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IDA GROUND-AIR MODEL I (IDAGAM I)

Volume 1: Comprehensive Description

Lowell Bruce Anderson
Jerome Bracken
James G. Healy
Mary J. Hutzler
Edward P. Kerlin

October 1974

INSTITUTE FOR DEFENSE ANALYSES.
PROGRAM ANALYSIS DIVISION

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The IDA Ground-Air Model I (IDAGAM I) is a deterministic, fully-automated, theater-level model of non-nuclear combat between two opposing forces. The report consists of five volumes, as follows: (1) Comprehensive Description, (2) Definitions of Variables, (3) Detailed Decription of Selected Portions, (4) Documentation, and (5) Testing (U) (SECRET).

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FOREWORD

IDAGAM I is a deterministic, fully automated model of non-nuclear combat between two opposing forces. The purpose of this report is to describe and document IDAGAM I. The report consists of five volumes, the contents of which are summarized as follows:

Volume 1 - Comprehensive Description

Volume 2 - Definitions of Variables

Volume 3 - Detailed Description of Selected Portions

Volume 4 - Documentation

I. STRUCTURE OF IDAGAM I

II. MACHINE CONVERSION

III. PREPARATION OF INPUTS

IV. DESCRIPTION OF OUTPUTS

Appendix A. SAMPLE OUTPUT

Appendix B. RELATIONSHIPS AMONG VARIABLES

Appendix C. VARIABLE SIZES AND LOCATIONS

Volume 5 - Testing

I. DESCRIPTION OF THE TEST PLAN

II. RESULTS OF TESTS

III. CONCLUSIONS

Appendix. SOURCES OF INPUT DATA

Volumes 1, 2, 3, and 4 are Unclassified; Volume 5 is Secret.

Since it would be much too unwieldy to include a copy of the code of the IDAGAM I computer program in this report, no such copy is included here. Copies of this code on appropriate media (tape, cards, etc.) can be obtained directly from the Institute for Defense Analyses.

CONTENTS OF VOLUME 1

FIGURES

INTRODUCTION

IDA Ground-Air Model I (IDAGAM I) is a deterministic, fully automated model of nonnuclear combat between two opposing sides. The purpose of this volume is to provide a comprehensive description of IDAGAM I. Chapter I discusses the level of detail of IDAGAM I. A summary description of the logic and a precise statement of the attrition equations used in IDAGAM I are given in Chapter II. Limitations of IDAGAM I and suggestions for further research are given in Chapter III. Preceding the description of IDAGAM I, some background information is given in this introduction.

The reader interested only in a summary description of IDAGAM I (but not in the details of the attrition equations) should read only Chapter I and Sections A, B.1, C.1, and D.1 of Chapter II of this volume.

IDAGAM I is the latest in what might loosely be called a family of models that began with ATLAS (Kerlin and Cole [19]) and led first to GACAM (Bracken, et al. [12]), then to GACAM II (Bracken, et al. [11]), and then to IDAGAM I. This is a "loose" family because the only thing that really ties it together is that the developers of each succeeding model, before developing that model, looked closely at the advantages and limitations of its predecessors. These models were not developed at the same place or under the same funding; and, with the exception of Edward Kerlin, they were not developed by the same people.

ATLAS had its own forerunners, but they were not fully automated models. GACAM was developed with a goal of simplifying the ATLAS ground-combat process, so that a more elaborate air-combat process could be played and so that the total computer running time of GACAM would be small enough to allow it to be used as part of higher-level models, such as GANCAM and the World-Wide Integrating Model (Bracken, et al. [12]). This goal was accomplished, but in the process the GACAM ground-combat process was, according to some critics, too oversimplified.

This criticism led to GACAM II, which attempted to increase the level of detail of the GACAM ground process by introducing sectors, weapons by type, and other details. However, GACAM II still retained the basic firepower structure of ATLAS and GACAM. This defect was felt to be severe and led to IDAGAM I. In addition to the "omission" characteristic of not being a firepower model, we believe that IDAGAM I has many positive characteristics that make it generally preferable to its predecessors. Indeed, the objective of this effort was not to develop a model designed to address a particular set of questions, but rather it was to improve in general upon previous theater-level models of nonnuclear combat. One purpose of this report is to describe IDAGAM I in sufficient detail that potential users can decide whether this is an appropriate model to address the particular problems that the potential user might have.

The authors would like to acknowledge thorough reviews of this report by Joseph Bruner (of General Research Corporation) and by Seymour Deitchman (as the responsible IDA reviewer). The authors would also like to acknowledge the sustained assistance of Mabel Iverstrom in typing and checking the accuracy of this report.

Chapter I

LEVEL OF DETAIL OF IDAGAM I

A. THEATER STRUCTURE

IDAGAM I is designed to be a theater-level model. If desired, only part of a theater can be played (and, with some restrictions, several theaters can be played simultaneously). For the purposes of this report, we will assume that IDAGAM I is being played at the theater level.

1. Geographical Sectors

The theater structure of IDAGAM I contains a series of nonintersecting geographical sectors that cover the theater area of interest. These geographical sectors are considered as avenues of advance that run the length of the theater, that are not necessarily of constant width, and that are separated from adjacent sectors by rough terrain, neutral countries, or other specifics of the theater area that would hinder combat across sector boundaries. Each geographical sector is assumed to consist of an integral number of intervals. Terrain-type variations, width-of-sector variations, defensive barriers, and other geographical characteristics will be played with these intervals. Intervals are assumed to be areas of constant width but variable depth, with the terrain or barrier assumed to be constant throughout the interval. (Additional information concerning the way that geography and defensive barriers are played in IDAGAM I is given in Chapter VI of Volume 3.)

2. Combat Sectors, Regions, and COMMZ

While geographical sectors run the length of the theater, ground combat will take place only in the portion of the geographical sector that is relatively near the line that separates the two sides. (This line is traditionally called the forward edge of the battle area, or FEBA.) If a unit is in the J^{th} geographical sector and is close enough to the FEBA to participate in combat, we will say that the unit is in the J^{th} combat sector (or, simply, in the J^{th} sector). If that unit is not close enough to participate in combat (i.e., it is in reserve), then we will say that the unit is in a region or in the COMMZ (defined below).

Regions consist of the rear portion of one or more geographical sectors. The left boundary of a region is the left boundary of the leftmost geographical sector contained in that region, and the right boundary of a region is the right boundary of the rightmost geographical sector contained in that region. Which geographical sectors are contained in which regions is an input to the model.[1] The grouping of geographical sectors into regions need not be the same for both sides. Thus, since the forward part of a geographical sector is a combat sector, each region boundary must coincide with the boundary of some combat sector. But region boundaries for one side need not correspond to region boundaries for the other side.

The number of regions and sectors played in IDAGAM I is an input. The maximum number of regions and sectors that can be played is part of the computer program, but this maximum number is easily changed. (A complete discussion of these maximums is given in Chapter I of Volume 3.)

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[1]However, if two geographical sectors are in a region, then all geographical sectors between them must be in the same region.

There is exactly one communications zone (COMMZ) for each side in the theater. The COMMZ for each side is located to the rear of all ground battle activity.

With the two opposing sides being denoted by Blue and Red, an example of theater structure with seven sectors, two Blue regions, and three Red regions is given in Figure 1.

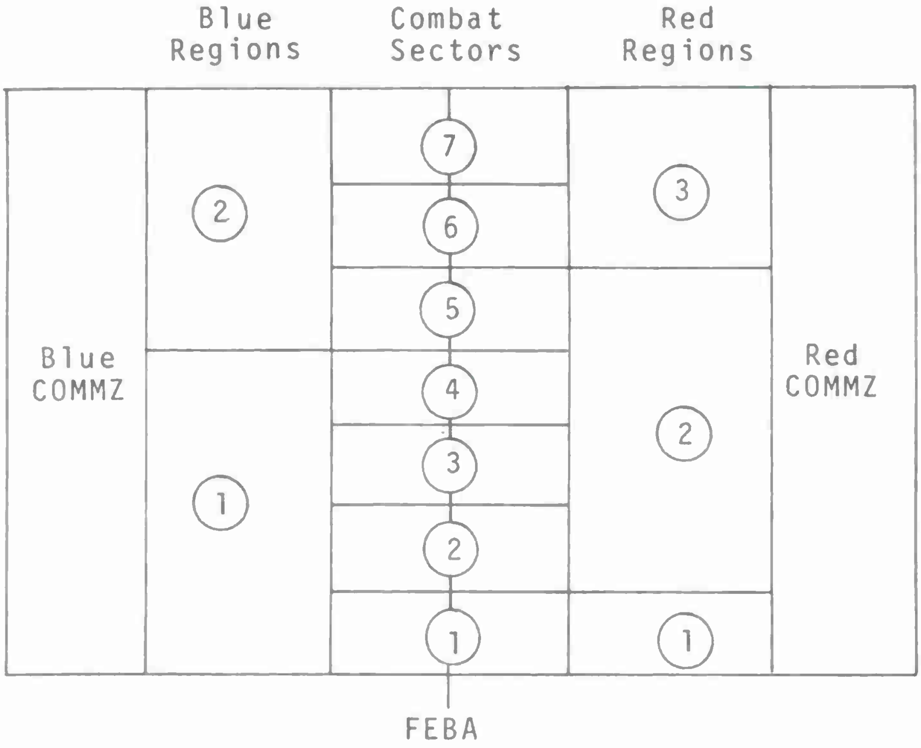

Figure 1. EXAMPLE OF THEATER STRUCTURE

B. RESOURCES

The two opposing sides, which are denoted by Blue and Red, are represented symmetrically in IDAGAM I. Each side can have ground resources and air resources.

1. Ground Resources

a. People

IDAGAM I accounts for exactly four categories of people (combat, combat support, service support, and theater support). However, the fourth category, theater-support personnel, must always be located in the COMMZ and cannot cause or suffer attrition (except for nonbattle-related casualties). They are considered only for accounting and supply-consumption purposes.

The distinction between the other three categories is also only for accounting purposes; however, the sum of the number of people in these three categories is significant in the attrition and FEBA-movement calculations. Thus, IDAGAM I actually "plays" only one type of people; and this type is the sum of the number of people in the first three personnel categories listed above.

b. Weapons

The number of types of weapons on each side that are played in IDAGAM I is an input.[1] For example, IDAGAM I could be played with 10 types of weapons on each side, where

Blue weapon type 1 = small arms;
Blue weapon type 2 = armored personnel carriers;
Blue weapon type 3 = tanks;
Blue weapon type 4 = antitank weapons;
Blue weapon type 5 = field artillery (155 mm or 8 in.);
Blue weapon type 6 = field artillery (175 mm);
Blue weapon type 7 = mortars;
Blue weapon type 8 = helicopters;
Blue weapon type 9 = antiaircraft artillery;

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[1]The maximum number of types of weapons that can be played is fixed in the computer program but can easily be changed. As many other quantities have this property, we will not make this statement each time such a quantity is introduced. (Instead, a complete list and full explanation is given in Chapter I of Volume 3.)

Blue weapon type 10 = surface-to-air missile systems;

and

Red weapon type 1 = small arms;
Red weapon type 2 = armored personnel carriers;
Red weapon type 3 = tanks;
Red weapon type 4 = antitank weapons;
Red weapon type 5 = field artillery (155 mm or 122 mm);
Red weapon type 6 = field artillery (130 mm);
Red weapon type 7 = mortars;
Red weapon type 8 = multiple rocket launchers;
Red weapon type 9 = antiaircraft artillery;
Red weapon type 10 = surface-to-air missile systems.

The last type of weapon on either side must be surface-to-air missile systems (SAMs) and, if a second type of ground-to-air weapon system (such as short-range SAMs or antiaircraft artillery (AAA)) is to be played, then it must be the second-to-last type of weapon.[1] Other than this, there is no formal restriction on what types of weapons can be played; and different types can be played on each side. However, some types of weapons (e.g., nuclear weapons or biological weapons) would not be appropriately modeled by IDAGAM I.

c. Minefields

Mines are not played as a type of weapon as described above. Instead, IDAGAM I plays preplanned minefields that are assumed

[1]IDAGAM I allows up to two types of ground-to-air weapons to be played, and one of these types is assumed to have shorter range than the other. In this report, we assume that these two types of weapons are SAMs and AAA. However, IDAGAM I can also be used to play two types of SAMs (long-range and short-range), provided either that AAA are not played or that they are grouped together with short-range SAMs as the second-to-last type of weapon.

If two types of ground-to-air weapons are not played, then certain parameters must be zeroed out, so that the second-to-last type of weapon is given no ground-to-air capability and no aircraft are sent on AAA-suppression missions.

to extend across one or more sectors and that can be of variable depth and density.[1] Minefields affect the movement and casualty rate of the attacker until the minefield is breached. (A defense using a minefield is treated essentially as a separate type of defensive posture, which is discussed further in Chapter VI of Volume 3.)

d. Divisions

The number of types of units that are played on each side is an input. However, these units must be separate units; one cannot be a part of another. Thus, IDAGAM I can play divisions and separate brigades, or brigades and separate battalions, but it cannot simultaneously play both divisions and the brigades that compose those divisions. Subject to this restriction, a unit of any size can be played; and units of various sizes can be played simultaneously.

IDAGAM I was designed with the idea of playing notional units (i.e., a type-i unit in sector J would be indistinguishable from any other type-i unit in sector J). (Units of different types in the same sector and units of the same type in different sectors are distinguishable.) However, it is possible under some severe restrictions to use IDAGAM I to play individual units. (This possibility is discussed at the end of Chapter I of Volume 3.) For simplicity, throughout this report we will assume that notional units are being played, which we will refer to as divisions.

As an example, IDAGAM I could be played with four types of notional divisions on each side, where

Blue division type 1 = U.S. armor division;

[1]If an actual minefield spans, say, one-half of a sector, then this partial span can be approximated by a minefield that spans the whole sector and has a depth of one-half (or less) of the actual minefield.

Blue division type 2 = U.S. mechanized infantry division;

Blue division type 3 = non-U.S. NATO armor division;

Blue division type 4 = non-U.S. NATO mechanized infantry division;

and

Red division type 1 = Soviet/Czech tank division;

Red division type 2 = Soviet/Czech motorized rifle division;

Red division type 3 = E. German/Polish tank division;

Red division type 4 = E. German/Polish motorized rifle division.

Each type of division has several characteristics that differentiate it from other types of divisions. For example, each type of division has its own--

(1) TOE (authorized number of people and weapons by type);

(2) actual number of people and weapons by type;

(3) effectiveness degradation function (this gives the percent degradation of effectiveness as a function of the percent strength of the division);

(4) reorganization rate;

(5) relative-movement rate;

(6) relative size;

(7) personnel strength level at which the division is withdrawn from combat; and

(8) personnel strength level required for the division to enter combat.

e. Location of Divisions, Weapons, and People

A divison can be located in any (combat) sector, in any of its side's regions, or in its side's COMMZ.[1] If a division is in a sector, then that division is in combat and can cause

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[1]This structure gives the maximum number of possible locations for each division. Fewer locations for specific types of divisions can be played. For example, if it is desired, only U.S. divisions in certain sectors and only non-U.S. NATO divisions in other sectors can be played.

and receive casualties. If a division is in a region or the COMMZ, then it is in reserve and cannot cause casualties. Divisions in the COMMZ cannot receive casualties, but divisions in regions can be attacked by enemy aircraft.[1]

Weapons can either be in a division (and so be located wherever the division is located) or be in the weapon-replacement pool in the COMMZ (there is one such pool for each side). There can never be more weapons in a division than the TOE of that division calls for (and, of course, the TOE of a particular type of division might call for no weapons of a particular type).

The first three types of people (combat, combat support, and service support) can either be in a division or be the personnel-replacement pool in the COMMZ. However, unlike weapons, people are not distinguished by type when they are in the replacement pool. They are distinguished by type only in divisions; and when people are moved from the replacement pool to a division, they are assumed to be distributed by type according to the needs of that division.

The fourth type of people (theater support) are located in the COMMZ and cannot be either in divisions or in the personnel-replacement pool.

f. Supplies

IDAGAM I plays exactly one type of supplies, which should be considered as general supplies and should include food, ammunition, and fuel for both ground forces and air forces (they are included under ground resources for convenience only). The unit of measure for these general supplies is considered to be tons, and the model assumes that each person (by type), each

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[1] IDAGAM I does not play enemy air attacks on resources in the COMMZ.
All divisions in reserve can suffer nonbattle-related casualties.

ground weapon (by type), and each aircraft (by type) consumes a certain number of tons of supply per time period (for people and ground weapons, this consumption rate is also a function of posture). (The way that supplies are played in IDAGAM I is discussed further in Section D.4, Chapter II, of this volume.)

g. Surface-To-Air Missile Systems

SAMs differ from the other types of weapons in the following respects: First, as noted above, SAMs have no "ground value" and are the only weapons with no such value. (We assume that AAA could be used against enemy ground troops.) Second the "ammunition" for the SAMs (i.e., the actual missiles themselves) are not played as part of the general supplies but are accounted for separately.[1] Finally, both SAMs and AAA can be used to defend airbases (as will be discussed in Subsection 2, which follows).

2. Air Resources

a. Aircraft

The number of types of aircraft on each side that are played in IDAGAM I is an input. (See Chapter I of Volume 3 for a discussion of the maximum number of types of aircraft that can be played.) For example, IDAGAM I could be played with seven types of Blue aircraft and three types of Red aircraft, where

[1]This was done because these missiles are, in a sense, weapons themselves and because the number of missiles fired is dependent not only on the number of launching systems but also on the number of enemy sorties that fly within range of these systems. Thus, the number of missiles expended per launcher is not constant; but, rather, it must be computed by the model. Given that the model has to do this computation, it seems worthwhile to keep track of the number of missiles expended.

Blue aircraft type 1 = F-4;
Blue aircraft type 2 = A-7;
Blue aircraft type 3 = F-105;
Blue aircraft type 4 = F-111;
Blue aircraft type 5 = F-100;
Blue aircraft type 6 = F-104;
Blue aircraft type 7 = F-5 or G-91;

and

Red aircraft type 1 = SU-7 or MIG-19;
Red aircraft type 2 = MIG-17 or MIG-21;
Red aircraft type 3 = MIG-23.

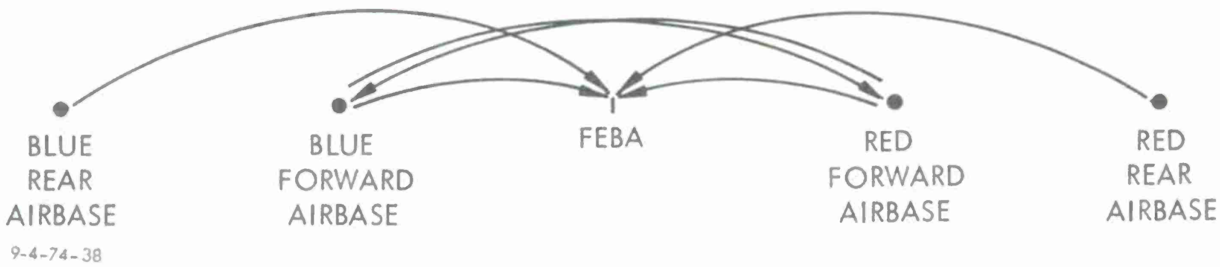
Parameters such as kill-effectiveness in an engagement with an enemy aircraft, effectiveness when attacking an enemy airbase, notional load of munitions (and hence effectiveness on close-air-support missions), sortie rate, range considerations, shelter priority, and occupancy factor (the percent of time that an aircraft is on the ground and so is vulnerable to an enemy air-borne attack)--these parameters are functions of type of aircraft. The input assignment of aircraft to missions is also a function of aircraft type (as will be explained in Subsection d, below).

b. Airbases

IDAGAM I plays two notional airbases in each region and one notional airbase in the COMMZ for each side. Unlike the number of types of weapons or types of aircraft, this number of notional airbases is fixed in the model and cannot easily be increased. (It would be easy to play fewer than this number of notional bases.) The rationale behind choosing this structure is as follows:

One notional airbase for each side was considered insufficient for the following reasons: First, certain aircraft may be able to fly close-air-support (CAS) missions only in certain sectors. For example, U.S. aircraft may be able to provide support only in sectors defended by U.S. divisions, and non-U.S. NATO aircraft may only be able to provide support in sectors defended by non-U.S. NATO divisions. These restrictions are handled by playing at least one notional airbase in each region.

However, allowing only one notional airbase for each region is still insufficient. To see this, consider the following simple case: Suppose each side had only one type of aircraft, and suppose that Blue's aircraft were identical in all respects to Red's aircraft. Suppose, further, that these aircraft had a fixed range--no matter what mission they flew. Then, aircraft that are assigned to attack an enemy airbase would have to be based close enough to the FEBA to reach that enemy airbase, whereas aircraft that are assigned to provide CAS need by only close enough to reach the FEBA. The two-sided picture is as follows:



Blue aircraft could attack the Red rear airbase only from the Blue forward airbase--and only if that airbase were very near the FEBA--and vice versa. But since an airbase very near the FEBA would be subject to attack by artillery (among other things), it is not reasonable to locate an airbase there. Thus, in this simple case, if a side wanted to use its aircraft on CAS missions only, it could assign all its aircraft to its (invulnerable) rear airbase. But any aircraft that are to fly airbase-attack (ABA) missions (or be flexible enough to fly either mission) must be stationed on the forward airbase, and these aircraft could attack only the enemy forward airbase.[1]

Of course, this case is oversimplified. Different types of aircraft have different maximum ranges, and there are range-payload trade-offs. But even in this simple case, if a model is to play both CAS and ABA missions properly, then the model should play at least two notional airbases--one forward and one rear. And if playing airbases in this way is required in the simple case, then in the more complex case, where actual ranges of the aircraft played are considered, a model would certainly be deficient if it did not play airbases at different ranges from the FEBA.

IDAGAM I can play airbases at three different ranges from the FEBA. The notional airbases closest to the FEBA are those in the forward part of the regions. The next closest are those in the rear parts of the regions. The notional airbase farthest from the FEBA is the one in the COMMZ. The forward and rear airbases in each region are meant to handle the problem described above, and the COMMZ airbase is meant to allow IDAGAM I to play long-range aircraft (like the F-111) or aircraft that could have a long range at the cost of a reduced payload.

In summary, the airbases played in IDAGAM I are notional airbases, each of which is considered to be composed of one or more actual airbases. IDAGAM I plays several notional airbases, partially in order to allow aircraft to provide CAS in certain sectors only, but primarily to play range considerations related to different types of missions as well as to different types of aircraft.

Each notional airbase is characterized by its location (the region number and forward or rear, or the COMMZ), by the number

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[1]This type of analysis has previously been used for naval aircraft that are based on aircraft carriers and are used to provide CAS.

of aircraft of each type based on it, and by the number of SAMs and AAA that are protecting it. These notional airbases can be pictured as moving as the FEBA moves. So if the FEBA were to move 200 km in the same direction in each sector, then each notional airbase can be thought of as moving 200 km in that direction; and the aircraft and SAMs and AAA would move along with their airbase. Aircraft based on each notional airbase can be protected by shelters, but these shelters are not "part" of the airbase because the shelters are in fixed positions and do not move as the airbase moves. (It is possible also to play "movable" shelters in IDAGAM I, under some restrictive assumptions.)

c. Aircraft Shelters

IDAGAM I plays one type of shelter for aircraft on each side. These shelters provide protection for the aircraft from enemy airbase attacks.

With the exception just noted, these shelters are considered to be in fixed positions. Thus, for example, a shelter may initially be so far from the FEBA that it cannot be used even by aircraft based in the COMMZ. If the FEBA were to move closer and closer to that shelter, the shelter could be used first by aircraft based on the COMMZ airbase, then by aircraft based on the rear-region airbase, and finally by aircraft based on the forward-region airbase. Then, if the FEBA were to move still closer, the shelter would be considered to be too close to the FEBA to be used even by aircraft based on the forward-region airbase. Finally, if the FEBA crosses the location of the shelter (i.e., the shelter is overrun by the enemy), the shelter is assumed to be destroyed. The initial location of the shelters and the distances from the FEBA (that determine which shelters can be used by which notional airbases) are input. New shelters can be added (i.e., built) at any time during the war by input.

Aircraft are sheltered according to a priority-sheltering scheme, so that if type-3 Blue aircraft have a higher priority for sheltering than type-2 Blue aircraft, all type-3 Blue aircraft are sheltered on an airbase before any type-2 Blue aircraft are sheltered. The priority for sheltering is input.

d. Missions for Aircraft

IDAGAM I plays seven primary missions and five secondary missions for aircraft. The seven primary missions are

(1) Close air support (CAS)
(2) Close-air-support escort (CASE)
(3) Battlefield defense (BD)
(4) Airbase attack (ABA)
(5) Airbase-attack escort (ABAE)
(6) Airbase defense (ABD)
(7) Interdiction of divisions in reserve (IDR).

Aircraft on CAS missions attempt to fire on enemy divisions in combat. Aircraft on BD missions attempt to intercept the enemy CAS aircraft before they can fire at friendly divisions. Aircraft on CASE missions attempt to engage the enemy BD aircraft before those aircraft intercept the CAS aircraft. Aircraft on ABA missions attempt to destroy enemy aircraft on the ground (i.e., on enemy airbases). Aircraft on ABD missions attempt to intercept enemy ABA aircraft before the latter can destroy friendly aircraft on the ground. Aircraft on ABAE missions attempt to engage the enemy ABD aircraft before those aircraft can intercept the ABA aircraft.[1] And aircraft on IDR missions attempt to fire on divisions in reserve in regions.

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[1] These interactions are not the only interactions possible. For example, since aircraft on ABA missions may have to pass over the battlefield in order to get to the enemy airbase, these aircraft might be engaged by enemy BD aircraft. Further details concerning these interactions are contained in the discussions about the air-combat model.

IDAGAM I requires as input the percent of aircraft of each type that are sent on each primary mission. (An input percentage could be zero for a mission if it is desired not to play that mission.) These input percentages can be changed over time by input, but IDAGAM I cannot internally calculate these percentages. Based on these input percentages and on other input parameters, IDAGAM I calculates which airbases the CAS missions come from and, based on where they come from, into which sectors they attack. Similarly, IDAGAM I calculates which airbases aircraft on the other missions come from and where they go to perform those missions (i.e., which enemy airbase ABA aircraft attack, which airbase ABD aircraft defend, etc.). (The way these calculations are made is discussed in Chapter II of Volume 3.)

The five secondary missions are

(1) Supply-interdiction
(2) SAM-suppression against SAMs in combat sectors
(3) AAA-suppression against AAA in combat sectors
(4) SAM-suppression against SAMs defending airbases
(5) AAA-suppression against AAA defending airbases.

Aircraft on supply-interdiction missions attempt to block or destroy enemy supplies en route to the combat sector. (Supplies already in combat sectors can be destroyed by aircraft on CAS missions.) Aircraft on SAM-suppression missions attempt to destroy enemy SAMs in the appropriate locations, and aircraft on AAA-suppression missions attempt to destroy enemy AAA in the appropriate locations.

A percentage of the aircraft on CAS missions are diverted from these missions and are assigned to fly SAM-suppression missions and AAA-suppression missions against the SAMs and AAA defending in the sector in which those aircraft are attacking. What has just been specified for CAS missions holds for aircraft on ABA missions and for SAMs and AAA defending the airbases that those aircraft are attacking. These percentages are not fixed, but are based on input parameters, the number of sorties flown, the number of SAMs, and the number of AAA systems that are defending the appropriate target. An input percentage of the remaining aircraft on CAS missions and an input percentage of the aircraft on IDR missions are diverted from those missions and assigned to fly supply-interdiction missions. These five types of missions are called secondary missions because the number of aircraft assigned to them are determined as a percentage of aircraft assigned to primary missions.

Two types of aircraft missions that are not played in IDAGAM I are deep-supply-interdiction missions (for example, aircraft cannot be assigned to destroy supplies in the enemy's COMMZ or ports) and attack missions against replacements or divisions in reserve in the COMMZ.

e. Air Munitions

The number of types of air munitions that are considered in IDAGAM I is an input. (See Chapter I of Volume 3 for a discussion of the maximum number of types of air munitions that can be considered.) For example, IDAGAM I could be played with nine types of Blue air munitions and five types of Red air munitions, where

Blue air munition type 1 = cannon;
Blue air munition type 2 = 2.75 FFAR (rockets);
Blue air munition type 3 = M-82 (iron bombs);
Blue air munition type 4 = napalm;
Blue air munition type 5 = cluster-bomb units of type 1;
Blue air munition type 6 = cluster-bomb units of type 2;
Blue air munition type 7 = rockeyes;
Blue air munition type 8 = cluster-bomb units of type 3;

Blue air munition type 9 = MK-84EO (smart bombs);

and

Red air munition type 1 = cannon;

Red air munition type 2 = iron bombs;

Red air munition type 3 = S-24 (rockets);

Red air munition type 4 = antipersonnel cluster-bomb units;

Red air munition type 5 = antiarmor cluster-bomb units.

Air munitions are used to determine the effectiveness of aircraft on CAS missions. This determination is made in a different way than for ground weapons. Each type of ground weapon has certain allocation and effectiveness parameters associated with it, and the types of ammunition for the ground weapons are not explicitly accounted for. However, the corresponding allocation and effectiveness parameters for air are associated with the various types of air munitions rather than with the types of aircraft. A notional load of the various types of munitions is input for each type of aircraft. (These notional loads can depend on whether the side is attacking or defending.) This notional load should be the weighted average of actual loads that aircraft of that type are likely to carry. The effectiveness of a CAS sortie of a particular type of aircraft is determined by the notional load of that type of aircraft and the effectiveness of each type of munition in that notional load. Just as ground ammunition is not played explicitly, neither are these air munitions played explicitly--in the sense that IDAGAM I does not keep track of a stockpile for each type of munition to determine when that type of munition is exhausted. (IDAGAM I accounts for consumption of supplies--which can include munitions--associated with aircraft, as described in Subsection 1.f, above.)

3. Allocation of Resources

IDAGAM I is not an optimization model. Some of the resources played in IDAGAM I must be directly allocated by user-supplied inputs; others can be allocated either in this way or by the automated decision logic in IDAGAM I (along with user-supplied parameters for this logic). A few allocations must be made by the automated decision logic with user-supplied parameters. (This characteristic of IDAGAM I is discussed further in Chapter III.)

Chapter II

DESCRIPTION OF IDAGAM I

A. MODEL STRUCTURE

IDAGAM I is a fixed time-step model of a war between two sides. (It is possible to vary the size of the time-steps through sufficiently complicated inputs, but for all practical purposes IDAGAM I has fixed time-steps.) IDAGAM I does not model the events that might occur before the start of the war or the events that might occur after the end of the war, nor does it model how long the war will last. The number of time periods to be played must be input, and IDAGAM I will play this number of time periods regardless of what happens to either side. (Of course, the user could ask IDAGAM I to play, say, 90 time periods, then judge from the output that the war would have ended after the 45th time period, and then ignore all outputs after the 45th time period--a procedure that would give the same output as asking IDAGAM I to play 45 time periods.)

IDAGAM I places no formal restriction on how long these time periods should be. However, there are some logical difficulties in playing time periods shorter than one day (for example, there is no automatic way for IDAGAM I to distinguish daylight from darkness). (Further details on using IDAGAM I to play time periods shorter than one day are discussed in Chapter I of Volume 3.) Accordingly, for the rest of this report we will assume that the time period played in IDAGAM I is one day (24 hours). (If desired, IDAGAM I can be played with 2-day, 3-day, or longer time periods.)

In addition to the number of time periods (days) played, the user must input the conditions (forces and parameters) at the start of the first day and all new forces that are to be added to each side during the war. (The user can also change parameters during the war, if he so chooses.)

The basic operation of the model has the following nine steps:

(1) Read initial forces and parameters.

(2) Set up the theater controls for the first day of combat.

(3) Set the "day being played" counter equal to 1.

(4) Do the air-combat model for that day.

(5) Do the ground-combat model for that day.

(6) Test to see if the "day being played" counter equals the input number of days to be played. If so, stop; if not, continue.

(7) Do the theater-control model for that day.

(8) Read new parameters, read force increments, and read user-directed divisional moves as appropriate.

(9) Add 1 to the "day being played" counter and go back to Step 4.

Each of these steps is now discussed briefly and in turn.

Step 1. Except for the number of days to be played and a few "bookkeeping" inputs that are read into the model in the main control program (which is called MAIN in the IDAGAM I computer program), the initial inputs to IDAGAM I are read into the model in one of the following four subroutines:

Computer Program Name	Short Definition
(1) RCD	Read Campaign Description;
(2) RFTZ	Read Forces at Time Zero;
(3) RPTZ	Read (ground-combat and theater-control) Parameters at Time Zero;
(4) RPAC	Read Parameters for Air Combat (at time zero).

RCD reads the number of weapon types, division types, aircraft types, sectors, regions, etc., to be played in that run of the model. RFTZ reads the initial forces (both ground and air) into the model. RPTZ reads the parameters for the ground-combat model and the theater-control model. RPAC reads the parameters for the air-combat model. To read in these initial values, these subroutines are called only once.

Step 2. Before the first day of battle can take place, certain quantities must be calculated from the input parameters. This calculation is done in the Theater Control at Time Zero subroutine (which is called the TCTZ subroutine in the computer program). The quantities computed in TCTZ are of two types: First, there are some quantities (such as terrain and width of sector) that are determined in the theater-control model for the next day's battle; they are determined for the first day of battle in the TCTZ subroutine.[1] Second, there are some quantities (such as the value of a weapon against a standard force) that need to be computed only once for the whole war. Like the subroutines described in Step 1, the TCTZ subroutine is called only once per run of IDAGAM I.

Step 3. The model sets up the iterative procedure at this step. The "day being played" counter is initially set equal to 1 and, at the end of each iteration, 1 is added to this counter. The model will stop when this counter reads the input number of days to be played as described in Step 6.

Step 4. The air-combat model (called the AC program overlay in the computer program) calculates all air-to-air

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[1]The theater-control model was not used here because it performs many other operations (besides determining these quantities) that do not need to be calculated before the first day of battle.

and ground-to-air attrition, the attrition of SAMs and AAA due to suppression missions, the attrition of aircraft on the ground due to ABA missions, and the attrition of personnel and weapons in divisions in reserve due to IDR missions. The number of surviving aircraft by type on CAS missions calculated by this model are used as input to the ground-combat model. The air-combat model is called on each day of the war.

Step 5. The ground-combat model (called the GC program overlay in the computer program) calculates the rest of the air-to-ground losses and all ground-to-ground losses (i.e., it calculates the attrition of people, weapons, and supplies in combat sectors due to enemy ground fire and enemy CAS missions). The ground-combat model also calculates the capture of territory, which is denoted by the movement of the FEBA in each sector. Like the air-combat model, this model is called on each day of the war.

Step 6. At this point in the program, the model compares the "day being played" counter and the input, which gives the number of days to be played in that particular run. If the counter and this input are equal, then the model stops. If they are not equal (i.e., if the number of days already played is less than the input number of days to be played), then the model continues. In a normal run, this is the only step at which the model can terminate.

Step 7. The theater-control model is divided (for computer purposes only) into two program overlays (called TC1 and TC2). The main functions of the theater-control model are to add replacement people and replacement weapons to divisions, to move reinforcement divisions from the COMMZ to regions and from regions to sectors, and to withdraw divisions from sectors to regions, as appropriate, for each side. In addition to playing the replacements and reserves for each side, the theater-control model makes a number of other computations (such as moving supplies and determining geographical quantities). Since the theater-control model is not used before the first day of combat, and since it is not needed on the last day of combat, it is called one less time than the total number of days played.

Step 8. If the user so desires, on any day of the war he can read new values for parameters and new force increments into the model; and he can direct divisions to be moved in the model. New parameters (both ground and air) are read in the RP subroutine. If a new value for a parameter is read into the model, it replaces the old parameter. Force increments (both ground and air) are read in the RF program overlay. If a force increment is read into the model, it does not replace (but, rather, adds to) the force already there. For example, if 10 aircraft of a particular type are added to a certain airbase that already has 20 aircraft of that type stationed on it, the model will start the next day with 30 aircraft of that type on that airbase. The theater-control model automatically moves divisions between regions and sectors according to certain rules. However, whenever the user would like to direct particular divisional moves, he can do so by reading in a negative value for the number of divisions, the appropriate negative number of people and weapons by type for the old location, and the corresponding positive numbers for the new location. But he can also move forces by using the MF subroutine. This subroutine requires only the number and type of divisions to be moved and the location from which and to which they are to be moved. The MF subroutine then automatically computes the number of people and weapons by type to be moved.

Step 9. At this step, the model increments the "day being played" counter by 1 and starts the next day of combat by sending control of the program back to Step 4.

(Steps 1, 3, 6, 8, and 9 are fairly self-explanatory and will not be discussed further in this volume. Steps 4, 5, and 7--the air-combat, ground-combat, and theater-control models--are discussed below and in Chapters II, III, and IV of Volume 3. Step 2, Theater Control at Time Zero, is discussed in Chapter V of Volume 3.)

B. SUMMARY DESCRIPTION OF THE AIR-COMBAT MODEL

This section contains an outline of the air-combat model and a description of its attrition processes in algebraic notation. Chapter II of Volume 3 contains a detailed description of the logic used by the air-combat model in the same mnemonic notation used in the IDAGAM I computer program. The reason for proceeding this way is that the attrition equations seem to be more easily understood when they are presented in algebraic notation and when the context in which they are used is described. Then, given this summary description and an understanding of the attrition equations, the detailed logic (as presented in Chapter II of Volume 3) can be followed more easily.

1. Structure of the Air-Combat Model

The air-combat model performs the following steps:

Step 1. The first operation performed by the air-combat model is to determine the number of shelters associated with each notional airbase. This determination is made by comparing the locations of the FEBA (which are input for the first day and are moved in each sector according to the ground-combat

(Note: this page is misbound. skip to next page and return to this point following pg 26 (page numbers are correct)

model on each day of the war) to the fixed locations of the remaining shelters (which are input).

Step 2. The second step is to compute the number of supplies consumed by the aircraft and associated men and equipment. If enough supplies are not available, the number of sorties that can be flown that day is reduced according to the shortage of supplies.

Step 3. The third step is to compute the assignments of aircraft by location. IDAGAM I requires as input the percent of aircraft of each type that are to be assigned to each primary mission. Based on these input percentages, the situation of ground forces, the number of aircraft of each type on each airbase, and other parameters, the model computes which airbases the aircraft on CAS missions come from and which sectors they attack; and it computes which airbases the aircraft on ABA missions come from, which enemy airbases they attack, and so on, for the other five primary missions.

Step 4. The fourth step is to divert a percentage of the aircraft on CAS missions in each sector from those missions to suppress the enemy SAMs and AAA that are defending in that sector. The percentage of aircraft that perform these suppression missions depends on the number of aircraft on CAS missions in that sector, the number of SAMs in that sector, and the number of AAA in that sector. Also, a percentage of aircraft on ABA missions against each airbase is diverted from those missions to suppress the SAMs and AAA that are defending that airbase. This percentage also depends on the numbers of aircraft on ABA missions against that airbase, SAMs defending that airbase, and AAA defending that airbase.

Step 5. In this step, all air-to-air attrition is computed. The results of this step are the number (by type and by mission) of aircraft killed and the number surviving from each airbase. The number of surviving aircraft on CAS missions is input to the ground-combat model. The number of surviving aircraft on ABA missions is used in Step 6 to compute aircraft losses on the ground. The attrition of SAMs and AAA by aircraft on suppression missions is also computed in this step. When do you compute A/C losses to AAA/SAM? how?

Step 6. The attrition of aircraft on the ground by enemy aircraft on ABA missions is computed in this step.

Step 7. The attrition to divisions in regions due to IDR missions is calculated in this step.

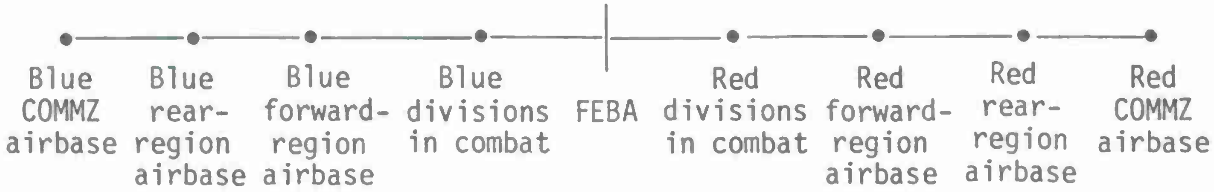
(A more detailed discussion of Steps 5 and 6 is given in the next three subsections. A more detailed discussion of Step 7 is given in Section C.2.f, below, because the discussion is more easily understood after ground attrition has been explained. Details on all the steps can be found in Chapter II of Volume 3.)

2. Air-To-Air and Suppression Interactions

As discussed earlier, IDAGAM I plays several notional airbases; and these airbases have certain spatial characteristics. For example, there are not just two notional airbases in each region. Rather, there are two notional airbases, one of which is closer to the enemy than the other; and the closer airbase should be pictured as being in between the farther one and the enemy. Further, the COMMZ airbase should be pictured as being behind all the region airbases. As explained above, to picture these airbases in any other way and yet still to play CAS and ABA missions would be inconsistent, unless there were no actual airbases in these locations. (If an airplane with a certain payload can fly ABA missions, then it certainly

(note: pg 27-28 bound between pg 22-23)

can move back, directly away from the enemy, and fly CAS missions with the same payload, providing that there is an airbase farther back for it to use.) Thus, a cross section of the theater would look like the following diagram:



To see the possible number of interactions that can take place, assume that each airbase is defended by aircraft on ABD missions and by SAMs and AAA. And suppose that the divisions in combat are defended by aircraft on BD missions and by SAMs and AAA. In IDAGAM I, it is assumed that AAA have such limited capability that they can kill only enemy aircraft that are attacking either the guns themselves or the targets they are defending. However, IDAGAM I allows the SAMs, BD aircraft, and ABD aircraft to have the possibility of detecting and killing enemy aircraft that are flying over the target they are defending to reach targets farther to the rear.[1] Thus, an aircraft on a CAS mission can be killed by enemy BD aircraft, enemy SAMs in combat divisions, and enemy AAA in combat divisions. An aircraft on an ABA mission against a forward airbase can be killed by enemy BD aircraft, enemy SAMs in combat divisions, and enemy ABD aircraft, SAMs, and AAA

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[1]As discussed further in the next section, certain parameters that govern the attrition process in an area can depend on whether the enemy aircraft are attempting to attack targets in that area or are attempting to fly to targets farther to the rear.

defending the forward-region airbase (and so on for aircraft attacking rear-region airbases or the COMMZ airbase).[1]

The order in which these interactions take place is as follows: All aircraft on CAS missions, CASE missions, SAM-suppression missions in sectors, and AAA-suppression missions in sectors are assumed to cross the FEBA each day before any aircraft on ABA-related missions. A more realistic assumption might be that missions of all types are spread throughout the day. But this assumption would, in a sense, require subdividing each day in the air model into finer time intervals. This subdivision would be more realistic, but it is not clear how much more accurate it would be; and it would increase the complexity of the model. For simplicity, IDAGAM I assumes that all missions of the same type that go to the same location cross the FEBA at the same time.[2] Given this assumption, it seems

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[1]The largest number of possible interactions occur when an aircraft attempts to attack the COMMZ airbase, for then it can be killed by enemy BD aircraft, SAMs in combat divisions, ABD forward-region aircraft, SAMs in the forward region, ABD rear-region aircraft, SAMs in the rear region, ABD COMMZ aircraft, SAMs in the COMMZ, and AAA in the COMMZ--a total of nine possible interactions.

IDAGAM I does not explicitly play the engagements that can occur as attackers, suppressors, and escorts return to their home airbase. Attrition to these aircraft on the way home is assumed to be a fraction of the attrition they suffered on their way to their targets.

[2]One effect of this assumption concerns sortie rates greater than 1. Suppose that the sortie rate for a particular type of aircraft on a particular mission is 2. Then the model approximates the number of kills made by one aircraft of that type with a sortie rate of 2 by the number of kills that would be made by two aircraft of that type but with a sortie rate of 1. (For sortie rates greater than 1, the number of aircraft of that type killed on that mission is assumed to be equal to the number of "sorties killed" divided by the sortie rate. No such approximation is necessary for sortie rates less than or equal to 1.)

reasonable to assume that the aircraft on CAS-related missions cross the FEBA first.[1]

Similarly, IDAGAM I assumes that all aircraft on ABA-related missions against forward-region airbases cross into the forward region before aircraft on ABA-related missions against rear-region airbases or the COMMZ airbase. And aircraft on ABA-related missions against the rear-region airbases cross into the rear region before aircraft on ABA-related missions against the COMMZ airbase.

Within a mission-related group, the escorts are assumed to attempt to engage enemy defenders before these defenders can attempt to engage the other attacking aircraft. Realistically, escorts can be used in two ways: Either they can "go first" and "sweep out" the air space ahead of the attacking aircraft, or they can fly alongside the attacking aircraft and attempt to engage those enemy aircraft that attempt to intercept this attacker-escort group. IDAGAM I models escorts as being used to sweep out air space.[2] — this ignores GCI tactics which might subvert this tactic.

After the escort-defender interactions, the attackers and suppressors interact with the remaining defenders. If this group of aircraft are to attack targets farther to the rear, the SAMs are allowed to shoot at the remaining attackers and suppressors, after which the remaining aircraft continue toward their target. If this group of aircraft is to attack targets

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[1]If there are no interactions allowed between BD aircraft and ABA and ABAE aircraft, etc., then it makes no difference who crosses first. However, if such interactions are possible, then whoever crosses first has a chance of engaging the BD aircraft--thus leaving fewer BD aircraft on station to intercept the group of aircraft that crosses the FEBA second.

[2]The second way of using escorts is equivalent to the first way if certain assumptions are made (one of which would be that the escort-defender interactions always occur before the defender can engage the attacker). Thus, IDAGAM I can also be considered as modeling the "fly alongside" use of escorts, providing that additional assumptions are made.

in this area, the SAMs are allowed to shoot at the remaining aircraft on suppression missions; then the remaining SAM-suppression aircraft can shoot at the SAMs, and then the remaining AAA-suppression aircraft interact with the AAA. Finally, the remaining SAMs and AAA can shoot at the remaining attack aircraft, and the attack aircraft that penetrate the SAMs and AAA can then attack their targets.

← what about AAA attrition of AAA suppression A/c?

3. Interactions Concerning the Attack of Aircraft on the Ground

One result of the interactions already discussed is the number of aircraft by type that successfully attack each enemy airbase. The targets for this attack are the number of aircraft (by type), sheltered and unsheltered, that are on the airbase when this attack occurs. Realistically, the aircraft stationed on an airbase that survived the previous day's combat will not all be actually on the airbase when this attack occurs (some will be out flying their own missions). Further, the percent of aircraft on the ground when an attack occurs may vary throughout the day. In IDAGAM I, the percent of aircraft actually on the ground when an attack occurs is approximated by a constant input percentage, which is a function of aircraft type. (This input percentage must be constant throughout the day, but the user can vary it day by day if he desires.) Applying this percentage to the number of surviving aircraft (by type) stationed on the airbase (after considering air-to-air and ground-to-air attrition) gives the number of aircraft (by type) that are assumed to be actually on the airbase when the attack occurs. (There is at most one attack on an airbase each day, and all the successful aircraft on ABA missions against that airbase are added together to form this attack.)

Those aircraft actually on the airbase when the attack occurs can be either in aircraft shelters or in the open. The method used to determine the number of aircraft shelters on each airbase has already been outlined. The priority for the sheltering of aircraft (by type) is an input to IDAGAM I. All aircraft of the type given the highest priority are sheltered before any aircraft of the second-highest priority and so on--until either all aircraft are sheltered or all the shelters on the airbase have been used. (It is assumed that each shelter can hold one aircraft, no matter what the type of that aircraft is.)

Attacking aircraft are assumed to split their attack between sheltered and nonsheltered aircraft in the following way: An input to IDAGAM I is the number of Blue aircraft that attack Red nonsheltered aircraft per Red nonsheltered aircraft before any Blue aircraft attack Red sheltered aircraft (and the same for Red attacking Blue). This input times the number of Red nonsheltered aircraft gives the "breakpoint" number of Blue aircraft that attack Red nonsheltered aircraft before any attacks are made on shelters. If the actual number of attacking Blue aircraft is less than or equal to this breakpoint number, all Blue aircraft attack nonsheltered Red aircraft. If the number of attacking Blue aircraft is greater than this breakpoint number, then this breakpoint number of Blue aircraft attack nonsheltered aircraft; and the remainder of the Blue attackers are split between nonsheltered Red aircraft and sheltered Red aircraft in the same ratio as the ratio of the number of nonsheltered Red aircraft to the number of sheltered Red aircraft.

Note that this last ratio is the same as the ratio of the number of nonsheltered Red aircraft to the number of Red shelters, since each shelter can hold one aircraft and all shelters are filled before any aircraft are left in the open. This observation is important because IDAGAM I assumes that the attacker cannot distinguish an occupied shelter from an unoccupied shelter. In the case where there are more shelters on a notional airbase than there are aircraft (so that there are no unsheltered aircraft), then IDAGAM I assumes that the aircraft are uniformly distributed among the shelters. In this case, the number of targets for the attacking aircraft is the number of shelters. If the attrition calculations (which are discussed in the next section) say that 10 percent of the shelters are "killed," then the model assumes that 10 percent of the sheltered aircraft are killed. IDAGAM I assumes that a fixed (input) percent of the shelters that are attacked are completely destroyed and that any damage to the remaining shelters can be repaired overnight.

4. Attrition Calculations

a. General Structure

Many methods for calculating attrition are available in the war-gaming literature. Some examples are the various Lanchester equations, binomial equations, and exponential equations. (See Anderson [2] for a review of the air-to-air attrition equations used in several other theater-level models.) Recent research at IDA has led to a better understanding of some of the attrition processes leading to these equations and has uncovered apparently fruitful areas for further research. However, until this research is accomplished, we will have to use the equations we have now, along with our understanding of the characterizations of the processes that lead to these equations.

Each of the interactions described in the previous two sections must have an attrition equation and appropriate parameters associated with it. IDAGAM I allows the user a choice among six attrition equations. The user can select any one of the six equations for all the interactions, or he can select different equations for different interactions.[1]

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[1]The interactions are divided into eight groups--namely,
(1) Defender-Escort interactions,
(2) Defender-Attacker interactions, (continued on next page)

Of the six attrition equations that are available in the air model, two are binomial, two are exponential, one is a Lanchester square equation, and one is a Lanchester linear equation. Before discussing these equations in detail, we will discuss the general structure in which these equations are used.

The weapons capable of causing attrition in the air model are divided into the following categories:

(1) attackers (including suppressors when appropriate),
(2) escorts,
(3) defenders,
(4) SAM suppressors,
(5) AAA suppressors,
(6) SAMs, and
(7) AAA.

The first five categories consist of aircraft, and these categories are further subdivided into the various types of aircraft played. Since IDAGAM I plays only one type of SAM and one type of AAA, the last two categories are not subdivided. Any weapon in these categories can be a target as well as a shooter. In addition, there are two more categories--namely,

(8) sheltered aircraft on the ground, and
(9) nonsheltered aircraft on the ground--

that can only be targets. (Losses to aircraft, by type, on the ground is determined by prorating the total losses among

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(cont'd)
(3) SAM-to-Air interactions,
(4) Air-to-SAM interactions,
(5) AAA-to-Air interactions,
(6) Air-to-AAA interactions,
(7) Attacker-to-Sheltered Aircraft interactions, and
(8) Attacker-to-Nonsheltered Aircraft interactions.
The attrition equations used must be the same for each group (it can be any of the six), but different attrition equations can be used for different groups.

the various types of aircraft. Thus, in effect, these two categories are not further subdivided into aircraft by type.)

An attrition calculation involves a number of shooting weapons by type from one (and only one) category shooting at a number of target weapons by type from one (and only one) other category. Suppose an attrition calculation involves weapons from category s ($1 \le s \le 7$) shooting at weapons from category t ($1 \le t \le 9$). (Not all combinations are possible; for example, attackers never shoot at their escorts, nor do SAMs shoot at AAA.) Let

S_i^s = the number of type-i shooters ($1 \le i \le M$)[1]

T_j^t = the number of type-j targets ($1 \le j \le N$); and

$\dot{T}_j^t$ = the number of type-j targets killed ($1 \le j \le N$).

(Note that M = 1 if s = 6 or 7 and that N = 1 if t = 6, 7, 8 or 9.) Then the attrition process calculates $\dot{T}_j^t$ as a function of $S_1^s, \ldots, S_M^s, T_1^t, \ldots, T_N^t$, and certain parameters.

Each of the six attrition equations in the air model uses two parameters: the probability of detection and the probability of kill, given detection.

The probability of detection is assumed to be independent of the particular types of aircraft involved, but it can depend on the categories of the shooting and target (s and t) weapons and on the following two properties of the interaction:

Let ℓ denote the location where the interaction is taking place (e.g., $\ell = 1$ denotes combat sector, $\ell = 2$ denotes forward region, $\ell = 3$ denotes rear region, and $\ell = 4$ denotes COMMZ).

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[1]In the case where airbase attackers are attacking aircraft on the ground (s = 1, and t = 8 or 9), multiple passes for the attacking aircraft are allowed. If a type-i aircraft can make p passes, then pS_i^1 is used in place of S_i^1 for these attrition calculations.

If the shooting weapon is on the attacking side (i.e., categories 1, 2, 4, or 5), then its detection capability is assumed to be independent of location. But if the shooting weapon is on the defending side (categories 3, 6, and 7), then its detection probability can depend on its location. This structure allows weapons located farther to the rear to have a higher probability of detecting attackers than weapons located closer to the front. This concept is important. IDAGAM I has a definite geographical structure, and certain defending weapons are behind other defending weapons; therefore, the weapons that are farther to the rear should be able to have higher detection capabilities than weapons of the same type that are closer to the FEBA.

The other characteristic of the interaction relates to the location of the mission of the aircraft on the attacker's side. Let m = 1 denote the situation where an aircraft of category 1, 2, 4, or 5 is trying to fly by a weapon of category 3, 6, or 7 in order to reach a target farther to the rear. Let m = 2 denote all other interactions (i.e., interactions that occur in the same location as the location of the primary target for the attacker's aircraft).

With this structure, the probability of detection is assumed to be independent of i and j but to depend on s, t, ℓ, and m. (A method that could be used in IDAGAM I to reflect different detection capabilities of different types of aircraft will be mentioned later.) This probability is denoted by

$D_{\ell m}^{st}$ = the probability that a shooter in category s detects a target in category t, given that the location of the interaction is given by ℓ and that m is as described above.[1]

[1]Just as not all combinations of the s's and t's are possible; given s and t, not all combinations of the ℓ's and m's are possible.

The probability of kill, given detection, depends on the categories of weapons involved (s and t) and on the types of weapons (i and j) in those categories, where appropriate. But given that a detection is made, it is assumed that the probability of kill, given detection, is independent of ℓ and m. Denote this probability of kill by

K_{ij}^{st} = the probability that a type-i shooter in category s kills a type-j defender in category t, given that the shooter has detected the target.

Finally, let

$$\bar{T}^t = \sum_{j=1}^{N} T_j^t$$

be the total number of targets. The equations used in the air model have the property that $\dot{T}_j^t$ depends on T_j^t and on $\bar{T}^t$, but $\dot{T}_j^t$ depends on $T_{j'}^t$ for $j' \neq j$ only through $\bar{T}^t$. Thus, the general form of the attrition equations used in the air model to determine $\dot{T}_j^t$ is

$$\dot{T}_j^t = f(S_1^s, \ldots, S_M^s, T_j^t, \bar{T}^t, D_{\ell m}^{st}, K_{1j}^{st}, \ldots, K_{Mj}^{st}) .$$

The particular forms for this equation available in the air model are described in the next sections.

b. A Single-Engagement Binomial Attrition Equation

The TAC CONTENDER model (described in [23]) uses an exponential attrition equation that is supposedly based on certain assumptions concerning an attrition process. Karr [16] has specified these assumptions and rigorously derived the binomial equation based on them. One of these assumptions is that, if a shooter detects several targets, he can engage only one of them (hence, the name "single-engagement binomial"). (For further details on the assumptions leading to this equation, as well as for the derivation and properties of this equation, the reader is referred to Karr.)

As discussed by Karr [16], the exact expression for the heterogeneous form of this equation is quite complex. However, this expression is simplified if the detection probabilities do not depend on the type of target. Since we assumed that the detection probabilities are independent of both the type of shooter and the type of target, we can take advantage of this simplification to give the following form for the single-engagement binomial attrition equation:

$$\dot{T}_j^t = T_j^t\left(1 - \prod_{i=1}^{M}\left[1 - \frac{K_{ij}^{st}}{\bar{T}^t}\left(1 - (1 - D_{\ell m}^{st})^{\bar{T}^t}\right)\right]^{S_i^s}\right) . \qquad (1)$$

c. <u>An Exponential Approximation</u>

Since the exact form of the equation based on the appropriate set of assumptions is the binomial Equation (1), it is not clear why one would want to approximate the binomial structure by an exponential structure. But since other models (e.g., TAC CONTENDER) have made this approximation, we allow, for purposes of comparison, the exponential approximation as an alternative attrition equation in the air model. The form of this exponential approximation is

$$\dot{T}_j^t = T_j^t\left(1 - \exp\left[-\frac{\sum_{i=1}^{M} S_i^s K_{ij}^{st}(1 - \exp(-D_{\ell m}^{st}\bar{T}^t))}{\bar{T}^t}\right]\right). \qquad (2)$$

d. <u>Another Exponential Attrition Equation</u>

Some models, such as the preliminary versions of IDAGAM I, have used another form of exponential equation. Again, we include this form solely for purposes of comparison. This form of an exponential attrition equation is

$$\dot{T}_j^t = T_j^t \left(1 - \exp \left[- \frac{\sum_{i=1}^{M} S_i^s K_{ij}^{st} D_{\ell m}^{st}}{\overline{T}^t} \right] \right) . \quad \underline{1}/ \tag{3}$$

e. A Multiple-Engagement Binomial Attrition Equation

Suppose all the assumptions used to derive Equation (1) are retained, except for the assumption that, if a shooter detects several targets, he can only engage one of them; and suppose this assumption is replaced by the assumption that the shooter (at no loss of effectiveness) can engage all the targets he detects. Then Karr [16] shows that the exact form for the equation resulting from these assumptions is

$$\dot{T}_j^t = T_j^t \left(1 - \prod_{i=1}^{M} \left[1 - D_{\ell m}^{st} K_{ij}^{st} \right]^{S_i^s} \right) . \tag{4}$$

Note that this equation is independent of $\overline{T}^t$.

f. A Lanchester Square Equation

Karr [15] develops a set of assumptions that lead, in the homogeneous case, to the standard homogeneous Lanchester square equation. However, Karr shows that these assumptions when extended to the heterogeneous case do not lead to the standard heterogeneous Lanchester square equation. The air model in IDAGAM I does not use the standard heterogeneous Lanchester square equation; instead, it uses the following discrete time form for the Lanchester square equation:

$$\dot{T}_j^t = \frac{T_j^t}{\overline{T}^t} \sum_{i=1}^{M} D_{\ell m}^{st} K_{ij}^{st} S_i^s \ . \quad \underline{1}/ \tag{5}$$

Note that if N = 1, then $T_1^t = \overline{T}^t$ and Equation (5) reduces to the standard discrete time form of the Lanchester square equation. (For an explanation of this form for N > 1, the reader is

[1] $D_{\ell m}^{st}$ is not a probability of detection, but is defined for Equations (3) and (5) as the number of targets that an average shooter engages.

referred to Karr.)

g. A Lanchester Linear Equation

There has been some discussion concerning what the proper form for the heterogeneous Lanchester linear equation should be. This discussion arose because there was no *pro forma* way to extend Lanchester linear equations from the homogeneous case to the heterogeneous case similar to the *pro forma* way that was available for extending the Lanchester square equation. Karr has also developed a set of assumptions that lead to the homogeneous Lanchester linear equation. Karr [15] shows that, based on these assumptions, there is a direct way to extend the Lanchester linear equation to the heterogeneous case. The discrete time form (for the heterogeneous Lanchester linear equation based on this extension) that is used in the air model is

$$\dot{T}_j^t = T_j^t \sum_{i=1}^{M} D_{\ell m}^{st} K_{ij}^{st} S_i^s \ . \tag{6}$$

Note that if M = N = 1, then Equation (6) reduces to the standard discrete time homogeneous Lanchester linear equation. Note also that this equation does not depend on $\overline{T}^t$.

h. A Comment on Detection Probabilities

If one wishes to play that the probability of detection depends on the type of shooter and type of target and if Equations (4) or (6) are to be used, then this dependence can be played in the air model by redefining $D_{\ell m}^{st}$ and K_{ij}^{st}. In this event, K_{ij}^{st} should be defined as the probability of detection of a type-j weapon by a type-i weapon times the probability of kill, given detection, of the type-j weapon by the type-i weapon. The parameter $D_{\ell m}^{st}$ should then be defined as a factor that accounts for variations in the detection probabilities due to ℓ and m. This method of accounting for detection as a function of weapon type would be a purely artificial attachment to Equations (1), (2), (3), and (5), as it would violate the basic assumptions that lead to these equations.

If it is desired to play that the probability of detection depends on the type of shooter and type of target and if Equations (1) or (2) are to be used, then a change in the IDAGAM I computer program could accomplish this. $D^{st}_{\ell m}$ should be replaced by $D^{st}_{ij\ell m}$, which would be defined as follows:

$D^{st}_{ij\ell m}$ = the probability that a type-i shooter in category s detects a type-j target in category t, given that the location of the interaction is given by ℓ and that m is as described above.

The working variable $\bar{D}^{st}_{i\ell m}$ should be defined by

$$\bar{D}^{st}_{i\ell m} = \sum_{j=1}^{N} T^t_j D^{st}_{ij\ell m} \Big/ \bar{T}^t .$$

Then Equations (1) and (2) should be changed so that $\bar{D}^{st}_{i\ell m}$ replaces $D^{st}_{\ell m}$ where it occurs in those equations. Note that if $D^{st}_{ij\ell m}$ is independent of j, then Equation (1) would still produce the true expected number of kills, based on the assumptions leading to that equation. But if $D^{st}_{ij\ell m}$ depends on j, then the value produced by Equation (1) is an approximation.[1]

One of the assumptions that lead either to Equation (3) or to Equation (5) is that a fixed (input) number of targets are engaged per shooter; thus, the probability that a particular shooter detects a particular target plays no role in these assumptions. For Equations (3) and (5), $D^{st}_{\ell m}$ should be defined as the number of targets in category t engaged by an average shooter in category s given ℓ and m. If it is desired to play that the proportioning of the number of engagements to each type of target depends on the type of shooter and types of targets, and if a Lanchester-square structure is desired, then Equation (5) should be replaced by an equation similar to those given in Section C.2.b (below), with the proportioning of engagements playing a role analogous to allocation of fire.

[1]The correct form for Equation (1) for the case where $D^{st}_{ij\ell m}$ depends on j is given by Karr [16], but it is quite complex and would use too much computer time to be useful in a model like IDAGAM I.

C. SUMMARY DESCRIPTION OF THE GROUND-COMBAT MODEL

This section contains an outline of the ground-combat model and a description of its attrition processes and FEBA-movement calculations in algebraic notation. (Chapter III of Volume 3 contains a detailed description of the logic used by the ground-combat model in the same mnemonic notation used in the IDAGAM I computer program.)

All other sections in this volume describe the calculations in the same order as they are performed in the computer program. However, in this section the calculations will not be described in the precise order in which they are performed. The basic logic of the description given here, as well as the end results is the same as that of the computer program; but the order in which the calculations are described is changed.[1]

1. Structure of the Ground-Combat Model

The ground-combat model performs the following steps (though not necessarily in this order):

Step 1. The attrition and FEBA-movement calculations need to be made each day for each combat sector. The first step in the ground-combat model is to set the sector index equal to 1. This index will be incremented each time through the attrition and FEBA-movement calculations until all sectors have been considered.

Step 2. Step 2 examines each of the opposing forces in the sector under consideration and degrades a force if it is

[1]The reason that we do so is that certain quantities in the ground model can be computed in any order without changing the results, and the particular order used by the computer program is not necessarily the one that best facilitates understanding the model.

not at least roughly balanced. (A force is not "balanced" if it has too many of certain types of weapons--e.g., artillery--and not enough of other types of weapons.)

Step 3. In Step 3, the ground model determines who will be on attack in the sector. (A side might be on defense in general, but might attack in a particular sector if it is strong enough in that sector.) If neither side is strong enough to attack, then a holding posture is said to exist in the sector.

Step 4. The ground model computes separately by weapon type a potential number of weapons lost and the number of casualties per weapon lost, and the total number of casualties. Based on these three quantities, the model computes by weapon type the (actual) number of weapons lost.

Step 5. In Step 5, the weapon losses are prorated by type to the divisions in the sectors; and the total number of casualties are divided into combat, combat-support, and service-support categories and applied to the divisions in the sector.

Step 6. The movement of the FEBA in the sector under consideration is calculated in Step 6.

Step 7. The amount of supplies in a sector can be reduced by consumption and by losses to enemy (air and ground) fire. Step 7 computes the amount of supplies consumed and the amount of supplies lost to enemy fire. (See Section D.4 of this chapter, below, for a discussion of how the calculations made in this step can be used to play air attacks on supplies in sectors, and how this relates to the supply-interdiction mission.)

Step 8. A certain percentage of the weapons lost is assumed to be only damaged and capable of repair. The percentage can depend on the side losing the weapons, on whether that side is on attack or defense in the sector, or posture, and on weapon type. This percentage is applied by weapon type to the number of weapons lost, to determine the number of weapons to be added to a weapon's "recovered and repairable" pool. Each day an input percentage of the weapons in the "recovered and repairable" pool are assumed to be repaired, and they are added to the weapon replacement pool in the COMMZ (this percentage can depend on weapon type).

Step 9. Nonbattle casualties are computed in this step for people in the sector. (The way that this computation is done is described in Subsection 2.e, below.)

Step 10. The sector index is compared to the total number of sectors in this step. If the sector index is less than the number of sectors, then the sector index is incremented by 1; and control of the model is sent back to Step 2. If all sectors have been considered, the model continues to Step 11.

Step 11. The final step of the ground model is to adjust the FEBA for front-to-flank considerations. The way that this adjustment is made is by first considering the attacker's front-to-flank ratios. If the attacker cannot sustain these ratios (as determined by various input-parameters and decision rules), then the FEBA is moved back in appropriate sectors until a sustainable position is reached. Then the defender's front-to-flank ratios are considered, and the defender is forced to withdraw in those sectors where his front-to-flank ratios are not sustainable.

2. Attrition Calculations

This section describes Step 4 of the ground-combat model (i.e., it describes the formulas for computing the attrition to people and weapons that occurs during one day of battle in the sector under consideration). Throughout this section, we assume that Red is on attack in the sector and that Blue is defending in posture-type k in that sector. (The case where Blue is on attack is handled symmetrically.)

a. Basic Structure

The basic structure of the attrition calculation in the ground-combat model is to compute three quantities--

(1) By weapon type, the potential number of weapons lost,

(2) By weapon type, the casualties per weapon lost, and

(3) The total number of casualties--

and then to compute by weapon type the actual number of weapons lost, based on these quantities. In this subsection, we will describe how the model uses these quantities to compute by weapon type the weapons lost. In the subsequent three subsections, we will show how each of these quantities is computed from the inputs.

For notational purposes, let

$\dot{B}_j$ = the actual number of Blue type-j ground weapons lost during the day in the sector;

$\dot{B}_j^p$ = the potential number of Blue type-j ground weapons lost during the day in the sector;

C_j^b = the number of Blue casualties associated with Blue type-j weapons lost;

$\dot{B}_0$ = the total number of Blue casualties that occurred during the day's battle in the sector;

and let $\dot{R}_j$, $\dot{R}_j^p$, C_j^r, and $\dot{R}_0$ be defined similarly for Red.

Assume for the time being that $\dot{B}_j^p$, C_j^b, and $\dot{B}_0$ are known. Then IDAGAM I makes two assumptions in order to calculate $\dot{B}_j$: IDAGAM I assumes--

(1) That the actual number of weapons lost is proportional to the potential number of weapons lost (i.e., that

$$\dot{B}_j = \alpha^b \dot{B}_j^p$$

for some α^b and for all types of Blue weapons).

(2) That the actual number of weapons lost (by type) times the casualties per weapon lost (by type), when summed over all types of weapons, gives the total number of casualties--i.e., that

$$\sum_j \dot{B}_j C_j^b = \dot{B}_0$$

(this assumption really just assumes that the definition of C_j^b is consistent).

These assumptions imply that

$$\sum_j \alpha^b \dot{B}_j^p C_j^b = \dot{B}_0$$

or

$$\alpha^b = \frac{\dot{B}_0}{\sum_j \dot{B}_j^p C_j^b} .$$

Thus, once $\dot{B}_j^p$, C_j^b, and $\dot{B}_0$ are known, IDAGAM I calculates by weapon type the actual number of weapons killed as

$$\dot{B}_j = \frac{\dot{B}_0}{\sum_{j'} \dot{B}_{j'}^p C_{j'}^b} \dot{B}_j^p .$$

The corresponding equation for Red is

$$\dot{R}_j = \frac{\dot{R}_0}{\sum_{j'} \dot{R}_{j'}^p C_{j'}^r} \dot{R}_j^p .$$

These equations are fairly straightforward. The real problem is how to calculate $\dot{B}_j^p$, C_j^b, and $\dot{B}_0$ (and the corresponding quantities for Red). Briefly, the calculations are made as follows: $\dot{B}_j^p$ is calculated using a Lanchester-type equation ($\dot{B}_j^p$ does not depend either on the casualties-per-weapon-lost variables or on force ratios). C_j^b is calculated directly as a weighted average of certain inputs. $\dot{B}_0$ is calculated using force ratios and historical data. Thus, $\dot{B}_j$ depends on force ratios only through the calculations for $\dot{B}_0$.

A characteristic of this approach is as follows: An advantage of heterogeneous Lanchester equations is that, since they *are* heterogeneous, they can be used to determine by weapon type the weapon losses. But their use has been limited partially by the fact that they require absolute measures of weapon-on-weapon effectiveness, data for which are hard to obtain (i.e., used alone, heterogeneous Lanchester equations need to know more than that an antitank weapon has 10 times the potential to kill enemy tanks that an artillery piece has). When used alone, these equations need as input the actual rate of enemy tank kills per antitank weapon. These absolute data are hard to obtain from laboratory tests or field exercises, and historical data rich enough to give by type of shooter and target the weapon losses are not available. Relative measures of weapon-on-weapon effectiveness could be used as data for heterogeneous Lanchester equations to obtain by weapon type a potential number of weapons lost, but then this potential number would have to be scaled to obtain by weapon type the actual number of weapons lost.

On the other hand, one reason for the relatively frequent use of force ratios for determining casualties is their capability of verification based on historical data. IDAGAM I combines the advantages of each method to compute by weapon type the potential number of weapons lost, using Lanchester equations and relative weapon-on-weapon data and then scaling this potential number, based on the total number of casualties (computed by using force ratios and historical data).

The inputs and method used to compute $\dot{B}_j^p$, C_j^b, and $\dot{B}_0$ are discussed below. The computation of $\dot{B}_0$ is of particular importance, since the number of casualties suffered is a primary output by itself (as well as being used to calculate by weapon type the number of weapons lost).

b. Potential Number of Weapons Lost (by Type)

The definitions of the inputs required to calculate by weapon type the potential number of weapons lost are as follows:

Let B_i = the number of Blue type-i ground weapons in the sector (this is initially input and is adjusted daily according to attrition, replacements, balance, etc.);

B_i^* = the number of Blue type-i ground weapons in a standard force (this is input);

B_c^a = the number of Blue type-c aircraft on CAS missions that successfully deliver their ordnance that day in the sector (this is calculated by the air-combat model);

A_{ij}^{*bgd} = the fraction of Blue type-i ground weapons that would fire at Red type-j weapons if Blue were on defense and the Red target force were the standard force (this is input);

P_{ijk}^{bgd} = the potential number of Red type-j ground weapons killed by each Blue type-i ground weapon per day if the Blue type-i weapon were to allocate all its fire to Red type-j weapons and if Blue were on defense in posture-type k (this is input);

L_{cm}^{bad} = the number of Blue type-m air munitions carried in a notional load of Blue type-c aircraft if Blue were on defense (this is input);

A_{mj}^{*bad} = the fraction of Blue type-m air munitions that would be fired at Red type-j weapons if Blue were on defense and the Red target force were the standard force (this is input);

P_{mj}^{bad} = the potential number of Red type-j ground weapons killed by each Blue type-m air munition if the Blue type-m air munition were to be fired solely at Red type-j weapons and if Blue were on defense (this is input);

and let R_i, R_i^*, R_c^a, A_{ij}^{*rga}, P_{ijk}^{rga}, L_{cm}^{raa}, A_{mj}^{*raa}, and P_{mj}^{raa} be defined similarly for Red on attack.

The ground model performs the following calculations (for Blue defending in posture-type k):

$$\dot{R}_j^p = \sum_i B_i \left(\frac{A_{ij}^{*bgd} R_j / R_j^*}{\sum_{j'} A_{ij'}^{*bgd} R_{j'} / R_{j'}^*} \right) P_{ijk}^{bgd}$$

$$+ \sum_c B_c^a \left(\sum_m L_{cm}^{bad} \left(\frac{A_{mj}^{*bad} R_j / R_j^*}{\sum_{j'} A_{mj'}^{*bad} R_{j'} / R_{j'}^*} \right) P_{mj}^{bad} \right) ,$$

$$\dot{B}_j^p = \sum_i R_i \left(\frac{A_{ij}^{*rga} B_j / B_j^*}{\sum_{j'} A_{ij'}^{*rga} B_{j'} / B_{j'}^*} \right) P_{ijk}^{rga}$$

$$+ \sum_c R_c^a \left(\sum_m L_{cm}^{raa} \left(\frac{A_{mj}^{*raa} B_j / B_j^*}{\sum_{j'} A_{mj'}^{*raa} B_{j'} / B_{j'}^*} \right) P_{mj}^{raa} \right) .$$

These equations have a natural interpretation. For example, in the first equation, B_i is the number of Blue (shooting) type-i ground weapons, the fraction

$$\frac{A_{ij}^{*bgd} R_j / R_j^*}{\sum_{j'} A_{ij'}^{*bgd} R_{j'} / R_{j'}^*}$$

gives the allocation of fire of Blue type-i weapons against Red type-j weapons, and P_{ijk}^{bgd} is the potential number of Red type-j weapons killed for each Blue type-i weapon that allocates its fire to Red type-j weapons. Summing this over i and adding in the similar terms for air-to-ground attrition gives the total potential number of Red type-j weapons killed.

Note that these equations would be the discrete version of the standard heterogeneous Lanchester-square equations if the allocation of fire were independent of the number of target weapons. However, it does not seem reasonable to make allocation of fire independent of the number of target weapons. (For an initial explanation of these equations, see Anderson [4] and Karr [15]. Forthcoming papers will explain this allocation in further detail.)

Note that nowhere in these computations are force ratios, numbers of casualties, or the number of casualties per weapon used.

c. Casualties per Weapon Lost (by Type)

The definitions of the inputs required to calculate the number of casualties per weapon lost (by type) are as follows:

Let D_{ij}^{bgd} = the number of Red casualties associated with each Red type-j weapon killed by a Blue type-i weapon if Blue is on defense (this is input);

D_{mj}^{bad} = the number of Red casualties associated with each Red type-j weapon killed by a Blue type-m air munition if Blue is on defense (this is input);

and let D_{ij}^{rga} and D_{mj}^{raa} be defined similarly for Red on attack killing Blue.

The difference between the D's (which are input) and the C's (which are required for the computation) is that the D's can depend on the type of shooting weapons, while the C's cannot. Clearly, the C's should be computed as a weighted average of the D's. The weighting factors that the ground model uses are calculated as follows:

$$\text{Let } Q_{ij}^{bgd} = B_i \left(\frac{A_{ij}^{*bgd} R_j / R_j^*}{\sum_{j'} A_{ij'}^{*bgd} R_{j'} / R_{j'}^*} \right) P_{ijk}^{bgd}$$

= the potential number of Red type-j weapons killed by Blue type-i weapons considering allocation of fire, if Blue were on defense in posture-type k;

$$Q_{mj}^{bad} = \sum_c B_c^a L_{cm}^{bad} \left(\frac{A_{mj}^{*bad} R_j / R_j^*}{\sum_{j'} A_{mj'}^{*bad} R_{j'} / R_{j'}^*} \right) P_{mj}^{bad}$$

= the potential number of Red type-j weapons killed by Blue type-m air munitions from all Blue aircraft, considering allocation of fire, if Blue were on defense;

and let Q_{ij}^{rga} abd Q_{mj}^{raa} be defined similarly for Red killing Blue. Note that these variables are not inputs; they are working variables calculated from inputs. Note also that

$$\sum_i Q_{ij}^{bgd} + \sum_m Q_{mj}^{bad} = \dot{R}_j^p$$

and

$$\sum_i Q_{ij}^{rga} + \sum_m Q_{mj}^{raa} = \dot{B}_j^p .$$

The weighting factors used to average the D_{ij}^{bgd}'s and D_{mj}^{bad}'s are

$\dfrac{Q_{ij}^{bgd}}{\dot{R}_j^p}$ = the ratio of the potential number of Red type-j weapons killed by Blue type-i ground weapons to the potential number of Red weapons killed by all Blue (ground and air) weapons; and

$\dfrac{Q_{mj}^{bad}}{\dot{R}_j^p}$ = the ratio of the potential number of Red type-j weapons killed by Blue type-m air munitions to the potential number of Red weapons killed by all Blue (ground and air) weapons.

With these weighting factors, we have

$$C_j^r = \sum_i \frac{Q_{ij}^{bgd}}{\dot{R}_j^p} D_{ij}^{bgd} + \sum_m \frac{Q_{mj}^{bad}}{\dot{R}_j^p} D_{mj}^{bad} ;$$

and, similarly, for Red killing Blue, we have

$$C_j^b = \sum_i \frac{Q_{ij}^{rga}}{\dot{B}_j^p} D_{ij}^{rga} + \sum_m \frac{Q_{mj}^{raa}}{\dot{B}_j^p} D_{mj}^{raa} .$$

d. Total Number of Casualties

(1) Basic Approach

A method to calculate the number of casualties has been available for some time (for example, it is used in the ATLAS model). IDAGAM I uses this method with certain major modifications. The ATLAS method uses historical data, force ratios, and firepower scores. IDAGAM I has not improved the historical data base; and thus, until better data are available, the standard functional relationships between force ratios and percent casualties must still be used. IDAGAM I has modified the concept of force ratios, and it avoids a problem that can occur in standard force-ratio models. IDAGAM I does not use firepower scores.[1]

In addition to these changes, IDAGAM I considers degradation of effectiveness due to shortages of supplies in a more appropriate way than ATLAS does; IDAGAM I directly considers degradation in effectiveness due to a shortage weapons (which ATLAS considers, at best, implicitly); IDAGAM I allows individual replacements to build up over time to full effectiveness (ATLAS assumes essentially that replacements are immediately fully effective); and IDAGAM I allows divisions to reorganize over time (which ATLAS does not allow).

[1]By a "firepower score" we mean any number (whether it is calculated based on lethal area per round times expected number of rounds fired, or not) that is assigned to each weapon type, such that each weapon of that type in combat contributes (linearly) that number to its side's total value--no matter what types of target weapons it is shooting at.

The reason that allowing divisions to reorganize is important is as follows: The ATLAS method assumes that the percent of effectiveness of a division is not necessarily equal to its percent strength. In particular, a division may be at 70-percent strength but only at 30 percent of its effectiveness. This assumption seems quite reasonable if a division at 100-percent strength suffered enough casualties in a battle to reduce its strength to 70 percent, for then the division might be so disorganized that it would be only 30-percent effective. But if the division suffered no additional casualties (and received no replacements) and it were not short of weapons or supplies, then the division should be able to reorganize itself over time so that being at 70 percent of its original strength would mean that it could reach 70 percent of its original effectiveness through reorganization.

The steps used to calculate the total number of casualties are as follows:

(1) Calculate degradation factor due to shortage of supplies.

(2) Calculate Blue ground value (for each type of division) based on number of Blue weapons in the sector.

(3) Calculate Blue ground value (for each type of division) based on Blue personnel strength in the sector.

(4) Calculate Blue ground effectiveness, considering weapons, personnel strength, and supplies.

(5) Calculate Blue air effectiveness in the sector.

(6) Do Steps 2, 3, 4, and 5 for Red.

(7) Form force ratios and calculate percent casualties for Blue and Red.

(8) Calculate number of casualties to Blue and Red.

After the input variables have been defined, each of these steps will be described in turn.

We are still assuming that Red is on attack and Blue is defending in posture-type k. (Symmetric calculations are made if Blue is on attack.) The notation already defined will be used here when needed. New inputs are as follows:

B^s = the number of tons of supplies that Blue has in the sector under consideration (this is initially input and is adjusted daily according to attrition, consumption, and resupply).[1]

S_i^{cb} = planned supply consumption rate for Blue type-i weapons (this is input).

S_0^{cb} = planned supply consumption rate for Blue people (this is input). (In the computer program, this quantity is also a function of the category of Blue personnel--combat, combat support, or service support. But for simplicity of notation, we will not denote this functional relationship here.)

$f^{sb}(x)$ = the effectiveness factor for supply degradation if Blue has x days of supply on hand at the beginning of the day. If Blue would have an effectiveness of y before considering possible supply shortages and if Blue had x days of supply on hand (both x and y are calculated by the model), then we assume that Blue's effectiveness is $yf^{sb}(x)$. The function f^{sb} is an input via a piecewise linear structure.

B_{id} = the number of Blue type-i ground weapons in all type-d divisions in the sector (this is initially input and is adjusted daily according to attrition, replacements, balance, etc.). Note that B_i, as previously defined, is calculated from the B_{id}'s by

$$B_i = \sum_d B_{id} .$$

B_{id}^t = the TOE[2] authorized number of Blue type-i ground weapons in a Blue type-d division (this is input).

B_{0d} = the number of Blue people in all type-d divisions in the sector (this is initially input and is adjusted daily according to attrition, replacements, etc.).

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[1] In Figure 2 (p. 114), the notation $B^{ss}(J)$, where J denotes the sector, is used in place of B^s, in order to show explicitly the dependence on sectors.

[2] We use TOE (Table of Organization and Equipment) here when defining the authorized number of people or weapons in a full-strength division.

B^t_{0d} = the TOE authorized number of people in a type-d division (this is input).

N^b_d = the number of Blue type-d divisions in the sector. (This is initially input and is adjusted daily according to reinforcements and withdrawals. This number is not adjusted by the actual strength of the divisions. For example, if there are two type-d divisions in the sector, each at 75-percent strength, then $N^b_d = 2$, not 1.5.)

$f^{bed}_d(x)$ = the reduction of the effectiveness of a Blue type-d division on defense if it were at full strength and was reduced to a fractional strength of x (before any reorganization). (In the example above, $f^{bed}_d(.70) = .30$.) This is input via a piecewise linear structure.

G^{bd}_d = the reorganization rate for a Blue type-d division if Blue is on defense (this is input).

L^b_d = the balance factor for Blue personnel in a type-d division (this is input, and its purpose will be explained below).

$f^{bcd}_k(x)$ = the percent of balanced Blue people that become casualties per day if Blue is on defense in posture-type k and the force ratio is x (this function is input via a piecewise linear structure).

Let R^s, S^{cr}_i, S^{cr}_0, $f^{sr}(x)$, R_{id}, R^t_{id}, R_{0d}, R^t_{0d}, N^r_d, $f^{rea}_d(x)$, G^{ra}_d, L^r_d, and $f^{rca}_k(x)$ be defined similarly for Red on attack.[1] Working variables will be introduced and defined as needed.

IDAGAM I considers three different methods of applying the general description (above) and variable definitions to determine the total number of casualties. Since these three methods are three different ways of calculating ground-weapon effectiveness and air effectiveness, Steps 1, 3, 4, 7, and 8 are the same for each method (with one exception, described below). Only Steps 2 and 5, and the corresponding calculations for Red in

[1] In Figure 2 (p. 114), the notation $R^{ss}(J)$, where J denotes sector, is used in place of R^s, in order to show explicitly the dependence on sectors.

Step 6, are varied. In Subsection (2), below, we will describe all the steps and will describe one method for computing weapon and aircraft effectiveness. This method is based on the potential number of casualties that a weapon or aircraft can inflict. A second method will be described in Subsection (3). This method is based on the potential of a weapon or aircraft to destroy the potential of the enemy's weapons--and vice versa. A third method, based simply on linear weights, is described in Subsection (4).[1]

(2) Potential Casualties

Steps 1, 3, 4, 7, and 8 (as described below) apply to each method of computing casualties. Steps 2, 5, and 6 are based on the potential number of casualties that a weapon or aircraft can cause.

Step 1. Effectiveness degradation due to a shortage of supplies is assumed to be a function of the number of days of supply on hand. Let

D^{shb} = days of supply on hand for Blue

= (Blue supplies in sector)/(Blue supplies consumed per day)

$$= \frac{B^{s}}{B_0 S_0^{cb} + \sum_i B_i S_i^{cb}} .$$

Then

E^{sb} = fractional effectiveness due to supply shortage for Blue

$$= f^{sb}(D^{shb}) .$$

[1]This order should not be taken as giving our recommendation of which method should be used. These three methods are presented in this order for expository purposes only.

Similarly,

E^{sr} = fractional effectiveness due to supply shortage for Red

$= f^{sr}(D^{shr})$,

where $D^{shr} = \dfrac{R^s}{R_0 S_0^{cr} + \sum_i R_i S_i^{cr}}$.

Step 2. The value of each Blue weapon is assumed to be equal to the potential number of Red casualties it can cause. That is,

V_{ik}^{lbwd} = the value of an individual Blue type-i ground weapon on defense in posture-type k against the particular Red force in the sector

$= \sum_j$ (potential number of Red type-j weapons that can be killed by a Blue type-i weapon defending in posture-type k, considering allocation of fire) × (the number of Red casualties associated with each kill of a Red type-j weapon by a Blue type-i weapon)

$$= \sum_j \left(\frac{A_{ij}^{*bgd} R_j / R_j^*}{\sum_{j'} A_{ij'}^{*bgd} R_{j'} / R_{j'}^*} \right) P_{ijk}^{bgd} D_{ij}^{bgd} \; . \; ^{1/}$$

Thus,

V_{kd}^{bwd} = the value of all weapons in a Blue type-d division on defense in posture-type k against the particular Red force in the sector

$= \sum_i V_{ik}^{lbwd} B_{id}$.

---

[1]Except for the allocation of fire, this value is quite similar to a firepower score. The major difference is in how this value is used. This value does not attempt to measure the overall effectiveness of a weapon (as in firepower models), but only to determine a scale factor. And since this scale factor is only one part of the attrition structure, the value computed here represents only one part of the effectiveness of a weapon.

Note that the sum (over all types of divisions) of V_{kd}^{bwd} equals $\sum_i \sum_j Q_{ij}^{bgd} D_{ij}^{bgd}$. Since Q_{ij}^{bgd} is the potential number of Red type-j weapons killed by all Blue type-i ground weapons, the sum (over all types of divisions) of V_{kd}^{bwd} is the total potential number of Red casualties caused by all Blue ground weapons, considering all types of Red target weapons.

More detail on the rationale for using this method to compute the weapon value of a division is given by Anderson [6].

Step 3. Blue ground effectiveness based on personnel strength for each type of division is assumed to be a function of the actual strength divided by the TOE strength for that type of division. This function gives a factor that is applied to the weapon value that the division would have if it were at full TOE strength. Let

V_{kd}^{tbwd} = the weapon value of a Blue type-d division at full TOE strength on defense in posture-type k

= $\sum_i$ (value of an individual Blue type-i weapon on defense in posture-type k) × (number of Blue type-i weapons in the TOE of a type-d division)

$$= \sum_i V_{ik}^{lbwd} B_{id}^{t}$$

Let B_d^{fs} = the fractional strength of a Blue type-d division in the sector

= [actual number of people in all Blue type-d divisions in the sector] ÷ [(the TOE number of people in one Blue type-d division) × (the number of Blue type-d divisions in the sector)]

$$= \frac{B_{0d}}{B_{0d}^{t} N_d^{b}} .$$

Then

E_d^{bpo} = the fractional effectiveness based on personnel strength of a Blue type-d division in the sector if there were no reorganization

$$= f_d^{bed}(B_d^{fs}) \ .$$

To account for reorganization, let

B_d^{fsy} = the fractional strength of a Blue type-d division in the sector yesterday (B_d^{fsy} on day n+1 equals B_d^{fs} on day n; and B_d^{fsy} = 1 on day 1);

E_d^{bpy} = the fractional effectiveness based on personnel strength of a Blue type-d division in the sector after reorganization yesterday (E_d^{bpy} on day n+1 equals E_d^{bp} on day n--E_d^{bp} will be defined below--and E_d^{bpy} = 1 on day 1); and

E_d^{bpoy} = the fractional effectiveness based on personnel strength of a type-d division in the sector yesterday if there were no reorganization

$$= f_d^{bed}(B_d^{fsy}) \ .$$

Accordingly, a Blue type-d division would have had yesterday a fractional effectiveness of E_d^{bpoy} had it never reorganized, but it actually had yesterday a fractional effectiveness of E_d^{bpy}. If $E_d^{bpy} - E_d^{bpoy} > 0$, then, to increase the actual effectiveness, some reorganization must have taken place yesterday or before. The most reorganization that can take place today is the difference between the fractional strength B_d^{fs} and the fractional effectiveness without reorganization E_d^{bpo}. Let

G_d^{bdy} = the ratio of the reorganization that has already been made before today to the maximum amount of reorganization that can be made today

$$= \frac{E_d^{bpy} - E_d^{bpoy}}{B_d^{fs} - E_d^{bpo}} \ .$$

Thus, adding in the reorganization that can be made today gives that the reorganization up through today is the minimum of $G_d^{bdy} + G_d^{bd}$ and 1.0.

The above arguments give that

E_d^{bp} = the fractional effectiveness based on personnel strength of a Blue type-d division in the sector considering reorganization

= [the fractional effectiveness based on personnel strength without reorganization] + [(the reorganization through today) × (the maximum amount of reorganization that can take place)]

$$= E_d^{bpo} + [(\min\{G_d^{bdy} + G_d^{bd}, 1.0\})(B_d^{fs} - E_d^{bpo})] .$$

Note that if no reorganization is allowed ($G_d^{bd} = 0$), then

$$E_d^{bp} = E_d^{bpo} ,$$

while if full reorganization can take place overnight ($G_d^{bd} = 1$), then

$$E_d^{bp} = B_d^{fs} .$$

The reason that the expression for E_d^{bp} in the intermediate cases is somewhat complicated is in order to consider adequately the effect of replacements or further attrition on reorganization.

With the above structure, the ground value of a division based on personnel strength can be calculated as

V_{kd}^{bpd} = Blue value based on personnel strength of a type-d division on defense in posture-type k

= (fractional effectiveness based on personnel strength of a type-d division on defense) × (the weapon value of a Blue division at full TOE strength on defense in posture-type k)

$$= E_d^{bp} V_{kd}^{tbwd} .$$

Note that V_{kd}^{tbwd} is the potential number of casualties that can be caused by a full-strength Blue type-d division on defense in posture-type k against the particular Red force in the sector (V_{kd}^{tbwd} is a function of R_j). E_d^{bp} is a degradation factor if the division is not at full personnel strength. Thus, V_{kd}^{bpd} can be interpreted as the potential number of casualties that can be caused by the Blue division, considering its actual personnel strength.

Step 4. The total Blue ground value then is

$$V_k^{bgd} = \text{total Blue ground value on defense in posture-type k}$$

$$= \sum_d (\min \{V_{kd}^{bwd}, V_{kd}^{bpd}\}) N_d^b E^{sb} .$$

The interpretation here is that if V_{kd}^{bwd} is the potential number of casualties, considering the actual number of weapons in the division, and if V_{kd}^{bpd} is the potential number of casualties, considering the actual number of people in the division then, since the division needs both people and weapons to inflict casualties, the potential number of casualties that the division can inflict, considering both its strength and its weapons, is the minimum of V_{kd}^{bwd} and V_{kd}^{bpd} .

Thus,

$$\sum_d (\min \{V_{kd}^{bwd}, V_{kd}^{bpd}\}) N_d^b$$

is the potential number of casualties that the entire Blue ground force in the sector can inflict, considering people and weapons. This would be Blue's potential if Blue had no shortage of supplies. Since a force needs supplies (as well as people and weapons) to inflict casualties, a degradation factor if Blue is short of supplies must also be considered, which is the role played by E^{sb}. Accordingly, V_k^{bgd} can be interpreted as the potential number of casualties that the Blue ground force on defense in posture-type k can inflict on the particular Red force in the sector, considering the actual number of weapons, the actual number of people, and the number of supplies Blue has in the sector.

Step 5. The total Blue air value is calculated as

V^{bad} = total Blue air value on defense

$$= \sum_j \sum_c B_c^a \left(\sum_m L_{cm}^{bad} \left(\frac{A_{mj}^{*bad} R_j / R_j^*}{\sum_{j'} A_{mj'}^{*bad} R_{j'} / R_{j'}^*} \right) P_{mj}^{bad} D_{mj}^{bad} \right)$$

$$= \sum_m \sum_j Q_{mj}^{bad} D_{mj}^{bad} \ .$$

Note that since Q_{mj}^{bad} is the potential number of Red type-j weapons killed by all Blue type-m air munitions and since D_{mj}^{bad} is the number of Red casualties per type-j weapon killed by a Blue type-m air munition, V^{bad} is the total potential number of Red casualties caused by all Blue air munitions, considering all types of target weapons in the particular Red force in the sector.

Step 6. V_{kd}^{rwa} and V_{kd}^{rpa} are calculated in a manner similar to that for V_{kd}^{bwd} and V_{kd}^{bpd} for a type-d division on attack against Blue defending in posture-type k, and

V_k^{rga} = total Red ground value on attack against Blue defending in posture-type k

$$= \sum_d (\min \{V_{kd}^{rwa}, V_{kd}^{rpa}\} N_d^r) E^{sr} \ .$$

V^{raa} is calculated in the analogous way--i.e.,

V^{raa} = total Red air value on attack

$$= \sum_j \sum_c R_c^a \left(\sum_m L_{cm}^{raa} \left(\frac{A_{mj}^{*raa} B_j / B_j^*}{\sum_{j'} A_{mj'}^{*raa} B_{j'} / B_{j'}^*} \right) P_{mj}^{raa} D_{mj}^{raa} \right.$$

$$= \sum_m \sum_j Q_{mj}^{raa} D_{mj}^{raa} \ .$$

Step 7. This step converts the values (or, equivalently, the potential number of casualties) into percent casualties which, when applied to the appropriate number of people, gives the actual number of casualties. The way that this calculation is done is to form force ratios from these values and then to use historical data to compute percent casualties as a function of force ratio.

Traditionally, force ratios have been written as the form of attacker over defender, so that if no CAS sorties were flown on either side the force ratio would be

$$\frac{V_k^{rga}}{V_k^{bgd}},$$

since we are assuming that Red is on attack in the sector. The historically based functions f_k^{bcd} and f_k^{rca} would then give that

$f_k^{bcd}\left(\frac{V_k^{rga}}{V_k^{bgd}}\right)$ = the percent casualties to Blue on defense in posture-type k in the sector that day;

and

$f_k^{rca}\left(\frac{V_k^{rga}}{V_k^{bgd}}\right)$ = the percent casualties to Red on attack against Blue defending in posture-type k in the sector that day.

This structure has been used in other models, most notably in ATLAS (see Kerlin and Cole [19]); and it is used in IDAGAM I for the case where neither side flies any CAS sorties.[1]

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[1]At first glance, it may appear inconsistent that the force ratio defined above is linear in the attacker's value but is nonlinear in the defender's value. But this is not a real inconsistency, because the functions that convert the force ratios to percent casualties depend on whether the side is on attack or defense. For example, if one defined the function $\bar{f}_k^{bcd}(x)$ by

$$\bar{f}_k^{bcd}(x) = f_k^{bcd}\left(\frac{1}{x}\right)$$

(continued on next page)

When both sides fly CAS sorties, the force ratio has traditionally been written as

$$\frac{V_k^{rga} + V^{raa}}{V_k^{bgd} + V^{bad}}$$

for Red on attack and Blue on defense in posture-type k. The force ratio in this form can be useful for several purposes, such as a one-parameter comparison of the total Red force versus the total Blue force. However, it has a severe defect if it is used to compute percent casualties to both the attacker and the defender. A complete discussion of this defect is given by Anderson [5]. Based on the arguments in that reference, IDAGAM I forms two force ratios--one for computing percent casualties to the attacker and one for computing percent casualties to the defender. Let

F^{cd} = force ratio for computing percent casualties to the defender in IDAGAM I

$$= \frac{V_k^{rga} + V^{raa}}{V_k^{bgd}} \text{ , and}$$

F^{ca} = force ratio for computing percent casualties to the attacker in IDAGAM I

$$= \frac{V_k^{rga}}{V_k^{bgd} + V^{bad}} \text{ ,}$$

(cont'd) and then computed casualties to the defender as

$$\bar{f}_k^{bcd}\left(\frac{V_k^{bgd}}{V_k^{rga}}\right) ,$$

one would obtain identically the same result as IDAGAM I obtains.

since we are assuming that Red is on attack and Blue is on defense in the sector.[1] Then

$$P^{cd} = \text{the percent casualties to the defender}$$

$$= f_k^{bcd}(F^{cd})$$

$$= f_k^{bcd}\left(\frac{V_k^{rga} + V^{raa}}{V_k^{bgd}}\right) ,$$

and

$$P^{ca} = \text{the percent casualties to the attacker}$$

$$= f_k^{rca}(F^{ca})$$

$$= f_k^{rca}\left(\frac{V_k^{rga}}{V_k^{bgd} + V^{bad}}\right) .$$

For comparison purposes, IDAGAM I allows the option of computing force ratios for casualties in the traditional way--i.e., it allows the option of computing F^{cd} and F^{ca} as

$$F^{cd} = F^{ca} = \frac{V_k^{rga} + V^{raa}}{V_k^{bgd} + V^{bad}} .$$

Step 8. To determine the number of casualties, ATLAS computes percent casualties and applies that percent to the number of people in each division, but not to any support

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[1] Analogous ratios are used if Blue is on attack and Red on defense. If a holding posture exists in the sector, then the percent casualties to Blue is given by

$$f^{bch}\left(\frac{V_k^{rgd} + V^{rad}}{V_k^{bgd}}\right) ,$$

where $f^{bch}(x)$ is an input function, and the percent casualties to Red is given by the same expression with "b" and "r" interchanged.

units attached to the division. IDAGAM I cannot attach units to other units, and a division should be thought of as consisting of its basic TOE units and all support units attached to it. We feel that this structure is an advantage of IDAGAM I, for the following reason: ATLAS, in effect, assumes that support units cannot suffer casualties. In particular, ATLAS assumes that corps artillery attached to a division is always invulnerable. Not only is this assumption inconsistent, since division artillery is not invulnerable, but also it can lead to an analysis that unjustifiably favors corps artillery over all other weapons. It may well be that if corps artillery is attached to a full-strength division, then the corps artillery will suffer very few casualties. But if the division has suffered significant attrition, then the corps artillery will begin to suffer attrition.

However, to apply the percent casualties directly to the IDAGAM I strength of a division (the division plus its supporting units) would increase the number of casualties if the division were at full TOE strength, which is not what is desired. The structure used in IDAGAM I is to multiply the number of people in the IDAGAM I-size division by a balance factor before applying the percent casualties. This balance factor can be taken as the ratio of the full strength of a type-d division in ATLAS (without supporting units) to the full strength of a type-d division in IDAGAM I (with supporting units). Denote this balance factor by L_d^b for Blue type-d divisions and L_d^r for Red type-d divisions. If all else were equal, this structure would cause IDAGAM I to calculate the same number of casualties as ATLAS if the divisions were at full strength, but the number of casualties would differ if divisions were under-strength.

A second modification to the number of people is based on supply shortage. Suppose two sides facing each other have a force ratio of 1 to 1. Suppose further that each side is so short of supplies that each is at .01 percent of its full effectiveness. Then the resulting force ratio would still be 1 to 1 and, unless this supply shortage is considered some other way, the same number of people on each side would be killed as would have been killed if each side were at full effectiveness, which is clearly unreasonable. To consider this mutual shortage of supplies, let

$$E^{sm} = \max \{E^{sb}, E^{sr}\} .$$

Note that $E^{sm} = 1.0$ if either side does not have a shortage of supplies. But if both sides are short of supplies, which is what causes this anomaly, then $E^{sm} < 1.0$.

With these two factors, the number of people associated with each type-d division to which the percent casualties is applied is

$$B_{0d} L_d^b E^{sm} .$$

With percent casualties computed in Step 7, the total number of Blue casualties is given by

$$\dot{B}_0 = P^{cd} \left(\sum_d N_d^b B_{0d} L_d^b \right) E^{sm} .$$

Similarly, for Red casualties,

$$\dot{R}_0 = P^{ca} \left(\sum_d N_d^r R_{0d} L_d^r \right) E^{sm} .$$

(3) Antipotential Potential

A method to compute the value of a weapon based on that weapon's capability to destroy the value of the enemy's weapons was discovered independently by Spudich [21] (his work is also contained in [24]), by Dare and James [13], and by Thrall [22]. Anderson [1] first noted that these researchers were using essentially the same method and showed the relationship between their work. Holter [14] has extended this work to the stage where it appears to be an extremely appealing method to determine weapon values for force ratios. In this subsection, we will give the equations used to implement this method. (The reader is referred to the references for the rationale behind these equations.)

Using the variables already defined, let K^{bw} be the matrix whose $(i,j)^{th}$ element is given by

$$K_{ij}^{bw} = \left(\frac{A_{ij}^{*bgd} R_j / R_j^*}{\sum_{j'} A_{ij'}^{*bgd} R_{j'} / R_{j'}^*} \right) P_{ijk}^{bgd} \ ;$$

K^{ba} be the matrix whose $(c,j)^{th}$ element is given by

$$K_{cj}^{ba} = \sum_m L_{cm}^{bad} \left(\frac{A_{mj}^{*bad} R_j / R_j^*}{\sum_{j'} A_{mj'}^{*bad} R_{j'} / R_{j'}^*} \right) P_{mj}^{bad} \ ;$$

and define the matrices K^{rw} and K^{ra} similarly for Red. Note that K^{bw} and K^{rw} are dependent on the type of posture and that all four matrices are dependent on which side is attacking. But, for convenience, we will drop the notation for this dependence and assume throughout that Red is on attack and Blue is defending in posture-type k.

Let $\bar{K}^{bw}$ be the matrix product of K^{bw} and K^{rw}, with the last row and column deleted. (This deletion is necessary because the last weapon type--SAMs--are assumed to have no ground value.) By our assumptions, $\bar{K}^{bw}$ is nonnegative. Assume it is also irreducible, so that $\bar{K}^{bw}$ has a maximal real eigenvalue that is uniquely determined up to a scaling constant.[1] Let

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[1]Thrall [22] gives a rationale for using the vectors that result from the numerical technique used in IDAGAM I, even when these matrices are reducible. If this "eigenvector" approach were used when considering isolated battles, it is possible that these matrices could be reducible. However, when considering sectorwide battles (as does IDAGAM I), it seems quite unlikely that these matrices could be reducible, and even if they are reducible, the arguments given by Thrall can be made to support their use.

the inverse of this eigenvalue be λ. Let i_b be a particular Blue weapon type (determined by input) and, for each posture-type k, define V_{ik}^{lbwd} to be the unique eigenvector corresponding to $1/\lambda$ such that $V_{i_b k}^{lbwd} = 1$. Let V_{ik}^{lrwa} be given by

$$V_{ik}^{lrwa} = \sqrt{\lambda} \sum_j K_{ij}^{rw} V_{jk}^{lbwd} \ .$$

These computations give the values for the Blue ground weapons (V_{ik}^{lbwd}) and the Red ground weapons (V_{ik}^{lrwa}) except for the SAMs on each side. Since it is assumed that SAMs cannot kill ground weapons, SAMs can be given a value of 0. Again, the reader is referred to the references (esp. to Holter [14]) for the logic behind this approach.

The air values cannot be determined directly by using this method, since it is assumed that ground weapons cannot kill aircraft; and, so, a matrix containing both ground weapons and aircraft would not be irreducible. However, as described in Dare and James [13] and Anderson [1], a consistent way to determine the air values V_c^{lbad} and V_c^{lraa} is as follows:

$$V_c^{lbad} = \sqrt{\lambda} \sum_j K_{cj}^{ba} V_{j1}^{lrwa}$$

and

$$V_c^{lraa} = \sqrt{\lambda} \sum_j K_{cj}^{ra} V_{j1}^{lbwd} \ .$$ [1]

The goal of Step 2 (above) is to calculate V_{kd}^{bwd}. To use the antipotential potentials determined here, the calculation of V_{kd}^{bwd} is identical to that of Step 2--i.e.,

$$V_{kd}^{bwd} = \sum_i V_{ik}^{lbwd} B_{id} \ ,$$

---

[1] Since we arbitrarily decided not to allow air values to be a function of ground posture, we used here posture-type 1 (normal attack/delay) as a typical ground posture for determining air values.

except that V_{ik}^{lbwd} is as defined just above, instead of as it was defined in Step 2. Similarly, the goal of Step 5 (above) is to calculate V^{bad}; and to use the antipotential potentials determined here, that calculation is replaced by

$$V^{bad} = \sum_c V_c^{lbad} B_c^a ,$$

where V_c^{lbad} is as defined above. All other calculations for Blue are performed exactly as in Subsection (2), above. The only difference between the potential-casualty method and the anti-potential-potential method is the way that V_{kd}^{bwd} and V^{bad} (and the corresponding values for Red) are calculated.

(4) Linear Weights

Both of the methods described above are relatively complex. The goal of this method is to provide a simple way to compute force ratios directly from inputs. Let

W_i^{bgd} = the linear weight of a Blue type-i weapon on defense;

W_m^{bad} = the linear weight of a Blue type-m air munition on defense;

and let W_i^{rga} and W_m^{raa} be defined similarly for Red on attack. Then this method computes V_{kd}^{bwd} and V^{bad} as

$$V_{kd}^{bwd} = \sum_i W_i^{bgd} B_{id} \; ^{1/}$$

and

$$V^{bad} = \sum_c B_c^a \sum_m L_{cm} W_m^{bad} .$$

If this were all that IDAGAM I did with the linear-weights method, then this method would be exactly consistent with the other methods. But since the goal of this method is to compute force ratios directly from inputs, and since the force ratios use the quantity V_k^{bgd} (not V_{kd}^{bwd}), V_k^{bgd} should be computed directly

[1]See footnote on next page.

from inputs. Accordingly, this method goes on to eliminate Step 3 and replace Step 4 (which computes V_k^{bgd}) with the computation

$$V_k^{bgd} = \sum_d V_{kd}^{bwd} N_d^b \; .$$[1]

That is, this method computes V_k^{bgd} directly as

$$V_k^{bgd} = \sum_i W_i^{bgd} B_i \; .$$

Similar computations are made for Red. Thus, the linear-weights method computes V_k^{bga}, V^{baa}, V_k^{rga}, and V^{raa} as described here and then computes total casualties as described in Steps 7 and 8 of Subsection (2), above.

The linear-weights method is simple and straightforward, and it is essentially the same method used by ATLAS, GACAM I, and GACAM II. The problem with this method is that it is too simple; it oversimplifies the complex process of ground combat; and it is included in IDAGAM I solely for purposes of comparison.

e. <u>Nonbattle Casualties</u>

The total number of casualties computed above represents the number of battlefield casualties. IDAGAM I can also play nonbattle casualties. An input percentage (which is a function of whether the division is on attack or defense and of what its posture is) is applied to all people in divisions in sectors to determine the number of nonbattle casualties that occur in sectors. Another input percentage is applied to all people in regions to determine the number of nonbattle casualties that occur in regions. And a third input percentage is applied to all people in the COMMZ to determine the number of nonbattle casualties that occur in the COMMZ. No weapon losses are associated with nonbattle casualties.

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[1] So V_{kd}^{bwd} and V_k^{bgd} are independent of k by this method.

f. Attrition to Reserve Divisions

The attrition to divisions in regions due to aircraft on IDR missions is calculated in IDAGAM I in the air-combat model. We have deferred its explanation to this subsection because the attrition equations are based on the logic described in Section C.2.b, above.

It is desirable to model the way that aircraft cause attrition to divisions in reserve consistently with the way that they cause attrition to divisions in combat. However, it is not possible to have identical attrition processes for these two cases in IDAGAM I, because IDAGAM I uses a force-ratio structure to scale potential weapons losses for combat attrition; and there is no force ratio when reserves are attacked by aircraft. But as discussed above, the force-ratio structure is used to determine casualties and to scale potential weapons losses. The potential numbers of weapons lost (by type) are determined from Lanchester-type equations independently of force ratios. If the inputs to these Lanchester-type equations were actual kills (rather than potential kills), these same equations could be used to determine by weapon type actual numbers of weapons lost. The casualties-per-weapon-lost data could then be used to calculate total casualties, which is what IDAGAM I does.

The inputs required are

K_{mj}^{bar} = the actual number of Red type-j ground weapons killed by each Blue type-m air munition if the Blue type-m munitions were fired at Red type-j weapons from Blue aircraft on missions,

and K_{mj}^{rar} for Red aircraft on IDR missions.

Let

B_c^{ar} = the number of Blue type-c aircraft on IDR missions that successfully deliver their ordnance that day in the region (this is calculated by the air-combat model);

R^r_j = the number of Red type-j ground weapons in reserve in regions;

R^r_0 = the number of Red people in reserve in regions;

and let R^{ar}_c, B^r_j, and B^r_0 be the same for Red air-interdiction of Blue divisions in reserve. (R^r_j, R^r_0, B^r_j, and B^r_0 are input on day 1 and are adjusted for each succeeding day--based on attrition, reinforcements, replacements, etc.). Finally, let

$\dot{R}^r_j$ = the number of Red type-j weapons lost to Blue aircraft on IDR missions;

$\dot{R}^r_0$ = the number of Red people lost to Blue aircraft on IDR missions;

and let $\dot{B}^r_j$ and $\dot{B}^r_0$ be the same for Blue losses (these are the quantities to be calculated). Then the model computes

$$\dot{R}^r_j = \sum_c B^{ar}_c \left(\sum_m L^{baa}_{cm} \left(\frac{A^{*baa}_{mj} R^r_j / R^*_j}{\sum_{j'} A^{*baa}_{mj'} R^r_{j'} / R^*_{j'}} \right) K^{bar}_{mj} \right),$$

$$\dot{R}^r_0 = \sum_j \sum_c B^{ar}_c \left(\sum_m L^{baa}_{cm} \left(\frac{A^{*baa}_{mj} R^r_j / R^*_j}{\sum_{j'} A^{*baa}_{mj'} R^r_{j'} / R^*_{j'}} \right) K^{bar}_{mj} D^{baa}_{mj} \right),$$

$$\dot{B}^r_j = \sum_c R^{ar}_c \left(\sum_m L^{raa}_{cm} \left(\frac{A^{*raa}_{mj} B^r_j / B^*_j}{\sum_{j'} A^{*raa}_{mj'} B^r_{j'} / B^*_{j'}} \right) K^{rar}_{mj} \right),$$

and

$$\dot{B}^r_0 = \sum_j \sum_c R^{ar}_c \left(\sum_m L^{raa}_{cm} \left(\frac{A^{*raa}_{mj} B^r_j / B^*_j}{\sum_{j'} A^{*raa}_{mj'} B^r_{j'} / B^*_{j'}} \right) K^{rar}_{mj} D^{raa}_{mj} \right).$$

Note that the notional loads, the allocation of air munitions, and the number of casualties per weapon lost are assumed to be the same as for CAS missions if the side were on attack in the sector.

3. FEBA-Movement Calculations

This section describes Step 6 of the ground-combat model, in which the movement of the FEBA in the sector under consideration is calculated. Throughout this section, we assume that Red is on attack, that Blue is defending in posture-type k, and that the terrain is of type t in the sector.[1] (The case where Blue is on attack is handled symmetrically.)

a. Structure and Underlying Assumptions

The ground-combat model assumes that FEBA movement is a function of force ratio, type of defender posture, type of terrain, inherent mobility of the attacker's divisions, and use of air forces.

The primary assumption here is that FEBA movement is a function of force ratio. (Posture and terrain are considered by allowing separate curves, which give FEBA movement as a function of force ratio for each combination of posture and terrain type.) The secondary assumptions concern how IDAGAM I models the inherent mobility of ground forces and the use of air forces.

Concerning the primary assumption, IDAGAM I does not model the maneuver of ground units. For example, it does not model a company or battalion breaking through enemy lines, disrupting the enemy's lines of communication, and forcing the enemy to withdraw in some part of a sector. Nor does it model maneuvers to gain a better attacking position, nor does it model envelopments by ground units that might take place within a sector.

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[1] If the movement of the FEBA as described below causes the FEBA to reach an internal boundary, then the posture, terrain, and width of the sector can change. In this case, the movement of the FEBA in the new interval is determined by the posture, terrain, and width of the sector in the new interval. (This is explained in greater detail in Chapter III of Volume 3.)

These maneuvers are all important events in combat but are, we believe, too detailed for a theater-level model like IDAGAM I. Instead, these maneuvers are considered as being part of the events that contribute to the FEBA-movement/force-ratio curves. In actual combat, the movement of the FEBA would not be the same all along the sector front; and, in various parts of the sector, different types of maneuvers may affect the FEBA position. But, with the two exceptions noted below, we assume that across the whole sector there is some average movement, which can be given by a FEBA-movement/force-ratio curve.

The two exceptions to this general assumption concern the secondary assumptions about mobility of ground forces and the use of air forces. Concerning the inherent mobility of ground forces, suppose an attacker had two ground forces, either one of which would give the same force ratio against a particular defender's force, but one of these forces was inherently more mobile than the other (perhaps due to the posture and terrain, as well as to the composition of these forces). Then it is reasonable to assume that the FEBA would move farther if the attacker used his more mobile ground force than it would if he used his less mobile ground force. The way that IDAGAM I reflects the inherent mobility of different types of attacking divisions (in the various postures and terrain types) is described in Subsection b below.

The second exception concerns the use of air forces. The differences between the mobility of high-performance aircraft and of ground units are so great that it is difficult to compare them. Yet both contribute to FEBA movement, and it would be inappropriate not to model the mobility of such aircraft or to model it in exactly the same way that mobility is modeled for ground units. The way that IDAGAM I attempts to model these differences in mobility is to allow explicitly the attacker's air forces to concentrate their attack in various parts of a sector, in order to create a greater force-ratio advantage in those parts (the defender can also concentrate his air forces to minimize this advantage to the attacker). Thus, the mobility of ground forces is modeled by directly making the FEBA move faster or slower across the whole sector, while the mobility of air forces is modeled by allowing the concentration of air forces in parts of the sector.[1] Three ways in which the attacker can concentrate his air forces will be discussed in Subsections c, d, and e. The results from these three subsections will be combined (in Subsection f) to give the movement of the FEBA.

New inputs required for this section are as follows:

M^{cmr} = method for computing mobility if Red is the attacker (allowable values for this input will be discussed below).

M^{r}_{dkt} = mobility factor for a Red type-d division in terrain-type t on attack against a defender in posture-type k.

S^{r}_{d} = relative size of a Red type-d division. Let the average Red division have a relative size of 1.0. Then if a Red type-d division is 0.8 as large as the average Red division, then $S^{r}_{d} = 0.8$.

$f^{r}_{kt}(x)$ = the basic movement of the FEBA which, when multiplied by the mobility factor, gives the actual movement of the FEBA for the case under consideration. $f^{r}_{kt}(x)$ is this basic movement of the FEBA if Red is on attack, Blue is defending in posture-type k, the terrain type is t, and the appropriate force ratio for the case under consideration is x. (The function f^{r}_{kt} is input via a piecewise linear structure.)

W^{ra} = the minimum width in which Red can effectively concentrate his air forces to help the Red ground forces create and hold a salient.

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[1]The basic assumption here is that it is much easier for a side to concentrate all his air attacks in certain parts of a sector (without the enemy's reacting by moving his ground forces) than it is for that side to concentrate all his ground forces (without enemy reaction); and that the concentration of ground units that normally occurs in combat is considered in the FEBA-movement/force-ratio curves.

W = the width of the sector at the location of the FEBA on the beginning of the day. (W is calculated from geographical inputs each day and based on the location of the FEBA.)

The only quantities defined in Section C.2 (above) that are needed here are the total ground and air values for Blue and Red (V_k^{bgd}, V^{bad}, V_k^{rga}, and V^{raa}) and the number of divisions of each type in the sector (N_d^b and N_d^r). Working variables for this section will be defined as needed.

b. Mobility Factor

Since different types of divisions have different inherent mobilities, some method must be used to determine the overall mobility of a force consisting of several different types of divisions. IDAGAM I allows the user to select one of four methods for computing the overall mobility factor. The method used depends on the input M^{cmr}:

If $M^{cmr} = 1$, then the overall mobility factor = 1 (i.e., mobility is not based on Red division type; it is based only on posture and terrain).

If $M^{cmr} = 2$, then the overall mobility factor is the minimum divisional mobility factor over the Red divisions in the sector, for the given posture and terrain.

If $M^{cmr} = 3$, then the overall mobility factor is the maximum divisional mobility factor over the Red divisions in the sector, for the given posture and terrain.

If $M^{cmr} = 4$, then the overall mobility factor is the weighted average of the divisional mobility factors, where the weighting is by number of divisions by type in the sector times the relative size of that type of division, for the given posture and terrain.

Formally, let

M_{kt} = the (overall) mobility factor if the defender is in posture-type k and the terrain is type t.

Then

$$M_{kt} = \begin{cases} 1.0 \ , & \text{if } M^{cmr} = 1; \\ \min\limits_{d} \{M^r_{dkt} : N^r_d > 0\} \ , & \text{if } M^{cmr} = 2; \\ \max\limits_{d} \{M^r_{dkt} : N^r_d > 0\} \ , & \text{if } M^{cmr} = 3; \\ \dfrac{\sum\limits_{d} M^r_{dkt} N^r_d S^r_d}{\sum\limits_{d'} N^r_{d'} S^r_{d'}} \ , & \text{if } M^{cmr} = 4 \ . \end{cases}$$

Thus, in each of the cases discussed below, if the appropriate force ratio is x, then the FEBA is assumed to move a distance of $M_{kt} f^r_{kt}(x)$.

c. Attacker's Air Advantage Greater Than His Ground Advantage

In this and the next two subsections, we will discuss three ways that the attacker can use his air forces.

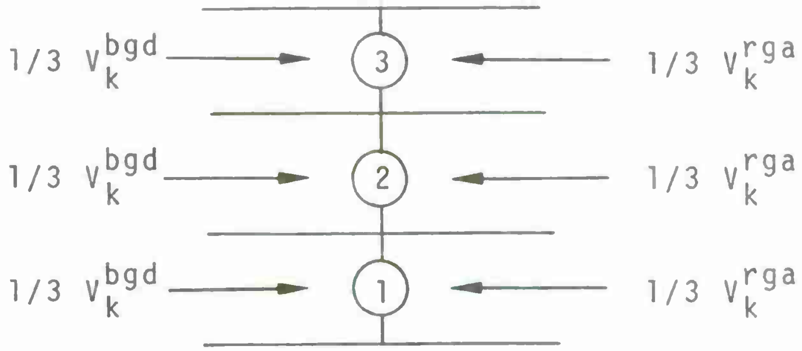
The first case we will consider is that where the attacker has a ground advantage ($V^{rga}_k > V^{bgd}_k$) and an air advantage ($V^{raa} > V^{bad}$), and his air advantage is greater than his ground advantage ($[V^{raa}/V^{bad}] > [V^{rga}_k/V^{bgd}_k]$). In this case, if the attacker were to concentrate his air forces in one part of this sector, then that part of the sector would have a higher force ratio and would move faster than the other parts of the sector (and the attacker will not be at a ground disadvantage in the remaining parts of the sector). This relationship remains true even if the defender were to know in which part the attacker would concentrate his air forces and if the defender were to fly his air forces to that part. Thus, in this case, the attacker can use his air forces to create one or more salients across the sector front.

It does not seem reasonable to assume that the attacker ignores this characteristic and that he spreads his air forces uniformly across the sector. This advantage for the attacker should be explicitly considered.

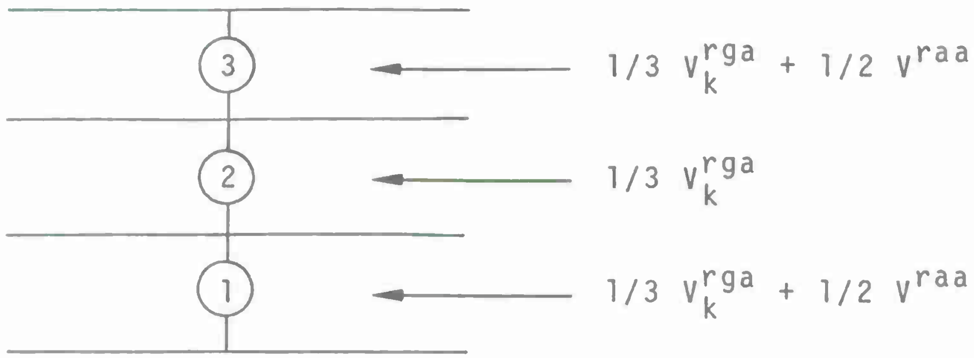
The attacker could concentrate all his air forces to create one salient whose width is W^{ra}, and this salient would move more quickly than the rest of the sector. But if the attacker does so, sooner or later he will have front-to-flank ratio problems with that salient. One way the attacker could effectively use this advantage is as follows:

Suppose that $W^{ra} = 1/3\ W$. Then we could picture the sector as follows:

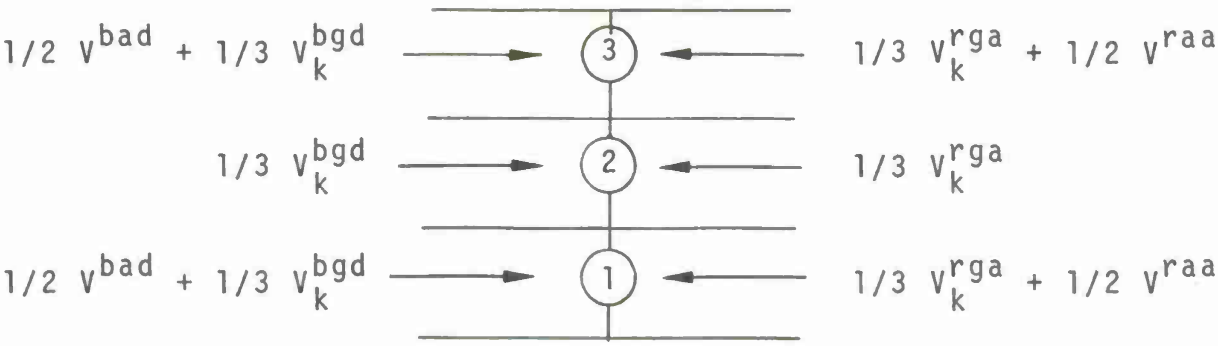
Denote each of these parts of the whole sector as minor sectors. Then, since $W^{ra} = 1/3\ W$, there are three minor sectors in the whole sector. The assumption concerning ground forces gives that each minor sector contains 1/3 of each side's ground forces. Numbering the minor sectors from 1 to 3 gives the following:



If the attacker were to concentrate his air forces in minor sectors 1 and 3, then the FEBA would move faster in these minor sectors than in minor sector 2. Then the "pinch" would be on the defender in minor sector 2. Minor sectors 1 and 3 each have only one exposed flank in the sector, and the sector boundaries are assumed to be geographical characteristics that limit combat across the boundaries. On the other hand, the defender would eventually be surrounded in minor sector 2 if he did not withdraw first. This concentration of the attacker's air forces can be pictured as follows:



If the attacker were to concentrate his air forces this way day after day (and if the air and ground advantages remain as described above day after day), then sooner or later, to avoid being surrounded, the defender would have to withdraw in minor sector 2 at the same rate that the attacker is advancing in minor sectors 1 and 3. This statement is true no matter how the defender uses his air forces, and so it is to the defender's advantage to use his air forces to slow down sectors 1 and 3--resulting in the following picture:



As described above, the defender will eventually be forced to withdraw from minor sector 2 at the same rate as the FEBA is moving in minor sectors 1 and 3. Accordingly, the average FEBA movement in the whole sector is approximated by the FEBA movement in either minor sector 1 or minor sector 3 (they are equal, since the force ratio, posture, and terrain are the same). Let

F_1 = the FEBA movement in the whole sector if the air and ground advantages are as described above.

Then we assume that

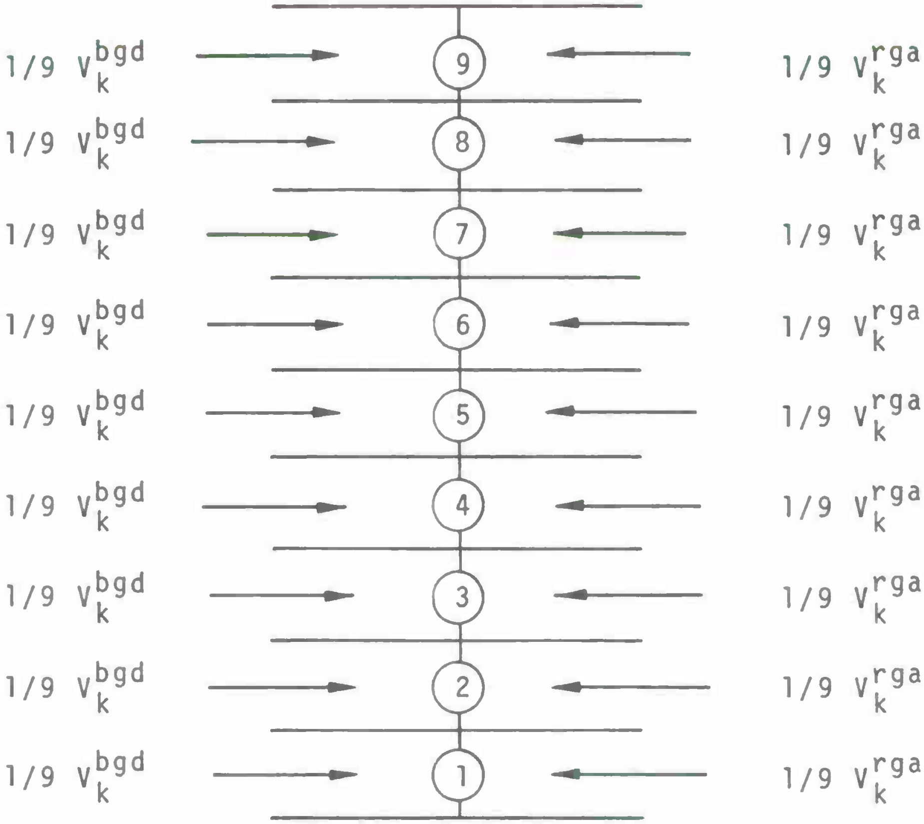
F_1 = the FEBA movement in a minor sector in which the attacker is concentrating his air forces.

For the case where $W^{ra} = 1/3\ W$, this concentration of air forces gives that

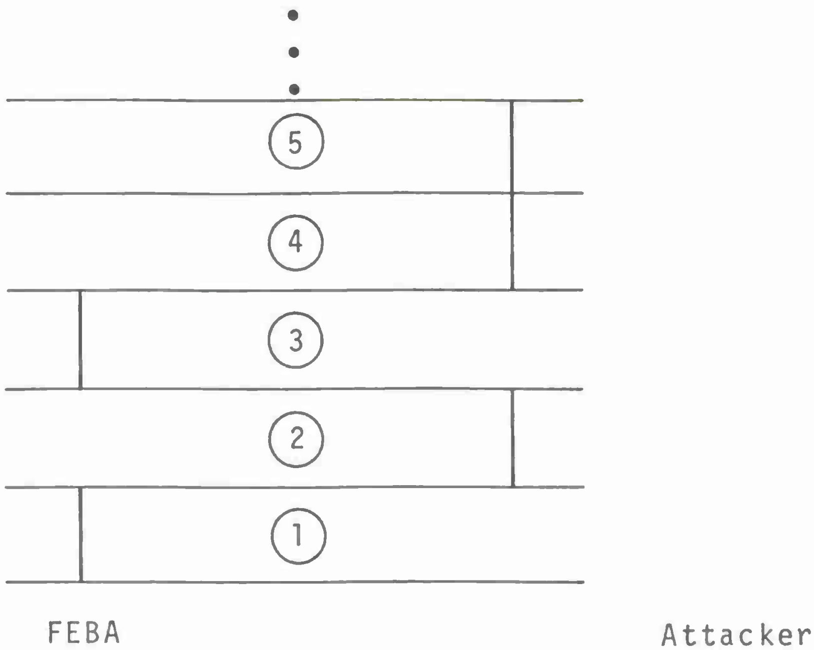
$$F_1 = M_{kt} f_{kt}^r \left(\frac{\frac{1}{3}\ V_k^{rga} + \frac{1}{2}\ V^{raa}}{\frac{1}{3}\ V_k^{bgd} + \frac{1}{2}\ V^{bad}} \right)$$

$$= M_{kt} f_{kt}^r \left(\frac{V_k^{rga} + \frac{3}{2}\ V^{raa}}{V_k^{bgd} + \frac{3}{2}\ V^{bad}} \right) .$$

Now suppose that $W^{ra} = 1/9\ W$. Then the ground-force picture would look like the following:



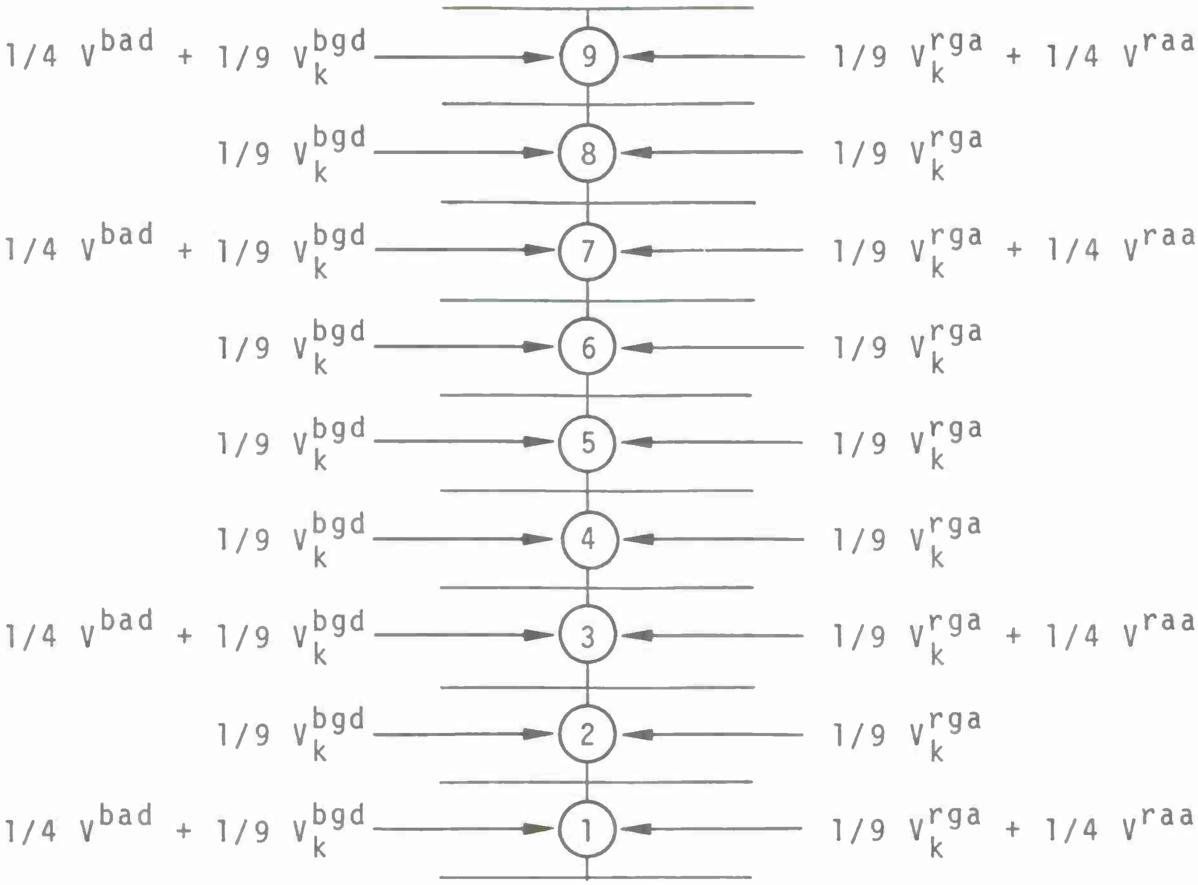
Suppose the attacker were to concentrate his air forces in minor sectors 1, 3, 7, and 9; and let us look closely at minor sectors 2 and 3. If the defender were also to concentrate his air forces in those sectors, then, after a while, the FEBA would look like the following:



What happens if the attacker presses on in minor sectors 1 and 2? The attacker in minor sector 3 is outflanked on two sides. On one side, the defender is in a "stable" position, but on the other side, in minor sector 2, the defender is also outflanked. And the situation is symmetric, in the sense that the defender in minor sector 2 is in the same position--outflanked on one side by the attacker in a "stable" position and outflanked on the other side by the attacker in minor sector 3. Either the attacker's advance in minor sectors 1 and 3 causes the defender to withdraw in minor sector 2, or the defensive resistance in minor sectors 2 and 4 causes the attacker to slow down in minor sector 3. While the situation is symmetric, the forces are not. The attacker is the attacker because he is stronger (in some sense); and so we assume that, in this case, the defender will withdraw in minor sector 2.

The same situation holds in minor sectors 6, 7, 8, and 9; and so we assume that the defender withdraws in minor sector 8. This assumption means that minor sectors 1, 2, 3, 7, 8, and 9 are all moving at the faster rate (due to concentration of the attacker's air forces). Thus, these minor sectors are outflanking minor sectors 4, 5, and 6; and so the defender must eventually withdraw in these sectors at the same rate that the attacker is advancing in minor sectors 1, 4, 7, and 9.

Accordingly, with air forces concentrated as described above, the picture is as follows:



And the definition for F_1 gives, for the case where $W^{ra} = 1/9\ W$, that

$$F_1 = M_{kt} f^r_{kt} \left(\frac{\frac{1}{9} V^{rga}_k + \frac{1}{4} V^{raa}}{\frac{1}{9} V^{bgd}_k + \frac{1}{4} V^{bad}} \right)$$

$$= M_{kt} f^r_{kt} \left(\frac{V^{rga}_k + \frac{9}{4} V^{raa}}{V^{bgd}_k + \frac{9}{4} V^{bad}} \right) .$$

The case where $W^{ra} = 1/27\ W$ can be described in a similar manner, with the attacker and defender concentrating their air forces in minor sectors 1, 3, 7, 9, 19, 21, 25, and 27. And so, in this case,

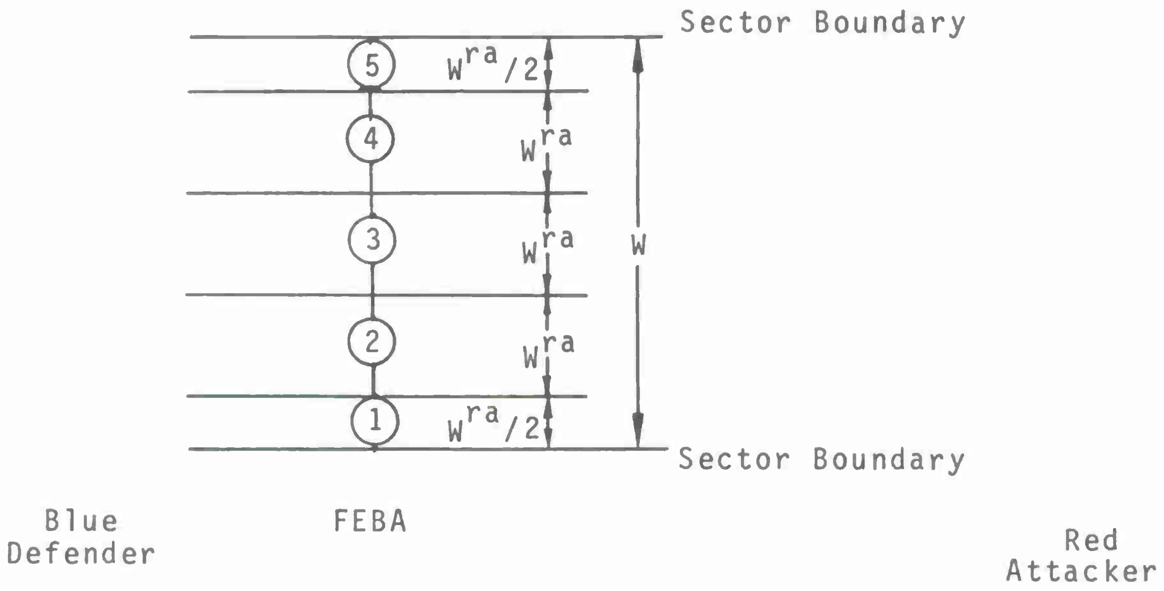
$$F_1 = M_{kt} f^r_{kt} \left(\frac{\frac{1}{27} V^{rga}_k + \frac{1}{8} V^{raa}}{\frac{1}{27} V^{bgd}_k + \frac{1}{8} V^{bad}} \right)$$

$$= M_{kt} f^r_{kt} \left(\frac{V^{rga}_k + \frac{27}{8} V^{raa}}{V^{bgd}_k + \frac{27}{8} V^{bad}} \right) .$$

The extension to any case $W^{ra} = 1/3^n\ W$ for any integer $(n \geq 0)$ is clear. In particular, if $W^{ra} = W$ $(n=0)$, then no concentration of air forces is possible and

$$F_1 = M_{kt} f^r_{kt} \left(\frac{V^{rga}_k + V^{raa}}{V^{bgd}_k + V^{bad}} \right) .$$

Now the problem is to determine what should happen if $1/3^n\ W < W^{ra} < 1/3^{n+1}\ W$ for some n. A simple approach would be to "round down" and use the formula for the case where $W^{ra} = 1/3^n\ W$. This approach would be reasonable if the attacker were constrained to line up his leftmost minor sector with his left boundary and his rightmost minor sector with his rightmost boundary. But the attacker is not constrained to do so. For example, suppose that $W^{ra} = 1/4\ W$. Then the attacker could view the whole sector as follows:



Since minor sectors 1 and 5 are only $1/2\ W^{ra}$ wide, the attacker cannot concentrate his air forces there; but if he concentrated his air forces in minor sectors 2 and 4, then the defender might be forced to withdraw in 1 and 5 more quickly because of the narrow width of these minor sectors; and then, with minor sectors 1, 2, 4, and 5 moving, the defender would be forced to withdraw in minor sector 3. The point here is that the sector boundaries play an important role. Rather than attempt to formulate a precise set of rules concerning the sector boundaries, IDAGAM I uses a smooth curve to handle the case where $1/3^n\ W < W^{ra} < 1/3^{n+1}\ W$.

This smooth curve is constructed as follows. Note that if $W^{ra} = 1/3^n\ W$, then each minor sector has $1/3^n$ of the ground forces and each minor sector that receives air support has $1/2^n$ of the air forces, where the examples are as follows:

n	3^n	2^n
0	1	1
1	3	2
2	9	4
3	27	8

Now suppose that this relationship holds for any real x (instead of just integer-valued n's). That is, suppose that if $W^{ra} = 1/3^x\ W$ then each minor sector that receives air support has $1/3^x$ of the ground forces and $1/2^x$ of the air forces. With this supposition, if $y = 1/3^x$ then each minor sector receiving air support will have y of the ground forces and $2^{((\log y)/\log 3)}$ of the air forces. For the examples, we have the following:

y	$2^{(\log y/\log 3)}$
1	1
1/3	1/2
1/9	1/4
1/27	1/8

Using this structure to calculate F_1 in general gives

$$F_1 = M_{kt} f^r_{kt} \left(\frac{yV_k^{rga} + 2^{((\log y)/\log 3)}\ V^{raa}}{yV_k^{bgd} + 2^{((\log y)/\log 3)}\ V^{bad}} \right)$$

where $y = W^{ra}/W$.

Equivalently, if we set

$$z = \frac{2^{((\log W^{ra}/W)/\log 3)}}{W^{ra}/W} ,$$

then

$$F_1 = M_{kt} f^r_{kt} \left(\frac{V^{rga}_k + zV^{raa}}{V^{bgd}_k + zV^{bad}} \right).$$

d. Attacker's Air Advantage Less Than His Ground Advantage

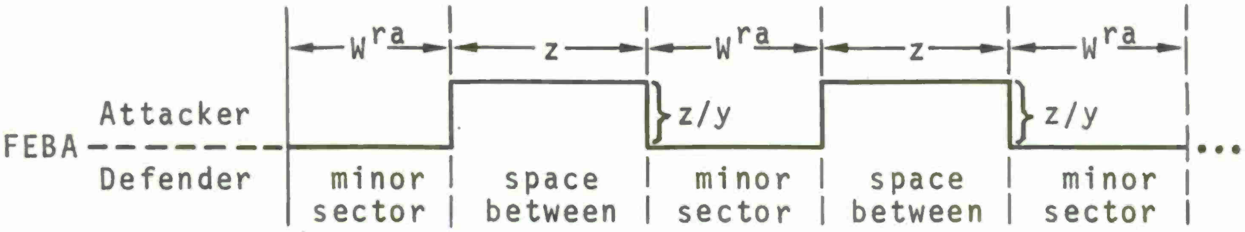
One characteristic of the procedure described above is that the attacker concentrates his air forces in the same minor sectors day after day, so that the defender must withdraw in the other minor sectors once the defender's front-to-flank ratio becomes too small in those sectors. The attacker concentrates his air forces in this way because the attacker's air advantage was greater than his ground advantage, and so the minor sectors in which he concentrated his air forces moved faster than the other minor sectors, even when the defender concentrated all his air force in those same minor sectors.

Now suppose the attacker has a ground advantage and a total advantage, but that either he has an air disadvantage or he has an air advantage that is less than his ground advantage (i.e., suppose that $V^{rga}_k > V^{bgd}_k$; $V^{rga}_k + V^{raa} > V^{bgd}_k + V^{bad}$; $(V^{raa}/V^{bad}) < (V^{rga}_k/V^{bgd}_k)$. If the attacker were to concentrate his air forces in the same minor sector day after day, then (as described above) the defender could do the same. But here this concentration would result in a lower overall force ratio in those minor sectors and so would impede the attacker's movement. Accordingly, in this case, it is not to the attacker's advantage to concentrate his air forces in the same minor sectors day after day.

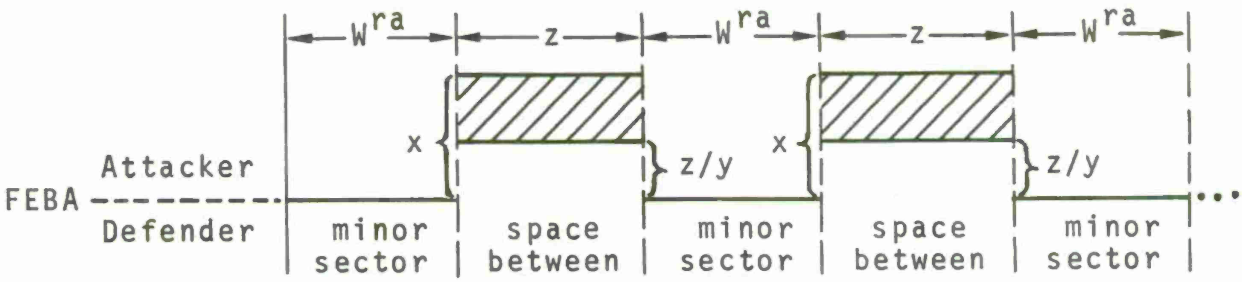
For this case, we will describe another method for the attacker to use his air forces. This method will create salients at the beginning of the day, but will flatten the

FEBA by the end of the day. Then, at the beginning of each day, the FEBA will be flat; and, so, the defender will not know where the attacker will use his air forces that day. Accordingly, this method assumes that the defender spreads his air forces uniformly across the whole sector front. The way that the attacker uses his air forces is described next:

Let the maximum front-to-flank ratio that the defender will accept in a minor sector be y, which may be pictured (turned sideways from the previous pictures, in order to simplify the notation here) as follows:



Thus, if the attacker concentrates his air forces in the minor sectors and not in the spaces between, and if the spaces between have width z, then the ground gained in each space between in a day by the attacker due to defender withdrawal to preserve the front-to-flank ratio is z(x - z/y)--the shaded area in the following picture:



The value of x is the difference between the advance in the minor sectors (in which the attacker's air forces are concentrated) and the advance in the spaces between (where the attacker flew no CAS sorties).

Assume that the attacker wishes to choose the width z to maximize the shaded area. The value of z that maximizes this area is

$$z^* = \frac{xy}{2}.$$

Rather than input a value for y and a corresponding value for the attacker's maximum front-to-flank ratio, we assume that

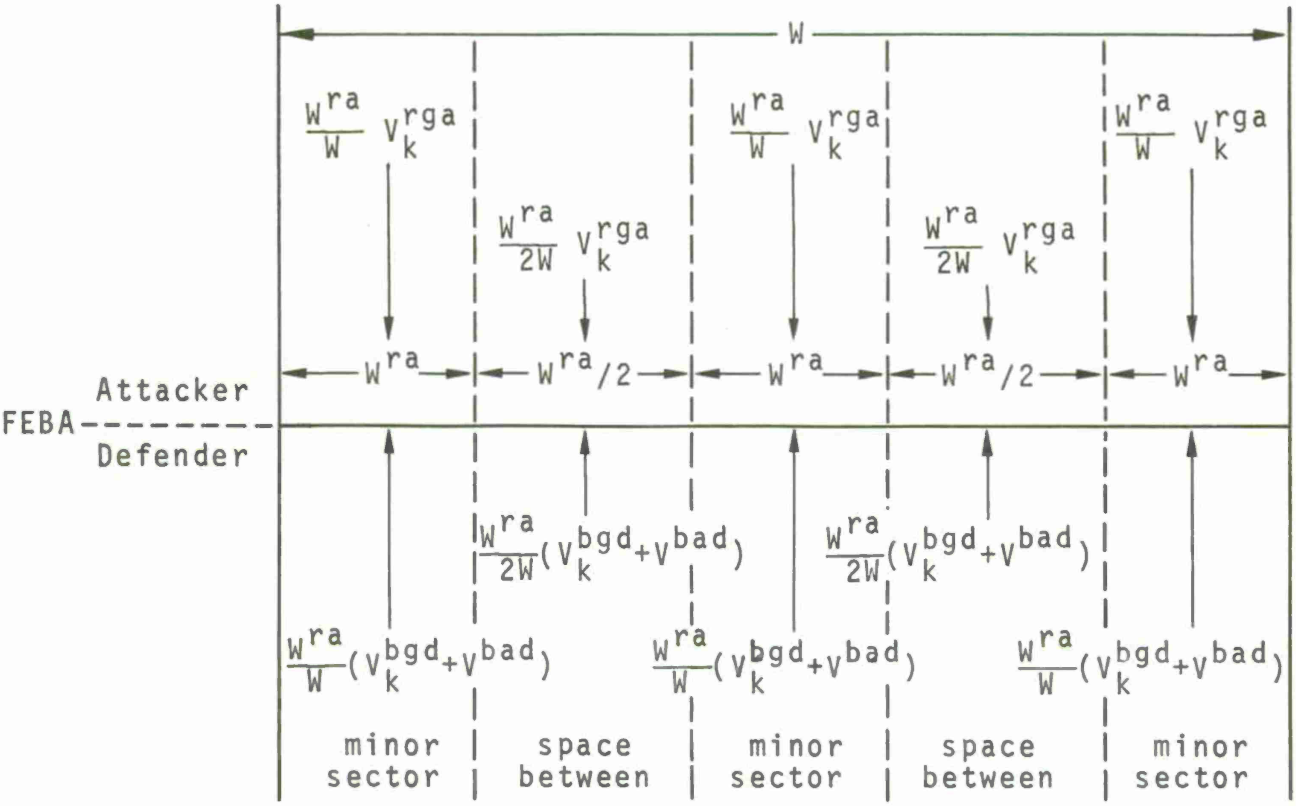
$$y = \frac{W^{ra}}{x} .$$

Note that, since the attacker's front-to-flank ratio in the minor sectors before the defender withdraws is W^{ra}/x, this ratio is the front-to-flank ratio that the attacker is actually accepting. Thus, the assumption that $y = W^{ra}/x$ is the same as assuming that the maximum front-to-flank ratio that the defender will accept in the minor sectors equals the front-to-flank ratio that the attacker actually accepts in the minor sectors. This assumption gives

$$z^* = \frac{xy}{2} = \frac{W^{ra}}{2} .$$

The structure of the attack is as follows: Suppose that $W = W^{ra} + n(\frac{3}{2} W^{ra})$ for some $n \geq 0$. Then the whole sector is divided into alternating minor sectors of width W^{ra} and spaces between of width $W^{ra}/2$. The attacker and defender spread their ground forces uniformly across the front, so that each minor sector will have W^{ra}/W of the ground forces and each space between will have $W^{ra}/2W$ of the ground forces. By our previous assumptions, the defender spreads his air forces uniformly across the front, so that the defender's air forces will split into these same proportions throughout the day.

For example, if n = 2 so that there are three minor sectors and two spaces between, then the defender's air and ground forces (and the attacker's ground forces) are assumed to be distributed as in the following picture (note that this picture, and succeeding pictures, are not drawn to scale):

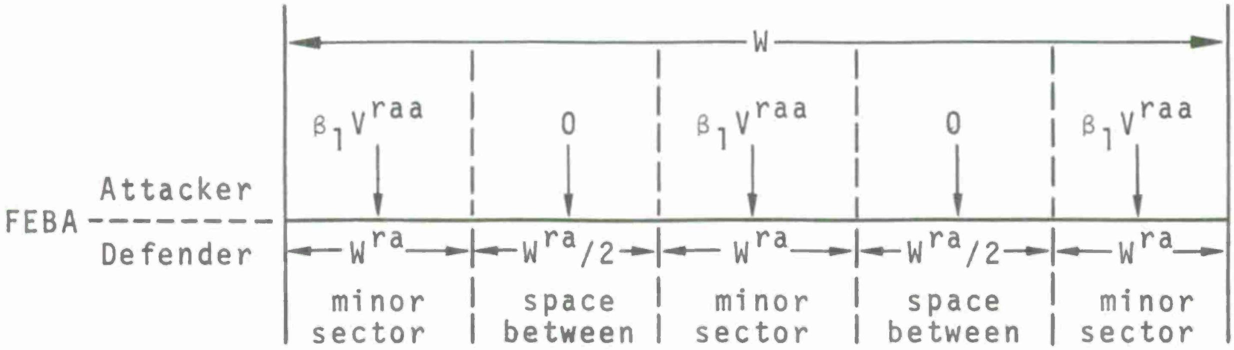


The attacker is assumed to use his air forces as follows: The attacker will first concentrate his air forces in the minor sectors, in order to create salients and force the defender to withdraw in the spaces between the minor sectors. Then the attacker will shift his air forces to concentrate on the spaces between to flatten out the FEBA.[1] Thus, for the first part of

[1]The spaces between the minor sectors have width less than W^{ra}, which is defined as the minimum width in which the attacker can effectively concentrate his air forces to create salients. But here the attacker is not creating a salient; he is using his air forces to attack the defender, who is surrounded on three sides.

the day, the attacker will allocate his air forces equally to each minor sector and will allocate no air forces to the spaces between. Then, near the end of the day, the attacker will shift his air forces and allocate them equally to the spaces between, giving no air support to the minor sectors.

In the example above (three minor sectors), the attacker will allocate his air forces at the beginning of the day as follows:



,

where $\beta_1 = 1/3$. The general formula for β_1, when $W = W^{ra} + n(\frac{3}{2} W^{ra})$ for any $n \geq 0$, is

$$\beta_1 = \frac{W^{ra}}{\frac{2}{3}(W - W^{ra}) + W^{ra}} = \frac{3W^{ra}}{2W + W^{ra}} ,$$

independent of n. (Note that if $n = 2$, so that $W = 4W^{ra}$, then $\beta_1 = 1/3$--as pictured above.)

Let ρ denote the fraction of the day that the attacker concentrates his air forces in the minor sectors (this fraction will be computed below), and let

F_{21} = the FEBA movement in a minor sector during the part of the day that the attacker concentrates his air forces in the minor sectors, assuming that the ground and air forces are as described above.

Then

$$F_{21} = \rho M_{kt} f^r_{kt} \left(\frac{\frac{W^{ra}}{W} V^{rga}_k + \frac{3W^{ra}}{2W + W^{ra}} V^{raa}}{\frac{W^{ra}}{W} V^{bgd}_k + \frac{W^{ra}}{W} V^{bad}} \right)$$

$$= \rho M_{kt} f^r_{kt} \left(\frac{V^{rga}_k + \frac{3W}{2W + W^{ra}} V^{raa}}{V^{bgd}_k + V^{bad}} \right);$$

or

$$F_{21} = \rho \overline{F}_{21} ,$$

where $\overline{F}_{21} = M_{kt} f^r_{kt} \left(\frac{V^{rga}_k + \frac{3W}{2W + W^{ra}} V^{raa}}{V^{bgd}_k + V^{bad}} \right).$

Note that $\overline{F}_{21}$ does not depend on ρ; and we will use $\overline{F}_{21}$ to calculate ρ below.

Let

F_{22} = the FEBA movement in a space between the minor sectors during the part of the day that the attacker is concentrating his air forces in the minor sectors, assuming that the ground and air forces are as described above.

Then

$$F_{22} = \rho M_{kt} f^r_{kt} \left(\frac{\frac{W^{ra}}{2W} V^{rga}_k}{\frac{W^{ra}}{2W} V^{bgd}_k + \frac{W^{ra}}{2W} V^{bad}} \right)$$

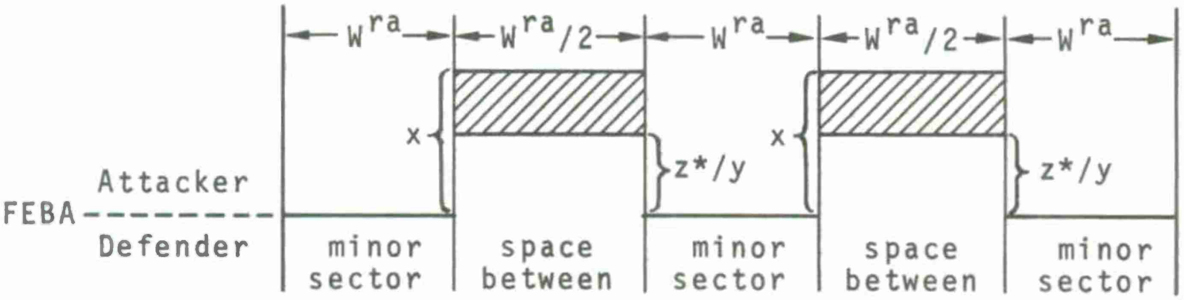
$$= \rho M_{kt} f^r_{kt} \left(\frac{V^{rga}_k}{V^{bgd}_k + V^{bad}} \right),$$

or

$$F_{22} = \rho \overline{F}_{22} ,$$

where $\overline{F}_{22} = M_{kt} f^{r}_{kt} \left(\dfrac{V_k^{rga}}{V_k^{bgd} + V^{bad}} \right)$.

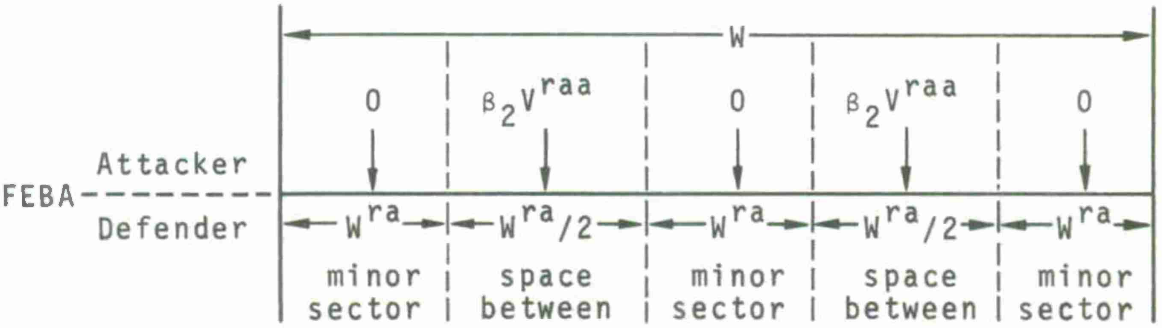
In the example, the FEBA position just before the attacker switches his air forces to the space between will look like the following:



,

where $x = F_{21} - F_{22}$ and $z^*/y = x/2 = \frac{1}{2}(F_{21} - F_{22})$.

In the remaining $(1 - \rho)^{th}$ of the day, the attacker concentrates his air forces in the spaces between. In the example, the attacker will allocate his air forces as follows:



,

where $\beta_2 = 1/2$. The general formula for β_2 is

$$\beta_2 = \frac{W^{ra}/2}{(W - W^{ra})/3} = \frac{3W^{ra}}{2(W - W^{ra})} .$$

Let

F_{23} = the FEBA movement in a space between the minor sectors during the part of the day that the attacker is concentrating his air forces in the spaces between the minor sectors, assuming that the ground and air forces are as described above.

Then

$$F_{23} = (1 - \rho)M_{kt}f^{r}_{kt}\left(\frac{\frac{W^{ra}}{2W} V^{rga}_{k} + \frac{3W^{ra}}{2(W - W^{ra})} V^{raa}}{\frac{W^{ra}}{2W} V^{bgd}_{k} + \frac{W^{ra}}{2W} V^{bad}}\right)$$

$$= (1 - \rho)M_{kt}f^{r}_{kt}\left(\frac{V^{rga}_{k} + \frac{3W}{W - W^{ra}} V^{raa}}{V^{bgd}_{k} + V^{bad}}\right).$$

or

$$F_{23} = (1 - \rho)\overline{F}_{23} ,$$

$$\text{where } \overline{F}_{23} = M_{kt}f^{r}_{kt}\left(\frac{V^{rga}_{k} + \frac{3W}{W - W^{ra}} V^{raa}}{V^{bgd}_{k} + V^{bad}}\right) .$$

Let

F_{24} = the FEBA movement in a minor sector during the part of the day that the attacker is concentrating his air forces in the spaces between minor sectors, assuming the ground and air forces as described above.

Then

$$F_{24} = (1 - \rho)M_{kt}f^{r}_{kt}\left(\frac{\frac{W^{ra}}{W} V^{rga}_{k}}{\frac{W^{ra}}{W} V^{bgd}_{k} + \frac{W^{ra}}{W} V^{bad}}\right)$$

$$= (1 - \rho)M_{kt}f^{r}_{kt}\left(\frac{V^{rga}_{k}}{V^{bgd}_{k} + V^{bad}}\right) ;$$

or

$$F_{24} = (1 - \rho)\overline{F}_{22} ,$$

where $\overline{F}_{22}$ is as defined above.

If the FEBA starts the day flat, then, in order for it to end the day flat, the attacker's advance in the minor sectors must equal his advance in the spaces between. The attacker's advance for the whole day in the minor sectors is given by $F_{21} + F_{24}$. During the first part of the day, the attacker's advance in the spaces between is given by the advance that the attacker achieves directly plus the distance that the defender is forced to withdraw, the sum of which is

$$\begin{aligned} F_{22} + (x - \frac{z^*}{y}) &= F_{22} + (F_{21} - F_{22} - \frac{1}{2} (F_{21} - F_{22})) \\ &= \frac{1}{2} (F_{21} + F_{22}) . \end{aligned}$$

During the second part of the day, the attacker's advance in the spaces between is given by F_{23}. Thus the attacker's advance for the whole day in the spaces between is given by $\frac{1}{2} (F_{21} + F_{22}) + F_{23}$. The attacker must time the shift in concentration of his air forces so that these quantities are equal--i.e., ρ must be such that

$$F_{21} + F_{24} = \frac{1}{2} (F_{21} + F_{22}) + F_{23} .$$

Making the appropriate substitutions gives

$$\rho\overline{F}_{21} + (1 - \rho)\overline{F}_{22} = \frac{1}{2} (\rho\overline{F}_{21} + (1 - \rho)\overline{F}_{22}) + (1 - \rho)\overline{F}_{23} ,$$

or

$$\frac{1}{2} \rho\overline{F}_{21} = (1 - \rho)(\overline{F}_{23} - \frac{1}{2} \overline{F}_{22}) .$$

Solving for ρ gives

$$\rho = \frac{2\overline{F}_{23} - \overline{F}_{22}}{\overline{F}_{21} + 2\overline{F}_{23} - \overline{F}_{22}} \quad .$$

Now let

F_2 = the overall FEBA movement in the whole sector during the whole day, assuming that the ground and air forces are as described above.

Then

$$F_2 = \rho\overline{F}_{21} + (1 - \rho)\overline{F}_{22} \; .$$

This formula was developed assuming that $W = W^{ra} + n(\frac{3}{2} W^{ra})$ for some $n \geq 0$. Based on arguments similar to those given for using a "smooth curve" in the first case considered above, we use this same formula here for all W -- i.e., we will also use it if $W^{ra} + n(\frac{3}{2} W^{ra}) < W < W^{ra} + (n + 1)(\frac{3}{2} W^{ra})$ for some $n \geq 0$. (Note that this formula is independent of n, so that it can be used directly even if $W^{ra} + n(\frac{3}{2} W^{ra}) < W < W^{ra} + (n + 1)(\frac{3}{2} W^{ra})$ for some $n \geq 0$.)

e. Attacker's Ground Disadvantage

Clearly, one way that the attacker can use his air forces is to spread them uniformly across the front just as the ground forces are assumed to be. If the attacker has a ground disadvantage (but is the attacker, because of a significant air advantage), then we assume that he cannot concentrate his air forces to create salients. In this case, we assume he must use his air forces to support his (outnumbered) ground forces uniformly across the front. Since this will not procure salients for the defender to attack, we assume that the defender also spends his air forces uniformly across the front. For this case, let

F_3 = the FEBA movement in the whole sector during the day, given that all forces are spread uniformly across the sector front.

Then

$$F_3 = M_{kt} f^r_{kt} \left(\frac{V^{rga}_k + V^{raa}}{V^{bgd}_k + V^{bad}} \right) .$$

f. Use of Above Computations

Three methods in which the attacker can use his air forces have been computed (above, Subsections c, d, and e). As described in Subsection e above, we assume the attacker must spread his air forces uniformly across the sector if he has a ground disadvantage. However, if the attacker has a ground advantage, then we assume that the attacker can use any of the three methods and that he will use the method that produces the largest FEBA advance. Let

F = the FEBA movement in the sector during one day of combat.

Then

$$F = \begin{cases} \max \{F_1, F_2, F_3\} , & \text{if } V^{rga}_k \geq V^{bgd}_k ; \\ F_3 , & \text{otherwise} . \end{cases}$$

Applying the FEBA movement F to the position of the FEBA at the beginning of the day gives the position of the FEBA at the beginning of the next day.

Finally, the point of these calculations and the assumptions leading up to them is not that the attacker would always follow the strategies described above, nor that these strategies would always work perfectly, nor that these strategies are optimal. The point here is that there is an intrinsic difference in mobility between jets and tanks, and a model of warfare should be able to represent this difference. IDAGAM I does not attempt explicitly to model the particular degrees of mobility for each type of weapon. Instead, it attempts to represent the gross difference in mobility between air forces and ground forces.

Then, with this difference in mobility, the attacker should, under some circumstances, be able to use his air forces in a better way than just spreading them uniformly across the sector front. The calculations given above allow the attacker a more reasonable way to use his air forces.

D. SUMMARY DESCRIPTION OF THE THEATER-CONTROL MODEL

This section contains an outline of the theater-control model. (Chapter IV of Volume 3 contains additional details on the logic used by the theater-control model.) Unlike the air-combat and ground-combat models, no battle-related attrition of people, weapons, or aircraft is played in the theater-control model.

1. Structure of the Theater-Control Model

The theater-control model consists of the following steps:

(1) Compute supply consumption in regions and COMMZ.

(2) Compute nonbattle casualties in regions and COMMZ.

(3) Add daily replacement people, weapons, and supplies to the replacement pools in the COMMZ.

(4) Compute personnel and weapon replacements for divisions in sectors and regions.

(5) Consider delayed effectiveness of personnel replacements.

(6) Compute theater attacker and sectors of main attack.

(7) Compute geographical quantities.

(8) Compute and move reinforcing divisions from COMMZ to regions.

(9) Compute and move reinforcing divisions from regions to sectors and withdrawing divisions from sectors to regions.

(10) Compute shipment of supplies and supply losses due to enemy aircraft on supply-interdiction missions.

Each of these steps is now discussed briefly and in turn.

Step 1. Supply consumption for divisions in sectors was computed in the ground-combat model. This step computes supply consumption for divisions in regions and the COMMZ, for people and weapons in replacement pools in the COMMZ, and for theater-support personnel in the COMMZ.

Step 2. Nonbattle casualties are computed here for people in regions and the COMMZ (as described in Section C.2.e).

Step 3. Additional people, weapons, and supplies can be read into the appropriate replacement pools using the RF (Read Forces) subroutine. But to simulate daily arrivals of these quantities into the theater using the RF subroutine would require calling this subroutine each day of the war. Since replacement people, weapons, and supplies are likely to arrive on a regular basis, IDAGAM I allows each of these pools to be incremented on a daily basis, and the size of the increment can vary linearly over time. These daily increments are made in this step.

Step 4. Replacement personnel and weapons are added into under-strength divisions in regions and sectors in this step--as described in Section 2, below. (Replacements are not automatically added to divisions in the COMMZ.)

Step 5. Delayed effectiveness of personnel replacements is considered as follows: Suppose that according to Step 4 a division is to receive r replacements on day d. Suppose that a replacement is assumed to have a fractional effectiveness of x_1 on his first day with the division, a fractional effectiveness of $x_1 + x_2$ on his second day with the division, and so on--until he reaches full effectiveness on his n^{th} day with the division. (The values of n and $x_1, \ldots, x_{n-1}$ are input; and $x_1, \ldots, x_{n-1}$ must satisfy the conditions that $x_i \geq 0$ for all $i \leq n - 1$ and $x_1 + \ldots + x_{n-1} \leq 1$.) IDAGAM I plays this by adding $x_1 r$ replacements to the division on day d, $x_2 r$ replacements on day d + 1, and so on--until it adds $(1 - x_1 - \ldots - x_{n-1})r$ replacements to the division on day d + n - 1.

Step 6. The side that is the theater attacker is determined by input on the first day of the war. For each succeeding day, the model computes the force ratio formed by the total Red attack value (e.g., if MCFR = 5, the total antipotential potential on attack against a standard Blue force summed over all Red divisions and all Red aircraft in the theater) divided by the total Blue defense value; and the model computes the force ratio formed by the total Blue attack value divided by the total Red defense value. The side with the higher force ratio on attack is assumed to be the theater attacker that day. The sectors of main attack for the side that is the theater attacker are then computed. These sectors of main attack can either be determined by input or be computed by the model. If they are to be computed by the model, the model will compute one sector of main attack in each region and, at the user's option, this can be either the sector with maximum penetration or the sector with minimum penetration.[1]

Step 7. Geographically related quantities (such as terrain, posture, and width of sector) are determined for each sector in this step by comparing the current location of the FEBA in the sector to inputs which give these geographical quantities as a function of FEBA position.

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[1]The idea here is that if the side that was initially the theater defender became, through reinforcements, the theater attacker, then that side might want to push the enemy out of his territory before he attempted a deep penetration into the enemy's territory. This strategy is accomplished by selecting the sector of main attack to be the one with the minimum penetration. Once the enemy is pushed back of the original FEBA position in all sectors, the model will automatically begin to compute the sector of main attack as the one with the maximum penetration.

Step 8. Reinforcing divisions are moved from the COMMZ to region in Step 8, as follows: A number of parallel phase lines that run the width of the theater and that can be of variable distance from each other are inputs to the model for each side. Associated with each space between these phase lines is an input fraction between 0.0 and 1.0. The sector in the theater with maximum penetration is determined, and the phase lines (between which the FEBA location in this sector falls) are found.[1] The input fraction associated with the space between these phase lines determines the fraction of the side's total ground force that the side wants to keep in reserve in the COMMZ (there are different phase lines and different fractions for each side). If the side has more than that fraction of his force in the COMMZ, the remainder is sent forward to the region that contains the sector of maximum (or, if appropriate, minimum) penetration.

For the purpose of these calculations, ground forces are measured as follows: Both the side that is the theater attacker and the posture in the sector of maximum (or minimum) penetration have already been determined (in Steps 5 and 6)--based on which the value of each weapon against a standard enemy force is determined. The value of each division is assumed to be the sum (over the types of weapons) of the number of weapons (of that type) in the division times the value of that weapon.

If the value of all the divisions in the COMMZ divided by the value of all the divisions in the theater is less than the fraction determined above, then no divisions are moved. If it is greater than or equal to this fraction, then the division with the largest value in the COMMZ is tested. If this division

[1]If the user has opted to have the model select the sector of main attack according to the sector with minimum penetration, then (for the attacker only) the sector with minimum penetration is used (instead of the sector with maximum penetration).

can be moved and the resulting ratio of divisions remaining in the COMMZ to all divisions is still greater than or equal to this fraction, then the division is moved. If not (i.e., if moving that division would cause the ratio to fall below the fraction), then it is not moved. Whether or not that division is moved, the division with the second-largest value in the COMMZ is then tested against the resulting ratio; and the second-largest division is then moved (or not), following the same rule. This procedure continues (for both sides) until the smallest division in the COMMZ has been tested.

If the user of IDAGAM I desires to direct the movement of divisions by input rather than by the logic described above, he can do so. To do so here, the user can input the number 1.0 for the fractions associated with all spaces between phase lines, which would mean that no divisions could be automatically moved from the COMMZ to regions. The user could then direct the movement of divisions by input using the MF (Move Forces) subroutine.

Step 9. Reinforcing divisions are moved from regions to sectors, and withdrawing divisions are moved from sectors to regions in Step 9--described in Section 3, below. (Since divisions are in reserve whether they are in regions or in the COMMZ, the model does not automatically withdraw divisions from regions to the COMMZ.)

Step 10. Supplies can be located in each sector, in two places in each region, and in two places in the COMMZ. The amount of supplies shipped to each location and supply losses due to enemy aircraft on supply-interdiction missions are computed in Step 10--as described in Section 4 (below), where a discussion of how enemy ground fire and aircraft on CAS missions destroy supplies in sectors is also given.

2. Personnel and Weapon Replacements

Personnel and weapon replacements are related, in that personnel replacements should not be sent to a division if there is such a shortage of weapons that the replacements would have no weapons to man. Likewise, weapon replacements should not be sent to a division if there are not enough people to man them. This restriction is important: For example, if people are sent to the front without weapons, then these people contribute nothing to their side's effectiveness; yet some of these people will become casualties.

The way that IDAGAM I computes replacements is as follows: First, a trial number of personnel replacements is calculated. This number is assumed to be the minimum of {the number of people in the replacement pool, the number of people needed by all divisions in all sectors and all regions, and the number of people already in these divisions times an input "pipeline" factor}. This trial number of replacements is then prorated to the divisions according to need.

Next, a weapon replacement rate (by weapon type and by division) is calculated. This rate is the number of weapons of that type needed by those divisions divided by the number of people needed. The idea here is that if a division needs 10 weapons of a certain type and 100 people, and if it receives 50 people as replacements, then, all things being equal, it should receive no more than 5 weapons of that type as replacements (5 equals the weapons replacement rate (10/100) times the number of personnel replacements).

A trial number of replacement weapons by type and by division is then computed. This trial number of replacement weapons is the minimum of both the weapon replacement rate times the number of personnel replacements and the number of weapons of that type needed by the divisions.

If the number of weapons of that type in the weapon replacement pool is less than the sum over all divisions of this trial number, then this trial number of replacement weapons of that type is reduced proportionately until the sum equals the number in the pool. If this trial number is reduced for one type of weapon, then the trial numbers of replacements for other types of replacements for other types of weapons are proportionately increased (the proportionality is determined by the weapon values). The trial numbers of replacements for the other types of weapons are increased until either (1) the divisions do not need any more weapons of that type, (2) the weapon replacement pool runs out of weapons of that type, or (3) the divisions receive enough other weapons so that the total value of these weapons (evaluated against their most effective target in a standard enemy force) equals the value of weapons (evaluated against their least effective target in a standard enemy force) that were not replaced because of the shortage in the weapon replacement pool. The number of replacement weapons by type, considering these calculations, is the actual number of replacement weapons of that type sent to the divisions.

This actual number of replacement weapons by type is used to determine the actual number of personnel replacements as follows: Let B_{id}, B^t_{id}, N^b_d, P^{bga}_{ijk}, and P^{bgd}_{ijk} be as defined in Section C.2 of this chapter (above); and let

B^r_{id} = the actual number of Blue type-i replacement weapons sent to type-d divisions, as already computed;

B^{tr}_{0d} = the trial number of Blue personnel replacements to be sent to type-d divisions, as already computed;

and define P^{bgn}_i and P^{bgx}_i as follows:

$$P^{bgn}_i = \text{the minimum value of a Blue type-i ground weapon}$$

$$= \min_{j,k} \{P^{bga}_{ijk}, P^{bgd}_{ijk}\};$$

P_i^{bgx} = the maximum value of a Blue type-i ground weapon

$$= \max_{j,k} \{P_{ijk}^{bga}, P_{ijk}^{bgd}\}.$$

Then

B_{0d}^r = the actual number of Blue personnel replacements sent to type-d divisions

is computed as $B_{0d}^r =$

$$\max\left\{0.0,\ \min\left\{B_{0d}^{tr},\ \left[B_{0d}^t N_d^b \sum_i \frac{(B_{id} + B_{id}^r) P_i^{bgx}}{B_{id}^t N_d^b P_i^{bgx} + \sum\limits_{i' \neq i} B_{i'd}^t N^b P_{i'}^{bgn}}\right] - B_{0d}\right\}\right\}$$

The idea here is that the actual number of Blue replacements should not make the percent personnel strength of the divisions greater than a kind of "maximum" weapons strength, where this "maximum" weapons strength is the sum over i of the value of type-i weapons actually in the divisions divided by the total value of all weapons in full TOE-strength divisions (where the type-i weapons are counted at their maximum value and all other weapon types are counted at their minimum value). This somewhat complicated expression has the property that, no matter what the composition of the Red force is, the Blue replacements do not make the Blue percent personnel strength greater than the Blue percent weapons strength (which depends on the composition of the Red forces), and the number of Blue replacement goes to zero as the total number of Blue weapons goes to zero.

Similar calculations are made for Red.

3. Reinforcements and Withdrawals of Divisions

The basic idea behind this part of IDAGAM I is as follows: The attacker will move reinforcing divisions from a region to the sectors in that region if he has "enough" divisions in the region ("enough" will be defined later) and if--

(1) The force ratio has fallen below a minimum acceptable value in any sector,

(2) He is being constrained by front-to-flank ratio in a sector of main attack, or

(3) The force ratio in a sector of main attack is below a desired value.[1]

The attacker will withdraw divisions from a sector to its region if--

(1) The sector is a sector of main attack and it is being constrained by front-to-flank ratio,

(2) The personnel strength of a division is below a minimum acceptable value, or

(3) The force ratio in a sector that is not a sector of main attack is above a desired value and the attacker's reserve level is too low.

The defender will move reinforcing divisions from a region to the sectors in that region if he has "enough" divisions in the region and if--

(1) The force ratio in the sector of maximum penetration in the region is above a desired value, or

(2) The force ratio in any sector is above a maximum acceptable value.[1]

The defender will withdraw divisions from a sector to its region if--

(1) The personnel strength of a division is below a minimum acceptable value, or

(2) The force ratio is below a desired value in a sector that is not a sector either of maximum penetration or in which the defender is in a defensive position, and the defender's reserve level is too low.

---

[1]Neither side will reinforce a sector with a division if the personnel strength of that division is below an input minimum.

Let the term "reserve level" for a region mean the ratio of the weapons value of the divisions in reserve in the region to the total of both the weapons value of the division in reserve and the divisions committed to all the sectors contained in the region. Some inputs and working variables required for this part of IDAGAM I are as follows: For Blue on attack in the theater, let

F_1^{bra} = the minimum acceptable force ratio in any sector for Blue on attack (this is input);

L_1^{bra} = the reserve level that Blue will not go below in attempting to achieve a force ratio of F_1^{bra}

= 0.0 (i.e., L_1^{bra} is a working variable, and it is assumed that the attacker will always attempt to achieve this force ratio--no matter what reserve level this gives);

F_2^{bra} = the force ratio that Blue will attempt to achieve in a sector flanking a sector of main attack that is constrained by front-to-flank ratio (this is a working variable and is set equal to the current force ratio in the corresponding sector of main attack);

L_2^{bra} = the reserve level that Blue will not go below in attempting to achieve a force ratio of F_2^{bra} (this is input);

F_{3k}^{bra} = the force ratio that Blue will attempt to achieve in a sector of main attack if Red is defending in posture-type k (this is input);

L_{3k}^{bra} = the reserve level that Blue will not go below in attempting to achieve a force ratio of F_{3k}^{bra} (this is input);

F_{1k}^{bwa} = the force ratio that Blue will not go below in attempting to achieve a reserve level of L_1^{bwa} (defined below) in a sector of main attack if Blue is in posture-type k in that sector and is withdrawing divisions from that sector to support in flanking sectors, due to front-to-flank ratio constraints (this is input);

L_1^{bwa} = the reserve level that Blue is attempting to achieve if he is withdrawing divisions from a sector of main attack due to front-to-flank ratio constraints

= 1.0 (i.e., L_1^{bwa} is a working variable, and it is assumed that the attacker will withdraw divisions from the sector of main attack--no matter what reserve level this gives, providing that a force ratio of F_{1k}^{bwa} is maintained);

F_3^{bwa} = the force ratio that Blue will not go below in attempting to achieve a reserve level of L_{3k}^{bwa} (this is input); and

L_{3k}^{bwa} = the average reserve level that Blue attempts to achieve when considering whether to withdraw divisions from a sector that is not a sector either of main attack or in which Blue is in a defensive position

= L_{3k}^{bra} (i.e., it is assumed that this average reserve level equals the reserve level that Blue will not go below in attempting to achieve a force ratio of F_{3k}^{bra} in Blue's sector of main attack).

For Blue on defense in the theater, let

F_{1k}^{brd} = the force ratio that Blue will attempt to achieve in the sector of maximum penetration if Blue is defending in posture-type k (this is input);

L_{1k}^{brd} = the reserve level that Blue will not go below in attempting to achieve a force ratio of F_{1k}^{brd} (this is input);

F_2^{brd} = the desired force ratio in any sector for Blue on defense (this is input);

L_2^{brd} = the reserve level that Blue will not go below in attempting to achieve a force ratio of F_2^{brd} (this is input);

F_2^{bwd} = the force ratio that Blue will not go above in attempting to achieve a reserve level of L_2^{bwd} (this is input); and

L_2^{bwd} = the average reserve level that Blue attempts to achieve when considering whether to withdraw divisions from a sector that is not a sector either of maximum penetration or in which the defender is in a defensive position

= L_2^{brd} (i.e., it is assumed that this average reserve level equals L_2^{brd}).

Similar definitions are made for Red.

These definitions provide a force ratio F and a reserve level L for each of the reasons for reinforcing or withdrawing divisions, with the exception that no force ratio or reserve level is defined for withdrawing under-strength divisions (i.e., F_2^{bwa}, L_2^{bwa}, F_1^{bwd}, and L_1^{bwd} are not defined). The way that under-strength divisions are withdrawn is as follows: Inputs are

S_d^{ba} = the minimum fractional strength for a Blue type-d division to be in combat if Blue is on attack;

S_d^{bd} = the minimum fractional strength for a Blue type-d division to be in combat if Blue is on defense;

and similar definitions are made for Red. If Blue is on attack and the fractional strength of a type-d division in combat is below S_d^{ba}, then that division is withdrawn from the sector to the region.[1]

Each of the reasons given above for reinforcing or withdrawing is considered in a particular order. The order is as follows: First, the attacker withdraws divisions if constrained by front-to-flank ratio. Then the attacker reinforces, considering the reasons in the order listed above. Then the defender reinforces in the order listed above. Then the attacker withdraws divisions in the order listed above, except for front-to-flank ratio (which he has already considered). Then the defender withdraws divisions in the order listed above.

Reinforcements are made in the following manner: For each reason for reinforcing, a force ratio and reserve level are defined above. The reserve level, essentially, says whether the side has "enough" divisions in reserve to be able to reinforce. The force ratio gives the goal that the side is attempting to achieve by reinforcing. If the side already has that force ratio, then it does not reinforce (for the reason

---

[1]If IDAGAM I is being used to play notional divisions by type, then all divisions of the same type in the same sector are assumed to be at the same strength. Thus, if one type-d division in a sector is under-strength, all are; and so all are withdrawn at the same time.

being considered). If it does not have that force ratio, then the side first considers its largest division (according to TOE weapons value) in the region. If that division can be moved forward and still stay within the bounds set by the reserve level, then that division is moved. If not, it is not moved. Then the second-largest division is considered. This procedure continues either until the force-ratio goal is achieved (in which case no more divisions are moved for the reason being considered) or until moving even the smallest division in the region would violate the bounds set by the reserve level.

Withdrawals of under-strength divisions have already been described. All other reasons for withdrawing divisions are considered in a way similar to that for reinforcing divisions, but with the roles of force ratio and reserve level reversed. The divisions are considered in order from the largest to the smallest; and if a division can be withdrawn without violating the force-ratio constraint, it is withdrawn--until either the reserve-level goal is achieved or until all the divisions have been considered.[1]

Finally, if the user wants to input all divisional moves rather than let the model's logic move divisions, he can do so by setting the force ratio inputs to appropriate levels (zero or very large, depending on the situation), and no division will be automatically moved. The user can then direct all divisional moves by using the MF subroutine.[2]

---

[1]In withdrawing divisions, IDAGAM I looks only at the sector under consideration and its region. The average reserve level is defined to be the reserve level that the region would have if all sectors had the same number of divisions as the sector being considered.

[2]Accordingly, IDAGAM I can be used to play no individual replacements (with complete "unit replacements"), complete individual replacements, or anything in between. However, one must be careful in attempting to use a general model (such as IDAGAM I) to evaluate individual replacement policies versus unit replacement policies. (For further details, the reader is referred to Anderson [3] and to Kerlin [17] and [18].)

4. Supplies

a. Location and Shipment of Supplies

In IDAGAM I, supplies can be located in sectors, in regions, and in the COMMZ. Supplies in a sector are there for use by divisions in the sector. Supplies in a region are accounted for in two places: (1) those supplies in the region for use by divisions in the region and (2) those supplies in the region that form a supply pool that feeds supplies to the divisions in the region and to the divisions in the sectors contained in the region. Similarly, supplies in the COMMZ are accounted for in two places: (1) those supplies in the COMMZ for use by divisions (and replacements and support personnel) in the COMMZ and (2) those supplies in the COMMZ that form a supply pool which feeds supplies to the divisions in the COMMZ and to the various region supply pools. (Supplies consumed by aircraft are taken directly from the COMMZ or region supply pools.)

Based on need, IDAGAM I automatically computes demands and ships supplies forward from the COMMZ to regions and from regions to sectors, and it ships supplies back and forth from the COMMZ and region pools to the divisions in those areas. (IDAGAM I cannot automatically ship supplies backwards from sectors to regions or from regions to the COMMZ.) A picture showing these various locations for supplies and the directions in which supplies can be shipped (for an example case of five sectors and two regions on each side) is given in Figure 2 (below).

For each region, IDAGAM I computes the quantity of supplies demanded by each sector in the region by comparing the days of supply on hand in each sector and to an input desired number of days of supply. A similar computation is made for supplies

Notation

$B^{ss}(J)$ = Blue supplies in sector J.

$B^{srp}(I)$ = Blue supplies in supply pool in region I.

$B^{srd}(I)$ = Blue supplies in region I for use by division in the region.

B^{szp} = Blue supplies in supply pool in COMMZ.

B^{szd} = Blue supplies in COMMZ for use by divisions in COMMZ.

(Similar notation holds for Red.)

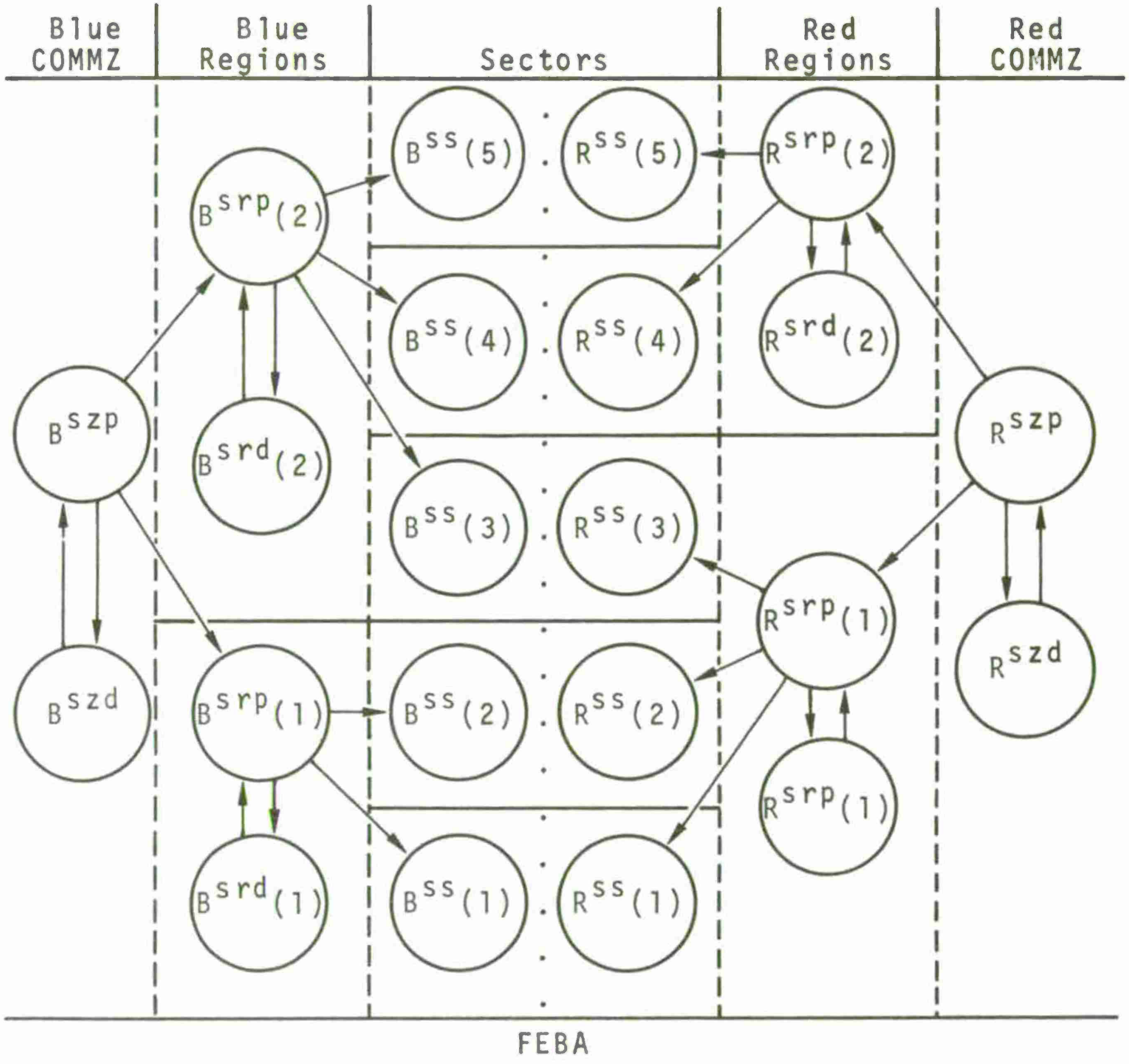

Figure 2. SUPPLY LOCATIONS FOR AN EXAMPLE OF FIVE SECTORS AND TWO REGIONS ON EACH SIDE

demanded by divisions located in the region.[1] These demands are added together and compared to the quantity of supplies in that pool to determine the quantity of supplies demanded by that region from the COMMZ supply pool. These comparisons are made for each region and, together with the supplies demanded by divisions (and replacements and support personnel) in the COMMZ, they form the total demand on the COMMZ supply pool. (Supplies can be returned to the COMMZ pool if the divisions in the COMMZ have too many supplies.) Supplies are then sent forward from the COMMZ to regions and then from regions to sectors. At each location, if the total demand can be met, then each individual demand is filled; and if the total demand cannot be met, then whatever supplies are available are prorated according to the amount demanded. This procedure determines the amount of supplies demanded by each location and the amount actually shipped to each location.

b. Interdiction and Destruction of Supplies

IDAGAM I plays that supplies can be destroyed by enemy aircraft in three places. They can be destroyed (1) in sectors by aircraft on CAS missions, (2) en route from regions to sectors by aircraft diverted to supply-interdiction from CAS missions, and (3) in regions by aircraft diverted to supply-interdiction from IDR missions. (IDAGAM I does not play the attack of supplies in the COMMZ or in supply depots or ports that might be located behind the COMMZ.) In a sense, IDAGAM I actually plays three different kinds of supply-interdiction missions, which will be described in this subsection.

[1]If the divisions in the region have too many supplies (which is possible if they had the right number of supplies at the beginning of the day and some divisions from the region were sent into combat in Step 8), then supplies are returned to the region supply pool.

In addition to destruction of supplies by aircraft, IDAGAM I plays that ground weapons can destroy enemy supplies in sectors in the regular course of fire on the enemy. This destruction is played by assuming that each weapon of a particular type in a sector destroys a fixed (input) amount of enemy supplies in the sector. For example, this amount of supplies destroyed might be zero for small arms but nonzero for artillery.

For aircraft, IDAGAM I plays that if a type-c aircraft successfully delivers its ordnance on a CAS mission, then, in addition to the enemy ground people and weapons it kills, it destroys X_c tons of supply (where X_c is input). This feature of IDAGAM I can be used in two ways. First, it can be used in the same sense as the destruction of supplies by ground weapons (i.e., it can be used to play the amount of supplies that are destroyed incidental to attacks on people and weapons). Second, this feature can be used to play direct attacks on supplies in sectors--as follows: Let L_{cm} be the number of type-m munitions in a notional load carried by a type-c aircraft, and suppose that P_c of the type-c aircraft attack weapons and people while $(1 - P_c)$ of these aircraft directly attack supplies in the sector. Suppose that, if a type-c aircraft directly attacks supplies, it can destroy Y_c tons of supplies per sortie. Then, by inputting P_cL_{cm} in place of L_{cm} and inputting $(1 - P_c)Y_c$ in place of X_c, IDAGAM I can play the direct attack of supplies in sectors by $(1 - P_c)$ of the type-c aircraft assigned to CAS missions.

As described above, IDAGAM I computes the amount of supplies demanded by and shipped to each location. All shipments of supplies are assumed to arrive at the desired location intact, except for supplies shipped from regions to sectors. These supplies can be destroyed or blocked by aircraft diverted from CAS missions to supply-interdiction missions, in the following way:

Aircraft diverted from CAS missions to supply-interdiction missions are assumed to attack supplies en route from regions to sectors. Let $B_c^1(J)$ be the number of successful Blue type-c aircraft sent on CAS missions in sector J before the fraction that do instead supply interdiction are subtracted, let F_c^{bsi} be the fraction of Blue type-c aircraft that are sent instead to interdict supplies en route to sectors (this is an input), and let $B_c^{si}(J)$ be the number of Blue type-c aircraft that interdict supplies en route to sector J. Then

$$B_c^{si}(J) = F_c^{bsi} B_c^1(J)$$

and the number of Blue type-c aircraft that actually do CAS missions in sector J is $(1 - F_c^{bsi})B_c^1(J)$. Each Blue type-c aircraft that successfully delivers its ordnance on missions to interdict supplies en route to sector J is assumed to destroy U_c^b tons of supplies and to block $V^b(J)$ of the remaining supplies from reaching the sector--where U_c^b and $V^b(J)$ are input. For example, suppose that Red is attempting to ship $R^{srs}(J)$ tons of supplies from a region to sector J, then the number of supplies en route to sector J that are assumed to be destroyed is given by

$$\dot{R}^{srs}(J) = \min \{R^{srs}(J), \sum_c U_c^b B_c^{si}(J)\} ,$$

and the number of supplies that are assumed to be blocked is given by

$$(R^{srs}(J) - \dot{R}^{srs}(J)) \min \{\sum_c V^b(J) B_c^{si}(J), 1.0\}$$

(i.e., this amount of supplies is returned to the region supply pool instead of being able to reach the divisions in sector J). Similar calculations are made for Red interdicting Blue supplies.

The number of aircraft that do supply interdiction in regions instead of IDR missions is calculated in a manner analogous to the way that aircraft are diverted from CAS missions to interdict supplies en route to sectors. If $B_c^7(IR)$ is the number of successful Blue type-c aircraft sent on IDR missions in region IR, then $F_c^{bir}B_c^7(IR)$ aircraft interdict supplies in that region instead, where F_c^{bir} is an input; and each of these aircraft is assumed to distribute U_c^{br} tons of supply. The destruction of supplies in a region is prorated between the region supply pool and the supplies in the region for use by divisions in the region.

The interdiction and destruction of supplies reduces either the number of supplies in sectors or the capability to resupply the sectors. As the number of supplies in sectors is reduced (through direct destruction and through consumption without resupply), the effectiveness of the divisions in the sectors is reduced (as described in Subsection (2) of Section C.2.d.

Chapter III

LIMITATIONS OF IDAGAM I AND SUGGESTIONS FOR FURTHER RESEARCH

Section A of this chapter describes some limitations in the scope of IDAGAM I. Limitations in the factors of combat that are modeled in IDAGAM I are the subject of Sections B, C, and D. Suggestions for further research are given in Section E.

A war game can be thought of as accounting for various resources on each side, allocating these resources to various combat missions or situations, and computing attrition to these resources. Limitations in the attrition processes of IDAGAM I are discussed in Section B, limitations of the allocation processes are discussed in Section C, and limitations of the accounting processes are discussed in Section D.

A. LIMITATIONS IN SCOPE

Some factors of combat that are not explicitly modeled in IDAGAM I are listed below:

(1) A complete theater-level general-purpose forces model would consider interactions of the resources and missions played in IDAGAM I with some resources and missions that are not played (such as merchant shipping, aircraft carriers, and antisubmarine warfare).

(2) Depending on the scenario, a conventional war might lead to the use of tactical or strategic nuclear weapons. IDAGAM I does not account for this possibility in any way.

(3) IDAGAM I does not play detailed logistics, the maneuver of individual units, road networks, intelligence, command and control, leadership, morale, prisoners of war, or refugees.

(4) IDAGAM I does not play the capture of cities or other quasi-political events that could shape a war.

(5) IDAGAM I does not explicitly play the effects of range, line of sight, or mobility for ground weapons. Not playing these effects may be acceptable for a theater-level model (since these factors can be considered implicitly when constructing the input parameters), but it is still important to note.

(6) IDAGAM I includes no cost structure.

B. LIMITATIONS OF THE ATTRITION PROCESSES

1. Primary Limitation of IDAGAM I

The primary limitation of IDAGAM I is that it is a deterministic model. This limitation affects more than just the attrition processes. (For example, IDAGAM I cannot send all aircraft on one mission with probability 0.6 and on another mission with probability 0.4. It must send 60 percent of the aircraft on the first mission and 40 percent on the second. Nor can IDAGAM I properly play weather or other stochastic phenomena.) But the most obvious impact of this limitation concerns attrition calculations.

Attrition is not a deterministic process. There are probabilities of detection or engagement and probabilities of kill, given engagement. Using expected values in place of probabilities, as IDAGAM I does, yields expected values of attrition only if the model is purely linear. And IDAGAM I certainly is not linear. As a result, each attrition calculation for each day does not yield the correct expected value. Without a model that computes expected values for attrition correctly, it is not possible to compute exactly how great this error is or how this error compounds over, say, a 90-day war.

A rough estimate of the scope of the error due to deterministic limitations is as follows: Suppose that IDAGAM I is being used to model a scenario in which there are a fixed number of divisions available to each side (this number may be increased day by day if new divisions enter the theater, but on any given day the number is fixed), and suppose that each side has enough soldiers and weapons to replace all combat losses that occur to these divisions (but that these replacements cannot be used to form new divisions). Suppose also that the same assumption holds for air wings. That is, suppose there are a fixed number of air wings (i.e., aircraft "slots") available to each side, and that each side has enough pilots and aircraft to replace all losses (i.e., all aircraft "slots" are always filled). But suppose these replacement pilots and aircraft cannot be brought into the theater except as one-for-one replacements for combat losses.

With these assumptions, the results of a deterministic attrition process (such as that in IDAGAM I) would seem to be close to the probabilistically correct results, because all losses are replaced; and, thus, stochastic variations in losses on one day will not affect the results of the next day.

But now suppose that these assumptions do not hold. That is, suppose that there are limited numbers of replacement tanks, aircraft, soldiers, etc., so that a side might run short (due to combat losses). Or suppose that the number of divisions and air wings in the theater is not fixed, so that a side has the option of using some of its replacements to form new divisions or new air wings at the risk of later running short on replacements. If either or both of these assumptions hold, then shortages might occur; and it is when shortages occur that stochastic variations are most significant. (For example, the derivation of the deterministic Lanchester square equation from probabilistic assumptions requires dismissing as negligible the probability that the number of weapons of any type on either side is ever close to zero.)

Requiring the first assumption above (that the number of divisions and air wings in the theater is fixed but that the number of replacements is unlimited) is a severe defect of any deterministic model and, accordingly, is a severe defect of IDAGAM I.

2. Other Limitations of the Attrition Processes in IDAGAM I

The limitations listed below are roughly in order of their importance.

a. IDAGAM I allows air forces to interdict enemy divisions in reserve, to destroy supplies, and to interdict the shipment of supplies. But these missions, plus CAS and suppression missions, are the only missions that IDAGAM I plays for aircraft that directly affect the enemy ground forces. There may be other significant missions for aircraft. For example, divisions on the move may be much more vulnerable to air attack than are stationary divisions in reserve or divisions actively engaged in combat. But IDAGAM I does not allow air forces selectively to attack moving divisions with increased effectiveness. Other such examples can be constructed. It may be that significant improvements can be made in the way that the contribution of air forces to the ground war is modeled.

b. Weapons (and aircraft) in IDAGAM I either destroy or damage enemy weapons (and aircraft) or leave them unharmed and fully effective. IDAGAM I does not allow ground weapons to be suppressed, nor does it allow attacking aircraft to be forced to drop their ordnance yet return home safely. Since suppression (both air and ground) may be a significant factor in combat, the fact that IDAGAM I cannot directly play suppression may be a significant limitation.

c. The Lanchester equation used by the ground-combat model (and available as an option in the air-combat model) is a first-order approximation to a correct expression for the expected attrition based on certain assumptions (see Karr [15]). Perhaps a first-order approximation is not sufficient and, if so, higher order approximations should be considered. In addition, those assumptions do not allow shooting weapons to acquire "dead" enemy weapons as targets and to "waste" their fire on these dead targets. Perhaps this wasting of fire should be allowed. Finally, further research is needed to determine whether the method for allocating fire used in IDAGAM I should be improved.

The assumptions behind the binomial equations in the air-combat model are given in Karr [16]. These assumptions seem fairly realistic, except for the assumptions that the same form of equation holds for both sides and that all the aircraft on both sides must shoot at each other simultaneously. It might seem more reasonable (1) to assume that there are (a) defenders that form a barrier and (b) penetrators that try to get through the barrier to attack targets on the other side and (2) to develop an attrition process based on the asymmetric role of the defenders and penetrators. (Work on such an attrition process has begun at IDA, but this process is not incorporated into IDAGAM I.)

d. Attrition equations for ground combat in IDAGAM I calculate potential numbers of weapons killed and then scale these potentials, based on force ratios and total casualties, to obtain actual numbers of weapons killed. A reasonable variation of this structure would be to redefine the inputs so that the results of the attrition equations could be interpreted directly as the actual numbers of weapons killed (by type), thus avoiding the scaling based on force ratios and casualties. This new attrition process would be identical to the way that attrition to divisions in reserve is now computed in the air-combat model. There are advantages and limitations to such an attrition process (compared to the one now used by IDAGAM I), and it would be an improvement to IDAGAM I to have this process available as an option.

e. IDAGAM I can play synergistic effects in the sense that weapons of different types have different capabilities against different types of targets. Also, weapons of one type might be needed to help protect weapons of another type on the same side.[1] But IDAGAM I cannot play that weapons of one type have an effectiveness that is increased in some way by the presence of other types of weapons on the same side. (It is not clear whether this type of synergism holds or, if it holds, whether it is significant. But if further studies indicate that it does hold and is significant, then not playing it would prove to be a limitation of IDAGAM I that severely affects the use of IDAGAM I to make weapons trade-offs.)

f. IDAGAM I essentially assesses attrition once each day. Thus, it uses a one-day discrete time approximation for attrition that, in reality, is closer to being a continuous process than it is to being a step process with 24-hour steps. This may not be a significant limitation for most uses of IDAGAM I, but this limitation could be considered in further research.

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[1]For example, tanks might be relatively ineffective without infantry to protect them from enemy hand-held antitank weapons, and artillery might be ineffective without infantry and tanks to protect it from direct assault. IDAGAM I allows this type of synergism to be played (see Chapter III of Volume 3 for further details).

C. LIMITATIONS OF THE ALLOCATION PROCESS

There are several levels of allocation of resources that can be considered in a theater-level model of combat. There are detailed allocations, such as allocation of ground fire. There are tactical allocations that a theater commander may have direct control of, such as where aircraft should be based, what missions aircraft should fly, where these missions should be flown, in which sectors ground forces should be concentrated, in which sectors a side should attack, and what the reserve policy should be. And there are force-mix allocations (e.g., how many artillery, tanks, interceptors, and fighter/bombers a side should have--assuming, say, a fixed budget for general-purpose forces) that must be made well before a war begins.

IDAGAM I treats the allocation of fire against an arbitrary force as a function of the mix of that force and an input allocation against a particular (standard) force.

IDAGAM I requires that the force-mix allocation be made directly by the user. That is, the numbers of all resources and the times when the resources become available must be entered into the model by the user. IDAGAM I cannot take a budget and determine what the force mix should be to satisfy some criteria.

Roughly speaking, there are three ways to make allocations: by user input, by an optimization algorithm,[1] or by an algorithm that attempts to produce a reasonable (in some sense) allocation but that does not attempt to solve a formal optimization problem. Some of the tactical allocations can be made in IDAGAM I only by direct user input; one can only be made by a nonoptimization algorithm; and the rest can be

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[1]By an "optimization algorithm" we mean an algorithm that attempts to give the optimal strategies for both sides of a two-person, zero-sum game.

made either by input or by nonoptimization algorithms (in all cases, these algorithms are based on user-supplied input parameters). IDAGAM I cannot solve a formal optimization problem to make any allocation.

The reason that we did not attempt to use an optimization algorithm to make allocations is as follows: Finding an algorithm that produces optimal strategies for a game in which the strategy space includes all the tactical decisions listed above would be a difficult research task. Indeed, finding an algorithm that optimizes over one of these tactical allocations (assigning aircraft to missions) is difficult (see Bracken [10], for example). And even if such an algorithm could be found, the result would be suboptimal when it is considered in light of all the tactical allocations that a theater commander must make.

As an alternative to an optimization algorithm, we first considered allowing each allocation to be made either by input or by a nonoptimization algorithm. But one allocation problem--the allocation of aircraft to target areas (i.e., in which sectors CAS aircraft should attack, against which airbases ABA aircraft should attack, etc.)--was tedious to input and required constant revision (if the number of aircraft on an airbase dropped to zero, then the attacker would want to attack a different airbase with his ABA aircraft). Further, to allow for both user input and a nonoptimization algorithm to handle this allocation would require a considerable additional amount of storage space in the computer. Accordingly, only a nonoptimization algorithm is available to handle this particular allocation.

Two tactical allocations can be made only by user input: the allocations that determine (1) how many of each type of aircraft are based on each airbase on each day of the war and (2) what missions these aircraft fly on each day.[1]

The movement of divisions within the theater (including the time of movement, where the move takes place, and which divisions are moved) and whether or not the theater attacker attacks in a particular sector are decisions that can be made either by user input or by nonoptimization algorithms.

D. LIMITATIONS OF THE ACCOUNTING PROCESS

A theater-level model cannot play, practically, all the details of combat; and each detail such a model does not play is a limitation of that model. For example, IDAGAM I does not allow sortie rates for aircraft to be a function of where the aircraft are based, nor does it allow the potential number of enemy weapons that a weapon can kill to be a function of the type of division that the weapon is in. Many more such examples of omitted details could be constructed, and one could question how important each of these details is. But a more important question is whether the overall level of detail is proper. Given that IDAGAM I as a deterministic model cannot play significant stochastic variations, a limitation of IDAGAM I may be that it already plays too many details. In Section E.1 below, we suggest that further research should consider building a model that can handle stochastic variations, perhaps at the expense of some of the details currently played in IDAGAM I.

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[1]There are several models that can be used to provide inputs for the allocation of aircraft to missions. TAC CONTENDER (U.S. Air Force [23]) and OPTSA (Bracken [10]) are optimization models, and the GACAM model (Bracken et al. [12]) contains a nonoptimization algorithm to allocate aircraft to missions.

E. SUGGESTIONS FOR FURTHER RESEARCH

1. The Current Most Important Research Problem

Currently, the most important research problem in the construction of a theater-level nonnuclear model of combat is reasonably to incorporate stochastic phenonema into such a model. We believe that this problem is most important for the following reasons:

Stochastic variations can be very important in determining the outcome of combat. The primary reason for this importance is that, as discussed above, stochastic considerations in attrition processes are important when there is a possibility of a shortage of weapons or aircraft. The basic issue concerns the likelihood of shortages of some types of weapons or aircraft. A deterministic approximation to stochastic attrition would be reasonable if there were no possibility of shortage--either because there will always be enough replacements (which, for some reason, cannot be used except as replacements) or because attrition to all types of weapons is always small (e.g., small enough so that neither side loses more than 10 percent of its tanks or of its attack aircraft over the course of the war, which for a 90-day war is a loss rate of about 0.1 percent per day). But if there is a possibility of shortage (or, equivalently, if weapons or aircraft have to be diverted from their primary missions in order to manage losses), then stochastic considerations are important and a deterministic model is inadequate.

A second reason that the stochastic variations are important is that weather is a stochastic phenonemon; and weather can have a considerable influence on combat, especially on air combat. Also, a deterministic model cannot be used to play mixed strategies. So, if mixed strategies are important (and some game-theoretic models indicate that they are), then this is another reason that is is important to play stochastic variations.

For these reasons, it appears that stochastic variations are important in modeling combat. But this importance alone would not make it the most important research problem at this time. Two other questions need to be considered: How important are the other "unsolved" research problems in this area? And is there a reasonable possibility that stochastic variations can be adequately incorporated into these combat models (without these models becoming mathematically intractable)?

Other significant research problems concerning the construction of models of theater-level, nonnuclear combat are--(1) developing better descriptions of attrition processes, (2) optimizing allocation of resources, and (3) playing more details. First, understanding and adequately playing stochastic variations in attrition processes is, we believe, the best improvement that can be made to the way that attrition is considered in theater-level combat models. Second, optimizing the allocations made in a deterministic model that inadequately describes combat may have some merit, but it is not as important as developing a model that does adequately describe combat (and then, perhaps, optimizing the allocations for that model in some way). Third, since some of the details that IDAGAM I now plays are not very significant (either because the outcomes are relatively insensitive to variations in them or because data are not available to that level of detail), it would be reasonable to *give up* some of the details already played (if necessary) to be able to play significant stochastic variations--not to mention attempting to *add* additional details to a model that cannot play stochastic variations.

Finally, a drawback to developing a truly stochastic model has been that computing exactly all the stochastic variations is an intractable mathematical problem, while using Monte Carlo techniques has led to models whose running times are too long to make a reasonable number of iterations.

There are two solutions to these difficulties. First, a Monte Carlo model could be developed that plays only the most significant details of combat (a deterministic model such as IDAGAM I could be used to determine which details are the most significant) and that randomizes only the most appropriate variables. By cutting down on the details played, IDAGAM I can run about 4 minutes of CPU time. Thus, if it were a Monte Carlo model with relatively few random variables, 15 iterations could be run in about 60 minutes of CPU time.

A second solution is to develop a model that approximates the distribution of random variables in such a way that the model is mathematically tractable. Such a model would not give the precisely correct stochastic variations, but it would be able to reflect stochastic variations and would give better approximations than deterministic models to the mathematically correct results. Advantages of such a model over a Monte Carlo model are that it would have to be run only once and that it would produce the same result if it were run twice with the same data. A relative disadvantage of such a model is that it would use more computer space and take longer to run than would one iteration of an equivalent Monte Carlo model.

We believe that it is possible to develop a model of theater-level combat that approximates the distribution of stochastic events in a reasonable yet tractable way. But whether this approach or the Monte Carlo approach or some other approach is taken, we believe that the most important research problem at this time in the construction of a theater-level, nonnuclear model of combat is reasonably to incorporate stochastic variations into such a model.

2. Short-Run Improvements

In view of the comments above, we do not recommend spending a significant effort now to optimize the allocations made by IDAGAM I or to add additional details to the model. Use of the model may indicate ways in which the nonoptimization algorithms can be significantly improved, in which case these improvements should be made. Also, there may be cases where a small effort to add a few particular details may add significantly to the usefulness of the model for a particular study. If making minor changes in either the logic or output of IDAGAM I would enhance its usefulness to a study, then such changes should be made at the computer facility that is running IDAGAM I for that study. In addition, there are a few changes to IDAGAM I that we feel would be generally useful, would add significantly to capabilities of the model, and would require only a minor effort to incorporate into IDAGAM I. These changes are listed below roughly in order of their importance.

a. Notional loads of munitions for aircraft on CAS missions and the probabilities of kill of sheltered and non-sheltered aircraft by aircraft on ABA missions should be a function of the distance the aircraft have to fly. For example, an aircraft flying a CAS mission from the COMMZ would have a smaller notional load than the same type of aircraft flying a CAS mission from the forward airbase in a region.

b. Aircraft on ABA missions should be able preferentially to attack enemy forward-region airbases rather than simply attacking the enemy airbases in proportion to the weighted number of aircraft on each airbase. This capability should be allowed because, in general, aircraft are more likely to be shot down on ABA missions to rearward airbases than they are on ABA missions to forward airbases.

c. With one exception, IDAGAM I first moves reinforcement divisions from regions to sectors and then withdraws divisions from sectors to regions. There are many cases where this order is irrelevant; but if a side is being constrained by the maximum number of divisions allowed in the sectors, then a better general rule would be to make the withdrawals first and then reinforce the sectors as appropriate.

d. Divisions are withdrawn if their personnel strength falls below an input minimum. It is reasonable to withdraw a division if its effectiveness is too low, but personnel strength is only one factor that determines the effectiveness of a division. Level of reorganization and weapons strength should also be considered in determining whether to withdraw a division for low effectiveness. These same points apply to the determination of whether an under-strength division in reserve is effective enough to be used to reinforce sectors.

e. If the TOE of a division includes a divisional slice for artillery (and, perhaps, SAMs), then the artillery will be withdrawn if the division is withdrawn. As an option, it would be reasonable to allow one type of division to be an artillery unit that would automatically be be withdrawn. Artillery then could be put into this type of unit instead of into a division slice, and this option would reflect a doctrine of not withdrawing artillery.

f. It would be reasonable to allow, as an option, more divisions to be deployed in a sector automatically as casualties reduce the size of the divisions already there (rather than have a maximum number of divisions in each sector that can be increased only by user input).

These and other improvements are currently being implemented at IDA, and prospective users of IDAGAM I should consult IDA concerning the status of these improvements.

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