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COMBAT PROCESSES AND MATHEMATICAL MODELS OF ATTRITION

Alan F. Karr

September 1975



INSTITUTE FOR DEFENSE ANALYSES
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PREFACE

There are two chapters in this paper. The first presents a rather general and heuristic scheme for classifying, from the standpoint of attrition modeling, physical combat processes, while the second describes what we feel to be preferred mathematical models of attrition for a number of combat situations. The general classification scheme clarifies, we hope, the distinctions that can be made among different combat situations, as well as suggesting for inclusion in attrition models factors not currently treated. In effect, each of the large family of combat situations delineated by our classification scheme should be described by its own attrition model (otherwise there would be no reason, in the modeling context, to distinguish such situations). Unfortunately, our knowledge of theater-level attrition-modeling is currently such that only one of the several classifications presented is actually operative. Chapter II presents groupings of a number of combat situations according to one of the classifications of Chapter I and presents a proposed attrition model for each group.

The context of this study is that of computerized, theater-level combat models such as IDAGAM I (Reference [1]), in which the primary outputs of interest are casualties (to personnel and to equipment) and FEBA position. In particular, this context influences the level of resolution of the distinctions made in the characterization scheme. Further, only nonnuclear combat is considered. We have sought, however, to make the characterization structure sufficiently general to include ground, air, and naval combat.

The author is indebted to Drs. Lowell Bruce Anderson, Jerome Bracken, and Jerry Blankenship of IDA for many helpful discussions during the preparation of this paper. Their suggestions have improved its content.

Chapter I

CLASSIFICATION OF COMBAT SITUATIONS

In this chapter, we present a scheme for the classification and characterization of combat processes. The categories are largely heuristic and connote mostly an attempt to clarify the ways in which one can think about combat processes. Some secondary purposes are to elucidate the assumptions made in choosing a mathematical model of combat attrition, to suggest areas for further research, and to stimulate development of more precise classifications. Indeed, a good classification of combat processes is the key step in choosing attrition models on the basis of their underlying assumptions.

The main purpose of mathematical models of attrition is to describe--as accurately as possible, subject to the constraints of the context within which we are working--the time history of the position, size, and structure of the two opposing forces. Seen in this light, actual losses of personnel, equipment, and supplies are an almost secondary output of the model.

Following tradition, we refer to the two opposing sides as Red and Blue. The FEBA represents the line separating the opposing forces and is presumed to be a simple, continuous curve. The calculation of FEBA position (and its variation over time) is a principal goal of many combat models. (One can also be interested in the modeling of combat interactions--guerrilla warfare and air-to-air combat being examples--in which there exists no FEBA at all; the present comments, but not the whole paper, are not relevant to this case). In IDAGAM I, orthogonal to the FEBA there exists for each side a partition of its territory into some sectors (not

necessarily the same number on both sides), and the level of resolution of the model is to maintain time evolving totals of the number of each type of resource in each sector. Though the existence of rear regions of some sort is also possible, this is of less interest and importance in terms of computation of the immediate effects of combat interactions.

The role of sectors is nontrivial: they are--as surrogates for more detailed positional information--determinants of *interaction eligibility*. That is, the sectors provide as inputs to mathematical models the numbers of resources participating in interactions that are not (in general) further divisible, except possibly for purposes of modeling and computation. The interactions determined by these sectors proceed independently of one another. Combat losses are then summed over the various sectors to obtain total losses. Movement of the FEBA is computed separately in each sector, independently of the others, but subject to smoothing constraints.

For the purposes of characterizing combat situations, we limit attention to indecomposable interactions of the type discussed in the preceding paragraph. Hence, there is no explicit representation at all of the positions of the resources involved in the interaction, and all resources present are assumed to be vulnerable (to some equal extent) to one another.

In Figure 1, we present a general set of descriptive characteristics of combat processes. (A discussion follows the figure.) The choice of these characteristics is based largely on their use in determining the possible applicability of certain mathematical models of attrition to the physical processes being studied and, in particular, that of the four models discussed in Chapter II.

We remark that the classification scheme is also intended to be valid for processes (e.g., naval combat) not treated explicitly in IDAGAM I, provided that the levels of resolution and aggregation be the same.

- I. Qualitative Characteristics
 - A. Relative positions of the two opposing forces
 - 1. On opposite sides of a well-defined FEBA
 - 2. Essentially completely intermingled
 - B. Nature of the interaction
 - 1. Each side searching to find the other
 - 2. One side maintaining a barrier through which the other attempts to penetrate
 - 3. One side attempting to destroy passive targets on the other
 - C. Objectives of the two sides
 - 1. Infliction of casualties
 - 2. FEBA movement
 - 3. Limitation of losses and loss rates of own resources
- II. Principal Quantifiable Characteristics
 - A. Scale
 - 1. Numbers of objects of each type participating in the interaction and numbers of newly arriving objects as functions of time
 - 2. Geographic dimensions over which the interaction takes place
 - 3. Length of time
 - B. Interaction independent characteristics of weapons
 - 1. Range
 - 2. Mobility
 - 3. Method and rate of engagement initiation
 - 4. Effect of shots: single- or multiple-kill and (in the latter case) how many; also various suppressive effects
 - 5. Personnel requirements
 - 6. Supply requirements

(concluded on next page)

Figure 1. CHARACTERIZATION OF COMBAT PROCESSES

Figure 1 (concluded)

- III. Important factors that are possibly neither quantifiable nor easy to deal with qualitatively
 - A. Organizational characteristics
 - 1. Command and control
 - 2. Communications
 - 3. Morale
 - B. Environmental factors
 - 1. Weather
 - 2. Terrain
 - C. Synergistic effects
 - D. Exogenous events

Some remarks on the classification scheme are in order. It seems rather clear that combat situations in which the two sides are separated by a clearly defined FEBA are qualitatively different from those in which they are quite intermingled (e.g., air-to-air combat or guerrilla warfare). No models yet developed, however, appear to do a good job of making this distinction. In the theater-level context (for land combat at least), a FEBA generally exists.

Category I.B ("Nature of the interaction") is crucial. In fact, it forms the basis for deciding which of the four models discussed in Chapter II is applicable to a given combat process. In other words, the state of mathematical models of attrition is such that only this part of the classification is operative.

Situation (I.B.1) is characterized by symmetry: elements of each side are seeking to engage those of the other. It is also an ongoing process, as opposed to that of barrier penetration (I.B.2), which occurs over a relatively short period of time and in which one side seeks to evade the other (which is

seeking to detect and engage it). The attack of passive targets (I.B.3) is self-explanatory.

Though objectives of the two sides are important, we seem not to know how to incorporate them into models.

The quantifiable characteristics listed under II.A ("Scale") are self-explanatory. Geographical scale is relevant even in the absence of explicit information concerning positions (indeed, more so in this case); it clearly affects the values of certain input parameters.

"Interaction independent characteristics of weapons" is intended to connote those properties that are the same for all interactions in which a given (type of) weapon is involved and can be measured in a noncombat situation. In some sense, a weapon possesses no truly interaction independent characteristics except physical dimensions; yet it seems reasonable that the characteristics listed (except possibly 3) are also roughly interaction independent. It turns out that the present state of attrition modeling requires that method and rate of engagement initiation be taken to be interaction independent (see Chapter II for details).

The factors in Category III are important, but no model yet devised treats any of them adequately. IDAGAM I makes at least an attempt to include synergistic effects. Weather is clearly a stochastic factor that cannot be handled accurately until more rigorous stochastic models are constructed. The same is true of exogenous events (e.g., political events or combat results in another theater). There exists at present only the alternative of treating such events (whose occurrence has a decisive and possibly overwhelming effect on the outcome of the combat) parametrically.

Chapter II

SOME RECOMMENDED ATTRITION MODELS

We now classify a number of combat situations into the three categories :

- (I.B.1) Search and engagement by both sides
- (I.B.2) Barrier penetration
- (I.B.3) Attack of passive target

and present a recommended form of attrition model for each of them.

The combat situations are classified and presented in Figure 2, which is not complete and to which we hope others will add.

For processes of class (I.B.1), we recommend a mixed-mode stochastic Lanchester process of the type described in Reference [5] (see also Reference [3]). For barrier processes, we recommend the new model derived in Reference [2]. For processes of attack of passive targets, we recommend either single-shot or multiple-shot attrition models of binomial form, as discussed in Reference [4]. All these models share the feature of being derived from unambiguous probabilistic hypotheses in a rigorous manner. Each model provides at least an approximation to the true expected attrition of each type of weapon, although none properly handles the replacement of random variables by their expectations for use as inputs in iterative calculations--except possibly the barrier penetration model, which is not yet implemented in a computerized simulation.

- (I.B.1) Processes of mutual search and engagement
 - (1) Ground combat with objectives of either FEBA movement or casualties
 - (2) Ground-air combat
 - (3) Aircraft combat for maintenance of air superiority
 - (4) Combat among submarines
- (I.B.2) Processes of barrier penetration
 - (1) Bomber aircraft through interceptors
 - (2) Aircraft through SAMs and AAA
 - (3) Submarines through minefields
 - (4) Submarines through submarine barriers
 - (5) Submarines through search aircraft
 - (6) Submarines through convoy escort ships
- (I.B.3) Processes of attack of passive targets
 - (1) Attacks of various targets by aircraft
 - (2) Attacks of merchant ships by submarines

Figure 2. CLASSIFICATION OF CERTAIN COMBAT SITUATIONS

The analysis leading to the mixed-mode (or, as we have called it, generalized) Lanchester attrition process (described in Reference [5]) is relevant to the previously rather obscure distinction between Lanchester square-law and Lanchester linear-law combat. Lanchester (Reference [6]) himself sought to distinguish the two according as the numerically superior side is able to make full use of its superiority (square law) or not (linear law). This is one interpretation of the distinction as we have come to understand it, at least in the case of homogeneous forces. For heterogeneous forces, however, the problem is more subtle; and we feel that the explanation to be presented below is the most cogent yet constructed.

In Reference [5], weapons are grouped into four classes on the basis of two qualitative distinctions. One of these

is a single-kill or multiple-kill (per shot) distinction (with which we shall not concern ourselves further here; the reader is referred to Reference [5] for more information). The second distinction is based on the qualitative nature of the rates of engagement initiation (i.e., rates at which shots are fired). A particular type of weapon is said to have *square-law engagement initiation* if the mean total rate at which it engages opposition weapons is independent of the numerical size of the opposition force (as well as of its precise structure). In other words, the mean engagement rate depends only on the type of firing weapon. Physical assumptions compatible with this behavior are discussed in References [3, pp. 29-40] and [5]. The possibility that the rates at which particular types of opposition weapons are engaged may depend on the numbers of various kinds of opposition weapons that are present is not inconsistent with our earlier statement, which concerned only the mean total rate of engagement.

On the other hand, a type of weapon is said to possess *linear-law engagement initiation* if the mean total rate at which it engages elements of target force $y = (y_1, \dots, y_n)$ --assuming the opposition has n different types of weapons--is of the form $\sum_{j=1}^n \alpha_j y_j$, where the α_j are nonnegative constants. A word of warning is in order here concerning our usage of the term "mean rate." We intend that it be interpreted in the sense of the infinitesimal generator of a continuous time Markov process (i.e., the instantaneous rate at which the event in question tends to occur--given the current state of the process). We refer the reader to Reference [3] for further details. A physical interpretation of linear-law engagement initiation can be found in Reference [3, pp. 41-48, 101-122]. In particular, the rate at which opposition weapons of any given type are engaged is directly proportional to the number of weapons of that type currently surviving.

It is also true, however, that for square-law engagement the rate of engagement *of a particular* type j of opposition weapon is proportional to the number of weapons of that type present. But in this case the constant of proportionality is of the form β/y^* where y^* is the total number of opposition weapons present, whereas in the linear-law case it is simply a constant α_j . Note that β is independent of the type j of the target weapon; both β and the α_j , of course, depend on the type of the engaging weapon. Table 1 clarifies this rather important distinction: y_j is the number of opposition type- j weapons present; and y^* is the total number of opposition weapons present.

Table 1. ENGAGEMENT RATES FOR LANCHESTER PROCESSES

Engagement Initiation	Type- j Weapons	All Opposition Weapons
Linear Law	$\alpha_j y_j$	$\sum_{j=1}^n \alpha_j y_j$
Square Law	$\beta \frac{y_j}{y^*}$	β

From this analysis, the first--and possibly most important--conclusion to be drawn is that the square-law/linear-law distinction *applies not to the combat as a whole but to individual types of weapons*. In other words, while a particular type of weapon may be said to have square- or linear-law engagement initiation, the combat itself cannot be said to possess either property. Even if all weapons present belong to one of the two classes (in this case, one would have essentially Process S3 or Process L3--see Reference [3]), it is still inaccurate to say that the combat itself is of that type. The distinction simply is not of that nature; it applies only to individual types of weapons.

It then becomes quite important to determine whether a particular type of weapon possesses square-law or linear-law engagement initiation. We believe that Lanchester's original differentiation is fairly close to the truth. A weapon type has square-law engagement initiation if all weapons of that type are able simultaneously to bring their fire to bear on the opposition. Two ways in which this simultaneity can be envisioned are that (1) shots are fired, at a rate determined only by the shooting weapon, at an area in which the opposition is known (or thought) to be located; or (2) weapons are mobile and push forward in such a way as to maintain a rate of contacts with the enemy which is independent of the number of enemy weapons present. In Reference [3], the former interpretation is adopted; the latter is due to L. B. Anderson.

On the other hand, linear-law engagement initiation arises when weapons of the type under consideration must engage opposition weapons essentially on a one-on-one basis. Probably the easiest way to envision this form of engagement is that of Reference [3]: each given opposition weapon requires an exponentially distributed random time to detect, different opposition weapons are detected independently, and an engagement occurs if and only if an opposition weapon is detected. (See Process L3 in Reference [3, pp. 47-48, and 115-122] for details and further interpretations; Processes L1 and L2 of Reference [3] are also relevant.)

The problem of deciding whether a particular type of weapon has square-law or linear-law engagement initiation seems to us, in terms of attrition modeling, crucial and difficult. The implications in terms of computed levels of attrition and FEBA movement are likely to be substantial--as is confirmed by experiments with simplified homogeneous models (essentially discretized versions of Lanchester's original differential equations). Hence this classification should not be undertaken

lightly or carelessly, as it may have overwhelming effects on the outputs of combat simulation models. For certain types of weapons--e.g., artillery (square law) or small arms (linear law)--the choice seems fairly clear. But for some other types (e.g., tanks), the choice is not at all clear. For example, it appears that to which category a tank belongs may be the result of tactical decisions by the two sides, may change during the course of a battle, and is more properly an output of the attrition model rather than a prescribed input. We cannot refute these criticisms except by noting that no attrition model yet devised addresses any of the difficulties raised. The possibility that engagement initiation is (in its qualitative nature) the result of tactical decisions is particularly interesting, however; the weaker side would seek to create linear-law engagement initiation, and the stronger side would desire square-law initiation.

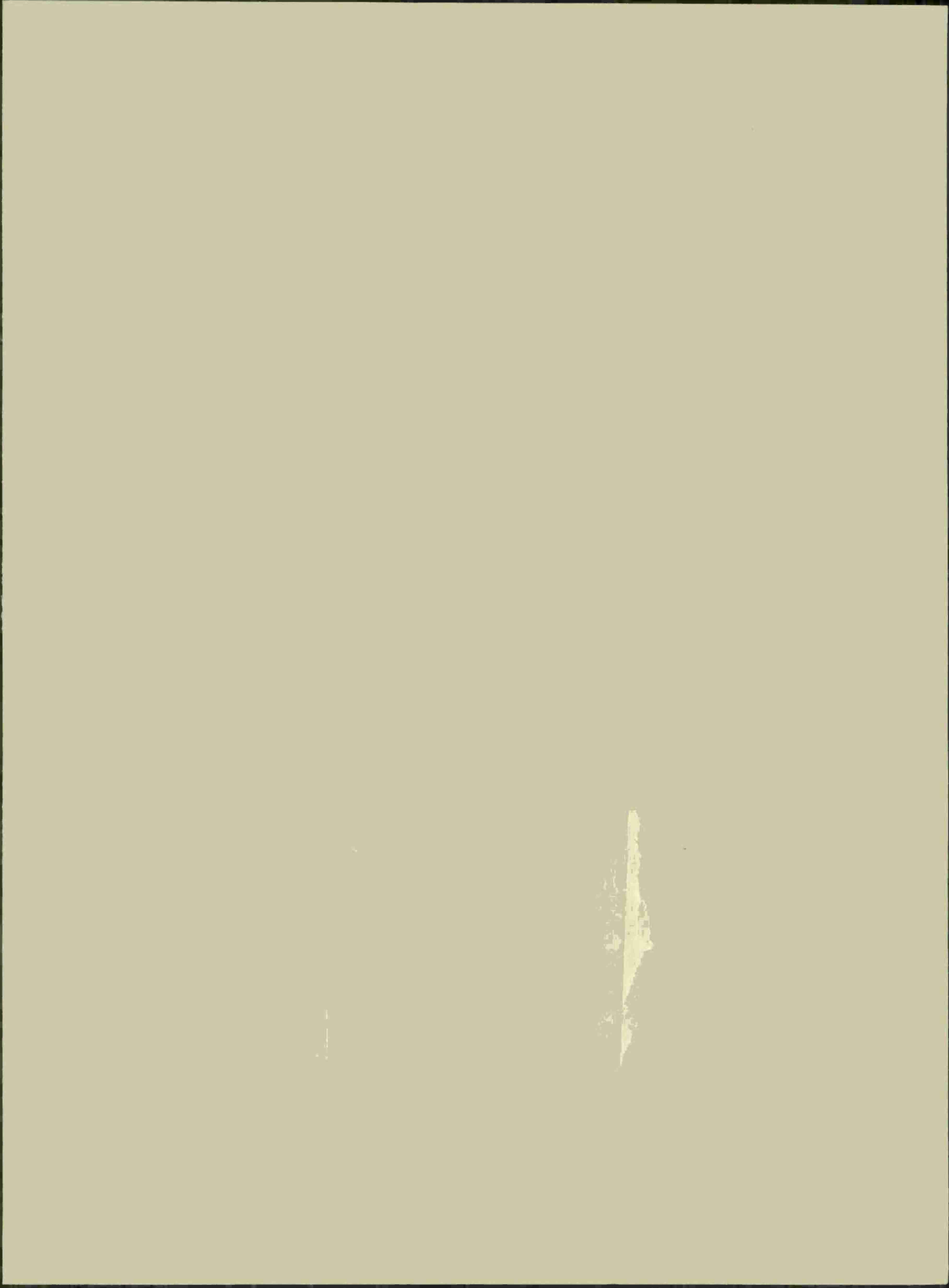
We conclude with some general remarks on the modeling of attrition in combat processes. It has always been our contention that no attrition model should be used that cannot be rigorously derived from a set of precise and unambiguous assumptions. There are several reasons for desiring rigorous models. First of all, assumptions are required to determine which model "best" fits a given physical situation--especially in the absence of data for testing models in terms of predictive capabilities. Clearly stated probabilistic assumptions can be tested statistically as well as judgmentally, though such testing involves the same data problems as testing predictive capability.

Moreover, one can understand the model itself only by understanding the underlying assumptions. In addition, the assumptions determine the qualitative nature of the input parameters required for computational implementation.

Finally, clearly stated assumptions force the model user to be aware of the approximations and errors implicit in the use of any mathematical model.

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