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EFFECT OF SLEWING ON THE FREQUENCY SPECTRUM OF A LASER BEAM IN A TURBULENT MEDIUM

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In this report, we have studied the effect of beam slewing on the amplitude and phase spectra of a laser beam propagating in a turbulent medium.

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Effect of Slewing on the Frequency Spectrum of a Laser Beam in a Turbulent Medium

1. INTRODUCTION

In this report, we study the effect of slewing on the temporal frequency spectrum of the scintillations of a laser beam propagating through a turbulent medium. We shall assume that the turbulent medium is characterized by the index of refraction

$$n(\underline{r}, t) = 1 + n_1(\underline{r}, t) \ , \qquad (1)$$

where the fluctuation $n_1 << 1$. It is further assumed that n_1 varies sufficiently slowly in comparison with the time required for the signal traversal.

2. SLEWING LASER BEAM ANALYSIS

Let us consider a laser beam propagating in the x direction in a turbulent medium which, in the absence of any slewing of the beam, has a transverse wind velocity given by $\underline{V}(\underline{x})$. If we assume that the flow of the turbulence is frozen, so that the medium doesn't change within the time of the signal propagation through it—except for the shift due to the wind—it is easily seen that

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$$n_1(\underline{r}, t) = (\underline{r} - \underline{V}t, 0) \ . \qquad (2)$$

Now let us suppose that the beam is slewed at a constant angular rate ω_s. Then, in a coordinate system moving with the slewing beam, it appears that the turbulent eddies are moving relative to the beam with a transverse velocity

$$v_e = V(x) \cos \theta + \omega_s x \ , \qquad (3)$$

where θ is defined in Figure 1 and ω_s is positive when the beam is slewing in a direction <u>opposite</u> to that of the ambient wind. In this case, for a coordinate system moving with the slewing beam we have

$$n_1(\underline{r}, t) = n_1(\underline{r} - \underline{v_e}t, 0) \ . \qquad (4)$$

The electric field in a coordinate system moving with the slewing beam can be written quite generally as

$$E(\underline{r}, t) = A \, e^{\chi(\underline{r}, t) + iS(\underline{r}, t)} \ , \qquad (5)$$

where χ is the logarithm of the amplitude, S is the phase fluctuation, and A is a determinate quantity, which is independent of the turbulent fluctuations. The

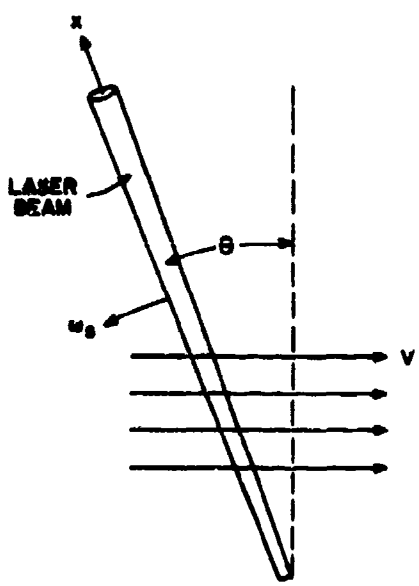

Figure 1. Geometry of a Slewing Laser Beam

correlation functions $B_\chi(\underline{r}_1, \underline{r}_2, \tau)$, $B_s(\underline{r}_1, \underline{r}_2, \tau)$ of the log amplitude and phase fluctuations are defined as

$$B_\chi(\underline{r}_1, \underline{r}_2, \tau) = \langle \chi(\underline{r}_1, t)\, \chi(\underline{r}_2, t+\tau) \rangle \qquad (6)$$

and

$$B_s(\underline{r}_1, \underline{r}_2, \tau) = \langle S(\underline{r}_1, t) S(\underline{r}_2, t+\tau) \rangle \ , \qquad (7)$$

where $\langle\ \rangle$ denotes an ensemble average. We assume that the transmitter field at x = 0 is given by (collimated beam)

$$E(x = 0, \underline{\rho}) = e^{-(\rho^2/W_o^2)} \qquad (8)$$

where $\underline{\rho} = (y, z)$. It can then be show . that for a point located at the center of the beam at a distance x = L from the transmitter, the correlation functions B_χ and B_s are given by

$$B_{\chi \atop s}(\tau) = \pi^2 \int_0^L d\eta \int_0^\infty \kappa d\kappa\, J_0(\kappa |\underline{v}_e \tau|) \left[2|H|^2 \pm H^2 \pm H^{*2}\right] \Phi(\kappa) \ , \qquad (9)$$

where the upper signs are for B_χ and the lower for B_s. Also

$$|H|^2 = k^2 \exp\left\{\frac{-\gamma_2 (L-\eta)\kappa^2}{k}\right\},$$

$$H^2 = -k^2 \exp\left\{-i(\gamma_1 - i\gamma_2)\ \frac{(L-\eta)\kappa^2}{k}\right\},$$

$$\gamma_1 = \frac{1 + \alpha_1^2 L\eta}{1 + \alpha_1^2 L^2} \ ,$$

$$\gamma_2 = \frac{\alpha_1 (L-\eta)}{1 + \alpha_1^2 L^2} \ ,$$

$$\alpha_1 = \frac{\lambda}{\pi W_o^2} \ ,$$

1. Ishimaru, A. (1969) Fluctuations of a focused beam wave for atmospheric turbulence probing, Proc. IEEE 57:407-414.

λ = signal wavelength,

and

$k = 2\pi/\lambda$.

The function $\Phi(\kappa)$ is the wavenumber spectrum of the index-of-refraction fluctuations and is given by

$$\Phi(\kappa) = \frac{0.033\, C_n^2 \exp^{(-\kappa^2 \ell_o^2)}}{[\kappa^2 + L_o^{-2}]^{11/6}} , \tag{10}$$

where L_o is the outer scale size of the turbulent eddies, $\ell_o = 2\pi\ell_o$ is the inner scale size of the eddies, and C_n^2 is the index of refraction structure constant. Equation (9) is valid only when the turbulence is weak. In particular, we require that

$$\sigma_1^2 = 1.23\, k^{7/6} C_n^2 L^{11/6} << 1 .$$

We further require that $L << c/\omega_s$ where c is the speed of light.

By observing the result in Eq. (9) it is evident that for frozen-in turbulence, the variance of the log-amplitude and phase fluctuations is independent of both the windspeed and the slewing rate. That is $\langle \chi^2 \rangle = B_\chi(0)$ and $\langle S^2 \rangle = B_s(0)$ are independent of $\underline{v}_e$ as is seen by setting $\tau = 0$ in Eq. (9). The temporal frequency spectrum of the fluctuations, however, does depend on both the wind velocity and the slewing rate. The temporal frequency spectra $W_\chi(\omega)$ and $W_s(\omega)$ are given by

$$\begin{Bmatrix} W_\chi(\omega) \\ W_s(\omega) \end{Bmatrix} = \int_0^\infty d\tau \cos \omega\tau \begin{Bmatrix} B_\chi(\tau) \\ B_s(\tau) \end{Bmatrix} . \tag{11}$$

If Eq. (9) is used in (11) we get, after performing the integration on τ, and assuming $\underline{V}(\underline{x})$ is independent of position and that the turbulence is homogeneous

$$\left\{\begin{matrix} W_\chi(\Omega) \\ W_s(\Omega) \end{matrix}\right\} = A_o \int_0^1 d\xi \int_{\left(\frac{\Omega}{1+\gamma\xi}\right)^2}^{\infty} \frac{ds \exp\left[-s(1-\xi)^2\left(\frac{a}{1+a^2}\right)\right]}{[s+F]^{11/6}\ [(1+\gamma\xi)^2 s + \Omega^2]^{1/2}}$$

$$\times \left\{\begin{matrix} \sin^2\left[\frac{s(1-\xi)}{2}\left(\frac{1+a^2\xi}{1+a^2}\right)\right] \\ \cos^2\left[\frac{s(1-\xi)}{2}\left(\frac{1+a^2\xi}{1+a^2}\right)\right] \end{matrix}\right. , \qquad (12)$$

where

$$v_o = V \cos\theta ,$$

$$F = \frac{L}{kL_o^2},$$

$$a = \frac{\lambda L}{\pi W_o^2},$$

$$\Omega = \frac{\omega}{v_o}\left(\frac{L}{k}\right)^{1/2} ,$$

$$A_o = 0.528\ \sigma_1^2 \left(\frac{L}{kv_o^2}\right)^{1/2} ,$$

and

$$\gamma = \frac{\omega_s L}{v_o}$$

Equation (12) has been evaluated numerically for the frequency spectra $W_\chi(\Omega)$ and $W_s(\Omega)$ for a number of different values of the slewing parameter γ; the results are presented in Figures 2 and 3. We note, as expected, that the spectral width of the amplitude and phase fluctuations increase as the slewing rate in the direction opposite to the turbulent flow is increased. Therefore, even though slewing does not affect the values of

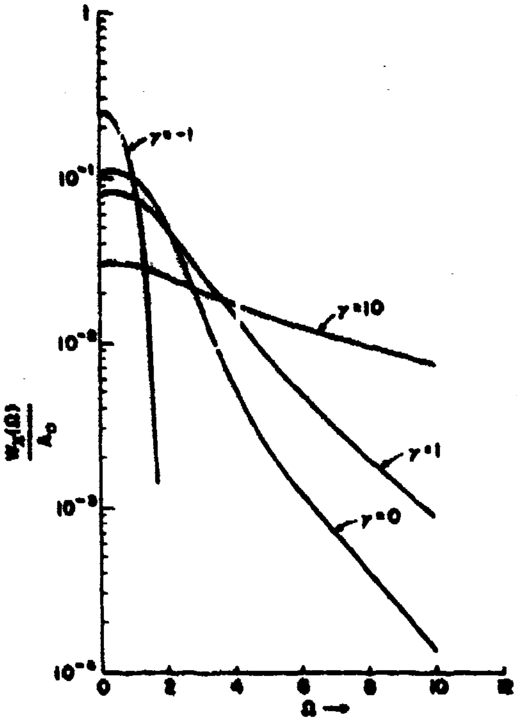

Figure 2. Temporal Frequency Spectrum of the Amplitude Fluctuations at the Center of a Laser Beam Propagating in Turbulence for the Case When $F = 10^{-7}$ and $a = 1$. Note that γ is positive when the beam is slewed in the direction opposite to V

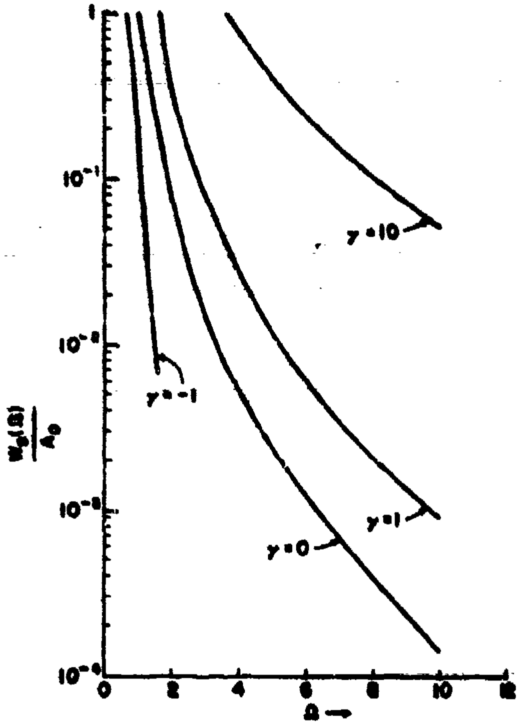

Figure 3. Temporal Frequency Spectrum of the Phase Fluctuations at the Center of a Laser Beam Propagating in a Turbulent Medium for the Case When $F = 10^{-7}$ and $a = 1$. Note that γ is positive when the beam is slewed in the direction opposite to V

$$\langle \chi^2 \rangle = \frac{1}{\pi} \int_{-\infty}^{\infty} W_\chi(\omega) d\omega$$

and

$$\langle S^2 \rangle = \frac{1}{\pi} \int_{-\infty}^{\infty} W_s(\omega) d\omega ,$$

it does affect the shape of W_χ and W_s.