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A NOTE ON THE OBJECTIVE JUNCTION FOR THE PILOT MODEL

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BY

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It is shown that if the objective for a linear programming model is chosen to be the weighted sum of people at different consumption levels, where the weights are equal to consumption levels measured in base-year prices, then the prices of the physical flow model will be comparable to base year prices in that people given the same income less taxes and savings will be able to purchase the same bill of goods; moreover prices will remain invariant if they use their demand functions to buy a different bill of goods. It is also shown that if industries use their respective production to change their inputs to maximize production, this will also increase the objective gross national cunsumption.

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A Note on the Objective Function for the PILOT Model

by

George B. Dantzig

Description of the PILOT model for studying the interaction of the detailed Energy Sector with the Economy is found in [1], [2].

In Part I, below, the model is stated without either population or labor constraints. The effect of adding these constraints is developed in Parts II and III; the use of weighted income distributions is discussed in Part IV.

PART I

Resource Capacity Constraints (1), Consumption Constraints (2):

Find production levels $X \geq 0$ and sizes of consumer groups $u \geq 0$:

		PRICES
(1)	$DX \leq r$	ρ
(2)	$AX - Cu \geq b$	$-\pi$
(3)	$Mu = z\,(\text{Max})$	

where $Cu = \Sigma C^j u$. C^j is the consumption profile, u_j is the size, M_j the consumption level (in base year prices) of the j-th consumer group, and z is the Gross National Consumption (GNC). If P is the total population, the above relations omit the population constraint

(4) $$\Sigma u_j = P \;.$$

It <u>assumes</u> that the optimal solution will satisfy $\Sigma u_j > P$. If this turns out to be the case the solution is interpreted that the consumption bundles are each increased by the ratio of $(\Sigma u_j/P)$. However if it turns out $\Sigma u_j < P$, then (4) would be imposed.

Because of the capacity restrictions (1), one expects many consumer groups to have $u_j > 0$ in an optimal solution. Indeed, in general, if there are k tight capacity constraints, then there will be $k+1$ consumption patterns j such that the optimal prices π satisfy

$$\pi C^j = M^j \quad . \tag{5}$$

Thus prices π have about the same meaning as base year prices in terms of what money can buy. The net money flow (GNC) satisfies

$$GNC = (\rho r - \pi b) = \Sigma M_j u_j \quad . \tag{6}$$

We <u>assume</u> GNC is redistributed by some mechanism so that each consumer class j gets its share $M_j u_j$. See Part IV where a consumption income distribution mechanism is imposed.

<u>Production Function:</u> Suppose at some iteration of the Simplex Method $X_k > 0$ represents the level of production of industry k. It produces one unit of item k at price π_k. Since the k-th column is basic, it "prices-out" to zero or

(7) $$\pi_k = \sum_{j \neq k} (-a_{ik})\pi_j + \rho D^k, \qquad a_{kk} = 1,\ a_{ik} \leq 0,\ i \neq k\ .$$

The industry can be expected to substitute process $(\bar{D}^k, \bar{A}^k)$ for (D^k, A^k), if it can find one, such that

(8) $$\pi_k > \sum_{j \neq k} (-\bar{a}_{ik})\pi_j + \rho\bar{D}^k$$

because this will bring it more profits. But this criterion also states that new column k prices out positive and so would be a candidate to introduce into the basis in order to increase GNC. Thus profit maximization by industry leads to maximization of GNC by the economy.

Demand Function: Similarly the consumer j faced with prices π and income for consumption M_j will substitute $\bar{C}^j$ for C^j where

(9) $$\pi\bar{C}^j = M_j\ .$$

[It may also be possible to find substitute columns satisfying (9) for non-basic j even though for these j, $\pi C^j > M_j$.] If we bring a substitute column into the basis, however, it will cause no change in prices and no increase in GNC because the substitute column prices-out to zero, i.e., GNC remains at a maximum.

Utility Function: The above suggests that once Max GNC is achieved, a secondary objective can be introduced to increase utility within a consumer class.

Generalized Programming: The introduction of substitute columns $(\bar{D}^k, \bar{A}^k)$ or $\bar{C}^j$ to increase profits or utility (once Max GNC has been achieved) is a column generating scheme (i.e., generalized programming). Convergence proofs (based on the DW Decomposition Principle) exist.

PART II

Suppose the population constraint is tight or it is unacceptable to proportionally increase a consumption profile as assumed in I. In this case we impose

		PRICE
(4)'	$+\sum u_j = +P$	$+\pi_p$.

The price on (4)' could have either sign. [Thus if the optimal price on population $\pi_p > 0$, then allowing the population to decrease, would increase GNC. If $\pi_p < 0$, then allowing population to increase would increase GNC.]

The effect of a price π_p on the population constraint is to change consumption income from M_j to $M_j - \pi_p$. Assuming again that money for consumption is redistributed so that a person in these consumer groups j receive $(M_j - \pi_p)$ instead of M_j, we can use the demand function to generate a substitute column satisfying

$$(9)' \qquad \pi\bar{C}^j = M_j - \pi_p \quad .$$

PART III

If we impose in addition, a labor constraint

$$(10) \qquad -\ell X + \sum u_j \geq 0 \ , \qquad\qquad -\pi_\ell$$

and labor is tight so that $\pi_\ell > 0$, then the effect is to change consumption income from M_j to $M_j - \pi_p + \pi_\ell$. In this case the demand function is used to generate the substitute column $\bar{C}^j$ satisfying

$$(9)'' \qquad \pi\bar{C}^j = M_j - \pi_p + \pi_\ell \quad .$$

The adjustment $\pi_\ell - \pi_p$ to consumption income, can be interpreted as either inflationary of deflationary depending whether labor pressure or population pressure is more dominant; moreover the lower consumption groups see a greater change in prices relative to their income for consumption than do the higher ones.

PART IV

The model has the weakness in that it fails to reflect the fact that in the real economy the distribution of consumption incomes are likely to presist into the future due to salary structures within organizations. It may be worthwhile to replace C^1 by a weighted distribution $\hat{C}^1 = \Sigma C^j w_j$ and then to successively translate the distribution to the right, i.e., replace C^2 by $\hat{C}^2 = \Sigma C^{j+1} w_j$ and C^{k+1} by $\hat{C}^{k+1} = \Sigma C^{j+k} w_j$. The objective in this case would be replaced by

$$\text{(11)} \qquad \text{Max } \Sigma \hat{M}_k \hat{u}_k$$

where $\hat{M}_k$ is the average consumption level of $\hat{C}^k$. This allows consumption income to move upward for all consumption groups but with a more likely spread of consumption income. Demand functions could then be applied directly to the newly defined groups (using their consumption income averages) or preferably they could be applied to the previously defined groups and then weight w_j used to generate the substitute consumption patterns.

References

[1] George B. Dantzig, "Formulating a PILOT Model for Energy in Relation to the National Economy," SOL 75-10, Department of Operations Research, Stanford University, April 1975.

[2] George B. Dantzig and S.C. Parikh, "On a PILOT Linear Programming Model for Assessing Physical Impact on the Economy of a Changing Energy Picture," SOL 75-14R, Department of Operations Research, Stanford University, August 1975.