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READINESS ANALYSIS FOR SHIPS WHERE BOTH MODERNIZATION AND REPLACEMENT ARE USED TO INCREASE FLEET READINESS

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13 ABSTRACT

A previous paper concerned itself with an optimal replacement policy model for a group of ships when the expected number of ships suffering readiness degradations greater than a given value was not permitted to be above a specified fraction of ships in service. Only complete replacement was considered. The present paper considers the additional possibility of modernization. The problem is analyzed for two different stochastic degradation processes and for "first order sequences". The basis for a simulation approach to determine optimal policies is presented.

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Introduction

In a previous paper, [2], a model dealing with an optimal replacement policy for a group of ships was developed and analyzed. This model was based on a paper by Kalman [1], and involved the determination of the optimal replacement interval under conditions where the expected number of ships suffering utility (readiness) degradations greater than a given value was not permitted to be above a specified fraction of ships in service. Ship readiness was associated with utility, where utility was measured on an ordinal-valued scale.

One of the purposes of [2] was to demonstrate how the optimal replacement interval may be shortened in the presence of constraints on allowable readiness, and a numerical example was given to illustrate this point. However, this work only considered replacement and did not consider the possibility of modernization as a means of boosting readiness to acceptable levels. Modernization which will be specifically considered in this paper, represents an alternative to complete replacement and may be more feasible from an economic standpoint in certain situations.

Modernization

The term "modernization" will be used in the sense of rehabilitating or replacing physical equipment or facilities of a ship which have either (1) suffered physical deterioration through the passage of time or usage or (2) have become technologically obsolescent in terms of new offensive or defensive capabilities of a potential adversary. As equipment deteriorates or as obsolescence grows, the readiness level of a ship will decline. In [2], it was assumed that the decrease in ship readiness was a function of calendar time from the time the ship was initially put into service. The function describing the magnitude of these decreases at a future time was assumed to be a random variable which combined both the physical deterioration and obsolescence effects. In a broader sense decreases in readiness will also occur because of decreases in the quantity and level of competence of trained manpower, and modernization will generally involve retraining of personnel to become sophisticated in the use of new facilities and equipment as well as the actual equipment replacement.

In the present paper, it will be assumed that the decision maker has a choice among a discrete set of alternatives representing different levels of modernization. One of these, the greatest degree of modernization, represents the complete scrapping and replacement of the existing ship. Although replacement can be considered as a level of modernization, there is a difference. With actual modernization ships must be taken out of service in order to be rehabilitated whereas with complete replacement, assuming that the new ship is available, it can take over as soon as the incumbent is scrapped.

To allow for the modernization time, Kalman introduced functions $\gamma_j^i\ (L_j, t_j)$ representing the time required to accomplish a modernization for the j^{th} ship in the sequence of replacements for the i^{th} initial incumbent ship, where L_j is the age of the j^{th} replacement at which modernization commences and t_j is the calendar time at which modernization starts.

Although Kalman distinguishes between modernization and replacement in his equation for the expected number of ships in each utility class at time t, we can treat both simultaneously by assuming that $\gamma_j^i\ (L_j, t_j) = 0$ if a ship is to be replaced completely and that $\gamma_j^i\ (L_j, t_j) > 0$ if a "lower" level of modernization is to take place. Naturally, the increases in cost occurring as a result of complete replacement rather than the next lowest level of modernization will be large. Thus, in a sense, we are paying for the replacement option (the one having the greatest increase in readiness) by a much larger cost outlay. This analysis, among other things, will investigate the conditions under which it is desirable to choose this option.*

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*If replacement is chosen, a sufficient lead time to construct the new ship must be estimated. Also, it is not known during construction exactly what the readiness level of the replacement will be since the enemy and environmental factors may make the replacement more or less ready than originally anticipated. In this first analysis we assume that the replacement will always be available when required and that the utility level of the replacement is known with certainty.

It will be assumed that a modernization cost function can be developed which relates the increase in readiness or utility level to expenditures. In reality this relationship will depend on the actual level of utility at which modernization begins, but initially it will be assumed that the function is independent of the actual utility level.

The above function could reasonably be approximated from cost data on equipment, facilities, labor for removal, installations, and redesign, etc., plus estimates on how the ordinal value of utility will increase if such sums were expended. Presumably, such a function would not be continuous but would be of a step-function type (as shown in Fig. 1) since different levels of modernization would be based on specific concepts, each requiring a certain capital investment. In other words there would be a list of specific investment proposals each described by an expenditure level and an expected amount of utility level increase. From this information a function of the type described could be generated.

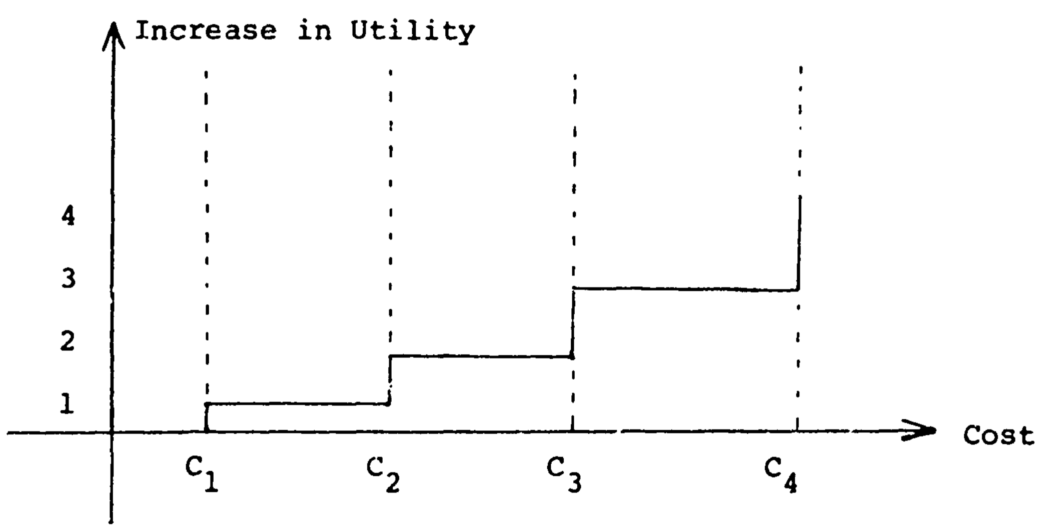

Fig. 1: Possible Relationship Between Capital Expenditures and Increases in Ship Readiness. Cost C_4 Represents Complete Replacement.

Readiness Degradation Process

In [2] it was assumed that the process by which the utility (readiness) of a ship decreased after it was put into service was a function of time. Furthermore it was assumed that the process was a random one where the probability of a degradation of j utility levels in an interval T could be described by a Poisson distribution with

$$\text{prob}\{\text{degradation} = j\} = p_j(T) = \frac{(\lambda T)^j e^{-\lambda T}}{j!}$$

Another model for the degradation process which involves discrete time and allows somewhat greater flexibility in being state dependent is a Markov process model with transition probabilities p_{ij} defined as the probability that if a ship is initially in state i (readiness level i) it will have a decrease of j levels of readiness in any unit time interval.

Here time t is discrete valued $(t = 0,1,2,\ldots)$ and the possible readiness states may be defined to range from zero (highest possible state of readiness) to N (lowest possible state). Thus, i can range in the interval (O,N) and $j = 0,1,\ldots,i$. Also $\sum_{j=0}^{i} p_{ij} = 1$. We assume that there is no possibility that the readiness of a ship can increase merely through the passage of time. As mentioned in [2], the Navy FORSTAT system does use ordinal-valued information to describe the readiness status for ships at the present time so that an ordinal valued utility function has some relation to present practice.

Our model represents a situation where a ship is initially at some state of readiness. In the absence of any major modernization or complete replacement its readiness will either stay the same with the passage of time or decrease. At some point there is a probability greater than zero that its readiness will fall below some minimum acceptable level. At this point a decision has to be made which depends on the resources (primarily funds) available. The ship can be permitted to stay at this undesirable state and to be further degraded. Or, the ship can be taken out of service, systems overhauled and modernized, and be put back into service. Or, finally, the ship can be scrapped and either replaced by another or not replaced at all. If either modernization or scrappage with replacement takes place a capital investment is required.

When there are many ships many alternate combinations of the above decisions are possible. Since the readiness degradation is a random process, different ships arrive at unacceptable levels at different times and the available funds at each time plus the changing costs of modernization and replacement may play important roles in any actual decision-making process.

In order to facilitate any analysis, several types of approximations can be considered. The first of these assumes that all ships of a certain class have the same probability distribution for readiness degradation, are in the same state initially and are degraded independently. Thus, the expected number of initial ships suffering a specified level of utility degradation at any time t may be found by multiplying the probability of degradation by N, the total number of ships.

Constraints

One of the principal differences between the nature of the constraints of [1] and [2] is that in [1] it is required that the expected number of ships in utility class "a" $(a = 1,2,\ldots,q)$ be equal to or greater than some specified value. Thus, a separate constraint is required for each utility class. In Kalman's notation, $Xa(t) \geq \hat{X}a(t)$, when $\hat{X}a(t)$ is the required number of ships in utility class "a" and $Xa(t)$ is the expected number. At any time the expected number of ships summed over all utility classes must be equal to the total number of ships initially put into service, since the model does not allow for scrapping a ship and not replacing it. Thus any ships out of service for modernization purposes can be considered to be in the lowest utility class q and the $\hat{X}a(t)$ cannot be chosen arbitrarily but must satisfy the additional requirement that $\sum_{a=1}^{q} \hat{X}a(t) = N$. At any rate these constraints are more restrictive than those of [2]. In fact they may be so restrictive that no feasible policies are possible. In [2] the approach has been to impose the single constraint that the expected number of ships suffering utility degradations (decrease in readiness) below some given level at any time must never be less than a given value. This second approach has the advantage of greater mathematical simplicity. Furthermore, it would seem that there would be much difficulty in determining consistent inputs among Navy commanders on the percentage of ships of a given type which should be at different levels of readiness, whereas the input data required for the constraints of [2] seems intuitively

e. . .er to obtain.

The formulation of [2] led to a single constraint on the service life of the incumbent ships and a single constraint on the service life of the replacement ships. These were of the form

$$L_o \leq \tilde{L}_o \quad \text{and} \quad L \leq \tilde{L}$$

where L_o represents the life of the incumbent ship after which the expected degradation in utility will be greater than desired, and L represents a similar life for the replacement ships. The $\tilde{L}_o$ and $\tilde{L}$ are determined as the minimum values of a set of numbers $L_o^{(r)}$ and $L^{(r)}$ which may have to be found by numerical approximation.

Let us now examine the process involved for a single ship assuming that t is discrete and that the analysis is confined to a finite time horizon T. Many alternate sequences of events are possible.

Figures 2-4 illustrate some possibilities:

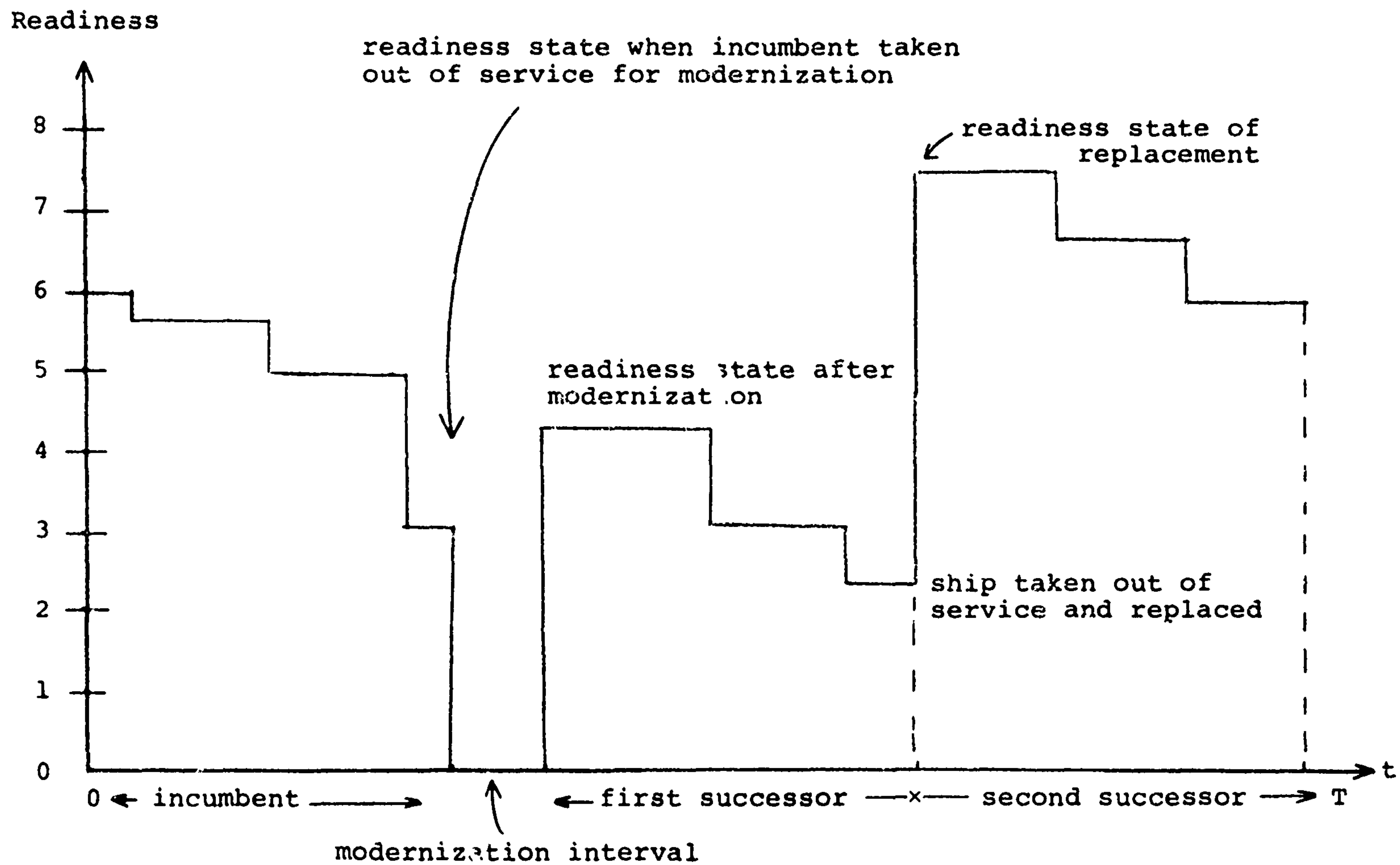

Figure 2: Modernization followed by replacement

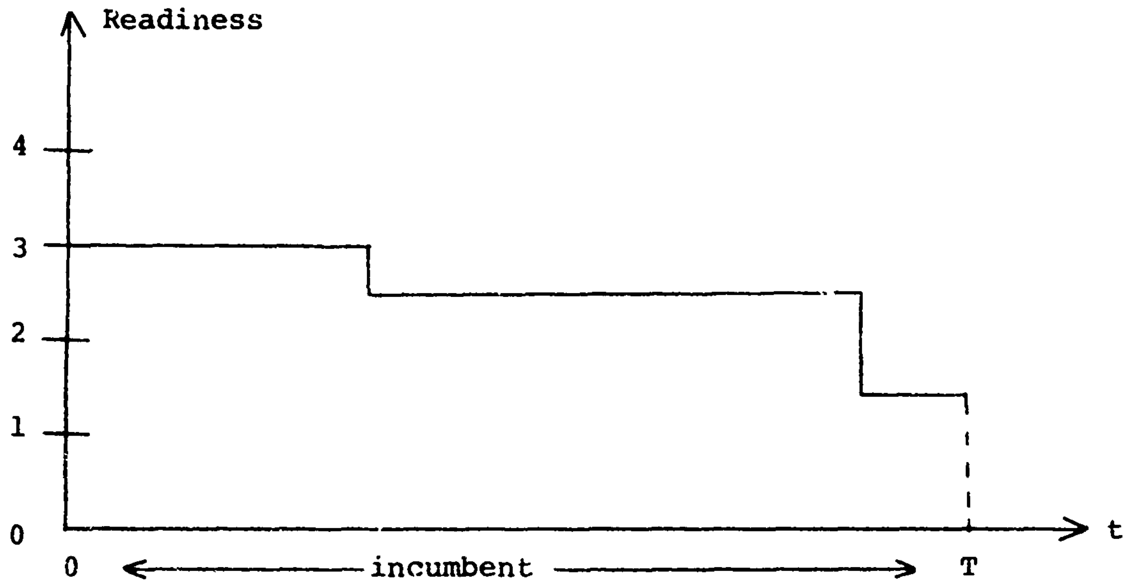

Figure 3: No Modernizations or Replacements

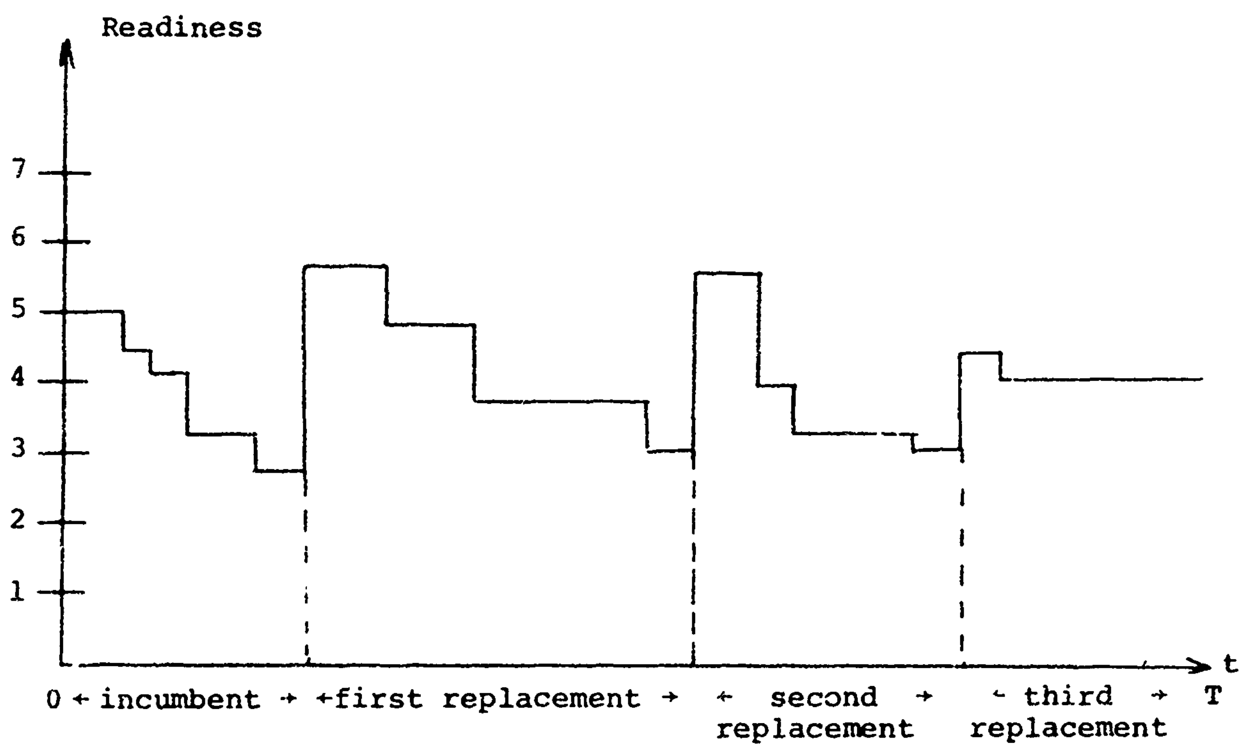

Figure 4: Successive Replacements

As observed above, for discrete t there are a large number of possibilities which can occur in (0,T). In order to reduce the problem to a form where analysis is possible we shall refer to n^{th} order sequences when an n^{th} order sequence is defined to be one involving only the incumbent and exactly n of its successors, which may be either replacements and/or modernizations. Thus, a first order sequence involves only the incumbent ship and its immediate successor (whether a modernization or replacement). A second order sequence would only involve the incumbent, its possible successor, and any possible successor to it. Figure 2 illustrates a second order sequence in (0,T). Figure 3 is a zeroth order sequence and Figure 4 is a third order sequence in (0,T). The basis of our analysis is to determine an optimal policy by the minimization of the present value of cost subject to a constraint on allowable readiness. Other things being equal, events occurring far into the future affect the present value less than events in the immediate future because of the discounting process. Thus, it seems reasonable in an initial analysis to consider only zeroth and first order replacement and/or modernization sequences. These considerations follow.

Analysis for First Order Sequences

The possible sequence of events if only the incumbent ship and its possible immediate successor are considered are:

(1) Retain incumbent for $t = T$ without replacement or modernization (zeroth order sequence).

(2) Retain incumbent for $t = t_1$ after making decision to replace it with a new ship at $t = t_o$, $t_o \leq t_1$. Retain replacement until $t = T$.

(3) Retain incumbent until $t = t_2$ and take it out of service at $t = t_2$ after which modernization takes place and the ship resumes service at $t = t_3$, $t_3 \geq t_2$. Retain modernized incumbent until $t = T$.

In (1) the readiness (utility level) will tend downward with time. In (2) there will be a downward trend in readiness followed by a rise at $t = t_1$, when the replacement takes over, followed by a declining trend, and in (3) there will be a downward trend until $t = t_2$, when readiness will drop to the lowest possible level while being modernized, after which it will rise to a higher level at $t = t_3$ before starting to decline again.

With respect to investment costs there are none in (1) whereas those of (2) and (3) depend on the level of modernization as shown in Fig. 1.

In order to focus the analysis on the optimal choice of policy subject to the constraint on readiness and to make the problem mathematically tractible we shall assume that the operating cost function of any ship is constant over its future lifetime as assumed in [1]. However, it will be assumed that the operating cost functions of any replacement to the incumbent (or of the modernized incumbent) are different from that of the incumbent. In particular, let:

ϕ_o = operating cost rate of the incumbent

ϕ_1 = operating cost rate of any modernized incumbent after modernization

ϕ_2 = operating cost rate of any replacement ship after replacement

C = readiness level increase obtained because of modernization or replacement

$\bar{W}_T(C)$ = cost of a replacement ship which will obtain increase in readiness of C

$W_T(C)$ = cost of modernization necessary to obtain increase in readiness of C

w = present value of total costs of any alternative

Now if we consider a fixed horizon T we can determine the present value of total costs of each of the three possibilities using a nominal interest rate r which is assumed to be compounded continuously.

First Alternative: Do not modernize or replace and continue to use incumbent over the entire time interval (0,T)

$$w = \int_0^T \phi_o e^{-r\tau} d\tau \quad = \quad \frac{\phi_o}{r}\left(1 - e^{-rT}\right)$$

Note that there is no investment cost of any kind because the initial investment in the incumbent is a sunk cost and is not a valid part of any analysis. Also, the salvage value is assumed to be insignificant in its contribution to the present value of total cost. Zero salvage values will also be assumed in the other two alternatives, approximations which should not produce serious errors.

Second Alternative: Make a decision to replace incumbent at t_o and replace it at $t_1 \geq t_o$. Use the replacement in the interval (t_1,T)

$$w = \int_0^{t_1} \phi_o e^{-r\tau} d\tau + \bar{W}_T(C) e^{-r(t_o+t_1)/2} + \int_{t_1}^{T} \phi_2 e^{-r\tau} d\tau$$

$$= \frac{\phi_o}{r}\left(1 - e^{-rt_1}\right) + \bar{W}_T(C) e^{-r(t_o+t_1)/2} + \frac{\phi_2}{r}\left(e^{-rt_1} - e^{-rT}\right)$$

Note that the time t_o at which a decision to replace is made does affect the present value of total cost. This is because the replacement cost $\bar{W}_T(C)$ will be assumed to occur in the interval (t_o, t_1) where a replacement has to be built. For convenience, we assume that this entire cost occurs at the mid-point of the interval. However, t_o will not affect the readiness level of the incumbent or replacement since the incumbent continues in use until it is replaced.

If a ship is taken out of service for modernization at t_2 this will affect the readiness. The ship will be out of service in the interval (t_2, t_3) (assuming that the ship is taken out of service at the same time that a decision to do so is made) and its readiness will drop to the lowest possible level assumed. The time t_3 represents the time when the modernized ship re-enters service, and this represents the third alternative.

Third Alternative: Make a decision to modernize incumbent at t_2, take it out of service at t_2, modernize the ship, put it back into service at t_3 and use the modernized version in the interval (t_3, T)

$$w = \int_{o}^{t_2} \phi_o e^{-r\tau} d\tau + W(C) e^{-r(t_2+t_3)/2} + \int_{t_3}^{T} \phi_1 e^{-r\tau} d\tau$$

$$w = \frac{\phi_o}{r} (1 - e^{-rt_2}) + W(C) e^{-r(t_2+t_3)/2} + \frac{\phi_1}{r} (e^{-rt_3} - e^{-rT})$$

We assume that the modernization cost is incurred uniformly over the modernization interval (t_2, t_3) and approximate the present value of this cost by assuming it all was incurred at the mid-point of this interval.

We must now bring in the constraints on readiness. Instead of requiring that the fraction of ships having utility degradations $\geq r$ levels be less than a specified value for all t in $0 \leq t \leq T$, suppose this requirement is imposed on only a finite set of times (t, 2t, 3t, ---nt) in this interval. Furthermore suppose that modernization or replacements can only be initiated at such times and that the modernization time when a modernized ship will be out of service is equal to kt, $k < n$. That is, $kt = t_3 - t_2$.

Consider the constraints at some rt, $0 < rt < T$. Suppose

N = the total number of ships initially in service at $t = 0$.

$N_1(rt)$ = the number of incumbent ships in service at rt.

$N_2^{(1)}(rt)$ = the number of incumbent ships that we decide to begin modernizing at rt and take out of action at this time

$N_2^{(2)}(rt)$ = the number of already modernized ships in existence at rt and in action

$N_3(rt)$ = the number of ships at rt which are replacements of incumbents.

We have the following constraints which must hold for all rt:

$$N_1(rt) = N - N_3(rt) - \sum_{m=0}^{r} N_2^{(1)}(mt)$$

$$N_2^{(2)}(rt) = \sum_{m=0}^{r-k} N_2^{(1)}(mt) , \qquad r \geq k$$

$$N = N_1 + N_2^{(1)} + N_2^{(2)} + N_3 \quad \text{for all } rt,\ rt \leq T .$$

Plan for Simulation

It is obvious that for first order sequences when (1) following a set of N initial incumbent ships during the interval (0,T), (2) allowing for modernizations and/or replacements at a discrete set of times t,2t,...nt in this interval and (3) satisfying constraints on readiness at each one of these times, the mathematical difficulty, even under the simplest assumptions on operating costs, becomes enormous. Therefore, we are proposing that a microsimulation of this problem be undertaken in order to obtain insights as to how some of the parameters of the model proposed will affect the optional modernization and replacement policy.

The model would operate as follows: Each incumbent ship will be assumed to start at some initial level of readiness. The readiness degradation process for each ship proceeds according to either a Markov process or the process assumed in [2]. Sequentially, at each time point rt ($r = 0,1,\ldots,n$) beginning at $t = 0$, the percentage of ships in each readiness category is determined and compared with required values (according to the way the single constraint is stated in [2]). If the constraint is satisfied, the degradation process can either continue to the next time point or one can start modernization or replacement decisions in anticipation of future situations where the constraint will not be satisfied. If the constraint is not satisfied, a definite choice must be made as to which incumbent ships will be upgraded, and a decision rule for choosing such ships has to be formulated. One possible choice is to select those ships having the greatest levels of degradation. Another is to select those with the lowest utility levels. After the choice of ships is made, decisions have to be made on which to modernize and which to replace. It is anticipated that the simulation will consider an experimental design where alternate combinations will be tried. The basic decision criterion is the present value of total cost subject to the satisfaction of the readiness constraint at each discrete time point.

In theory, since we shall start with a finite number of ships, make decisions at only a finite set of time periods, and have a finite set of alternatives at each time point there are only a finite set of alternate policies that are available.

However, as is readily observable, this number becomes large even for just a few ships and time periods. Thus, a complete enumeration of all possibilities does not seem feasible. However, we do feel that with an adequate sampling plan near optimal policies will be able to be discerned and the effect of changes in parameter values studied.

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[1] Peter J. Kalman, "A Stochastic Constrained Optimal Replacement Model: the Case of Ship Replacement," Operations Research 20, No. 2, March-April 1972.

[2] Seymour Kaplan, "A Note on a Constrained Replacement Model for Ships Subject to Degradations in Utility," Naval Research Logistics Quarterly, Vol. 21, No. 3, September 1974.