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ACOUSTIC ANALYSIS FOR A MOVING SOURCE WITH APPLICATION
TO AN ISOSPEED OCEAN

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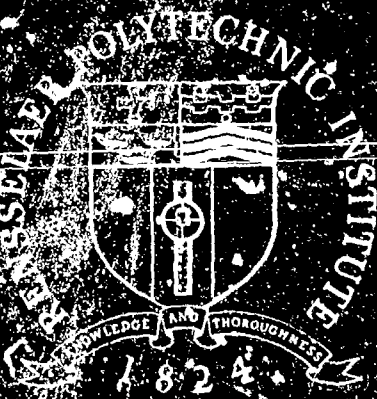Prepared for:

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1

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INTRODUCTION

Ray theory has been used extensively in the study of acoustic transmissions in an underwater channel, where the position of both a point source and point receiver are assumed fixed. Aspects of the moving source problem have been treated in relation to its effects on radar.[1,2,3] However, results are not generally applicable in an underwater environment because refraction, boundary reflection, the coherent summation of multiple arrivals, and other effects are not usually considered. Only recently have investigations been directed toward consideration of the moving source problem in underwater acoustics.[4,5] Computer results for total-field phase and amplitude were presented in the case of a source moving radially outward at fixed speed from a fixed receiver, when a specific deep-water sound-speed profile is assumed.

The intent of this paper is twofold. We wish to present an analytical technique whereby a stationary sound-source problem can be transformed into a moving source problem, including a careful explanation of Doppler effects for a cw sound source. In addition, to illustrate the technique, we consider a specific moving source problem in an isospeed channel. The velocity of the source is chosen constant, but its direction with respect to a fixed receiver is not restricted to the radial case. The total acoustic field is analyzed for both short-and long-range runs of the source.

In Sec. I, a general ray-theory treatment of the moving source problem is investigated, in which a general technique

is formulated for determining Doppler effects for a cw source moving with arbi'rary velocity over an arbitrary smooth curve in three-dimensional space. A discussion of the effects of source motion on ray geometry, boundary reflection theories, spreading loss, and travel time leads to total-field equations. In an illustration of the general approach, Sec. II deals with a specific moving source problem for the case of a constant sound-speed medium, bounded by a horizontal surface and bottom, where both the sound source and receiving point are located on the ocean bottom, and where the source is assumed to follow a straight-line path with constant speed. General equations developed in Sec. I are used to construct the total-field phase and amplitude as a function of receiver time. In Sec. III, certain restrictions are imposed, under which a linearized model of the relationship between acoustic phase and receiver time is developed and explored in detail. Phase variation for a source following a long straight-line path with constant speed v is examined in Sec. IV. In particular, a relation between phase and receiver time is developed and used in sampling phase along the entire source trajectory. In addition, an approximate expression for relative cumulative phase for all receiver times is derived. Section V summarizes the principal results of this paper.

I. GENERAL TREATMENT

We begin by taking a receiving point R to lie at the origin of a Cartesian system in which the x and y axes are horizontal and the z axis is vertical. Suppose that, at any

instant in time, a point sound source S also lies in the x-y plane and has velocity $\underset{\sim}{v}$ with corresponding speed denoted by v. It is well known that, for "line of sight" transmission, as shown in Fig. 1(a), the frequency f_R detected at the receiver differs from the frequency f emitted at the source.[6] If the distance between the source and receiver remains constant, the wavelength detected at R is $\lambda = c/f$, where c is the (constant) speed of sound. However, if the source moves as shown in Fig. 1(a), then at any instant in time it is effectively moving away from R with a speed $v \cos \psi$, when $0 \le \psi \le \pi/2$ rad. The motion is toward R for $\pi/2 \le \psi \le \pi$ rad. If $\underset{\sim}{R}$ describes the receiver-source position vector at any instant in time, where R is the corresponding distance, then the resultant wavelength detected at the receiver a time R/c later is equal to

$$\lambda_R = (c+v \cos\psi)/f, \tag{1}$$

and the resultant frequency is

$$f_R = c/\lambda_R = f[1 + (v/c)\cos\psi]^{-1} . \tag{2}$$

In the case of radial source motion, we find that

$$f_R = f(1 \underset{-}{+} v/c)^{-1} , \tag{3}$$

where the plus sign corresponds to $\psi = 0$ and the minus sign to $\psi = \pi$. In essence, the term $v \cos\psi$ can be interpreted as the scalar projection of the velocity $\underset{\sim}{v}$ onto the negative ray direction, where in this case the negative ray direction is

that of $\underset{\sim}{R}$.   Letting v_p denote this scalar projection, we may write

$$f_R = f(1 + v_p/c)^{-1} \quad . \tag{4}$$

The same technique can be applied when a moving source follows an arbitrary path Γ with arbitrary velocity in three-dimensional space where the sound speed is a function of depth, the source and receiver are at arbitrary depths, and the source depth may change with time. Let $\underset{\sim}{R}(\hat{t})$ denote the position vector from the receiver to the source at a time $\hat{t}$, and let $\underset{\sim}{v}(\hat{t}) = d\underset{\sim}{R}/d\hat{t}$ denote the corresponding velocity vector, as shown in Fig. 1(b). Even when sound speed is constant, the possibility of boundary reflections leads to a family of rays which propagate from the source to the receiver. If we let $\theta_n(\hat{t})$ denote the angle at the source of the nth ray, measured from the x' direction, and let $\underset{\sim}{u}_n(\hat{t})$ denote the unit vector associated with the negative nth-ray direction at time $\hat{t}$ as shown in Fig. 1(b), then

$$\underset{\sim}{u}_n(\hat{t}) = \cos \theta_n(\hat{t}) \underset{\sim}{i}' + \sin \theta_n(\hat{t}) \underset{\sim}{k} \quad , \tag{5}$$

where $\underset{\sim}{i}'$ and $\underset{\sim}{k}$ are unit vectors in the x' and z directions, respectively. Note that x' measures horizontal directed distance in the vertical plane containing the ray and that $\theta_n < 0 (\theta_n > 0)$ for a ray directed upward (downward) at the source. The scalar projection of $\underset{\sim}{v}(\hat{t})$ onto $\underset{\sim}{u}_n(\hat{t})$ is

$$v_p(\hat{t}) = \underset{\sim}{v}(\hat{t}) \cdot \underset{\sim}{u}_n(\hat{t}) = |\underset{\sim}{v}(\hat{t})| \cos \psi(\hat{t}) \quad , \tag{6}$$

where the angle $\psi(\hat{t})$ is determined by $\underset{\sim}{v}$ and $\underset{\sim}{u}_n$ when their

initial points coincide. Applying Eq. 5 to Eq. 6,

$$v_p(\hat{t}) = [v_x(\hat{t})\underset{\sim}{i}+v_y(\hat{t})\underset{\sim}{j}+v_z(\hat{t})\underset{\sim}{k}] \cdot [\cos\theta_n(\hat{t})\underset{\sim}{i}'+\sin\theta_n(\hat{t})\underset{\sim}{k}]$$

$$= [R_x(\hat{t})v_x(\hat{t})+R_y(\hat{t})v_y(\hat{t})]\cos\theta_n(\hat{t})\left[R_x^2(\hat{t})+R_y^2(\hat{t})\right]^{-1/2}$$

$$+v_z(\hat{t})\sin\theta_n(\hat{t}) , \tag{7}$$

where $\underset{\sim}{i}$, $\underset{\sim}{j}$, and $\underset{\sim}{k}$ are unit vectors in the x,y, and z directions, respectively, and

$$\underset{\sim}{i}' = \left[R_x(\hat{t})\underset{\sim}{i} + R_y(\hat{t})\underset{\sim}{j}\right]\left[R_x^2(\hat{t}) + R_y^2(\hat{t})\right]^{-1/2} \tag{8}$$

is the unit vector in the positive x' direction. A subscript
on R and v refers to the component of $\underset{\sim}{R}$ and $\underset{\sim}{v}$ in the corresponding
direction. When the source path Γ lies in the x-y plane,
$v_z = 0$ and Eq. 7 simplifies to

$$v_p(\hat{t}) = \underset{\sim}{r}(\hat{t}) \cdot \underset{\sim}{v}(\hat{t})\cos\theta_n(\hat{t}) , \tag{9}$$

where $\underset{\sim}{r}(\hat{t}) = \underset{\sim}{i}'$.

Since the source is moving, the frequency associated with
the nth ray detected by the receiver at a time t is not the
frequency corresponding to the source position at t. Rather,
it is that corresponding to the source position at an earlier
time. Letting t_n describe the time associated with the
continuous emission of the nth ray from the source, there
exists a relationship between source time t_n and receiver
time t given by

$$t_n = t - T_n(t_n), \tag{10}$$

where T_n is the travel time of the nth ray. Observing from Eq. 10 that t_n is an implicit function of t, we have

$$t_n = F_n(t) , \qquad (11)$$

so that $T_n(t_n)$ may be rewritten as $\tilde{T}_n(t)$. Clearly, the frequency of the nth ray detected at a time t is the frequency of the ray emitted at the time $t_n = t - \tilde{T}_n(t)$. Evaluating Eq. 7 at $\hat{t} = t_n$ and using this transformation,

$$v_p(t_n) = \tilde{v}_p(t) = \left[\tilde{R}_x(t)\tilde{v}_x(t) + \tilde{R}_y(t)\tilde{v}_y(t)\right]\cos\tilde{\theta}_n(t) \left[\tilde{R}_x^2(t) + \tilde{R}_y^2(t)\right]^{-1/2}$$
$$+ \tilde{v}_z(t)\sin\tilde{\theta}_n(t) , \qquad (12)$$

where $\tilde{R}_{x,y,z}(t) = R_{x,y,z}[F_n(t)]$, $\tilde{v}_{x,y,z}(t) = v_{x,y,z}[F_n(t)]$, and $\tilde{\theta}_n(t) = \theta_n[F_n(t)]$. The corresponding frequency at the receiver, analogous to Eq. 4, is

$$f_{Rn}(t) = f\left[1 + \{[\tilde{R}_x(t)\tilde{v}_x(t) + \tilde{R}_y(t)\tilde{v}_y(t)]\cos\tilde{\theta}_n(t)[\tilde{R}_x^2(t) + \tilde{R}_y^2(t)]^{-1/2}\right.$$
$$\left. + \tilde{v}_z(t)\sin\tilde{\theta}_n(t)\}\tilde{c}^{-1}(t)\right]^{-1} , \qquad (13)$$

where c(t) represents the sound speed at the source depth at receiver time t. If Eq. 9 is used in the case where Γ lies in the x-y plane, then Eq. 13 simplifies to

$$f_{Rn}(t) = f\left[1 + \tilde{\underset{\sim}{r}}(t) \cdot \tilde{\underset{\sim}{v}}(t)\Big| \cos\tilde{\theta}_n(t)\Big| \tilde{c}^{-1}(t)\right]^{-1} . \qquad (14)$$

At the receiver, the contribution of the nth ray at time t is

$$V_n(t) = A_n(t_n)\sin\Big\{\omega_{Rn}(t)t_n + S_n(t_n)\Big\}$$
$$= \tilde{A}_n(t)\sin\Big\{\omega_{Rn}(t)\left[t - \tilde{T}_n(t)\right] + \tilde{S}_n(t)\Big\} , \qquad (15)$$

where $t_n = t - \tilde{T}_n(t)$ is given by Eqs. 10 and 11, $\tilde{A}_n(t) = A_n[F_n(t)]$ is amplitude, $\tilde{S}_n(t) = S_n[F_n(t)]$ is phase change resulting from surface and/or bottom reflections, if any, and $\omega_{Rn}(t) = 2\pi f_{Rn}(t)$. If the nth ray experiences no boundary reflections, then $S_n(t) \equiv 0$. Substituting Eq. 13 into Eq. 15 with $\omega = 2\pi f$, we obtain

$$V_n(t) = \tilde{A}_n(t) \sin \left[\omega \left\{ 1 + [\tilde{R}_x(t)\tilde{v}_x(t) + \tilde{R}_y(t)\tilde{v}_y(t)]\tilde{c}^{-1}(t)\cos\tilde{\theta}_n(t) \right. \right.$$

$$\times \left[\tilde{R}_x^2(t) + \tilde{R}_y^2(t)\right]^{-1/2} + \tilde{v}_z(t)\tilde{c}^{-1}(t)\sin\tilde{\theta}_n(t) \bigg\}^{-1} \left[t - \tilde{T}_n(t)\right] + \tilde{S}_n(t) \bigg] .$$

$$(16)$$

We now turn to a brief discussion of the phase term $\tilde{S}_n$ and the amplitude $\tilde{A}_n$ in Eqs. 15 and 16. If Rayleigh reflection theory[7] is used, then, at any one reflection of the nth ray from a horizontal boundary,

$$B_n \exp(i\varepsilon_n) = \left\{ (\rho_m/\rho_w) - \left[(c_w/c_m)^2 - \cos^2\theta_w\right]^{1/2}\left[1 - \cos^2\theta_w\right]^{-1/2} \right\}$$

$$\times \left\{ (\rho_m/\rho_w) + \left[(c_w/c_m)^2 - \cos^2\theta_w\right]^{1/2}\left[1 - \cos^2\theta_w\right]^{-1/2} \right\}^{-1} . \quad (17)$$

In Eq. 17, the subscript w refers to the water while m refers to the medium on the opposite side of the reflecting boundary, either the ocean bottom or the atmosphere. The quantity B_n is the amplitude loss at a reflection of the nth ray, ε_n is the shift in phase there, and the symbol i represents the imaginary unit. By Snell's law,

$$\cos\theta_w = \left[c_w/c(t_n)\right]\cos\theta_n(t_n) = \left[c_w/\tilde{c}(t)\right]\cos\tilde{\theta}_n(t) . \quad (18)$$

When Eq. 18 is substituted into Eq. 17, it becomes apparent that

amplitude loss and phase shift at a boundary reflection, in general, are time-dependent. In addition, the number N of bottom reflections and the number M of surface reflections of the nth ray may be time-dependent, so that we should write $N=N_n(t_n)=\tilde{N}_n(t)$ and $M=M_n(t_n)=\tilde{M}_n(t)$. If we make the usual assumption of no amplitude loss and a π-rad phase shift at a surface reflection, then the contribution of all boundary reflections to $\tilde{A}_n(t)$ is $\exp\left[\tilde{N}_n(t)\ln\tilde{B}_n(t)\right]$ and $\tilde{S}_n(t)$ is given by

$$\tilde{S}_n(t) = \tilde{N}_n(t)\varepsilon_n(t)+\pi\tilde{M}_n(t), \tag{19}$$

where B_n and ε_n refer only to a bottom reflection. If the nth ray experiences no bottom reflections at time t, then $\tilde{N}_n(t)\equiv0$; if it does not reflect from the surface at time t, then $\tilde{M}_n(t)\equiv0$. Of course, other reflection models may be considered. If, for example, a frequency-dependent model[8] is employed, then the variation of frequency with time, as described in Eq. 13, must be included.

When the sound source and ocean medium are stationary, when sound speed is depth-dependent only, and when the ocean channel possesses horizontal boundaries, the contribution of spreading loss L to amplitude is $L^{-1/2}$, where L has been written as[9,10]

$$L = \left|R(\sin\theta_x/\cos\theta_o)\partial R/\partial\theta_o\right|. \tag{20}$$

In Eq. 20, θ_o is the ray angle at the source, θ_x is the ray angle at the receiving point, and R is the horizontal distance between source and receiving point. It has been shown[11] that

a non-stationary ocean requires corrections in the spreading loss, because a moving ocean causes an omnidirectional source to become directional and because rays are no longer normal to wavefronts. However, the corrections are negligible in an isospeed, uniform-current medium when the ratio of current speed to sound speed is small. Since there exist some analogies between a moving source and a moving medium problem, corrections in Eq. 20 are expected here also. However, in subsequent sections, source speed will be a small fraction of the speed of sound so that Eq. 20 will be assumed to be applicable. In using Eq. 20 for the nth ray, R should be replaced by $\tilde{R}(t) = R[F_n(t)]$ and θ_o by $\tilde{\theta}_n(t) = \theta_n[F_n(t)]$, where $F_n(t)$ is given by Eq. 11 and where $\tilde{R} = (\tilde{R}_x^2 + \tilde{R}_y^2)^{1/2}$. Further, from Snell's law, $|\sin\theta_x|$ should be replaced by $\{1 - [c_R/\tilde{c}(t)]^2 \cos^2\tilde{\theta}_n(t)\}^{1/2}$, where c_R is the sound speed at the fixed receiver. The contribution of spreading loss to amplitude is, therefore,

$$L_n^{-1/2}(t) = \left| \tilde{R}(t) \left\{1 - [c_R/\tilde{c}(t)]^2 \cos^2\tilde{\theta}_n(t)\right\}^{1/2} \left\{\cos\tilde{\theta}_n(t)\right\}^{-1} \right.$$
$$\left. \times \; \partial\tilde{R}(t)/\partial\tilde{\theta}_n(t) \right|^{-1/2} . \tag{21}$$

Thus the amplitude of the nth ray at the receiver is

$$\tilde{A}_n(t) = L_n^{-1/2}(t) \exp[\tilde{N}_n(t) \ln B_n(t)] , \tag{22}$$

where B_n is given by Eqs. 17 and 18, and $L_n^{-1/2}$ is given by Eq. 21.

With the phase term $\tilde{S}_n$ and amplitude $\tilde{A}_n$ known from Eqs. 19 and 22, respectively, the receiver arrival V_n associated with

the nth ray is known from Eq. 15 or 16. In order to determine
the total acoustic field at the receiver, we elect to add and
subtract the factor $\bar{\omega}t$ in the argument of the sine in Eq. 15.
The quantity $\bar{\omega}$ is any constant, but arbitrary, circular
frequency. Then

$$V_n(t) = \tilde{A}_n(t)\sin[\bar{\omega}t + \phi_n(t)] \ , \tag{23}$$

where

$$\phi_n(t) = \omega_{Rn}(t)t - \bar{\omega}t - \omega_{Rn}(t)\tilde{T}_n(t) + \tilde{S}_n(t) \ . \tag{24}$$

The total field may be expressed as

$$A(t)\sin[\bar{\omega}t + \Phi(t)] = \sum \tilde{A}_n(t)\sin[\bar{\omega}t + \phi_n(t)] \ , \tag{25}$$

where the amplitude A and phase Φ can be determined from the
following system of equations:

$$A^2(t) = \left[\sum \tilde{A}_n(t)\sin\phi_n(t)\right]^2 + \left[\sum \tilde{A}_n(t)\cos\phi_n(t)\right]^2 , \tag{26}$$

$$\sin \Phi(t) = A^{-1}(t)\sum \tilde{A}_n(t)\sin\phi_n(t) \ , \tag{27}$$

$$\cos \Phi(t) = A^{-1}(t)\sum \tilde{A}_n(t)\cos\phi_n(t) . \tag{28}$$

Since $A(t)$ and $\Phi(t)$ are time-dependent, it is apparent from
Eq. 25 that the total acoustic field at the receiver is not a
sinusoidal function of time. However, it may be interpreted
as a signal with constant circular frequency $\bar{\omega} = 2\pi\bar{f}$, having
time-dependent amplitude A and phase Φ. If $\bar{f}$ is chosen to be

the frequency emitted by the sound source, then $\bar{\omega}=\omega=2\pi f$ in Eqs. 23-28. A second procedure, similar to that of Ref. 4, is to average frequency over the first N' significant ray arrivals and over a time interval τ. If we define

$$\Delta\omega \equiv (N'\tau)^{-1} \sum_{n=0}^{N'-1} \int_{t'}^{t'+\tau} [\omega_{Rn}(t)-\omega]dt , \qquad (29)$$

where t' is some constant value of receiver time, then $\bar{\omega}$ should be replaced by $\omega+\Delta\omega$ in Eqs. 23-28.

The above results are too general to predict the presence of shadow and caustic boundaries at any instant of time. These boundaries are important, however, since they can be expected to lead to discontinuities in Doppler shift and diffraction effects. Once sound speed, receiver location, source path, and source velocity are selected for use in a particular application, these phenomena can be examined.

II. THE ISOSPEED CHANNEL

In some special situations, the moving source problem simplifies greatly. To illustrate the general treatment of Sec. I, we consider here the case of a constant sound-speed medium of depth H, bounded by a horizontal surface and bottom. Both the sound source and receiving point are located on the ocean bottom, and the source is assumed to follow a straight-

line path with constant speed v. Shadow and caustic boundaries cannot occur in an isospeed medium. Suppose, for definiteness, that the source $\mathcal{S}$ is located a distance R from the receiving point $\mathcal{R}$ at time t=0. Also, let the angle α in the ocean bottom describe the direction of the source trajectory where $\alpha = \psi(0)$. The source-receiver geometry appears in Fig. 2(a), while Fig. 2(b) shows the first two rays from the moving source to the receiving point. Note that the index n equals the number of bottom reflections of the nth ray, and that the source location changes with n.

From geometric considerations, the distance between $\mathcal{S}$ and $\mathcal{R}$ at time $\hat{t}$ is given by

$$R(\hat{t}) = |\underset{\sim}{R}(\hat{t})| = \left| R^2 + (v\hat{t})^2 + 2Rv\hat{t}\cos\alpha \right|^{1/2} . \tag{30}$$

The scalar projection of $\underset{\sim}{v}$ onto the negative direction of the nth ray (from receiver to source), given by Eq. 9, may be written as

$$v_p(\hat{t}) = \underset{\sim}{r}(\hat{t}) \cdot \underset{\sim}{v}(\hat{t}) \cos\theta_n(\hat{t}) = \beta v \left| [R^2(\hat{t}) - R^2\sin^2\alpha]^{1/2} / R(\hat{t}) \right| \cos\theta_n(\hat{t}) , \tag{31}$$

where $\beta = \text{sgn}[v\hat{t} + R\cos\alpha]$ and where $\theta_n(\hat{t})$ describes the angle at the source of the nth ray at time $\hat{t}$. As discussed in Sec. I, the frequency of the nth ray detected by the receiver at time t is not the frequency associated with the source position at t but, rather, is that corresponding to the source position at time $t_n = t - \tilde{T}_n(t)$, where $\tilde{T}_n(t) = T_n[F_n(t)]$ by application of Eq. 11. Under such a transformation, Eq. 31 with $\hat{t} = t_n$ may be expressed as

$$\tilde{v}_p(t) = \beta v \left| [\tilde{R}^2(t) - R^2 \sin^2 \alpha]^{1/2} / \tilde{R}(t) \right| \cos \tilde{\theta}_n(t), \tag{32}$$

where $\tilde{R}(t) = R[F_n(t)]$ and $\tilde{\theta}_n(t) = \theta_n[F_n(t)]$. The corresponding frequency at the receiver is, analogous to Eq. 14,

$$f_{Rn}(t) = f \left[1 + \beta v \left| [\tilde{R}^2(t) - R^2 \sin^2 \alpha]^{1/2} / \tilde{R}(t) \right| c^{-1} \cos \tilde{\theta}_n(t) \right]^{-1} ,$$

$$\tag{33}$$

where c is now a constant. Since the travel time of the nth ray is

$$T_n(t_n) = \left| R^2(t_n) + [2(n+1)H]^2 \right|^{1/2} c^{-1} , \tag{34}$$

$F_n(t)$, given by Eq. 11, can be determined explicitly. Specifically, from Eqs. 10,11,30, and 34,

$$t_n = F_n(t) = \left| [t + RMc^{-1}\cos\alpha] - [T_{no}^2 + 2RMtc^{-1}\cos\alpha + M^2(t^2 - T_{no}^2 \right.$$
$$\left. + R^2 c^{-2} \cos^2\alpha)]^{1/2} \right| (1-M^2)^{-1/2} , \tag{35}$$

where $M = v/c$ is called the Mach number and

$$T_{no} = \left| R^2/c^2 + [2(n+1)H/c]^2 \right|^{1/2} \tag{36}$$

is the travel time of the nth ray if the source were stationary at $(x,y) = (0,R)$. The launch angle $\tilde{\theta}_n(t)$ may be determined from

$$\cos \tilde{\theta}_n(t) = \tilde{R}(t) \left| \tilde{R}^2(t) + [2(n+1)H]^2 \right|^{-1/2} , \tag{37}$$

where $\tilde{R}(t)$ can be found from Eqs. 30 and 35 with $\hat{t} = t_n$. Thus, from Eq. 33, the corresponding circular frequency at the receiver is

$$\omega_{Rn}(t) = \omega \left[\left| T_{no}^2 + M^2 F_n^2(t) + 2RM(\cos\alpha)F_n(t)/c \right|^{1/2} \right| \left[T_{no}^2 + M^2 F_n^2(t) \right.$$

$$+ 2RM(\cos\alpha)F_n(t)/c]^{1/2} + \beta M[R^2(\cos\alpha)^2/c + M^2 F_n(t)$$

$$\left. + 2RM(\cos\alpha)F_n(t)/c]^{1/2}\left|^{-1} \right] , \tag{38}$$

where $F_n(t)$ is given in Eq. 35.

We now make the usual assumption of no amplitude loss and a π-rad phase shift at a surface reflection. If Rayleigh reflection theory is used at each reflection of the nth ray from the horizontal bottom, then the use of Eqs. 18 and 37 enables us to write Eq. 17 as

$$B_n(t)\exp[i\varepsilon_n(t)] = \left[(\rho_m/\rho_w) - \left| (c_m/c_w)^2 - [\tilde{R}(t)/2(n+1)H]^2 \right.\right.$$

$$\times \ [1-(c_w/c_m)^2]\left|^{1/2} \right] \left[(\rho_m/\rho_w) + \left| (c_m/c_w)^2 - [\tilde{R}(t)/2(n+1)H]^2 \right.\right.$$

$$\times \ [1-(c_w/c_m)^2]\left|^{1/2} \right]^{-1} . \tag{39}$$

Although this result could be used in Eqs. 19 and 22, we choose to make instead the simpler assumptions of a 2-dB loss and a π-rad phase shift at each bottom reflection. Then, the contribution of all boundary reflections to the amplitude $\tilde{A}_n$ of each ray arrival is $10^{-0.1n}$, and the phase $\tilde{S}_n$, given by Eq. 19, becomes

$$\tilde{S}_n(t) = n\pi + (n+1)\pi = \pi \ (\text{mod} \ 2\pi). \tag{40}$$

From Eqs. 30, 35, and 37, $\tilde{R}$ and $\tilde{\theta}_n$ are known and the contribution of spreading loss to amplitude is, from Eq. 21,

$$L_n^{-1/2}(t) = c^{-1} \left| T_{no}^2 + M^2 F_n^2(t) + 2RM(\cos\alpha) F_n(t)/c \right|^{-1/2} \quad , \quad (41)$$

where $F_n(t)$ is given by Eq. 35. Thus the amplitude of the nth arrival, given by Eq. 22, becomes

$$\tilde{A}_n(t) = 10^{-0.1n} \, L_n^{-1/2}(t). \tag{42}$$

With $\tilde{S}_n, \tilde{A}_n, \omega_{Rn}$, and $\tilde{T}_n$ known, the nth arrival V_n, given by Eqs. 23 and 24, is

$$V_n(t) = 10^{-0.1n} \, c^{-1} \left| T_{no}^2 + M^2 F_n^2(t) + 2RM(\cos\alpha) F_n(t)/c \right|^{-1/2}$$

$$\times \sin[\bar{\omega} t + \phi_n(t)] \quad , \tag{43}$$

where

$$\phi_n(t) = \omega \left[\left| T_{no}^2 + M^2 F_n^2(t) + 2RM(\cos\alpha) F_n(t)/c \right|^{1/2} \left\{ \left[T_{no}^2 + M^2 F_n^2(t) \right. \right. \right.$$

$$+ 2RM(\cos\alpha) F_n(t)/c]^{1/2} + \beta M[R^2 \cos^2\alpha/c + M^2 F_n^2(t) + 2RM(\cos\alpha)$$

$$\times F_n(t)/c]^{1/2} \right\}^{-1} \left[t - \left| T_{no}^2 + M^2 F_n^2(t) + 2RM(\cos\alpha) F_n(t)/c \right|^{1/2} \right] - \bar{\omega} t + \pi.$$

$$(44)$$

The total field is now known from Eq. 25, and its amplitude and phase from Eqs. 26-28.

III. LINEARIZED MODEL

Under certain assumptions, it is possible to expand Eqs. 43 and 44 in powers of the Mach number M. In particular, an expansion that keeps only linear terms in M greatly simplifies these equations while preserving the effect of source motion.

When the ratio of source speed to sound speed is small ($M \ll 1$), such a linearization is valid when restrictions are placed on time duration, and, subsequently, source movement.

If one performs a linearization in M of Eq. 44, then

$$\phi_n(t) \approx 2M\eta_n\omega(\cos\alpha)T_{no} - 2M\eta_n\omega(\cos\alpha)t - \omega T_{no} + (\omega - \bar{\omega})t + \pi , \quad (45)$$

where

$$\eta_n = R/T_{no}c = \left| 1 + [2(n+1)H/R]^2 \right|^{-1/2} . \quad (46)$$

A similar linearization of $\tilde{A}_n(t)$, given in Eq. 42, leads to

$$\tilde{A}_n(t) \approx 10^{-0.1n} R^{-1} \eta_n [1 - M\eta_n(\cos\alpha)(t/T_{no} - 1)] . \quad (47)$$

A further simplification of Eqs. 45 and 47 is possible if $H \ll R$ and n is not too large. In such a case, $\eta_n \approx 1 - \delta$ and $T_{no} \approx (R/c)(1+\delta)$ where $\delta = 2[(n+1)H/R]^2 \ll 1$. If η_n is replaced by unity and T_{no} by R/c only in those terms containing M in Eqs. 45 and 47, we obtain

$$\phi_n(t) \approx 2M\omega Rc^{-1}\cos\alpha - 2M\omega(\cos\alpha)t - \omega T_{no} + (\omega - \bar{\omega})t + \pi , \quad (48)$$

and

$$\tilde{A}_n(t) \approx 10^{-0.1n} R^{-1} \eta_n [1 - M(\cos\alpha)(ct/R - 1)] . \quad (49)$$

Of course, the remaining η_n and T_{no} terms may be approximated also, if desired. The corresponding total acoustic field, given in Eq. 25, may now be expressed as the imaginary part of

$$A(t)\exp i[\bar{\omega}t+\phi(t)]\simeq[1-M(\cos\alpha)(ct/R-1)]\exp i[\bar{\omega}t+2M\omega Rc^{-1}$$

$$\times\cos\alpha-2M\omega(\cos\alpha)t+(\omega-\bar{\omega})t]\sum_{n=0}^{N'-1}10^{-0.1n}R^{-1}{}_{n}\eta_{n}\exp i(-\omega T_{no}+\pi).$$

$$(50)$$

Although the summation in Eq. 25 contains an infinity of terms, its rapid convergence permits its termination. In Eq. 50 we have kept N' terms where, for example, N' might be chosen so that the amplitudes of subsequent arrivals are less than 0.01 that of the $n = 0$ ray. The termination justifies the earlier assumption that δ is small when H<<R. The summation in Eq. 50 represents the approximate total field for a stationary source at $(x,y)=(0,R)$. Since this location corresponds to $t=0$, we shall write the corresponding amplitude and phase as $A_o(0)$ and $\phi_o(0)$, respectively. Thus,

$$A_o(0)\exp i\ \phi_o(0)\simeq\sum_{n=0}^{N'-1}10^{-0.1n}\ R^{-1}{}_{n}\eta_{n}\exp i(-\omega T_{no}+\pi).\qquad(51)$$

The reason for our selective approximations in Eqs. 48 and 49 is now clear: we are able to express the complex form of the total field as the product of a source-motion factor and the total field for a stationary source, where each term in the latter is exact. If a term $\exp i\bar{\omega}t$ is cancelled on each side of Eq. 50 and if Eq. 51 is employed, a comparison of the moduli and arguments in the resulting equation leads to the following linearized approximations for the total-field phase and amplitude:

$$\phi(t) \simeq \phi_0(0) \cdots 2M\omega(\cos\alpha)(t-R/c)+(\omega-\bar{\omega})t \ , \qquad (52)$$

and

$$A(t) \simeq A_0(0) \ [1-M(\cos\alpha)(ct/R-1)] \ . \qquad (53)$$

Since ω is usually large, $M\omega$ is not small even for a slowly moving source. Thus Eq. 52 suggests that the effect of source motion on phase variation is significant, in general. However, the effect on amplitude variation is negligible if $Mc/R \ll 1$.

If $\bar{\omega}$ is chosen to be the circular frequency ω at the source, then Eq. 52 simplifies to

$$\phi(t) = \phi_1(t) \simeq \phi_0(0)-2M\omega(\cos\alpha)(t-R/c). \qquad (54)$$

A second procedure, outlined in Sec. I, is to average frequency over the first N' significant ray arrivals and over a time interval τ. When we carry out an average analogous to Eq. 29, we obtain

$$\Delta\omega \simeq - M\omega(\cos\alpha) \ , \qquad (55)$$

where the right-hand side is just the Doppler shift associated with the n=0 ray. If we take $\bar{\omega}=\omega+\Delta\omega$, with $\Delta\omega$ given by Eq. 55, then Eq. 52 gives

$$\phi(t) = \phi_2(t) \simeq \phi_0(0) - 2M\omega(\cos\alpha)(t/2-R/c). \qquad (56)$$

We observe from Eqs. 54 and 56 that both ϕ_1 and ϕ_2 are approximately linear in time, where we have assumed that $|t|$

is small. Furthermore, the total-field phase is the same for $\alpha = \alpha_o$ and $\alpha = -\alpha_o$, so that it is necessary to examine phase only for $0 \leq \alpha \leq \pi$ rad. Differentiation of Eq. 54 with respect to time leads to the observation

$$\phi_1'(t) \Big|_{\alpha=\alpha_o} \approx - \phi_1'(t) \Big|_{\alpha=\pi-\alpha_o} \tag{57}$$

Thus, the slopes of the ϕ_1 versus time curves for $\alpha=\alpha_o$ and $\alpha=\pi-\alpha_o$ are equal in magnitude but opposite in sign. In other words, source trajectories with a component approaching toward or receding from the receiving point at the same rate give rise to the same magnitude of phase-rate. These comments apply to ϕ_2 as well. If Eq. 56 is differentiated with respect to receiver time and compared with the derivative of Eq. 54, we see that

$$\phi_1'(t) \approx 2\phi_2'(t) \ . \tag{58}$$

Therefore, the rate of change of phase with averaged frequency, ϕ_2, is only half that of the phase with unaveraged frequency, ϕ_1.

The relative phases $\phi_1-\phi_o$ and $\phi_2-\phi_o$ appear versus time in Figs. 3 and 4 for selected values of the source-trajectory angle α in deg. For these figures we selected the parameter values R=10 km, $c = 1500$ msec^{-1}, $v = 10$ kn, and $f = 350$ hz. Although the water depth H does not appear in Eqs. 52-58, it will be recalled that our results assume that H/R is small.

With the above parameter values and H = 100m, Eqs. 52, 54, and 56 were found to be in excellent agreement with the exact phase given by Eqs. 26-28 and Eqs. 43-44, when the time interval was restricted to approximately $-5.0 \leq t \leq 5.0$ min. However, Figs. 3 and 4 show a time interval of half this amount. The lower horizontal axes indicate the directed distance r in km traveled by the source from its position at time t = 0. Figure 3 illustrates the linearity of phase over a relatively short time interval and the fact that the slopes of the phase curves for $\alpha = \alpha_o{}^o$ and $\alpha = (180 - \alpha_o)^o$ are equal in magnitude but opposite in sign. Note in Fig. 3 that, over the 5.0 min interval shown, the phase variation is as much as 730 cycles, where the maximum variation occurs when the sound source follows a radial path ($\alpha = 0°, 180°$). When $\alpha = 90°$, the source trajectory is tangent to the circumferential direction at t = 0 and the approximate phase variation is zero as expected, since the source-receiver range is virtually unchanged. Figure 4 is similiar to Fig. 3, but shows the relative phase $\phi_2 - \phi_o$ when average Doppler shift is included. In agreement with Eq. 58, curves in Fig. 4 have only half the slope of corresponding curves in Fig. 3.

Primary phase is phase modulo one cycle, and will be indicated in this paper by the subscript p. Both the primary relative phases $[\phi_1 - \phi_o]_p$ and $[\phi_2 - \phi_o]_p$ appear in Fig. 5 over the time interval $-5.0 \leq t \leq 5.0$ sec when the source moves radially outward from the fixed receiver ($\alpha = 0°$). As in Figs. 3 and 4,

it is observed that phase variation is decreased by one-half upon inclusion of an averaged Doppler-shift frequency. The two curves are qualitatively similiar to corresponding results in Ref. 4, even though phase variation in that reference was calculated for a deep-water sound-speed profile and for different parameter values.

IV. THE LONG TRAJECTORY

Under the assumptions of the previous section, we now consider the source to follow a long straight-line path with constant speed v. We let d denote the distance of closest approach of source to receiver and let $t=0$ describe the time at which S is the distance d from R . Suppose now that the source is a distance $R(t_o)$ from the receiver where $\alpha(t_o)$ is the corresponding angle subtended by the source-path and receiver-source directions, as shown in Fig. 6. In Secs. II and III, R represented the initial distance between source and receiving point at $t=0$, and $\phi_o(0)$ described the phase of a stationary source at a distance R from the receiver at time $t=0$ in Eq. 54. In that equation, $\bar{\omega}$ was chosen to be the circular frequency at the source, and we shall restrict ourselves to this case here. The analog of Eq. 54, for ϕ expanded about t_o rather than zero, is

$$\phi(t) \simeq \phi_o(t_o) - 2M\omega[\cos\alpha(t_o)][(t-t_o)-R(t_o)/c], \quad (59)$$

where $|t-t_o|<<1$ and $\phi_o(t_o)$ is the phase of a stationary source at time $t=t_o$. Now, let t_o be the time required for S to travel

from the point P of closest approach to $\mathcal{R}$ to the generic

position shown in Fig. 6. If the distance traveled is r and

if the speed of $\mathcal{S}$ is v, then $r=vt_o$. Both $R(t_o)$ and $\alpha(t_o)$

are found from the following expressions:

$$R(t_o) = [d^2+v^2t_o^2]^{1/2} = c[(d/c)^2+M^2t_o^2]^{1/2} , \qquad (60)$$

and

$$\cos\alpha(t_o) = vt_o[d^2+v^2t_o^2]^{-1/2} = Mt_o[(d/c)^2+M^2t_o^2]^{-1/2} . \qquad (61)$$

Using Eqs. 60 and 61, Eq. 59 may now be expressed as

$$\Phi(t) \simeq \Phi_o(t_o)+2M^2\omega t_o-2M^2\omega t_o[(d/c)^2+M^2t_o^2]^{-1/2}(t-t_o) \qquad (62)$$

for $|t-t_o| \leq 5$ min and $H/R<<1$. If we let $\hat{\Phi}(t,t_o)\equiv\Phi(t)-\Phi_o(t_o)$

denote the phase relative to that of a stationary source at the

position corresponding to $t=t_o$, then Eq. 62 can be rewritten

as

$$\hat{\Phi}(t,t_o) \simeq -2M^2\omega t_o\left\{[(d/c)^2+M^2t_o^2]^{-1/2}(t-t_o)-1\right\} . \qquad (63)$$

In Fig. 6, primary phase $[\hat{\Phi}(t,t_o)]_P$ is sampled for seven

20 sec intervals over a period of 2 hours or a source travel of

approximately 37 km, for $d = 10$ km. When $t_o\simeq0$, it is seen

that the effect of a slowly moving source on phase variation

is negligible. Such a result is consistent for a source path

tangent to the circumferential direction, as discussed in the

previous section and shown by the $\alpha=90°$ curve in Fig. 3. As

$|t_o|$ increases from zero, a steady increase in the magnitude of phase variation is observed. For large values of $|t_o|$, the source is effectively moving along a radial path from the fixed receiving point, where maximum phase variation was predicted in Fig. 3 by the $\alpha=0°$ and $\alpha=180°$ curves.

A continuous representation of approximate relative cumulative phase, rather than primary phase, can be formulated since it is reasonable to assume that all time derivatives of the phase exist in some neighborhood of each value of t_o. Then, to within the order of the approximations made in the previous section, Eq. 62 represents the first two terms of the Taylor series for $\phi(t)$ about the point $t=t_o$. Since the Taylor coefficient of $(t-t_o)$ is $\phi'(t_o)$, we have from Eq. 62 that

$$\phi'(t_o) \approx - 2M^2\omega t_o [(d/c)^2 + M^2 t_o^2]^{-1/2} \quad . \tag{64}$$

Integration of this result with respect to t_o gives

$$\phi(t_o) \approx -2\omega[(d/c)^2 + M^2 t_o^2]^{1/2} + C, \tag{65}$$

where C is a constant of integration. Because we are interested only in relative cumulative phase, we select $C = 2\omega d/c$ in order to make $\phi(0) = 0$. In addition, we write t in place of t_o to obtain

$$\phi(t) \approx -2\omega[(d/c)^2 + M^2 t^2]^{1/2} + 2\omega d/c \quad . \tag{66}$$

For large values of $|t|$, Eq. 66 becomes

$$\Phi_A(t) \approx -2M\omega|t| + 2\omega d/c . \tag{67}$$

Thus the magnitude of the rate of phase variation is approximately constant for large $|t|$, as observed in Eq. 62 or in Fig. 3 as α approaches $0°$ or $180°$. Further, Eq. 67 may be construed as an upper bound on the cumulative phase.

The approximate cumulative phase, given by Eq. 66, is plotted as the solid curve in Fig. 7 over an eight-hour time period or a corresponding source-travel distance of 148 km. The phase is observed to be hyperbolic and its asymptotes, given by Eq. 67, are graphed as dashed lines. The figure reiterates previous conclusions. For $t \approx 0$, the source is near its point of closest approach to the receiver, and phase variation differs little from zero. As $|t|$ increases, a rapid increase in variation is observed, which approaches a constant when the source trajectory is effectively radial relative to the receiving point. The cumulative phase, Eq. 66, is a global result that was obtained from a local result, Eq. 62. Since the latter is only approximate, Eqs. 66 and 67 need not be accurate for very large values of $|t|$.

V. SUMMARY

This paper presents an analysis for the treatment of the effects of sound-source motion on the total acoustic field at a fixed receiving point in the ocean. The source emits a cw signal, and may follow an arbitrary path in three-dimensional space with arbitrary velocity. Sound speed is

taken to be depth-dependent. Using ray theory, it is explained
how source motion affects ray geometry, frequency, travel time
and spreading loss, and boundary loss and phase shift. It is
shown how the movement of the sound source leads to a received
signal that may be interpreted as a waveform of arbitrary
frequency whose amplitude and phase are time-dependent.

Application of the general theory is made to a constant
sound-speed channel with horizontal boundaries in which the
sound source and receiving point are located on the ocean
bottom. The source is assumed to follow a straight-line
path with constant speed. A simplified linearized acoustic
model is developed when source speed is much less than sound
speed and when the ocean depth is much smaller than the source-
receiver range. The results are valid over a sufficiently short
time interval, and the effect of source motion on acoustic phase
variation is shown to be highly significant. Both primary and
cumulative phase are examined as functions of time for various
directions of the source trajectory. The same magnitude of
phase-rate is obtained for source trajectories with a component
approaching toward or receding from the receiver at the same
rate. Maximum phase change occurs for radial source motion,
while minimum change occurs for circumferential motion. For
selected parameter values, acoustic phase is shown to change by
hundreds of cycles over a time interval of only a few minutes.
When the arbitrary frequency, discussed above, is taken to be

the average Doppler-shift frequency associated with the first ray arrival, the phase variation at the receiver is only one-half the variation when the frequency is chosen to be that of the signal emitted at the source.

The above phase results, which are local in time, are extended to the global case in which the sound source follows a long straight-line path at constant speed. When the source is near its point of closest approach to the receiver, phase variation is small. However, a steady increase in variation is observed as the source departs from this point. An approximate formula for cumulative phase is developed with the result that this phase is a hyperbolic function of time. The asymptotes of the hyperbola provide a bound for the approximate cumulative phase and an estimate to it when the sound-source location is far from its point of closest approach to the receiver.

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FIGURE LEGENDS

FIG. 1. Doppler effect and geometry for a moving source and
fixed receiver.

FIG. 2. Geometry for linear, constant-speed source motion and
two typical ray paths.

FIG. 3. Relative phase versus receiver time and source travel
for selected source angles: $\bar{\omega}=\omega$, R=10km, c=1500msec^{-1},
v=10kn, and f=350hz.

FIG. 4. Relative phase versus receiver time and source travel
for selected source angles: $\bar{\omega}=\omega+\Delta\omega$, R=10km, c=1500msec^{-1},
v=10kn, and f=350hz.

FIG. 5. Primary relative phase versus receiver time and source
travel for the source angle $\alpha=0^{\circ}$ and $\omega=\bar{\omega}$ and $\bar{\omega}=\omega+\Delta\omega$,
respectively. Other parameters as in Fig. 3.

FIG. 6. Primary relative phase versus receiver time and source
travel sampled over a 2 hr interval for $\bar{\omega}=\omega$ and d=10 km.
Other parameters as in Fig. 3.

FIG. 7. Cumulative relative phase versus time and source travel
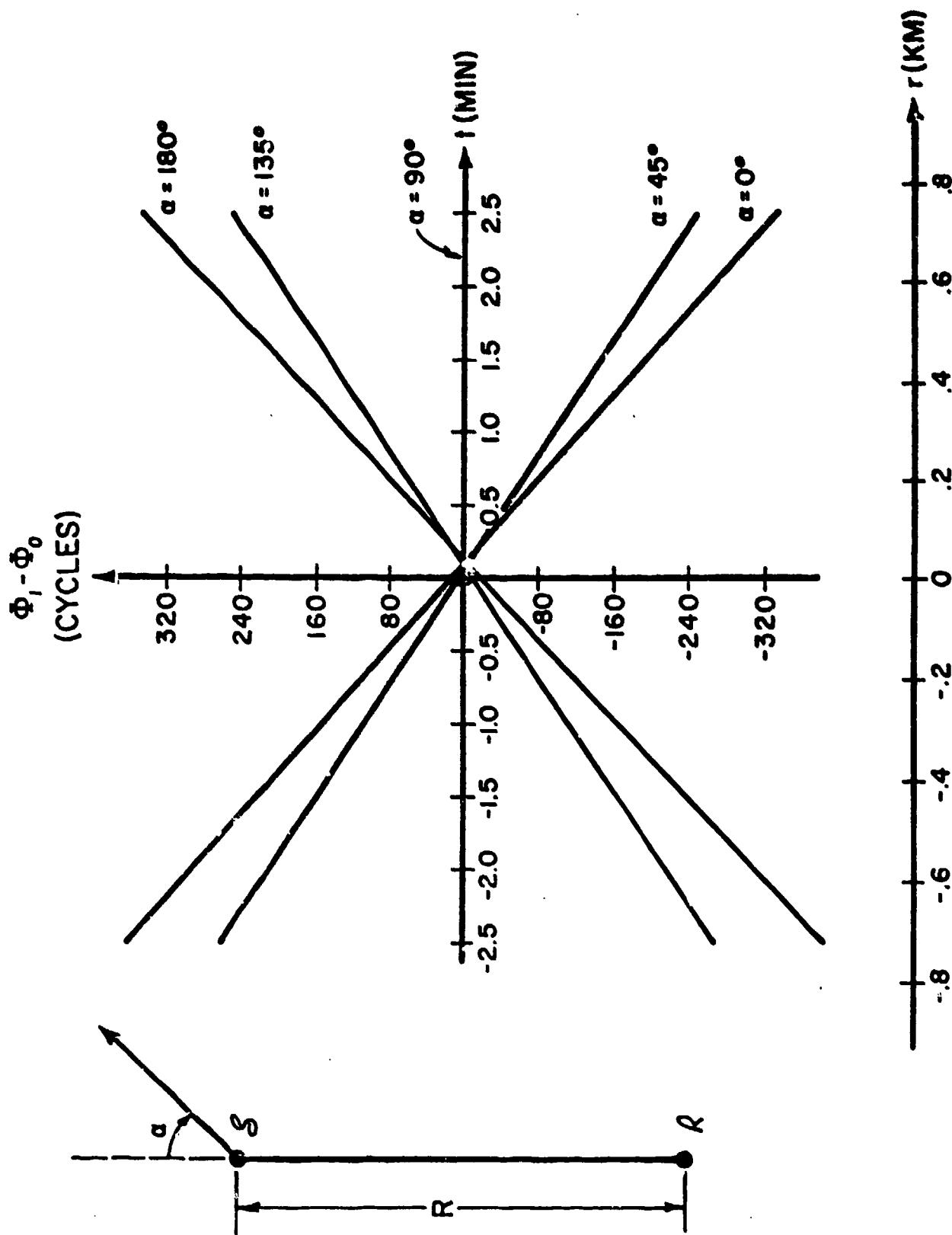over an 8 hr interval for $\bar{\omega}=\omega$ and d=10 km. Other
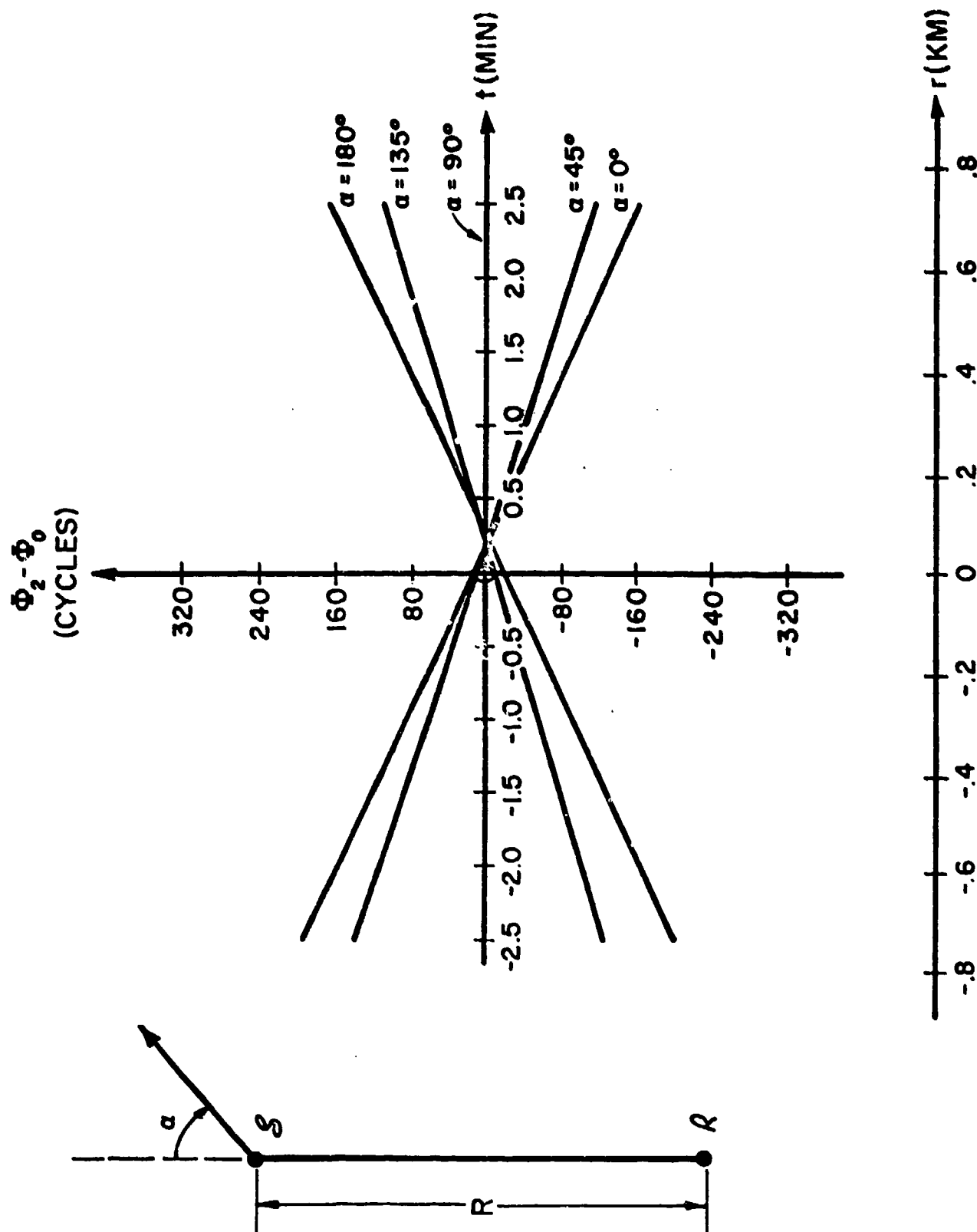parameters as in Fig. 3.
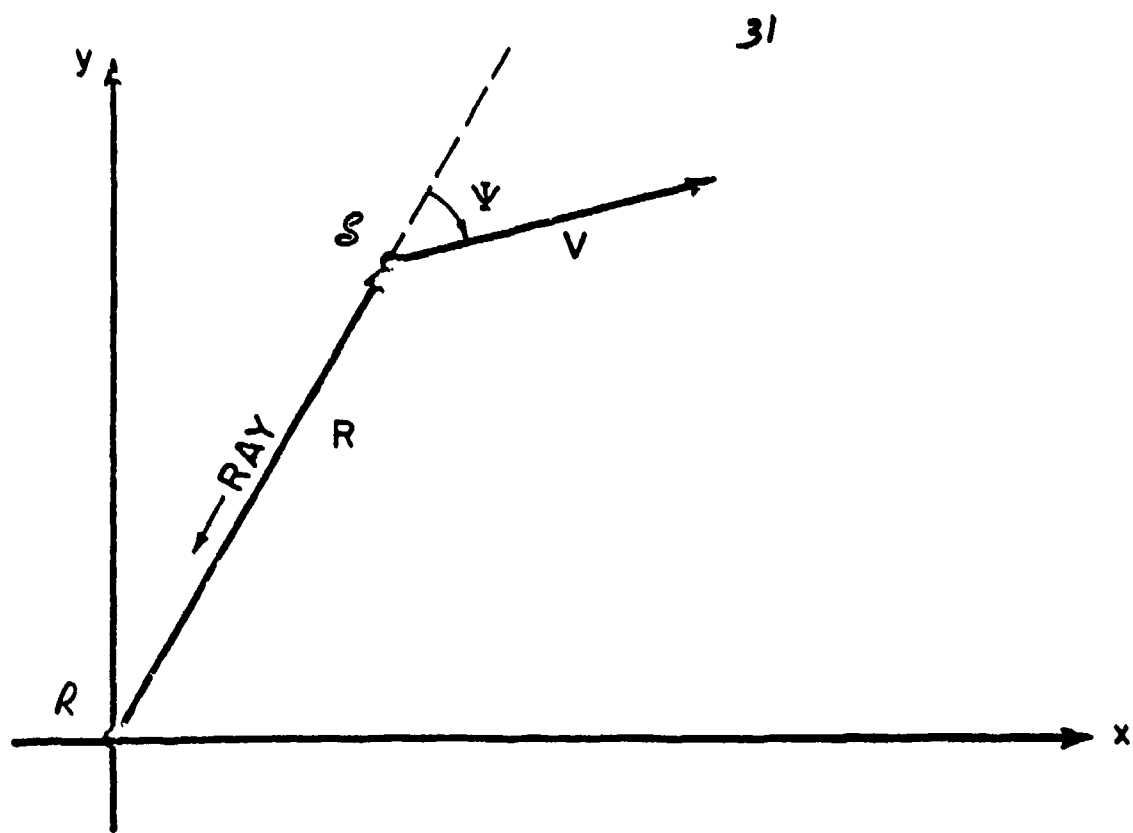
(a)
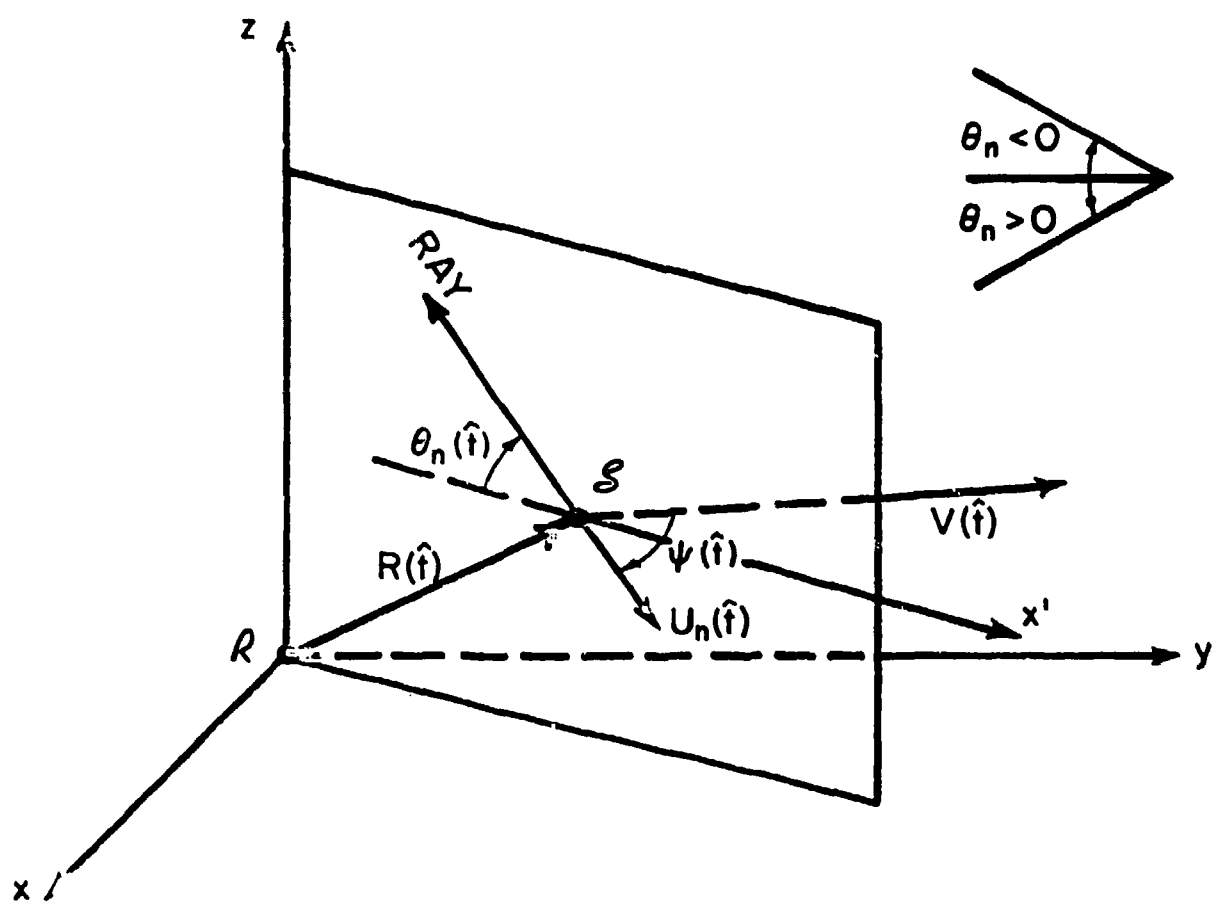
(b)

FIGURE 1
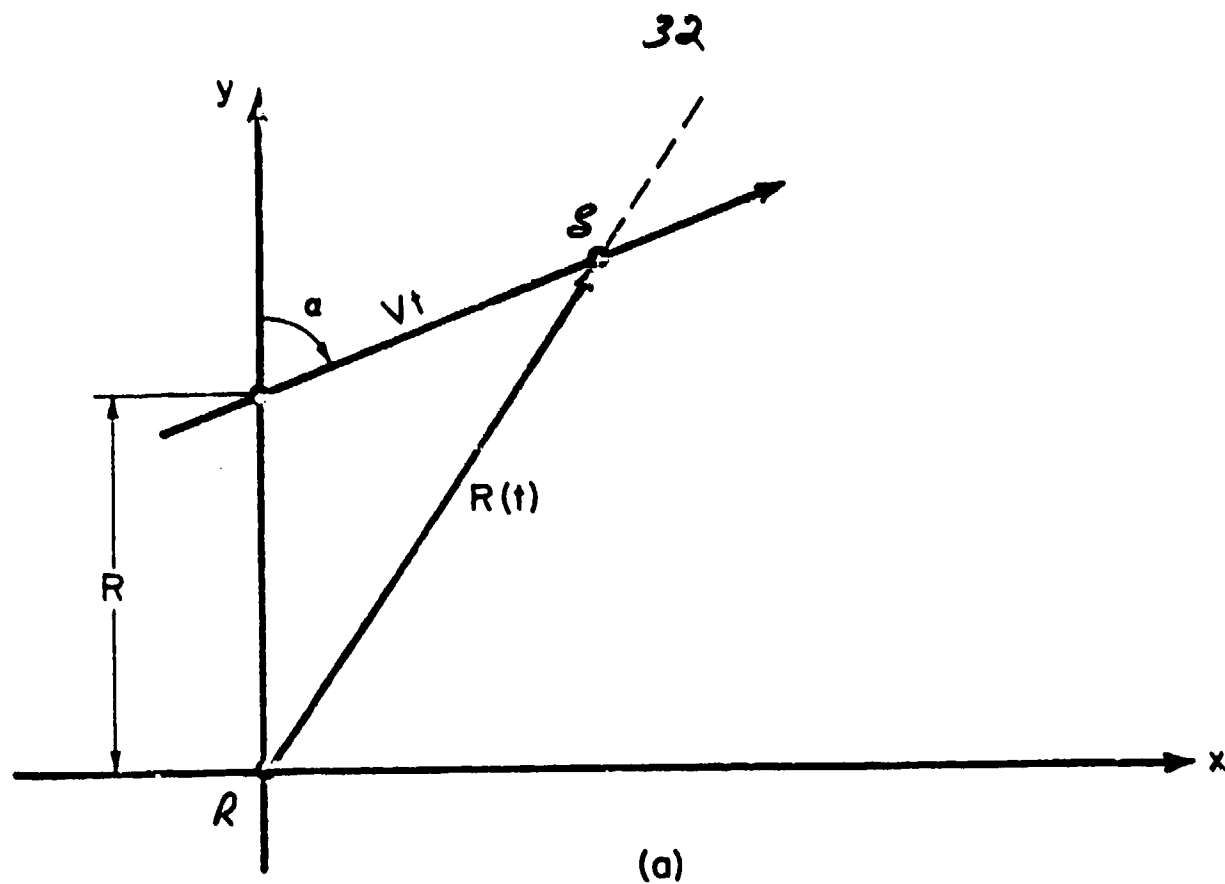
(a)
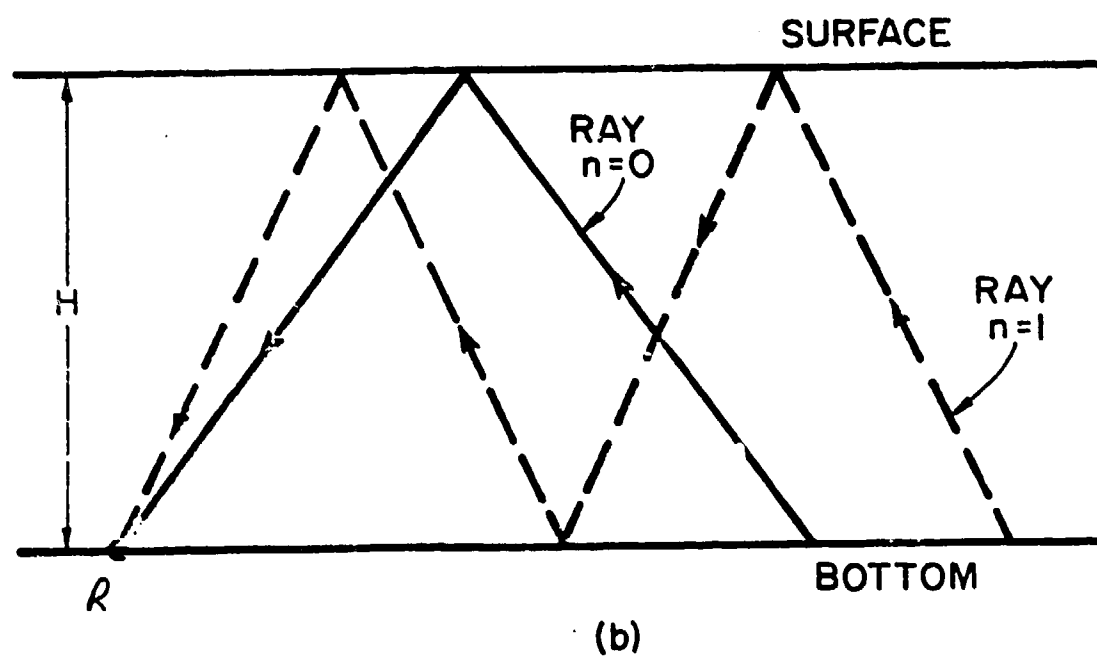
(b)

FIGURE 2

FIGURE 3

FIGURE 4

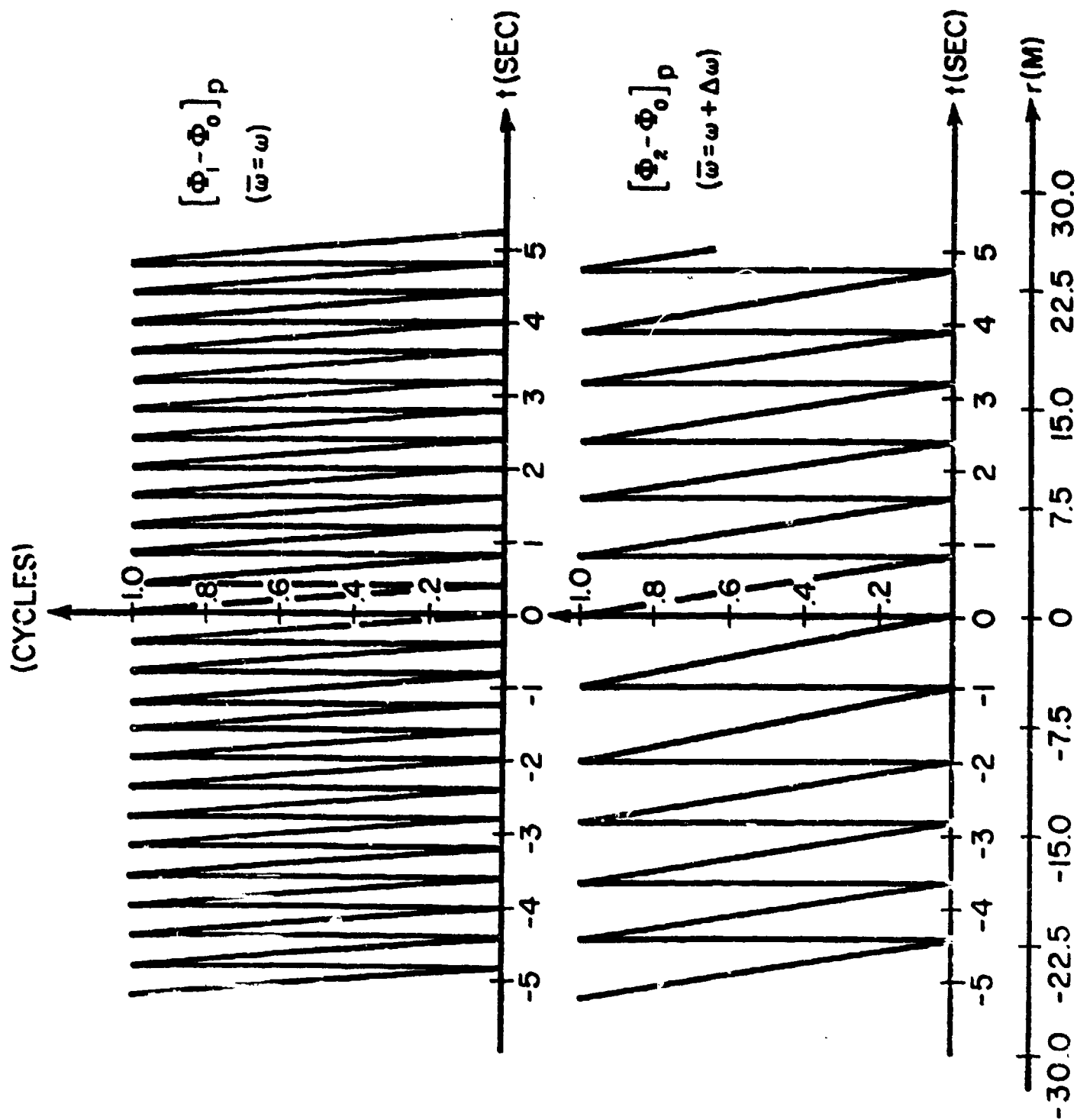FIGURE 5
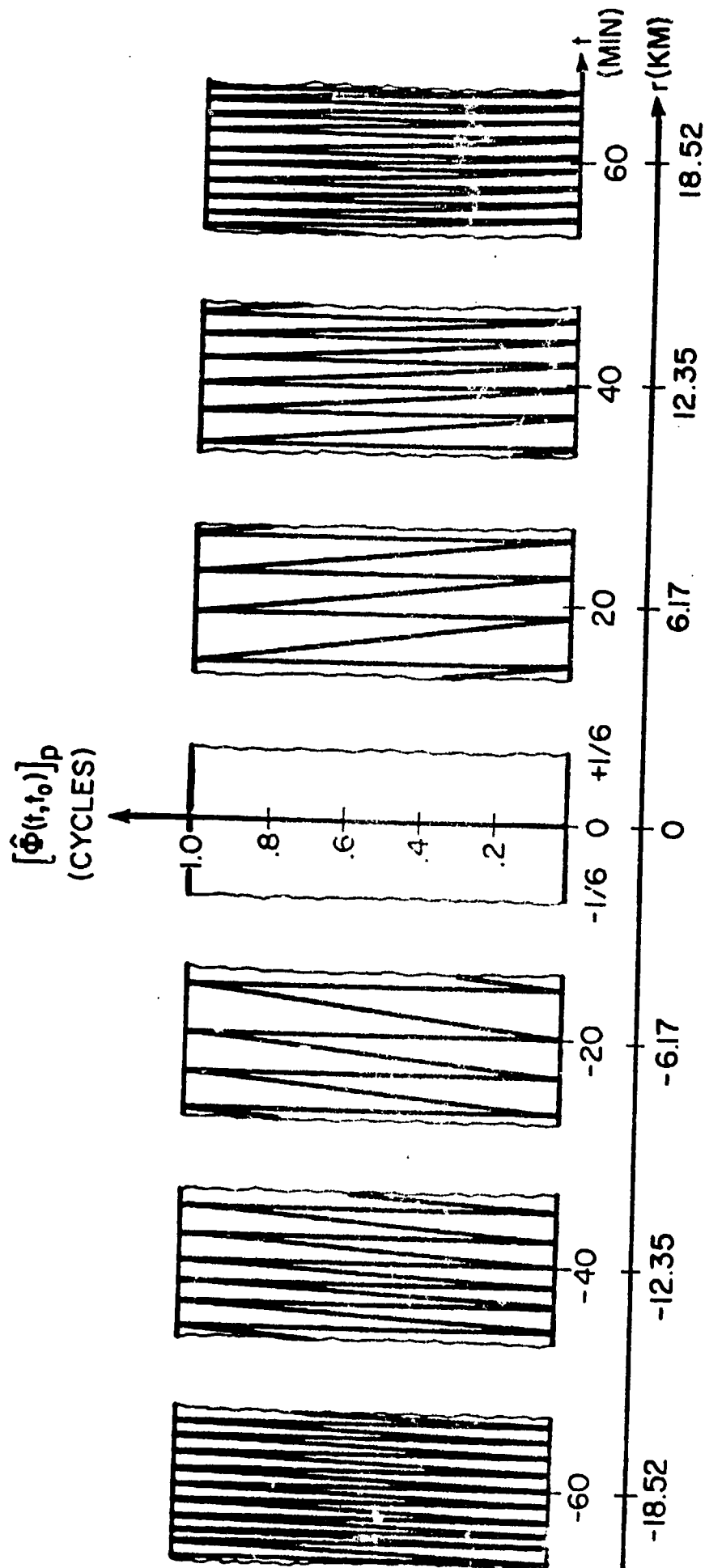
FIGURE 6

FIGURE 7