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LOW-FREQUENCY SOUND PROPAGATION IN A FLUCTUATING INFINITE OCEAN--II

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I INTRODUCTION

In an earlier report,^{1*} we described an approximation scheme, called the "supereikonal approximation," for calculating fluctuations produced by a specified spectrum of sound-speed fluctuations, in pressure signals from a CW source in the ocean. In that report, our analysis was confined to the case where the fluctuations were superimposed on a homogeneous isotropic ocean.

Here we should like to extend and improve on the earlier version in two major ways. First, we should like to rewrite the results obtained in Ref. 1 for the homogeneous case in a much simpler form, in which the geometrical optics limit valid for short ranges is transparent, and in which the corrections to this limit that became important at large ranges are easily visualized. We should also like to derive formulas for the fluctuations in the quantities of most immediate experimental interest--namely, the mean square phase of the received pressure, and the mean square logarithm of the amplitude of the received pressure.

Second, we should like to extend our treatment to the realistic case in which the background ocean is not isotropic and homogeneous, but contains a sound channel. As we shall see, we can handle this situation as well, with only a modest increase in complexity, as long as the fluctuations and ranges are such that separate ray paths are still identifiable. Our result will then apply to fluctuations in the signal associated with a given ray path.

Numerical results, and specific models of fluctuation spectra, will be treated in a separate report.

*References are listed at the end of the report.

II REVIEW OF SUPEREIKONAL APPROXIMATION

Let us begin our discussion by neglecting the effects of the sound channel. The problem of sound propagation in the presence of fluctuations superimposed on a homogeneous isotropic background is easier to set up and to visualize than when the fluctuations are superimposed on an inhomogeneous background, so it is conceptually advantageous to work out this case first. Then, as we shall see later, when the inhomogeneous background representing the sound channel is introduced, the analysis can be carried out very much as in the homogeneous case, and the resulting formulas, while geometrically more complex, are entirely analogous to those obtained in the simpler example.

Our analysis will be based on the supereikonal approximation, and it will be convenient at this point to give a brief review of the results of this method. The problem is to evaluate the pressure $p(\vec{x})$ at a point $\vec{x}$ produced by a point source (which, for convenience, we can take to have unit strength) located at the origin. The sound propagation from the source to the point $\vec{x}$ is through an isotropic homogeneous ocean, in which the sound speed is c , on which is superimposed a fluctuation in sound speed $\delta c(\vec{x})$ that is, of course, very small compared to c . Mathematically, then, the pressure satisfies the wave equation

$$(\nabla^2 + k^2)p(\vec{x}) = V(\vec{x})p(\vec{x}) \quad (2.1)$$

where $k = \omega/c$ for a source emitting sound of frequency ω , and where

$$V(\vec{x}) = 2k^2 \delta c(\vec{x})/c \quad (2.2)$$

The boundary condition associated with Eq. (2.1) is that as $\vec{x} \rightarrow 0$,

$$p(\vec{x}) \rightarrow \frac{1}{4\pi x} \quad (2.3)$$

(we shall denote $|\vec{x}|$ simply by x)

(We may comment that in the case of an inhomogeneous background, in Eq. (2.1), k is simply replaced by $k(\vec{x}) = \omega/c(\vec{x})$, where $c(\vec{x})$ is the sound speed in the inhomogeneous background.)

Equation (2.1) may be cast into integral form through the use of the outgoing-wave Green's function

$$(\nabla^2 + k^2)\Delta(\vec{x} - \vec{y}) = \delta^3(\vec{x} - \vec{y}) \quad ; \quad (2.4)$$

explicitly, we have

$$\Delta(\vec{x}) = \frac{1}{4\pi x} e^{ikx} \quad (2.5)$$

Then we may write, in place of Eq. (2.1),

$$p(\vec{x}) = \Delta(\vec{x}) + \int d^3y \Delta(\vec{x} - \vec{y}) V(\vec{y}) p(\vec{y}) \quad (2.6)$$

Iteration of this integral equation generates the perturbation series for $p(\vec{x})$, which is more convenient to write in Fourier-transformed form, as follows:

$$\begin{aligned} p(\vec{q}) = & \Delta(\vec{q}) + \Delta(\vec{q}) \int \frac{d^3\vec{q}_1}{(2\pi)^2} V(\vec{q}_1) \Delta(\vec{q} - \vec{q}_1) \\ & + \Delta(\vec{q}) \int \frac{d^3\vec{q}_1}{(2\pi)^3} \int \frac{d^3\vec{q}_2}{(2\pi)^3} V(\vec{q}_1) \Delta(\vec{q} - \vec{q}_1) V(\vec{q}_2) \Delta(\vec{q} - \vec{q}_1 - \vec{q}_2) \\ & + \dots \end{aligned} \quad (2.7)$$

where

$$\Delta(\vec{q}) = \frac{1}{q^2 - k^2 + i\epsilon}, \quad (2.8)$$

and, of course, where

$$p(\vec{q}) = \int d^3\vec{x} e^{-i\vec{q}\cdot\vec{x}} p(\vec{x}). \quad (2.9)$$

The supereikonal approximation now consists of neglecting all momentum transfer correlations in the perturbation series. That is, we approximate $(q - q_1 - q_2 - \dots - q_n)^2 - k^2 + i\epsilon$ by $q^2 - 2q \cdot (q_1 + q_2 + \dots + q_n)^2 + q_1^2 + q_2^2 + \dots + q_n^2 - k^2 + i\epsilon$, and neglect all terms of the form $\vec{q}_i \cdot \vec{q}_j$ when $i \neq j$. Note that the first approximation occurs in the second-order term in V . Once this simplification is made, the perturbation series can be summed exactly, and one obtains the result

$$p(\vec{x}) = \frac{\sqrt{-i\pi}}{8\pi^2} \int_0^\infty \frac{d\beta}{\beta^{3/2}} e^{i[\beta k^2 + \frac{x^2}{4\beta} + \beta I(\beta, \vec{x}) + i\epsilon]} \quad (2.10)$$

where

$$I(\beta, \vec{x}) = \int \frac{d^3\vec{q}}{(2\pi)^3} V(\vec{q}) \int_0^1 ds e^{-i[sq \cdot x + \beta s(1-s)q^2]} \quad (2.11)$$

This expression constitutes the supereikonal approximation to the pressure. The conditions under which it is valid are

$$k x \gg 1, \quad k L \gg 1$$

and

$$x \ll \frac{1}{k^2 L} \left(\frac{c}{\delta c} \right)^2. \quad (2.12)$$

(In fact, the last condition may well be too stringent.) Here L is the correlation length of the sound-speed fluctuations--i.e., the correlation function $C(\vec{x} - \vec{y}) \equiv \langle V(\vec{x}) V(\vec{y}) \rangle$ vanishes when $|\vec{x} - \vec{y}| \gtrsim L$.

It is worth noting that if, in Eq. (2.11), the $\theta s(1-s)q^2$ term is omitted from the exponent, we obtain

$$p(\vec{x}) = \frac{1}{4\pi x} c^{i \left(\sqrt{k^2 + \int_0^1 V(s\vec{x}) ds} \right) x}$$

which is the conventional WKB, or eikonal, or geometrical optics, approximation to the pressure. The primary virtue of the supereikonal form, therefore, is that it contains as limiting cases both the conventional eikonal and complete first-order perturbation-theory approximations.

While Eqs. (2.10) and (2.11) do constitute a closed-form solution for the pressure, the expressions are still a bit unwieldy, and further simplification of them is useful. To this end, let us evaluate the integral in Eq. (2.10) by stationary phase, keeping in mind that x and k are both large. The stationary phase point is β_0 , where

$$k^2 - \frac{x^2}{4\beta_0^2} + I(\beta_0, \vec{x}) + \beta_0 \frac{\partial}{\partial \beta} I(\beta, \vec{x}) \Big|_{\beta=\beta_0} - 3/2\beta_0 = 0.$$

From Eq. (2.11), we may estimate that

$$\beta_0 \frac{\partial}{\partial \beta} I(\beta, \vec{x}) \Big|_{\beta=\beta_0} \sim \frac{k^2}{L^2} \cdot \frac{\delta c}{c} \cdot \beta_0 ;$$

hence, if

$$x < k L^2 \frac{c}{\delta c} , \quad (2.13)$$

the stationary phase point is accurately given by the solution of the simpler equation

$$k^2 - x^2/4\beta_0^2 = 0$$

and is located at $\beta_0 = x/2k$. Thus we find

$$p(x) = \frac{e^{ikx}}{4\pi x} e^{\frac{ix}{2k} I\left(\frac{x}{2k}, \vec{x}\right)} . \quad (2.14)$$

To approximately evaluate the integral I in this expression, we recall that the supereikonal approximation should be exact to first order in V . This requirement then yields the result

$$p(\vec{x}) = \Delta(\vec{x}) \exp \frac{1}{\Delta(\vec{x})} \int d^3\vec{y} \Delta(\vec{x}-\vec{y}) V(\vec{y}) \Delta(\vec{y}) . \quad (2.15)$$

This expression is known as Rytov's approximation to the pressure. A direct derivation of it may be made by replacing the wave equation (2.1) by an equation for $\log[p(\vec{x})/\Delta(\vec{x})]$ and solving this to first order in V .² However the justification for the approximation is somewhat obscure in

this direct derivation; in the approach via the supereikonal technique, what is being left out is more clearly visualized.

In any event, depending on the validity of the criterion (2.13), one may use either the supereikonal expression (2.10) or the Rytov expression (2.15) to proceed further.* We shall use (2.15).

Let us write, then,

$$p(\vec{x}) = \Delta(\vec{x}) e^{X(\vec{x})} \quad (2.16)$$

where

$$X(\vec{x}) = \frac{1}{\Delta(\vec{x})} \int d^3\vec{x}' \Delta(\vec{x}-\vec{x}') V(\vec{x}') \Delta(\vec{x}') \quad (2.17)$$

For our homogeneous background case, where the Green's function Δ is given by Eq. (2.5), we have

$$X(\vec{x}) = \frac{1}{4\pi} \int d^3\vec{x}' \frac{x}{x' |\vec{x}-\vec{x}'|} e^{ik(x' + |\vec{x}-\vec{x}'| - x)} V(\vec{x}') \quad (2.18)$$

In concluding this section, and as an aside, let us comment on the geometrical optics limit of this expression. This limit results from an evaluation of $X(\vec{x})$ by the method of stationary phase. Provided that the Fresnel condition

$$x < k L^2$$

*

All of these statements, we remind the reader, are subject to the range constraints under which the supereikonal expression was derived in the first place.

is met, the stationary phase path in Eq. (2.18) is the straight line joining 0 to $\vec{x}$, and the stationary phase value of $X(\vec{x})$ is just

$$X(x,0,0) = \frac{i}{2k} \int_0^x dx' V(x',0,0) \quad , \quad (2.19)$$

which is immediately recognized as the correct geometrical optics expression for the phase. The analogous expression for the amplitude in geometrical optics is obtained by keeping the second-order transverse derivatives in V as well.¹

III FLUCTUATIONS IN A HOMOGENEOUS ISOTROPIC OCEAN

The quantities that it will be of interest to compute are the statistical averages $\langle X^2(\vec{x}) \rangle$ and $\langle |X(\vec{x})|^2 \rangle$. (We note that $\langle X(\vec{x}) \rangle = 0$, of course.) These are connected to phase and amplitude fluctuations $\langle \phi^2 \rangle$ and $\langle |\log p/p_0|^2 \rangle \equiv \langle A^2 \rangle$ by the relations

$$\langle \phi^2 \rangle = 1/2 \left(\langle |X|^2 \rangle - \text{Re} \langle X^2 \rangle \right) \quad (3.1)$$

and

$$\langle A^2 \rangle = 1/2 \left(\langle |X|^2 \rangle + \text{Re} \langle X^2 \rangle \right) \quad (3.2)$$

We will, in addition, obtain the cross correlation $\text{Im} \langle X^2 \rangle$.

Insofar as the sound-speed fluctuations, and hence the fluctuations in X , are gaussian, the pressure fluctuations are related to the statistical averages as well. We have

$$\langle p \rangle = p_0 e^{1/2 \langle X^2 \rangle}$$

$$\langle p^2 \rangle = p_0^2 e^{2 \langle X^2 \rangle}$$

$$\langle |p|^2 \rangle = |p_0|^2 e^{1/2 \langle (X+X^*)^2 \rangle}$$

where we write

$$p_0(\vec{x}) \equiv \Delta(\vec{x}) = \frac{e^{ikx}}{4\pi x}$$

as the received pressure from the unit point source in the absence of fluctuations.

Eventually, we will also be interested in correlations; these will involve averages such as $\langle X(\vec{x}_1)X(\vec{x}_2) \rangle$, etc., but we shall ignore these for now.

Let us first evaluate $\langle |X(\vec{x})|^2 \rangle$. For convenience, we shall choose $\vec{x}$ to lie along the x axis, so that $\vec{x} = (x, 0, 0)$. From the definition, Eq. (2.18), we have

$$\begin{aligned} \langle |X(\vec{x})|^2 \rangle &= \left(\frac{1}{4\pi} \right)^2 \int d^3\vec{y}_1 \int d^3\vec{y}_2 \frac{x^2}{y_1 |x - \vec{y}_1| y_2 |x - \vec{y}_2|} \\ &\quad e^{ik(y_1 + |x - \vec{y}_1| - x)} e^{-ik(y_2 + |x - \vec{y}_2| - x)} \\ &\quad c(\vec{y}_1 - \vec{y}_2) \end{aligned} \quad (3.3)$$

where we have introduced the correlation function

$$c(\vec{y}_1 - \vec{y}_2) \equiv \langle v(\vec{y}_1) v(\vec{y}_2) \rangle .$$

We assume C to be independent of $(y_1 + y_2)/2$ for this case of a homogeneous background.

It is convenient in Eq. (3.3) to shift to relative and center-of-mass coordinates. We define

$$\vec{y} = y_1 - y_2, \quad \vec{Y} = \frac{y_1 + y_2}{2} .$$

Then, if we assume that $C(\vec{y})$ cuts off for values of $y \geq L$ where $L \ll x$, we may expand in y/Y . Thus, Eq. (3.3) becomes

$$\langle |X(\vec{x})|^2 \rangle = \left(\frac{1}{4\pi} \right)^2 \int d^3\vec{Y} \frac{x^2}{Y^2 |Y - x|^2} \int d^3\vec{y} C(\vec{y}) \exp ik \vec{y} \cdot [\hat{Y} - (x \hat{Y})] \quad (3.4)$$

Here $\hat{Y}$ and $x \hat{Y}$ stand for unit vectors in the direction of $\vec{Y}$ and $x \vec{Y}$, respectively, and we have written $|Y \pm y/2| \approx Y$, $|x - Y \pm y/2| \approx |x \vec{Y}|$ in the geometrical factors multiplying the exponentials. This approximation introduces a negligible error.

The integral over $d^3\vec{Y}$ may now be evaluated by stationary phase. The stationary phase path is the straight line joining 0 to $\vec{x}$, and the result is

$$\langle |X(\vec{x})|^2 \rangle = \frac{x}{4k} \int dy_{||} C(y_{||}, 0) \quad (3.5)$$

where $y_{||}$ refers to the component of $\vec{y}$ in the direction parallel to $\vec{x}$.

Introducing the Fourier transform of the correlation function

$$\tilde{C}(\vec{q}) = \int d^3\vec{r} e^{-i\vec{q} \cdot \vec{r}} C(\vec{r}) \quad (3.6)$$

permits us to rewrite Eq. (3.5) in the sometimes more convenient form

$$\langle |X(\vec{x})|^2 \rangle = \left(\frac{1}{4\pi k} \right)^2 x \int d^2\vec{q}_{\perp} \tilde{C}(0, \vec{q}_{\perp}) \quad (3.7)$$

where " $\perp$ " refers to the directions perpendicular to $\vec{x}$.

Next let us turn to $\langle X^2 \rangle$. We now have, instead of Eq. (3.3), the expression

$$\begin{aligned} \langle X(\vec{x})^2 \rangle &= \left(\frac{1}{4\pi} \right)^2 \int d^3 \vec{y}_1 \int d^3 \vec{y}_2 \frac{x^2}{y_1 |\vec{x} - \vec{y}_1| y_2 |\vec{x} - \vec{y}_2|} \\ &\quad e^{ik(y_1 + |\vec{x} - \vec{y}_1| - x)} e^{ik(y_2 + |\vec{x} - \vec{y}_2| - x)} \\ &\quad C(y_1 - y_2) \end{aligned} \quad (3.8)$$

We again shift to the variables $\vec{Y}$ and $\vec{y}$, and appeal to the vanishing of $C(\vec{y})$ for $y \gtrsim L$ to justify expanding in y/Y and $y/|\vec{x} - \vec{Y}|$. We obtain

$$\begin{aligned} \langle X(\vec{x})^2 \rangle &= \left(\frac{1}{4\pi} \right)^2 \int d^3 \vec{Y} \frac{x^2}{Y^2 |\vec{x} - \vec{Y}|^2} e^{2ik(Y + |\vec{x} - \vec{Y}| - x)} \int d^3 \vec{y} C(\vec{y}) \\ &\quad \exp \frac{ik}{4} \left(\frac{y^2 - (\vec{y} \cdot \hat{Y})^2}{Y} + \frac{y^2 - [\vec{y} \cdot (\vec{x} - \vec{Y})]^2}{|\vec{x} - \vec{Y}|} \right). \end{aligned} \quad (3.9)$$

As before, we may evaluate the integral over $d^3 Y$ by stationary phase. This yields

$$\begin{aligned} \langle X(\vec{x})^2 \rangle &= \left(\frac{1}{4\pi} \right)^2 \int_0^x ds \int dy_{||} \int d^2 \vec{y}_{\perp} C(y_{||}, y_{\perp}) \\ &\quad \frac{x^2}{s^2 (x - s)^2} e^{\frac{ik}{4} \left(\frac{1}{s} + \frac{1}{x-s} \right) y_{\perp}^2} \end{aligned} \quad (3.10)$$

where, again, " $||$ " and " $\perp$ " refer to directions parallel and perpendicular to $\vec{x}$.

At this point it is convenient to express $C(y_{\parallel}, \vec{y}_{\perp})$ in terms of its Fourier transform, as given by Eq. (3.6). The integral over $dy_{\parallel} d^2y_{\perp}$ can then be carried out, and we finally obtain the relatively simple expression

$$\tilde{C} \langle X(\vec{x})^2 \rangle = - \left(\frac{1}{4\pi k} \right)^2 \int d^2\vec{q}_{\perp} \tilde{C}(0, \vec{q}_{\perp}) \int_0^x ds e^{i \frac{q_{\perp}^2}{k} \frac{(s-x)s}{x}} \quad (3.11)$$

Equations (3.7) and 3.11) constitute our central results. They express the quantities of interest as integrals along unperturbed ray paths (in this case straight lines) of the Fourier transform of the correlation function $C(\vec{q})$ times rather simple geometrical factors. As we shall see later, entirely parallel expressions obtain in the more difficult case of an inhomogeneous background medium.

The expression for $\langle |X(\vec{x})|^2 \rangle$, Eq. (3.7), is precisely the same result for this quantity obtained by using geometrical optics to compute $X(\vec{x})$ itself, and then calculating $\langle |X(\vec{x})|^2 \rangle$ from this. [This is easily seen by referring back to Eq. (2.19).] In contrast, Eq. (3.11) is not what one obtains for $\langle X(\vec{x})^2 \rangle$ from geometrical optics. Geometrical optics for this quantity is recovered if one expands the exponential in Eq. (3.11), a procedure that evidently is valid only if

$$\frac{q_{\perp}^2}{k} \frac{(s-x)s}{x} \ll 1$$

Since $q_{\perp} \sim 1/L$ and $s, x-s \sim x$, this condition can be more familiarly written as

$$x \ll k L^2$$

which we recognize as the Fresnel condition under which the geometrical optics approximation for $X(\vec{x})$ itself was valid in the first place.

Thus, Eq. (3.11) constitutes an improvement over geometrical optics, while Eq. (3.7) coincides with geometrical optics. Conversely, geometrical optics for $\langle |X|^2 \rangle$ is valid out to a very large range, while geometrical optics for $\langle X^2 \rangle$ is valid only within the range $x < k L^2$.

It is of interest to study Eq. (3.11) in the limit of very long range. As $x \rightarrow \infty$, the integral over ds can be approximately evaluated, and we find¹

$$\langle X^2(\vec{x}) \rangle \approx \frac{iC(0)}{8\pi k} (\gamma + \log 4kx - i\pi/2) \quad (3.12)$$

where $\gamma = 0.577\dots$ is Euler's constant. Thus, for small x satisfying the Fresnel condition, we have

$$\langle X^2(\vec{x}) \rangle \approx - \left(\frac{1}{4\pi k} \right)^2 \int d^2 q_{\perp} \tilde{C}(0, \vec{q}_{\perp}) \left[x + \frac{i q_{\perp}^2}{6k} x^2 + \dots \right], \quad (3.13)$$

and for very large x we have $\langle X^2(\vec{x}) \rangle \sim i \log x$ as given by Eq. (3.12). Between these two extremes, of geometrical optics and of very long ranges, Eq. (3.11) provides a smooth interpolation.

Equation (3.7), on the other hand, gives $\langle |X|^2 \rangle$ for all values of x , large and small, and simply says that $\langle |X|^2 \rangle$ is proportional to the range x everywhere.

IV RYTOV'S METHOD IN AN INHOMOGENEOUS OCEAN

Now let us turn to the effects of the sound channel. That is, we must replace the nonfluctuating sound speed c in the homogeneous case by a (specified) function of position $c(\vec{x})$. In fact, for the ocean, $c(\vec{x}) = c(z)$ is a function of depth only. A reasonably explicit form for $c(z)$ is the relatively simple expression³

$$c(z) = c_A [1 + \epsilon(e^{-\eta} + \eta - 1)]$$

where $\eta = (z - z_A)/\frac{1}{2}B$, where z_A is the sound-channel depth, and ϵ and B are parameters. We shall, however, write our formulas for a general $c(\vec{x})$ until the time comes to make explicit numerical estimates.

The wave equation for the pressure, which is our starting point, now becomes altered from Eq. (2.1) to the equation

$$[\nabla^2 + k^2(\vec{x})]p(\vec{x}) = V(\vec{x})p(\vec{x}) \quad , \quad (4.1)$$

still with the same boundary condition that

$$p(\vec{x}) \rightarrow \frac{1}{4\pi x}$$

as $\vec{x} \rightarrow 0$, corresponding to an isotropic point source at the origin, where now $k(\vec{x}) = \omega/c(\vec{x})$.

We must first study the nonfluctuating part of the problem, to evaluate the Green's function in the presence of the sound channel. This satisfies

$$[\nabla^2 + k^2(\vec{x})]\Delta(\vec{x}, \vec{y}) = \delta^3(\vec{x} - \vec{y}) \quad (4.2)$$

Note that it is no longer a function only of $\vec{x} - \vec{y}$ as it was in the homogeneous case. We shall assume that geometrical optics provides a good approximation to the nonfluctuating sound-channel problem. This means that we can represent $\Delta(\vec{x}, \vec{y})$ as a sum of contributions from each ray joining $\vec{x}$ and $\vec{y}$. To be specific, we may write

$$\Delta(\vec{x}, \vec{y}) = \sum_{i=1}^{n(\vec{x}, \vec{y})} \Delta_i(\vec{x}, \vec{y}) \quad (4.3)$$

where $n(\vec{x}, \vec{y})$ is the number of rays and Δ_i is the contribution of the i^{th} ray. We have, in particular for rays joining the origin and $\vec{x}$,

$$\Delta_i(\vec{x}, 0) = K_i(\vec{x}, 0) \exp i \int_0^{\vec{x}} ds k(\vec{x}_i(s)) \quad , \quad i = 1 \dots n(\vec{x}), \quad (4.4)$$

where ds is an element of path length along the ray, $\vec{x}_i(s)$ is the i^{th} ray joining 0 to $\vec{x}$, and K_i is a normalization factor.

Now when the fluctuations are turned on, the signals traveling on each of the rays joining the origin to the point of observation $\vec{x}$ are subject to small-angle scatterings by the perturbing potential $V(\vec{x})$. The signals are thus deflected slightly from the undisturbed rays by each interaction with V . The repeated action of V thus produces, on each ray, a sort of random walk of the signal away from the original ray. When we average over an ensemble of perturbations V , the disturbed signals will fill up a tube surrounding the undisturbed ray. Provided that these tubes

around each of the original rays do not overlap, the received pressure will be a sum of contributions from each ray tube.

We may estimate the radius of a ray tube as follows. The mean free path d between interactions of the signal traveling along a given ray with the perturbing potential V is of the order of $kc/\delta c$. Hence, over a range x the number of scatterings is $n = x/d$. The average deflection angle due to each scattering is of the order of $1/kL_V$ vertically and $1/kL_H$ horizontally, where L_V and L_H are the vertical and horizontal correlation lengths of the sound-speed fluctuations. Since the process is a random walk, the net displacement due to n collisions is proportional to $\sqrt{n}$, and hence the vertical and horizontal extents of the tube are, roughly,

$$r_V \sim \sqrt{\frac{x}{k}} \frac{1}{kL_V} \sqrt{\frac{c}{\delta c}}$$

and

$$r_H \sim \sqrt{\frac{x}{k}} \frac{1}{kL_H} \sqrt{\frac{c}{\delta c}} .$$

Let us assume that the vertical extent of the tubes is small enough so that the tubes remain distinct. Then the pressure at $\vec{x}$ is the sum of contributions from each tube,

$$p(\vec{x}) = \sum_{i=1}^{n(\vec{x})} p_i(\vec{x}) , \quad (4.5)$$

where $n(\vec{x})$ is the number of unperturbed rays joining the source to the point $\vec{x}$. We shall be interested in $p_i(\vec{x})$.

We note that $p_i(\vec{x})$ is the pressure that would be received at $\vec{x}$ if the source were not isotropic, but rather emitted all its energy in the direction of the i^{th} unperturbed ray. Thus $p_i(\vec{x})$ must satisfy the wave equation (4.1) but with an anisotropic boundary condition that itself depends on $\vec{x}$. To make this more precise, let us define $p_i(\vec{y};\vec{x})$ to be the pressure at $\vec{y}$ from a source at the origin that emits only within a small solid angle* around the direction of the i^{th} unperturbed ray joining the origin to $\vec{x}$. Thus $p_i(\vec{x}) = p_i(\vec{x};\vec{x})$, and furthermore $p_i(\vec{y},\vec{x})$ vanishes unless y is inside the i^{th} ray tube. Then

$$[\nabla_{\vec{y}}^2 + k^2(\vec{y})]p_i(\vec{y};\vec{x}) = V(\vec{y})p_i(\vec{y};\vec{x}), \quad i = 1 \dots n(\vec{x}) \quad (4.6)$$

In analogy with this definition of $p_i(\vec{y};\vec{x})$, we may also define an "unperturbed" Green's function $\Delta_i(\vec{y};\vec{x},0)$, $i = 1 \dots n(\vec{x})$, to satisfy

$$[\nabla_{\vec{y}}^2 + k^2(\vec{y})]\Delta_i(\vec{y};\vec{x},0) = 0, \quad (4.7)$$

again with the same boundary condition. This function, also, will vanish except when $\vec{y}$ is near the i^{th} unperturbed ray.

We may now directly derive the analogue of Eq. (2.15) by computing the quantity $\log p_i(\vec{y};\vec{x})/\Delta_i(\vec{y};\vec{x},0)$ in perturbation theory, and using Eq. (4.2). We find, setting $\vec{y} = \vec{x}$, that

$$p_i(\vec{x}) = \Delta_i(\vec{x};\vec{x},0) e^{\frac{1}{\Delta_i(\vec{x};\vec{x},0)} \int_{i^{\text{th}} \text{ ray tube}} d^3\vec{x}' \Delta(\vec{x},\vec{x}') V(\vec{x}') \Delta(\vec{x}',0)} \quad (4.9)$$

We have here replaced $\Delta_i(\vec{x};\vec{x},0)$ simply by $\Delta_i(\vec{x},0)$. Equation (4.9) is evidently the generalization of the Rytov formula (2.15) to the situation of an inhomogeneous background and many rays. The expression clearly

fails if the range is so large that the ray tubes overlap; otherwise the validity conditions are the same as those in the homogeneous-background case.

V FLUCTUATIONS IN AN INHOMOGENEOUS OCEAN

In this section we shall use Eq. (4.9) to calculate the various averages of interest for the contribution of a single ray tube to the pressure in the presence of the sound channel. We shall, for simplicity, drop the index i , though we should keep in mind that when there are several unperturbed raypaths their contributions are to be added to obtain the total pressure. Our interest, then will be in the statistical fluctuations of the contributions of a single ray, or rather a single ray tube.

As in the homogeneous case, we define

$$X(\vec{x}) = \frac{1}{\Delta(\vec{x}, 0)} \int d^3\vec{y} \Delta(\vec{x}, \vec{y}) V(\vec{y}) \Delta(\vec{y}, 0) \quad (5.1)$$

and we wish to compute $\langle X^2 \rangle$ and $\langle |X|^2 \rangle$. We recall, from Section IV, that assuming geometrical optics to be a valid approximation for the nonfluctuating background permits us to write the Green's function as

$$\Delta(\vec{x}, \vec{y}) = K(\vec{x}, \vec{y}) e^{ikS(\vec{x}, \vec{y})} \quad (5.2)$$

where

$$kS(\vec{x}, \vec{y}) = \int_{\vec{x}}^{\vec{y}} ds \, k(\vec{x}(s)) \quad (5.3)$$

and where the normalization factor is

$$K(\vec{x}, \vec{y}) = \frac{1}{4\pi} \sqrt{\det \frac{\partial}{\partial x_{\perp i}} \frac{\partial}{\partial y_{\perp j}} S(\vec{x}, \vec{y})} \Big|_{\vec{x}_{\perp} = \vec{y}_{\perp} = 0}.$$

Here, $\perp$ refers to directions perpendicular to the ray. (An excellent approximation to this, for our purposes, is to write simply $K(\vec{x}, \vec{y}) = (4\pi|\vec{x}-\vec{y}|)^{-1}$, as in the homogeneous-ocean case; we need to be careful about deviations from homogeneity only in the phases.) In Eq. (5.3) the line integral is along the ray of interest joining the points $\vec{x}$ and $\vec{y}$.

To repeat our earlier calculations requires us to introduce the correlation function $\langle V(\vec{y}_1) V(\vec{y}_2) \rangle$. In the homogeneous case, this quantity depended only on the separation $y_1 - y_2$. Now, however, because of the background inhomogeneities, it will also depend on $(y_1 + y_2)/2$ (actually it will depend only on the mean depth $(z_1 + z_2)/2$ since the inhomogeneities depend only on depth). Thus we must now define the correlation function by

$$V(\vec{y}_1, \vec{y}_2) \equiv C\left(y_1 - y_2, \frac{y_1 + y_2}{2}\right).$$

As before, let us look first at $\langle |X|^2 \rangle$. We have

$$\begin{aligned} \langle |X(x)|^2 \rangle &= \int d^3 \vec{Y} \frac{K(\vec{x}, \vec{Y}) K(\vec{Y}, 0)}{K(\vec{x}, 0)} \int d^2 \vec{y} C(\vec{y}, \vec{Y}) \\ &\quad \exp i k [S(\vec{x}, \vec{Y} + \vec{y}/2) - S(\vec{x}, \vec{Y} - \vec{y}/2) \\ &\quad + S(\vec{Y} + \vec{y}/2, 0) - S(\vec{Y} - \vec{y}/2, 0)] \end{aligned} \quad (5.4)$$

and we must keep in mind that we are to integrate only over the ray tube surrounding the unperturbed ray of interest. In the homogeneous-background case we expanded the exponent in powers of $\vec{y}$, because $C(\vec{y})$ vanished for large $|\vec{y}|$. We may do the same here. Thus,

$$\langle |X(\vec{x})|^2 \rangle = \int d^3\vec{Y} \frac{K(\vec{x}, \vec{Y})K(\vec{Y}, 0)}{K(\vec{x}, 0)} \int d^3\vec{y} C(\vec{y}, \vec{Y}) \exp i k \vec{y} \cdot \vec{\nabla}_Y [S(\vec{x}, \vec{Y}) + S(\vec{Y}, 0)] \quad (5.5)$$

The integral on $d^3\vec{Y}$ is again to be evaluated by stationary phase. The stationary phase path is evidently the unperturbed ray joining the origin to the observation point $\vec{x}$. Hence, we may write

$$\langle |X(\vec{x})|^2 \rangle = \left(\frac{1}{4\pi k} \right)^2 \int_0^{\vec{x}} ds \int d^2\vec{q}_\perp(s) \tilde{C}(\vec{q}_\perp(s), \vec{Y}(s)) \quad (5.6)$$

in complete parallel to the homogeneous case. Here the line integral on ds is along the unperturbed ray, $\vec{q}_\perp(s)$ refers to the component of $\vec{q}$ perpendicular to the ray at s , $\vec{Y}(s)$ is a point on the ray at s , and

$$\tilde{C}(\vec{q}, \vec{Y}) \equiv \int d^3\vec{y} e^{i\vec{q} \cdot \vec{y}} C(\vec{y}, \vec{Y}) \quad (5.7)$$

Next we turn to $\langle X^2 \rangle$:

$$\begin{aligned}
\langle X(\vec{x})^2 \rangle = & \int d^3\vec{Y} \frac{K(\vec{x},\vec{Y})K(\vec{Y},0)}{K(\vec{x},0)} \int d^3\vec{y} C(\vec{y},\vec{Y}) \\
& \exp i k \left[S(\vec{x},\vec{Y} + \vec{y}/2) + S(\vec{x},\vec{Y} - \vec{y}/2) \right. \\
& + S(\vec{Y} + \vec{y}/2,0) + S(\vec{Y} - \vec{y}/2,0) \\
& \left. - 2 S(\vec{x},0) \right] .
\end{aligned} \tag{5.8}$$

Now when we expand the exponent in powers of $\vec{y}$ the linear terms vanish, so that we have

$$\begin{aligned}
\langle X(\vec{x})^2 \rangle = & \int d^3\vec{Y} \frac{K(\vec{x},\vec{Y})K(\vec{Y},0)}{K(\vec{x},0)} e^{2ik(S(\vec{x},\vec{Y})+S(\vec{Y},0)-S(\vec{x},0))} \\
& \int d^3\vec{y} C(\vec{y},\vec{Y}) \exp ik/4 y_i y_j A_{ij}(\vec{Y})
\end{aligned} \tag{5.9}$$

where we define

$$A_{ij}(\vec{Y}) = \frac{\partial}{\partial Y_i} \frac{\partial}{\partial Y_j} \left[S(\vec{x},\vec{Y}) + S(\vec{Y},0) \right] . \tag{5.10}$$

Evaluation of the integral on $d^3\vec{Y}$ by stationary phase again selects as the stationary phase path the unperturbed ray joining 0 to $\vec{x}$. The integral on $d^3\vec{y}$ can then be done by introducing the Fourier transform $\tilde{C}(\vec{q},\vec{Y})$ as in Eq. (5.7). Finally, we obtain

$$\langle X(\vec{x})^2 \rangle = - \left(\frac{1}{4\pi k} \right)^2 \int_0^{\vec{x}} ds \int d^2 q_{\perp}(s) \tilde{C}(\vec{q}_{\perp}(s), \vec{Y}(s)) e^{\frac{i}{k} q_{\perp i}(s) q_{\perp j}(s) A^{-1}(\vec{Y}(s))_{ij}} \quad (5.11)$$

The notation is as in Eq. (5.6), and the result is again in complete analogy to the homogeneous case.

Most of the comments we made in Section III concerning the results in the homogeneous background apply here as well. The expression for $\langle |X|^2 \rangle$ is again just that obtained in the geometrical optics approximation, but that for $\langle X^2 \rangle$ is not. Geometrical optics for $\langle X^2 \rangle$ is valid provided that

$$\frac{i}{k} q_{\perp i}(s) q_{\perp j}(s) A^{-1}(\vec{Y}(s))_{ij} \ll 1 \quad (5.12)$$

This is the analogue of the Fresnel condition.

In the homogeneous case, this condition boiled down to the requirement that

$$x < k L^2 \quad .$$

In the presence of a sound channel, the restriction (5.12) on the range is less severe; the condition (5.12) reduces approximately to

$$\frac{x}{k L_H^2} + \frac{x \tan^2 \theta}{k L_V^2} \ll 1 \quad (5.13)$$

where $\tan \theta$ is the maximum ray inclination to the horizontal. Since L_H is much larger than L_V , and since $\tan \theta$ is small, this restriction is

easier to meet than $x/k L_V^2 \ll 1$. Thus, in the presence of a sound channel, geometrical optics should be valid to a greater range than would be the case with a uniform background sound speed.

VI CONCLUSION

We have presented relatively simple formulas for phase and amplitude fluctuations of pressure signals in the ocean in the presence of a specified spectrum of sound-speed fluctuations for both a homogeneous ocean and one with a sound channel. These formulas reduce to the geometrical optics approximation for short ranges, but for long ranges they give results far less divergent with range than does geometrical optics.

For the case of a sound channel, where there are in general many rays, our results apply to fluctuations in the contribution of a single ray to the received pressure; thus the approximation is limited to ranges at which rays are still separable. Fluctuations in the total received pressure are much greater, due to interference between different rays, and are insensitive to details of the fluctuations in a single ray.⁴

The next step to be undertaken is to use the results outlined here, together with a semi-empirical fluctuation spectrum, to make numerical estimates of the fluctuations for the purpose of comparison with experiments. This will be described elsewhere.

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