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ADDENDUM TO: BAND MODEL FORMULATION FOR INHOMOGENEOUS OPTICAL PATHS

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Addendum to: Band Model Formulation for Inhomogeneous Optical Paths

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20. ABSTRACT (Continue on reverse side if necessary and identify by block number)

The statistical band model formulation for highly inhomogeneous optical paths presented in a previous paper is extended to the case where the strengths of lines in $\Delta\nu$ are distributed according to a modified inverse probability function.

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In a previous paper,[1] the mean equivalent width derivative functions $y(x, \rho)$ for bands of randomly arranged Lorentz or Doppler lines with a constant or exponential distribution of line strengths were derived and tabulated. Here, an extension to the exponential-tailed inverse line strength distribution of Malkmus[2] is made. Familiarity on the reader's part with the contents of Ref. 1 is assumed.

The exponential-tailed inverse distribution is an approximation to the purely inverse distribution but is more mathematically convenient to use than the latter, at least when employed in conjunction with the Lorentz line shape. According to this distribution, the probability that a line in $\Delta\nu$ has strength S $(0 \leq S \leq \infty)$ in dS is

$$P(S)dS = \frac{1}{S \ln R}\left[\exp\left(-\frac{S}{S_M}\right) - \exp\left(-\frac{RS}{S_M}\right)\right]dS \qquad (1)$$

where S_M is the maximum line strength cut-off and S_M/R the minimum cut-off that would apply to the purely inverse distribution. The mean line strength for the distribution is

$$\overline{S} = \frac{R-1}{R \ln R} S_M. \qquad (2)$$

The approximation to the inverse distribution results as $R \to \infty$. In application, P(S) in the form of Eq. (1) is used to compute the desired result in terms of R, and then the limit $R \to \infty$ is taken. In most cases, the functional form of the result is independent of R.

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The equivalent width for an array of Lorentz lines with this strength distribution has the simple form[2]

$$\frac{\overline{W}_L}{\delta} = \beta_L f(x) \tag{3}$$

where

$$f(x) = \sqrt{1 + 2x} - 1. \tag{4}$$

For an array of Doppler lines, the result is

$$\frac{\overline{W}_D}{\delta} = \beta_D H(x) \tag{5}$$

where

$$H(x) = \frac{2}{\sqrt{\pi}} \int_0^{\infty} \ln\left[1 + x \exp(-z^2)\right] dz. \tag{6}$$

The curve of growth function H(x) has been considered by Malkmus,[3] who has given a series solution for small x and an asymptotic expansion for large x.

The equivalent width derivative function $y(x, \rho)$ for an array of Lorentz lines and in the Lindquist-Simmons approximation is obtained by using P(S) of Eq. (1) with Eqs. (14) and (18) of Ref. 1. The result is

$$y(x, \rho) = \frac{2\rho}{\pi} \int_0^{\pi} \frac{d\theta}{\left[(\rho^2 + 1) + (\rho^2 - 1)\cos\theta\right]\left[1 + ax(1 + \cos\theta)\right]\left[1 + bx(1 + \cos\theta)\right]} \tag{7}$$

where $a = R/(\sqrt{R} + 1)^2$ and $b = a/R$. The limit $R \to \infty$ yields

$$y(x,\rho) = \frac{2\rho}{\pi} \int_0^{\pi} \frac{d\theta}{[(\rho^2 + 1) + (\rho^2 - 1)\cos\theta][1 + x(1 + \cos\theta)]}. \tag{8}$$

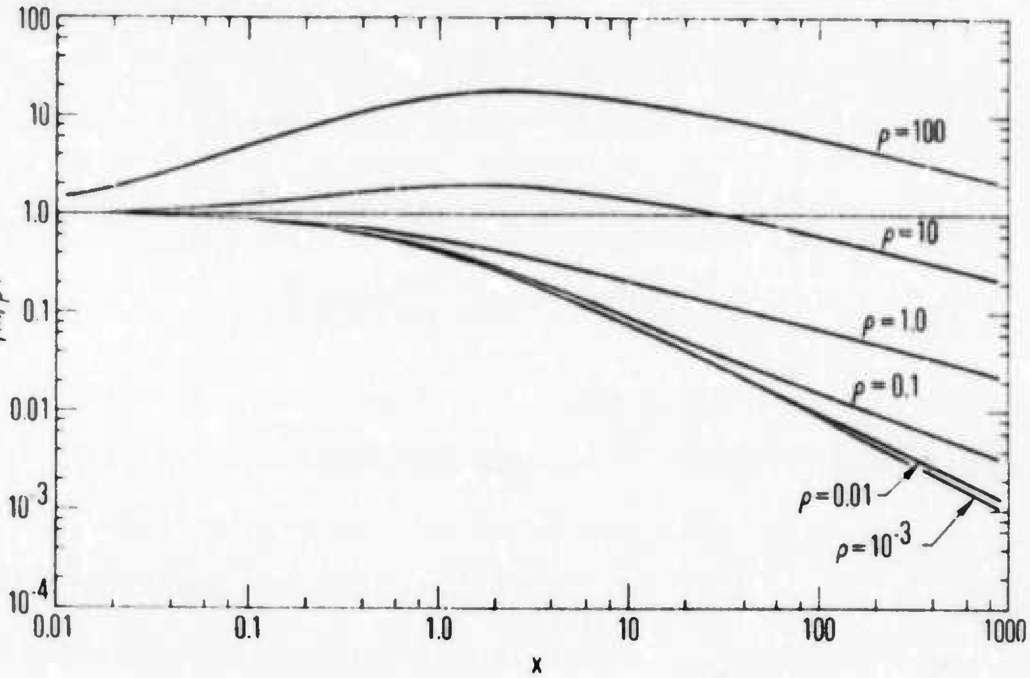
This result is very similar in form to that obtained in Ref. 1 for the exponential distribution. The only difference is that the exponent on the function $1 + x(1 + \cos\theta)$ in the denominator has been reduced from 2 to 1 and is clearly a result of the S^{-1} factor in the distribution function. This reduction in power is significant since now, when the transformation to the complex z plane by $z = e^{i\theta}$ is made in order to perform the integration of Eq. (8) by the method of residues, the order of the poles at z_3 and z_4 [Eqs. (27c) and (27d) of Ref. 1] is also reduced from 2 to 1. A straightforward evaluation of the residues yields the relatively simple expression

$$y(x,\rho) = \frac{2\rho(1 + x) + (1 + \rho^2)\sqrt{1 + 2x}}{\sqrt{1 + 2x}\,[\rho + \sqrt{1 + 2x}]^2}. \tag{9}$$

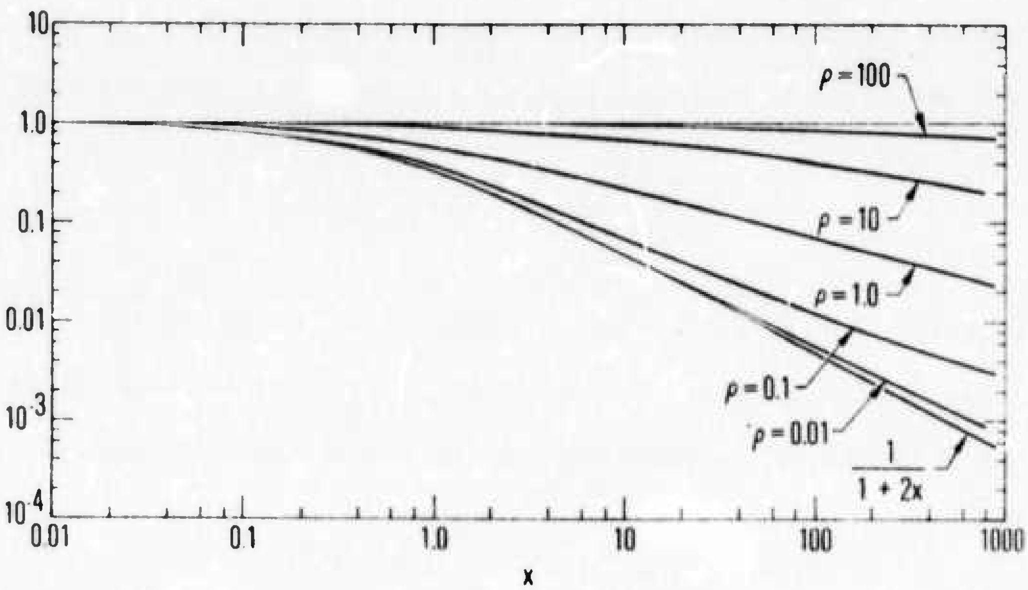
For a homogeneous path ($\rho = 1$), Eq. (9) reduces to $y(x,1) = (1 + 2x)^{-1/2}$, which is $df(x)/dx$ as required. Curves of $y(x,\rho)$ vs x for several values of ρ according to Eq. (9) and according to the Curtis-Godson expression

$$y(x,\rho) = (2 - \rho)\frac{df(x)}{dx} + (\rho - 1)\frac{f(x)}{x} \tag{10}$$

are shown in Fig. 1. The discussion of these curves is similar to that given for the constant and exponential distribution curves in Ref. 1. The simplicity in form of the result of Eq. (9) over the results for the constant



(a)



(b)

Fig. 1. Equivalent Width Derivative Function for an Exponential-Tailed Inverse Intensity Distribution of Lorentz Lines: (a) Curtis-Godson Approximation; (b) Lindquist-Simmons Approximation

and exponential cases is enhanced by the fact that an inverse distribution is a more realistic representation for many molecular species than either of the other two.

When Eqs. (16) and (18) of Ref. 1 are used along with P(S) of Eq. 1, the derivative function for an array of Doppler lines in the Lindquist-Simmons approximation is found to be (after $R \to \infty$)

$$y(x, \rho) = \frac{2}{\sqrt{\pi}} \int_0^{\infty} \frac{e^{-z^2}}{[1 + x \exp(-\rho^2 z^2)]} dz. \tag{11}$$

Again, this result is similar to that obtained for an exponential distribution of Doppler lines in Ref. 1. The only change is a reduction in the power of the denominator from 2 to 1. Note that $y(x, 1) = dH(x)/dx$ as required for a homogeneous path. A series representation for $x < 1$ is

$$y(x, \rho) = \sum_{n=0}^{\infty} \frac{(-x)^n}{\sqrt{1 + n\rho^2}}, \tag{12}$$

and a useful approximation ($\lesssim$ 0.8%) for $\rho \lesssim 0.5$ is

$$y(x, \rho) = \frac{1}{\sqrt{1 + x}\,[1 - x(\rho^2 - 1)]^{1/2}}. \tag{13}$$

The most useful representation of $y(x, \rho)$ is a table of values for a wide range of x and ρ. The entries of Table 1 were computed by a numerical integration of Eq. (11) in a manner similar to that of Ref. 1 except that

Table 1

Equivalent Width Derivative Function for an Exponential-Tailed Inverse Intensity Distribution of Doppler Lines in the Lindquist-Simmons Approximation

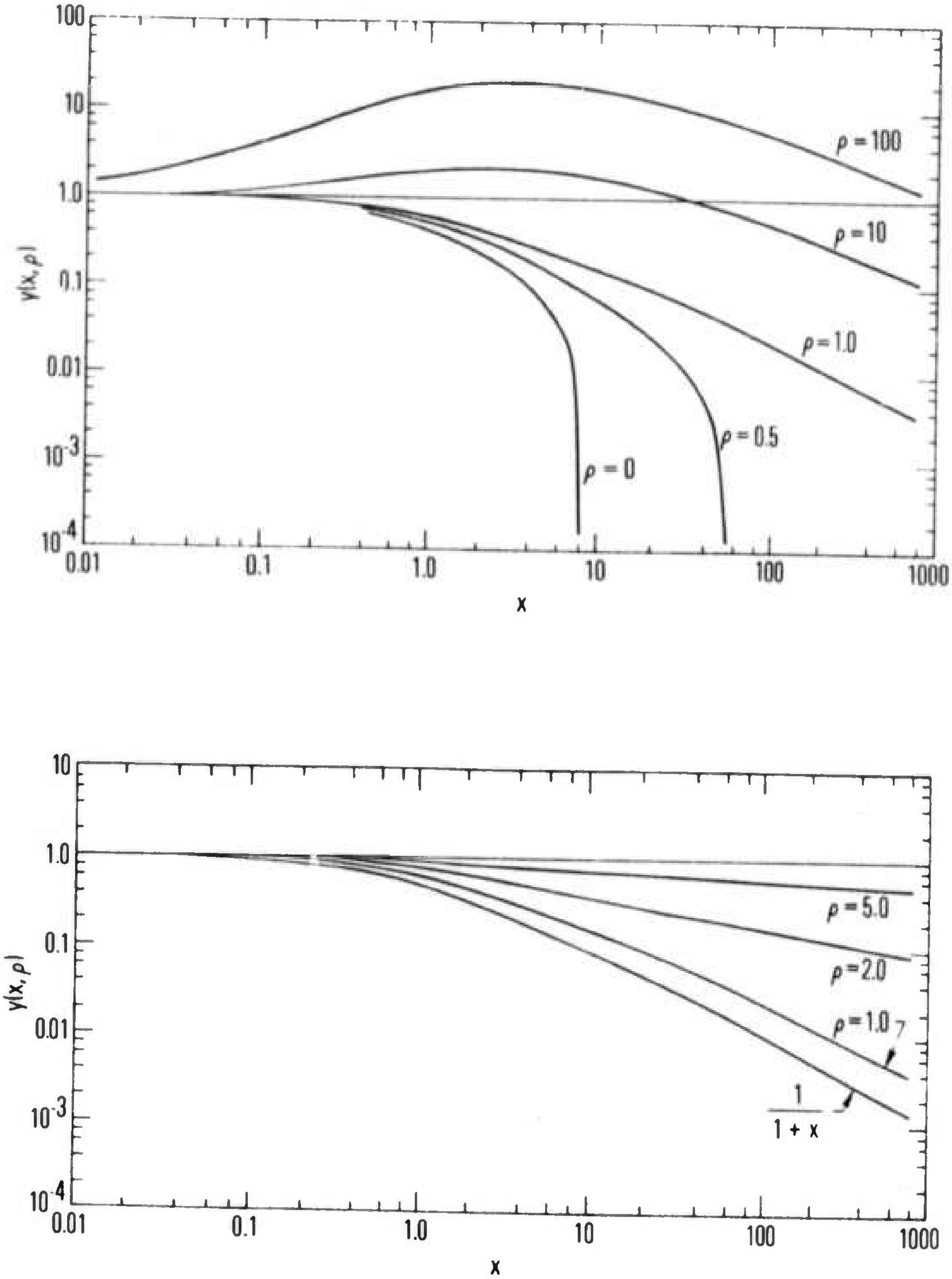
x	ρ						
	5.00E-1	1.00E+0	1.25E+0	1.50E+0	2.00E+0	3.00E+0	5.00E+0
1.0E-2	0.99114E+0	0.99299E+0	0.99380E+0	0.99450E+0	0.99556E-0	0.99686E-0	0.99805E-0
2.0E-2	0.98243E+0	0.98608E+0	0.98770E+0	0.98907E+0	0.99119E-0	0.99377E-0	0.99613E-0
5.0E-2	0.95723E+0	0.96603E+0	0.96995E+0	0.97329E+0	0.97844E-0	0.98474E-0	0.99053E-0
1.0E-1	0.91803E+0	0.93460E+0	0.94207E+0	0.94846E+0	0.95836E-0	0.97050E-0	0.98168E-0
2.0E-1	0.84868E+0	0.87828E+0	0.89190E+0	0.90367E+0	0.92200E-0	0.94464E-0	0.96560E-0
5.0E-1	0.69248E+0	0.74750E+0	0.77433E+0	0.79799E+0	0.83553E-0	0.88273E-0	0.92692E-0
1.0E+0	0.53069E+0	0.60490E+0	0.64392E+0	0.67937E+0	0.73706E-0	0.81132E-0	0.88198E-0
2.0E+0	0.36239E+0	0.44564E+0	0.49455E+0	0.54100E+0	0.61957E-0	0.72444E-0	0.82669E-0
5.0E+0	0.18622E+0	0.25945E+0	0.31182E+0	0.36593E+0	0.46452E-0	0.60548E-0	0.74932E-0
1.0E+1	0.10301E+0	0.15883E+0	0.20643E+0	0.25986E+0	0.36450E-0	0.52442E-0	0.69490E-0
2.0E+1	0.54426E-1	0.92598E-1	0.13195E+0	0.18053E+0	0.28411E-0	0.45506E-0	0.64664E-0
5.0E+1	0.22544E-1	0.43173E-1	0.70728E-1	0.10995E+0	0.20490E-0	0.38046E-0	0.59211E-0
1.0E+2	0.11408E-1	0.23642E-1	0.43542E-1	0.75384E-1	0.16102E-0	0.33469E-0	0.55672E-0
2.0E+2	0.57384E-2	0.12759E-1	0.26649E-1	0.51787E-1	0.12735E-0	0.29611E-0	0.52535E-0
5.0E+2	0.23038E-2	0.55566E-2	0.13871E-1	0.31709E-1	0.94277E-1	0.25368E-0	0.48873E-0
1.0E+3	0.11533E-2	0.29368E-2	0.84594E-2	0.21992E-1	0.75569E-1	0.22668E-0	0.46396E-0
2.0E+3	0.57700E-3	0.15432E-2	0.51628E-2	0.15322E-1	0.60853E-1	0.20319E-0	0.44126E-0
5.0E+3	0.23088E-3	0.65458E-3	0.26937E-2	0.95640E-2	0.45971E-1	0.17650E-0	0.41390E-0
1.0E+4	0.11546E-3	0.34067E-3	0.16503E-2	0.67263E-2	0.37322E-1	0.15906E-0	0.39491E-0
2.0E+4	0.57731E-4	0.17676E-3	0.10132E-2	0.47468E-2	0.30380E-1	0.14358E-0	0.37718E-0
5.0E+4	0.23093E-4	0.73960E-4	0.53344E-3	0.30083E-2	0.23224E-1	0.12570E-0	0.35546E-0
1.0E+5	0.11547E-4	0.38163E-4	0.32916E-3	0.21370E-2	0.18996E-1	0.11383E-0	0.34018E-0

a Gaussian quadrature formula was used in place of the Wedle formula. The values are accurate to 1 part in 10^5. Curves of $y(x, \rho)$ according to the Lindquist-Simmons result [Eq. (11)] and the Curtis-Godson expression

$$y(x, \rho) = (2 - \rho) \frac{dH(x)}{dx} + (\rho - 1) \frac{H(x)}{x} \tag{14}$$

are displayed in Fig. 2.

For both the Lorentz and Doppler cases, the transition from the small x to the large x behavior of $y(x, \rho)$ is more gradual for the modified inverse distribution than for the exponential distribution (compare Fig. 1 of this paper with Fig. 2 of Ref. 1 for the Lorentz case and Fig. 2 of this paper with Fig. 4 of Ref. 1 for the Doppler case). This effect is due to the greater weight placed on weak lines by the inverse distribution than by the exponential distribution.



(b)

Fig. 2. Equivalent Width Derivative Function for an Exponential-Tailed Inverse Intensity Distribution of Doppler Lines: (a) Curtis-Godson Approximation; (b) Lindquist-Simmons Approximation

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