here that the difference between the actual and ideal power-transmitting capacity must be due to misalignment and dynamic effects.

Series A2. Series A2 differs from Series A1 only in that the test gears have rougher surfaces. Following the same procedure as illustrated above, the revised data for Series A2 tests are presented in Table E-3. Scoring first occurred in Test 118 at 209 hp, at which time $T_s = 265^\circ\text{F}$, $\Delta T = 28^\circ\text{F}$, and $T_c = 293^\circ\text{F}$. Yet, the true critical temperature in this case is

$$T_{cr} = 540 - 3.80 \delta_i = 426^\circ\text{F}$$

which is considerably higher than 293°F .

Series A3. Series A3 differs from Series A1 and A2 in that the test gears have even rougher surfaces. Following the same procedure as illustrated above, the revised data for Series A3 tests are given in Table E-4. Scoring first occurred in Test 110 at 170 hp, at which time $T_s = 257^\circ\text{F}$, $\Delta T = 26^\circ\text{F}$, and $T_c = 283^\circ\text{F}$. The true critical temperature in this case is

TABLE E-2. REVISED DATA FOR SERIES A1 TESTS

Test no.	P, hp	ϕ_{av} , Btu/sec	T _j , °F	T _s , °F	$\Delta T(\text{AGMA})$, °F	ΔT , °F	T _c , °F	Remarks
82	329	0.4356	200	289	125	36	325	
96	348	0.4608	200	293	131	37	330	
92	355	0.4700	200	294	133	38	332	
101	361	0.4780	200	295	135	38	333	
87	362	0.4793	200	296	135	38	334	Scored
80	377	0.4991	200	299	139	40	339	Scored
102	384	0.5084	200	300	141	40	340	
97	391	0.5177	200	302	143	41	343	
106	393	0.5203	200	302	143	41	343	Scored
107	436	0.5773	200	311	155	44	355	Scored
103	438	0.5799	200	312	155	44	356	Scored
104	465	0.6157	200	317	163	46	363	Scored
105	474	0.6276	200	319	165	47	366	Scored

Conversion equations

$$\delta_i = 3(8 + 9)/4 = 12.8 \mu\text{in. AA}$$

$$f = 0.0154 + 0.00007 \delta_i = 0.0163$$

$$\phi_{av} = 0.001324 P$$

$$T_s = T_j + 173 \phi_{av}^{0.80}$$

$$\Delta T = 20.65 f (1 - 0.018 S') \Delta T(\text{AGMA})$$

$$= 0.285 \Delta T(\text{AGMA})$$

$$T_{cr} = 540 - 3.80 \delta_i = 491^\circ\text{F}$$

TABLE E-3. REVISED DATA FOR SERIES A2 TESTS

<u>Test no.</u>	<u>P, hp</u>	<u>ϕ_{av}, Btu/sec</u>	<u>T_j, °F</u>	<u>T_s, °F</u>	<u>$\Delta T(\text{AGMA})$, °F</u>	<u>ΔT, °F</u>	<u>T_c, °F</u>	<u>Remarks</u>
115	193	0.2794	200	261	117	27	288	
113	201	0.2858	200	263	120	27	290	
118	209	0.2972	200	265	123	28	293	Scored
119	223	0.3171	200	269	130	30	299	Scored
116	232	0.3299	200	271	134	31	301	
120	239	0.3399	200	273	136	31	304	Scored
114	270	0.3839	200	280	150	35	315	Scored
117	271	0.3854	200	280	150	35	315	Scored

Conversion equations

$$\delta_i = 3 (20 + 20)/4 = 30.0 \text{ } \mu\text{in. AA}$$

$$f = 0.0154 + 0.00007 \delta_i = 0.0175$$

$$\phi_{av} = 0.001422 P$$

$$T_s = T_j + 173 \phi_{av}^{0.80}$$

$$\begin{aligned} \Delta T &= 20.65 f (1 - 0.018 S') \Delta T(\text{AGMA}) \\ &= 0.231 \Delta T(\text{AGMA}) \end{aligned}$$

$$T_{cr} = 540 - 3.80 \delta_i = 426^\circ \text{F}$$

TABLE E-4. REVISED DATA FOR SERIES A3 TESTS

<u>Test no.</u>	<u>P, hp</u>	<u>ϕ_{av}, Btu/sec</u>	<u>T_j, °F</u>	<u>T_s, °F</u>	<u>$\Delta T(AGMA)$, °F</u>	<u>ΔT, °F</u>	<u>T_c, °F</u>	<u>Remarks</u>
110	170	0.2501	200	257	132	26	283	Scored
111	199	0.2927	200	265	149	30	295	Scored
108	207	0.3045	200	267	153	30	297	Scored
109	207	0.3045	200	267	153	30	297	Scored
112	217	0.3192	200	269	159	32	301	Scored

Conversion equations

$$\delta_i = 3 (25 + 27)/4 = 39.0 \mu\text{in. AA}$$

$$f = 0.0154 + 0.00007 \delta_i = 0.0181$$

$$\phi_{av} = 0.001471 P$$

$$T_s = T_j + 173 \phi_{av}^{0.80}$$

$$\begin{aligned} \Delta T &= 20.65 f (1 - 0.018 S') \Delta T(AGMA) \\ &= 0.199 \Delta T(AGMA) \end{aligned}$$

$$T_{cr} = 540 - 3.80 \delta_i = 392^\circ\text{F}$$

$$T_{cr} = 540 - 3.80 \delta_i = 392^\circ\text{F}$$

which is again considerably higher than 283°F .

Series B. Series B tests were conducted at 3 different power levels, with the oil flow rate reduced to approach scoring. Neither the oil jet temperature nor the oil flow rate was reported. However, T_s was measured and reported. Since T_s is known in this case, all that is needed is to revise the ΔT , in order to obtain T_c . The revised results are presented in Table E-5.

Note that scoring occurred in Test 272 at 516 hp, at which time $T_s = 402^\circ\text{F}$, $\Delta T = 20^\circ\text{F}$, and $T_c = 422^\circ\text{F}$; and also in Test 273 at 688 hp, $T_s = 409^\circ\text{F}$, $\Delta T = 24^\circ\text{F}$, and $T_c = 433^\circ\text{F}$. The true critical temperature in this case is

$$T_{cr} = 540 - 3.80 \delta_i = 483^\circ\text{F}$$

which is considerably higher than either 422°F or 433°F .

The quantity ϕ_{av} is not required in the data conversion, but it will be required in the estimate of the ideal scoring-limited power-transmitting capacity. This quantity is thus also furnished in Table E-5.

Series C. Series C was conducted with AISI 9310 steel gears and MIL-L-23699 oil (Oil E), in a similar manner as Series A1, A2, and A3. Hence a similar data conversion process may be used.

It was noted in Chapter VI, Section B, that the coefficient of friction for AISI 9310 steel disks was nearly the same with Oil E and Oil F, thus the same f equation is used in this case as before. The other data manipulations require no further comments. The revised results are presented in Table E-6.

Note that scoring first occurred in Test 10 at 254 hp, $T_s = 261^\circ\text{F}$, $\Delta T = 38^\circ\text{F}$, and $T_c = 299^\circ\text{F}$. This is considerably lower than the true critical temperature for this metal-oil combination, which, from Equation (52), is

$$T_{cr} = 515 - 3.80 \delta_i = 447^\circ\text{F}$$

TABLE E-5. REVISED DATA FOR SERIES B TESTS

Test no.	P, hp	ϕ_{av} , Btu/sec	T _j , °F	T _s , °F	$\Delta T(AGMA)$, °F	ΔT , °F	T _c , °F	Remarks
265	344	0.4259	—	171	52	15	186	
268	344	0.4259	—	280	52	15	295	
271	344	0.4259	—	398	52	15	413	
266	516	0.6388	—	171	70	20	191	
269	516	0.6388	—	305	70	20	325	
272	516	0.6388	—	402	70	20	422	Scored
267	688	0.8517	—	187	87	24	211	
264	688	0.8517	—	195	87	24	219	
270	688	0.8517	—	303	87	24	327	
273	688	0.8517	—	409	87	24	433	Scored

Conversion equations

$$\delta_i = 3(10 + 10)/4 = 15.0 \mu\text{in. AA}$$

$$f = 0.0154 + 0.00007 \delta_i = 0.0165$$

$$\phi_{av} = 0.001238 P$$

$$T_s = \text{measured}$$

$$\begin{aligned} \Delta T &= 20.65 f (1 - 0.018 S') \Delta T(AGMA) \\ &= 0.279 \Delta T(AGMA) \end{aligned}$$

$$T_{cr} = 540 - 3.80 \delta_i = 483^\circ\text{F}$$

TABLE E-6. REVISED DATA FOR SERIES C TESTS

Test no.	P, hp	ϕ_{av} , Btu/sec	T_j , °F	T_s , °F	$\Delta T(\text{AGMA})$, °F	ΔT , °F	T_c , °F	Remarks
1	63	0.0673	200	220	50	14	234	
4	190	0.2031	200	248	114	31	279	
7	222	0.2373	200	255	128	35	290	
10	254	0.2715	200	261	141	38	299	Scored
13	286	0.3057	200	267	154	42	309	Scored
16	317	0.3389	200	273	167	45	318	Scored
18	349	0.3731	200	278	179	48	326	Scored
20	381	0.4073	200	284	191	52	336	Scored

Conversion equations

$$\delta_i = 3(12 + 12)/4 = 18.0 \mu\text{in. AA}$$

$$f = 0.0154 + 0.00007 \delta_i = 0.0167$$

$$\phi_{av} = 0.001069 P$$

$$T_s = T_j + 173 \phi_{av}^{0.80}$$

$$\begin{aligned} \Delta T &= 20.65 f (1 - 0.018 S') \Delta T(\text{AGMA}) \\ &= 0.270 \Delta T(\text{AGMA}) \end{aligned}$$

$$T_{cr} = 515 - 3.80 \delta_i = 447^\circ \text{F}$$

Estimation of Ideal Scoring-Limited Power Capacity

As defined in Chapter VII, Section A, the ideal scoring-limited power-transmitting capacity, P_I , of a set of gears is that for the ideal case in the absence of tooth misalignment and dynamic load. This quantity may be predicted for the cases examined by the computer, as explained in Chapter VII, Section B. However, it can also be done by relatively simple calculations as illustrated below.

Series A1. At constant speed and constant T_j , Table E-2 gives

$$T_s = 200 + 173 \phi_{av}^{0.80}$$

Table E-2 also gives $T_{cr} = 491^\circ\text{F}$. Thus the general expression for ΔT is

$$\Delta T = 491 - T_s = 291 - 173 \phi_{av}^{0.80} \quad (\text{E-11})$$

Now, at constant speed and constant f , Equation (E-5) states that ΔT must also be proportional to $w^{0.75}$ or $P^{0.75}$. Take Test 87 in Table E-2, $\Delta T = 38^\circ\text{F}$ at $P = 362$ hp. Thus

$$\Delta T = 38 \left(\frac{P}{362} \right)^{0.75} \quad (\text{E-12})$$

By the critical temperature hypothesis, scoring would occur when ΔT from Equation (E-11) equals ΔT from Equation (E-12). To obtain this ΔT , the two ΔT curves from the two above equations may be easily calculated, and plotted vs. P . The intersection of these two curves then gives P_I on the abscissa and the corresponding ΔT on the ordinate. From Equation (E-11), the corresponding T_s is simply $T_s = 491 - \Delta T$.

Using the above procedure, it can be shown that, for Series A1 conditions,

$$P_I = 973 \text{ hp}$$

$$\Delta T = 80^\circ\text{F}$$

$$T_s = 411^\circ\text{F}$$

These results are tabulated in Table E-7, along with other pertinent data.

Series A2, A3, and C. An identical procedure may be used for these test series. The results obtained are presented in Table E-7 for comparison.

Series B. Series B requires a different treatment, since T_j in this case is not known. It is proposed to estimate the ideal performance for this case at two assumed values of T_j , and examine the results.

Consider Test 272 in Table E-5, which scored at 516 hp, at measured $T_s = 402^\circ\text{F}$. The value of T_j is not known. Moreover, scoring was obtained by reducing the oil flow rate, which is also not known. In the absence of such information, let it first be assumed that $T_j = 200^\circ\text{F}$. This then permits an estimate for the value of C' in Equation (E-10), thus:

$$\begin{aligned} C' &= (T_s - T_j) / \phi_{av}^{0.80} \\ &= (402 - 200) / (0.6388)^{0.80} = 289.1 \end{aligned}$$

Then under the assumed conditions

$$T_s - 200 = 289.1 \phi_{av}^{0.80}$$

Since $T_{cr} = 483^\circ\text{F}$, therefore

$$\Delta T = 483 - T_s = 283 - 289.1 \phi_{av}^{0.80} \quad (\text{E-13})$$

The other ΔT expression is

$$\Delta T = 20 \left(\frac{P}{516} \right)^{0.75} \quad (\text{E-14})$$

TABLE E-7. COMPARISON OF ACTUAL AND IDEAL PERFORMANCE

	<u>Series B</u> <u>(Test 272)</u>		<u>Series B</u> <u>(Test 273)</u>		<u>Series</u> <u>A1</u>	<u>Series</u> <u>A2</u>	<u>Series</u> <u>A3</u>	<u>Series</u> <u>C</u>
	<u>Case I</u>	<u>Case II</u>	<u>Case III</u>	<u>Case IV</u>	<u>(Test 87)</u>	<u>(Test 118)</u>	<u>(Test 110)</u>	<u>(Test 10)</u>
$\delta_i, \mu\text{in.}$	15.0	15.0	15.0	15.0	12.8	30.0	39.0	18.0
$T_j, ^\circ\text{F}$	200	250	200	250	200	200	200	200
V_t, fpm	4760	4760	4760	4760	5749	5749	5749	11968
f	0.0165	0.0165	0.0165	0.0165	0.0163	0.0175	0.0181	0.0167
<u>Actual Performance</u>								
$T_s, ^\circ\text{F}$	402	402	409	409	296	265	257	261
$\Delta T, ^\circ\text{F}$	20	20	24	24	38	28	26	38
$T_c, ^\circ\text{F}$	422	422	433	433	334	293	283	299
P_A, hp	516	516	688	688	362	209	170	254
<u>Ideal Performance</u>								
$T_s, ^\circ\text{F}$	458	456	454	453	411	361	334	355
$\Delta T, ^\circ\text{F}$	25	27	29	30	80	65	58	92
$T_{cr}, ^\circ\text{F}$	483	483	483	483	491	426	392	447
P_I, hp	700	756	879	933	973	644	496	821
<u>Comparison</u>								
P_A/P_I	0.74	0.68	0.78	0.74	0.37	0.32	0.34	0.31

Solving Equations (E-13) and (E-14) simultaneously as before, one obtains

$$P_I = 700 \text{ hp}$$

$$\Delta T = 25^\circ \text{F}$$

$$T_s = 458^\circ \text{F}$$

These values are tabulated in Table E-7 as Case I.

For Case II, it is assumed that $T_j = 250^\circ \text{F}$. In that case

$$C' = (402 - 250)/(0.6388)^{0.80} = 217.5$$

Thus, similar to the above,

$$\Delta T = 483 - T_s = 233 - 217.5 \phi_{av}^{0.80} \quad (\text{E-15})$$

and Equation (E-14) remains applicable in this case.

Solving Equations (E-14) and (E-15) simultaneously as before, then

$$P_I = 756 \text{ hp}$$

$$\Delta T = 27^\circ \text{F}$$

$$T_s = 456^\circ \text{F}$$

as tabulated in Table E-7 as Case II.

Note from Table E-5 that scoring was also obtained in Test 273 at 688 hp, at measured $T_s = 409^\circ \text{F}$. By similar calculations, the ideal performance based on Test 273 at $T_j = 200^\circ \text{F}$ and 250°F can also be estimated. These results are presented in Table E-7 as Case III and Case IV, respectively.

Comparison of Actual and Ideal Performance

Table E-7 compares the actual performance with the ideal

performance for the 5 test series examined. A quantity of key interest is the ratio of the actual to ideal scoring-limited power-transmitting capacity, P_A/P_I , which is given on the last line of the table.

Note for Series B that the value of P_A/P_I for Cases I, II, III, and IV, based upon two different tests each with two assumed T_j values, do not vary greatly. The average of this ratio for Series B is 0.74; the standard deviation is 0.04, or only 5.5 percent.

As defined in Chapter VII, Section A, the fact that P_A/P_I is less than unity is due to the effects of tooth misalignment and dynamic load. The deduced data so far do not directly provide an indication of the relative contributions of the misalignment and dynamic effects. However, the influence of dynamic effect is unmistakably reflected by the steady decrease of P_A/P_I in Table E-7 as the pitchline velocity, V_t , is increased. An attempt to separate the misalignment and dynamic effects is presented in Chapter VII, Section C.

APPENDIX F

SUMMARY OF SPUR GEAR TEST DATA

The spur gear test program consisted of replicate tests on five sets of ground spur gears and five sets of honed spur gears of same design and material, tested for scoring at the same speed and oil jet temperature. These tests were performed at BHC under subcontract to SwRI. BHC Report 299-097-005, "Results of Gear Tooth Scoring Investigation Conducted on 31 x 76 Spur Gears" by R. Battles, R. T. Jenkins, and C. E. Braddock, dated October 24, 1974, was submitted to SwRI on December 2, 1974. Copies of the full report are on file at USAAMRDL, BHC, and SwRI. The following is an abstract of the report, plus analysis and interpretations made by SwRI personnel.

Test Gears

A description of the spur gears is given in Table 7 in Chapter VIII, Section B. Briefly, the pinion had 31 teeth, a pitch diameter of 3.4671 in., and a face width of 1.375 in.; and the gear had 76 teeth, a pitch diameter of 8.9412 in., and a face width of 1.250 in. The diametral pitch was 8.5 in.⁻¹, and the pressure angle was 22.0°. The gears were made of AISI 9310 CEVM steel, carburized to a case thickness of 0.030-0.040 in., a case hardness of 60-63 R_C, and a core hardness of 33-41 R_C—essentially the same as the test disks employed in the disk test program (App. D).

The surface finishes of the gears were originally specified as about 17 μ in. AA for the ground gears and about 7 μ in. AA for the honed gears. However, the gears as received from the vendor were found to be considerably smoother. In order to expedite the test program, five sets of ground spur gears as received were selected for the spur gear test program, and the remaining five sets were rehoned at BHC.

The surface finishes of all pinions and gears were measured by BHC in both profile and lead directions before break-in, after break-in, and after test termination. These measurements were made on two teeth by three instruments at different times; but not by the same instrument at all times. In order to facilitate comparisons on the same basis, the reported measurements by BHC were converted to one instrument base by a constant multiplier. The revised initial surface roughness values for the gears are presented in Table F-1. The revised surface roughness values after break-in are presented in

TABLE F-1. INITIAL SURFACE ROUGHNESS OF SPUR GEARS

Test no.	Pinion, μ in. AA			Gear, μ in. AA			δ_i , μ in. AA (Pinion & gear)
	Profile	Lead	Composite	Profile	Lead	Composite	
<u>Ground gears</u>							
G1	10.1	4.5	7.3	7.6	4.2	5.9	13.2
G2	9.7	3.5	6.6	9.0	3.2	6.1	12.7
G3	8.8	4.9	6.9	8.8	2.8	5.3	12.7
G4	9.8	4.2	7.0	9.1	5.6	7.4	14.4
G5	<u>9.8</u>	<u>3.9</u>	<u>6.9</u>	<u>9.8</u>	<u>7.7</u>	<u>8.7</u>	<u>15.6</u>
Avg.	9.6	4.2	6.9	8.9	4.7	6.8	13.7
<u>Honed gears</u>							
H1	6.8	7.0	6.9	9.0	10.0	9.5	16.4
H2	7.0	6.5	6.8	8.5	9.0	8.8	15.6
H3	7.0	7.5	7.3	8.5	8.0	8.3	15.6
H4	7.0	7.3	7.2	9.5	10.5	10.0	17.2
H5	<u>7.0</u>	<u>6.3</u>	<u>6.7</u>	<u>7.5</u>	<u>7.5</u>	<u>7.5</u>	<u>14.2</u>
Avg.	7.0	6.9	7.0	8.6	9.0	8.8	15.8

Table F-2. Surface roughness values after test termination, being generally very high and erratic and not pertinent to this study, are not presented herein.

In Tables F-1 and F-2, the test numbers with prefix "G" refer to the ground gear sets, while those with prefix "H" refer to the honed gear sets. Each tabulated surface roughness value in the profile or lead direction is the revised value of the average of the two readings taken on the two teeth. The composite surface roughness of either the pinion or gear is calculated by using Equation (B-1) in Appendix B. The composite surface roughness of the gear set is then calculated by using Equation (B-2).

Referring now to the data for the ground gears shown in Table F-1, it will be noted that the initial composite surface roughness of the gear set, δ_i , varied from 12.7 $\mu\text{in. AA}$ to 15.6 $\mu\text{in. AA}$, with an average of 13.7 $\mu\text{in. AA}$. Note further that the ratio of surface roughness in the profile direction to that in the lead direction is fairly close to 2, which is characteristic of most ground surfaces (App. B). The initial composite surface roughness, δ_i , of the honed gear sets varied from 14.2 $\mu\text{in. AA}$ to 17.2 $\mu\text{in. AA}$, with an average of 15.8 $\mu\text{in. AA}$.

Table F-2 shows similar surface roughness data after break-in. Note that the break-in process reduced the average composite surface roughness of the ground gear sets by about 21 percent and that of the honed gear sets by about 27 percent—generally in line with the trends previously reported for sliding-rolling disks.²⁵

Test Oil

The test oil used was Oil F, the same MIL-L-7808G oil used in the disk test program (App. D).

Test Equipment

A 4-square regenerative test rig shown in Figure F-1 was used in testing. Note that the test gears were mounted on vertical shafts, with the gear as the driver. In addition to the conventional measurements of gear speed, gear torque, and oil jet temperature, temperature measurements were also made on the test pinion by means of four welded thermocouples at locations shown in Figure F-2.

Dimensional inspection of the test section housing, support bearings, and bearing retainers revealed a basic rig misalignment of

TABLE F-2. SURFACE ROUGHNESS OF SPUR GEARS
AFTER BREAK-IN

Test no.	Pinion, μ in. AA			Gear, μ in. AA			δ_i , μ in. AA (Pinion & gear)
	Profile	Lead	Composite	Profile	Lead	Composite	
<u>Ground gears</u>							
G1	7.0	5.0	6.0	5.6	5.0	5.3	11.3
G2	6.0	3.2	4.6	5.6	2.8	4.2	8.8
G3	6.7	4.2	5.5	8.3	5.6	7.0	12.5
G4	7.4	4.2	5.8	8.7	3.5	6.1	11.9
G5	<u>5.6</u>	<u>2.5</u>	<u>4.1</u>	<u>6.3</u>	<u>4.6</u>	<u>5.5</u>	<u>9.6</u>
Avg.	6.5	3.8	5.2	6.9	4.3	5.6	10.8
<u>Honed gears</u>							
H1	5.0	5.5	5.3	4.5	6.8	5.7	11.0
H2	5.3	5.5	5.4	5.5	6.5	6.0	11.4
H3	4.8	6.5	5.7	7.0	8.0	7.5	13.2
H4	2.5	5.0	3.8	6.5	7.0	6.8	10.6
H5	<u>4.8</u>	<u>5.5</u>	<u>5.2</u>	<u>6.0</u>	<u>6.8</u>	<u>6.4</u>	<u>11.6</u>
Avg.	4.5	5.6	5.1	5.9	7.0	6.5	11.6

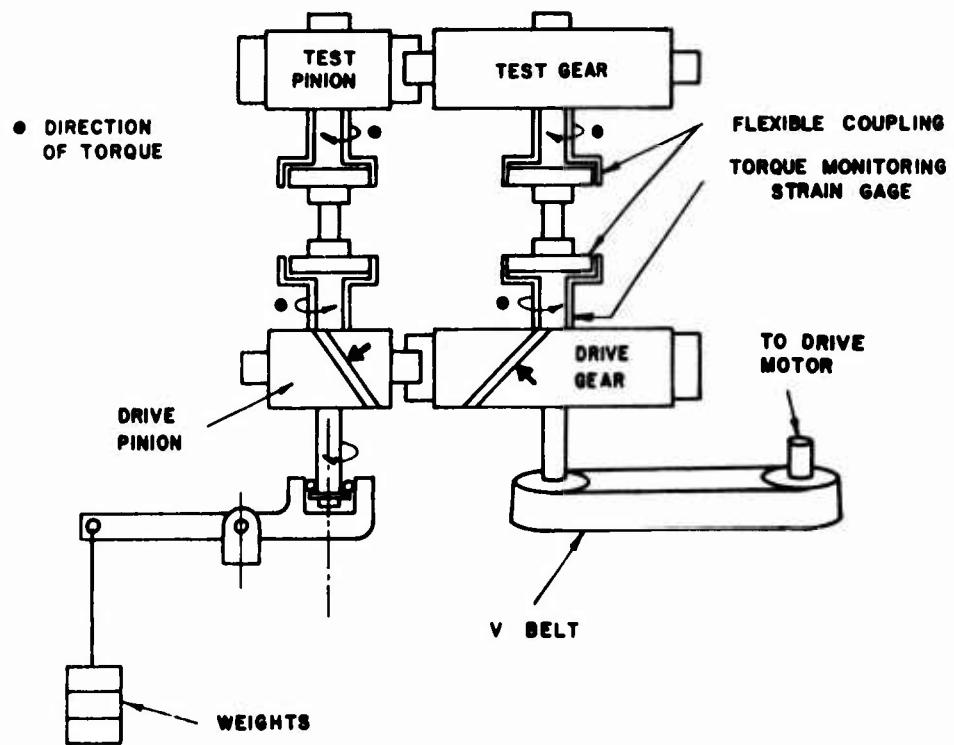


Figure F-1. Schematic of spur gear test rig

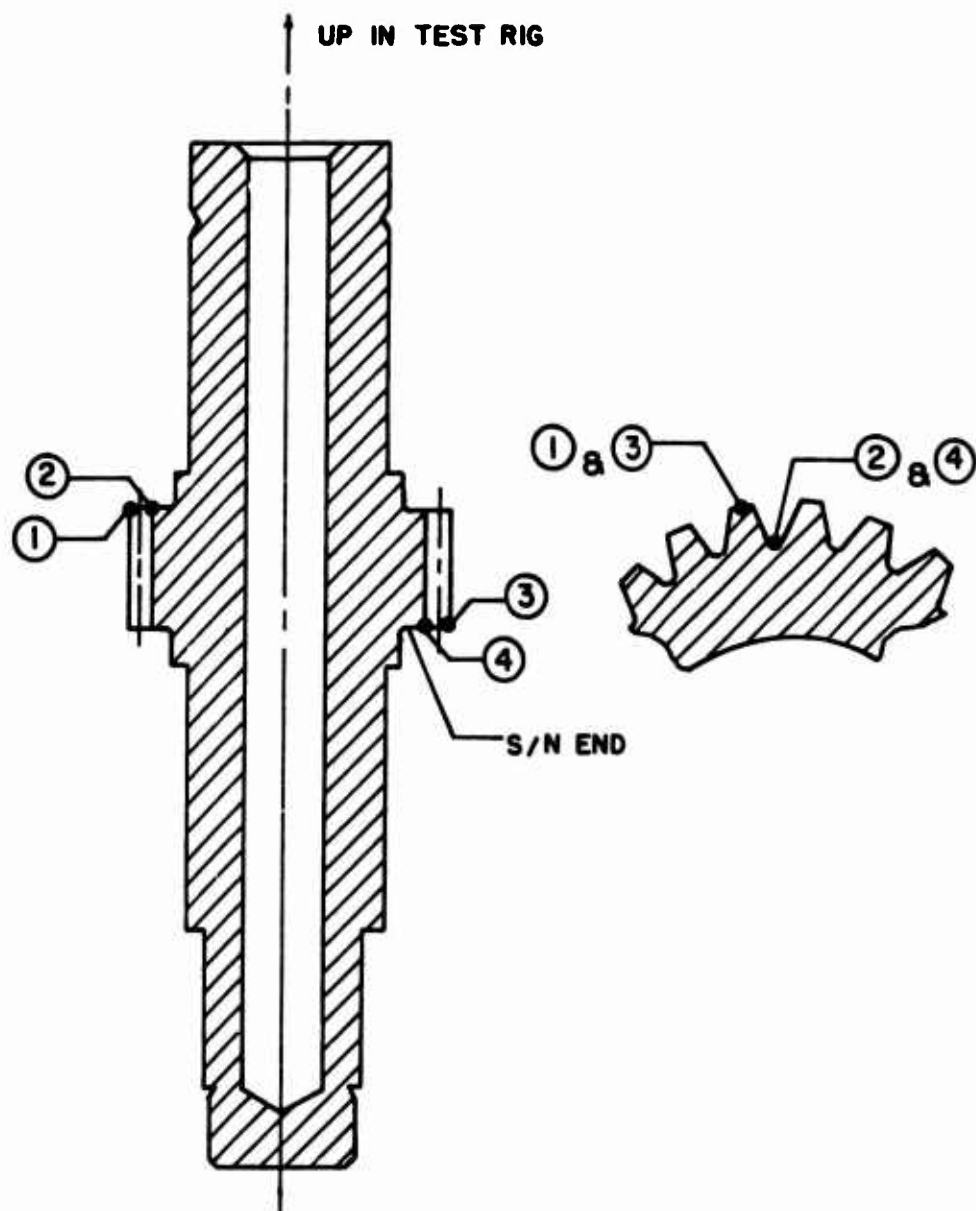


Figure F-2. Thermocouple locations on spur gear pinion

0.0007 rad. between the pinion and gear, with the lower end of the tooth (the S/N end, Fig. F-2) being the more heavily loaded than the upper end.

It was also found that the pinions and gears themselves had some lead errors. An attempt was made to match the pinion and gear in each set to compensate for the lead errors as much as possible. The net lead errors on the five sets of ground gears averaged 0.00006 rad. The net lead errors on the five sets of honed gears averaged 0.00014 rad. The variations from these average values from one matched set to another was less than 0.00008 rad. standard deviation.

The total misalignment of the gear teeth is the sum of the misalignments in the test rig and the gear sets. The total misalignment thus averaged 0.00076 rad. for the ground gear sets, and 0.00084 rad. for the honed gear sets.

Test Procedure

The gear sets were broken in by the following schedule: First, at an oil jet temperature of 120°F and a pinion speed of 4,385 rpm (55% of test speed), each gear set was operated for 30 min. at each of four load levels, viz., 500, 1000, 1500, and 2000 ppi. The oil jet temperature was then raised to 190°F, and each gear set was operated for 30 min. at each of two load levels, viz., 1000 and 2000 ppi.

The scoring test was conducted at an oil jet temperature of 250°F and a pinion speed of 7,992 rpm. The tooth load was increased from an initial value of 1000 ppi, in 150-ppi steps of 10 min. each, until either scoring was obtained or the test rig capacity (2950 ppi) was reached. After each load step, the rig was stopped and a visual inspection made to detect scoring. Following the inspection, the previous load step was repeated before proceeding to the next load step.

The test gears were lubricated by cascading oil and by an oil jet directed into the gear mesh. During the scoring test sequence, the oil jet temperature was 250°F and the flow rate was 0.28 gpm. Neither the temperature nor the flow rate of the cascading oil was measured. However, the cascading oil temperature was estimated by BHC to be close to 250°F, since the oil was drawn from the same heated reservoir as that supplied to the oil jet. The cascading oil flow rate was estimated by BHC to be about 2-3 times the oil jet flow rate, or about $2.5 \times 0.28 = 0.7$ gpm.

Scoring-Limited Power-Transmitting Capacity

The test results on the 10 spur gear sets are summarized in Table F-3.

With the ground gears, two of the tests did not result in scoring at the maximum rig capacity of 791 hp. The average scoring power was therefore in excess of 679 hp by an unknown margin. A statistically more meaningful scoring power, deduced from Weibull analysis at 10-percent scoring probability, is 507 hp, and its 90-percent confidence limits are 334 and 770 hp.

With the honed gears, all 5 tests produced scoring. The average scoring power was 606 hp. A statistically more meaningful scoring power, deduced from Weibull analysis at 10-percent scoring probability, is 425 hp, and its 90-percent confidence limits are 270 and 688 hp.

It is interesting to note that the ground gears were smoother than the honed gears, hence the ground gears would be expected to score at a higher power level than the honed gears. This trend is indeed reflected by both the average and statistically-deduced scoring power. However, the statistically-deduced values are more meaningful. The poor confidence limits were due to the inherent scatter of the scoring phenomenon and the very few tests performed.

Pinion Surface Temperature

The pinion surface temperature was measured by means of welded thermocouples at four locations as shown in Figure F-2. The readings at scoring or at test termination (i. e., at 791 hp) are presented in Table F-3. Note that the temperature at the upper end of the pinion was much lower than that at the lower end. Much of this difference was of course due to the effect of tooth misalignment discussed earlier, which placed less load on the upper end of the gear teeth than on the lower end. The other reason might be that the cascading oil, which must go through the cooler upper support bearing, was actually at a lower temperature than 250° F as it hit the upper gear surfaces, and became heated as it progressed downward. On the other hand, temperature measurements of this kind are difficult to make at best; and it is suspected that all reported pinion surface temperature readings are probably on the low side. There is presently no realistic way to evaluate the overall flow and heat transfer effects being encountered.

TABLE F-3. SUMMARY OF SPUR GEAR TEST RESULTS

Test no.	Scoring load, ppi	Scoring torque, in.-lb	Scoring power, hp	Pinion surface temperature, °F			
				Upper end		Lower end	
				Tooth tip	Tooth root	Toothtip	Tooth root
<u>Ground gears</u>							
G1	2050	10622	550	219	234	285	278
G2	>2950	>15285	>791	>227	>245	>269	>264
G3	2650	13731	711	235	246	280	275
G4	2050	10622	550	231	244	256	255
G5	>2950	>15285	>791	>237	>251	>270	>271
<u>Honed gears</u>							
H1	2800	14508	751	245	240	300	285
H2	2500	12954	671	236	245	312	299
H3	1900	9845	510	237	239	278	—
H4	2350	12176	630	246	251	283	282
H5	1750	9067	469	228	235	263	267

APPENDIX G

SUMMARY OF SPIRAL BEVEL GEAR TEST DATA

The spiral bevel gear test program consisted of five replicate tests on a specific design of spiral bevel gears. BHC Report 299-097-004, "Results of Gear Tooth Scoring Investigation Conducted on 22 x 23 Spiral Bevel Gears" by R. Battles, R. T. Jenkins, and C. E. Braddock, dated June 27, 1974, was submitted to SwRI on October 31, 1974. Copies of the full report are on file at USAAMRDL, BHC, and SwRI. The following is an abstract of the report, plus analysis and interpretations made by SwRI personnel.

Test Gears

A description of the spiral bevel gears tested in this program is given in Table 11 in Chapter VIII, Section D. Briefly, the pinion had 22 teeth, a pitch diameter of 3.6000 in., and a face width of 0.866 in.; and the gear had 23 teeth, a pitch diameter of 3.7637 in., and a face width of 0.866 in. The diametral pitch was 6.111 in.⁻¹, the pressure angle was 22.5°, and the spiral angle was 35.0°. The gears were made of AISI 9310 CEVM steel, carburized to a case thickness of 0.030-0.040 in., a case hardness of 60-63 R_C, and a core hardness of 33-41 R_C—essentially the same as the test disks employed in the disk test program (App. D).

The surface finish of the gears was specified as 22 μ in. AA maximum. Attempts were made at BHC to measure the surface roughnesses of the pinions and gears; but without success. Therefore, no surface roughness readings were reported.

Test Oil

The test oil used was Oil F, the same MIL-L-7808G oil used in the disk test program (App. D).

Test Equipment

A standard transmission test stand available at BHC was modified for use in testing. As shown in Figure G-1, an electric motor was connected to a variable-speed magnetic coupling driving a speed-up gearbox, which drove the test gears via a slave transmission. To simplify the operation of the test stand, the main rotor mast was removed, and a water brake was used for power absorption.

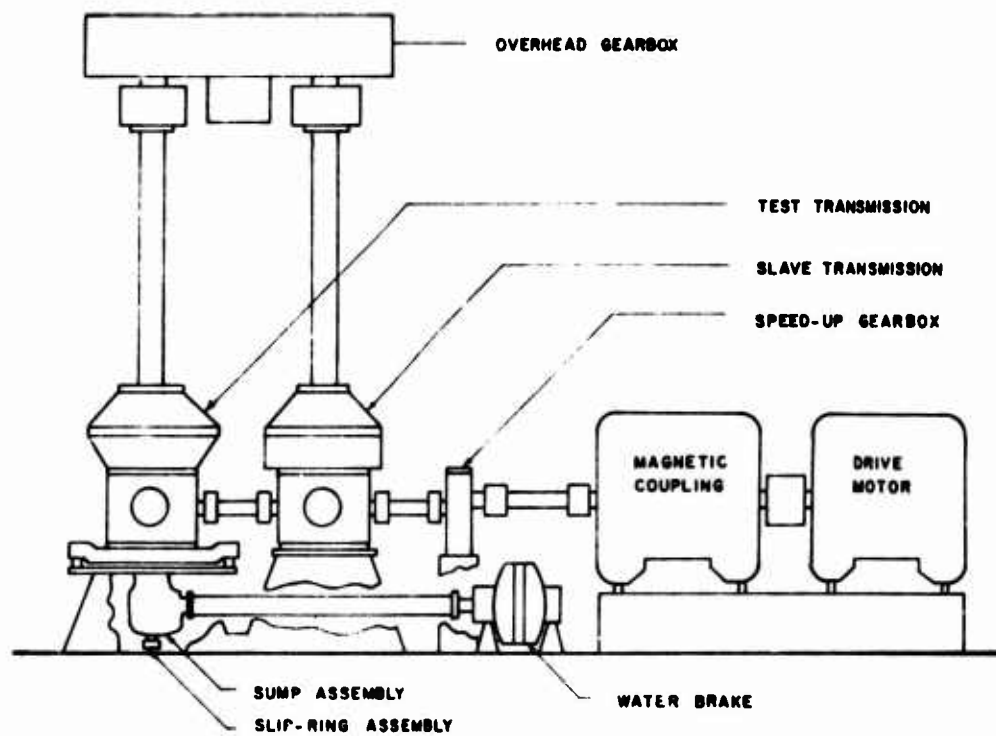


Figure G-1. Schematic of spiral bevel gear test rig

The test gears were installed in an overhung mounting with each gear supported by a duplex ball bearing and a roller bearing. The pinion was the driver.

In addition to the conventional measurements of gear speed, gear torque, and oil jet temperature, temperature measurements were also made on the test pinion by means of five imbedded or welded thermocouples at locations shown in Figure G-2.

No information on possible test gear misalignment was available.

Test Procedure

The gear sets were broken in by the following schedule: First, at an oil jet temperature of 120°F and a pinion speed of 2,700 rpm (60% of test speed), each gear set was operated for 30 min. at each of three load levels, viz., 517, 1035, and 1551 ppi. The oil jet temperature was then raised to 190°F, and each gear set was operated for 30 min. at each of two load levels, viz., 1035 and 2070 ppi.

The scoring test was conducted at an oil jet temperature of 190°F and a pinion speed of 4,500 rpm. The tooth load was increased from an initial value of 2070 ppi, in 207-ppi steps of 10 min. each, until scoring was obtained. After each load step, the rig was stopped for visual inspection of scoring. Following the inspection, the previous load step was repeated before proceeding to the next load step.

The test gears were lubricated by cascading oil and by an oil jet directed into the mesh. During the scoring test sequence, the oil jet temperature was 190°F and the flow rate was 0.45 gpm. Neither the temperature nor the flow rate of the cascading oil was measured. However, as in the case of spur gear tests (App. F), it is estimated that the cascading oil temperature was close to 190°F and the flow rate was probably on the order of 1.1 gpm.

Scoring-Limited Power-Transmitting Capacity

Six tests were run on the spiral bevel gears. However, one test was aborted because the test oil pump was inadvertently not turned on. The results of the five remaining valid tests are summarized in Table G-1. The average scoring power was 367 hp. A statistically more meaningful scoring power, derived from Weibull analysis at 10-percent scoring probability, is 346 hp, and its 90-percent confidence limits are 320 and 374 hp.

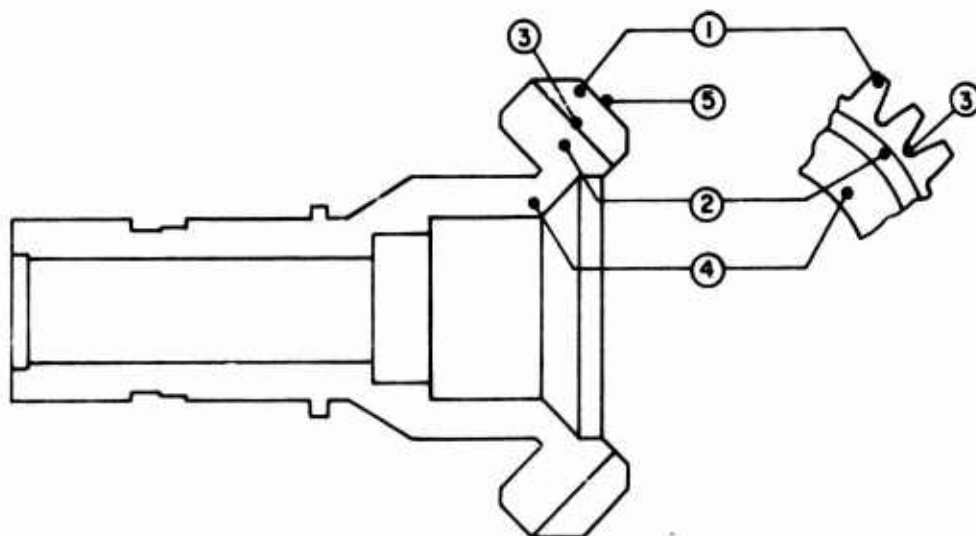


Figure G-2. Thermocouple locations on spiral bevel gear pinion

TABLE G-1. SUMMARY OF SPIRAL BEVEL GEAR
TEST RESULTS

Test no.	Scoring load, ppi	Scoring torque, in.-lb	Scoring power, hp	Pinion surface temperature, °F			Pinion blank temperature, °F	
				Tooth tip	Tooth root	Top land	Web	Hub
1	4140	5222	357	200	187	—	169	185
2	4347	5483	374	—	190*	—	—	—
3	4554	5744	392	—	213*	220	—	—
4	4140	5222	357	—	224*	275	—	—
5	4140	5222	357	—	217	235	—	—

* Average of readings from three equally spaced thermocouples
in position 3 (Fig. G-2).

Pinion Surface and Blank Temperatures

The pinion surface temperature was measured by means of imbedded or welded thermocouples at three locations shown in Figure G-2. The temperatures reached at scoring are presented in Table G-1. In Tests 2, 3, and 4, the reported temperatures at the tooth root were the averages of readings from three equally-spaced thermocouples in position 3 shown in Figure G-2. The individual thermocouple readings in each case differed from the reported average by about $\pm 10^{\circ}\text{F}$. Note that the tooth tip and top land temperatures were generally higher than the tooth root temperature, due apparently to the fact that the spiral bevel gear teeth were not modified and therefore carried more load at the tooth tip.

The pinion blank temperature was measured by means of imbedded thermocouples at two locations shown in Figure G-2. As seen in Table G-1, only one set of pinion blank temperatures was taken (Test 1). Note that the blank temperature was considerably lower than the surface temperature, and is therefore not a realistic quantity to use in critical temperature calculations.

APPENDIX H

SPUR GEAR COMPUTER PROGRAM

Program Goal

This program evaluates the scoring potential of spur gears. Pertinent gear data are entered into the program, and data relating to contact geometry, tooth contact loads, friction, instantaneous surface temperature, and instantaneous frictional power loss are determined for approximately 21 points in the mesh. Contact loads are determined from quasi-static conditions, considering the effect of tooth deflections by Walker's method.⁵⁰⁻⁵² A dynamic factor by Tuplin's method⁶³ is also calculated.

Program Language and Computer Type

The program is written in FORTRAN IV language for a CDC 6000 Series computer, using RUN compiler and SCOPE 3.4 system.

Input Cards

There are 11 data cards per set of data. Up to 4 additional cards may be used to study the effect of modifications other than the designed profile modification.

Data are entered according to the gear's function, driver or driven, instead of the more common but less descriptive appellation of pinion and gear.

Input data are entered on each card. In most cases up to 10 characters may be entered. All data are in inches unless noted otherwise. Card data and necessary explanations follow:

<u>Word</u>	<u>Column</u>	<u>Symbol</u>	<u>Description</u>
<u>Input Card 1</u>			
1	1-5	KARC	Use: 0 - if Columns 61 through 80 on Card 4 and Columns 1 through 20 on Card 5 are arc tooth thickness 1 - if Columns 61 through 80 on Card 4 and Columns 1 through 20 on Card 5 are chordal tooth thickness

<u>Word</u>	<u>Column</u>	<u>Symbol</u>	<u>Description</u>
<u>Input Card 2</u> (PRELIMINARY INPUT DATA)			
1	1-10	ANP	Number of driver teeth
2	11-20	ANG	Number of driven teeth
3	21-30	BP	Thermal constant (driver), $\text{lb}/^{\circ}\text{F-in.-sec}^{\frac{1}{2}}$
4	31-40	BG	Thermal constant (driven), $\text{lb}/^{\circ}\text{F-in.-sec}^{\frac{1}{2}}$
5	41-50	BRKP	Tip break radius, maximum (driver)
6	51-60	BRKG	Tip break radius, maximum (driven)
7	61-70	DOPMA	Outside diameter, maximum (driver)
8	71-80	DOGMA	Outside diameter, maximum (driven)
<u>Input Card 3</u>			
1	1-10	DOPMI	Outside diameter, minimum (driver)
2	11-20	DOGMI	Outside diameter, minimum (driven)
3	21-30	DRPMA	Root diameter, maximum (driver)
4	31-40	DRGMA	Root diameter, maximum (driven)
5	41-50	DRPMI	Root diameter, minimum (driver)
6	51-60	DRGMI	Root diameter, minimum (driven)
7	61-70	EP	Young's modulus (driver)
8	71-80	EG	Young's modulus (driven)

<u>Word</u>	<u>Column</u>	<u>Symbol</u>	<u>Description</u>
<u>Input Card 4</u>			
1	1-10	FMINP	Face width, minimum (driver)
2	11-20	FMING	Face width, minimum (driven)
3	21-30	PRP	Poisson's ratio (driver)
4	31-40	PRG	Poisson's ratio (driven)
5	41-50	RFMIP	Root fillet radius, minimum (driver)
6	51-60	RFMIG	Root fillet radius, minimum (driven)
7	61-70	TPMAS	Arc or chordal tooth thickness at standard pitch diameter, maximum (driver)
8	71-80	TGMAS	Arc or chordal tooth thickness at standard pitch diameter, maximum (driven)

Input Card 5

1	1-10	TPMIS	Arc or chordal tooth thickness at standard pitch diameter, minimum (driver)
2	11-20	TGMIS	Arc or chordal tooth thickness at standard pitch diameter, minimum (driven)
3	21-30	BMIN	Backlash, minimum
4	31-40	BMAX	Backlash, maximum
5	41-50	CNSTD	Use: 0 - if gears operate on standard centers

Value - if gears operate on non-standard centers

<u>Word</u>	<u>Column</u>	<u>Symbol</u>	<u>Description</u>

Input Card 5 (Cont'd)

6	51-60	PHIN	Pressure angle, deg.
7	61-70	PND	Diametral pitch

Input Card 6 (RPM AND HORSEPOWER LOOP)

1	1-10	RPMP	Rpm of driver, initial
2	11-20	RINC	Rpm increment. Blank if only one rpm
3	21-30	RPMX	Rpm maximum. Blank if only one rpm

The effect of several driver speeds on the gear set may be studied by using this loop. There are no limits on the number of times the speed is incremented, so long as the maximum rpm does not exceed 10 digits

4	31-40	HORSES	Horsepower, initial
5	41-50	DHP	Horsepower increment. Blank if only one horsepower
6	51-60	HMP	Horsepower maximum. Blank if only one horsepower

The effect of the application of several power levels for the gear set may be studied using this loop. Since this horsepower loop is inside the rpm loop, the program will analyze the complete range of horsepower for each rpm before moving on to the next rpm

<u>Word</u>	<u>Column</u>	<u>Symbol</u>	<u>Description</u>
<u>Input Card 7</u> (PROFILE MODIFICATIONS)			
1	1-10	MOD	Use: 0 - if profiles are not modified 1 - if pinion and gear tips are modified 2 - if pinion tip and root are modified 3 - if gear tip and root are modified
2	11-20	P1	Modification at break diameter for modified profiles in engagement from A to C If MOD = 0, leave blank If MOD = 1 or 3, this is gear tip modification If MOD = 2, this is pinion root modification
3	21-30	P2	Modification at break diameter for modified profiles in engagement from B to D If MOD = 0, leave blank If MOD = 1 or 2, this is pinion tip modification If MOD = 3, this is gear root modification
4	31-40	E1	Roll angle, deg., where modification ends If MOD = 0, leave blank

<u>Word</u>	<u>Column</u>	<u>Symbol</u>	<u>Description</u>
<u>Input Card 7 (Cont'd)</u>			
			If MOD = 1 or 3, this is gear roll angle
			If MOD = 2, this is pinion roll angle
5	41-50	E2	Roll angle, deg., where modification ends
			If MOD = 0, leave blank
			If MOD = 1 or 2, this is pinion roll angle
			If MOD = 3, this is gear roll angle
6	51-60	RDMORE	Use: 0 - if this is last modification card
			1 - if another modification card follows
			A separate card is used for each set of modifications, up to a total of 5 cards. Each card may use a different MOD so that the effect of various types of modification may be studied, as well as the effect of different amounts of modification. The program assumes a linear modification from the break radius to a radius having a roll angle of E1 or E2

Input Card 8 (METHOD CARD)

1	1-10	METH	If only static conditions are to be obtained, leave blank; otherwise put a 1 in Col. 10 to calculate a dynamic factor

<u>Word</u>	<u>Column</u>	<u>Symbol</u>	<u>Description</u>
<u>Input Card 9</u>			If METH is blank, omit this card; otherwise:
1	1-10	AP	Thickness of driver gear rim below the root. If gear is solid, this dimension will be the height the driver extends above the shaft surface.
2	11-20	AG	Thickness of driven gear rim below the root. If gear is solid, this dimension will be the height the driven gear extends above the shaft surface.
3	21-30	PEP	Pitch or spacing error (driver)
4	31-40	PEG	Pitch or spacing error (driven)
5	41-50	PMI	Mass moment of inertia (driver), lb-sec ² -in.
6	51-60	PMG	Mass moment of inertia (driven), lb-sec ² -in.
<u>Input Card 10</u>			If METH is blank, omit this card. If METH is 1 and values are unknown from other sources, leave card blank; otherwise:
1	1-10	NAT(1)	Lowest natural frequency of gear system, rpm
2	11-20	NAT(2)	Highest natural frequency of gear system, rpm
3	21-30	NAT(3)	Natural frequency of teeth acting as a spring, rpm

<u>Word</u>	<u>Column</u>	<u>Symbol</u>	<u>Description</u>

Input Card 10 (Cont'd)

These natural frequencies are those for a system consisting of the two gears under analysis, and the gears (or loads) on the ends of their shafts. These frequencies may be found by the Holzer method. If none of these frequencies are available for input, the program will calculate NAT(3) from the simple system consisting of the two gear blanks as masses and the contacting teeth as springs.

Input Card 11 (FRICTION AND TEMPERATURE)

1	1-10	FR1	Friction factor from Eq. (55), (57), (59), or (61)
2	11-20	FR2	Friction factor from Eq. (54), (56), (58), or (60)
3	21-30	TCON	Temperature difference factor from Eq. (69)
4	31-40	TEMP	Oil jet temperature, °F

A sample set of data cards for the ground spur gear design and operating conditions given in Chapter VIII, Section B, is shown in Figure H-1.

Since the tooth thickness is given as arc length on Cards 4 and 5, Card 1 contains a zero in Column 5. The pinion was the driver in this example, so pinion data were entered in the driver parts of the appropriate cards. From Card 6, it is noted that the effect of one speed level was studied at six power levels, 100 hp apart. The involute profiles of the pinion and gear were modified on the tips; hence Card 7 contains a 1 in Column 10 for MOD. Since only one set of modifications was considered, Column 60 for RDMORE is blank. Card 8 has a 1 in Column 10 indicating that a dynamic factor was calculated. Since a dynamic factor was calculated, Card 9 contains data. No vibration data were available; hence Card 10 is blank. Card 11

```

0
1
2
3
4
5
6
7
8
9
10
11

```

31.	76.	42.15	42.15	.0212	.0217	3.9078	9.1846
3.7068	9.1136	3.3590	8.6370	3.3430	8.6270	30.E+06	30.E+06
1.3750	1.2500	.3000	.3000	.0550	.0500	.18530	.17730
.18430	.17630	.009	.007	0	22.	8.5	
8000.	0.	0.	600.	100.	1100.		
1	.00045	.00020	25.00000	27.43000			
1							
.73200	.31600	.00020	.00020	.01670	.23664		
.0164	.0802	225.0	250.0				

Figure H-1. Sample ground spur gear computer program data cards

contains friction factors calculated for plain cross-ground surfaces with a composite surface roughness of $13.7 \mu\text{in. AA}$, using Equations (56) and (57).

Computer Program

Figure H-2 shows the listing of the computer program. Control cards are not included.

Sample Printout

Figure H-3 gives the data printout for the input data of Figure H-1. Results are shown for 600 hp only.

The first page of the printout lists the input data for reference purposes, plus miscellaneous geometric parameters. The second page provides additional input data listing.

The third page is the ROLL ANGLE SECTION in which roll angles are given for both the pinion and gear for the start and end of contact, and for the lowest point of single tooth contact (LPSTC) and the highest point of single tooth contact (HPSTC).

Two contact ratios are given. The larger represents the maximum attainable, ignoring tip rounding. The smaller value is the actual contact ratio, taking the rounding into account.

The body of the figure gives, for corresponding pinion and gear roll angles, the radii of curvature and the instantaneous sliding and sum velocity for several points throughout the mesh.

The next page is a continuation of the ROLL ANGLE SECTION, listing the dimensions of the uniform stress parabola inscribed within the tooth outline.

The fifth page is the LOAD FORCE SECTION, in which the quasi-static loads are determined as a function of the tooth deflections and the profile modifications at several points in the mesh cycle.

Roll angles at positions 7 and 8 and also positions 17 and 18 are duplicated. Positions 7 and 8 represent the point where double-tooth contact starts (LPSTC), and positions 17 and 18 represent the point where double-tooth contact ends (HPSTC). Two positions are needed to define conditions at each of these points, because as the gear

teeth go from double-tooth to single-tooth contact, or the reverse, the load theoretically changes instantaneously.

The last page is the FLASH TEMPERATURE SECTION where, for points throughout the mesh cycle, the parameters related to the operational behavior of the gears are given. The dynamic increment and corresponding dynamic factor are given at the bottom of the page.

The program enters the rpm loop first, then the hp loop, and finally the modification loop. Thus the program will perform all of the calculations in the LOAD FORCE SECTION and the FLASH TEMPERATURE SECTION for each set of modifications specified in the modification loop at the initial values of hp and rpm. When the modification loop has been completed, the hp will be incremented and the modification loop repeated.

```

C      PROGRAM SPURGH (INPUT,OUTPUT,TAPES=INPUT,TAPES=OUTPUT)
C      * EXTERNAL SPUR GEARS = FOR *
C      * EVALUATING DESIGN PARA- *
C      * METERS, PROGRAMMED BY- *
C      * MARY CARTER SMRI *
C      * . . . . . *
C
C      REAL NAT,NATFU
C      INTEGER RDMORE
C      DIMENSION ID(2),ID1(6),IMD(5),IMD1(4),IMD2(3)
C      * IMD(7),IM(4),IMI(6),IMZ(3),IP(2),AMOD(6,4),
C      * FP(40),FG(40),FPCO(40),FGCO(40),MP(40),MG(40),MOD(5)
C      * NAT(3),DFSP(40),DECP(40),DECG(40),RMOP(40),
C      * HMOP(40),ROLLRAY(40,2),VS(40),VT(40),DFBG(40),
C      * FRIC(40),PHE(40),TP(40),TG(40),XOIMP(40),XOIMG(40)
C      * NIFO(40,2)
C      DATA IN/10M FORCE = /,
C      * ID1/10M LOAD FORC,10MFACTOR BY,10MTUPLIN MET,
C      * 10ME,10M,10MHOD
C      DATA IMD/10M SECTION,10MVARIAULPS,10MUSED IN,...,10M LOOP
C      * 10M CALCS,/,
C      * IMD1/10M NORMAL,10MHORSEPOWER,10M MODIF,10M METHOD
C      * IMD2/10M ROLL,10M LOAD,10MDYNAMIC,10M HERTZ
C      * 10M FLASH/
C      * IMD3/10M ANGLES,10M FORCE,10MFRICITION,10M STRESS
C      * 10M TEMP F,10M CTEMP,10M PHE/
C      COMMON JA,JB,JC,JD,KAMP,PI,RN,RN,RPMP,LOU,LIN,ANP,ANG,ORP,DBG,
C      * 10DRG,CNSTO,PEP,PEG,ALPHA,OPA,PI,P2,IXT,BET
C      LIN == INPUT LOGICAL UNIT NUMBER
C      LOU == OUTPUT LOGICAL UNIT NUMBER
C      RN == NO. OF ROWS OF STORAGE ARRAY...(ROLLRAY)
C      RN == CONVERT FROM DEGREES TO RADIANS
C      DEGR == CONVERT FROM RADIANS TO DEGREES
C
C      KRW=40
C      LIN=5
C      LOU=6
C      RNS,N17451293
C      DEGR=57.2957795131
C      PI=3.14159265358979
C
C      INPUT DATA SECTION
C
C      $ READ (LIN,1000) KARC
C      IF (EOF,LIN) 700,10
C      10 WRITE (LOU,1100)
C      READ (LIN,1020) ANP,ANG,BP,BG,BRKP,BRKG,DOPMA,DOGMA,
C      * DOPMI,DOGMI,DRPMA,DRGMA,DRPMI,DRGMI,EP,EG,
C      * FMINP,FMING,PRP,PRG,RPMIP,RPMIG,TPMAS,TCMAS,
C      * TPMIS,TGMIS,BMIN,BMAX,CNSTO,PHIN,PNO
C      WRITE (LOU,1120) ANP,ANG,BP,BG,BRKP,BRKG,DOPMA,DOGMA,
C      * DOPMI,DOGMI,DRPMA,DRGMA,DRPMI,DRGMI,EP,EG
C      WRITE (LOU,1126) FMINP,FMING,PRP,PRG,RPMIP,RPMIG,TPMAS,TCMAS,
C      * TPMIS,TGMIS,BMIN,BMAX,CNSTO,PHIN,PNO
C      PERFORM PRELIMINARY CALCULATIONS
C      FNRABPMIN=RN
C      FNRSTIN(FNRA)
C      FNRCONCS(FNRA)
C      FNRANSI/FNCO
C      CSTIN=(ANP+ANG)/(2.*PNO)
C      IF (CNSTO) 110,109,110
C      109 CNSTO=CNSTO

```

Figure H-2. Listing of ground spur gear computer program

```

110 POX=(ANP+ANG)/(2.*CNSTD)
FX=(CSTD*FNCO)/CNSTD
FXRA=ATAN(SQRT(1.-(FX)**2)/FX)
FXSI=SIN(FXRA)
FXCI=COS(FXRA)
FXTA=FXSI/FXCI

C
ZFN=FNTA*FNRA
ZFX=FXTA*FXRA
ANG=ANG/ANP
RMG=ANP/ANG

C
DBP=ANP/PND
DBP=DP*FNCO
DXP=ANP/PDX
DG=ANG/PND
DHG=DG*FNCO
DXG=ANG/PDX
DODBP=DOBP*(2.*BRKP)
DODHG=DOHG*(2.*BRKG)
ZA=.5*(SQRT((DODHG*DODBP)-(DHG*DBG))-SQRT((DXG*DXG)-(DBG*DBG)))
ZR=.5*(SQRT((DODBP*DODBP)-(DBP*DBP))-SQRT((DXP*DXP)-(DBP*DBP)))
VO=.5*(SQRT((DXP*DXP)-(DBP*DBP)))
*U=.5*(SQRT((DXG*DXG)-(DBG*DBG)))
VA=VO-ZA
PB=P-DBP/ANP
CPN=PI*DXP/ANP
ER=1./(((1.-(PRP*PRP))/EP+((1.-(PRG*PRG))/EG))/E.)
WDP=DOBP-URPMA
WOG=DOHG-DRGMA
ADP=(DODBP*DXP)/2.
ADG=(DODHG*DXG)/2.
FNRA=FNRA/RN
FXRA=FXRA/RN
WRITE (LOU,1200) ADP,ADG,DBP,DBG,DODBP,DODHG,DP,DG,
* DXP,DXG,WDP,WOG
* WRITE (LOU,1201) ANG,CNSTD,CPN,FNRA,FXRA,ER,PB,
* POX,RMG,VO,WO,ZA,ZFN,ZFX,ZR

C
FNRA=FNRA*RN
FXRA=FXRA*RN
IF (KARC,EQ,0) GO TO 116

C
C CALCULATE ARC TOOTH THK, FROM CHORDAL THK.
C
ANP=(5*TPMIS)/(5*DP)
ANRATAN(AN/(SQRT(1.-(AN)**2)))
TPMIS=AN*DP
ANP=(5*TPMAS)/(5*DP)
ANRATAN(AN/(SQRT(1.-(AN)**2)))
TPMAS=AN*DP
ANP=(5*TGMIS)/(5*DG)
ANRATAN(AN/(SQRT(1.-(AN)**2)))
TGMIS=AN*DG
ANP=(5*TGMA5)/(5*DG)
ANRATAN(AN/(SQRT(1.-(AN)**2)))
TGMA5=AN*DG
WRITE (LOU,1205) TPMAS,TGMA5,TPMIS,TGMIS

C
C CALCULATE ARC TOOTH THK, AT THE OPERATING PITCH DIA, (DXP)
C
116 TPMIN=DXP*(((TPMIS/DP)+ZFN)+ZFX)

```

```

SPGR0640+0063
SPGR0650+0064
SPGR0660+0065
SPGR0670+0066
SPGR0680+0067
SPGR0690+0068
SPGR0700+0069
SPGR0710+0070
SPGR0720+0071
SPGR0730+0072
SPGR0740+0073
SPGR0750+0074
SPGR0760+0075
SPGR0770+0076
SPGR0780+0077
SPGR0790+0078
SPGR0800+0079
SPGR0810+0080
SPGR0820+0081
SPGR0830+0082
SPGR0840+0083
SPGR0850+0084
SPGR0860+0085
SPGR0870+0086
SPGR0880+0087
SPGR0890+0088
SPGR0900+0089
SPGR0910+0090
SPGR0920+0091
SPGR0930+0092
SPGR0940+0093
SPGR0950+0094
SPGR0960+0095
SPGR0970+0096
SPGR0980+0097
SPGR0990+0098
SPGR1000+0099
SPGR1010+0100
SPGR1020+0101
SPGR1030+0102
SPGR1040+0103
SPGR1050+0104
SPGR1060+0105
SPGR1070+0106
SPGR1080+0107
SPGR1090+0108
SPGR1100+0109
SPGR1110+0110
SPGR1120+0111
SPGR1130+0112
SPGR1140+0113
SPGR1150+0114
SPGR1160+0115
SPGR1170+0116
SPGR1180+0117
SPGR1190+0118
SPGR1200+0119
SPGR1210+0120
SPGR1220+0121
SPGR1230+0122
SPGR1240+0123
SPGR1250+0124

```

Figure H-2. Listing of ground spur gear computer program (cont'd)

TPMAX=DXP*((TPH48/DP)+ZFX)	SPGR1260=0125
TGMIN=DXG*((TGH48/DG)+ZFX)	SPGR1270=0126
TGMAX=DXG*((TGH48/DG)+ZFX)	SPGR1280=0127
PK=(TPMIN/DXP)+ZFX	SPGR1290=0128
GK=(TGMIN/DXG)+ZFX	SPGR1300=0129
WRITE (LOU,1206) TPMAX,TGMAX,TPMIN,TGMIN,PK,GK	SPGR1310=0130
READ (LIN,1040) RMPM,RINC,RPMX,MORSES,DMP,MPM	SPGR1320=0131
1040 FORMAT (6F10.2)	SPGR1330=0132
WRITE (LOU,1127) RMPM	SPGR1340=0133
IF (RINC,EQ,0) GO TO 12	SPGR1350=0134
WRITE (LOU,1128) RINC,RPMX	SPGR1360=0135
C	SPGR1365=0136
C READ VARIABLES FOR HORSEPOWER LOOP	SPGR1370=0137
17 WRITE (LOU,1115) IMD(2),IMD(3),IMD(4),IMD(5)	SPGR1380=0138
WRITE (LOU,1140) MORSES	SPGR1390=0139
IF (DMP,EQ,0) GO TO 24	SPGR1400=0140
WRITE (LOU,1141) DMP,MPM	SPGR1410=0141
C	SPGR1420=0142
24 I=1	SPGR1430=0143
25 READ (LIN,1046) MOD(I),(AMOD(I,J),J=1,4),NDMORE	SPGR1440=0144
IF (MOD(I),EQ,0) GO TO 24	SPGR1450=0145
IF (I,GT,1) GO TO 26	SPGR1460=0146
WRITE (LOU,1115) IMD(2),IMD(3),IMD(4),IMD(5)	SPGR1470=0147
26 IM1=IM2(1)	SPGR1480=0148
IM2=IM2(2)	SPGR1490=0149
IF (MOD(I),GT,2) IM1=IM2(2)	SPGR1500=0150
IF (MOD(I),GT,1) IM2=IM2(3)	SPGR1510=0151
27 WRITE (LOU,1146) AMOD(I,1),AMOD(I,2)	SPGR1520=0152
WRITE (LOU,1147) AMOD(I,3),AMOD(I,4)	SPGR1530=0153
24 IF (NDMORE,EQ,0) GO TO 30	SPGR1540=0154
I=I+1	SPGR1550=0155
GO TO 25	SPGR1560=0156
30 NMOD=I	SPGR1570=0157
C	SPGR1580=0158
C READ AND PRINT DATA FOR DFORCE SECTION	SPGR1590=0159
C	SPGR1600=0160
READ (LIN,1050) METH	SPGR1610=0161
WRITE(LOU,18)	SPGR1620=0162
38 FORMAT(//42M METHOD USED FOR CALCULATING NORMAL FORCE=/ 1X,24M1 NORMAL FORCE = STATIC FORCE)	SPGR1630=0163
IF (METH,NE,1) GO TO 41	SPGR1640=0164
READ (LIN,1055) AP,AG,PEP,PEG,PMI,GMI,NAT	SPGR1650=0165
WRITE(LOU,39)	SPGR1660=0166
39 FORMAT(1X,44M2 DYNAMIC FACTOR CALCULATED BY TUPLIN METHOD)	SPGR1670=0167
WRITE (LOU,1155) AP,AG,PEP,PEG,PMI,GMI,NAT	SPGR1680=0168
C	SPGR1700=0170
C READ AND PRINT DATA FOR FLASH TEMP SECTION	SPGR1740=0171
41 READ (LIN,1060) FR1,FR2,TCON,TEMP	SPGR1800=0172
WRITE (LOU,1110) IMD(2),IMD(3),IMD(4),IMD(5),IMD(1)	SPGR1810=0173
45 WRITE (LOU,1160) FR1,FR2,TCON,TEMP	SPGR1820=0174
C	SPGR1830=0175
C	SPGR1840=0176
100 IPHI=PMIN	SPGR1850=0177
C	SPGR1860=0178
C F=PMINP	SPGR1870=0179
IF (FMING,LT,FMINP) F=FMING	SPGR1880=0180
C	SPGR1890=0181
C BEGIN RPM LOOP	SPGR1900=0182
C	SPGR1910=0183
MP1=MORSES	SPGR1920=0184
40 MORSES=MP1	SPGR1930=0185
IRECAL=1	SPGR1940=0186

Figure H-2. Listing of ground spur gear computer program (cont'd)


```

130 TOP = (63025, *HORSES) / RPMP
WTP = (2, *TOP) / DXF
RPMG = WTP * RMG
TQG = (63025, *HORSES) / RPMG
WYG = (2, *TQG) / DXG
WN = WTP / FX
MM = 1
KAMP = 1
IF (IRECAL, EQ, 0) GO TO 135
141 IRECAL = 0
C
C ROLL ANGLE SECTION
C
150 WRITE (LOU, 1210) IMD2(1), IMD3(1), IMD(1)
CALL ROLLANG (VO, ZA, ZR, NO, DOPMA, DOGMA, PB, FXTA
, ROLLRAY, RHOP, RMOG, DECP, DECG, VS, VT
, EBCP, EECF, MARK1, MARK3, KRW)
CALL PHI (DECP, FP, MF, TP, XDIMP, DFSP, DXF, DDP, DRPMI, PK, ANP,
RPMIP, EP, PI, RN, JO, FMIMP, FPCO, KRW)
CALL PHI (DECG, FG, MG, TG, YDIMP, DFSG, DXG, DBC, DRGMI, CK, ANG,
RPMIG, EG, PI, RN, JO, FMING, FGCO, KRW)
WRITE (LOU, 1215)
DO 155 I = 1, JO
T1 = TP(I) * 2,
T2 = TG(I) * 2,
WRITE (LOU, 1220) MP(I), MG(I), T1, T2, FPCO(I), FGCO(I)
155 CONTINUE
C
C LOAD SECTION
C BEGINNING OF MODIFICATION LOOP
135 IF (KAMP, EQ, 2) GO TO 145
IF (MOD(MM), EQ, 0) GO TO 145
140 P1 = MOD(MM, 1)
P2 = MOD(MM, 2)
E1 = MOD(MM, 3)
E2 = MOD(MM, 4)
145 WRITE (LOU, 1210) IMD2(2), IMD3(2), IMD(1), HORSES, RPMP, WN
MOD(MM)
CALL LDVROL (ROLLRAY, DFSP, DFSG, WYFO, MD,
E1, E2, MARK1, MARK3, EBCP, EECF, KRW, TDFES)
IF (KAMP, EQ, 2) IRECAL = 1
GO TO (140, 150) KAMP
C
C WHEN KAMP = 2 THE ROLL ANGLE AND LOAD CALCULATIONS ARE REPEATED
C
C DYNAMIC FACTOR SECTION
C MF...METHOD INDICATOR
C MF = 1 NORMAL FORCE/LOAD FORCE FOR EACH
C ROLL ANGLES POSITION
C MF = 2 CALL SUBROUTINE FORCE TO CALC.
C A DYNAMIC FACTOR
C
160 INT = 1
MF = 1
L1 = 1
L2 = 4
DF = 1
GO TO 210
165 IF (METH, EQ, 0) GO TO 510
MF = 2
GO TO 180
SPGR1960+0187
SPGR1960+0188
SPGR1970+0189
SPGR1980+0190
SPGR1990+0191
SPGR2000+0192
SPGR2010+0193
SPGR2020+0194
SPGR2030+0195
SPGR2040+0196
SPGR2050+0197
SPGR2060+0198
SPGR2070+0199
SPGR2080+0200
SPGR2090+0201
SPGR2100+0202
SPGR2110+0203
SPGR2120+0204
SPGR2130+0205
SPGR2140+0206
SPGR2150+0207
SPGR2160+0208
SPGR2170+0209
SPGR2180+0210
SPGR2190+0211
SPGR2200+0212
SPGR2210+0213
SPGR2220+0214
SPGR2230+0215
SPGR2240+0216
SPGR2250+0217
SPGR2260+0218
SPGR2270+0219
SPGR2280+0220
SPGR2290+0221
SPGR2300+0222
SPGR2310+0223
SPGR2320+0224
SPGR2330+0225
SPGR2340+0226
SPGR2350+0227
SPGR2360+0228
SPGR2370+0229
SPGR2380+0230
SPGR2390+0231
SPGR2400+0232
SPGR2410+0233
SPGR2420+0234
SPGR2430+0235
SPGR2440+0236
SPGR2450+0237
SPGR2460+0238
SPGR2470+0239
SPGR2480+0240
SPGR2490+0241
SPGR2500+0242
SPGR2510+0243
SPGR2520+0244
SPGR2530+0245
SPGR2540+0246
SPGR2550+0247
SPGR2560+0248

```

Figure H-2. Listing of ground spur gear computer program (cont'd)

```

100 LINE
   LPMF=0
   LPMF=1
   WRITE (LOU,1250) IMDS(3),ID1(2),ID1(L3),ID1(L2)
   MATFORMAT(3)
   CALL FORCE (AP,AG,CPN,DXP,DIG,EP,EG,FMINP,FMING,FNRA,
   *HORSES,NATPQ,PMI,SMI,PRP,PRG,TPMIS,TSMIS,VA,WOP,WOG,INT,TOFES)
   GO TO 510
C
C   FLAME TEMPERATURE SECTION
C
C   *FOR EACH ROLL ANGLE POSITION A FRICTION COEFFICIENT IS DETERMINED
C   *VALUES ARE THEN FOUND FOR FLAME TEMPERATURE,
C
210 CALL FRPME (TS,TEMP,MXFO,FRIC,PHE,KRW,JD,L1,V8,FH,ER,
   *PHEAV,FRI,FR2,TCON,JB,JC,ROLLRAY,PNET,JA,RPM,ANP)
   WRITE (LOU,1211) IMDS(5),IMDS(6),IMD(1),HORSES,RPM,WN,TS,PHEAV
   WRITE (LOU,1250) IMD1(1),ID(L1),ID1(MF),ID1(L2)
   WRITE (LOU,1230) IMDS(1),IMDS(2),(IMDS(N),N=4,5),(IMDS(M),M=1,7)
   DO 270 I=1,JD
     UFORCE=MXFO(I,L1)
     RHOE = (RHOP(I)+RHOG(I))/(RHOP(I)+RHOG(I))
     IF (DFORCE,NE,0.) GO TO 215
     DT = 0.
     MERTZ=0.
     CTE=PTS
     GO TO 255
215 MERTZ=3989*SQRT(ER/FH)*SQRT(DFORCE/RHOE)
     DT=201*(FRIC(I)*((DFORCE/FH)**(3./4.))+(LR**(.1./4.))/(RHOE**(.1./
     *4.)))+(ABS(RHOP(I)-(ANP/ANG+RHOG(I)))*SQRT(RPM)/(SP*SQRT(RHOP(I))
     *ANG*SQRT(ANP/ANG+RHOG(I))))
225 CTE=PTS+DT
255 WRITE (LOU,1235) I,ROLLRAY(I,1),DFORCE,FRIC(I),MERTZ,DT,
   ,CTEMP,PHE(I)
270 CONTINUE
C   INCREMENT METHOD LOOP
500 GO TO (165,510) MF
C
C   INCREMENT MODIFICATION LOOP
510 IF (MM,EQ,NMOD) GO TO 520
   MM=MM+1
   IF (IRECAL,EU,1) GO TO 151
   GO TO 135
C
C   INCREMENT HORSEPOWER LOOP
C
520 IF (HORSES,GE,MPH) GO TO 600
525 HORSES=HORSES+DMP
   GO TO 130
600 IF (RPM,GE,RPMX) GO TO 5
   RPM=RRPMP+RINC
   GO TO 90
C
1000 PRINT FORMATS
1001 FORMAT (815)
1020 FORMAT (8F10.4/6F10.4,2E10.2/8F10.5/7F10.5)
1045 FORMAT (5X,15,4F10.6,25X,16)
1050 FORMAT (110)
1055 FORMAT (6F10.4/3F10.2)
1060 FORMAT (2F10.4,2F10.1)
C   PRELIMINARY DATA OUTPUT FORMATS
1100 FORMAT (1M1)
1110 FORMAT (/4X,2A10,2A7,A10/)
SPGR2580+0244
SPGR2590+0250
SPGR2600+0251
SPGR2610+0252
SPGR2615+0253
SPGR2620+0254
SPGR2630+0255
SPGR2640+0256
SPGR2650+0257
SPGR2660+0258
SPGR2670+0259
SPGR2680+0260
SPGR2690+0261
SPGR2710+0262
SPGR2720+0263
SPGR2730+0264
SPGR2740+0265
SPGR2750+0266
SPGR2760+0267
SPGR2770+0268
SPGR2780+0269
SPGR2790+0270
SPGR2800+0271
SPGR2810+0272
SPGR2820+0273
SPGR2830+0274
SPGR2840+0275
SPGR2850+0276
SPGR2860+0277
SPGR2870+0278
SPGR2880+0279
SPGR2890+0280
SPGR2900+0281
SPGR2910+0282
SPGR2920+0283
SPGR2930+0284
SPGR2940+0285
SPGR2950+0286
SPGR2960+0287
SPGR2970+0288
SPGR2980+0289
SPGR2990+0290
SPGR3000+0291
SPGR3010+0292
SPGR3020+0293
SPGR3030+0294
SPGR3040+0295
SPGR3050+0296
SPGR3060+0297
SPGR3070+0298
SPGR3080+0299
SPGR3090+0300
SPGR3100+0301
SPGR3110+0302
SPGR3120+0303
SPGR3130+0304
SPGR3140+0305
SPGR3150+0306
SPGR3160+0307
SPGR3170+0308
SPGR3180+0309
SPGR3190+0310

```

Figure H-2. Listing of ground spur gear computer program (cont'd)

```

1115 FORMAT (//1X,3A10,A7)
1120 FORMAT (4X,4NHVALUES AND DEFINITIONS OF PRIMARY INPUT VARIABLES
  * 30M (INCH UNITS UNLESS NOTED) /
  * 2X,16M (DRIVER) ,6X,14M (DRIVEN)
  *//11M ANP ,F10,5,5X,10MANG ,F10,5,5X,
  * 60M=NUMBER OF TEETH
  *//11M NP ,F10,5,5X,10MHG ,F10,5,5X,
  * 60M=THERMAL CONSTANT (LB, SQRT(SEC.)/IN, DEG. F)
  *//11M BRKP ,F10,5,5X,10MBRKG ,F10,5,5X,
  * 60M=MAXIMUM TIP BREAK
  *//11M OOPMA ,F10,5,5X,10MOOGMA ,F10,5,5X,
  * 60M=OUTSIDE DIAMETER, MAXIMUM
  *//11M OOPMI ,F10,5,5X,10MOOGMI ,F10,5,5X,
  * 60M=OUTSIDE DIAMETER, MINIMUM
  *//11M OOPMA ,F10,5,5X,10MOOGMA ,F10,5,5X,
  * 60M=ROOT DIAMETER, MAXIMUM
  *//11M OOPMI ,F10,5,5X,10MOOGMI ,F10,5,5X,
  * 60M=ROOT DIAMETER, MINIMUM
  *//11M EF ,F10,0,5X,10MEG ,F10,0,5X,
  * 60M=YOUNG'S MODULUS, PSI.
  *)
1125 FORMAT (
  * 11M FMINP ,F10,5,5X,10MFMING ,F10,5,5X,
  * 60M=FACE WIDTH, MINIMUM
  *//11M MRP ,F10,5,5X,10MRPG ,F10,5,5X,
  * 60M=POISSON'S RATIO
  *//11M RFMIP ,F10,5,5X,10RFMIG ,F10,5,5X,
  * 60M=ROOT FILLET RADIUS
  *//11M TMMAS ,F10,5,5X,10MTGMAS ,F10,5,5X,
  * 60M=ARC OR CHORDAL TOOTH THICKNESS, MAXIMUM
  *//11M TMMIS ,F10,5,5X,10MTGMIS ,F10,5,5X,
  * 60M=ARC OR CHORDAL TOOTH THICKNESS, MINIMUM
  *//11M RMIN ,F10,5,30X, 36M=RACKLASH MINIMUM
  *//11M RMAX ,F10,5,30X, 36M=RACKLASH MAXIMUM
  *//11M CNSTO ,F10,5,30X, 36M=CENTER DISTANCE, STAND, IF ZERO
  *//11M PMIN ,F10,5,30X, 36M=PRESSURE ANGLE (DEG.)
  *//11M PND ,F10,5,30X, 36M=DIAMETRAL PITCH
  **)
1200 FORMAT (//35X,4NHVALUES AND DEFINITIONS OF PRELIMINARY CALCULATED
  * 10M=VARIABLES /
  * 2X,16M (DRIVER) ,6X,14M (DRIVEN)
  *//11M ADP ,F10,5,5X,10MADG ,F10,5,5X,
  * 60M=ADDENDUM (IN.)
  *//11M ORP ,F10,5,5X,10MORG ,F10,5,5X,
  * 60M=BASE CIRCLE DIAMETER
  *//11M OODRP ,F10,5,5X,10MOODRG ,F10,5,5X,
  * 60M=OUTSIDE DIAMETER AREA
  *//11M OP ,F10,5,5X,10MOPG ,F10,5,5X,
  * 60M=STD. PITCH DIAMETER
  *//11M OXP ,F10,5,5X,10MOXP ,F10,5,5X,
  * 60M=NON. STD. PITCH DIAMETER
  *//11M WOP ,F10,5,5X,10MWOPG ,F10,5,5X,
  * 60M=HEIGHT OR WHOLE DEPTH
  *)
1201 FORMAT (11M ANG ,F10,5,30X,11M=GEAR RATIO
  *//11M CNSTO ,F10,5,30X, 36M=CALCULATED CENTER
  *//11M CON ,F10,5,30X, 36M=NORMAL CIRCLE PITCH
  *//11M PNRA ,F10,5,30X, 36M=PRESSURE ANGLE (DEG.)
  *//11M PNRA ,F10,5,30X, 36M=WORKING PRESSURE ANGLE (DEG.)
  *//11M EP ,F10,5,30X, 36M=REDUCED MODULUS (PSI.)
  *//11M PH ,F10,5,30X, 36M=BASE PITCH
  *//11M PHX ,F10,5,30X, 36M=DIAMETRAL PITCH, WORKING
  *)
SPGR3200=0311
SPGR3210=0312
SPGR3220=0313
SPGR3230=0314
SPGR3240=0315
SPGR3250=0316
SPGR3260=0317
SPGR3270=0318
SPGR3280=0319
SPGR3290=0320
SPGR3300=0321
SPGR3310=0322
SPGR3320=0323
SPGR3330=0324
SPGR3340=0325
SPGR3350=0326
SPGR3360=0327
SPGR3370=0328
SPGR3380=0329
SPGR3390=0330
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SPGR3420=0333
SPGR3430=0334
SPGR3440=0335
SPGR3450=0336
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SPGR3470=0338
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SPGR3490=0340
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SPGR3680=0359
SPGR3690=0360
SPGR3700=0361
SPGR3710=0362
SPGR3720=0363
SPGR3730=0364
SPGR3740=0365
SPGR3750=0366
SPGR3760=0367
SPGR3770=0368
SPGR3780=0369
SPGR3790=0370
SPGR3800=0371
SPGR3810=0372

```

Figure H-2. Listing of ground spur gear computer program (cont'd)

```

*//11M RMG      = ,F10.5,30X, 36M=1/GEAR RATIO
*//11M VO       = ,F10.5,30X,
*63M-DISTANCE FROM INITIAL INTERFERENCE POINT TO PITCH POINT
*//11M WN       = ,F10.5,30X,
*63M-DISTANCE FROM SECOND INTERFERENCE POINT TO PITCH POINT
*//11M ZA       = ,F10.5,30X, 36M=PATH OF CONTACT IN APPROACH
*//11M ZFN      = ,F10.5,30X, 36M=INVOLUTE PHI FOR STD. CENTERS TO PITCH
* ,20MTCM CIRCLE (RAD.)
*//11M ZFX      = ,F10.5,30X, 36M=INVOLUTE PHI FOR NON-STD. CENTERS TO PITCH
* ,30MO PITCH CIRCLE (RAD.)
*//11M ZR       = ,F10.5,30X, 36M=PATH OF CONTACT IN RECESS
1205 FORMAT (/3X,36MARC TOOTH THK. FROM CHORDAL TOOTH THK./
* 2X,16M (DRIVER) ,6X,16M (DRIVEN)
*//11M YPMIS    = ,F10.5,5X,10MTGMIS = ,F10.5,5X,
*//11M YPMIS    = ,F10.5,5X,10MTGMIS = ,F10.5,5X/
1206 FORMAT (/3X,42MOPERATING ARC TOOTH THK. AT PITCH DIAMETER/
* 2X,16M (DRIVER) ,6X,16M (DRIVEN)
*//11M YPMAX    = ,F10.5,5X,10MTGMAX = ,F10.5,5X, 36M=MAXIMUM
*//11M YPMIN    = ,F10.5,5X,10MTGMIN = ,F10.5,5X, 36M=MINIMUM
*//11M PK       = ,F10.5,5X,10MCK = ,F10.5,5X,
* ,40M=ANGLE FROM ORIGIN OF INVOLUTE TO CENTER LINE OF TOOTH (RAD)
1127 FORMAT (1M1//35X,40MINPUT VARIABLES FOR SPECIFIED LOOPS AND SECTIONS
*NS//1X,30MVARIBLES USED IN... RPM LOOP
*//11M RPM      = ,F10.2,30X, 36M=RPM DRIVER
*)
1128 FORMAT (
* 11M RINC      = ,F10.2,30X, 36M=RPM DRIVER INCREMENT
*//11M RPMX     = ,F10.2,30X, 36M=RPM DRIVER MAXIMUM
*)
1140 FORMAT (
*//11M HORSES   = ,F10.5,30X, 36M=HORSEPOWER
*)
1141 FORMAT (
* 11M DHP       = ,F10.5,30X, 36M=HORSEPOWER INCREMENT
*//11M MPH      = ,F10.5,30X, 36M=MAXIMUM HORSEPOWER
*)
1146 FORMAT (/11M P1 = ,F10.5,5X,10MP2 = ,F10.5)
1147 FORMAT ( 11M E1 = ,F10.5,5X,10ME2 = ,F10.5)
1155 FORMAT (
* 2X,16M (DRIVER) ,6X,16M (DRIVEN)
*//11M AP       = ,F10.5,5X,10MAG = ,F10.5,5X,
* ,60M=THICKNESS OF RIM BELOW ROOT
*//11M PEP      = ,F10.5,5X,10MP2G = ,F10.5,5X, 13M=PITCH ERROR
*//11M PMI      = ,F10.5,5X,10MGM1 = ,F10.5,5X,
* ,60M=MASS MOMENT OF INERTIA (LB,SEC,IN.)
*//11M NAT(1)   = ,F10.2,30X, 36M=1ST NATURAL FREQUENCY(RPM)
*//11M NAT(2)   = ,F10.2,30X, 36M=2ND NATURAL FREQUENCY(RPM)
*//11M NAT(3)   = ,F10.2,30X, 36M=3RD NATURAL FREQUENCY(RPM)
*)
1160 FORMAT (
*//11M FR1      = ,F4.4,10X, 30M=CONSTANT FRICTION FACTOR
*//11M FR2      = ,F4.4,10X, 30M=VARIABLE FRICTION FACTOR
*//11M TCON     = ,F4.1,10X, 30M=TEMP DIFFERENCE FACTOR
*//11M TEMP     = ,F4.1,10X, 30M=OIL JET TEMPERATURE,F
*)
1215 FORMAT (1M1/////7X,2MMP,12X,2MHG,12X,2MTP,12X,2MTG,11X,4MHFPCO,
* 11X,4MHFGCO/)
1210 FORMAT (1M1//1X,2A7,A10//F11.2,13M = HORSEPOWER/ F11.2,
* 16M = RPM DRIVER / F11.2,5M = MM)
1211 FORMAT (1M1//1X,2A7,A10//F11.2,13M = HORSEPOWER/F11.2,
* 16M = RPM DRIVER /F11.2,5M = MM/

```

Figure H-2. Listing of ground spur gear computer program (cont'd)

```

      * F11,2,7H = TS,F/F11,4,14H = PHEAV,8/SEC)
1230 FORMAT (///3X,2A10,11X,2A10/3X,7A10/)
1220 FORMAT (3X,F8,4,5(6X,F8,4))
1235 FORMAT (1X,12,2X,F5,2,3X,F7,1,1X,E10,4,2X,F7,0.5X,F6,2,3X,F6,2,
      * 3X,E9,3)
1250 FORMAT (///4X,A8,3A10/)
C
700 CONTINUE
END
SUBROUTINE LOYROL (ROLLRAY,DFSP,DFSG,NXFO,MOD,
      * E1,E2,MARK1,MARK2,EBCP,ECCP,KRW,TOPES)
      * DIMENSION ROLLRAY(KRW,2),TOP(40),DFSP(KRW),DFSG(KRW),PMP(30),
      * PMS(30),NX(40),PRMOD(40),NXFO(KRW,2),DPP(40),DFG(40)
      * DATA IM2,IM4,IM6,IM8 /4M = P1, 4M = P2, 4M = E1,
      * 4M = E2 /
      * COMMON JA,J8,JC,JD,KAMP,P1,RN,WN,RPMP,LOU,LIN,ANP,ANG,OBP,OBG,
      * MODBG,CNSTO,PEP,PEG,ALPHA,OPA,P1,P2,INT,BET
      * MARK1=J8+1
      * MARK2=J8+1
      * J=0
      * INUM=0,
C
C CALCULATE DEFLECTIONS, DPP(I) /G(I) AND TOTAL DEFLECTIONS TOP
C
DO I=1,JD
  DPP(I)=DFSP(I)+NH
  DFG(I)=DFSG(I)+NH
  TOP(I)=DPP(I)+DFG(I)
5 CONTINUE
MODIFICATIONS (N)=NORMAL CONTACT
C MOD=0 NO MODIFICATION
C MOD=1 MODIFY GEAR AND PINION TIP
C MOD=2 MODIFY PINION TIP AND ROOT
C MOD=3 MODIFY GEAR TIP AND ROOT
C
PMP(I) TIP MODIFICATIONS, A=C (N)
PMP(I) ROOT MODIFICATIONS, B=D (N)
(PMP(I) IS MOD, FOR ANGLE ONE BASE PITCH AWAY FROM
  ANGLE WHERE PMP(I) IS DETERMINED)
L2,L1 POINTERS TO ROLLRAY, PIN, ANGLES (COL 1) GEAR (COL 2)
C
GENERAL EQUATIONS,,, PMP(I) = P1*((EX -E1)/(EA+E1))
PMP(I) = P2*((EY -E2)/(ED+E2))
MODIFICATION FACTORS, P1,P2,F1,E2 ARE GIVEN
P1 = MODIFICATION AT START OF CONTACT
E1 = CORRESPONDING ROLL ANGLE
P2 = MODIFICATION AT END OF CONTACT
E2 = CORRESPONDING ROLL ANGLE
EX = ROLL ANGLES A=C (N)
EY = ROLL ANGLES B=D (N)
EA = GEAR OR PINION ANGLES AT POINT -A=, EBCP,ECCG
ED = GEAR OR PINION ANGLES AT POINT -D=, EECF,EECG
J POINTS TO ANGLES ONE BASE PITCH AWAY FROM -I-
C
I1=1
TOPES = TOP(J8)
IF (MOD,EO,0) GO TO 70
L1=0
L2=1
IF (MOD,EO,3) L2=2
IF (MOD,EO,2) L1=1
IF (KAMP,EO,2) GO TO 25

```

```

SPGR4450+0435
SPGR4460+0436
SPGR4470+0437
SPGR4480+0438
SPGR4490+0439
SPGR4500+0440
SPGR4510+0441
SPGR4520+0442
SPGR4530+0443
LOVR0000+0444
LOVR0010+0445
LOVR0020+0446
LOVR0030+0447
LOVR0040+0448
LOVR0050+0449
LOVR0060+0450
LOVR0070+0451
LOVR0080+0452
LOVR0090+0453
LOVR0100+0454
LOVR0110+0455
LOVR0120+0456
LOVR0130+0457
LOVR0140+0458
LOVR0150+0459
LOVR0160+0460
LOVR0170+0461
LOVR0180+0462
LOVR0190+0463
LOVR0200+0464
LOVR0210+0465
LOVR0220+0466
LOVR0230+0467
LOVR0240+0468
LOVR0250+0469
LOVR0260+0470
LOVR0270+0471
LOVR0280+0472
LOVR0290+0473
LOVR0300+0474
LOVR0310+0475
LOVR0320+0476
LOVR0330+0477
LOVR0340+0478
LOVR0350+0479
LOVR0360+0480
LOVR0370+0481
LOVR0380+0482
LOVR0390+0483
LOVR0400+0484
LOVR0410+0485
LOVR0420+0486
LOVR0430+0487
LOVR0440+0488
LOVR0450+0489
LOVR0460+0490
LOVR0470+0491
LOVR0480+0492
LOVR0490+0493
LOVR0500+0494
LOVR0510+0495
LOVR0520+0496

```

Figure H-2. Listing of ground spur gear computer program (cont'd)

LDVR0530=0547
LDVR0530=0548
LDVR0530=0549
LDVR0530=0550
LDVR0570=0551
LDVR0530=0552
LDVR0540=0553
LDVR0600=0554
LDVR0610=0555
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LDVR0730=0567
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LDVR0840=0578
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LDVR0860=0580
LDVR0870=0581
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LDVR0910=0585
LDVR0920=0586
LDVR0930=0587
LDVR0940=0588
LDVR0950=0589
LDVR0960=0590
LDVR0970=0591
LDVR0980=0592
LDVR0990=0593
LDVR1000=0594
LDVR1010=0595
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LDVR1030=0597
LDVR1040=0598
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LDVR1310=0625
LDVR1320=0626
LDVR1330=0627
LDVR1340=0628
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LDVR1680=0662
LDVR1690=0663
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LDVR1800=0674
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LDVR1890=0683
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LDVR2080=0702
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LDVR2100=0704
LDVR2110=0705
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LDVR2290=0723
LDVR2300=0724
LDVR2310=0725
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LDVR3510=0845
LDVR3520=0846
LDVR3530=0847
LDVR3540=0848
LDVR3550=0849
LDVR3560=0850
LDVR3570=0851
LDVR3580=0852
LDVR3590=0853
LDVR3600=0854
LDVR3610=0855
LDVR3620=0856
LDVR3630=0857
LDVR3640=0858
LDVR3650=0859
LDVR3660=0860
LDVR3670=0861

245

```

K2=JB-1
DO 85 I=K1,K2
  NX(I) = MN
  PRMOD(I)=0.
85 CONTINUE
IF (MOD(E0,0) GO TO 180
J=JA-1
DO 95 I=1,JC
  J=J+1
  PRMOD(I)=PMG(I)
  PRMOD(J)=PMF(I)
95 CONTINUE
127 WRITE (LOU,1002)
DO 130 I=1,JD
  WRITE (LOU,1003) I,(ROLLRAY(I,J),J=1,2), DFP(I), DFG(I),YDF(I),
    * NX(I),PRMOD(I)
130 CONTINUE
IF (MOD(CT,0) GO TO 205
180 WRITE (LOU,1000)
DO 190 I=1,JD
  WRITE (LOU,1001) I,(ROLLRAY(I,J),J=1,2), DFP(I), DFG(I),YDF(I),
    * NX(I)
190 CONTINUE
1000 FORMAT (///7X,14HROLL ANGLE8,10X,11HDEFLECTION8,10X,5HTOTAL,
  *7X,4HLOAD/6X,6HDRIVER,3X,6HDRIVEN,7X,6HDRIVER,6X,6HDRIVEN,6X,
  *11HDEFLECTION8,3X,5HFORCE/)
1001 FORMAT (1X,12, 3X,F5,2,4X,F5,2,3(4X,E9,3),3X,F8,1)
1002 FORMAT (1X,12, 4X,F5,2,4X,F5,2,3(4X,E9,3),3X,F8,1,2X,E9,3)
1003 FORMAT (///7X,14HROLL ANGLE8,10X,11HDEFLECTION8,10X,5HTOTAL,
  *7X,4HLOAD/6X,6HDRIVER,3X,6HDRIVEN,7X,6HDRIVER,6X,6HDRIVEN,6X,
  *11HDEFLECTION8,3X,5HFORCE,2X,13HMODIFICATIONS/)
205 IF (MARK1,E0,0,AND,MARK3,CT,JD) GO TO 240
215 IF (MARK1,CT,JA,OR,MARK3,LT,JD) GO TO 230
  NXZR=,0002*HN
  IF (MARK1,E0,0) GO TO 220
  IF (ABS(NX(JA)),GT,NXZR) GO TO 230
  IF (MARK3,CT,JD) GO TO 225
220 IF (ABS(NX(JD)),GT,NXZR) GO TO 230
  NX(JD)=0.
  IF (MARK1,E0,0) GO TO 240
225 NX(JA)=0.
  GO TO 240
230 KAMP=KAMP+1
  IF (MARK1,E0,0) GO TO 235
  IF (MARK1,CT,INT/2) GO TO 210
  EP1=ROLLRAY(MARK1,1) * HN
  EP = ROLLRAY(MARK1+1,1)*HN
  U1 = ABS(NX(MARK1)) * (EP -EP1)/(NX(MARK1+1)+ABS(NX(MARK1)))
  EDCP=EP1+U1
  IF (MARK3,CT,JD) GO TO 240
  K=JD-MARK3+1
  IF (K,GT,INT/2) GO TO 210
235 EP2=ROLLRAY(MARK3,1)*HN
  EP2=ROLLRAY(MARK3-1,1)*HN
  U2 = NX(MARK3-1) * (EP-EP2)/(NX(MARK3-1)+ABS(NX(MARK3)))
  EDCP=EP2+U2
  GO TO 240
210 WRITE (LOU,777)
777 FORMAT (////45H PROGRAM TERMINATES, NEG. LOADS EXCEED INT/2)
STOP
240 DO 250 I=1,JD
  NXFO(I,1)=NX(I)
  LDVR1150=0559
  LDVR1160=0560
  LDVR1170=0561
  LDVR1180=0562
  LDVR1190=0563
  LDVR1200=0564
  LDVR1210=0565
  LDVR1220=0566
  LDVR1230=0567
  LDVR1240=0568
  LDVR1250=0569
  LDVR1260=0570
  LDVR1270=0571
  LDVR1280=0572
  LDVR1290=0573
  LDVR1300=0574
  LDVR1310=0575
  LDVR1320=0576
  LDVR1330=0577
  LDVR1340=0578
  LDVR1350=0579
  LDVR1360=0580
  LDVR1370=0581
  LDVR1380=0582
  LDVR1390=0583
  LDVR1400=0584
  LDVR1410=0585
  LDVR1420=0586
  LDVR1430=0587
  LDVR1440=0588
  LDVR1450=0589
  LDVR1460=0590
  LDVR1470=0591
  LDVR1480=0592
  LDVR1490=0593
  LDVR1500=0594
  LDVR1510=0595
  LDVR1520=0596
  LDVR1530=0597
  LDVR1540=0598
  LDVR1550=0599
  LDVR1560=0600
  LDVR1570=0601
  LDVR1580=0602
  LDVR1590=0603
  LDVR1600=0604
  LDVR1610=0605
  LDVR1620=0606
  LDVR1630=0607
  LDVR1640=0608
  LDVR1650=0609
  LDVR1660=0610
  LDVR1670=0611
  LDVR1680=0612
  LDVR1690=0613
  LDVR1700=0614
  LDVR1710=0615
  LDVR1720=0616
  LDVR1730=0617
  LDVR1740=0618
  LDVR1750=0619
  LDVR1760=0620

```

Figure H-2. Listing of ground spur gear computer program (cont'd)

```

250 CONTINUE
RETURN
END
SUBROUTINE PHI (DEC,F ,M,T,XDIM,DFS,DX,DS,DR,PGK,AN,RPMI, EV, PHI
* PI,RN,JO,FMIN,PCO,KRW)
DIMENSION DEC(KRW),F(KRW),M(KRW),XDIM(KRW),T(KRW),DFS(KRW),
* DV(40),RV(40),PCO(40),DW(40)
DATA IGEAR /6HGEAR /
DO 10 I=1,JO
FF = DR/DEC(I)
FRA = A/AN(SORT(1,=(FF)**2)/FF)
FBI = SIN(FRA)
PCO(I)=COS(FRA)
FPT = FBI/PCO(I)
F(I) = FPT*PGK
DV(I) = DS/COS(F(I))
RV(I) = DV(I)*S
10 CONTINUE
C
C CALL SUBROUTINES XY AND WEAK WHEN...
C
40 CALL XY (DS,DR,RPMI,PGK,X,Y)
CALL WEAK (RV,T,M,XDIM,DW,X,Y,RPMI,KRW,JO)
50 DO 60 I=1,JO
T2=T(I)*2,
DFS(I)=19,0/(EV+FMIN)*(M(I)/T2) *COS(F(I))
60 CONTINUE
RETURN
END
SUBROUTINE WEAK (RV,T,M,XDIM,DW,X,Y,RPMI,KRW,JO)
DIMENSION T(KRW),M(KRW),RV(KRW),XDIM(KRW),DW(KRW),ALPHA(40)
C
C SUBROUTINE WEAK CALCS. THE DIA. OF THE WEAKEST SECTION (DW) BY
C INSCRIBING THE LARGEST PARABOLA THAT WILL FIT THE GEAR TOOTH SHAPE.
C
DO 50 I=1,JO
ALPHA(I)=.1
DELTA =.1
10 V = SIN(ALPHA(I))*RPMI
V1 = SORT((RPMI)**2-(V)**2)
C T = HALF CHORD AT THE WEAKEST SECTION
T(I)=X=V1
Y=V1/(SIN(ALPHA(I))/COS(ALPHA(I)))
C M = TOOTH HEIGHT FROM WEAKEST SECTION TO VERTEX OF PARABOLA
M(I)=(RV(I)-V)*V
YAP=YA+.S
IF (YAP=M(I)) 20,40,30
20 ALPHA(I)=ALPHA(I)+DELTA
DELTA=.1*DELTA
IF (.00000001-DELTA) 10,40,40
30 ALPHA(I)=ALPHA(I)+DELTA
GO TO 10
40 YB=Y-V
C DW = WEAKEST SECTION DIAMETER
DW(I)=SORT((YB)**2+(T(I))**2)*2,
Q=ATAN(M(I)/T(I))
Q=1.57079633-Q
C XDIM= X DIMENSION
XDIM(I)=T(I)*(SIN(Q)/COS(Q))
50 CONTINUE
RETURN
END
SUBROUTINE XY (DS,DR,RPMI,PGK,X,Y)
LOVR1770=0621
LOVR1780=0622
LOVR1790=0623
PHI 0000=0624
PHI 0010=0625
PHI 0020=0626
PHI 0030=0627
PHI 0040=0628
PHI 0050=0629
PHI 0060=0630
PHI 0070=0631
PHI 0080=0632
PHI 0090=0633
PHI 0100=0634
PHI 0110=0635
PHI 0120=0636
PHI 0130=0637
PHI 0140=0638
PHI 0150=0639
PHI 0160=0640
PHI 0170=0641
PHI 0180=0642
PHI 0190=0643
PHI 0200=0644
PHI 0210=0645
PHI 0220=0646
PHI 0230=0647
PHI 0240=0648
WEAK0000=0649
WEAK0010=0650
WEAK0020=0651
WEAK0030=0652
WEAK0040=0653
WEAK0050=0654
WEAK0060=0655
WEAK0070=0656
WEAK0080=0657
WEAK0090=0658
WEAK0100=0659
WEAK0110=0660
WEAK0120=0661
WEAK0130=0662
WEAK0140=0663
WEAK0150=0664
WEAK0160=0665
WEAK0170=0666
WEAK0180=0667
WEAK0190=0668
WEAK0200=0669
WEAK0210=0670
WEAK0220=0671
WEAK0230=0672
WEAK0240=0673
WEAK0250=0674
WEAK0260=0675
WEAK0270=0676
WEAK0280=0677
WEAK0290=0678
WEAK0300=0679
WEAK0310=0680
WEAK0320=0681
XY 0000=0682

```

Figure H-2. Listing of ground spur gear computer program (cont'd)


```

C
C SUBROUTINE XV ==CALCULATES COORDINATES TO CENTER OF FILLET RADIUS
C
  MM=(DR/2.)*RPMI
  MM=(DB/2.)*M
  IF (MM) 10,10,12
10 CPR=(DB/2.)/M
  CPARAMATAN(SORT(1.-(CPR)**2)/CPR)
  OPP=SQRT((M)**2-(DB/2.)**2)
  A=OPP/RPMI
  M1=SQRT((A)**2+(DB/2.)**2)
  CA=(DB/2.)/M1
  CARAMATAN(SORT(1.-(CA)**2)/CA)
  ZCA=(SIN(CARA)/COS(CARA))-CARA
  B=(CPR-CARA)-ZCA
  FAPRA=PBK+B
11 X=SQN(FAPRA)*M
  Y=COS(FAPRA)*M
  RETURN
12 XX=(DB/2.)*SQN(PBK)
  FAPBI=(XX/RPMI)/M
  FAPRA=ATAN(FAPBI/(SQRT(1.-(FAPBI)**2)))
  GO TO 11
END
SUBROUTINE ROLLANG (VO,ZA,ZR,WO,DOPMA,DOGMA,PB,FXTA
  ,ROLLRAY,RHOP,RHOG,DECP,DECG,VS,VT
  ,ECCP,EZCP,MARK1,MARK2,KRW)
  DIMENSION ROLLRAY(KRW,2),RHOP(KRW),RHOG(KRW),VS(KRW),VT(KRW),
  ,DECP(KRW),DECG(KRW)
  COMMON JA,JB,JC,JD,KAMP,PI,RN,WN,RPMI,LOU,LIN,ANP,ANG,OMP,ONG,
  ,ODDSG,CNSTD,PZP,PEG,ALPHA,UPA,P1,P2,IXT,BET
  SUBROUTINE ROLLANG
  CALCULATES,,1. EPSILON OD=MAX
  2. ROLL ANGLES
  3. PROFILE CONTACT RATIOS
  4. RADIUS OF CURVATURE FOR EACH ROLL ANGLE
  5. SLIDING VELOCITY FOR EACH ROLL ANGLE
  6. SUM VELOCITY FOR EACH ROLL ANGLE
  7. DIAMETER AT ENGAGEMENT CONDITIONS FOR EACH ROLL
  ANGLE
  METHOD OF CALCULATING ROLL ANGLES
  1. FOR KAMP=1 -CALCULATE PINION ROLL ANGLES AT MAIN POINTS
  FOR KAMP=2 -(ROLL ANGLES ARE TO BE RECALCULATED BECAUSE OF A
  OF A NEGATIVE LOAD) - CALCULATE MAIN
  POINTS WITH ADJUSTMENTS U1 AND/OR U2
  2. USING THESE VALUES CALCULATE ALL PINION AND GEAR ROLL ANGLE
  DEFINITIONS OF VARIABLES FOR SUBROUTINE ROLLANG
  KAMP = 1 , ROLL ANGLES CALCULATED WITH NORMAL CONDITIONS
  = 2 , ROLL ANGLES RECALCULATED WITH NORMAL CONDITIONS
  AMPMA -PROFILE CONTACT RATIO (MAXIMUM)
  AMPM1 -PROFILE CONTACT RATIO (MINIMUM)
  PINION GEAR
  FOPMA EOSMI -EPSILON OD MAX DRIVER
  FOPM1 EOSMA -EPSILON OD MAX DRIVEN
  ONEPB ONEPBG -ONE BASE PITCH
  ARRAY DESIGNATION
  ROLLRAY COL,1 -DRIVER ROLL ANGLES
  COL,2 -DRIVEN GEAR ROLL ANGLES
  VS -SLIDING VELOCITIES
  VT -SUM VELOCITIES

```

```

XV 0010+0683
XV 0020+0684
XV 0030+0685
XV 0040+0686
XV 0050+0687
XV 0060+0688
XV 0070+0689
XV 0080+0690
XV 0090+0691
XV 0100+0692
XV 0110+0693
XV 0120+0694
XV 0130+0695
XV 0140+0696
XV 0150+0697
XV 0160+0698
XV 0170+0699
XV 0180+0700
XV 0190+0701
XV 0200+0702
XV 0210+0703
XV 0220+0704
XV 0230+0705
XV 0240+0706
ROLL0000+0707
ROLL0010+0708
ROLL0020+0709
ROLL0030+0710
ROLL0040+0711
ROLL0050+0712
ROLL0060+0713
ROLL0070+0714
ROLL0080+0715
ROLL0090+0716
ROLL0100+0717
ROLL0110+0718
ROLL0120+0719
ROLL0130+0720
ROLL0140+0721
ROLL0150+0722
ROLL0160+0723
ROLL0170+0724
ROLL0180+0725
ROLL0190+0726
ROLL0200+0727
ROLL0210+0728
ROLL0220+0729
ROLL0230+0730
ROLL0240+0731
ROLL0250+0732
ROLL0260+0733
ROLL0270+0734
ROLL0280+0735
ROLL0290+0736
ROLL0300+0737
ROLL0310+0738
ROLL0320+0739
ROLL0330+0740
ROLL0340+0741
ROLL0350+0742
ROLL0360+0743
ROLL0370+0744

```

Figure H-2. Listing of ground spur gear computer program (cont'd)

Figure H-2. Listing of ground spur gear computer program (cont'd)

```

K2=K1+1
DO 35 I=1,K2
J=I-1
K=JC+1
ESTP = EESP + ((EESP-EESP)/(21-2*K1))*J
ESTG = (VO*NO)/(OEG/2) = ESTP*(ANP/ANG)
C
ROLLRAY(K,1)=ESTP/RN
ROLLRAY(K,2)=ESTG/RN
35 CONTINUE
EBCG=ROLLRAY(JA,2) * RN
AMPMA = ((ANG*(FJGMA*RN-PXTA))+(ANP*(EOPMA*RN-PXTA)))/(2*PI)
AMPMI = ((ANG*(EBCG-PXTA))+(ANP*(EBCP-PXTA)))/(2*PI)
WRITE (LOU,1006) (ROLLRAY(JA,I),I=1,2),(ROLLRAY(JC,J),J=1,2),
(ROLLRAY(JB,K),K=1,2),(ROLLRAY(JD,L),L=1,2),
EOPMA,EOPMI,EOPMA,AMPMA,AMPMI
C
70 WRITE (LOU,1008)
DO 40 I=1,JD
EP=ROLLRAY(I,1)*RN
EG=ROLLRAY(I,2)*RN
RMOP(I) = EP * (OEP/2)
RMOC(I) = (VO*NO) * RMOP(I)
DECP(I) = SORT(EP**2+.1) * OEP
DECC(I) = SORT(EG**2+.1) * OEG
VS(I) = (RMOP(I)-(ANP/ANG*RMOC(I)))/(PI*RPMP/30.)
VT(I) = (RMOC(I)-(ANP/ANG*RMOP(I)))/(PI*RPMP/30.)
WRITE (LOU,1009) I,(ROLLRAY(I,J),J=1,2),RMOP(I),RMOC(I),VS(I),
VT(I)
40 CONTINUE
1006 FORMAT (/IX,40HROLL ANGLES AT MAIN POINTS,,(IN DEGREES)//
*IX,F5,2,5H EBCP,5X,F5,2,5H EBCG,7X,18H= BEGIN CONTACT, A/
*IX,F5,2,5H EESP,5X,F5,2,5H EBCG,7X,28H= BEGIN SINGLE TOOTH CONTACT,
*, 12H, C (LPSTC)/
*IX,F5,2,5H EESP,5X,F5,2,5H EBCG,7X,28H= END SINGLE TOOTH CONTACT,
*, 10H (HPSTC)/
*IX,F5,2,5H EBCP,5X,F5,2,5H EBCG,7X,16H= END CONTACT, D/
*IX,F5,2,6H EOPMA,4X,F5,2,6H EOPMI,6X,15H= OD MAX DRIVER/
*IX,F5,2,6H EOPMA,4X,F5,2,6H EOPMI,6X,15H= OD MAX DRIVER/
*IX,F5,2,6H AMPMA,4X,F5,2,6H AMPMI,6X,23H= PROFILE CONTACT RATIO)
1008 FORMAT (////7X,12HROLL ANGLES,5X,19HRAIDUS OF CURVATURE,10X,
* 7HSLIDING,7X,3HSM/47X,2(6X,8HVELOCITY)/
* 5X,15HDRIVER DRIVEN,6X,16HDRIVER DRIVEN,
* 6X,2(6X,8H(IN/SEC))/)
1009 FORMAT (IX,I2 ,3X,F5,2,4X,F5,2,5X,F7,4,3X,F7,4,4X,2(7X,F7,2))
RETURN
END
SUBROUTINE FRPME (TS,TEMP,NXFO,FRIC,PHE,KRN,JD,L1,VS,FN,ER,
* PHEAV,FR1,FR2,YCON,JB,JC,ROLLRAY,PHEI,JA,RPMP,ANP)
DIMENSION FRIC(KRN),PHE(KRN),NXFO(KRN,2),VS(KRN),ROLLRAY(KRN,2)
DO 10 I=1,JD
DFORCE=NXFO(I,L1)
IF (DFORCE.NE.0.) GO TO 8
FRIC(I)=0.
GO TO 25
8 PVS=DFORCE*((ABS(VS(I))**.5)+(-1./3.))
IF (PVS.LT.200.) GO TO 15
FRIC(I)=PVS
GO TO 25
15 FRIC(I)=FRP*(PVS**(-.30))
25 PHE(I)=FRIC(I)*DFORCE*ABS(VS(I))/9336.
30 CONTINUE

```

```

ROLL1000=0807
ROLL1010=0808
ROLL1020=0809
ROLL1030=0810
ROLL1040=0811
ROLL1050=0812
ROLL1060=0813
ROLL1070=0814
ROLL1080=0815
ROLL1090=0816
ROLL1100=0817
ROLL1110=0818
ROLL1120=0819
ROLL1130=0820
ROLL1140=0821
ROLL1150=0822
ROLL1160=0823
ROLL1170=0824
ROLL1180=0825
ROLL1190=0826
ROLL1200=0827
ROLL1210=0828
ROLL1220=0829
ROLL1230=0830
ROLL1240=0831
ROLL1250=0832
ROLL1260=0833
ROLL1270=0834
ROLL1280=0835
ROLL1290=0836
ROLL1300=0837
ROLL1310=0838
ROLL1320=0839
ROLL1330=0840
ROLL1340=0841
ROLL1350=0842
ROLL1360=0843
ROLL1370=0844
ROLL1380=0845
ROLL1390=0846
ROLL1400=0847
ROLL1410=0848
ROLL1420=0849
ROLL1430=0850
ROLL1440=0851
ROLL1450=0852
ROLL1460=0853
FRPM0070=0854
FRPM0080=0855
FRPM0090=0856
FRPM0100=0857
FRPM0110=0858
FRPM0120=0859
FRPM0130=0860
FRPM0140=0861
FRPM0150=0862
FRPM0160=0863
FRPM0170=0864
FRPM0180=0865
FRPM0190=0866
FRPM0200=0867
FRPM0210=0868

```

Figure H-2. Listing of ground spur gear computer program (cont'd)

```

PHE1=0.
J=JC=1
DO 60 I=1,J
PHE1=PHE1+(PHE(I)*PHE(I+1))
60 CONTINUE
PHEAV1=PHE1*(ROLLRAY(JC,1)-ROLLRAY(JA,1))/(B,=J)
PHE2=0.
K1=JC=1
K2=JB=2
KX=K2-K1+1
DO 90 I=K1,K2
PHE2=PHE2+(PHE(I)*PHE(I+1))
90 CONTINUE
PHEAV2=K2*(ROLLRAY(JB,1)-ROLLRAY(JC,1))/(B,=KX)
PHE3=0.
J=JN=1
KX=JD=JB
DO 80 I=JB,J
PHE3=PHE3+(PHE(I)*PHE(I+1))
80 CONTINUE
PHEAV3=PHE3*(ROLLRAY(JD,1)-ROLLRAY(JB,1))/((KX)=2,=)
PHE4=(PHEAV1)+PHEAV2+PHEAV3)*ANP/(B,=RPMP)
PHE4V=PHE4*RPMP/B0.
T8=(TC0+(PHE4V)*,B0))*TEMP
RETURN
END
SUBROUTINE FORCE (AP,AG, CPH,DIP,DIG,EP,EG,FMINP,FMING,FNRA,
=MORSES,NATQ,PMI,GMI,PRP,PRG,TPM18,TGM18,VA,NOP,NDC,INT,YDFLS)
COMMON JA,JB,JC,JD,KAMP,PI,RN,RN,RPMP,LOU,LIN,ANP,ANG,DBP,DBG,
=NDDBG,CHSTD,PEP,PEG,ALPHA,OPA,PI,P2,INT,BET
PMP=RPMP*(ANP/ANG)
IF (RPMP,LT,PMP) PMP=RPMP
PM = PMI/((DIP/2,)*(DIP/2,))
GM = GMI/((DIG/2,)*(DIG/2,))
EM = (PM+GM)/(PM+GM)
CH = (1,/FMING) * (0,*(NDC/2,)*=3/(EG+TGM18*=3))*
(1,/FMINP) * (0,*(NOP/2,)*=3/(EP+TPM18*=3))
CH1 = (1,/FMING) * ((2,*(1,+(PRG)/EG)*((0,1/AC*(CPN*COB(FNRA)))*
,125))
CH2 = (1,/FMINP) * ((2,*(1,+(PRP)/EP)*((0,1/AP*(CPN*COB(FNRA)))*
,125))
CC1=(1,/FMING)*(ANG/(B,*(AG/(CPN*COB(FNRA))*2)*EG))
CC2=(1,/FMINP)*(ANP/(B,*(AP/(CPN*COB(FNRA))*2)*EP))
CC=CC1+CC2
CR=PI+CH2
CT=CB+CR+CC
TS=1,/CT
TGM=1000,=MORSES/PMMP
V=(PI+DIP/12,)*RPMP
IF (NATQ,NE,0,=) GO TO 10
T8P,=PI*SQRT(CT*EM)
NATQ=60,/T
GO TO 20
10 T8=,/NATQ
20 LEM=(PEP+PEG)*TOFES=PI
VPS=(PEP+PEG)*TOFES=(2,=PI)
100 V=(PI+DIP/12,)*RPMP
BT=CPN/12,*(B0,/V)
TT = AT/T
IF (TT,=,277) 120,120,110
110 ET=0,B16/SQRT(1,*(B,6*(TT+TT)))
GO TO 130

```

```

FRPH0150+0064
FRPH0160+0070
FRPH0170+0071
FRPH0180+0072
FRPH0190+0073
FRPH0200+0074
FRPH0210+0075
FRPH0220+0076
FRPH0230+0077
FRPH0240+0078
FRPH0250+0079
FRPH0260+0080
FRPH0270+0081
FRPH0280+0082
FRPH0290+0083
FRPH0300+0084
FRPH0310+0085
FRPH0320+0086
FRPH0330+0087
FRPH0340+0088
FRPH0350+0089
FRPH0360+0090
FRPH0370+0091
FRPH0380+0092
FRPH0390+0093
FRPH0400+0094
FORC0000+0095
FORC0010+0096
FORC0020+0097
FORC0030+0098
FORC0040+0099
FORC0050+0100
FORC0070+0101
FORC0080+0102
FORC0090+0103
FORC0100+0104
FORC0110+0105
FORC0120+0106
FORC0130+0107
FORC0140+0108
FORC0150+0109
FORC0160+0110
FORC0180+0111
FORC0200+0112
FORC0210+0113
FORC0220+0114
FORC0230+0115
FORC0240+0116
FORC0250+0117
FORC0260+0118
FORC0270+0119
FORC0280+0120
FORC0290+0121
FORC0300+0122
FORC0310+0123
FORC0320+0124
FORC0330+0125
FORC0340+0126
FORC0350+0127
FORC0360+0128
FORC0370+0129
FORC0380+0130

```

Figure H-2. Listing of ground spur gear computer program (cont'd)

```

120 FT=1./((1.+(b,b*(TY+TY)))
130 DI = EE+TY+TS
IF (RPM,GT,PMR) GO TO 135
DXG = DXF
135 TIMI=(DXG/2)
DF=YG/(YG+YI)
WRITE (LUU,1001) DI,DF
1001 FORMAT (E11.4,21H = DYNAMIC INCREMENT/E11.4,18H = DYNAMIC FACTOR)
*)
RETURN
END
FORC0390+0931
FORC0400+0932
FORC0410+0933
FORC0420+0934
FORC0430+0935
FORC0440+0936
FORC0450+0937
FORC0460+0938
FORC0470+0939
FORC0480+0940
FORC0490+0941
0942

```

Figure H-2. Listing of ground spur gear computer program (cont'd)


```

INPUT VARIABLES FOR SPECIFIED LOOPS AND SECTIONS

VARIABLES USED IN... RPM LOOP
RPM = 8000.00
-RPM DRIVER

VARIABLES USED IN...HORSEPOWER LOOP
HORSES = 600.00000
HHP = 100.00000
HPM = 1100.00000
-HORSEPOWER
-HORSEPOWER INCREMENT
-MAXIMUM HORSEPOWER

VARIABLES USED IN... MODIF. LOOP
P1 = .00045 P2 = .00035
E1 = 25.00000 E2 = 27.00000

METHOD USED FOR CALCULATING NORMAL FORCE-
1 NORMAL FORCE = STATIC FORCE
2 DYNAMIC FACTOR CALCULATED BY TUPLIN METHOD
(DRIVER)
AP = .79200 AC = .31600
PEP = .00020 PEC = .00020
PMI = .01678 CMI = .00060
NAT(1) = .00000
NAT(2) = .00000
NAT(3) = .00000
-THICKNESS OF RIM BELOW ROOT
-PITCH ERROR
-MASS MOMENT OF INERTIA (LB,SEC.SQ,IN.)
-1ST NATURAL FREQUENCY(RPM)
-2ND NATURAL FREQUENCY(RPM)
-3RD NATURAL FREQUENCY(RPM)

VARIABLES USED IN... FLASH TEMP SECTION
FR1 = .0100
FR2 = .0002
TCOM = 225.0
TEMP = 250.0
-CONSTANT FRICTION FACTOR
-VARIABLE FRICTION FACTOR
-TEMP DIFFERENCE FACTOR
-OIL JET TEMPERATURE,F

```

Figure H-3. Sample ground spur gear computer printout (cont'd)

ROLL ANGLE SECTION

ROLL ANGLES AT MAIN POINTS, (IN DEGREES)

19.68 ENCP 26.60 EDCG - BEGIN CONTACT, A
 20.00 ENCP 26.90 EDCG - BEGIN SINGLE TOOTH CONTACT, C (LPSTC)
 26.24 EDCG 21.07 EDCG - END SINGLE TOOTH CONTACT, B (MPSTC)
 31.70 EDCG 19.66 EDCG - END CONTACT, D
 33.14 EDCG 19.85 EDCG - OD MAX DRIVEN
 37.91 EDCG 27.32 EDCG - OD MAX DRIVEN
 1.75 AMPMA 1.07 AMPMA - PROFILE CONTACT RATIO

ROLL ANGLES		RADIUS OF CURVATURE		SLIDING VELOCITY (IN/SEC)	SUN VELOCITY (IN/SEC)
DRIVER	DRIVEN	DRIVER	DRIVEN		
1	19.68			-295.81	1020.57
2	19.54	1.932	1.9247	-262.96	1033.75
3	19.44	1.937	1.891	-232.12	1046.91
4	17.30	1.963	1.8715	-200.78	1060.12
5	16.28	1.965	1.8499	-169.93	1073.30
6	14.18	1.935	1.8189	-138.09	1086.38
7	20.08	1.940	1.7918	-106.74	1099.66
8	20.08	1.946	1.7652	-82.73	1109.76
9	20.77	1.930	1.7499	-58.71	1119.86
10	21.06	1.933	1.7295	-34.70	1129.96
11	22.15	1.937	1.7091	-10.69	1140.06
12	22.84	1.927	1.6838	13.33	1150.16
13	23.53	1.909	1.6631	37.34	1160.26
14	24.22	1.890	1.6431	61.35	1170.36
15	24.91	1.871	1.6227	85.37	1180.46
16	25.60	1.855	1.6023	109.38	1190.56
17	26.29	1.837	1.5820	133.39	1200.66
18	26.98	1.820	1.5620	157.40	1210.76
19	27.67	1.803	1.5420	181.41	1220.86
20	28.36	1.786	1.5220	205.42	1230.96
21	29.05	1.769	1.5020	229.43	1241.06
22	29.74	1.752	1.4820	253.44	1251.16
23	30.43	1.735	1.4620	277.45	1261.26
24	31.12	1.718	1.4420		

Figure H-3. Sample ground spur gear computer printout (cont'd)

MP	MC	TP	TC	PPCO	PCCO
.057	.2042	.2056	.2597	.9687	.9020
.0997	.1920	.2015	.2603	.9650	.9082
.0539	.1802	.2070	.2610	.9610	.9114
.0505	.1687	.2030	.2616	.9569	.9136
.0635	.1576	.2045	.2627	.9527	.9158
.0608	.1460	.2050	.2630	.9483	.9179
.0746	.1360	.2022	.2651	.9437	.9200
.0794	.1286	.2029	.2651	.9437	.9200
.0848	.1211	.2070	.2652	.9401	.9212
.0898	.1138	.2052	.2675	.9365	.9233
.0755	.1067	.2030	.2689	.9327	.9254
.1016	.0948	.2519	.2705	.9289	.9265
.1079	.0931	.2519	.2729	.9250	.9281
.1197	.0867	.2097	.2745	.9211	.9296
.1219	.0809	.2082	.2769	.9171	.9312
.1299	.0793	.2068	.2795	.9130	.9327
.1394	.0743	.2055	.2828	.9089	.9343
.1449	.0665	.2055	.2825	.9049	.9363
.1511	.0591	.2041	.2870	.9039	.9382
.1630	.0518	.2020	.2921	.8979	.9402
.1759	.0446	.2009	.3055	.8923	.9421
.1891	.0375	.2001	.3117	.8866	.9440
.2029	.0303	.2000	.3145	.8800	.9459

Figure H-3. Sample ground spur gear computer printout (cont'd)

LOAD FORCE SECTION

600.00 = WORKPOWER
 8000.00 = RPM DRIVER
 2795.73 = IN
 .00000 = P1
 25.00000 = F1
 .00035 = P2
 27.00000 = F2

	ROLL DRIVER	ANGLES DRIVER	DEFLECTIONS DRIVER	DEFLECTIONS DRIVER	TOTAL DEFLECTIONS	LOAD FORCE	MODIFICATIONS
1	19.64	26.60	1.043E-04	7.977E-04	8.020E-04	968.8	9.500E-04
2	15.58	26.24	1.640E-04	7.036E-04	8.676E-04	668.9	2.069E-04
3	16.98	25.87	1.881E-04	6.804E-04	8.685E-04	668.9	2.070E-04
4	17.38	25.50	1.975E-04	6.182E-04	8.158E-04	1313.0	1.000E-04
5	18.04	25.13	2.166E-04	5.772E-04	7.938E-04	1669.7	3.777E-05
6	19.18	24.77	2.372E-04	5.367E-04	7.740E-04	1402.2	0.
7	20.00	24.40	2.594E-04	4.975E-04	7.570E-04	868.9	0.
8	20.00	24.40	2.594E-04	4.975E-04	7.570E-04	2795.7	0.
9	20.74	24.12	2.777E-04	4.682E-04	7.460E-04	2795.7	0.
10	21.76	23.84	2.970E-04	4.396E-04	7.366E-04	2795.7	0.
11	22.19	23.66	3.174E-04	4.116E-04	7.290E-04	2795.7	0.
12	22.84	23.27	3.380E-04	3.893E-04	7.271E-04	2795.7	0.
13	23.53	22.94	3.610E-04	3.677E-04	7.287E-04	2795.7	0.
14	24.27	22.71	3.850E-04	3.474E-04	7.324E-04	2795.7	0.
15	24.91	22.43	4.094E-04	3.287E-04	7.381E-04	2795.7	0.
16	25.60	22.15	4.353E-04	3.120E-04	7.473E-04	2795.7	0.
17	26.24	21.87	4.624E-04	2.984E-04	7.608E-04	2795.7	0.
18	26.84	21.60	4.905E-04	2.885E-04	7.790E-04	2364.9	0.
19	27.14	21.33	5.191E-04	2.807E-04	8.000E-04	2187.9	0.
20	28.04	20.77	5.800E-04	1.997E-04	7.797E-04	1820.4	5.017E-05
21	28.04	20.77	5.800E-04	1.997E-04	7.797E-04	1682.8	1.251E-04
22	29.84	20.48	6.224E-04	1.953E-04	8.177E-04	1130.1	2.001E-04
23	30.74	20.03	6.674E-04	1.945E-04	8.619E-04	893.8	2.750E-04
24	31.70	19.66	7.141E-04	1.951E-05	8.336E-04	786.8	3.500E-04

Figure H-3. Sample ground spur gear computer printout (cont'd)

PLASH TIME SECTION
 600.00 = NONRESPONDER
 8000.00 = RPM DRIVER
 2745.73 = IN
 394.78 = 13.6
 .5743 = PHTAU, 8/SEC

NORMAL FORCE = LOAD FORCE

ROLL ANGLES	LOAD FORCE	FRICTION	HERTZ STRESS	PLASH TEMP F	CTEMP	PHE
1 19.68	660.8	2.2374E-02	74593	18.22	913.00	3.713E-01
2 15.58	660.8	1.9895E-02	87056	18.50	913.29	3.752E-01
3 16.98	964.4	1.7546E-02	102421	18.63	913.91	4.221E-01
4 17.38	1313.0	1.6900E-02	117135	18.54	913.38	4.031E-01
5 18.28	1665.7	1.6900E-02	124622	18.48	913.23	4.057E-01
6 19.18	1902.2	1.6900E-02	136228	18.38	911.19	4.614E-01
7 20.08	2068.4	1.6900E-02	134891	13.27	908.03	3.874E-01
8 20.08	2745.7	1.6900E-02	162619	16.63	911.91	5.202E-01
9 20.77	2745.7	1.6900E-02	160825	12.75	907.53	4.063E-01
10 21.46	2745.7	1.6900E-02	159150	8.46	903.74	2.884E-01
11 22.15	2745.7	1.6900E-02	157585	5.24	900.03	1.704E-01
12 22.84	2745.7	1.6900E-02	156123	1.60	346.38	5.249E-02
13 23.53	2745.7	1.6900E-02	154755	1.98	346.76	6.545E-02
14 24.22	2745.7	1.6900E-02	153472	5.50	900.48	1.834E-01
15 24.91	2745.7	1.6900E-02	152284	8.46	903.74	3.013E-01
16 25.60	2745.7	1.6900E-02	151194	12.37	907.15	4.183E-01
17 26.29	2745.7	1.6900E-02	150104	15.73	910.52	5.372E-01
18 26.98	2745.7	1.6900E-02	149014	13.71	908.94	4.971E-01
19 27.67	2127.9	1.6900E-02	124465	16.34	911.13	5.254E-01
20 28.36	1830.4	1.6900E-02	114558	17.71	912.94	5.534E-01
21 29.04	1482.8	1.6900E-02	106842	17.73	912.51	5.298E-01
22 29.74	1130.1	1.6900E-02	92481	16.47	911.75	4.774E-01
23 30.74	893.6	1.8253E-02	81441	17.40	912.14	4.644E-01
24 31.70	126.8	1.9637E-02	73523	17.81	912.54	4.548E-01

DYNAMIC FACTOR BY TUPLIN METHOD

2.7035E+02 = DYNAMIC INCREMENT
 9.0989E+01 = DYNAMIC FACTOR

Figure H-3. Sample ground spur gear computer printout (cont'd)

APPENDIX I

HELICAL GEAR COMPUTER PROGRAM

Program Goal

This program examines parallel-axis helical gears for scoring potential. The analysis follows a pair of teeth from the time that they come into contact until they disengage, examining the conditions in the gear mesh at many intervals (usually 16) during the mesh cycle.

At any interval, the instantaneous line of contact for which the mesh conditions are being examined is divided into several segments. The number of segments may range from two to seven, depending upon the relative length of the instantaneous line of contact. Each segment is then considered, for purposes of analysis, as a narrow spur gear in the normal plane. Loads, velocities, and radii of curvature are found for the center of this narrow spur gear, from which the instantaneous conjunction surface temperature is determined.

Operating parameters are calculated on the basis of static load. These parameters may then be corrected for dynamic loads by applying a dynamic factor, evaluated by a method suggested by Tuplin.⁶³

Program Language and Computer Type

The program is written in FORTRAN IV language for a CDC 6000 Series computer, using RUN compiler and SCOPE 3.4 system.

Input Cards

There are nine data cards per set of data. Data sets may be stacked. The program contains a horsepower loop, so the effect on the gears of several equally spaced horsepower levels may be studied.

The program will handle either the pinion or the gear as the driving member. However, the data must be put onto the cards with the driver parameters in the "pinion" categories, regardless of which member is actually the driver.

A description of the cards follows. All units are in inches unless otherwise noted.

Use driver gear parameters for "pinion"

Use driven gear parameters for "gear"

<u>Word</u>	<u>Column</u>	<u>Symbol</u>	<u>Description</u>
<u>Input Card 1</u> (PRELIMINARY INPUT DATA)			
1	1-10	BDP	Base diameter (pinion)
2	11-20	BDG	Base diameter (gear)
3	21-30	BP	Thermal constant (pinion), $\text{lb}/^\circ\text{F-in.-sec}^{\frac{1}{2}}$
4	31-40	BG	Thermal constant (gear), $\text{lb}/^\circ\text{F-in.-sec}^{\frac{1}{2}}$
5	41-50	CTP	Normal circular tooth thickness (pinion)
6	51-60	CTG	Normal circular tooth thickness (gear)
7	61-70	EP	Young's modulus (pinion)
8	71-80	EG	Young's modulus (gear)
<u>Input Card 2</u>			
1	1-10	FWP	Face width (pinion)
2	11-20	FWG	Face width (gear)
3	21-30	ODP	Outside diameter (pinion)
4	31-40	ODG	Outside diameter (gear)
5	41-50	PRP	Poisson's ratio (pinion)
6	51-60	PRG	Poisson's ratio (gear)
7	61-70	RNP	Number of teeth (pinion)
8	71-80	RNG	Number of teeth (gear)

<u>Word</u>	<u>Column</u>	<u>Symbol</u>	<u>Description</u>

Input Card 3

1	1-10	WDP	Tooth height or whole depth (pinion)
2	11-20	WDG	Tooth height or whole depth (gear)
3	21-30	C	Center distance
4	31-40	CPN	Normal circular pitch
5	41-50	DPN	Normal diametral pitch
6	51-60	HA	Helix angle, deg.
7	61-70	PAN	Normal pressure angle, deg.

Input Card 4 (TEST CONDITIONS)

1	1-10	RPMP	Rpm of pinion (driver)
2	11-20	TEMP	Oil jet temperature, °F
3	21-25	IT	Number of positions of the line of contact that will be studied, usually 16

Input Card 5 (FRICTION DATA)

1	1-10	FR1	Friction factor from Eq. (55), (57), (59), or (61)
2	11-20	FR2	Friction factor from Eq. (54), (56), (58), or (60)
3	21-30	TCON	Temperature difference factor from Eq. (69)

<u>Word</u>	<u>Column</u>	<u>Symbol</u>	<u>Description</u>
<u>Input Card 6</u> (HORSEPOWER LOOP)			
1	1-10	HP	Initial horsepower input
2	11-20	DHP	Horsepower increment. If only one power level is to be used, this must be zero
3	21-30	HPM	Maximum horsepower. If only one power level is to be used, this must be zero
<u>Input Card 7</u> (METHOD CARD)			
1	1-10	METH	If only static conditions are to be obtained, leave blank; otherwise put a 1 in Col. 10 to calculate a dynamic factor
<u>Input Card 8</u>			If METH is blank, leave this card blank; otherwise:
1	1-10	AP	Thickness of pinion rim below the root. If pinion is solid, this dimension will be the height the pinion extends above the shaft surface.
2	11-20	AG	Thickness of gear rim below the root. If gear is solid, this dimension will be the height the gear extends above the shaft surface.
3	21-30	PEP	Pitch or spacing error (pinion)
4	31-40	PEG	Pitch or spacing error (gear)
5	41-50	PMI	Mass moment of inertia (pinion), lb-sec ² -in.
6	51-60	PMG	Mass moment of inertia (gear), lb-sec ² -in.
7	61-70	PMP	Tip profile modification (pinion)
8	71-80	PMG	Tip profile modification (gear)

<u>Word</u>	<u>Column</u>	<u>Symbol</u>	<u>Description</u>
<u>Input Card 9</u>			If METH is blank, leave this card blank. If values are unknown from other sources, leave card blank; otherwise:
1	1-10	RN(1)	Lowest natural frequency of gear system, rpm
2	11-20	RN(2)	Highest natural frequency of gear system, rpm
3	21-30	RN(3)	Natural frequency of teeth acting as a spring, rpm

A sample set of data cards for the helical gear design and operating conditions given in Chapter VIII, Section C, is shown in Figure I-1. The pinion was the driver in this example. The friction factors on Card 5 were determined for plain surfaces, Equations (107) and (108), with a composite surface roughness of 33 μ in. AA. Card 6 shows that nine power levels were run, starting at 600 hp, and increasing by 100-hp increments, to a maximum of 1400 hp. Card 7 indicates that a dynamic factor was calculated. As a consequence, Card 8 contains data. No vibration analysis data were available, so Card 9 was left blank.

Computer Program

Figure I-2 shows the listing of the computer program. Control cards are not included.

Sample Printout

Figure I-3 gives the data printout for the input data of Figure I-1. Results are shown for 600 hp only.

The first page of the printout lists the input data for reference purposes. The second page gives some miscellaneous geometry parameters and a listing of the length of the simultaneous lines of contact as the gears pass through mesh; the last column of this list is the sum of all lines of contact at any one position. The next three pages give the details of the behavior in the gear mesh for a single power level. For each value of EFF (designated by f in Fig. 7, Chap. IV), data are displayed for that position of the line of contact. For

example, at $EFF(3) = 0.150$, the instantaneous line of contact has been divided into 4 segments. At the center of each segment, the roll angle, radii of curvature, sliding and sum velocities, friction, maximum Hertz stress, instantaneous surface temperature rise, instantaneous surface temperature, and instantaneous frictional power loss have been determined. The instantaneous surface temperature, for example, at the center of the first segment is 323.2°F , the maximum for that line of contact. At the bottom of the last page, the natural frequency of the tooth pairs as cantilever beams is given. This value should be well removed from the operating speed for satisfactory dynamic behavior. The dynamic increment and resulting dynamic factor are also given.

3.5345	15.7344	42.2	42.2	.1791	.1791	3.0E+07	3.0E+07
2.500	2.380	4.052	17.2903	.3	.3	31.	136.
2							
.2521	.2520	10.47055	.3636	6.5	18.2366	22.	
3							
5000.	190.	16					
4							
.0185	.0322	312.5					
5							
600.00	100.00	1400.00					
6							
1							
7							
.640	.500	.0002	.0003	.0343	6.2523	.00002	.00001
8							
9							

Figure I-1. Sample helical gear computer program data cards

```

PROGRAM HELICAL (INPUT,OUTPUT,TAPE%INPUT,TAPE%OUTPUT)
REAL NATFB
DIMENSION ROLLRAY(11,14,16),XLRV(16),XLPND(16,6),
      RN(3),METH(1)
COMMON AG,AP,ADG,ODP,C,CPN,CTG,CTP,EG,EP,FNG,FNP,GMI,HAR,HP,
1      INT,IN,NOU,ODS,PANR,PDS,PPD,PES,PEP,PMS,PI,PMI,PMP,RNG,
2      PMPR,PRS,PNP,RNP,RNPP,VA,WDG,WDP,DFORCE,MN,NATFB
DATA IS/4MGZAR/
IN=8
NOU=6
JMN=1
JRN=2
JRP=3
JRG=4
JVS=5
JVT=6
JFR=7
JME=8
JDT=9
JTC=10
JPM=11
PI=3.14159265
RADIAN=PI/180.

DEFINITIONS OF INPUT VARIABLES FOR HELICAL GEAR (INCH UNITS)
USE DRIVER VALUES FOR (PINION) ITEMS
USE DRIVER VALUES FOR (GEAR) ITEMS

      BDP  = BASE DIAMETER (PINION)
      ADG  = BASE DIAMETER (GEAR)
      AP   = THERMAL CONSTANT (PINION),LB./SQRT(SEC.)IN,DEG.F
      AG   = THERMAL CONSTANT (GEAR),LB./SQRT(SEC.)IN,DEG.F
      CTP  = NORMAL CIRCULAR TOOTH THICKNESS (PINION)
      CTG  = NORMAL CIRCULAR TOOTH THICKNESS (GEAR)
      EP   = YOUNGS MODULUS (PINION)
      EG   = YOUNGS MODULUS (GEAR)
      FNP  = FACE WIDTH (PINION)
      FNG  = FACE WIDTH (GEAR)
      ODP  = OUTSIDE DIAMETER (PINION)
      ODG  = OUTSIDE DIAMETER (GEAR)
      PRP  = POISSONS RATIO (PINION)
      PRG  = POISSONS RATIO (GEAR)
      RNP  = NUMBER OF PINION (DRIVER) TEETH
      RNG  = NUMBER OF GEAR TEETH
      WDP  = HEIGHT OR WHOLE DEPTH (PINION)
      WDG  = HEIGHT OR WHOLE DEPTH (GEAR)
      C     = CENTER DISTANCE
      CPN  = NORMAL CIRCULAR PITCH
      OPH  = DIAMETRAL PITCH, NORMAL
      HA   = HELIX ANGLE, DEG.
      PAN  = PRESSURE ANGLE, NORMAL, DEG.
      RPNP = DRIVER RPM
      TFMP = OIL JET TEMPERATURE, DEG.F
      IT   = INCREMENTS OF EFF (USUALLY 16)
      FR1  = CONSTANT FRICTION FACTOR
      FR2  = VARIABLE FRICTION FACTOR
      TCON = TEMP. DIFFERENCE FACTOR
      HP   = INITIAL POWER LEVEL, HP
      DNP  = POWER INCREMENT
      UPN  = UPPER LIMIT ON HORSEPOWER
      METH = USED IF DYNAMIC FACTOR REQ'D.

```

```

MEL10000+0001
MEL10010+0002
MEL10020+0003
MEL10030+0004
MEL10040+0005
MEL10050+0006
MEL10060+0007
MEL10070+0008
MEL10080+0009
MEL10090+0010
MEL10100+0011
MEL10110+0012
MEL10120+0013
MEL10130+0014
MEL10140+0015
MEL10150+0016
MEL10160+0017
MEL10170+0018
MEL10180+0019
MEL10190+0020
MEL10200+0021
MEL10210+0022
MEL10220+0023
MEL10230+0024
MEL10240+0025
MEL10250+0026
MEL10260+0027
MEL10270+0028
MEL10280+0029
MEL10290+0030
MEL10300+0031
MEL10310+0032
MEL10320+0033
MEL10330+0034
MEL10340+0035
MEL10350+0036
MEL10360+0037
MEL10370+0038
MEL10380+0039
MEL10390+0040
MEL10400+0041
MEL10410+0042
MEL10420+0043
MEL10430+0044
MEL10440+0045
MEL10450+0046
MEL10460+0047
MEL10470+0048
MEL10480+0049
MEL10490+0050
MEL10500+0051
MEL10510+0052
MEL10520+0053
MEL10530+0054
MEL10540+0055
MEL10550+0056
MEL10560+0057
MEL10570+0058
MEL10580+0059
MEL10590+0060
MEL10600+0061
MEL10610+0062

```

Figure I-2. Listing of helical gear computer program


```

*//114 BDP      = ,F10,5,5X,10MBDG      = ,F10,5,5X,
*   60M=BASE CIRCLE DIAMETER
*//114 BP       = ,F10,5,5X,10MBG       = ,F10,5,5X,
*   60M=THERMAL CONSTANT (LB./SQRT(SEC.)IN,DEG.F)
*//114 CTP      = ,F10,5,5X,10MCTG      = ,F10,5,5X,
*   60M=NORMAL CIRCLE TOOTH THICKNESS
*//114 EP       = ,E10,0,5X,10MEG      = ,E10,0,5X,
*   60M=YOUNG'S MODULUS, PSI
*//114 FMP      = ,F10,5,5X,10MPMG      = ,F10,5,5X,
*   60M=FACE WIDTH
*//114 ODP      = ,F10,5,5X,10MODG      = ,F10,5,5X,
*   60M=OUTSIDE DIAMETER
*//114 PRP      = ,F10,5,5X,10MPRG      = ,F10,5,5X,
*   60M=POISSON'S RATIO
*//114 RNP      = ,F10,5,5X,10MRNG      = ,F10,5,5X,
*   60M=NUMBER OF TEETH
*)
  WRITE (NOUT,1006) NDP,NDG,C,CPN,DPN,HA,PAN,RPMP,TEMP,IT,
    PR1,PR2,TCOIN
1006 FORMAT (
*//114 NDP      = ,F10,5,5X,10MBDG      = ,F10,5,5X,
*   60M=HEIGHT OR WHOLE DEPTH
*//114 C       = ,F10,5,30X, 36M=CENTER DISTANCE
*//114 CPN     = ,F10,5,30X, 36M=NORMAL CIRCLE PITCH
*//114 DPN     = ,F10,5,30X, 36M=DIAMETERAL PITCH
*//114 HA      = ,F10,5,30X, 36M=HELIX ANGLE (DEG.)
*//114 PAN     = ,F10,5,30X, 36M=PRESSURE ANGLE (DEG.)
*//114 RPMP    = ,F10,2,30X, 36M=RPM DRIVER
*//114 TEMP    = ,F10,2,30X, 36M=OIL JET TEMPERATURE (DEG.F)
*//114 IT      = ,F10, 30X,36M=INCREMENTS OF EFF
*//114 PR1     = ,F10,4,30X,36M=CONSTANT FRICTION FACTOR
*//114 PR2     = ,F10,4,30X,36M=VARIABLE FRICTION FACTOR
*//114 TCOIN   = ,F10,1,30X,36M=TEMP DIFFERENCE FACTOR
*)
  WRITE (NOUT,1007) MP,DMP,MPM
1007 FORMAT (//36M INPUT VARIABLES FOR HORSEPOWER LOOP/
*//114 MP       = ,F10,3,30X, 36M=INITIAL HORSEPOWER
*//114 DMP      = ,F10,3,30X, 36M=HORSEPOWER INCREMENT
*//114 MPM      = ,F10,3,30X, 36M=MAXIMUM HORSEPOWER
*)
  WRITE (NOUT,1008)
1008 FORMAT (//42M METHOD USED FOR CALCULATING NORMAL FORCE=)
  I=1
  WRITE (NOUT,1009) I
1009 FORMAT (IX,I2,26M NORMAL FORCE=STATIC FORCE)
  I=2
  IF (METH,ME,1) GO TO 20
  WRITE (NOUT,1010) I
1010 FORMAT (IX,I2,43M DYNAMIC FACTOR CALCULATED BY TUPLIN METHOD)
  WRITE (NOUT,1012) AP,AG,PEP,PEG,PMI,GMI,PMP,PMG,RN(1),RN(2),RN(3)
1012 FORMAT (//47M INPUT VARIABLES FOR DYNAMIC FORCE CALCULATIONS/
*//114 (DRIVEN)      ,6X,14M      (DRIVEN)
*//114 AP           = ,F10,5,5X,10MAG      = ,F10,5,5X,
*   60M=THICKNESS OF RIM BELOW ROOT
*//114 PEP         = ,F10,5,5X,10MPEG      = ,F10,5,5X, 13M=PITCH ERROR
*//114 PMI         = ,F10,5,5X,10MGMI      = ,F10,5,5X,
*   60M=MASS MOMENT OF INERTIA (LB.SEC.SQ.IN.)
*//114 PMP         = ,F10,5,5X,10MPMG      = ,F10,5,5X,
*   60M=PROFILE MODIFICATION
*//114 RN(1)       = ,F10,2,30X, 36M=1ST NATURAL FREQUENCY (RPM)
*//114 RN(2)       = ,F10,2,30X, 36M=2ND NATURAL FREQUENCY (RPM)
*//114 RN(3)       = ,F10,2,30X, 36M=3RD NATURAL FREQUENCY (RPM)

```

```

MEL11240+0125
MEL11250+0126
MEL11260+0127
MEL11270+0128
MEL11280+0129
MEL11290+0130
MEL11300+0131
MEL11310+0132
MEL11320+0133
MEL11330+0134
MEL11340+0135
MEL11350+0136
MEL11360+0137
MEL11370+0138
MEL11380+0139
MEL11390+0140
MEL11400+0141
MEL11410+0142
MEL11420+0143
MEL11430+0144
MEL11440+0145
MEL11450+0146
MEL11460+0147
MEL11470+0148
MEL11480+0149
MEL11490+0150
MEL11500+0151
MEL11510+0152
MEL11520+0153
MEL11530+0154
MEL11540+0155
MEL11550+0156
MEL11560+0157
MEL11570+0158
MEL11580+0159
MEL11590+0160
MEL11600+0161
MEL11610+0162
MEL11620+0163
MEL11630+0164
MEL11640+0165
MEL11650+0166
MEL11660+0167
MEL11670+0168
MEL11680+0169
MEL11690+0170
MEL11700+0171
MEL11710+0172
MEL11720+0173
MEL11730+0174
MEL11740+0175
MEL11750+0176
MEL11760+0177
MEL11770+0178
MEL11780+0179
MEL11790+0180
MEL11800+0181
MEL11810+0182
MEL11820+0183
MEL11830+0184
MEL11840+0185
MEL11850+0186

```

Figure I-2. Listing of helical gear computer program (cont'd)

```

C      *)
C      PRELIMINARY CALCULATIONS FOR HELICAL GEAR TEST.
C
20  F=HAM/4*(FNP,FNG)
    PANR=PANR/RADIAN
    MAR=MAR/RADIAN
    POP=2*(C/(1+RNG/RNP))
    PDG=2*(C-POP)
    ADP=(ODP+POP)/2,
    ADG=(ODG+PDG)/2,
    ZA = (SQRT((ODG+ODG)-(BDG+BDG))-SQRT((PDG+PDG)-(BDG+BDG))) / 2,
    VO = SQRT((POP+POP)-(BDP+BDP)) / 2,
    VA = VO-ZA
    MA = (SQRT(ODG+ODG-(BDG+BDG)))/2,
    BHAR= 4314(SIN(MAR)*COS(PANR))
    ZR = (SQRT((ODP+ODP)-(BDP+BDP))-SQRT((PDG+PDG)-(BDP+BDP))) / 2,
    Z = ZA+ZR
    BPN = CPN * COS(PANR)
    BPT = PI*BDP/RNP
    ZO = BPT-ZR
    ZC = BPT-ZA
    ER = 1. / (((1-(PRP+PRP))/EP+((1-(PRG+PRG))/EG))'.2)
    WRITE (NOUT,1000)
    WRITE (NOUT,1013) ADP,ADG,POP,PDG,BHAR,BPN,BPT,ER
1013 FORMAT (//35X,44HVALUES AND DEFINITIONS OF PRELIMINARY CALCULATED
    * 10HVARIALES /
    * 1X,26H(INCH UNITS UNLESS NOTED) /
    * 2X,16H (DRIVER) ,6X,14H (DRIVEN)
    *//11H ADP = ,F10.5,5X,10HADG = ,F10.5,5X,
    * 60H=ADDENDUM (IN.)
    *//11H POP = ,F10.5,5X,10HPDG = ,F10.5,5X,
    * 60H=PITCH DIAMETER
    *//11H BHAR = ,F10.6,30X, 36H=BASE HELIX ANGLE (RADIAN)
    *//11H BPN = ,F10.6,30X, 36H=NORMAL BASE PITCH
    *//11H BPT = ,F10.6,30X, 36H=TRANSVERSE BASE PITCH
    *//11H ER = ,F10.3,30X, 36H=REDUCED MODULUS (PSI)
    WRITE (NOUT,1014) VA,MA,Z,ZA,ZC,ZO,ZR
1014 FORMAT (
    * 11H VA = ,F10.6,30X, 36H=DISTANCE FROM FIRST INTERFERENCE
    * INT TO START OF CONTACT
    *//11H MA = ,F10.6,30X, 60H=DISTANCE FROM SECOND INTERFERENCE
    * OINT TO START OF CONTACT
    *//11H Z = ,F10.5,30X, 36H=PATH OF CONTACT
    *//11H ZO = ,F10.5,30X, 36H=PATH OF CONTACT IN APPROACH
    *//11H ZC = ,F10.5,30X, 36H=DISTANCE FROM PITCH POINT TO HPBTC
    *//11H ZO = ,F10.5,30X, 36H=DISTANCE FROM PITCH POINT TO LPBTC
    *//11H ZR = ,F10.5,30X, 36H=PATH OF CONTACT IN RECESS
    WRITE (NOUT,4090)
4090 FORMAT (//////1X,40HCALCULATIONS OF XL FOR EACH VALUE OF EFF//)
    ICASE=1
    RANGPL = (VO-ZD)/(ROP/2)/RADIAN
    RANGPM = (VO-ZC)/(ROP/2)/RADIAN
    FNP = FN * TAN(MAR)/(CPN/COS(MAR))
    TMP = Z/(BPN/COS(MAR))
    FSIN = FN*SIN(BHAR)
    FCOS = FN * COS(BHAR)
    ZCOS = Z * COS(BHAR)
    IF (Z,LT,FSIN) ICASE=ICASE+1
C
C      DEFINITIONS FOR T LOOP VARIABLES
C      EFF = INDEPENDENT VARIABLE

```

```

HEL1186000187
HEL1187000188
HEL1188000189
HEL1189000190
HEL1190000191
HEL1191000192
HEL1192000193
HEL1193000194
HEL1194000195
HEL1195000196
HEL1196000197
HEL1197000198
HEL1198000199
HEL1199000200
HEL1200000201
HEL1201000202
HEL1202000203
HEL1203000204
HEL1204000205
HEL1205000206
HEL1206000207
HEL1207000208
HEL1208000209
HEL1209000210
HEL1210000211
HEL1211000212
HEL1212000213
HEL1213000214
HEL1214000215
HEL1215000216
HEL1216000217
HEL1217000218
HEL1218000219
HEL1219000220
HEL1220000221
HEL1221000222
HEL1222000223
HEL1223000224
HEL1224000225
HEL1225000226
HEL1226000227
HEL1227000228
HEL1228000229
HEL1229000230
HEL1230000231
HEL1231000232
HEL1232000233
HEL1233000234
HEL1234000235
HEL1235000236
HEL1236000237
HEL1237000238
HEL1238000239
HEL1239000240
HEL1240000241
HEL1241000242
HEL1242000243
HEL1243000244
HEL1244000245
HEL1245000246
HEL1246000247
HEL1247000248

```

Figure I-2. Listing of helical gear computer program (cont'd)

```

C          (EFF WILL BE INCREMENTED BY DEF AND Y  IT  TIMES
C          WHERE Y=0,...(IT-1))
C          XLRY = ARRAY THAT STORES VALUES OF XL
C          BETA = INCREMENT OF BPN
C          BINC = POSITIVE AND NEGATIVE INCREMENT OF BETA
C          UBETA = UPPER AND LOWER LIMITS OF BETA
C
C          TEST FOR THE NO. OF INCREMENTS OF BPN
C
C          BETA=0,
C          TEST=0,
C          30 BETA=BETA+1,
C          TEST=0, BETA=BPN
C          IF (TEST.GT.(ZCOS+FSIN)) GO TO 35
C          GO TO 30
C          35 XBETA=BETA-1,
C          DEF = (ZCOS+FSIN)/(IT-1)
C
C          START Y LOOP
C          DO 130 L=1,IT
C          LXL=0
C          BETA=0,
C          BINC=1,
C
C          A, CHOOSE F
C
C          TL=1,
C          T=1,
C          EFF=DEF*Y
C          45 GO TO (50,55) ICASE
C
C          B, (CASE I)
C
C          50 A0=0,
C          A1=FSIN
C          A2=ZCOS
C          A1A2=A1+A2
C          GO TO 60
C
C          B, (CASE II)
C
C          55 A0=0,
C          A1=ZCOS
C          A2=FSIN
C          A1A2=A1+A2
C          60 IF (EFF.GT.A0.AND.EFF.LT.A1A2) GO TO 62
C          XL=0,
C          GO TO 70
C          62 IF (EFF.GT.A1) GO TO 64
C          XL = EFF*((1./TAN(BHAR))+TAN(BHAR))
C          GO TO 70
C          64 IF (EFF.GT.A2.OR.EFF.EQ.A2) GO TO 66
C          IF (ICASE,EQ,2) GO TO 65
C          XL = FM * (COS(BHAR)+(SIN(BHAR)*TAN(BHAR)))
C          GO TO 70
C          65 XL = Z*(SIN(BHAR)+(COS(BHAR)*(1./TAN(BHAR))))
C          GO TO 70
C          66 XL = FM * (COS(BHAR)+SIN(BHAR)*TAN(BHAR))+(Z*(SIN(BHAR)+COS(BHAR)
C          )*(1./TAN(BHAR)))-(EFF*(TAN(BHAR)*(1./TAN(BHAR))))
C
C          C, STORE VALUE OF XL

```

Figure I-2. Listing of helical gear computer program (cont'd)

```

70 LXL=LXL+1
   XLRY(LXL) =XL
   IF (BETA,NE,0.) GO TO 105
C
C   D. EVALUATE XL AGAINST...      AND CHOOSE APPROPRIATE K
C                                   K MUST BE AN EVEN NO.
C
   KKK=2
   K=0
   KKK=12
   IF (XL,LT,(FM/S,)) K=KKK
   IF (XL,GT,(FM/P,)) K=KKK
C
C   E. CALCULATE MO,M,M(N)
C
80 MO = (EFF-FW*SIN(BMAR))/COS(BMAR)
   IF (MO,LT,0.) MO=0.
   M =EFF/COS(BMAR)
   IF (M,GT,2) M=2
   K2=K/2
   DO 90 N=1,K2
   MN=N-1
   MV = MO*((M-MO)/K)*(2+MN+1)
C
C   F. CALCULATE FOR EACH VALUE OF M(N)....
C
   RANGLP = (VA+MN)/(BDP/2.)
   RMOP = (1./COS(BMAR))*(VA+MN)
   RMOG = (1./COS(BMAR))*(MA+MN)
   VP = RMOP*RPMP*(PI/30.)
   VG = RMOG*RPMP*(RNP/RMG)*(PI/30.)
   VS=ABS(VP-VG)
   VTEVP+VG
C
C   G. STORE
C
   ROLLRAY(N,JMN,L) = MN
   ROLLRAY(N,JRN,L) = RANGLP/RADIAN
   ROLLRAY(N,JRP,L) = RMOP
   ROLLRAY(N,JRG,L) = RMOG
   ROLLRAY(N,JVS,L) = VS
   ROLLRAY(N,JVT,L) = VT
90 CONTINUE
   XLFNO(L,1)=K2
   XLFNO(L,2)=EFF
   XLFNO(L,3)=XL
C
C   H. CALCULATE A NEW EFF
C
105 IF (XBETA,EQ,BETA) GO TO 107
106 BETA=BETA+RINC
   EFF = XLFNO(L,2)+BETA*BPW
   GO TO 60
107 IF (XBETA,LT,0.) GO TO 110
   BINC=BINC
   BETA=0.
   XBETA=-XBETA
   GO TO 106
C
C   I. TOTAL ALL XLS
C
110 XBETA=ABS(XBETA)

```

```

MEL13100+0311
MEL13110+0312
MEL13120+0313
MEL13130+0314
MEL13140+0315
MEL13150+0316
MEL13160+0317
MEL13170+0318
MEL13180+0319
MEL13190+0320
MEL13200+0321
MEL13210+0322
MEL13220+0323
MEL13230+0324
MEL13240+0325
MEL13250+0326
MEL13260+0327
MEL13270+0328
MEL13280+0329
MEL13290+0330
MEL13300+0331
MEL13310+0332
MEL13320+0333
MEL13330+0334
MEL13340+0335
MEL13350+0336
MEL13360+0337
MEL13370+0338
MEL13380+0339
MEL13390+0340
MEL13400+0341
MEL13410+0342
MEL13420+0343
MEL13430+0344
MEL13440+0345
MEL13450+0346
MEL13460+0347
MEL13470+0348
MEL13480+0349
MEL13490+0350
MEL13500+0351
MEL13510+0352
MEL13520+0353
MEL13530+0354
MEL13540+0355
MEL13550+0356
MEL13560+0357
MEL13570+0358
MEL13580+0359
MEL13590+0360
MEL13600+0361
MEL13610+0362
MEL13620+0363
MEL13630+0364
MEL13640+0365
MEL13650+0366
MEL13660+0367
MEL13670+0368
MEL13680+0369
MEL13690+0370
MEL13700+0371
MEL13710+0372

```

Figure I-2. Listing of helical gear computer program (cont'd)


```

      XLTY=0
      IF (LXL,NE,1) GO TO 111
      XLTY=XLRY(LXL)
      GO TO 118
111 DO 115 I=1,LXL
      XLTY=XLTY+XLRY(I)
115 CONTINUE
      WRITE (NOUT,9091) (XLRY(I),I=1,LXL),XLTY
9091 FORMAT (16(2X,F6.3))
C
C      J. STORE XLTOT
C
118 XLFNO(L,3)=XLTY
120 CONTINUE
C
C      START HP LOOP
C
135 NT = 126000.*HP/(RPM*PDP)
      NN = NT/(COS(MAR)*COS(PANR))
      IMT=1
      DFORCE=NN
      GO TO 145
138 I = METH,NE,1) GO TO 200
      I = 2
      NATFQ=RN(3)
      CALL FORCE
      GO TO 200
145 CALL SHFR (JRP,JRG,JVS,JFR,JPM,XLFNO,ROLLRAY,ER,IT,DFORCE,
      * RNP,Z,RDP,PI, FR1,FR2,TCN,TEMP,TS,PHEAV,FRIC)
      DO 160 L=1,17
      XL=XLFNO(L,4)
      K2=XLFNO(L,1)
      K2=K2
      XLTOT=XLFNO(L,3)
166 DO 150 N=1,K2
      RHOP=ROLLRAY(N,JRP,L)
      RHOG=ROLLRAY(N,JRG,L)
      RHOF = ABS(RHOP+RHOG)/(RHOP+RHOG))
      V3=ROLLRAY(N,JVS,L)
      VT=ROLLRAY(N,JVT,L)
      FRIC = ROLLRAY(N,JFR,L)
      IF (XLTOT,GT,0.) GO TO 167
      WRITE (NOUT,9999) XLTOT
9999 FORMAT (9H XLTOT = ,E13.4)
      ROLLRAY(N,JDT,L) = 9.
      GO TO 150
167 IF (DFORCE,GT,0.) GO TO 168
      WRITE (NOUT,8888) DFORCE
8888 FORMAT (9H DFORCE = ,E13.4)
      ROLLRAY(N,JDT,L) = 8.
      GO TO 150
168 CONTINUE
      IF (L,EO,1,OR,L,EO,17) GO TO 165
      DT = .201 * (FRIC*((DFORCE/XLTOT)**(3./4.))*(ER**(1./4.)))/
      1 (RHOE**(1./4.)) * (ABS(RHOP - (RNP/RNG+RHOG))*SQRT(RPM)/
      2 (RP+SQRT(RHOP)+(BG+SQRT(RNP/RNG+RHOG))) * (COS(9MAR)/
      3 SQRT(COS(MAR))))
      GO TO 164
165 DT=0.
164 CT=TEMP+DT
      HERTZ = .3989*SQRT((DFORCE*XL)/(K/2*XLTOT)*(ER/RHOE))
      ROLLRAY(N,JDT,L) = DT
      MEL137200+0373
      MEL137300+0374
      MEL137400+0375
      MEL137500+0376
      MEL137600+0377
      MEL137700+0378
      MEL137800+0379
      MEL137900+0380
      MEL138000+0381
      MEL138100+0382
      MEL138200+0383
      MEL138300+0384
      MEL138400+0385
      MEL138500+0386
      MEL138600+0387
      MEL138700+0388
      MEL138800+0389
      MEL138900+0390
      MEL139000+0391
      MEL139100+0392
      MEL139200+0393
      MEL139300+0394
      MEL139400+0395
      MEL139500+0396
      MEL139600+0397
      MEL139700+0398
      MEL139800+0399
      MEL139900+0400
      MEL140000+0401
      MEL140100+0402
      MEL140200+0403
      MEL140300+0404
      MEL140400+0405
      MEL140500+0406
      MEL140600+0407
      MEL140700+0408
      MEL140800+0409
      MEL140900+0410
      MEL141000+0411
      MEL141100+0412
      MEL141200+0413
      MEL141300+0414
      MEL141400+0415
      MEL141500+0416
      MEL141600+0417
      MEL141700+0418
      MEL141800+0419
      MEL141900+0420
      MEL142000+0421
      MEL142100+0422
      MEL142200+0423
      MEL142300+0424
      MEL142400+0425
      MEL142500+0426
      MEL142600+0427
      MEL142700+0428
      MEL142800+0429
      MEL142900+0430
      MEL143000+0431
      MEL143100+0432
      MEL143200+0433
      MEL143300+0434

```

Figure I-2. Listing of helical gear computer program (cont'd)

```

      ROLLRAY(N,JTC,L)=CTEMP
      ROLLRAY(N,JME,L)=MERTZ
150  CONTINUE
160  CONTINUE
C
C      START PRINTING LOOP
C
      KZTABJ
170  DO 190 L=1,IT
      KZ=XLFNO(L,1)
      KZTABKZ+KZ+5
      IF (KZT,L,58) GO TO 179
      WRITE (NOUT,101) MP,FMP,RANGPL,T8,RMP,TMP,RANGPH,PHEAV,NH
      KZTABKZ+5
179  WRITE (NOUT,1015) L,(XLFNO(L,J),J=2,3),XLFNO(L,5)
      DO 180 N=1,KZ
      WRITE (NOUT,1017) N,(ROLLRAY(N,J,L),J=1,11)
180  CONTINUE
190  CONTINUE
199  GO TO (170,200) IMT
200  MP=MP+RMP
      IF (MP,LE,MPH) GO TO 175
      GO TO 1
1000 FORMAT (1H1)
1015 FORMAT (//4X,4HEFF(,12,4H) = ,F6,3,7X,7H1TOT = ,F8,4,5X,
1      1 2PMNORMAL TOOTH LOAD = ,F12,2,7X,4MH(N),3X,6HRANGLP,3X,4HRMOP,
2      4X,4HRHOG,3X,7HVB(IPB),2X,7HVT(IPB),3X,4HFRIC,4X,4HMERTZ(MX),
3      3X,5HOT(F),2X,5HCTEMP,2X,10HPHE(B/SEC))
1017 FORMAT (2X,1P,2X,F5,3,3(3X,F5,P),2X,F7,2,2X,F8,P,1X,F6,4,3X,F8,1,
12(3X,F5,1),3X,F8,4,4X,F6,2)
1021 FORMAT (1H1,13MHNRSEPOWER = ,F8,2,5X,6HMP = ,F8,3,5X,
4      4HRANGPL = ,F5,2,8X,11HTR,DEG F = ,F5,1,14H RPM DRIVER = ,F8,2,
5      5X,6HMP = ,F8,3,5X,4HRANGPH = ,F5,2,5X,14HPHEAV,B/SEC = ,F7,4,
6      6 14H NH,LB. = ,F8,2)
220  CONTINUE
      END
      SUBROUTINE SBFR (JRP,JRG,JVB,JFR,JPH,XLFNO,ROLLRAY,ER,IT,OPORCE,
4      4 RNP,Z,BOP,PI, FR1,FR2,TCON,TEMP,T8,PHEAV,FRIC)
      DIMENSION ROLLRAY(11,14,16),XLFNO(16,5)
      DO 20 L=1,IT
      KZ=XLFNO(L,1)
      KZTABKZ
      XLTOT=XLFNO(L,3)
      XL=XLFNO(L,4)
      PMET=0.
      DO 10 N=1,KZ
      RMOP = ROLLRAY(N,JRP,L)
      RHOG = ROLLRAY(N,JRG,L)
      RMDE = ABS((RMOP+RHOG)/(RMOP+RHOG))
      VSB=ROLLRAY(N,JVB,L)
      IF (L,EQ,1.OR,L,EQ,IT) GO TO 11
      PVS=(OPORCE*XL)/( XLTOT)*(ABS(VS))*(-1./3.))
      IF(PVS,1.,200.) GO TO 15
      FRIC=FR1
      GO TO 25
15  FRIC=FR2*(PVS*(-.30))
      GO TO 25
11  FRIC=0.
25  IF (L,EQ,1.OR,L,EQ,IT) GO TO 17
      PMF=FRIC*(OPORCE*XL)/(K/2*XLTOT)*VSB*(1./9336.)
      GO TO 27
17  PHEAV,

```

```

HEL14340000036
HEL14350000036
HEL14360000037
HEL14370000038
HEL14380000039
HEL14390000040
HEL14400000041
HEL14410000042
HEL14420000043
HEL14430000044
HEL14440000045
HEL14450000046
HEL14460000047
HEL14470000048
HEL14480000049
HEL14490000050
HEL14500000051
HEL14510000052
HEL14520000053
HEL14530000054
HEL14540000055
HEL14550000056
HEL14560000057
HEL14570000058
HEL14580000059
HEL14590000060
HEL14600000061
HEL14610000062
HEL14620000063
HEL14630000064
HEL14640000065
HEL14650000066
HEL14660000067
HEL14670000068
HEL14680000069
HEL14690000070
HEL14700000071
HEL14710000072
HEL14720000073
HEL14730000074
HEL14740000075
HEL14750000076
HEL14760000077
HEL14770000078
HEL14780000079
HEL14790000080
HEL14800000081
HEL14810000082
HEL14820000083
HEL14830000084
HEL14840000085
HEL14850000086
HEL14860000087
HEL14870000088
HEL14880000089
HEL14890000090
HEL14900000091
HEL14910000092
HEL14920000093
HEL14930000094
HEL14940000095
HEL14950000096

```

Figure I-2. Listing of helical gear computer program (cont'd)


```

CC = CC1+CC2
CT = C0+CR+CC
TS = 1./CT
TG = 67000.*HP / PMP
V = (PI*PDP/12.) * PMP
EE = PEP + PFG + PMG
PP = (PEP+PEG)-(2.*PMG)
IF (NATF7,NE,0.) GO TO 10
T3=2.*PI*SQRT(CT*EM)
NATF0060./T3
WRITE (4OUT,1023) NATF0
1023 FORMAT (//1X,14H100TH NAT, FREQ. = ,F10,1,4H RPM)
GO TO 150
10 1460./NATF0
C
C CALCULATION OF DYNAMIC FACTOR BY TIMPLIN METHOD
C
150 HT = CMN *(1./COS(MAR))/12. * (60./V)
TY = HT/T3
IF (TY-.277) 370,370,360
360 ET = 7.815 / SQRT(1.+(6.6*(TY-TY)))
GO TO 380
370 ET = 1./((1.+(6.6*(TY-TY)))
380 D1 = EE+ET*TS
IF (PMP,CT,PMP) GO TO 381
PDP=PDP
381 TIND1=(PDP/2.)
DF=TG/(TG+T1)
WRITE (4OUT,1022) D1,DF
1022 FORMAT (/E11,5,20H = DYNAMIC INCREMENT, LB./E11,0,
* 15H = DYNAMIC FACTOR)
RETURN
END
FORC0410+0554
FORC0420+0560
FORC0430+0561
FORC0440+0562
FORC0450+0563
FORC0460+0564
FORC0470+0565
FORC0480+0566
FORC0490+0567
FORC0500+0568
FORC0510+0569
FORC0520+0570
FORC0530+0571
FORC0540+0572
FORC0550+0573
FORC0560+0574
FORC0570+0575
FORC0580+0576
FORC0590+0577
FORC0600+0578
FORC0610+0579
FORC0620+0580
FORC0630+0581
FORC0640+0582
FORC0650+0583
FORC0660+0584
FORC0670+0585
FORC0680+0586
FORC0690+0587
FORC0700+0588
FORC0710+0589
FORC0720+0590
FORC0730+0591
+0592

```

Figure I-2. Listing of helical gear computer program (cont'd)

VALUES AND DEFINITIONS OF PRIMARY INPUT VARIABLES

(INCH UNITS UNLESS NOTED)		(DRIVEN)	
BDP	= 3.53950	BDG	= 15.73990
BP	= 42.20000	BC	= 42.20000
CTP	= 1.7910	CTG	= 1.7910
EG	= 3E+07	EG	= 3E+07
PMG	= 2.50000	PMG	= 2.50000
ODP	= 4.03200	ODG	= 17.24030
PMG	= .30000	PMG	= .30000
PMG	= 31.00000	PMG	= 138.00000
ODP	= 4.56210	ODG	= .25200
C	= 10.470350		
CPH	= 1.26460		
DPH	= 8.500000		
MA	= 18.2E+60		
PAN	= 22.20000		
RPMP	= 5.000.00		
TEMP	= 190.00		
IT	= 16		
PHI	= .0188		
PHI	= .0422		
TCOM	= 312.5		

-BASE CIRCLE DIAMETER
-THERMAL CONSTANT (LB./SQRT(SEC.))IN,DEG.F
-NORMAL CIRCLE TOOTH THICKNESS
-YOUNG'S MODULUS, PSI.
-FACE WIDTH
-OUTSIDE DIAMETER
-POISSON'S RATIO
-NUMBER OF TEETH
-WEIGHT OR WHOLE DEPTH
-CENTER DISTANCE
-NORMAL CIRCLE PITCH
-DIAMETER PITCH
-HELIX ANGLE (DEG.)
-PRESSURE ANGLE (DEG.)
-RPM DRIVER
-OIL JET TEMPERATURE (DEG.F)
-INCREMENTS OF EFF
-CONSTANT FRICTION FACTOR
-VARIABLE FRICTION FACTOR
-TEMP DIFFERENCE FACTOR

-INITIAL HORSEPOWER
-HORSEPOWER INCREMENT
-MAXIMUM HORSEPOWER

INPUT VARIABLES FOR HORSEPOWER LOOP

HP	= 507.000
DMP	= 100.300
HPM	= 1400.000

METHOD USED FOR CALCULATING NORMAL FORCE=

1 NORMAL FORCE/STATIC FORCE

2 DYNAMIC FACTOR CALCULATED BY TUPLIN METHOD

INPUT VARIABLES FOR DYNAMIC FORCE CALCULATIONS

AP	= .50000	AG	= .50000
PEP	= .03020	PEG	= .00030
PHI	= .03930	GHI	= 6.25240
PMG	= .00008	PMG	= .00008
RN(1)	= -0.00		
RN(2)	= -0.00		
RN(3)	= -0.00		

-THICKNESS OF RIM BELOW ROOT
-PITCH ERROR
-MASS MOMENT OF INERTIA (LB.SEC.SQ.IN.)
-PROFILE MODIFICATION
-1ST NATURAL FREQUENCY (RPM)
-2ND NATURAL FREQUENCY (RPM)
-3RD NATURAL FREQUENCY (RPM)

Figure I-3. Sample helical gear computer printout

(INCH UNITS UNLESS NOTED)
 (DRIVER) (DRIVEN)
 ADP = .09537 ADG = .04523
 PDP = 3.84122 POG = 12.04983
 BMAR = .245352
 BPM = 3.32887
 BPT = 3.58142
 ER = 3.297E+02
 VA = .915873
 WA = 3.584101
 Z = .45422
 ZA = .23623
 ZC = .12146
 ZD = .14020
 ZR = .21344

-ADDENDUM (IN.)
 -PITCH DIAMETER
 -BASE HELIX ANGLE (RADIAN)
 -NORMAL BASE PITCH
 -TRANSVERSE BASE PITCH
 -REDUCED MODULUS (PSI)
 -DISTANCE FROM FIRST INTERFERENCE POINT TO START OF CONTACT
 -DISTANCE FROM SECOND INTERFERENCE POINT TO START OF CONTACT
 -PATH OF CONTACT
 -PATH OF CONTACT IN APPROACH
 -DISTANCE FROM PITCH POINT TO MBTC
 -DISTANCE FROM PITCH POINT TO LPSTC
 -PATH OF CONTACT IN RECESS

CALCULATIONS OF XL FOR EACH VALUE OF EFF

0.000	1.231	1.560	.356	0.000	0.000	0.000	0.000	3.148
.270	1.500	1.317	.087	0.000	0.000	0.000	0.000	3.174
.540	1.560	1.049	0.000	0.000	0.000	0.000	0.000	3.148
.810	1.560	.777	0.000	0.000	0.000	0.000	0.000	3.148
1.080	1.560	.507	0.000	0.000	0.000	0.000	0.000	3.148
1.344	1.448	.238	0.000	.114	0.000	0.000	0.000	3.174
1.560	1.198	0.000	0.000	.384	0.000	0.000	0.000	3.148
1.560	.928	0.000	0.000	.654	0.000	0.000	0.000	3.148
1.560	.654	0.000	0.000	.928	0.000	0.000	0.000	3.148
1.560	.384	0.000	0.000	1.198	0.000	0.000	0.000	3.174
1.344	.114	0.000	0.000	1.448	.238	0.000	0.000	3.174
1.080	0.000	0.000	0.000	1.560	.507	0.000	0.000	3.148
.810	0.000	0.000	0.000	1.560	.777	0.000	0.000	3.148
.540	0.000	0.000	0.000	1.560	1.049	0.000	0.000	3.148
.270	0.000	0.000	0.000	1.500	1.317	.087	0.000	3.174
0.000	0.000	0.000	0.000	1.231	1.560	.356	0.000	3.148

Figure I-3. Sample helical gear computer printout (cont'd)

```

MORSEPOWER = 500.00      RPM = 2.022      RANGPL = 14.84      15 DEG F = 299.3
RPM DRIVER = 5000.00     TAP = 1.298      RANGPM = 20.34      PHEAV(B/SEC) = .2500
MWLB.

```



```

EFF( 1 ) = 0.000      NORMAL TOOTH LOAD = 0.00      CTMP      PHE(B/SEC)
M(N)      RANGLP      VT(IPB)  PRIC  HERTZ(MX)  DT(F)      CTMP      PHE(B/SEC)
1  0.000  16.73      782.98  0.0000      0.0  0.0  299.3  0.0000

```



```

EFF( 2 ) = .075      NORMAL TOOTH LOAD = 380.14      CTMP      PHE(B/SEC)
M(N)      RANGLP      VT(IPB)  PRIC  HERTZ(MX)  DT(F)      CTMP      PHE(B/SEC)
1  .034  18.00      759.65  .0255      380.14  0.1  305.6  .1004

```



```

EFF( 3 ) = .150      NORMAL TOOTH LOAD = 766.79      CTMP      PHE(B/SEC)
M(N)      RANGLP      VT(IPB)  PRIC  HERTZ(MX)  DT(F)      CTMP      PHE(B/SEC)
1  .020  17.26      731.31  .0207      766.79  0.2  323.2  .0616
2  .084  18.54      747.48  .0203      766.79  0.7  318.0  .0094
3  .096  19.91      745.65  .0198      766.79  1.8  313.1  .0276
4  .137  21.18      781.31  .0191      766.79  4.3  308.5  .0260

```



```

EFF( 4 ) = .225      NORMAL TOOTH LOAD = 1150.19      CTMP      PHE(B/SEC)
M(N)      RANGLP      VT(IPB)  PRIC  HERTZ(MX)  DT(F)      CTMP      PHE(B/SEC)
1  .024  17.68      735.98  .0188      1150.19  0.4  319.9  .0802
2  .088  19.54      765.98  .0188      1150.19  1.2  313.5  .0574
3  .119  21.50      785.48  .0188      1150.19  8.2  307.5  .0395
4  .206  23.51      810.48  .0188      1150.19  8.7  302.0  .0116

```



```

EFF( 5 ) = .301      NORMAL TOOTH LOAD = 1533.58      CTMP      PHE(B/SEC)
M(N)      RANGLP      VT(IPB)  PRIC  HERTZ(MX)  DT(F)      CTMP      PHE(B/SEC)
1  .034  18.00      739.65  .0188      1533.58  0.8  318.8  .1014
2  .110  20.55      770.48  .0188      1533.58  11.1  310.9  .0612
3  .146  23.04      805.32  .0188      1533.58  3.6  302.9  .0204
4  .276  25.64      839.65  .0188      1533.58  3.3  302.7  .0201

```



```

EFF( 6 ) = .376      NORMAL TOOTH LOAD = 1900.95      CTMP      PHE(B/SEC)
M(N)      RANGLP      VT(IPB)  PRIC  HERTZ(MX)  DT(F)      CTMP      PHE(B/SEC)
1  .034  17.74      739.65  .0188      1900.95  0.8  319.9  .0870
2  .090  19.91      765.65  .0188      1900.95  13.1  312.9  .0590
3  .164  22.03      792.93  .0188      1900.95  6.6  305.4  .0310
4  .224  24.15      820.21  .0188      1900.95  7.6  299.4  .0230
5  .295  26.28      847.99  .0188      1900.95  5.8  304.3  .0250
6  .364  28.50      876.77  .0188      1900.95  10.2  309.5  .0530

```

Figure I-3. Sample helical gear computer printout (cont'd)

HORSEPOWER = 600.00 FMP = 2.022 RANGPL = 19.8% TS, DEG F = 249.3
 RPM DRIVER = 5000.00 TMP = 1.268 RANGPM = 28.3% PHEAV, B/SEC = .2690
 MM, LB.

EFF(7) = .551	LMOT = 3.1975	NORMAL TOOTH LOAD = 2216.85	PHE(B/SEC)
M(N)	VS(198)	FRIC	DT(F)
1 .038 17.45	739.04	.0188	CTEMP 319.0
2 .014 20.91	771.17	.0188	CTEMP 310.4
3 .084 22.86	803.24	.0188	CTEMP 303.6
4 .065 25.32	835.32	.0188	CTEMP 301.8
5 .031 27.77	867.54	.0188	CTEMP 308.1
6 .016 30.22	899.67	.0188	CTEMP 313.4

EFF(8) = .524	LMOT = 3.1975	NORMAL TOOTH LOAD = 2216.85	PHE(B/SEC)
M(N)	VS(198)	FRIC	DT(F)
1 .038 17.45	739.04	.0188	CTEMP 319.0
2 .014 20.91	771.17	.0188	CTEMP 310.4
3 .084 22.86	803.24	.0188	CTEMP 303.6
4 .065 25.32	835.32	.0188	CTEMP 301.8
5 .031 27.77	867.54	.0188	CTEMP 308.1
6 .016 30.22	899.67	.0188	CTEMP 313.4

EFF(9) = .601	LMOT = 3.1975	NORMAL TOOTH LOAD = 2216.85	PHE(B/SEC)
M(N)	VS(198)	FRIC	DT(F)
1 .038 17.45	739.04	.0188	CTEMP 319.0
2 .014 20.91	771.17	.0188	CTEMP 310.4
3 .084 22.86	803.24	.0188	CTEMP 303.6
4 .065 25.32	835.32	.0188	CTEMP 301.8
5 .031 27.77	867.54	.0188	CTEMP 308.1
6 .016 30.22	899.67	.0188	CTEMP 313.4

EFF(10) = .674	LMOT = 3.1975	NORMAL TOOTH LOAD = 2216.85	PHE(B/SEC)
M(N)	VS(198)	FRIC	DT(F)
1 .038 17.45	739.04	.0188	CTEMP 319.0
2 .014 20.91	771.17	.0188	CTEMP 310.4
3 .084 22.86	803.24	.0188	CTEMP 303.6
4 .065 25.32	835.32	.0188	CTEMP 301.8
5 .031 27.77	867.54	.0188	CTEMP 308.1
6 .016 30.22	899.67	.0188	CTEMP 313.4

EFF(11) = .752	LMOT = 3.1740	NORMAL TOOTH LOAD = 1900.45	PHE(B/SEC)
M(N)	VS(198)	FRIC	DT(F)
1 .044 19.78	762.44	.0188	CTEMP 312.8
2 .020 21.40	794.57	.0188	CTEMP 306.3
3 .084 23.02	826.70	.0188	CTEMP 300.3
4 .065 24.64	858.83	.0188	CTEMP 294.3
5 .031 26.26	890.96	.0188	CTEMP 288.3
6 .016 27.88	923.09	.0188	CTEMP 282.3

Figure I-3. Sample helical gear computer printout (cont'd)


```

MORSEPOWER = 500.00      FMP = 2.072      RANGPL = 15.84      TS.DEG F = 244.3
RPM DRIVER = 5000.00     TMP = 1.250      RANGPM = 20.30      PHEAV.B/SEC = .2640
WHLB.

EFF(12) = .827
M(N)
1 .174 22.54  .73  3.56  30.15  744.05 .0188  57732.4  5.2  305.5  CTEMP  PHE(B/SEC)
2 .250 25.00  .81  3.48  14.80  832.34 .0188  55355.5  1.4  301.2  .0112
3 .236 27.63  .84  3.34  67.18  855.22 .0188  53386.9  8.4  307.7  .0518
4 .315 30.10  .47  3.31  114.81  894.06 .0188  51733.3  14.5  313.8  .0425

NORMAL TOOTH LOAD = 1537.50
HERTZ(MN) DT(F)
PHE(B/SEC)

EFF(13) = .402
M(N)
1 .248 24.27  .80  3.44  7.42  828.22 .0188  48189.1  1.8  308.4  .0054
2 .307 26.68  .86  3.43  47.41  853.22 .0188  46842.4  6.0  305.3  .0278
3 .266 28.84  .42  3.36  86.50  878.23 .0188  45662.6  10.7  310.8  .0603
4 .326 30.50  .48  3.30  126.34  903.23 .0188  44614.8  16.2  314.5  .0732

NORMAL TOOTH LOAD = 1150.14
HERTZ(MN) DT(F)
PHE(B/SEC)

EFF(14) = .477
M(N)
1 .317 26.44  .87  3.42  53.44  857.44 .0188  38077.3  4.8  305.1  .0208
2 .286 28.27  .91  3.37  80.32  874.86 .0146  37435.7  10.2  305.4  .0321
3 .298 29.54  .48  3.33  104.64  890.73 .0281  36844.8  13.4  313.8  .0434
4 .325 30.81  .44  3.24  132.47  907.44 .0205  36244.7  17.4  316.7  .0550

NORMAL TOOTH LOAD = 766.74
HERTZ(MN) DT(F)
PHE(B/SEC)

EFF(15) = 1.057
M(N)
1 .315 20.10  .47  3.31  114.81  894.06 .0250  51046.7  14.2  318.5  .1222

NORMAL TOOTH LOAD = 380.14
HERTZ(MN) DT(F)
PHE(B/SEC)

EFF(16) = 1.127
M(N)
1 .354 31.95  1.01  3.27  146.13  915.23 0.0800  0.0  0.0  0.0  CTEMP  PHE(B/SEC)

NORMAL TOOTH LOAD = 0.00
HERTZ(MN) DT(F)
PHE(B/SEC)

TOOTH MAT. FREQ. = 113671.5 RPM
1.9071E00P = DYNAMIC INCREMENT 1.0.
9.5374E-01 = DYNAMIC FACTOR

```

Figure I-3. Sample helical gear computer printout (cont'd)

APPENDIX J

SPIRAL BEVEL GEAR COMPUTER PROGRAM

Program Goal

This program examines spiral bevel gears for scoring potential. The analysis follows a pair of teeth from the time they engage until they separate, examining the conditions in the gear mesh at a number of intervals (usually 20) during the mesh cycle. The area of contact is assumed to be elliptical. Geometric parameters, velocities, loads, and temperatures are evaluated at the center of the contact ellipse. The method of analysis generally follows that of Coleman.⁶⁶

Operating parameters are calculated on the basis of static load. These parameters may be corrected by applying a dynamic factor.

Program Language and Computer Type

The program is written in FORTRAN IV language for a CDC 6000 Series computer, using RUN compiler and SCOPE 3.4 system.

Input Cards

There are six data cards per set of data. Data sets may be stacked. The program contains both rpm and a horsepower loop, so the effect of changes in these operating variables on gear behavior may be studied.

Either the gear or the pinion may be the driver. However, data must be entered as applicable to the pinion or the gear, regardless of which drives. The program will evaluate conditions for either left- or right-hand rotation of the driver or, in the case of a reversing gear drive, both directions.

A description of the cards follows. All units are in inches unless otherwise noted.

<u>Word</u>	<u>Column</u>	<u>Symbol</u>	<u>Description</u>

Input Card 1 (PRELIMINARY INPUT DATA)

1	1-10	RNP	Number of pinion teeth
2	11-20	RNG	Number of gear teeth

<u>Word</u>	<u>Column</u>	<u>Symbol</u>	<u>Description</u>
<u>Input Card 1 (Cont'd)</u>			
3	21-30	AP	Pinion addendum
4	31-40	AG	Gear addendum
5	41-50	BP	Pinion thermal constant, lb/°F-in.-sec ²
6	51-60	BG	Gear thermal constant, lb/°F-in.-sec ²
7	61-70	DEDP	Pinion dedendum angle, deg
8	71-80	DEDG	Gear dedendum angle, deg
<u>Input Card 2</u>			
1	1-10	EP	Young's modulus, pinion
2	11-20	EG	Young's modulus, gear
3	21-30	PRP	Poisson's ratio, pinion
4	31-40	PRG	Poisson's ratio, gear
5	41-50	PA	Pressure angle, deg
6	51-60	SA	Spiral angle, deg
7	61-70	SIGMA	Shaft angle, deg
8	71-80	FW	Face width
<u>Input Card 3</u>			
1	1-10	PD	Diametral pitch
2	11-20	DRIVE	Driving gear; use: PIN = pinion GEAR = gear

<u>Word</u>	<u>Column</u>	<u>Symbol</u>	<u>Description</u>
<u>Input Card 3 (Cont'd)</u>			
3	21-30	ROT	Rotation of driver as seen looking toward apex; use: CW = clockwise CCW = counterclockwise REV = reverse
4	31-40	HAND	Hand of driver spiral; use: RH = right-hand LH = left-hand
5	41-50	M	Number of divisions of the line of contact that will be studied, usually 20

Input Card 4 (FRICTION AND TEMPERATURE)

1	1-10	TEMP	Oil jet temperature, °F
2	11-20	FR1	Friction factor from Eq. (55), (57), (59), or (61)
3	21-30	FR2	Friction factor from Eq. (54), (56), (58), or (60)
4	31-40	TCON	Temperature difference factor from Eq. (69)

Input Card 5 (RPM LOOP)

1	1-10	RPMP	Pinion rpm
2	11-20	RINC	Rpm pinion increment. Blank or zero if only one rpm
3	21-30	RPMX	Maximum pinion rpm. Blank or zero if only one rpm

Input Card 6 (HORSEPOWER LOOP)

1	1-10	HPO	Initial horsepower

<u>Word</u>	<u>Column</u>	<u>Symbol</u>	<u>Description</u>
<u>Input Card 6 (Cont'd)</u>			
2	11-20	HINC	Horsepower increment. Blank or zero if only one horsepower
3	21-30	HPMX	Maximum horsepower. Blank or zero if only one horsepower

A sample set of data cards for the spiral bevel gear design and operating conditions given in Chapter VIII, Section D, is shown in Figure J-1. The pinion is the driver, turns CW, and has a left-hand spiral, as shown on Card 3. Card 4 contains friction factors calculated for plain surfaces with a composite surface roughness of $33 \mu\text{in. AA}$, using Equations (111) and (112). Only one rpm was required as shown by Card 5, and Card 6 shows that 10 hp levels were run, starting at 300 hp to a maximum of 1200 hp.

Computer Program

Figure J-2 shows a listing of the computer program. Control cards are not included.

Sample Printout

Figure J-3 gives the data printout for the data input of Figure J-1. Results are shown for 600 hp only.

The first page lists the input data for reference purposes. The second page gives the results of the preliminary calculations. The third page lists, for $M+1$ positions across the plane of action, those variables that are independent of the power level. These are velocities and some geometric parameters. The next page gives, again for $M+1$ positions across the plane of action, those parameters such as load, instantaneous surface temperature, and instantaneous frictional power loss that are power-dependent.

22.	23.	.134	.124	42.150	42.150	2.283	2.517

1

30.0E+06	30.0E+06	.3	.3	22.5	35.0	90.0	.871

2

6.111	FIN	CW	LH	20

3

130.0	.0201	.0983	675.0

4

4500.	0.	0.

5

300.	100.	1200.

6

Figure J-1. Sample spiral bevel gear computer program data cards


```

C      CP      = MATERIAL LOAD FACTOR
C      CR      = MODIFIED CONTACT RATIO
C      DP      = PITCH DIAMETER (PINION)
C      DG      = PITCH DIAMETER (GEAR)
C      RRMNDP  = MEAN RADIUS OF CURVATURE (PINION)
C      RRMNDG  = MEAN RADIUS OF CURVATURE (GEAR)
C      PTASQ   = A PARAMETER
C      PAC     = A FACTOR
C      FI      = INERTIA FACTOR
C      FMP     = FACE CONTACT RATIO
C      GAMP    = PITCH ANGLE (PINION), RADIANS
C      GANG    = PITCH ANGLE (GEAR), RADIANS
C      GMEGP   = PINION SPEED, RAD/SEC
C      GN      = MEAN NORMAL BASE PITCH
C      RNP     = PITCH RADIUS IN MEAN NORMAL SECTION (PINION)
C      RNG     = PITCH RADIUS IN MEAN NORMAL SECTION (GEAR)
C      P2      = A PARAMETER
C      RNP     = BASE RADIUS IN MEAN NORMAL SECTION (PINION)
C      RNG     = BASE RADIUS IN MEAN NORMAL SECTION (GEAR)
C      RP      = PITCH RADIUS IN MEAN TRANSVERSE SECTION (PINION)
C      RG      = PITCH RADIUS IN MEAN TRANSVERSE SECTION (GEAR)
C      ROP     = OUTSIDE RADIUS IN MEAN NORMAL SECTION (PINION)
C      ROG     = OUTSIDE RADIUS IN MEAN NORMAL SECTION (GEAR)
C      TMP     = TRANSVERSE CONTACT RATIO
C      ZN      = LENGTH OF ACTION IN NORMAL PLANE
C
C      READ INPUT VARIABLES FOR SPIRAL BEVEL GEARS
C
100 READ (IN,4004) RNP,RNG,AP,AG,DP,DG,DEDP,DEDC,
2      EP,EG,PRP,PRG,PA,SA,SIGMA,FN,
3      PD,DRIVE,ROT,MAND,M,
4      TEMP,FRI,FR2,TCON
4004 FORMAT (2F10.0,4F10.3,2E10.2,5F10.2,F10.3,3A10,11D/
1      1F10.1,3F10.4)
IF (FOP,IN) 600,150
150 READ (IN,4005) RMP,PRC,RPMX
READ (IN,4005) MPD,MINC,MPMX
4005 FORMAT (3F10.2)
WRITE (NOUT,4010) RNP,RNG,AP,AG,DP,DG,DEDP,DEDC,EP,EG,PRP,PRG
WRITE (NOUT,4011) PA,SA,SIGMA,FN,PD,DRIVE,ROT,MAND,M,
TEMP,FRI,FR2,TCON
4010 FORMAT (1M1,///34X,41MVALUES AND DEFINITIONS OF INPUT VARIABLES,
* 26M (INCH UNITS UNLESS NOTED)/
* 2X,16M (PINION), 6X,10M (GEAR),
*/11M RNP = ,F10.4,5X,10M RNG = ,F10.4,5X,
* 40M=NUMBER OF TEETH
*/11M AP = ,F10.4,5X,10M AG = ,F10.4,5X,
* 40M=ADDENDUM
*/11M RP = ,F10.4,5X,10M RG = ,F10.4,5X,
* 40M=THERMAL CONSTANT, LB./SQRT(SEC.) IN, DEG, F
*/11M DEDP = ,F10.4,5X,10M DEDG = ,F10.4,5X,
* 40M=DEDENDUM ANGLE, DEG,
*/11M EP = ,E10.2,5X,10M EG = ,E10.2,5X,
* 40M=YOUNG'S MODULUS, PSI
*/11M PRP = ,F10.4,5X,10M PRG = ,F10.4,5X,
* 40M=POISSON'S RATIO
*)
4011 FORMAT (
* 11M PA = ,F10.2,30X,36M=NORMAL PRESSURE ANGLE, DEG,
*/11M SA = ,F10.2,30X,36M=SPIRAL ANGLE, DEG,
*/11M SIGMA = ,F10.2,30X,36M=SHAFT ANGLE, DEG,

```

Figure J-2. Listing of spiral bevel gear computer program (cont'd)


```

C      RM02  = RADIUS OF PROFILE CURVATURE ON GEAR, IN.          88EV1860+0187
C      SG    = LENGTH OF LINE OF CONTACT THRU CRITICAL POINT, IN. 88EV1870+0188
C      SR    = LOAD SHARING RATIO                                88EV1880+0189
C      VF    = LENGTHWISE SLIDING VELOCITY, IN/SEC.             88EV1890+0190
C      VFG   = LENGTHWISE SLIDING VELOCITY OF GEAR, IN/SEC.     88EV1900+0191
C      VFP   = LENGTHWISE SLIDING VELOCITY OF PINION, IN/SEC.   88EV1910+0192
C      VFT   = LENGTHWISE SUM VELOCITY, IN/SEC.                 88EV1920+0193
C      VPT   = PROFILE SUM VELOCITY, IN/SEC.                   88EV1930+0194
C      VT    = RESULTANT SUM VELOCITY, IN/SEC.                 88EV1940+0195
C      VN    = TANGENTIAL VELOCITY VECTOR, IN/SEC.            88EV1950+0196
C      VP    = PROFILE SLIDING VELOCITY, IN/SEC.              88EV1960+0197
C      VPG   = PROFILE SLIDING VELOCITY OF GEAR, IN/SEC.     88EV1970+0198
C      VPP   = PROFILE SLIDING VELOCITY OF PINION, IN/SEC.    88EV1980+0199
C      VS    = RESULTANT SLIDING VELOCITY, IN/SEC.            88EV1990+0200
C      VI    = RESULTANT ABSOLUTE VELOCITY OF PINION, IN/SEC. 88EV2000+0201
C      VP    = RESULTANT ABSOLUTE VELOCITY OF GEAR, IN/SEC.    88EV2010+0202
C      VO    = DYNAMIC LOAD, LB.                               88EV2020+0203
C      VT    = MAXIMUM TANGENTIAL LOAD AT WHEEL PITCH POINT, LB. 88EV2030+0204
C      ZN    = DISTANCE FROM PITCH LINE TO CRITICAL POINT IN THE MEAN 88EV2040+0205
C              NORMAL SECTION, IN.                             88EV2050+0206
C
C      PERFORM PRELIMINARY CALCULATIONS
C
C      253 SIGMA=SIGMA+RADIAN
C      PAR=PAR+RADIAN
C      SAR=SAR+RADIAN
C      ALFG = DFG
C      ALFP = DFP
C      ALFR=ALFP+RADIAN
C      ALGR=ALFG+RADIAN
C      NP = RNP/PD
C      NG = RNG/PD
C      ANGLE = (180.-SIGMA)+RADIAN
C      GAMG=ATAN(SIN(SIGMA)/((RNG/RNP)+COS(SIGMA)))
C      270 GAMG = SIGMA+GAMG
C      IF (ABS(GAMG-PI/2.),GT,.0015) GO TO 274
C      272 GAMG=1000,
C      GAMGC=.0001
C      GO TO 278
C      274 GAMGT=TAN(GAMG)
C      GAMGCT=1/(GAMG)
C      276 AU = DG/(2.*SIN(GAMG))
C      A = AO-FN/2,
C      RP = (A/AO)*(NP/(2.*COS(GAMG)))
C      RG = (A/AO)*(DG/(2.*GAMGC))
C      AMP = AP-(FN/2.)*TAN(ALFR)
C      AMG = AG-(FN/2.)*TAN(ALGR)
C      PRNP = NP*((1./COS(SAR))+(1./COS(SAR)))
C      PRNG = RG*((1./COS(SAR))+(1./COS(SAR)))
C      RPNP = PRNP+COS(PAR)
C      RBNG = PRNG+COS(PAR)
C      RPNP = PRNP+AMP
C      RBNG = PRNG+AMG
C      DRHNP = SQRT((RPNP+RPNP)-(RPNP+RPNP))=PRNP*SIN(PAR)
C      DRHNG = SQRT((RBNG+RBNG)-(RBNG+RBNG))=PRNG*SIN(PAR)
C      ZN = DRHNP+DRHNG
C      YK2=FN/AO*((2.-FN/AO)/(2.*(1.-FN/AO)))
C      FNP=AN+PD/PT*((YK2*TAN(SAR))-(1./2.)*((YK2*TAN(SAR))**3))
C      PZ = (A/AO)*((PI+COS(SAR))/(COS(PAR))*((COS(SAP)+COS(SAR))
C              +((TAN(PAR)+TAN(PAR))))
C

```

Figure J-2. Listing of spiral bevel gear computer program (cont'd)

```

TMP = (ZM*PD)/P2
CN = SQRT(TMP*TMP+PMP*PMP)
R3A=ACOS(COS(PAR)*SQRT(COS(SAR)*COS(SAR)+TAN(PAR)*TAN(PAR)))
R3M=BSA/RADIAN
P = (A/AN)*(PI/PD)*COS(SAR)*COS(PAR)
IF (FAC,4E,0.) GO TO 286
FAC = (RNG-RNP)/(1.2+RNG+.8*RNP)
285 ETASO = (ZM*ZN+COS(BSA)*.4)*(PW+PN)*(SIN(BSA)*SIN(BSA))
CP = SQRT(1.5/(PI*((1.-PRP*PRP)/EP)+((1.-PRG*PRG)/EG)))
20 F1=1.0
10 G1=GANP/RADIAN
C2=GANP/RADIAN
R3=BSA/RADIAN
WRITE (NOUT,9002) ALFP,ALPG,AMP,ANG,OP,DG,DRHOP,DRHOG,G1,G2,
PRNP,PRNG,RBND,RBNG,RONP,RONG
WRITE (NOUT,9003) RP,RG,A,AD,BSP,CP,ETASO,FAC,PI,P2,PN,ZN,
TMP,FMP,CR
9002 FORMAT (1H1,////45X,35H0 U T P U T D A T A S E C T I O N////
1X,6H(PINION),16X,6H(GEAR)
11M ALFP = ,F10,5,5X,10HALFG = ,F10,5,5X,
13M=ADDENDUM ANGLE (DEG.)
11M AMP = ,F10,5,5X,10MANG = ,F10,5,5X,
13M=MEAN ADDENDUM (IN.)
11M OP = ,F10,5,5X,10MDG = ,F10,5,5X,
13M=PITCH DIAMETER (IN.)
11M DRHOP = ,F10,5,5X,10DRHOG = ,F10,5,5X,
13M=MEAN RADIUS OF CURVATURE (IN.)
11M GANP = ,F10,5,5X,10GANP = ,F10,5,5X,
13M=PITCH ANGLE (DEG.)
11M PRNP = ,F10,5,5X,10MPNG = ,F10,5,5X,
13M=PITCH RADIUS IN MEAN NORMAL SECTION (IN.)
11M RBND = ,F10,5,5X,10MRNG = ,F10,5,5X,
13M=BASE RADIUS IN MEAN NORMAL SECTION (IN.)
11M RONP = ,F10,5,5X,10MRNG = ,F10,5,5X,
13M=OUTSIDE RADIUS IN MEAN NORMAL SECTION (IN.)
)
9003 FORMAT (
11M RP = ,F10,5,5X,10MRG = ,F10,5,5X,
13M=PITCH RADIUS IN MEAN TRANSVERSE SECTION (IN.)
11M A = ,F10,5,30X,37M=MEAN CONE DISTANCE (IN.)
11M AD = ,F10,5,30X,37M=CONE DISTANCE (IN.)
11M BSA = ,F10,5,30X,37M=MEAN BASE SPIRAL ANGLE (DEG.)
11M CP = ,F10,5,30X,37M=MATERIAL LOAD FACTOR (SQRT(LG,)/IN.)
11M ETASO = ,F10,5,30X,37M=SQUARE OF TOTAL LENGTH OF ACTION IN
15HNORMAL SECTION /
52X,37MWITHIN CONTACT ELLIPSE, (SQUARE IN.)
11M FAC = ,F10,5,30X,37M=A FACTOR
11M FI = ,F10,5,30X,37M=INERTIA FACTOR
11M P2 = ,F10,5,30X,37M=FACTOR IN TRANSVERSE CONTACT RATIO
11M PN = ,F10,5,30X,37M=MEAN NORMAL BASE PITCH (IN.)
11M ZN = ,F10,5,30X,37M=LENGTH OF ACTION IN NORMAL PLANE
11M TMP = ,F10,5,30X,37M=TRANSVERSE CONTACT RATIO
11M FMP = ,F10,5,30X,37M=FACE CONTACT RATIO
11M CR = ,F10,5,30X,37M=MODIFIED CONTACT RATIO
)
C
C
C BEGIN NONSEPOWER LOOP FOR SPIRAL BEVEL GEAR,
RPL00P=RPMP
240 RPMP=RPLO0P
245 RPMP=PO
WRITE (NOUT,9013)

```

Figure J-2. Listing of spiral bevel gear computer program (cont'd)

```

297 NUN=N
  AT = (MP+63000.)/(RMP*(DP/2.))
C
C BEGIN LOOP FOR THE INDEPENDENT VARIABLE EFF,
C
  I=1
  X=1.
  IF (I*NT,EO,I*NE) GO TO 280
  I=I+1
  IF (I*NT,EO,I*GEAR) X=X+X
  IF (I*HND,EO,I*H) X=X+X
  IF (I*HNT,EO,I*CN) X=X+X
280 N=1
  DO 300 I=1,N
    J=1
    FFF = (SQRT(ETASQ)/P.)*(SQRT(ETASQ)/2.)*2./N
    EFF=V(I,J)*EFF
    RAD = ETASQ+4.*(EFF*EFF)
    IF (RAD) 300,300,100
  300 ETA1 = SQRT(RAD)
    FFRGT = ((Z4+Z4*EFF+(COS(BSA)+COS(PBA)))/ETASQ)+Z4/2.
    SECONO = ((P4+Z4*ETA1+SIN(BSA))/ETASQ)
  310 ZO = FIRST*(X+PAC*SECONO)-ORHOG
  320 VNP=RPMP/30.*A*SIN(GAMP)*COS(SAR)
    VFP = VN*(TAN(SAR)+ARS(ZO)*SIN(PAR)*(TAN(SAR)/(A*TAN(GAMP))))-
      ARS(ZO)*COS(PAR)*(1./(A*COS(SAR)))
    VFG = V4*(TAN(SAR)-ABS(ZO)*SIN(PAR)*(TAN(SAR)/(A*GAMGT))-
      ABS(ZO)*COS(PAR)*(1./(A*COS(SAR))))
    VF = VFP-VFG
    VPP = VN*(SIN(PAR)+ABS(ZO)*(1./(A*TAN(GAMP))))
    VPG = VN*(SIN(PAR)-ABS(ZO)*(1./(A*GAMGT)))
    VP = VPP-VPG
    VFT = VFP+VFG
    VPT = VPP+VPG
    VT = SQRT(VFT*VFT+VPT*VPT)
    VS = SQRT(VF*VF+VP*VP)
    EFF=V(I,JVS)*VS
    EFF=V(I,JVT)*VT
    V1 = SQRT(VFP+VFP+VPP+VPP)
    V2 = SQRT(VFG+VFG+VPG+VPG)
    SUM = N,N
    DO 340 K=1,100
      RAD = (ETA1+ETA1+4.*K*PN*(K*PN+2.*EFF))*3
      IF (RAD) 350,330,330
  330 SUM = SUM+SQRT(RAD)
  340 CONTINUE
    WRITE (NOUT,9016) SUM
    STOP
  350 SUM1 = 0.
    DO 370 K=1,100
      RAD = (ETA1+ETA1+4.*K*PN*(K*PN+2.*EFF))*3
      IF (RAD) 380,360,360
  360 SUM1 = SUM1+SQRT(RAD)
  370 CONTINUE
    WRITE (NOUT,9016) SUM1
    STOP
  380 IF (ETA1,NE,0.) GO TO 381
    ETA1=.00001
  381 ETICUB=ETA1*3
    ET2CUB = ET1CUB+SUM+SUM1
    SR = ETICUB/ET2CUB
    WND=ABS(A/AO)*3R*PI/(COS(PAR)*COS(SAR))

```

```

SBFV3100+0311
SBFV3110+0312
SBFV3120+0313
SBFV3130+0314
SBFV3140+0315
SBFV3150+0316
SBFV3160+0317
SBFV3170+0318
SBFV3180+0319
SBFV3190+0320
SBFV3200+0321
SBFV3210+0322
SBFV3220+0323
SBFV3230+0324
SBFV3240+0325
SBFV3250+0326
SBFV3260+0327
SBFV3270+0328
SBFV3280+0329
SBFV3290+0330
SBFV3300+0331
SBFV3310+0332
SBFV3320+0333
SBFV3330+0334
SBFV3340+0335
SBFV3350+0336
SBFV3360+0337
SBFV3370+0338
SBFV3380+0339
SBFV3390+0340
SBFV3400+0341
SBFV3410+0342
SBFV3420+0343
SBFV3430+0344
SBFV3440+0345
SBFV3450+0346
SBFV3460+0347
SBFV3470+0348
SBFV3480+0349
SBFV3490+0350
SBFV3500+0351
SBFV3510+0352
SBFV3520+0353
SBFV3530+0354
SBFV3540+0355
SBFV3550+0356
SBFV3560+0357
SBFV3570+0358
SBFV3580+0359
SBFV3590+0360
SBFV3600+0361
SBFV3610+0362
SBFV3620+0363
SBFV3630+0364
SBFV3640+0365
SBFV3650+0366
SBFV3660+0367
SBFV3670+0368
SBFV3680+0369
SBFV3690+0370
SBFV3700+0371
SBFV3710+0372

```

Figure J-2. Listing of spiral bevel gear computer program (cont'd)

```

302 SG = FN*ZN*COB(BSA)*(ETA1/ETAB0)
EFFRY(I,J0)=MM
RMOB = (RP*BIN(PAR))/(COS(RSA)+COS(BSA))
RMOG = (RG*BIN(PAR))/(COS(BSA)+COS(BSA))
RMO1 = RMOB*ZO
RMO2 = RMOG*ZO
RMOF = ABS((RMO1+RMO2)/(RMO1+RMO2))
304 Q = CP+SQRT((MM)/(RMOF+SG))
OMEG = ATAN(BIN(PAR)*ATAN(BAR))
ALF1 = ATAN(VPP/VDP)
ALF2 = ATAN(VPG/VFG)
AS = SG/2.
RS = (1./(PI*CP+CP))+Q+RMOF
D1 = 2.*SQRT((AS+AS+RS+BS)/(AS+AS+(BIN(ALF1+OMEG))+2+BS+BS
      +(COS(ALF1+OMEG))+2))
D2 = 2.*SQRT((AS+AS+BS+BS)/(AS+AS+(BIN(ALF2+OMEG))+2+BS+BS
      +(COS(ALF2+OMEG))+2))
BOTARP=SQRT(V1/D1)+SG*SQRT(V2/D2)
305 EFFRY(I,J0T)=ROT
EFFRY(I,JH)=RMOF
EFFRY(I,J0)=0
EFFRY(I,JR)=BR
IF (MPO,NE,MP) GO TO 303
NUM=NUM+1
WRITE (NOUT,4014) NUM,EFF,VS,VT,V1,V2,ZO,BR,RMOF
303 CONTINUE
NUM=N
CALL FRCN (EFFRY,MM,JVS,JFR,JPH,JND,TS,PHEAV,TEMP,FRI,FR2,TCN,
      ZN,RP,BSA,M,RMP,RADIAN,GAMP)
WRITE (NOUT,4017) MP,MT,PHEAV,TS,RMP
WRITE (NOUT,4018)
DO 430 J=1,MM
EFFRY(I,JFF)
VSEFFRY(I,JVS)
VTEFFRY(I,JVT)
FRTCEFFRY(I,JFR)
PHEEFFRY(I,JPH)
HNEFFRY(I,JND)
SREFFRY(I,JR)
BOTEFFRY(I,J0T)
QREFFRY(I,J0)
RMOFEFFRY(I,JH)
IF (Q,GT,10.,OR,Q,EQ,10.) GO TO 304
FRCO.
PHEO.
QO.
304 DELT = (.89+Q*FRC+VS)/BOT
302 CTMP=TS+DELT
420 NUM=NUM+1
WRITE (NOUT,4019) NUM,EFF,DELT,CTMP,MM,Q,FRC,PHE
430 CONTINUE
440 CONTINUE
IF (TXREV,EQ,1) GO TO 500
IXRLV=1
IXM=-IXM
GO TO 200
500 MP=MP+MINC
IF (MP,LE,MPMX) GO TO 297
MP=MP+RMP+RINC
IF (RMP,LE,RPME) GO TO 295
GO TO 100
600 WRITE (NOUT,4022)

```

Figure J-2. Listing of spiral bevel gear computer program (cont'd)

```

      STOP
9013 FORMAT (1H)////1X,40H VARIABLES CALCULATED INDEPENDENT OF MORSEPOSS8BEV434000035
      4014 (INCHES AND IN./SEC)////9X,3HEFF,11X,2MVS,11X,2MVT,11X,2MV1,8BEV436000037
      01X,2MVS,11X,2MZ0,11X,2MSR,10X,4HMOE//8BEV437000038
9015 FORMAT (1X,12,1M,,2X,9(E10,3,3X))8BEV438000039
9016 FORMAT (50H VALUE OF INDEX KN IN CALCULATION OF ETAP HAS EXCEEDED,8BEV439000040
      1 50H ALPHA LIMIT OF 100 = SERIES SUM OF 100 TERMS = ,8BEV440000041
      2 50H )8BEV441000042
9017 FORMAT (50H VALUE OF INDEX KN IN CALCULATION OF ETAP HAS EXCEEDED,8BEV442000043
      1 50H BETA LIMIT OF 100 = SERIES SUM OF 100 TERMS = ,8BEV443000044
      2 50H )8BEV444000045
9018 FORMAT (1H)////1X,FB,1,10H = MORSEPOWER/8BEV445000046
      0 1X,F10,1,11H = NY, LB,/8BEV446000047
      0 1X,E10,3,10H = PMEAV, BTU/SEC,/8BEV447000048
      0 1X,F10,2,10H = TS, DEG, °/8BEV448000049
      0 1X,F10,1,10H = PINION RPM/8BEV449000050
9019 FORMAT (5X,40H HERTZ,16X,3MPE/9X,3HEFF,9X,4MDT,5,5X,8BEV450000051
      0 3MCTEMP,5,7X,4MAN,LR,4X,10HSTRESS,PSI,4X,4MFRIC,8BEV451000052
      0 4X,7MRTU/SEC/)8BEV452000053
9020 FORMAT (22HEND OF INPUT PROBLEMS)8BEV453000054
9019 FORMAT (1X,12,1M,,2X,E10,3,4X,FS,1,6X,FS,1,6X,FB,1,3X,E9,3,4X,8BEV454000055
      0 5,0,4X,E9,3)8BEV455000056
      END8BEV456000057
      SUBROUTINE FRCYN (EFFRY,MH,JVS,JFR,JPH,JND,TS,PMEAV,TEMP,8BEV457000058
      1 FRI,FRZ,TCON,ZN,RP,BSA,M,RNP,RADIAN,GAMP)8BEV458000059
      DIMENSION EFFRY(30,10)8BEV459000060
      DO 10 I=1,MH8BEV460000061
      WNOEFFRY(I,JND)8BEV461000062
      IF (4H,4P,0) GO TO 58BEV462000063
      FRCIBU,8BEV463000064
      GO TO 278BEV464000065
      5 V90(EFFRY(I,JVS))8BEV465000066
      NVS000(VS00(0,1,3,))8BEV466000067
      IF (4H,4LT,200) GO TO 158BEV467000068
      FRCIBFW8BEV468000069
      GO TO 218BEV469000070
      15 FRCIBFRZ0(NVS00(0,10))8BEV470000071
      27 PHE = FRCIBFWVS/9336,8BEV471000072
      EFFRY(I,JPH)=PHE8BEV472000073
      EFFRY(I,JFR)=FRCIB8BEV473000074
      30 CONTINUE8BEV474000075
      PHE = 0,8BEV475000076
      DO 15 I=1,M8BEV476000077
      PHE10PHE10(EFFRY(I,JPH)+EFFRY(I+1,JPH))8BEV477000078
      15 CONTINUE8BEV478000079
      PMEAV = (PHE10(2,04)10RNP*ZN/COS(BSA)/(RP+RADIAN*360,0COS(GAMP))8BEV479000080
      TS=(TCON+(PMEAV)*0,00)*TEMP8BEV480000081
      RETRN8BEV481000082
      END8BEV482000083
      000008BEV483000084

```

Figure J-2. Listing of spiral bevel gear computer program (cont'd)

OUTPUT DATA SECTION

ALPP	=	2.51700	(PI-VION)	ALPG	=	2.28300	-ADDENDUM ANGLE (DEG.)
AMP	=	1.1486		AMC	=	1.0664	-MEAN ADDENDUM (IN.)
DP	=	1.50003		DC	=	2.76370	-PITCH DIAMETER (IN.)
DRMOP	=	1.2238		DRMOC	=	1.25793	-MEAN RADIUS OF CURVATURE (IN.)
DRMP	=	1.22647		DRMOC	=	1.25793	-PITCH ANGLE (DEG.)
PRMP	=	3.04137		PRMC	=	3.37874	-PITCH RADIUS IN MEAN NORMAL SECTION (IN.)
RRMP	=	2.85605		RRMC	=	3.12160	-BASE RADIUS IN MEAN NORMAL SECTION (IN.)
RRMP	=	2.85623		RRMC	=	3.12160	-OUTSIDE RADIUS IN MEAN NORMAL SECTION (IN.)
RP	=	2.07439		RG	=	2.26720	-PITCH RADIUS IN MEAN TRANSVERSE SECTION (IN.)
A	=	2.16863					-MEAN CONE DISTANCE (IN.)
AO	=	2.80413					-CONE DISTANCE (IN.)
BA	=	21.44478					-MEAN BASE SPIRAL ANGLE (DEG.)
CP	=	2801.405					-MATERIAL LOAD FACTOR (80MTLBS./IN.)
ETAS-	=	1.35410					-SQUARE OF TOTAL LENGTH OF ACTION IN NORMAL SECTION WITHIN CONTACT ELLIPSE, (SQUARE IN.)
FAC	=	1.00619					-A FACTOR
F1	=	1.00000					-INERTIA FACTOR
P2	=	2.25302					-FACTOR IN TRANSVERSE CONTACT RATIO
PH	=	1.2400					-MEAN NORMAL BASE PITCH (IN.)
ZN	=	1.33141					-LENGTH OF ACTION IN NORMAL PLANE
THP	=	1.17460					-TRANSVERSE CONTACT RATIO
PHP	=	1.00140					-FACE CONTACT RATIO
BR	=	1.06					-MODIFIED CONTACT RATIO

Figure J-3. Sample spiral bevel gear computer printout (cont'd)

VARIABLES CALCULATED INDEPENDENT OF HORSEPOWER (INCHES AND IN./SEC)

	EXP	V0	V1	V2	Z0	DR	RHOE
1.	0.000E-01	0.000E+01	0.000E+02	0.000E+02	0.000E+01	0.000E+01	0.000E-01
2.	0.000E-01	0.000E+01	0.000E+02	0.000E+02	0.000E+01	0.000E+01	0.000E-01
3.	0.000E-01	0.000E+01	0.000E+02	0.000E+02	0.000E+01	0.000E+01	0.000E-01
4.	0.000E-01	0.000E+01	0.000E+02	0.000E+02	0.000E+01	0.000E+01	0.000E-01
5.	0.000E-01	0.000E+01	0.000E+02	0.000E+02	0.000E+01	0.000E+01	0.000E-01
6.	0.000E-01	0.000E+01	0.000E+02	0.000E+02	0.000E+01	0.000E+01	0.000E-01
7.	0.000E-01	0.000E+01	0.000E+02	0.000E+02	0.000E+01	0.000E+01	0.000E-01
8.	0.000E-01	0.000E+01	0.000E+02	0.000E+02	0.000E+01	0.000E+01	0.000E-01
9.	0.000E-01	0.000E+01	0.000E+02	0.000E+02	0.000E+01	0.000E+01	0.000E-01
10.	0.000E-01	0.000E+01	0.000E+02	0.000E+02	0.000E+01	0.000E+01	0.000E-01
11.	0.000E-01	0.000E+01	0.000E+02	0.000E+02	0.000E+01	0.000E+01	0.000E-01
12.	0.000E-01	0.000E+01	0.000E+02	0.000E+02	0.000E+01	0.000E+01	0.000E-01
13.	0.000E-01	0.000E+01	0.000E+02	0.000E+02	0.000E+01	0.000E+01	0.000E-01
14.	0.000E-01	0.000E+01	0.000E+02	0.000E+02	0.000E+01	0.000E+01	0.000E-01
15.	0.000E-01	0.000E+01	0.000E+02	0.000E+02	0.000E+01	0.000E+01	0.000E-01
16.	0.000E-01	0.000E+01	0.000E+02	0.000E+02	0.000E+01	0.000E+01	0.000E-01
17.	0.000E-01	0.000E+01	0.000E+02	0.000E+02	0.000E+01	0.000E+01	0.000E-01
18.	0.000E-01	0.000E+01	0.000E+02	0.000E+02	0.000E+01	0.000E+01	0.000E-01
19.	0.000E-01	0.000E+01	0.000E+02	0.000E+02	0.000E+01	0.000E+01	0.000E-01
20.	0.000E-01	0.000E+01	0.000E+02	0.000E+02	0.000E+01	0.000E+01	0.000E-01
21.	0.000E-01	0.000E+01	0.000E+02	0.000E+02	0.000E+01	0.000E+01	0.000E-01

Figure J-3. Sample spiral bevel gear computer printout (cont'd)

600.0 = HORSEPOWER
 666.6 = WT, LB.
 1.975E-01 = PMCAV, BTU/SEC.
 379.4 = TS, DEG. F
 660.0 = PINION RPM

	CFI	DT,F	CTEMP,F	MM, LB	MAX, MERTZ STRESS, PSI	PRIC	PME BTU/SEC
1.	-2.966E-01	8.0	379.4	0	0	.0000	0
2.	-2.967E-01	29.5	379.4	911.3	1.921E+05	.0000	0.723E-02
3.	-2.967E-01	25.5	379.4	1008.6	1.942E+05	.0000	1.310E-01
4.	-2.967E-01	27.5	379.4	1628.4	2.149E+05	.0001	0.044E-01
5.	-1.967E-01	26.5	379.4	2217.0	2.420E+05	.0201	2.132E-01
6.	-1.968E-01	24.5	379.4	3408.0	2.612E+05	.0201	2.482E-01
7.	-1.968E-01	20.5	379.4	4611.6	2.741E+05	.0201	2.316E-01
8.	-0.968E-02	16.0	379.4	6011.2	2.467E+05	.0201	1.917E-01
9.	-5.962E-02	10.0	379.4	8040.2	2.130E+05	.0201	1.792E-01
10.	-2.966E-02	5.0	379.4	1133.0	2.870E+05	.0201	3.732E-02
11.	0	0.0	379.4	0	0	.0000	0.000E+00
12.	2.966E-02	14.0	379.4	5040.2	2.866E+05	.0201	1.004E-01
13.	5.962E-02	10.3	379.4	6617.2	2.131E+05	.0201	2.456E-01
14.	0.964E-02	24.5	379.4	4011.6	2.450E+05	.0201	2.439E-01
15.	1.968E-01	20.5	379.4	3408.0	2.700E+05	.0201	3.175E-01
16.	1.968E-01	21.3	379.4	2807.0	2.549E+05	.0201	3.102E-01
17.	1.968E-01	32.0	379.4	2217.0	2.405E+05	.0201	2.607E-01
18.	2.967E-01	32.6	379.4	1620.6	2.170E+05	.0201	2.433E-01
19.	2.967E-01	27.5	379.4	1008.6	1.870E+05	.0201	1.796E-01
20.	2.967E-01	26.5	379.4	911.3	1.900E+05	.0253	4.408E-02
21.	2.966E-01	0.0	379.4	0	0	.0000	0

Figure J-3. Sample spiral bevel gear computer printout (cont'd)

APPENDIX K

CALCULATION OF AVERAGE FRICTIONAL POWER LOSS

As explained in Chapter VII, Section B, the average frictional power loss, ϕ_{av} , for a gear set is derived basically from Equation (72). However, due to the way the analyses were made (Chap. VII, Sect. D, E, and F), the detailed computing procedures differed for the three different gear types. These computations will now be illustrated by reference to the three specific gear types examined in Chapter VIII, the computer printouts for which at 600 hp were presented in Appendixes H, I, and J.

Spur Gears

For the ground spur gear set operating at 600 hp under the conditions stated in Chapter VIII, Section B, the instantaneous f , W , V_s , and ϕ' values vs. the pinion roll angle, ϵ , as given in Figure H-3, are summarized in Table K-1. Contact line positions 1 through 7 represent a double tooth contact region with $\Delta\epsilon_1 = 20.08 - 14.68 = 5.40^\circ$, divided into 6 equal intervals. Contact line positions 8 through 17 represent a single tooth contact region with $\Delta\epsilon_2 = 26.29 - 20.08 = 6.21^\circ$, divided into 9 equal intervals. Contact line positions 18 through 24 represent a double tooth contact region with $\Delta\epsilon_3 = 31.70 - 26.29 = 5.41^\circ$, divided into 6 equal intervals. Note that at any contact line position, $\phi' = f W V_s / 9336$ as tabulated.

Because the roll angle intervals into which $\Delta\epsilon_1$, $\Delta\epsilon_2$, and $\Delta\epsilon_3$ are divided are not of equal magnitude, the integration of the ϕ' vs. ϵ curve is made separately for each region and summed as implied in Equation (72), so

$$\begin{aligned} \sum \int \phi'(\epsilon) d\epsilon &= \text{Area}_1 + \text{Area}_2 + \text{Area}_3 \\ &= \bar{h}_1 \Delta\epsilon_1 + \bar{h}_2 \Delta\epsilon_2 + \bar{h}_3 \Delta\epsilon_3 \quad (\text{K-1}) \end{aligned}$$

where $\bar{h}_1$, $\bar{h}_2$, and $\bar{h}_3$ are the average heights of the ϕ' vs. ϵ curves in regions 1, 2, and 3. Using the trapezoidal rule for integration in the computer program, one may write

TABLE K-1. INSTANTANEOUS ϕ' VALUES
(GROUND SPUR GEARS, 600 hp)

Contact line position	ϵ , deg.	f	W, lb	V_s , ips	ϕ' , Btu/sec
1	14.63	0.022379	468.8	294.81	0.3313
2	15.58	0.019895	668.4	263.46	0.3752
3	16.48	0.017596	964.9	232.12	0.4221
4	17.38	0.016400	1313.0	200.78	0.4631
5	18.28	0.016400	1665.7	169.43	0.4957
6	19.18	0.016400	1902.2	138.09	0.4614
7	20.08	0.016400	2068.9	106.74	0.3879
8	20.08	0.016400	2795.7	106.74	0.5242
9	20.77	0.016400	2795.7	82.73	0.4063
10	21.46	0.016400	2795.7	58.71	0.2884
11	22.15	0.016400	2795.7	34.70	0.1704
12	22.84	0.016400	2795.7	10.69	0.0615
13	23.53	0.016400	2795.7	13.33	0.0655
14	24.22	0.016400	2795.7	37.34	0.1834
15	24.91	0.016400	2795.7	61.35	0.3013
16	25.60	0.016400	2795.7	85.37	0.4193
17	26.29	0.016400	2795.7	109.38	0.5372
18	26.29	0.016400	2326.9	109.38	0.4471
19	27.19	0.016400	2127.4	140.73	0.5259
20	28.09	0.016400	1830.9	172.07	0.5534
21	28.99	0.016400	1482.8	203.41	0.5298
22	29.89	0.016800	1130.1	234.76	0.4774
23	30.79	0.018253	893.6	266.10	0.4649
24	31.70	0.019637	726.8	297.45	0.4548

$$\bar{h} = \frac{1}{2n} \left[\sum_{i=1}^n (\phi'_i + \phi'_{i+1}) \right] \quad (K-2)$$

where n = number of elemental intervals

ϕ'_i = ϕ' value at the beginning of interval i

ϕ'_{i+1} = ϕ' value at the end of interval i

Applying Equation (K-2) to the ϕ' values in Table K-1, one obtains

$$\bar{h}_1 = \frac{1}{2 \times 6} (\phi'_1 + 2\phi'_2 + \dots + 2\phi'_6 + \phi'_7) = 0.42952$$

$$\bar{h}_2 = \frac{1}{2 \times 9} (\phi'_8 + 2\phi'_9 + \dots + 2\phi'_{16} + \phi'_{17}) = 0.26864$$

$$\bar{h}_3 = \frac{1}{2 \times 6} (\phi'_{18} + 2\phi'_{19} + \dots + 2\phi'_{23} + \phi'_{24}) = 0.50039$$

$$\text{and } A_1 = \bar{h}_1 \Delta\epsilon_1 = 0.42952 \times 5.40 = 2.31941$$

$$A_2 = \bar{h}_2 \Delta\epsilon_2 = 0.26864 \times 6.21 = 1.66825$$

$$A_3 = \bar{h}_3 \Delta\epsilon_3 = 0.50039 \times 5.41 = 2.70711$$

Thus, by Equation (K-1),

$$\sum \int \phi'(\epsilon) d\epsilon = A_1 + A_2 + A_3 = 6.69477$$

and by Equation (72) and noting that $N_p = 31$,

$$\phi_{av} = \left[\sum \int \phi'(\epsilon) d\epsilon \right] \frac{N_p}{360}$$

$$= 6.69477 \times 31/360$$

$$= 0.5765 \text{ Btu/sec}$$

which compares with $\phi_{av} = 0.5763 \text{ Btu/sec}$ given in the computer print-out (Fig. H-3) and tabulated in Table 8 (Chap. VIII); the slight numerical difference is due to the rounding off of significant places in the calculation given herein.

Helical Gears

For the helical gear set operating at 600 hp under the conditions stated in Chapter VIII, Section C, the instantaneous f , W , V_s , and ϕ'' values vs. the contact line position, taken from or calculated from data given in Figure I-3, are summarized in Table K-2. Note from Chapter VII, Section E, that in order to obtain the value of ϕ'' at any contact line position, the contributions of the several "elemental spur gears" on this contact line should be summed up. For example, referring to contact line position 3 in Figure I-3, this contact line is divided into four elemental spur gears at a total normal load of 766.79 lb, or at a normal load of $766.79/4 = 191.7 \text{ lb}$ for each elemental spur gear. Therefore, the elemental contribution to ϕ'' at this contact line is, from data given in Figure I-3,

$$\begin{aligned} \phi_3'' &= (0.0207 \times 191.70 \times 145.14)/9336 \\ &\quad + (0.0203 \times 191.70 \times 118.82)/9336 \\ &\quad + (0.0198 \times 191.70 \times 92.49)/9336 \\ &\quad + (0.0191 \times 191.70 \times 66.17)/9336 \\ &= 0.0616 + 0.0494 + 0.0375 + 0.0260 \\ &= 0.1745 \text{ Btu/sec} \end{aligned}$$

where the quantities 0.0616, 0.0494, 0.0375, and 0.0260 are given in the computer printout; but their sum, $\phi_3'' = 0.1745 \text{ Btu/sec}$, is stored in the computer program. If this process is repeated for all contact line positions, the appropriate values of ϕ'' will be as in Table K-2.

To calculate ϕ_{av} , it is convenient to transform Equation (85) to the form

TABLE K-2. INSTANTANEOUS ϕ'' VALUES
(HELICAL GEARS, 600 hp)

<u>Contact line position</u>	<u>f</u>	<u>W, lb</u>	<u>V_s, ips</u>	<u>ϕ'', Btu/sec</u>
1	0	0	158.31	0
2	0.0253	380.19	131.98	0.1359
3	-	766.79	-	0.1745
4	-	1150.19	-	0.1837
5	-	1533.59	-	0.2038
6	-	1900.95	-	0.2580
7	-	2216.85	-	0.3397
8	-	2216.85	-	0.3397
9	-	2216.85	-	0.3397
10	-	2216.85	-	0.3397
11	-	1900.95	-	0.2517
12	-	1533.58	-	0.1850
13	-	1150.19	-	0.1556
14	-	766.79	-	0.1528
15	0.0250	380.19	119.81	0.1222
16	0	0	146.13	0

$$\phi_{av} = (\bar{h} \Delta\epsilon) \frac{N_p}{360} \quad (K-3)$$

where $\bar{h}$, the average contribution for a pair of teeth as contact moves across the plane of action, is, by applying Equation (K-2),

$$\begin{aligned} \bar{h} &= \frac{1}{2n} \left[\sum_{i=1}^n (\phi_i'' + \phi_{i+1}'') \right] \\ &= \frac{1}{2 \times 15} (\phi_1'' + 2\phi_2'' + \dots + 2\phi_{15}'' + \phi_{16}'') \\ &= 0.2121 \text{ Btu/sec} \end{aligned}$$

Now, the angle through which the pinion turns as contact moves across the plane of action is, from Equation (86),

$$\begin{aligned} \Delta\epsilon &= \frac{2Z}{d_b} \cdot \frac{180}{\pi} \\ &= \frac{2 \times 0.45422}{3.53450} \cdot \frac{180}{\pi} = 14.7262^\circ \end{aligned}$$

Substituting the calculated $\bar{h}$ and $\Delta\epsilon$ into Equation (K-3), and noting that $N_p = 31$,

$$\begin{aligned} \phi_{av} &= 0.2121 \times 14.7262 \times 31/360 \\ &= 0.2690 \text{ Btu/sec} \end{aligned}$$

which is the value given in the computer printout (Fig. I-3) and tabulated in Table 10 (Chap. VIII).

Spiral Bevel Gears

For the spiral bevel gear set operating at 600 hp under the conditions stated in Chapter VIII, Section D, the instantaneous f , W , V_s , and ϕ'' values vs. the contact line position, as taken from Figure J-3, are summarized in Table K-3.

To calculate ϕ_{av} , it is convenient to transform Equation (88)

TABLE K-3. INSTANTANEOUS ϕ'' VALUES
(SPIRAL BEVEL GEARS, 600 hp)

<u>Contact line position</u>	<u>f</u>	<u>W, lb</u>	<u>V_s, ips</u>	<u>ϕ'', Btu/sec</u>
1	0	0	89.14	0
2	0.0250	411.3	79.16	0.0872
3	0.0201	1008.6	69.55	0.1510
4	0.0201	1620.4	60.02	0.2094
5	0.0201	2217.8	50.53	0.2413
6	0.0201	2807.0	41.07	0.2482
7	0.0201	3402.0	31.63	0.2316
8	0.0201	4011.6	22.20	0.1917
9	0.0201	4617.2	12.79	0.1272
10	0.0201	5098.2	3.40	0.0373
11	0.0201	5135.0	5.98	0.0661
12	0.0201	5098.2	15.34	0.1684
13	0.0201	4617.2	24.69	0.2455
14	0.0201	4011.6	34.03	0.2939
15	0.0201	3402.0	43.35	0.3175
16	0.0201	2807.0	52.65	0.3182
17	0.0201	2217.8	61.93	0.2957
18	0.0201	1620.4	71.18	0.2483
19	0.0201	1008.6	80.39	0.1746
20	0.0253	411.3	89.53	0.0999
21	0	0	98.29	0

again to the form of Equation (K-3) given previously for helical gears.
Thus

$$\begin{aligned}\bar{h} &= \frac{1}{2n} \left[\sum_{i=1}^n (\phi_i'' + \phi_{i+1}'') \right] \\ &= \frac{1}{2 \times 20} (\phi_1'' + 2\phi_2'' + \dots + 2\phi_{20}'' + \phi_{21}'') \\ &= 0.1877 \text{ Btu/sec}\end{aligned}$$

Now, the angle through which the pinion turns as contact moves across the plane of action is, from Equation (89),

$$\begin{aligned}\Delta\epsilon &= \frac{Z}{A \sin \gamma} \cdot \frac{180}{\pi} \\ &= \frac{ZN \cos \psi_b}{A \sin \gamma} \cdot \frac{180}{\pi} \\ &= \frac{0.53141 \cos (31.99975^\circ)}{2.16863 \sin (43.72697^\circ)} \cdot \frac{180}{\pi} = 17.2255^\circ\end{aligned}$$

Substituting the calculated $\bar{h}$ and $\Delta\epsilon$ into Equation (K-3), and noting that $N_p = 22$,

$$\begin{aligned}\phi_{av} &= 0.1877 \times 17.2255 \times 22/360 \\ &= 0.1975 \text{ Btu/sec}\end{aligned}$$

which is the value given in the computer printout (Fig. J-3) and tabulated in Table 12 (Chap. VIII).

LIST OF SYMBOLS

a	addendum, in.
a	major semiwidth of Hertzian ellipse, in.
B	minor width of Hertzian rectangle, in.
b	dedendum, in.
b	minor semiwidth of Hertzian ellipse, in.
C	center distance, in.
C	fitting constant in $(T_s - T_j)$ vs. ϕ equation for disks
C'	fitting constant in $(T_s - T_j)$ vs. ϕ_{av} equation for gears
C _o	fitting constant in $(T_o - T_j)$ vs. ϕ equation for disks
C' _o	fitting constant in $(T_o - T_j)$ vs. ϕ_{av} equation for gears
c	specific heat of gear material, in./°F
c	specific heat of oil, Btu/lb-°F
D	pitch diameter of gear, in.
d	pitch diameter of pinion, in.
D _b	base diameter of gear, in.
d _b	base diameter of pinion, in.
D _o	outside diameter of gear, in.
d _o	outside diameter of pinion, in.
E	Young's modulus, psi
* E	Equivalent Young's modulus, psi
e	angular misalignment, deg

LIST OF SYMBOLS (Cont'd)

e	pitch error, in.
e_a	apparent tooth error, in.
e_e	effective tooth error, in.
e_f	effective pitch error, in.
F	face width, in.
F_i	dynamic normal tooth load increment, lb
f	coefficient of friction
G	gear ratio
h_m	minimum EHD film thickness without side flow and inlet-shear thermal corrections, μ in.
h'_m	minimum EHD film thickness with side flow and inlet-shear thermal corrections, μ in.
K_i	inertia factor (AGMA)
K_v	dynamic factor (AGMA)
k	thermal conductivity of gear material, lb/°F-sec
k	thermal conductivity of oil, Btu/ft-°F-sec
L	total length of lines of contact for all tooth pairs simultaneously in contact, in.
l	elemental segment on a line of contact, in.
M	sliding-to-sum velocity ratio
m_c	transverse contact ratio
m_f	face contact ratio

LIST OF SYMBOLS (Cont'd)

m_N	load sharing ratio
m_o	modified contact ratio
m_t	total contact ratio
N_g	number of teeth in gear
N_p	number of teeth in pinion
n_g	rotative speed of gear, rpm
n_p	rotative speed of pinion, rpm
P	diametral pitch, in. ⁻¹
P	power transmitted, hp
P_A	actual scoring-limited power-transmitting capacity, hp
P_I	ideal scoring-limited power-transmitting capacity, hp
p	pressure, psi
p	transverse circular pitch, in.
p_b	transverse base pitch, in.
P_N	normal base pitch, in.
R	equivalent radius of curvature, in.
S	surface roughness, μ in. rms (AGMA)
S_d	dynamic factor
S_m	misalignment factor
T	temperature, °F
T	torque, in.-lb

LIST OF SYMBOLS (Cont'd)

T_c	maximum instantaneous conjunction surface temperature, °F
T_{cr}	critical temperature, °F
T_j	oil jet temperature, °F
T_o	conjunction-inlet oil temperature, °F
T_s	quasi-steady surface temperature, °F
ΔT	maximum instantaneous conjunction surface temperature rise, °F
V_1	surface velocity of body 1, ips
V_2	surface velocity of body 2, ips
V_g	surface velocity of gear, ips
V_p	surface velocity of pinion, ips
V_s	sliding velocity, ips
V_t	sum velocity, ips
V_t	pitchline velocity, fpm
W	normal tooth load, lb
w	unit normal tooth load, ppi
W_d	dynamic normal tooth load, lb
W_t	tangential tooth load, lb
w_t	unit tangential tooth load, lb
Z	length of path of contact in transverse plane, in.

LIST OF SYMBOLS (Cont'd)

α	pressure-viscosity coefficient of oil, psi^{-1}
β	Blok's thermal coefficient of gear material, $\text{lb}/^\circ\text{F-in.}-\text{sec}^{\frac{1}{2}}$
Γ	pitch angle of gear, deg
γ	pitch angle of pinion, deg
Δ	profile modification, in.
δ	composite surface roughness of one surface, $\mu\text{in. AA}$
δ_c	composite surface roughness of mating surfaces, $\mu\text{in. AA}$
δ_i	initial composite surface roughness of mating surfaces, $\mu\text{in. AA}$
δ_m	normal tooth deflection, in.
ϵ	roll angle, deg
θ	load angle, deg
Λ	EHD film thickness ratio
μ	absolute viscosity of oil, cp
ν	kinematic viscosity of oil, cs
ν	Poisson's ratio of gear material
ρ	density of gear material, lb/in.^3
ρ	density of oil, g/ml
ρ_g	radius of curvature of gear tooth, in.
ρ_p	radius of curvature of pinion tooth, in.
ϕ	pressure angle, deg

LIST OF SYMBOLS (Cont'd)

ϕ	frictional power loss, Btu/sec
ϕ_{av}	average frictional power loss, Btu/sec
ϕ_n	pressure angle in normal plane, deg
ψ	helix angle, deg
ψ	spiral angle, deg
ψ_b	base helix angle, deg
ψ_b	base spiral angle, deg
ω_g	angular velocity of gear, rad/sec
ω_p	angular velocity of pinion, rad/sec