AFOSR - TR - 75 - 1628

Qualified requestors may obtain additional copies from the Defense Documentation Center, all others should apply to the National Technical Information Service.

See 1473

AFOSR 74-2739

Conditions of Reproduction

Reproduction, translation, publication, use and disposal in whole or in part by or for the United States Government is permitted.

THE EFFECT OF TRANSVERSE SHEAR DEFORMATION ON THE ELASTIC STABILITY OF PLATES OF COMPOSITE MATERIALS*

Alton P. Smith, Jr.**

and

Jack R. Vinson***

DEPARTMENT OF MECHANICAL AND AEROSPACE ENGINEERING

UNIVERSITY OF DELAWARE

January 1975

* Research sponsored by the Air Force Office of Scientific Research, Office of Aerospace Research, United States Air Force, Grant Number AFOSR - 74 -, under the direction of William J. Walker. This document has been approved for public release; distribution is unlimited.

** Currently, Research Engineer, E. I. duPont de Nemours, Inc., Newark, Delaware. This report is in part the thesis of Mr. Smith for the Master of Mechanical and Aerospace Engineering Degree.

*** Professor and Chairman

ABSTRACT

Methods of analysis are developed for determining the critical in-plane buckling loads for plates of composite materials with a wide variety of boundary conditions. Although the expressions developed are restricted to specially orthotropic construction of one lamina, their use is more general, and can be used for laminated specially orthotropic plates that are symmetric with respect to the plate mid-surface (i.e. no bending-stretching coupling).

It is seen that the effects of transverse shear deformation can be significant, not only in altering the magnitude of the buckling loads but even in altering the wave number.

TABLE OF CONTENTS

			PAGE
I.		LITERATURE REVIEW	1
II.	A.	INTRODUCTION	7
	B.	NOMENCLATURE	9
	C.	FORMULATION OF GOVERNING EQUATIONS	11
	D.	ELASTIC STABILITY OF ORTHOTROPIC PLATES WITH X EDGES AND ONE Y EDGE SIMPLY SUPPORTED AND THE OTHER Y EDGE FREE	23
	E.	ELASTIC STABILITY OF ORTHOTROPIC PLATES WITH THE Y EDGES CLAMPED AND THE X EDGES SIMPLY SUPPORTED	30
	F.	ELASTIC STABILITY OF ORTHOTROPIC PLATES WITH THE X EDGES AND Y EDGES SIMPLY SUPPORTED	35
	G.	ELASTIC STABILITY OF ORTHOTROPIC PLATES WITH X EDGES AND ONE Y EDGE SIMPLY SUPPORTED AND THE OTHER Y EDGE CLAMPED	40
	H.	CONCLUSIONS AND RECOMMENDATIONS FOR FURTHER STUDY	45

CHAPTER I

LITERATURE REVIEW

The effect of transverse shear deformation has been studied extensively in the analysis of sandwich plates. However, much of the research was concentrated on plates with simply-supported boundary conditions.

In 1945, Eric Reissner developed a system of equations for the theory of the bending of thin isotropic elastic plates which took into account the transverse shear deformation of the plate [1]*. This theory was extended to include the elastic buckling of orthotropic sandwich plates using small deflection theory by Libove and Batdorf [2]. In 1950, Seide and Stowell presented a theory for the elastic and plastic buckling of simply supported solid-core sandwich plates in compression applying the theory of reference 2 [3]. In their analysis, the faces and core of the sandwich plate were assumed to be isotropic. Results were presented for plates having face materials of 24S-T3 aluminum alloy, 75S-T6 alclad aluminum alloy, and stainless steel.

* Number in brackets refers to the numbered references in bibliography.

In 1956, W. R. Kimel developed a theoretical analysis for the elastic buckling of a simply supported rectangular sandwich panel subjected to combined edgewise bending and compression [4]. The solution is based on the assumption that the sandwich panel is composed of isotropic facings and an orthotropic core, and utilized the Rayleigh-Ritz energy method. Equations are presented from which the critical buckling load can be obtained; however, these equations are reduced for use in the form of usable design curves by assuming membrane facings and infinite transverse modulus of elasticity in the core. The same author extended the above analysis to include the elastic buckling of the same panel subjected not only to combined edgewise bending and compression but also to shear [5].

Norris, in 1958, presented curves and formulas for use in computing the buckling of flat panels of sandwich construction under edgewise compressive loads [6]. In the computation of these curves, the bending stiffness of the individual facings was neglected, so that the values of the parameters for each edge condition could be represented by a single family of curves. Various boundary conditions were treated for sandwich panels with isotropic facings and honeycomb cores. A method was also included for determining from these curves the buckling stress of sandwich panels for which the stiffness of the individual facings is not neglected.

Two years later, Chang and Ebcioglu treated the instability of a simply supported rectangular sandwich panel with anisotropic cores and faces of different materials or thicknesses subjected to a uniform edge compression [7]. Their work was important because it was found that some of the simplifying assumptions made in earlier theories were found to be inadequate to explain the behavior of aircraft sandwich structures. Notably, the assumption of the isotropic core cannot be applied without introducing appreciable error particularly for a core weak in transverse shear. The deflection due to transverse shear is considered only for the core but not for the faces, and the solution is expressed in terms of a buckling coefficient given as the ratio of the critical lateral buckling load to the critical cylindrical buckling load which is a function of several non-dimensional parameters. A single family of curves shows the buckling behavior for all parameters.

The edgewise compressive buckling of flat sandwich panels was further studied by Zahn and Cheng in 1964 [8]. Panels with equal isotropic facings and orthotropic or isotropic cores were analyzed. The loaded panel edges were assumed to be simply supported and the sides hinged to simple beams that carry end loads. This reportedly assessed the influence of the elasticity of the supports. Curves of buckling coefficients were presented for sandwich panels with isotropic cores.

In January, 1969, Cheng and Testa considered the stability of unidirectional, equally spaced fibers in the plane of a composite plate formulated by treating the fibers as beams and the matrix by the theory of generalized plane stress [9]. Critical values of the axial loads and the corresponding buckling modes were evaluated numerically.

In S.A. Ambartsumyan's book - "Theory of Anistropic Plates" - he developed the equations for the elastic stability of simply supported (along the entire contour) orthotropic rectangular plates including the effects of transverse shear deformation [10]. The formulation is presented for plates in which the orthotropic principal axes coincide with the plate axes (i.e. specially orthotropic). The same theory is employed for a rectangular plate in which two sides are simply supported. Some numerical results were presented.

The above work by Ambartsumyan was extended by J. E. Ashton and M. E. Waddoups to include an energy formulation and solutions for the analysis of plane, generally anisotropic, rectangular plates [11]. The formulation includes stability analysis, the calculation of natural frequencies and mode shapes, and the analysis of displacements due to lateral loads. The stability analysis includes the total potential energy due to bending and in plane loads and then employs the Ritz method to minimize the equations. Various boundary conditions are considered but the effects of transverse shear deformation are neglected when plate stability is discussed.

In April, 1969, a paper by Ashton and Love was published in which an experimental study of the stability of rectangular boron epoxy laminated plates was presented [12]. The plates were clamped on loaded edges and either clamped or simply supported on the unloaded sides. The buckling loads were determined by the Southwell plots and then compared to the analytical results obtained in reference 11 with good agreement.

J. M. Whitney considered the effects of transverse deformation on the buckling of laminated plates [13]. He develops a bending theory, including transverse shear deformation for plates constructed of a high E/G ratio. Closed form solutions are obtained for transverse static loading, natural frequencies of flexural vibrations, and buckling loads for simply supported laminates of special orthotropic construction. His results indicate that for a square $\pm$ 45° plate with an infinite number of layers, the effect of transverse shear deformation on the buckling loads are quite pronounced for small values of a/h, and decrease as a/h increases.

E. J. Brunelle derived the modified Mindlin plate equations that include the effect of transverse isotropy (which accentuates the effects of shear deformation), initial stress, and initial displacements [14]. These plate equations are suitable for investigating static deflections, free and forced vibrations, and elastic stability. This paper, however, limited the analysis to the elastic stability of a rectangular plate with two parallel sides simply supported and the remaining two sides subjected to a variety of boundary conditions. A buckling coefficient is determined for various values of a/b for uniaxial compressive loads. His work shows that transverse isotropy induces decreases in the buckling loads. These decreases become larger as the boundary restraint is increased. Therefore, as the plate becomes more transversely isotropic, the addition of boundary restraints is progressively less effective in raising the buckling loads of this type of plate.

A recently published book by Ashton and Whitney deals with laminated plates [17]. A solution for a simply supported laminated plate including transverse shear deformation is developed.

CHAPTER II

A. INTRODUCTION

The effect of transverse shear deformation on the elastic stability of rectangular, orthotropic, homogeneous, single layer plates are analyzed using Reissner's variational theorem, minimum potential energy, and Raleigh methods.

Sandwich plates and moderately thick plates used extensively in the aerospace and maritime industries, are typical of plates in which the effects of transverse shear deformation are very important. Neglecting this type of deformation can lead to overestimating the buckling load resulting in serious errors. Work has been performed on simply support plates, including the effects of transverse shear deformation. It is the purpose of this report to extend the analysis to plates under in-plane loading for other types of boundary conditions. The results will be helpful to the designer in determining when the effects should be included in the elastic stability analysis of the aformentioned plates.

In the following, the general governing equations are derived with in-plane loading in the X and Y directions. Specific numerical examples are then presented for four different sets of boundary conditions with only the X direction in-plane load acting.

The results are presented in both graphic and tabular form. They compare the buckling load for a plate including and neglecting transverse shear deformation for various material properties.

B. NOMENCLATURE

A_{mn}	Variational Constant
a, A	Plate edge length in x-direction, in.
b, B	Plate edge length in y-direction, in.
D_x	Flexural rigidity, $\frac{E_x h^3}{12(1-\nu_{xy}\nu_{yx})}$, lb-in
D_y	Flexural rigidity, $\frac{E_y h^3}{12(1-\nu_{xy}\nu_{yx})}$, lb-in
D_{xy}	$\frac{G_{xy} h^3}{12}$, lb-in.
E_x, E_y	Young's moduli in the x and y directions respectively, psi
G_{xy}, G_{xz}, G_{yz}	Shear moduli, psi
h	Plate thickness, in.
M_x, M_y, M_{xy}	Stress couples, in-lb/in.
N_x, N_y	In-plane loads, lb/in.
$\overline{N}_x$	Non-dimensionalized N_x; $\overline{N}_x = \frac{N_x \pi^2}{G_{xz} h}$
Q_x, Q_y	Transverse stress resultants, lb/in.

R	Volume
S	Surface
T_i	Tractions, psi
μ	Displacement in x-direction, in.
V	Potential energy function
v	Displacement in y-direction, in.
$W(\sigma_{ij})$	Strain energy density function in terms of stresses
w	Displacement in z-direction, in.
α,β	Rotations in x and y directions respectively
$\epsilon_{ij},\sigma_{ij}$	Strains and stresses respectively
ν_{ij}	Poisson's ratio: the ratio of the compressive strain in j-direction to tensile strain in i-direction due to a tensile stress in the i-direction
$\bar{\nu}$	$1-\nu_{xy}\nu_{yx}$
ψ	Reissner's functional

C. FORMULATION OF GOVERNING EQUATIONS

The governing equations used to determine the critical buckling load for an orthotropic, homogeneous, single-layer plate with and without the effects of transverse shear deformation are formulated. The strain energy density function is formulated for use in Reissner's Variational Theorem [15]. Once this is done, the potential energy function can be formulated. Using the Minimum Potential Energy Theorem, all variables are then expressed in terms of displacements only. This lends itself to an equation for the potential energy which can be minimized to determine the critical buckling load once a suitable estimate for the lateral displacement is obtained.

Consider a specially orthotropic, homogeneous, single-layer plate including transverse shear deformation as shown in Figure 1. The plate is under in-plane compressive loads N_x and N_y per unit edge distance. Due to the plate's special orthotropy, there are three mutually perpendicular axes of elastic symmetry, two of which lie in the plane of the plate parallel to the geometric axes with the third axes being normal to the plane of the plate. Since the matrix of elastic coefficients must be symmetric, there exist six independent elastic constants. The linear strain displacement relationships can be used since they are sufficient to determine the critical buckling load. The deformation of linear elements originally perpendicular to the X-Y plane can be considered to be the sum of a translation and a rotation. The stress-strain relationships are:

$$\varepsilon_x = \frac{1}{E_x}\sigma_x - \frac{\nu_{yx}}{E_y}\sigma_y - \frac{\nu_{xz}}{E_x}\sigma_z$$

$$\varepsilon_y = \frac{-\nu_{xy}}{E_x}\sigma_x + \frac{1}{E_y}\sigma_y - \frac{\nu_{yz}}{E_y}\sigma_z$$

$$\varepsilon_z = 0$$

$$\varepsilon_{xy} = \frac{1}{2G_{xy}}\sigma_{xy}$$

$$\varepsilon_{yz} = \frac{1}{2G_{yz}}\sigma_{yz}$$

$$\varepsilon_{xz} = \frac{1}{2G_{xz}}\sigma_{xz}$$

Similarily, the strain-displacement relationships are:

$$\varepsilon_x = z\,\frac{\partial\alpha}{\partial x}$$

$$\varepsilon_y = z\frac{\partial\beta}{\partial y}$$

$$\varepsilon_z = 0$$

$$\varepsilon_{xy} = \frac{1}{2}\left(\frac{\partial\alpha}{\partial y} + \frac{\partial\beta}{\partial x}\right)$$

$$\varepsilon_{yz} = \frac{1}{2}\left(\frac{\partial w}{\partial y} + \beta\right)$$

$$\varepsilon_{xz} = \frac{1}{2}\left(\frac{\partial w}{\partial x} + \alpha\right)$$

where,

$$u = \alpha(x,y)z$$

$$v = \beta(x,y)z$$

Applying Reissner's Variational Theory as outlined in [15] the following expressions will be used.

$$(\sigma_x,\sigma_y,\sigma_{xy}) = \frac{12z}{h^3}(M_x,M_y,M_{xy})$$

$$(\sigma_{xz},\sigma_{yz}) = \frac{3}{2h}\left[1-\left(\frac{z}{h/2}\right)^2\right](Q_x,Q_y)$$

$$\sigma_z = 0$$

Utilizing Reissner's theory, the strain energy function for the formulated problem is given by the following relationship

$$\psi = \int_R F \, dR$$

where $$F = \sigma_{ij}\varepsilon_{ij} - W(\sigma_{ij})$$

and $$W(\sigma_{ij}) = \frac{1}{2} [(\frac{1}{E_x} \sigma_x^2 + \frac{1}{E_y} \sigma_y^2 + \frac{1}{E_z} \sigma_z^2)$$

$$-2(\frac{\nu_{xy}}{E_x} \sigma_x \sigma_y^2 + \frac{\nu_{yz}}{E_y} \sigma_y \sigma_z + \frac{\nu_{xz}}{E_x} \sigma_x \sigma_z)$$

$$+ \frac{1}{G_{yz}} \sigma_{yz}^2 + \frac{1}{G_{xz}} \sigma_{xz}^2 + \frac{1}{G_{xy}} \sigma_{xy}^2]$$

From minimum potential energy theory, it is postulated that the minimum potential energy of an elastic body is given by the following equation:

$$V = \int_R F \, dxdy\, dz - \int_R \bar{F}_i \mu_i \, dR - \int_{S_T} T_i \mu_i dS$$

F represents the strain energy function of the body, $\bar{F}$ denotes the body forces acting (which in this case, are neglected), and T_i are surface tractions which in this case are the compressive loads per unit edge distances, N_x and N_y. With this in mind, the resulting potential energy function for the problem as formulated is given by:

$$V = \int_R F \, dR - \int_S \frac{N_x}{2} \left(\frac{\partial w}{\partial x}\right)^2 dS - \int_S \frac{N_y}{2} \left(\frac{\partial w}{\partial y}\right)^2 dS$$

Using the expression for F, the stress-strain, stress-displacement, and Reissner relationships, and after integrating across the thickness, the potential energy function which has to be minimized is given by equation (1.1).

$$\begin{aligned} V = \int_0^a \int_0^b \Big[& - \frac{N_x}{2} \left(\frac{\partial w}{\partial x}\right)^2 - \frac{N_y}{2} \left(\frac{\partial w}{\partial y}\right)^2 + M_x \frac{\partial \alpha}{\partial x} + M_y \frac{\partial \beta}{\partial y} \\ & + M_{xy} \left(\frac{\partial \alpha}{\partial y} + \frac{\partial \beta}{\partial x}\right) + Q_x \left(\frac{\partial w}{\partial x} + \alpha\right) + Q_y \left(\frac{\partial w}{\partial y} + \beta\right) \\ & - \frac{6M_x^2}{E_x h^3} - \frac{6M_y^2}{E_y h^3} - \frac{6M_{xy}^2}{G_{xy} h^3} + \frac{12\nu_{xy}}{E_x h^3} M_x M_y \\ & - \frac{3}{5} \left(\frac{Q_x^2}{G_{xz} h} + \frac{Q_y^2}{G_{yz} h}\right) \Big] \, dx\,dy \end{aligned} \qquad (1.1)$$

In order to use the Rayleigh method to determine the critical buckling load for various boundary conditions, equation (1.1) must be manipulated to include only terms involving the deflection in the z-direction. This can be accomplished by taking variations of equation (1.1) with respect to the variables $w, \alpha, \beta, M_x, M_y, M_{xy}, Q_x$, and Q_y and setting the resulting equation equal to zero. Equations (1.2) results.

$$
\begin{aligned}
0 = & \int_0^a [M_y \delta\beta + M_{xy}\delta\alpha + Q_y \delta w]_0^b \, dx \\
& + \int_0^b [-N_x \frac{\partial w}{\partial x} \delta w + M_x \delta\alpha + M_{xy}\delta\beta + Q_x \delta w - N_y \frac{\partial w}{\partial y} \delta w]_0^a \, dy \\
& + \int_0^a \int_0^b \Bigg\{ [N_x \frac{\partial^2 w}{\partial x^2} + N_y \frac{\partial^2 w}{\partial y^2} - \frac{\partial Q_x}{\partial x} - \frac{\partial Q_y}{\partial y}] \delta w \\
& + [-\frac{\partial M_{xy}}{\partial y} + Q_x - \frac{\partial M_x}{\partial x}] \delta\alpha + [-\frac{\partial M_y}{\partial y} - \frac{\partial M_{xy}}{\partial x} + Q_y] \delta\beta \\
& + [\frac{\partial \alpha}{\partial x} - \frac{12 M_x}{E_x h^3} + \frac{12 \nu_{xy} M_y}{E_x h^3}] \delta M_x \\
& + [\frac{\partial \beta}{\partial y} - \frac{12 M_y}{E_y h^3} + \frac{12 \nu_{xy} M_x}{E_x h^3}] \delta M_y \\
& + [\frac{\partial \alpha}{\partial y} + \frac{\partial \beta}{\partial x} - \frac{12 M_{xy}}{G_{xy} h^3}] \delta M_{xy} \\
& + [\frac{\partial w}{\partial x} + \alpha - \frac{6}{5} \frac{Q_x}{G_{xz} h}] \delta Q_x \\
& + [\frac{\partial w}{\partial y} + \beta - \frac{6}{5} \frac{Q_y}{G_{yz} h}] \delta Q_y \Bigg\} dxdy \qquad (1.2)
\end{aligned}
$$

Setting the 1st two terms of equation (1.2) equal to zero, specifies the boundary conditions. The rest of terms when set equal to zero result in the Euler-Lagrange differential equations of the variational problem. They are of vital interest since from these equations, it will be possibile to express the variables in terms of w, α, and β. The Euler-Lagrange equations are:

$$N_x \frac{\partial^2 w}{\partial x^2} + N_y \frac{\partial^2 w}{\partial y^2} - \frac{\partial Q_x}{\partial x} - \frac{\partial Q_y}{\partial y} = 0$$

$$-\frac{\partial M_{xy}}{\partial y} + Q_x - \frac{\partial M_x}{\partial x} = 0$$

$$-\frac{\partial M_y}{\partial y} - \frac{\partial M_{xy}}{\partial x} + Q_y = 0$$

$$\frac{\partial \alpha}{\partial x} - \frac{12M_x}{E_x h^3} + \frac{12\nu_{xy} M_y}{E_x h^3} = 0 \qquad (1.3)$$

$$\frac{\partial \beta}{\partial y} - \frac{12M_y}{E_y h^3} + \frac{12\nu_{xy} M_x}{E_x h^3} = 0$$

$$\frac{\partial \alpha}{\partial y} + \frac{\partial \beta}{\partial x} - \frac{12M_{xy}}{G_{xy} h^3} = 0$$

$$\frac{\partial w}{\partial x} + \alpha - \frac{6}{5} \frac{Q_x}{G_{xz} h} = 0$$

$$\frac{\partial w}{\partial y} + \beta - \frac{6}{5} \frac{Q_y}{G_{yz} h} = 0$$

After some algebraic manipulation of equations (1.3), the variables M_x, M_y, M_{xy}, Q_x, and Q_y can be expressed as functions of w, α, and β. The results are shown in equations (1.4)

$$M_x = D_x\left(\frac{\partial \alpha}{\partial x} + \frac{\partial \beta}{\partial y} \nu_{yx}\right)$$

$$M_y = D_y\left(\frac{\partial \alpha}{\partial x} \nu_{xy} + \frac{\partial \beta}{\partial y}\right)$$

$$M_{xy} = \frac{G_{xy}h^3}{12}\left(\frac{\partial \alpha}{\partial y} + \frac{\partial \beta}{\partial x}\right) \tag{1.4}$$

$$Q_x = \frac{5G_{xz}h}{6}\left(\frac{\partial w}{\partial x} + \alpha\right)$$

$$Q_y = \frac{5G_{yz}h}{6}\left(\frac{\partial w}{\partial y} + \beta\right)$$

Equations (1.4) are now substituted into equation (1.1) and after some lengthy algebraic manipulations, equation (1.5) results.

$$V = \int_0^a\int_0^b \Bigg[\left(-\frac{N_x}{2} + \frac{5}{12}G_{xz}h\right)\left(\frac{\partial w}{\partial x}\right)^2 + \frac{5}{6}\alpha \frac{\partial w}{\partial x} G_{xz}h + \left(\frac{5}{12}G_{yz}h - \frac{N_y}{2}\right)\left(\frac{\partial w}{\partial y}\right)^2$$
$$+ \frac{5}{6}\beta \frac{\partial w}{\partial y}G_{yz}h + \frac{D_x}{2}\left(\frac{\partial \alpha}{\partial x}\right)^2 + D_x\nu_{yx} \frac{\partial \alpha}{\partial x}\frac{\partial \beta}{\partial y} + \frac{D_y}{2}\left(\frac{\partial \beta}{\partial y}\right)^2$$
$$+ \frac{G_{xy}h^3}{24}\left|\left(\frac{\partial \alpha}{\partial y}\right)^2 + 2\frac{\partial \alpha}{\partial y}\frac{\partial \beta}{\partial x} + \left(\frac{\partial \beta}{\partial x}\right)^2\right| + \frac{5}{12}G_{xz}h\alpha^2$$
$$+ \frac{5}{12}G_{yz}h\beta^2\Bigg]dxdy \tag{1.5}$$

Equation (1.5) is in terms of only the lateral deflection, rotation, the applied loads and the material properties peculiar to the plate being investigated. However, the Rayleigh method cannot be used until an expression for $\alpha(x,y)$ and $\beta(x,y)$ in terms of $w(x,y)$ is formulated. This can be accomplished by reducing equations (1.3) to those for a beam in the x-direction. The result is given by equations (1.6).

$$N_x \frac{d^2w}{dx^2} - \frac{dQ_x}{dx} = 0$$

$$-\frac{dM_x}{dx} + Q_x = 0 \qquad (1.6)$$

$$\frac{d\alpha}{dx} - \frac{12M_x}{E_x h^3} = 0$$

$$\frac{dw}{dx} + \alpha - \frac{6}{5}\frac{Q_x}{G_{xz}h} = 0$$

By manipulating equations (1.6), the following expression for $\alpha(x)$ results:

$$\alpha(x) = \frac{E_x h^2}{10G_{xz}} \frac{d^3w}{dx^3} \left(\frac{6N_x}{5G_{xz}h} - 1\right) - \frac{dw}{dx}$$

The above equation can then be generalized to include the y coordinate. The resulting expression as shown by equation (1.7) gives the desired result for $\alpha(x,y)$ as a function of $w(x,y)$ only.

$$\alpha(x,y) = \frac{E_x h^2}{10G_{xz}} \frac{\partial^3 w}{\partial x^3} \left(\frac{6N_x}{5G_{xz}h} - 1\right) - \frac{\partial w}{\partial x} \qquad (1.7)$$

Simiiarily, by reducing equations (1.3) to those of a beam in the y-direction, an expression for $\beta(x,y)$ in terms of $w(x,y)$ can be obtained.

$$\beta(x,y) = \frac{E_y h^2}{10G_{yz}} \frac{\partial^3 w}{\partial y^3} \left(\frac{6N_y}{5G_{yz}h} - 1\right) - \frac{\partial w}{\partial y} \qquad (1.8)$$

It should be noted that $\alpha(x,y)$ and $\beta(x,y)$ reduce to the classical values if transverse shear deformation is neglected - if we let G_{xz} and G_{yz} approach infinity.

By putting equations (1.7) and (1.8) into equation (1.5) and performing the differentiation indicated, the following equation for the potential energy results:

$$V = \int_0^a \int_0^b \left\{ - \frac{N_x}{2}\left(\frac{\partial w}{\partial x}\right)^2 - \frac{N_y}{2}\left(\frac{\partial w}{\partial y}\right)^2 + 2 D_{xy}\left(\frac{\partial^2 w}{\partial x \partial y}\right)^2 \right.$$

$$-2D_{xy} \frac{E_x h^2}{10G_{xz}}\left(\frac{6N_x}{5G_{xz}h} - 1\right)\frac{\partial^4 w}{\partial x^3 \partial y}\frac{\partial^2 w}{\partial x \partial y} - 2D_{xy}\frac{E_y h^2}{10G_{yz}}\left(\frac{6N_y}{5G_{yz}h} - 1\right)\frac{\partial^4 w}{\partial y^3 \partial x}\frac{\partial^2 w}{\partial x \partial y}$$

- - CONT'D

$$+\frac{D_{xy}}{2}\left[\frac{E_x h^2}{10G_{xz}}\frac{\partial^4 w}{\partial x^3 \partial y}\left(\frac{6N_x}{5G_{xz}h}-1\right)+\frac{E_y h^2}{10G_{yz}}\frac{\partial^4 w}{\partial y^3 \partial x}\left(\frac{6N_y}{5G_{yz}h}-1\right)\right]^2$$

$$+\frac{D_x}{2}\left[\frac{E_x h^2}{10G_{xz}}\frac{\partial^4 w}{\partial x^4}\left(\frac{6N_x}{5G_{xz}h}-1\right)-\frac{\partial^2 w}{\partial x^2}\right]^2$$

$$+D_x \nu_{yx}\left[\frac{E_x h^2}{10G_{xz}}\left(\frac{6N_x}{5G_{xz}h}-1\right)\frac{\partial^4 w}{\partial x^4}-\frac{\partial^2 w}{\partial x^2}\right]\left[\frac{E_y h^2}{10G_{yz}}\left(\frac{6N_y}{5G_{yz}h}-1\right)\frac{\partial^4 w}{\partial y^4}-\frac{\partial^2 w}{\partial y^2}\right]$$

$$+\frac{D_y}{2}\left[\frac{E_y h^2}{10G_{yz}}\left(\frac{6N_y}{5G_{yz}h}-1\right)\frac{\partial^4 w}{\partial y^4}-\frac{\partial^2 w}{\partial y^2}\right]^2$$

$$+\frac{5}{1200}\frac{E_x^2 h^5}{G_{xz}}\left(\frac{6N_x}{5G_{xz}h}-1\right)^2\left(\frac{\partial^3 w}{\partial x^3}\right)^2+\frac{5}{1200}\frac{E_y^2 h^5}{G_{yz}}\left(\frac{6N_y}{5G_{yz}h}-1\right)\left(\frac{\partial^3 w}{\partial y^3}\right)^2\Bigg\}dx\,dy \quad (1.9)$$

Equation (1.9) is the general form of the potential energy function to be minimized for a single layer specially orthotropic, homogeneous plate including transverse shear deformation under in-plane loads N_x and N_y. The equation is in terms of material properties and $w(x,y)$ which implies that the Rayleigh-Ritz method can now be employed to determine the critical buckling load for various boundary conditions.

Before proceeding, a similar expression to that shown in equation (1.9) can be derived for a plate neglecting transverse shear deformation. The resulting expression for the potential energy function neglecting transverse shear deformation is given by equation (1.10).

$$V = \int_0^a \int_0^b \left\{ \frac{D_x}{2} \left[\left(\frac{\partial^2 w}{\partial x^2}\right)^2 + 2\nu_{yx} \frac{\partial^2 w}{\partial y^2} \frac{\partial^2 w}{\partial x^2} + \frac{\nu_{yx}}{\nu_{xy}} \left(\frac{\partial^2 w}{\partial y^2}\right)^2 \right] + 2D_{xy} \left(\frac{\partial^2 w}{\partial x \partial y}\right)^2 \right\} dxdy - \frac{1}{2} \int_0^a \int_0^b \left[N_x \left(\frac{\partial w}{\partial x}\right)^2 + N_y \left(\frac{\partial w}{\partial y}\right)^2 \right] dxdy \qquad (1.10)$$

As the discussion proceeds, the critical buckling load determined by including transverse shear deformation (equation 1.9) will be compared to that value obtained when it is neglected (equation 1.10).

D. THE ELASTIC STABILITY OF SPECIALLY ORTHOTROPIC PLATES WITH X-EDGES AND ONE Y-EDGE SIMPLY SUPPORTED AND THE OTHER Y-EDGE FREE

Since the governing equations have been derived, the buckling load for an orthotropic plate can be determined. Consider the case of a plate in which the x edges are simply supported, the $y=0$ edge is simply supported and the $y=b$ edge is free. Furthermore, consider the plate to be acted upon by in-plane loads N_x and N_y. A suitable approximation for the vertical displacement $w(x,y)$ is

$$w(x,y) = Ay \sin \frac{n\pi x}{a} \tag{2.1}$$

where A is the amplitude. Substituting equation (2.1) into equation (1.9), the governing equation for the buckling load of an orthotropic place including transverse shear deformation, yields equation (2.2).

$$V = \int_0^a \int_0^b \left\{ -\frac{N_x}{2}\left(A \frac{N\pi}{a} y \cos \frac{N\pi x}{a}\right)^2 - \frac{N_y}{2}\left(A \sin \frac{N\pi x}{a}\right)^2\right.$$

-- CONT'D

$$+2D_{xy}\left(A\frac{N\pi}{a}\cos\frac{N\pi x}{a}\right)-2D_{xy}\frac{E_xh^2}{10G_{xz}}\left(\frac{6N_x}{5G_{xz}h}-1\right)\left(-A^2\frac{N^4\pi^4}{a^4}\cos^2\frac{N\pi x}{a}\right)$$

$$+\frac{D_{xy}}{2}\frac{E_x^2h^4}{100G_{xz}^2}\left(\frac{6N_x}{5G_{xz}}-1\right)\left(-A\frac{N^3\pi^3}{a^3}\cos\frac{N\pi x}{a}\right)^2$$

$$+\frac{D_x}{2}\left[\frac{E_x^2h^4}{100G_{xz}^2}\left(\frac{6N_x}{5G_{xz}h}-1\right)^2\left(A\frac{N^4\pi^4}{a^4}y\sin\frac{N\pi x}{a}\right)^2\right.$$

$$\left.-2\frac{E_xh^2}{10G_{xz}}\left(\frac{6N_x}{5G_{xz}h}-1\right)\left(-A^2\frac{N^6\pi^6}{a^6}y^2\sin^2\frac{N\pi x}{a}\right)+\left(-A\frac{N^2\pi^2}{a^2}y\sin\frac{N\pi x}{a}\right)^2\right]$$

$$+\frac{5}{1200}\frac{E_x^2h^5}{G_{xz}}\left(\frac{6N_x}{5G_{xz}h}-1\right)^2\left(-A\frac{N^3\pi^3}{a^3}y\cos\frac{N\pi x}{a}\right)^2\Bigg\}\quad (2.2)$$

Integrating equation (2.2) with respect to the variables x and y, taking variations with respect to A, setting the result equal to zero, and noting that;

$$\int_0^a\int_0^b \sin^2\frac{n\pi x}{a}\,dxdy = \int_0^a\int_0^b \cos^2\frac{n\pi x}{a} = \frac{ab}{2}$$

$$\int_0^a\int_0^b y^2\cos^2\frac{n\pi x}{a}\,dxdy = \int_0^a\int_0^b y^2\sin^2\frac{n\pi x}{a}\,dxdy = \frac{ab^3}{6},$$

yields equation (2.3).

$$0 = \frac{N_x}{12} \frac{\pi^2 n^2 b^3}{a} - \frac{N_y}{4} ab + D_{xy} \frac{\pi^2 n^2 b}{a}$$

$$+ \frac{D_{xy} E_x h^2}{10 G_{xz}} \frac{\pi^4 n^4 b}{a^3} \left(\frac{6N_x}{5G_{xz}h} - 1\right) + \frac{D_{xy} E_x^2 h^4}{400 G_{xz}^2} \frac{\pi^6 n^6 b}{a^5} \left(\frac{6N_x}{5G_{xz}h} - 1\right)^2$$

$$+ \frac{D_x E_x^2 h^4}{1200 G_{xz}^2} \frac{\pi^8 n^8 b^3}{a^7} \left(\frac{6N_x}{5G_{xz}h} - 1\right)^2 + \frac{D_x E_x h^2}{60 G_{xz}} \frac{\pi^6 n^6 b^3}{a^5} \left(\frac{6N_x}{5G_{xz}h} - 1\right)$$

$$+ \frac{D_x}{12} \frac{\pi^4 n^4 b^3}{a^3} + \frac{E_x^2 h^5}{1440 G_{xz}} \frac{\pi^6 n^6 b^3}{a^5} \qquad (2.3)$$

Equation (2.3) is the quadratic equation to solve to obtain the buckling load N_x (note that $N_y = \gamma N_x$ where γ is a specified constant in any problem). It can be non-dimensionalized by dividing each term by $G_{xz}h$. This does not yield the more conventional form of the non-dimensionalize critical buckling load, $\frac{N_x b^2}{D_x \pi^2}$, but it does result in reducing the number of material ratios that have to be evaluated. Non-dimensionalizing equation (2.3) yields equation (2.4).

$$0 = \bar{N}_x^2 \left[\frac{3}{50}\left(\frac{G_{xy}}{G_{xz}}\right)\left(\frac{E_x}{G_{xz}}\right)^2\left(\frac{h}{a}\right)^4 n^6 + \frac{1}{5}\left(\frac{E_x}{G_{xz}}\right)^2\left(\frac{h}{a}\right)^2\left(\frac{b}{a}\right)^2 n^6 \right.$$

$$\left. + \frac{3\pi^2}{50\bar{\nu}}\left(\frac{E_x}{G_{xz}}\right)^3\left(\frac{h}{a}\right)^4\left(\frac{b}{a}\right)^2 n^8 \right] + \bar{N}_x \left[2\left(\frac{G_{xy}}{G_{xz}}\right)\left(\frac{E_x}{G_{xz}}\right)\left(\frac{h}{a}\right)^2 n^4 \right.$$

--CONT'D

$$+ \frac{\pi^2}{3\bar{\nu}}\left(\frac{E_x}{G_{xz}}\right)^2\left(\frac{h}{a}\right)^2\left(\frac{b}{a}\right)^2 n^6 - \frac{3\pi^2}{10}\left(\frac{G_{xy}}{G_{xz}}\right)\left(\frac{E_x}{G_{xz}}\right)^2\left(\frac{h}{a}\right)^4 n^6$$

$$- \frac{\pi^4}{30\bar{\nu}}\left(\frac{E_x}{G_{xz}}\right)^3\left(\frac{h}{a}\right)^4\left(\frac{b}{a}\right)^2 n^8 - \frac{\pi^2}{3}\left(\frac{E_x}{G_{xz}}\right)^2\left(\frac{h}{a}\right)^2\left(\frac{b}{a}\right)^2 n^6$$

$$- \frac{50}{3\pi^2}\left(\frac{a}{h}\right)^2\left(\frac{b}{a}\right)^2 n^2 - \frac{50}{\pi^4}\left(\frac{a}{h}\right)^2 \gamma\Bigg]$$

$$+\Bigg[\frac{50}{3}\left(\frac{G_{xy}}{G_{xz}}\right) n^2 + \frac{25\pi^2}{18\bar{\nu}}\left(\frac{E_x}{G_{xz}}\right)\left(\frac{b}{a}\right)^2 n^4 - \frac{5\pi^2}{3}\left(\frac{G_{xy}}{G_{xz}}\right)\left(\frac{E_x}{G_{xz}}\right)\left(\frac{h}{a}\right)^2 n^4$$

$$- \frac{5\pi^4}{18\bar{\nu}}\left(\frac{E_x}{G_{xz}}\right)^2\left(\frac{h}{a}\right)^2\left(\frac{b}{a}\right)^2 n^6 + \frac{\pi^4}{24}\left(\frac{G_{xy}}{G_{xz}}\right)\left(\frac{E_x}{G_{xz}}\right)^2\left(\frac{h}{a}\right)^4 n^6$$

$$+ \frac{\pi^6}{72\bar{\nu}}\left(\frac{E_x}{G_{xz}}\right)^3\left(\frac{h}{a}\right)^4\left(\frac{b}{a}\right)^2 n^8 + \frac{5\pi^4}{36}\left(\frac{E_x}{G_{xz}}\right)^2\left(\frac{h}{a}\right)^2\left(\frac{b}{a}\right)^2 n^6\Bigg] \qquad (2.4)$$

If equation (2.4) is solved, it will yield the critical buckling load for an orthotropic, homogeneous plate including the effects of transverse shear deformation for the boundary conditions described.

Before obtaining specific results from equation (2.4) it is desirable to derive the equation for the buckling load of an orthotropic plate neglecting transverse shear deformation (classical buckling load). This is accomplished by substituting equation (2.1) into equation (1.10), the potential energy function for a plate neglecting transverse shear deformation. With $N_y = 0$, the resulting expression is:

$$N_{x_{classical}} = \frac{E_x}{G_{xz}} \frac{\pi^4}{12\bar{\nu}} \left(\frac{h}{a}\right)^2 n^2 + \frac{G_{xy}}{G_{xz}} \left(\frac{h}{a}\right)^2 \left(\frac{a}{b}\right)^2 \pi^2 \tag{2.5}$$

Since one is only interested in the lowest buckling load, inspection of equation (2.5) yields the desired result when $n = 1$. There is also no reason to suspect that including transverse shear deformation will change this fact. Therefore, let $n = 1$ in equation (2.4). The resulting equations can then be compared to determine the effects of transverse shear deformation on the elastic stability for plates or different material properties, aspect ratios, and b/a ratios.

A composite material plate in which the fibers are in the x-direction is chosen for numerical study. This lends itself to simplicity because $G_{xy} = G_{xz}$. Furthermore, one need only to evaluate the critical buckling load with N_x acting (i.e. $\gamma = 0$). Due to the simplicity of the problem described as compared to the other cases to be evaluated, the buckling load will be derived for varying aspect ratios a/b at each of three values of E_x/G_{xz}. These numerical results will be repeated for different ratios of h/a. The limits for these variables are given by following inequalities:

$$2 < E_x/G_{xz} < 60$$

$$1/3 < a/b < 3$$

$$\frac{1}{100} < h/a < \frac{1}{10}$$

Note that the limits on the ratio, E_x/G_{xz}, cover most of the materials in which the effects of transverse shear deformation are important.

Results are shown in Figures 2a, 2b, 2c, 3a, 3b, and 3c.

E. THE ELASTIC STABILITY OF SPECIALLY ORTHOTROPIC PLATES WITH THE Y EDGES CLAMPED AND THE X EDGES SIMPLY SUPPORTED

The next specially orthotropic plate to be evaluated is one in which the Y edges are clamped and the X edges are simply supported. A suitable estimate for the deflection - and one which satisfies all the boundary conditions at the edges - is given by equation (3.1).

$$w(x,y) = A_{mn} \sin \frac{m\pi x}{a}\left(1-\cos \frac{2n\pi y}{b}\right) \qquad (3.1)$$

The appropriate derivatives are given by equations (3.2). Following the same procedure as outlined in the previous problem and with only N_x acting, equations (3.2) and (3.3) result. They are respectively the elastic stability equations including and neglecting the effects of transverse shear deformation for the defined set of boundary conditions (Note that the equations have been non-dimensionalized by again dividing through by $G_{xz}h$).

$$0 = \bar{N}_x^2 \left[\frac{3\pi^2}{500}\left(\frac{G_{xy}}{G_{xz}}\right)\left(\frac{E_x}{G_{xz}}\right)^2\left(\frac{a}{b}\right)^2 n^2 + \frac{9\pi^2}{2000\,\bar{\nu}}\left(\frac{E_x}{G_{xz}}\right)^3 m^2 + \frac{9}{200}\left(\frac{E_x}{G_{xz}}\right)^2\left(\frac{a}{h}\right)^2\right]$$

-- CONT'D

$$+\bar{N}_x\Bigg[-\frac{15}{4\pi^2}\left(\frac{a}{h}\right)^6\frac{1}{m^4} + \frac{\pi^2}{5}\left(\frac{G_{xy}}{G_{xz}}\right)\left(\frac{E_x}{G_{xz}}\right)\left(\frac{a}{b}\right)^2\frac{n^2}{m^2} - \frac{\pi^4}{100}\left(\frac{G_{xy}}{G_{xz}}\right)\left(\frac{E_x}{G_{xz}}\right)^2\left(\frac{a}{b}\right)^2 n^2$$

$$-\frac{\pi^4}{25}\left(\frac{G_{xy}}{G_{xz}}\right)\left(\frac{E_x}{G_{xz}}\right)\left(\frac{E_y}{G_{yz}}\right)\left(\frac{a}{b}\right)^4\frac{n^4}{m^2} - \frac{3\pi^4}{400\bar{\nu}}\left(\frac{E_x}{G_{xz}}\right)^3 m^2 + \frac{3\pi^2}{40\bar{\nu}}\left(\frac{E_x}{G_{xz}}\right)^2\left(\frac{a}{h}\right)^2(1-\bar{\nu})$$

$$-\frac{\pi^4}{25\bar{\nu}}\nu_{yx}\left(\frac{E_x}{G_{xz}}\right)^2\left(\frac{E_y}{G_{yz}}\right)\left(\frac{a}{b}\right)^4\frac{n^4}{m^2} + \frac{\pi^2}{10\bar{\nu}}\nu_{yx}\left(\frac{E_x}{G_{xz}}\right)^2\left(\frac{a}{b}\right)^2\left(\frac{a}{h}\right)^2\frac{n^2}{m^2}\Bigg] + \Bigg[\frac{5\pi^2}{3}\left(\frac{G_{xy}}{G_{xz}}\right)\left(\frac{a}{b}\right)^2\left(\frac{a}{h}\right)^4\frac{n^2}{m^4}$$

$$-\frac{\pi^4}{6}\left(\frac{G_{xy}}{G_{xz}}\right)\left(\frac{E_x}{G_{xz}}\right)\left(\frac{a}{b}\right)^2\left(\frac{a}{h}\right)^2\frac{n^2}{m^2} - \frac{2\pi^4}{3}\left(\frac{G_{xy}}{G_{xz}}\right)\left(\frac{E_y}{G_{yz}}\right)\left(\frac{a}{b}\right)^4\left(\frac{a}{h}\right)^2\frac{n^4}{m^4} + \frac{\pi^6}{240}\left(\frac{G_{xy}}{G_{xz}}\right)\left(\frac{E_x}{G_{xz}}\right)^2\left(\frac{a}{b}\right)^2 n^2$$

$$+\frac{\pi^6}{30}\left(\frac{G_{xy}}{G_{xz}}\right)\left(\frac{E_x}{G_{xz}}\right)\left(\frac{E_y}{G_{yz}}\right)\left(\frac{a}{b}\right)^4\frac{n^4}{m^2} + \frac{\pi^6}{15}\left(\frac{G_{xy}}{G_{xz}}\right)\left(\frac{E_y}{G_{yz}}\right)^2\left(\frac{a}{b}\right)^6\frac{n^6}{m^4} + \frac{\pi^6}{320\bar{\nu}}\left(\frac{E_x}{G_{xz}}\right)^3 m^2 - \frac{\pi^4}{16\bar{\nu}}\left(\frac{E_x}{G_{xz}}\right)^2\left(\frac{a}{h}\right)^2$$

$$+\frac{5\pi^2}{16\bar{\nu}}\left(\frac{E_x}{G_{xz}}\right)\left(\frac{a}{h}\right)^4\frac{1}{m^2} + \frac{\pi^6}{30\bar{\nu}}\nu_{yx}\left(\frac{E_x}{G_{xz}}\right)^2\left(\frac{E_y}{G_{yz}}\right)\left(\frac{a}{b}\right)^4\frac{n^4}{m^2} - \frac{\pi^4}{12\bar{\nu}}\nu_{yx}\left(\frac{E_x}{G_{xz}}\right)^2\left(\frac{a}{b}\right)^2\left(\frac{a}{h}\right)^2\frac{n^2}{m^2}$$

$$-\frac{\pi^4}{3\bar{\nu}}\nu_{yx}\left(\frac{E_x}{G_{xz}}\right)\left(\frac{E_y}{G_{yz}}\right)\left(\frac{a}{b}\right)^4\left(\frac{a}{h}\right)^2\frac{n^4}{m^4} + \frac{5\pi^2}{6\bar{\nu}}\nu_{yx}\left(\frac{E_x}{G_{xz}}\right)\left(\frac{a}{b}\right)^2\left(\frac{a}{h}\right)^4\frac{n^2}{m^4} + \frac{\pi^4}{32}\left(\frac{E_x}{G_{xz}}\right)^2\left(\frac{a}{h}\right)^2$$

$$+\frac{4\pi^6}{15\bar{\nu}}\frac{\nu_{yx}}{\nu_{xy}}\left(\frac{E_x}{G_{xz}}\right)\left(\frac{E_y}{G_{yz}}\right)^2\left(\frac{a}{b}\right)^8\frac{n^8}{m^6} - \frac{4\pi^4}{3\bar{\nu}}\frac{\nu_{yx}}{\nu_{xy}}\left(\frac{E_x}{G_{xz}}\right)\left(\frac{E_y}{G_{yz}}\right)\left(\frac{a}{b}\right)^6\left(\frac{a}{h}\right)^2\frac{1}{m^6}$$

$$+\frac{5\pi^2}{3\bar{\nu}}\frac{\nu_{yx}}{\nu_{xy}}\left(\frac{E_x}{G_{xz}}\right)\left(\frac{a}{b}\right)^4\left(\frac{a}{h}\right)^4\frac{n^4}{m^6} + \frac{2\pi^4}{3}\frac{\nu_{yx}}{\nu_{xy}}\left(\frac{E_x}{G_{xz}}\right)\left(\frac{E_y}{G_{yz}}\right)\left(\frac{a}{b}\right)^6\left(\frac{a}{h}\right)^2\frac{n^6}{m^6}\Bigg] \qquad (3.2)$$

And neglecting transverse shear deformation,

$$\overline{N}_{x_{classical}} = \frac{\pi^4}{12\overline{\nu}} \left(\frac{E_x}{G_{xz}}\right) \left(\frac{h}{a}\right)^2 \left[m^2 + \frac{8}{3} \nu_{yx} \left(\frac{a}{b}\right)^2 n^2 + \frac{16}{3} \frac{\nu_{yx}}{\nu_{xy}} \left(\frac{a}{b}\right)^4 \frac{n^4}{m^2}\right]$$

$$+ \frac{4\pi^4}{9} \left(\frac{G_{xy}}{G_{xz}}\right) \left(\frac{h}{a}\right)^2 \left(\frac{a}{b}\right)^2 n2 \qquad (3.3)$$

By analyzing equation (3.3), it is clear that the minimum buckling load occurs when $n = 1$. This will also be the case for the more complex equation (3.2). The numerical analysis, then, will be carried out for the $n = 1$ case in both equations since the minimum buckling load is the only one of interest. However, nothing can yet be said about the value of m since it occurs in the numerator and denominator of both equations.

The second order equation is very complicated and will only be solved for a specific material whose properties are shown in equations (3.4). This material is typical of many unidirectional filamentary lamina in which the effects of transverse shear deformation are important.

$$\begin{aligned} E_x &= 32.5 \times 10^6 \text{ psi} \\ E_y &= 1.84 \times 10^6 \text{ psi} \qquad (3.4) \\ G_{xy} = G_{xz} &= .642 \times 10^6 \text{ psi} \end{aligned}$$

$$G_{yz} = .361 \times 10^6 \text{ psi}$$

$$\nu_{xy} = .256$$

$$\nu_{yx} = .0146$$

These material properties will be substituted into equations (3.2) and (3.3) and the buckling envelope of plate will be determined for m=1,2,3 at each of four values for $\frac{h}{a}(\frac{h}{a} = \frac{1}{10}, \frac{1}{25}, \frac{1}{50}, \frac{1}{100})$. The numerical results are presented in figures 4a, 4b, 4c, 4d, 5a, 5b, 5c, and 5d.

F. THE ELASTIC STABILITY OF SPECIALLY ORTHOTROPIC PLATES WITH ALL FOUR EDGES SIMPLY SUPPORTED

The third set of boundary conditions which will be evaluated is that of the plate simply supported on all edges. This case has been solved previously but not for the approximations of $\alpha(x,y)$ and $\beta(x,y)$ as formulated herein. The method of arriving at the equations is the same as for the previous two cases. A suitable selection for the deflection is given by equation (4.1).

$$w(x,y) = A_{mn} \sin\frac{m\pi x}{a} \sin\frac{n\pi y}{b} \qquad (4.1)$$

As was previously noted, the procedure is the same and the resulting non-dimensionalized equation for the buckling load of an orthotropic plate simply supported on all edges, with only N_x acting, and including transverse shear deformation is given equation (4.2).

$$0 = \bar{N}_X^2 \left[\frac{3\pi^2}{50n} \left(\frac{G_{xy}}{G_{yz}}\right) \left(\frac{E_x}{G_{xz}}\right)^2 \left(\frac{a}{b}\right)^2 n^2 + \frac{9\pi^2}{1500\,\bar{D}} \left(\frac{E_x}{G_{yz}}\right)^3 m^2 + \frac{3}{50} \left(\frac{E_x}{G_{yz}}\right)^2 \left(\frac{a}{h}\right)^2 \right]$$

-- CONT'D

$$+\bar{N}_x\left[-\frac{5}{\pi^2}\left(\frac{a}{h}\right)^6\frac{1}{m^4}+\frac{\pi^2}{5}\left(\frac{G_{xy}}{G_{xz}}\right)\left(\frac{E_x}{G_{xz}}\right)\left(\frac{a}{b}\right)^2\left(\frac{a}{h}\right)^2\frac{n^2}{m^2}-\frac{\pi^4}{100}\left(\frac{G_{xy}}{G_{xz}}\right)\left(\frac{E_x}{G_{xz}}\right)^2\left(\frac{a}{b}\right)^2 n^2\right.$$

$$-\frac{\pi^4}{100}\left(\frac{G_{xy}}{G_{xz}}\right)\left(\frac{E_x}{G_{xz}}\right)\left(\frac{E_y}{G_{yz}}\right)\left(\frac{a}{b}\right)^4\frac{n^4}{m^2}-\frac{\pi^4}{100\bar{\nu}}\left(\frac{E_x}{G_{xz}}\right)^3 m^2+\frac{\pi^2}{10\bar{\nu}}\left(\frac{E_x}{G_{xz}}\right)^2\left(\frac{a}{h}\right)^2$$

$$\left.-\frac{\pi^4}{100\bar{\nu}}\nu_{yx}\left(\frac{E_y}{G_{yz}}\right)\left(\frac{E_x}{G_{xz}}\right)\left(\frac{a}{b}\right)^4\frac{n^4}{m^2}+\frac{\pi^2}{10\bar{\nu}}\nu_{yx}\left(\frac{E_x}{G_{xz}}\right)^2\left(\frac{a}{b}\right)^2\left(\frac{a}{h}\right)^2\frac{n^2}{m^2}-\frac{\pi^2}{10}\left(\frac{E_x}{G_{xz}}\right)^2\left(\frac{a}{h}\right)^2\right]$$

$$+\left[\frac{5\pi^2}{3}\left(\frac{G_{xy}}{G_{xz}}\right)\left(\frac{a}{b}\right)^2\left(\frac{a}{h}\right)^4\frac{n^2}{m^4}-\frac{\pi^4}{6}\left(\frac{G_{xy}}{G_{xz}}\right)\left(\frac{E_x}{G_{xz}}\right)\left(\frac{a}{b}\right)^2\left(\frac{a}{h}\right)^2\frac{n^2}{m^2}\right.$$

$$-\frac{\pi^4}{6}\left(\frac{G_{xy}}{G_{xz}}\right)\left(\frac{E_y}{G_{yz}}\right)\left(\frac{a}{b}\right)^4\left(\frac{a}{h}\right)^2\frac{n^4}{m^4}+\frac{\pi^6}{240}\left(\frac{G_{xy}}{G_{xz}}\right)\left(\frac{E_x}{G_{xz}}\right)^2\left(\frac{a}{b}\right)^2 n^2$$

$$+\frac{\pi^6}{120}\left(\frac{G_{xy}}{G_{xz}}\right)\left(\frac{E_x}{G_{xz}}\right)\left(\frac{E_y}{G_{yz}}\right)\left(\frac{a}{b}\right)^4\frac{n^4}{m^2}+\frac{\pi^6}{240}\left(\frac{G_{xy}}{G_{xz}}\right)\left(\frac{E_y}{G_{yz}}\right)^2\left(\frac{a}{b}\right)^6\frac{n^6}{m^4}$$

$$+\frac{\pi^6}{240\bar{\nu}}\left(\frac{E_x}{G_{xz}}\right)^3 m^2-\frac{\pi^4}{12\bar{\nu}}\left(\frac{E_x}{G_{xz}}\right)^2\left(\frac{a}{h}\right)^2+\frac{5\pi^2}{12\bar{\nu}}\left(\frac{E_x}{G_{xz}}\right)\left(\frac{a}{h}\right)^4\frac{1}{m^2}$$

-- CONT'D

$$+\frac{\pi^6}{120\bar{\nu}}\nu_{yx}\left(\frac{E_x}{G_{xz}}\right)^2\left(\frac{E_y}{G_{yz}}\right)\left(\frac{a}{b}\right)^4\frac{n^4}{m^2}-\frac{\pi^4}{12\bar{\nu}}\nu_{yx}\left(\frac{E_x}{G_{xz}}\right)^2\left(\frac{a}{b}\right)^2\left(\frac{a}{h}\right)^2\frac{n^2}{m^2}$$

$$-\frac{\pi^4}{12\bar{\nu}}\nu_{yx}\left(\frac{E_x}{G_{xz}}\right)\left(\frac{E_y}{G_{yz}}\right)\left(\frac{a}{b}\right)^4\left(\frac{a}{h}\right)^2\frac{n^4}{m^4}+\frac{5\pi^2}{6\bar{\nu}}\nu_{yx}\left(\frac{E_x}{G_{xz}}\right)\left(\frac{a}{b}\right)^2\left(\frac{a}{h}\right)^4\frac{n^2}{m^4}$$

$$+\frac{\pi^6}{240\bar{\nu}}\frac{\nu_{yx}}{\nu_{xy}}\left(\frac{E_x}{G_{xz}}\right)\left(\frac{E_y}{G_{yz}}\right)^2\left(\frac{a}{b}\right)^8\frac{n^8}{m^6}-\frac{\pi^4}{12\bar{\nu}}\frac{\nu_{yx}}{\nu_{xy}}\left(\frac{E_x}{G_{xz}}\right)\left(\frac{E_y}{G_{yz}}\right)\left(\frac{a}{b}\right)^6\left(\frac{a}{h}\right)^2\frac{n^6}{m^6}$$

$$+\frac{5\pi^2}{12\bar{\nu}}\frac{\nu_{yx}}{\nu_{xy}}\left(\frac{E_x}{G_{xz}}\right)\left(\frac{a}{b}\right)^4\left(\frac{a}{h}\right)^4\frac{n^4}{m^6}+\frac{\pi^4}{24}\left(\frac{E_x}{G_{xz}}\right)^2\left(\frac{a}{h}\right)^2$$

$$+\frac{\pi^4}{24}\frac{\nu_{yx}}{\nu_{xy}}\left(\frac{E_x}{G_{xz}}\right)\left(\frac{E_y}{G_{yz}}\right)\left(\frac{a}{b}\right)^6\left(\frac{a}{h}\right)^2\frac{n^6}{m^6}\Bigg] \qquad (4.2)$$

Similarly, the equation for the buckling load of the same plate neglecting transverse shear deformation is given by equation (4. 3).

$$\bar{N}_{x\,\text{classical}} = \frac{\pi^4}{12\bar{\nu}}\left(\frac{E_x}{G_{xz}}\right)\left(\frac{h}{a}\right)^2\left[m^2 + 2\nu_{yx}\left(\frac{a}{b}\right)^2 n^2 + \frac{\nu_{yx}}{\nu_{xy}}\left(\frac{a}{b}\right)^4\frac{n^4}{m^2}\right] + \frac{\pi^4}{3}\left(\frac{G_{xy}}{G_{xz}}\right)\left(\frac{h}{a}\right)^2\left(\frac{a}{b}\right)^2 \qquad (4.3)$$

From equation (4.3), it is seen that the critical or minimum buckling load occurs when $n = 1$. Including transverse shear deformation should not affect this result, and for that reason, in the numerical example $n = 1$ is used in equation (4.2). Again, one cannot a priori state what value of m will produce the minimum buckling load since it is a function of the material properties.

Equations (4.2) and (4.3) are compared using the numerical example as detailed at the end of section E. The results are shown in figures 6a, 6b, 6c, 6d, 7a, 7b, 7c, and 7d.

G. THE ELASTIC STABILITY OF SPECIALLY ORTHOTROPIC PLATES WITH X EDGES AND ONE Y EDGE SIMPLY SUPPORTED AND THE OTHER Y EDGE CLAMPED

The last set of boundary conditions which will be evaluated is the case where both x edges and one y edge are simply supported and the other y edge is clamped. A suitable estimate for the lateral deflection is given by equation (5.1)

$$w(x,y) = p \sin \frac{m\pi x}{a} (yb^3 - 3y^3 b + 2y^4) \qquad (5.1)$$

Following the same procedure as detailed in the previous problems, the resulting non-dimensionalized equation for the buckling load of an orthotropic plate simply supported on both x edges and one y edge and clamped on the other y edge, with only N_x acting and including transverse shear deformation is given by (5.2).

$$0 = \bar{N}_x^2 \Big[.001 \left(\frac{G_{xy}}{G_{yz}}\right)\left(\frac{E_x}{G_{xz}}\right)^2 \left(\frac{h}{a}\right)^2 \left(\frac{a}{b}\right)^2 + \frac{.00087}{\bar{D}} \left(\frac{E_y}{G_{xz}}\right)^3 \left(\frac{h}{a}\right)^2 m^2$$

-- CONT'D

$$+.0085\left(\frac{E_x}{G_{xz}}\right)^2\left(\frac{h}{a}\right)^4\Bigg] + \bar{N}_x\Bigg[-.0075\left(\frac{a}{h}\right)^4\frac{1}{m^4} + .033\left(\frac{G_{xy}}{G_{xz}}\right)\left(\frac{E_x}{G_{xz}}\right)\left(\frac{a}{b}\right)^2\frac{1}{m^2}$$

$$-.0165\left(\frac{G_{xy}}{G_{xz}}\right)\left(\frac{E_x}{G_{xz}}\right)^2\left(\frac{h}{a}\right)^2\left(\frac{a}{b}\right)^2 - .0345\left(\frac{G_{xy}}{G_{xz}}\right)\left(\frac{E_x}{G_{xz}}\right)\left(\frac{E_y}{G_{yz}}\right)\left(\frac{h}{a}\right)^2\left(\frac{a}{b}\right)^4\frac{1}{m^2}$$

$$-\frac{.0143}{\bar{\nu}}\left(\frac{E_x}{G_{xz}}\right)^3\left(\frac{h}{a}\right)^2 m^2 + \frac{.0144}{\bar{\nu}}\left(\frac{E_x}{G_{xz}}\right)^2 - \frac{.0348}{\bar{\nu}}\left(\frac{E_x}{G_{xz}}\right)^2\left(\frac{E_y}{G_{yz}}\right)\nu_{yx}\left(\frac{h}{a}\right)^2\left(\frac{a}{b}\right)^4\frac{1}{m^2}$$

$$+\frac{.0168}{\bar{\nu}}\left(\frac{E_x}{G_{xz}}\right)^2\nu_{yx}\left(\frac{a}{b}\right)^2\frac{1}{m^2} - .0139\left(\frac{E_x}{G_{xz}}\right)^2\Bigg].$$

$$+\Bigg[.276\left(\frac{G_{xy}}{G_{xz}}\right)\left(\frac{a}{b}\right)^2\left(\frac{a}{h}\right)^2\frac{1}{m^4} - .272\left(\frac{G_{xy}}{G_{xz}}\right)\left(\frac{E_x}{G_{xz}}\right)\left(\frac{a}{b}\right)^2\frac{1}{m^2}$$

$$-.59\left(\frac{G_{xy}}{G_{xz}}\right)\left(\frac{E_y}{G_{yz}}\right)\left(\frac{a}{b}\right)^4\frac{1}{m^4} + .068\left(\frac{G_{xy}}{G_{xz}}\right)\left(\frac{E_x}{G_{xz}}\right)^2\left(\frac{h}{a}\right)^2\left(\frac{a}{b}\right)^2$$

$$+.29\left(\frac{G_{xy}}{G_{xz}}\right)\left(\frac{E_x}{G_{xz}}\right)\left(\frac{E_y}{G_{yz}}\right)\left(\frac{h}{a}\right)^2\left(\frac{a}{b}\right)^4\frac{1}{m^2} + .468\left(\frac{G_{xy}}{G_{xz}}\right)\left(\frac{E_y}{G_{yz}}\right)^2\left(\frac{h}{a}\right)^2\left(\frac{a}{b}\right)^6\frac{1}{m^4}$$

--CONT'D

$$- \frac{.0585}{\bar{\nu}} \left(\frac{E_x}{G_{xz}}\right)^3 \left(\frac{h}{a}\right)^2 m^2 - \frac{.118}{\bar{\nu}}\left(\frac{E_x}{G_{xz}}\right)^2 + \frac{.06}{\bar{\nu}}\left(\frac{E_x}{G_{xz}}\right)\left(\frac{a}{h}\right)^2 \frac{1}{m^2}$$

$$+ \frac{.285}{\bar{\nu}}\left(\frac{E_x}{G_{xz}}\right)^2\left(\frac{E_y}{G_{yz}}\right)\nu_{yx}\left(\frac{h}{a}\right)^2\left(\frac{a}{b}\right)^4\frac{1}{m^2} - \frac{.138}{\bar{\nu}}\nu_{yx}\left(\frac{E_x}{G_{xz}}\right)^2\left(\frac{a}{b}\right)^2\frac{1}{m^2}$$

$$-\frac{.3}{\bar{\nu}}\nu_{yx}\left(\frac{E_x}{G_{xz}}\right)\left(\frac{E_y}{G_{yz}}\right)\left(\frac{a}{b}\right)^4\frac{1}{m^4} + \frac{.14}{\bar{\nu}}\nu_{yx}\left(\frac{E_x}{G_{xz}}\right)\left(\frac{a}{b}\right)^2\left(\frac{a}{h}\right)^2\frac{1}{m^4} + .057\left(\frac{E_x}{G_{xz}}\right)^2$$

$$+\frac{.48}{\bar{\nu}}\frac{\nu_{yx}}{\nu_{xy}}\left(\frac{E_x}{G_{xz}}\right)\left(\frac{E_y}{G_{yz}}\right)^2\left(\frac{h}{a}\right)^2\left(\frac{a}{b}\right)^8\frac{1}{m^6} - \frac{.2}{\bar{\nu}}\frac{\nu_{yx}}{\nu_{xy}}\left(\frac{E_x}{G_{xz}}\right)\left(\frac{E_y}{G_{yz}}\right)\left(\frac{a}{b}\right)^6\frac{1}{m^6}$$

$$+\frac{.15}{\bar{\nu}}\frac{\nu_{yx}}{\nu_{xy}}\left(\frac{E_x}{G_{xz}}\right)\left(\frac{a}{h}\right)^2\left(\frac{a}{b}\right)^4\frac{1}{m^6} + .475\frac{\nu_{yx}}{\nu_{xy}}\left(\frac{E_x}{G_{xz}}\right)\left(\frac{E_y}{G_{yz}}\right)\left(\frac{a}{b}\right)^6\frac{1}{m^6} \qquad (5.2)$$

Similarly, the equation for the buckling load of the same plate neglecting transverse shear deformation is given by equation (5.3)

$$\bar{N}_{x_{cl}} = \frac{8.1}{\bar{\nu}}\left(\frac{E_x}{G_{xz}}\right)\left(\frac{h}{a}\right)^2\left[m^2 + 2.34\nu_{yx}\left(\frac{a}{b}\right)^2 + 2.5\frac{\nu_{yx}}{\nu_{xy}}\left(\frac{a}{b}\right)^4\frac{1}{m^2}\right]$$

-- CONT'D

$$+ 37.4 \frac{G_{xy}}{G_{xz}} \left(\frac{h}{a}\right)^2 \left(\frac{a}{b}\right)^2 \qquad (5.3).$$

Equations (5.3) and (5.4) will be compared using the numerical example as detailed at the end of section E. The results are shown in figures 8a, 8b, 8c, 8d, 9a, 9b, 9c, and 9d.

H. CONCLUSIONS AND RECOMMENDATIONS FOR FURTHER STUDY

The data show that when the effects of transverse shear deformation are neglected, the calculated buckling load will be higher. However, the percent difference varies depending on the thickness of the plate. For thin plates ($\frac{h}{a} = \frac{1}{50}$ and $\frac{h}{a} = \frac{1}{100}$), the effects are minimal (less than 10% in most cases). For moderately thick plates ($\frac{h}{a} = \frac{1}{10}$ and $\frac{h}{a} = \frac{1}{25}$), the effects are much more pronounced, and serious error will result unless transverse shear deformation is included in the analysis.

The above conclusion is very evident when the stability of the buckling load versus the aspect ratio is examined. For moderately thick plates ($\frac{h}{a} = \frac{1}{10}$), the shape of the curve including transverse shear deformation and that neglecting it are similar only in the first mode. As the aspect ratio is increased, the data including transverse shear deformation shows that the plate will buckle in the second mode well before the predicted value using classical thin plate theory. This result is repeated for $\frac{h}{a} = \frac{1}{25}$, but in this case, the envelopes are similar for the 1st and

2nd buckling modes and are not comparable thereafter. As a thin plate configuration is attained ($\frac{h}{a} = \frac{1}{50}$ and $\frac{h}{a} = \frac{1}{100}$), the envelopes are similar for all mode shapes studied, and as the curves imply, neglecting transverse shear deformation will not result in serious error.

It must be pointed out that the preceding conclusions were made for the particular material examined. However, the results will be the same for any moderately thick plate where the effects of transverse shear deformation are important. This is especially true for sandwich type plates.

CHAPTER III

FIGURES

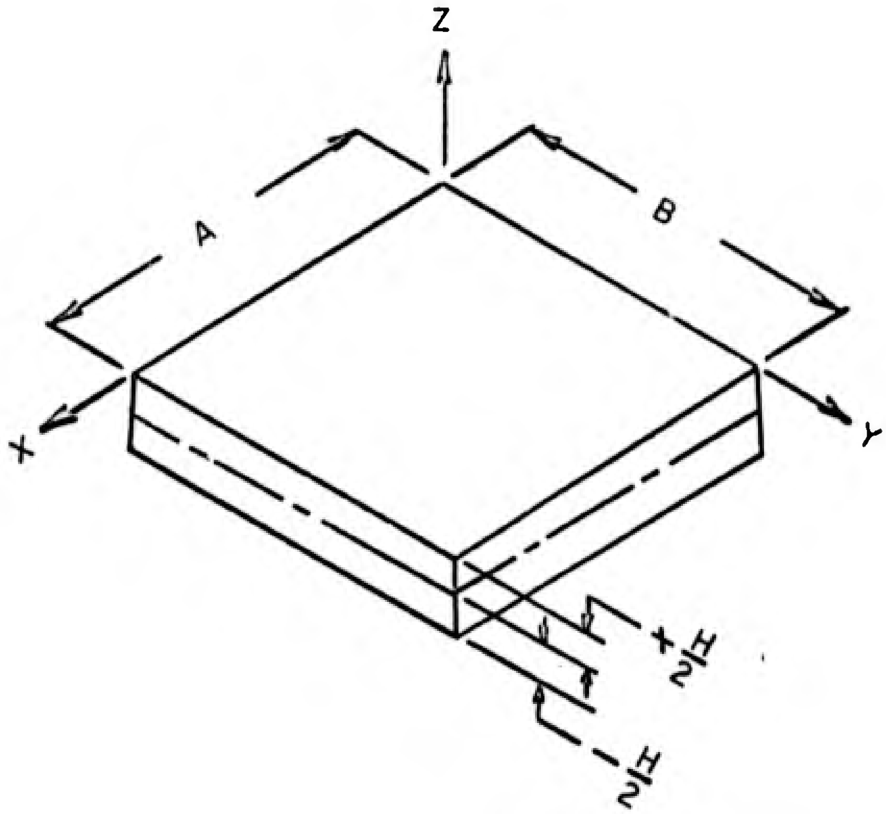

FIGURE I

THREE DIMENSIONAL REPRESENTATION OF THE PLATE

Buckling Load Neglecting T. S. Def. $\bar{N}_{x_{classical}}$	Buckling Load including T. S. Def. $\bar{N}_x$	a/b	E_x/G_{xz}
.173	.171	1/3	2
.260	.256	1	2
1.045	1.028	3	2
1.631	1.380	1/3	20
1.720	1.451	1	20
2.500	2.132	3	20
4.871	3.940	1/3	60
4.958	4.040	1	60
5.750	5.100	3	60

FIGURE 2A

Buckling loads for specially orthotropic plate with x-edges and one y-edge simply supported - other y-edge free

$$\frac{h}{a} = \frac{1}{10}$$

Buckling Load Neglecting T. S. Def. $\bar{N}_{x_{classical}}$	Buckling Load including T. S. Def. $\bar{N}_x$	a/b	E_x/G_{xz}
.00694	.00695	1/3	2
.01042	.0105	1	2
.0417	.0418	3	2
.0654	.0652	1/3	20
.0689	.0687	1	20
.100	.0996	3	20
.1954	.1917	1/3	60
.1989	.1950	1	60
.230	.225	3	60

FIGURE 2B

Buckling loads for specially orthotropic plate with x-edges and one y-edge simply supported and other y-edge free

$$\frac{h}{a} = \frac{1}{50}$$

Buckling Load Neglecting T. S. Def. $\bar{N}_{x_{classical}}$	Buckling Load including T. S. Def. $\bar{N}_x$	a/b	E_x/G_{xz}
.00173	.00180	1/3	2
.00263	.00266	1	2
.01038	.01063	3	2
.01631	.0164	1/3	20
.0172	.0172	1	20
.0250	.0250	3	20
.0487	.0488	1/3	60
.0496	.0496	1	60
.0573	.0574	3	60

FIGURE 2C

Buckling loads for orthotropic plate with x-edges and one y-edge simply supported and other y-edge free

$$\frac{h}{a} = \frac{1}{100}$$

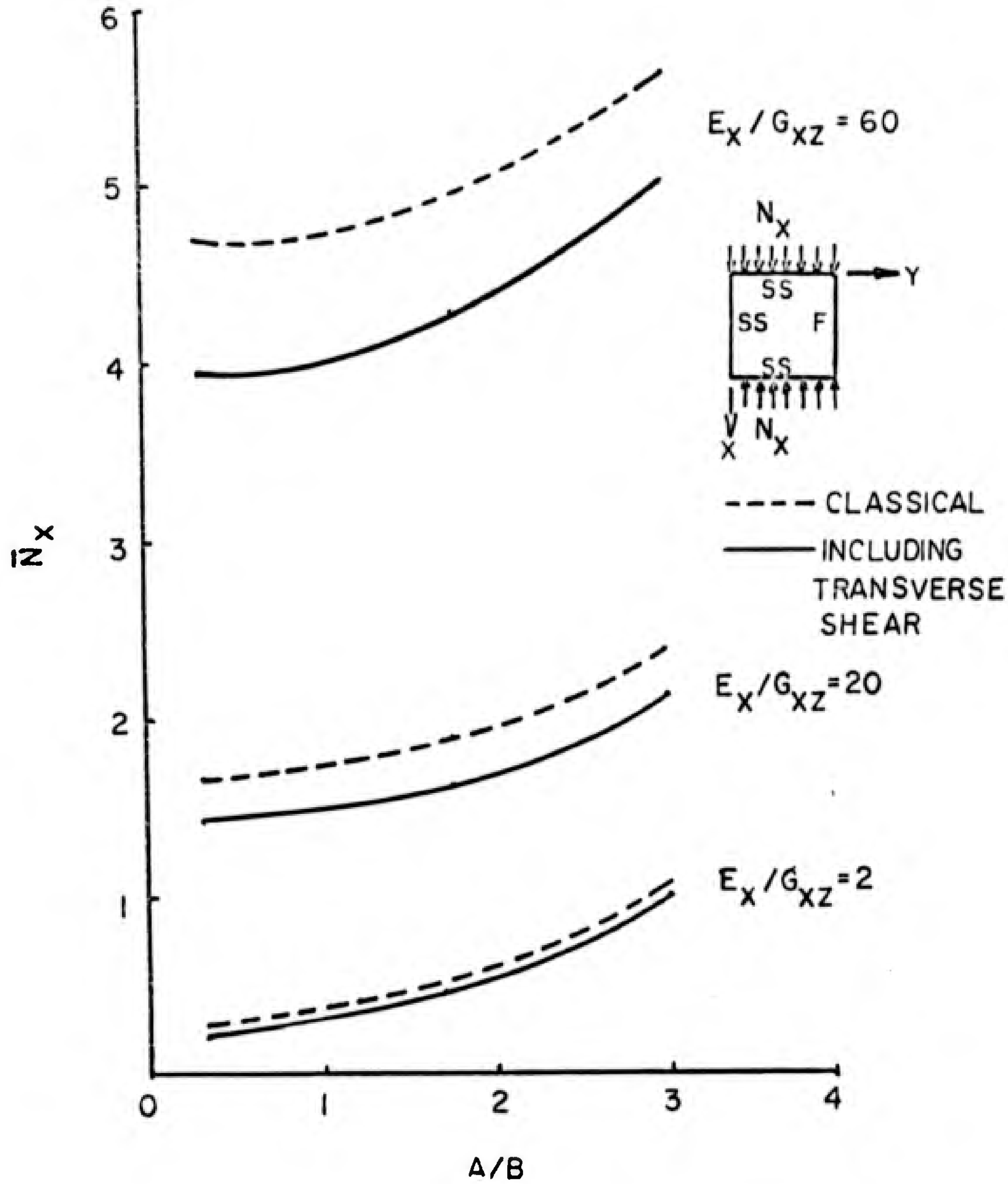

FIGURE 3A

BUCKLING LOAD VS. A/B FOR H/A=1/10

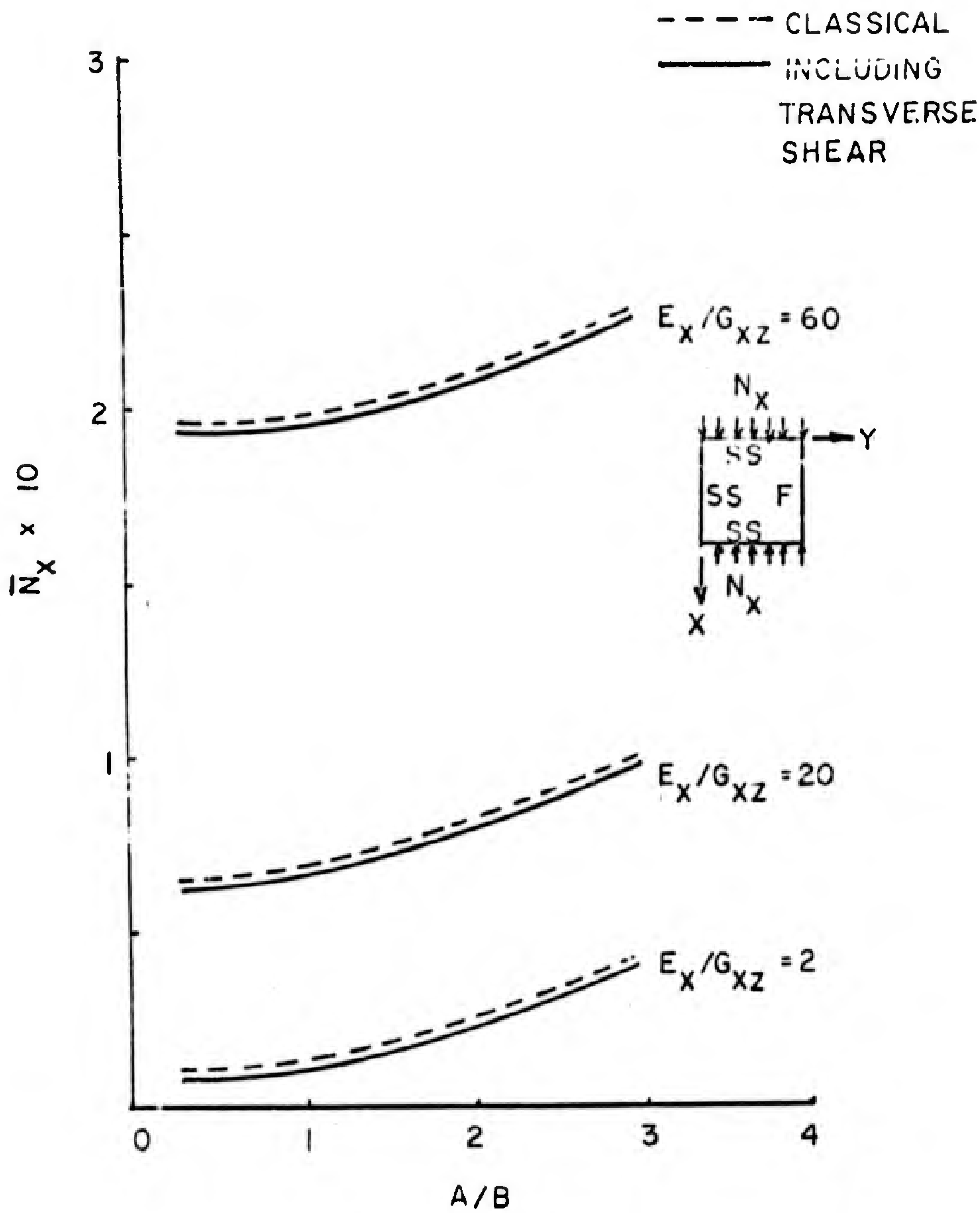

FIGURE 3B

BUCKLING LOAD VS. A/B FOR H/A = 1/50

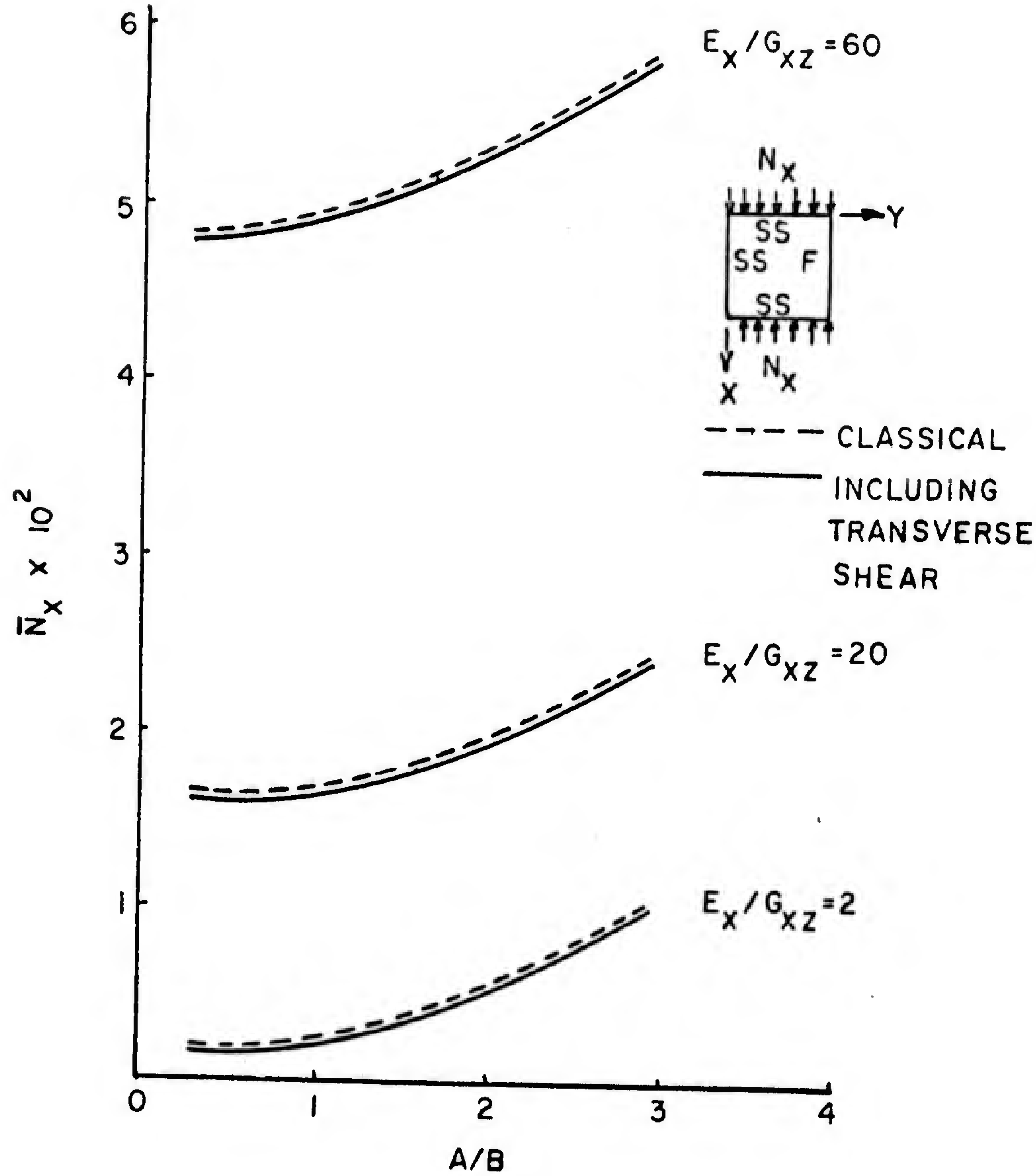

FIGURE 3C

BUCKLING LOAD VS. A/B FOR H/A=1/100

Buckling Load Neglecting T. S. Def. $\overline{N}_{x_{classical}}$	Buckling Load including T. S. Def. $\overline{N}_x$	a/b	m
4.15	2.84	1/3	1
4.32	2.95	1/2	1
5.90	4.30	1	1
8.41	6.60	1.28	1
11.77	11.00	1.50	1
16.47	5.40	1/3	2
–	5.43	1/2	2
–	5.45	1	2
–	5.58	1.28	2
–	5.65	1.50	2
–	5.84	1.75	2
37.63	5.75	1	3
–	5.77	1.28	3
–	5.79	1.50	3
–	5.80	1.75	3
–	5.83	2	3
–	6.55	2.75	3

FIGURE 4a

Buckling Load for specially orthotropic plate (56.1% boron fibers by volume in epoxy matrix) with x-edges simply supported and the y-edges clamped

$$\frac{h}{a} = \frac{1}{10}$$

Buckling Load Neglecting T. S. Def. $\overline{N}_{x_{classical}}$	Buckling Load including T. S. Def. $\overline{N}_x$	a/b	m
.6688	.6233	1/3	1
.6919	.6490	1/2	1
.9497	.8878	1	1
4.2236	3.7641	2	1
17.6561	14.2650	3	1
2.7681	2.1091	1	2
3.7988	2.9783	2	2
5.1604	4.1026	2.5	2
7.5105	6.0227	3	2
8.1292	8.1340	3.1	2
16.8944	15.6564	4	2
6.0205	3.4805	1	3
6.6364	3.9571	2	3
7.3565	4.5944	2.5	3
8.5428	6.0289	3	3
8.8511	6.6599	3.1	3

FIGURE 4b

Buckling load for specially orthotropic plate (56.1% boron fibers by volume in epoxy matrix) with x-edges simply supported and y-edges clamped

$$\frac{h}{a} = \frac{1}{25}$$

Buckling Load Neglecting T. S. Def. $\bar{N}_{x\,classical}$	Buckling Load including T. S. Def. $\bar{N}_x$	a/b	m
.167	.163	1/3	1
.238	.234	1	1
1.058	.978	2	1
4.426	4.110	3	1
13.342	11.755	4	1
.692	.645	1	2
.950	.901	2	2
1.881	1.734	3	2
4.234	3.681	4	2
8.946	7.648	5	2
1.658	1.423	2	3
2.134	1.847	3	3
3.264	2.828	4	3
5.506	4.646	5	3
9.446	8.952	6	3

FIGURE 4c

Buckling load for specially orthotropic plate (56.1% boron fibers by volume in epoxy matrix) with x-edges simply supported and y-edges clamped

$$\frac{h}{a} = \frac{1}{50}$$

Buckling Load Neglecting T. S. Def. $\bar{N}_{x\,classical}$	Buckling Load including T. S. Def. $\bar{N}_x$	a/b	m
.04330	.04320	1/2	1
.05936	.05916	1	1
.2640	.2628	2	1
1.1034	1.0854	3	1
.1730	.1708	1	2
.2374	.2362	2	2
.4694	.4637	3	2
1.0558	1.0296	4	2
2.2584	2.1854	5	2
.5342	.5154	3	3
.8117	.7871	4	3
1.3815	1.3131	5	3
2.3750	2.2176	6	3
3.9811	3.6416	7	3

FIGURE 4d

Buckling Load for specially orthotropic plate (56.1% boron fibers by volume in epoxy matrix) with x-edges simply supported and y-edges clamped

$$\frac{h}{a} = \frac{1}{100}$$

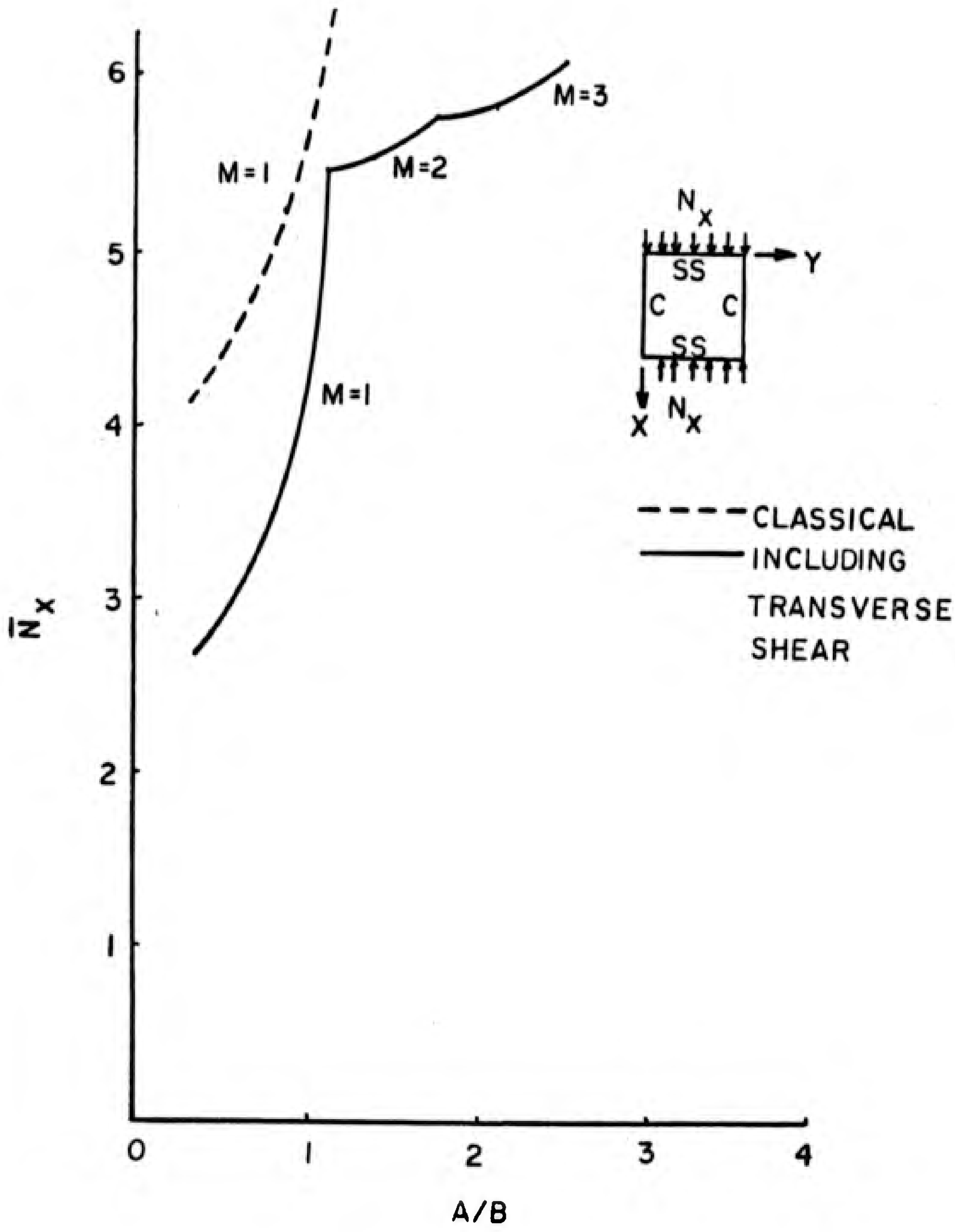

FIGURE 5A

BUCKLING LOAD VS. A/B FOR H/A=1/10

PLATE: 56.1% BORON FIBERS IN EPOXY MATRIX

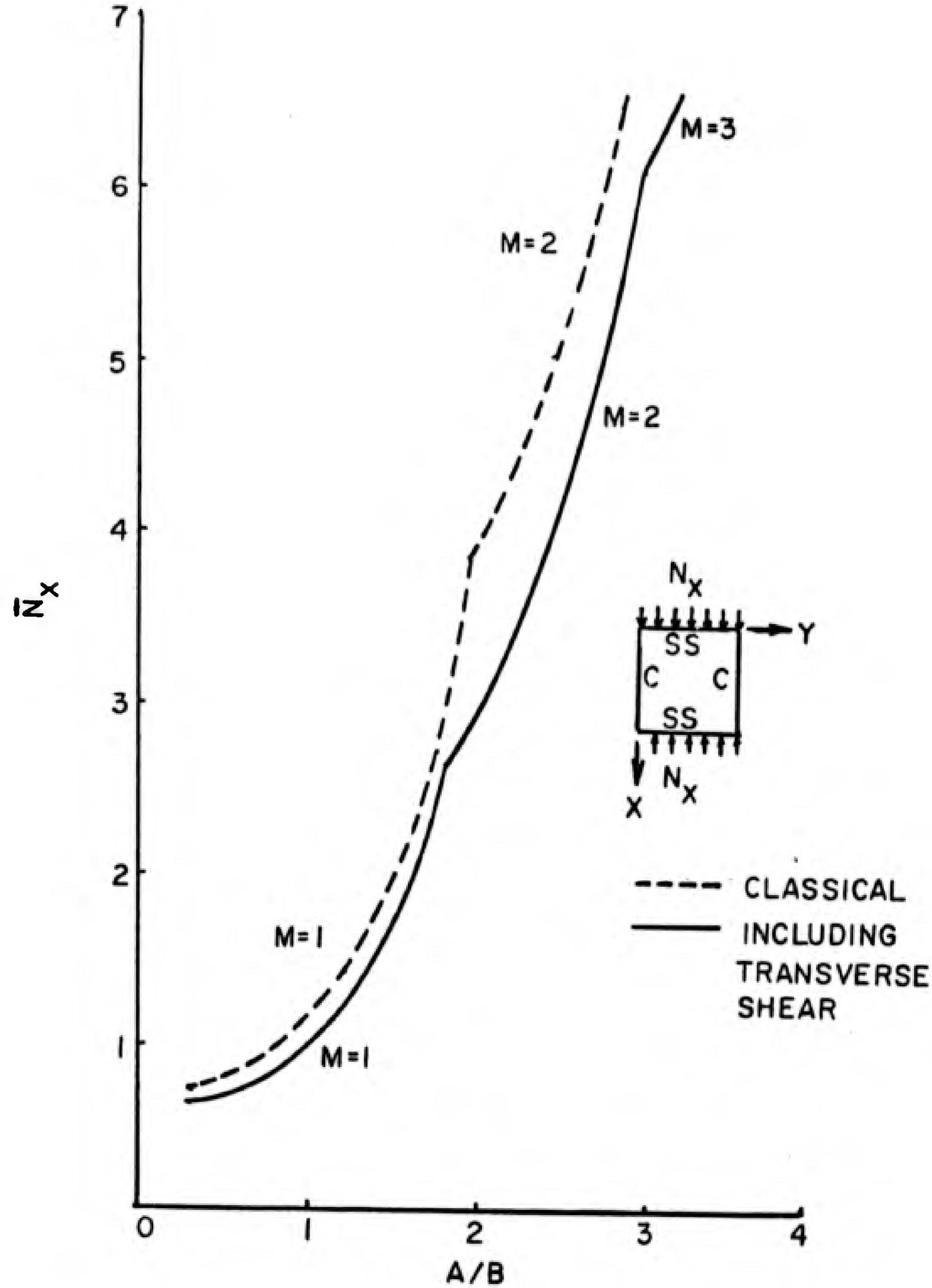

FIGURE 5B

BUCKLING LOAD VS. A/B FOR H/A=1/25

PLATE: 56.1% BORON FIBERS IN EPOXY MATRIX

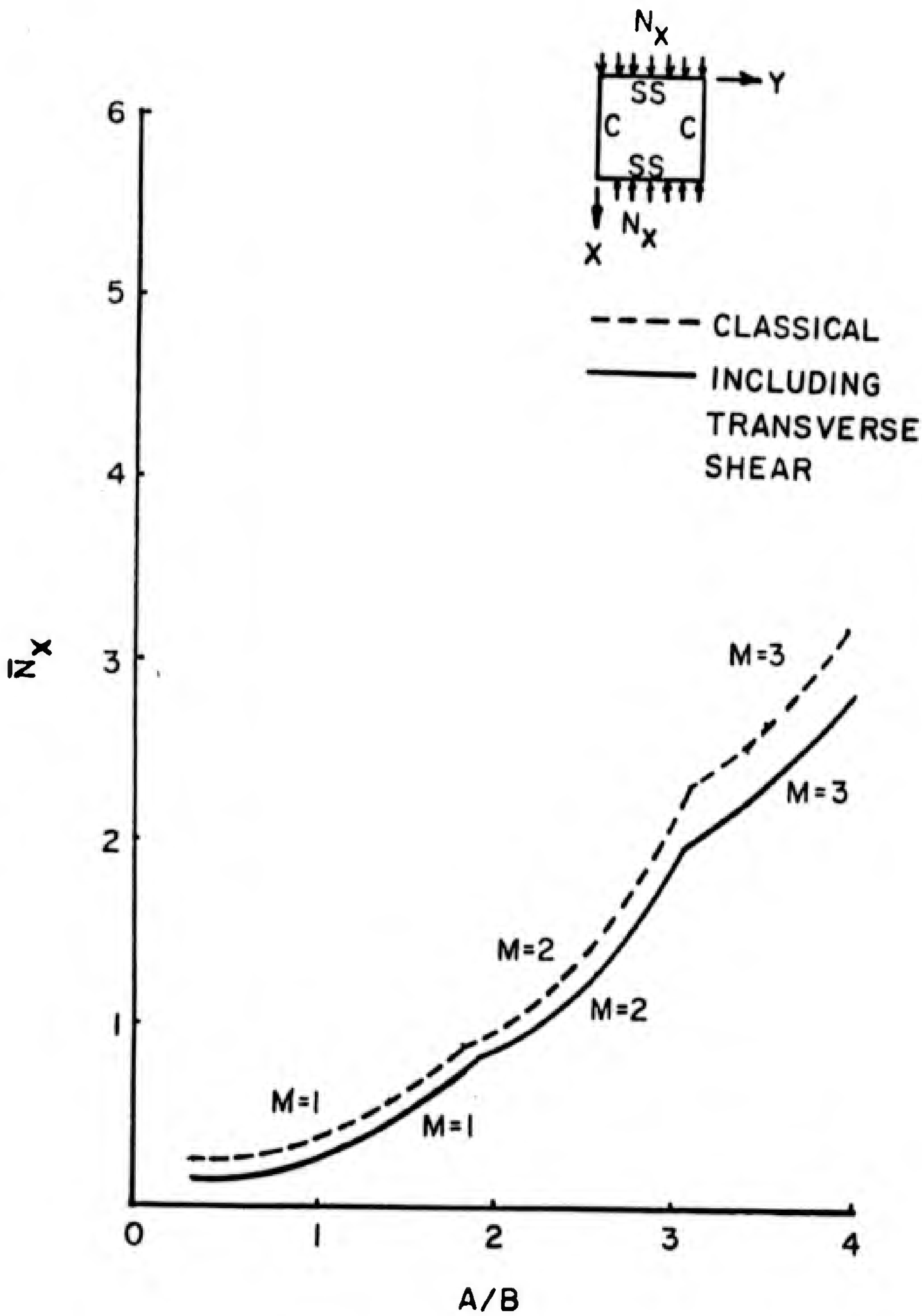

FIGURE 5C

BUCKLING LOAD VS. A/B FOR H/A=1/50

PLATE: 56.1% BORON FIBERS IN EPOXY MATRIX

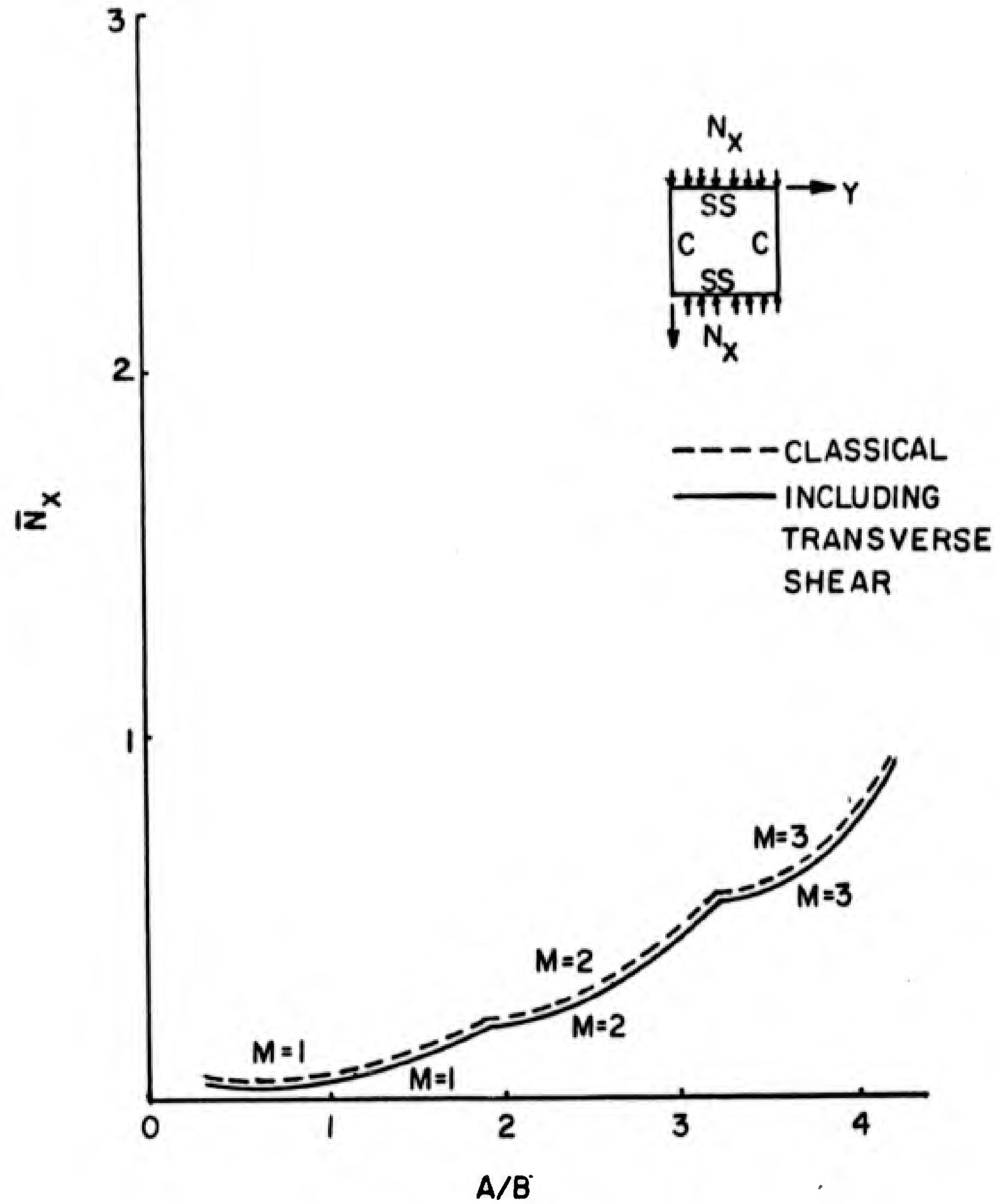

FIGURE 5D

BUCKLING LOAD VS. A/B FOR H/A=1/100

PLATE: 56.1% BORON FIBERS IN EPOXY MATRIX

Buckling Load Neglecting T. S. Def. $\bar{N}_{x_{classical}}$	Buckling Load Including T. S. Def. $\bar{N}_x$	a/b	m
4.15	2.76	1/3	1
4.22	2.80	1/2	1
4.78	3.32	1	1
5.45	3.92	1.28	1
6.28	4.95	1.50	1
9.60	9.20	2	1
–	4.64	1	2
–	4.70	1.28	2
–	4.73	1.50	2
–	5.08	2.00	2
–	5.35	2.25	2
–	5.53	2.40	2
–	5.37	1.28	3
–	5.38	1.50	3
–	5.40	2	3
–	5.44	2.4	3
–	5.45	3	3
–	5.46	4	3

FIGURE 6a

Buckling load for specially orthotropic plate (56.1% boron fibers by volume in epoxy matrix) with x-edges and y-edges simply supported

$$\frac{h}{a} = \frac{1}{10}$$

Buckling Load Neglecting T. S. Def. $\overline{N}_{x_{classical}}$	Buckling Load including T. S. Def. $\overline{N}_x$	a/b	m
.658	.615	1/3	1
.669	.625	1/2	1
.756	.712	1	1
1.518	1.482	2	1
4.268	4.231	3	1
2.678	2.051	1	2
3.022	2.343	2	2
3.958	3.244	3	2
6.034	5.124	4	2
6.649	6.200	4.2	2
6.795	4.144	3	3
7.978	4.654	4	3
8.311	6.160	4.2	3
8.494	6.669	4.3	3
8.648	7.640	4.4	3

FIGURE 6b

Buckling load for specially orthotropic plate (56.1% boron fibers by volume in epoxy matrix) with x-edges and y-edges simply supported

$$\frac{h}{a} = \frac{1}{25}$$

Buckling Load Neglecting T. S. Def. $\bar{N}_{x_{classical}}$	Buckling Load including T. S. Def. $\bar{N}_x$	a/b	m
.166	.160	1/3	1
.192	.190	1	1
.385	.344	2	1
1.079	1.060	3	1
2.948	2.750	4	1
.676	.637	1	2
.764	.721	2	2
.978	.950	3	2
1.540	1.430	4	2
2.560	2.380	5	2
4.336	4.000	6	2
2.020	1.750	4	3
2.565	2.385	5	3
3.460	3.100	6	3
4.810	4.450	7	3
6.860	6.300	8	3

FIGURE 6c

Buckling load for specially orthotropic plate (56.1% boron fibers by volume in epoxy matrix) with x-edges and y-edges simply supported

$$\frac{h}{a} = \frac{1}{50}$$

Buckling Load Neglecting T. S. Def. $\overline{N}_{x_{classical}}$	Buckling Load including T. S. Def. $\overline{N}_x$	a/b	m
.0422	.0420	1/2	1
.0476	.0474	1	1
.0952	.0949	2	1
.2664	.2519	3	1
.6992	.6796	4	1
.1893	.1719	2	2
.2501	.2484	3	2
.3818	.3780	4	2
.6350	.6289	5	2
1.0719	1.0571	6	2
.5050	.4872	4	3
.6400	.6162	5	3
.8621	.8330	6	3
1.2059	1.1014	7	3
1.7117	1.6800	8	3

FIGURE 6d

Buckling load for specially orthotropic plate (56.1% boron fibers by volume in epoxy matrix) with x-edges and y-edges simply supported

$$\frac{h}{a} = \frac{1}{100}$$

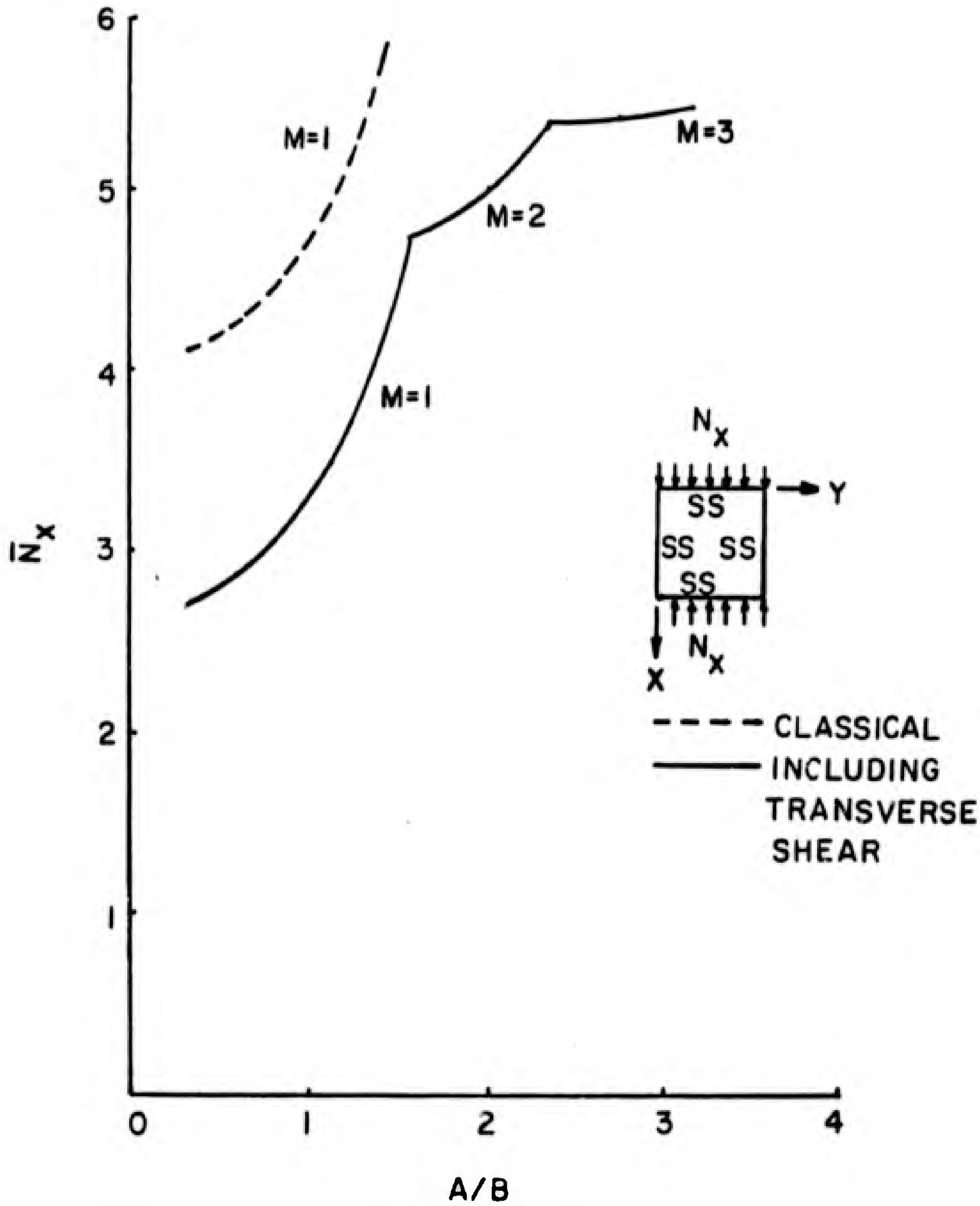

FIGURE 7A

BUCKLING LOAD VS. A/B FOR H/A=1/10

PLATE: 56.1% BORON FIBERS IN EPOXY MATRIX

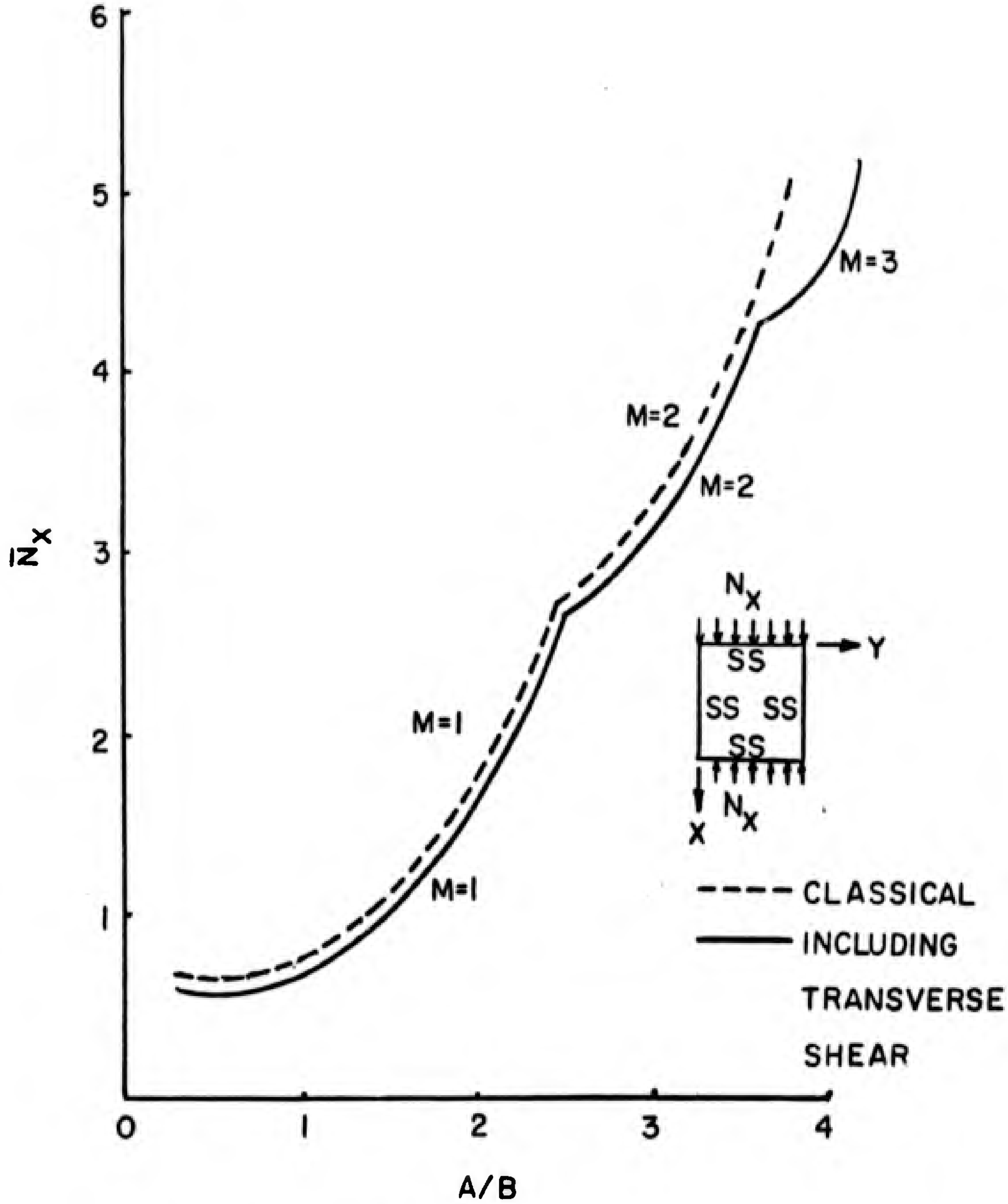

FIGURE 7B

BUCKLING LOAD VS. A/B FOR H/A=1/25

PLATE: 56.1% BORON FIBERS IN EPOXY MATRIX

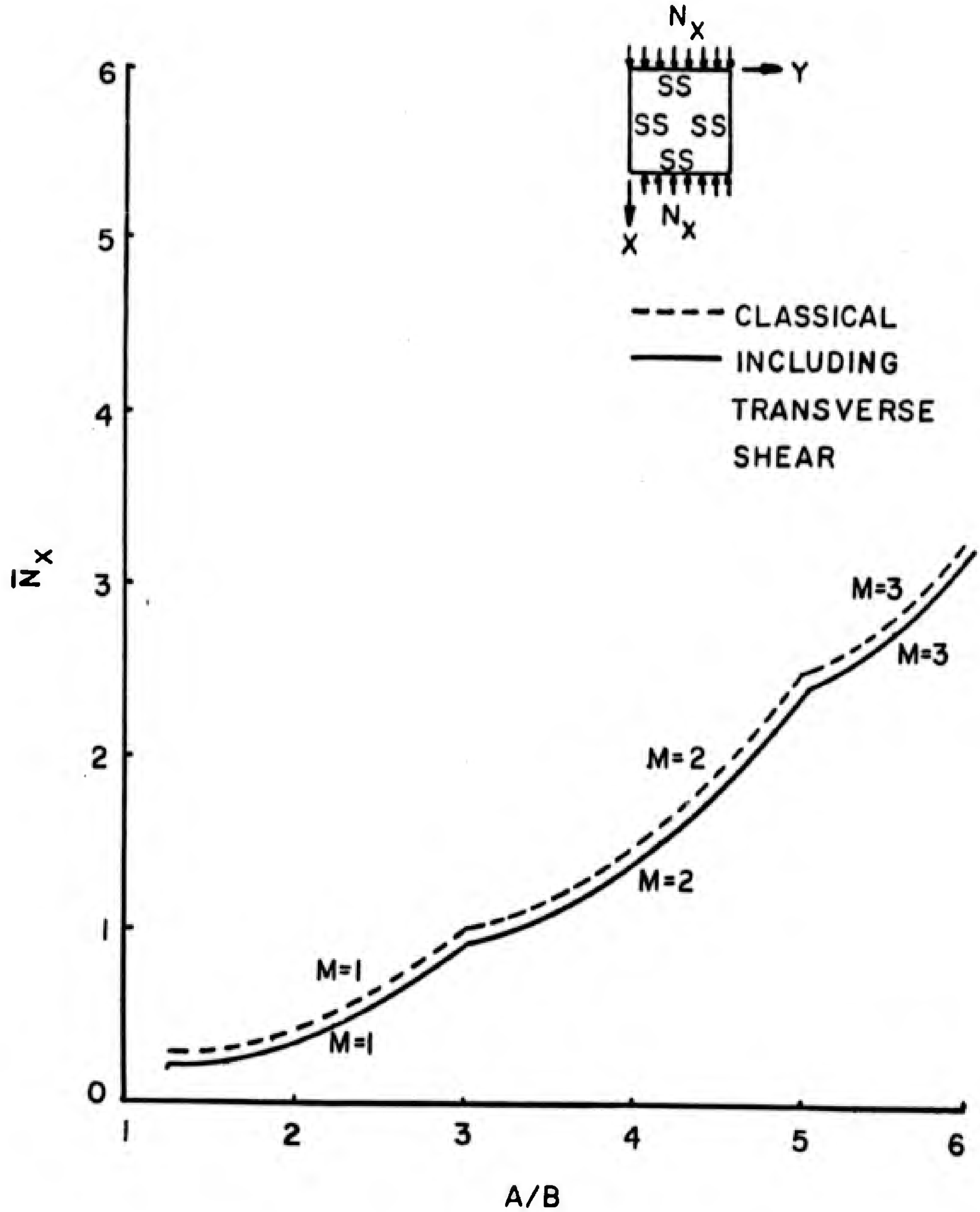

FIGURE 7C

BUCKLING LOAD VS. A/B FOR H/A = 1/50

PLATE: 56.1% BORON FIBERS IN EPOXY MATRIX

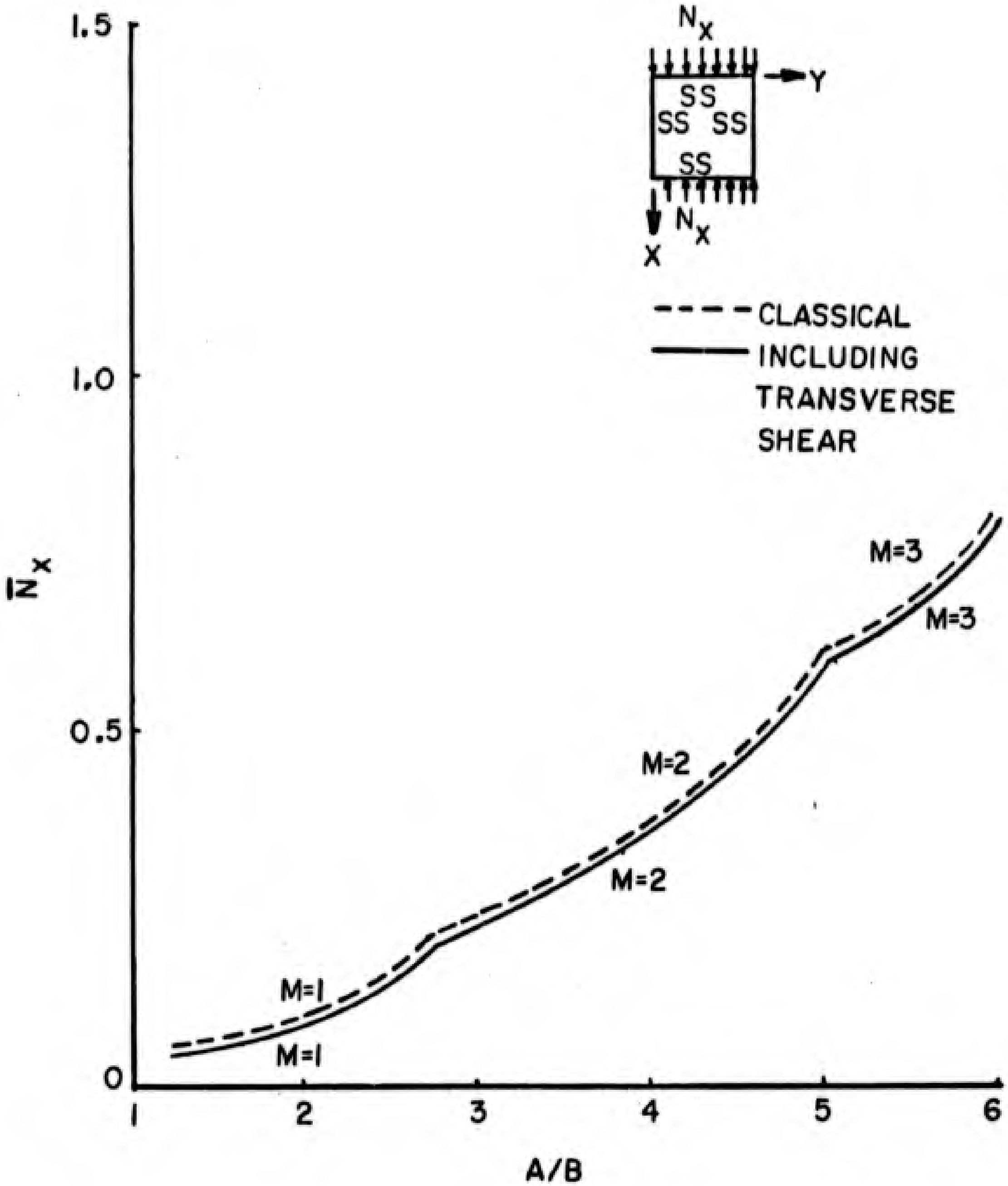

FIGURE 7D

BUCKLING LOAD VS. A/B FOR H/A=1/100

PLATE: 56.1% BORON FIBERS IN EPOXY MATRIX

Buckling Load Neglecting T. S. Def. $\bar{N}_{x_{classical}}$	Buckling Load including T. S. Def. $\bar{N}_x$	a/b	m
4.0736	2.8089	1/3	1
4.1733	2.8879	1/2	1
5.0930	3.7133	1.00	1
5.9305	4.8019	1.2	1
6.2702	5.3300	1.26	1
–	4.9992	1.00	2
–	5.0994	1.2	2
–	5.1902	1.26	2
–	5.2845	1.30	2
–	5.8102	1.50	2
–	5.7150	1.00	3
–	5.7180	1.30	3
–	5.7200	1.50	3
–	5.9200	2.00	3

FIGURE 8a

Buckling load for specially orthotropic plate (56.1% boron fibers by volume in epoxy matrix) with x-edges and one y-edge simply supported and other y-edge clamped

$$\frac{h}{a} = \frac{1}{10}$$

Buckling Load Neglecting T. S. Def. $\bar{N}_{x_{classical}}$	Buckling Load including T. S. Def. $\bar{N}_x$	a/b	m
.6622	.6177	1/3	1
.6863	.6255	1/2	1
.8165	.7437	1	1
2.4410	2.4226	2	1
8.8285	8.6127	3	1
3.2660	2.6505	2	2
5.1655	4.8650	3	2
5.4792	5.2780	3.1	2
6.1333	6.0120	3.3	2
6.7370	6.7012	3.4	2
7.3472	5.3040	3	3
7.5121	5.6500	3.1	3
7.8838	6.4445	3.3	3
8.6020	6.6982	3.4	3

FIGURE 8b

Buckling load for specially orthotropic plate (56.1% boron fibers by volume in epoxy matrix) with x-edges and one y-edge simply supported and other y-edge clamped

$$\frac{h}{a} = \frac{1}{25}$$

Buckling Load Neglecting T. S. Def. $\bar{N}_{x_{classical}}$	Buckling Load including T. S. Def. $\bar{N}_x$	a/b	m
.1628	.1624	1/3	1
.1668	.1660	1/2	1
.2036	.2030	1	1
.6080	.6078	2	1
2.1980	2.1970	3	1
.8144	.7819	2	2
1.2860	1.2659	3	2
2.4256	2.4056	4	2
4.7086	4.6875	5	2
1.8288	1.6239	3	3
2.4084	2.2305	4	3
3.5136	3.4669	5	3
5.4186	5.2174	6	3

FIGURE 8c

Buckling load for specially orthotropic plate (56.1% boron fibers by volume in epoxy matrix) with x-edges and one y-edge simply supported and other y-edge clamped

$$\frac{h}{a} = \frac{1}{50}$$

Buckling Load Neglecting T. S. Def. $\bar{N}_{x_{classical}}$	Buckling Load including T. S. Def. $\bar{N}_x$	a/b	m
.0406	.0403	1/3	1
.0416	.0414	1/2	1
.0509	.0504	1	1
.1522	.1516	2	1
.5490	.5478	3	1
.2038	.2027	2	2
.3225	.3211	3	2
.6074	.6061	4	2
1.1852	1.1830	5	2
.4571	.4468	3	3
.6038	.5952	4	3
.8839	.8758	5	3

FIGURE 8d

Buckling Load for specially orthotropic plate (56.1% boron fibers by volume in epoxy matrix) with x-edges and one y-edge simply supported and other y-edge clamped

$$\frac{h}{a} = \frac{1}{100}$$

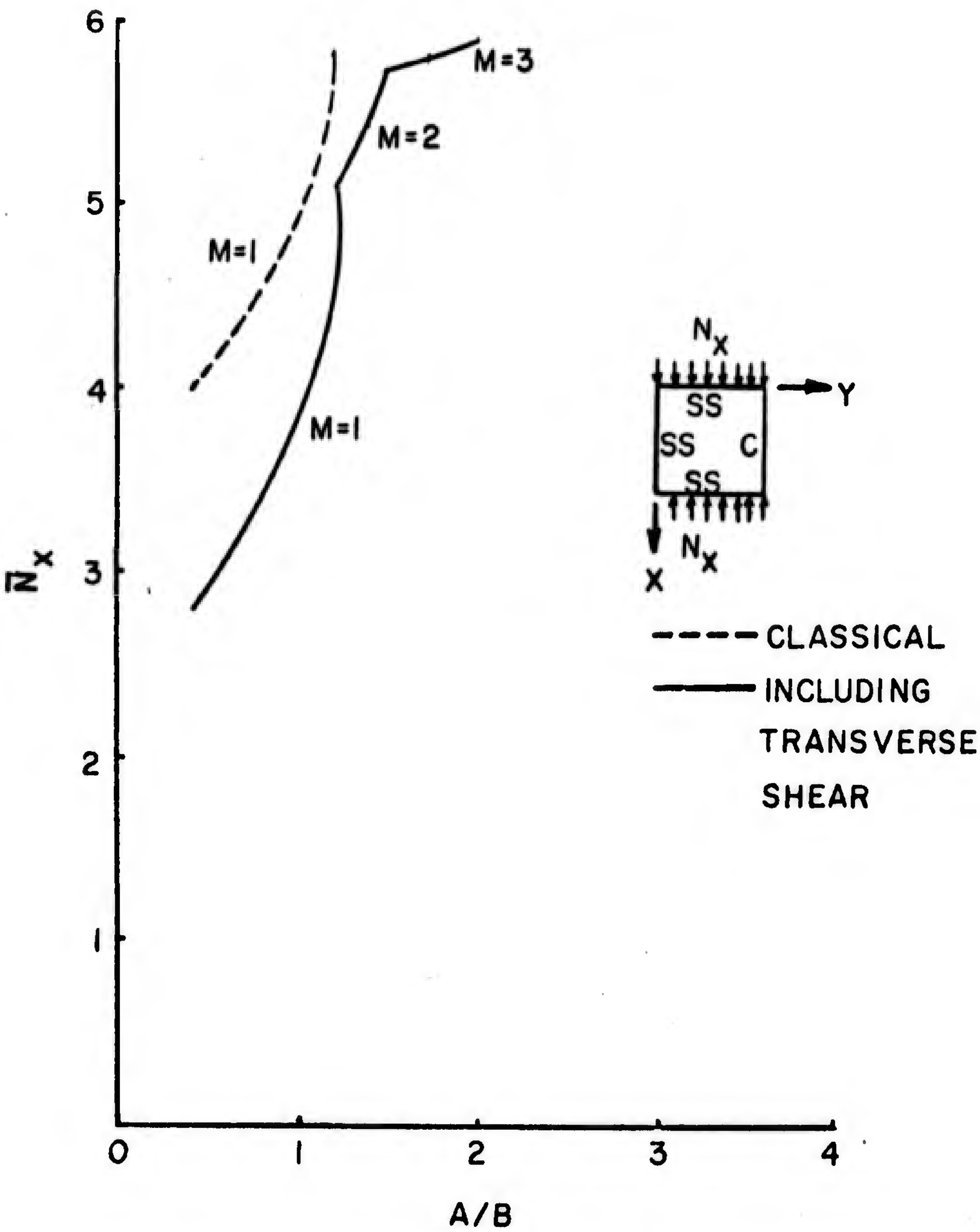

FIGURE 9A

BUCKLING LOAD VS. A/B FOR H/A=1/10

PLATE: 56.1% BORON FIBERS IN EPOXY MATRIX

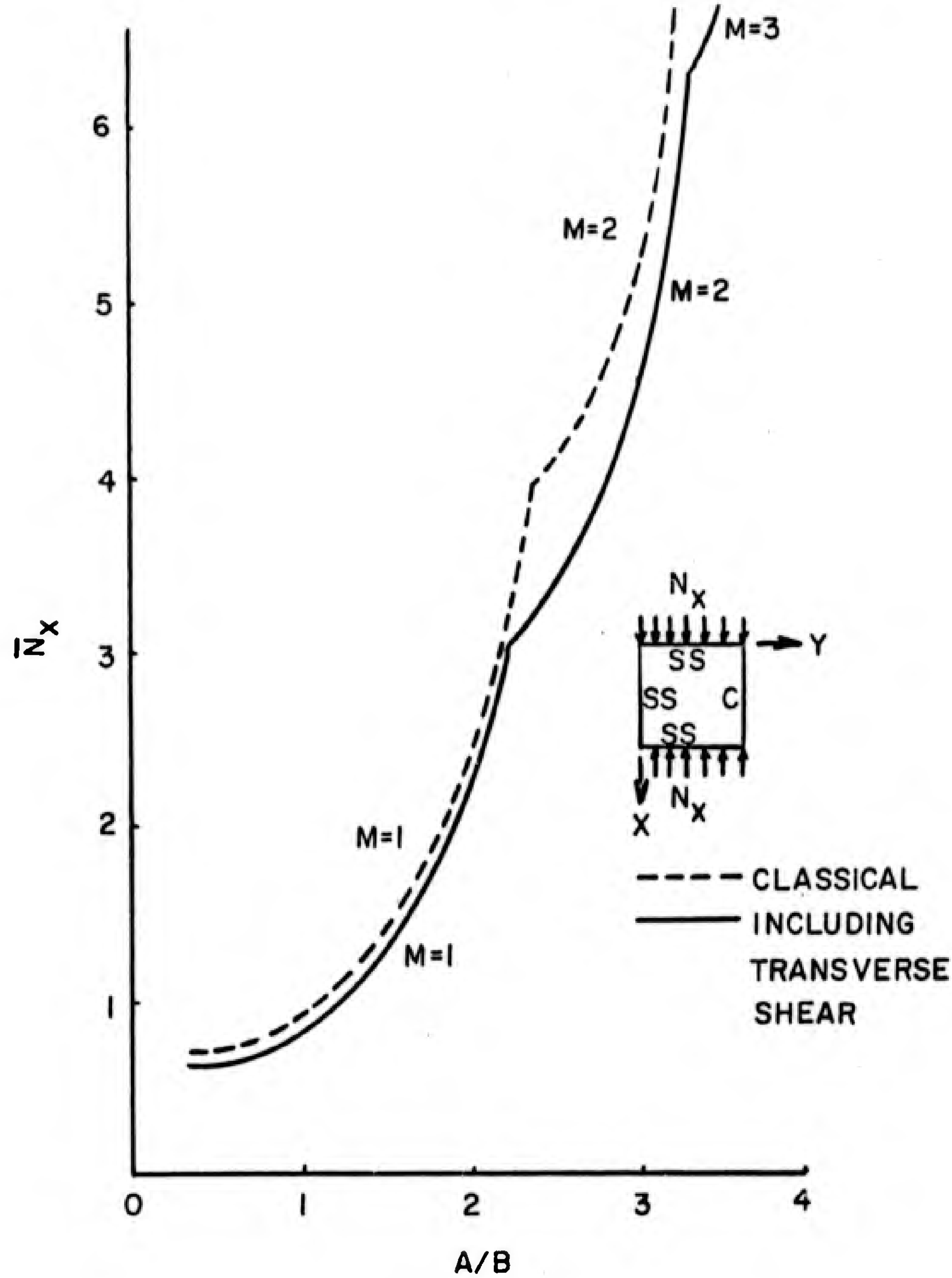

FIGURE 9B

BUCKLING LOAD VS. A/B FOR H/A=1/25

PLATE: 56.1% BORON FIBERS IN EPOXY MATRIX

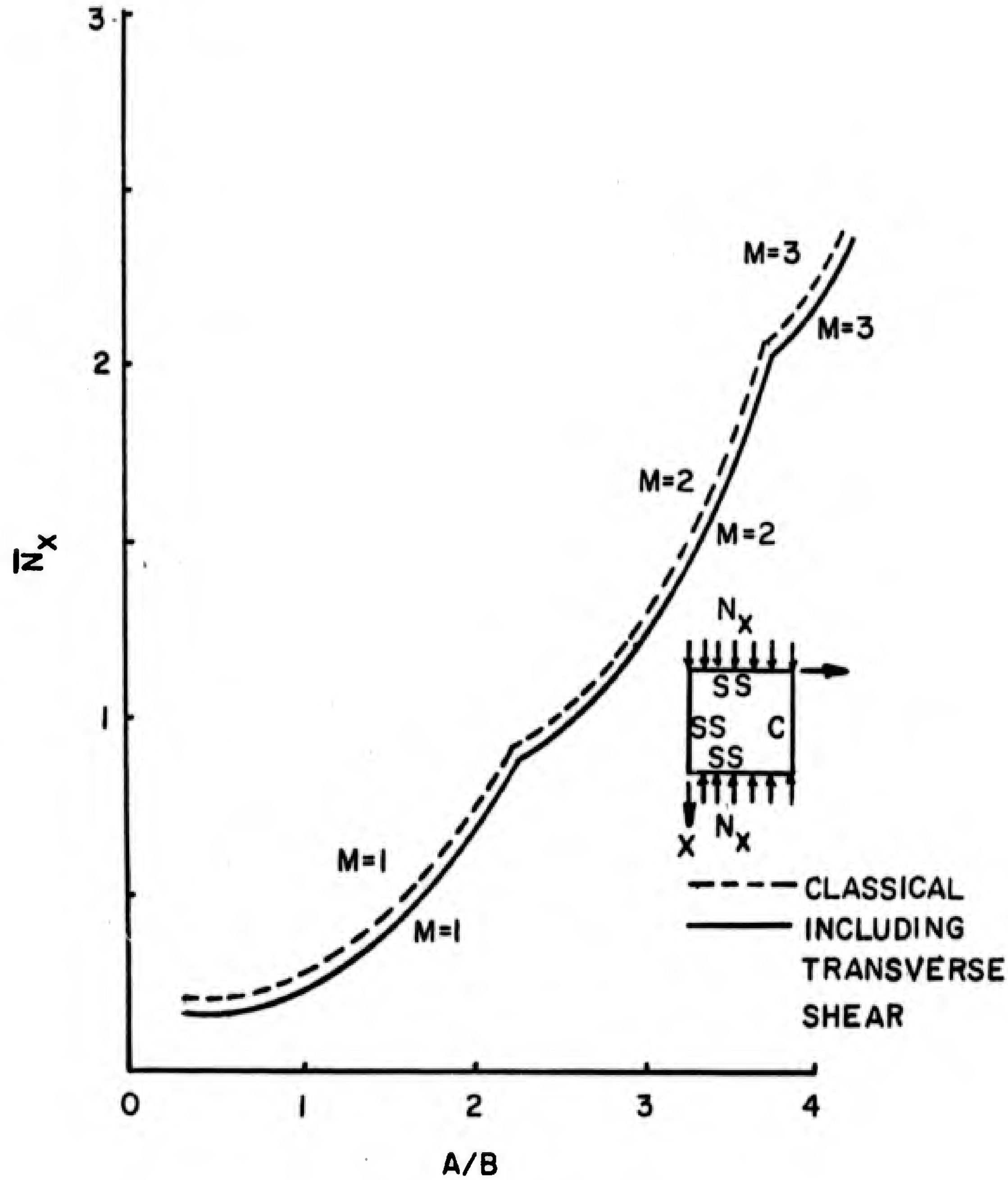

FIGURE 9C

BUCKLING LOAD VS. A/B FOR H/A=1/50

PLATE: 56.1% BORON FIBERS IN EPOXY MATRIX

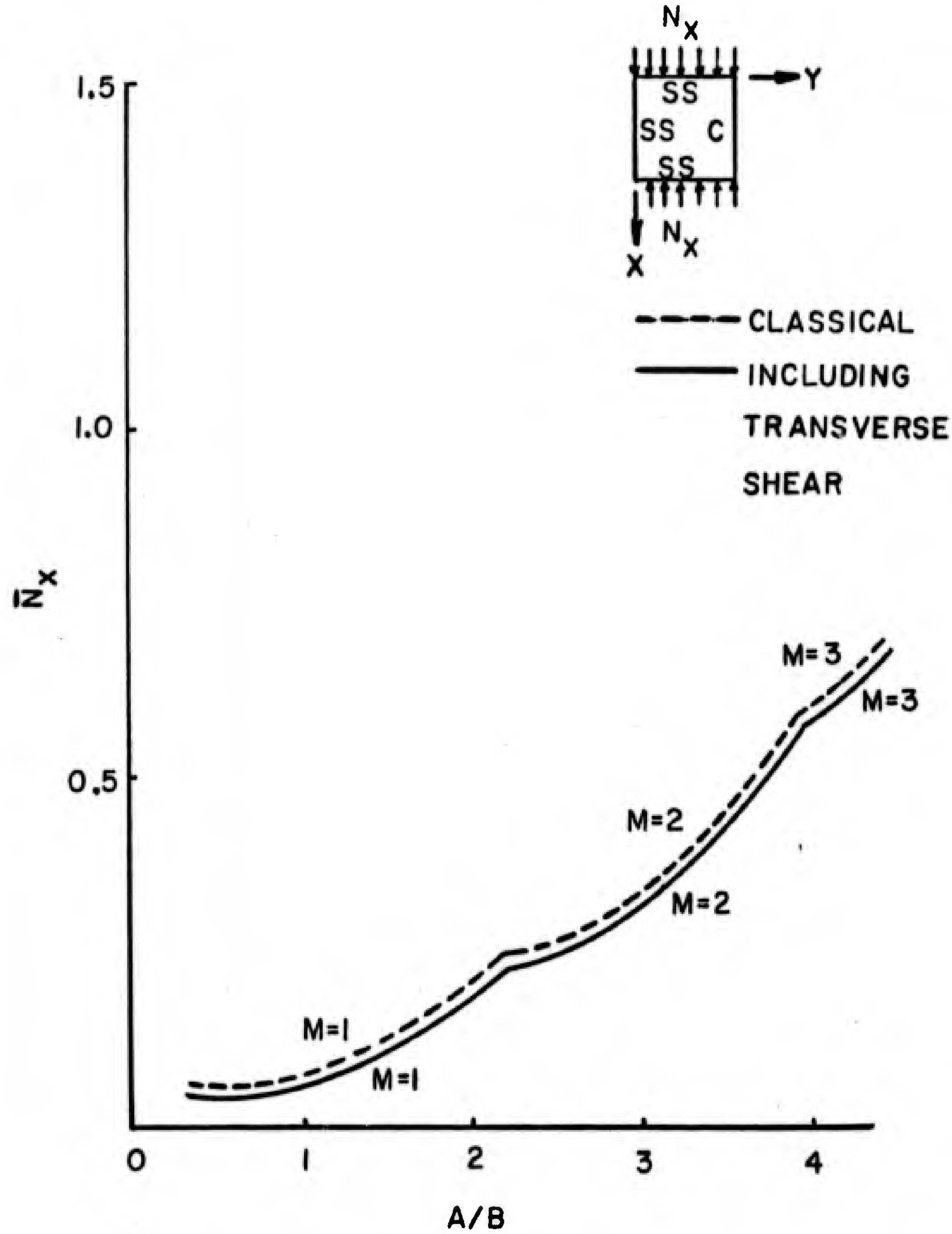

FIGURE 9D

BUCKLING LOAD VS. A/B FOR H/A=1/100

PLATE: 56.1% BORON FIBERS IN EPOXY MATRIX

BIBLIOGRAPHY

1. Reissner, Eric, "The Effect of Transverse Shear Deformation on the Bending of Elastic Plates," Journal of Applied Mechanics, Vol 12, No.2 (June 1945)

2. Libove, Charles, and Batdorf, S. B., "A General Small Deflection Theory for Flat Sandwich Plates," NACA Rep. 899 (1948)

3. Seide, Paul, and Stowell, E. Z., "Elastic and Plastic Buckling of Simply Supported Solid-core Sandwich Plates in Compression," NACA Rep. 967 (1950)

4. Kimel, W. R., "Elastic Buckling of a Simply Supported Rectangular Sandwich Panel Subjected to Combined Bending and Compression," ANC-23 Panel on Sandwich Construction, No. 1857 (1956)

5. Kimel, W. R., "Elastic Buckling of a Simply Supported Rectangular Sandwich Panel Subjected to Combined Edgewise Bending, Compression and Shear," ANC-23 Panel on Sandwich Construction, No. 1859 (1956)

6. Norris, C. B., "Compressive Buckling Curves for Flat Sandwich Panels with Isotropic Facings and Isotropic or Orthotropic Cores" _ANC-23 Panel on Sandwich Construction_, No. 1854 (1958)

7. Chang, C. C. and Ebcioglu, I. K., "Elastic Instability of Rectangular Sandwich Panel of Orthotropic Core with Different Face Thicknesses and Materials," _Journal of Applied Mechanics_ (September 1960)

8. Zahn, J. J. and Cheng, S., "Edgewise Compressive Buckling of Flat Sandwich Panels: Loaded Ends Simply Supported and Sides Supported by Beams," _MIL-HDBK-23 Working Group on Composite Construction for Flight Vehicles_, No. FPL-019 (February 1964)

9. Chung, Wen-Yi and Testa, R. B., "Elastic Stability of Fibers in a Composite Plate," _Journal of Composite Materials_, Vol. 3 (January 1969)

10. Ambartsumyan, S. A. "Theory of Anisotropic Plates," _Translated from Russian by T. Cheron and edited by J. E. Ashton, Technical, Publishing Co._ (1969)

11. Ashton, J. E. and Waddoups, M. E., "Analysis of Anisotropic Plates", Journal of Composite Materials, Vol. 3 (January 1969)

12. Ashton, J. E. and Love, T. S., "Experimental Study of the Stability of Composite Plates," Journal of Composite Materials, Vol. 3 (April 1969)

13. Whitney, J. M., "The Effect of Transverse Deformation on Bending of Laminated Plates," Journal of Composite Materials, Vol. 3 (July 1969)

14. Brunelle, E. J., "Buckling of Transversely Isotropic Mindlin Plates," AIAA Journal, Vol. 9, No. 6 (June 1971)

15. Reissner, E., "On a Variational Theorem in Elasticity," Journal of Math. and Phys., 1950

16. Timoshenko, S. P. and Gere, J. M., "Theory of Elastic Stability," McGraw-Hill Book Company, Inc. (1961)

17. Ashton, J.E. and Whitney, J.M., "Theory of Laminated Plates," Technomic Publishing Co., Inc. (1970)

BEFORE COMPLETING FORM

1. REPORT NUMBER: AFOSR - TR - 75 - 1628	2. GOVT ACCESSION NO.
3. RECIPIENT'S CATALOG NUMBER	
4. TITLE (and Subtitle): THE EFFECT OF TRANSVERSE SHEAR DEFORMATION ON THE ELASTIC STABILITY OF PLATES OF COMPOSITE MATERIALS	5. TYPE OF REPORT & PERIOD COVERED: INTERIM
	6. PERFORMING ORG. REPORT NUMBER: MAE-TR-180
7. AUTHOR(s): JACK R. VINSON, ALTON P. SMITH, JR	8. CONTRACT OR GRANT NUMBER(s): AFOSR 74-2739
9. PERFORMING ORGANIZATION NAME AND ADDRESS: UNIVERSITY OF DELAWARE, DEPARTMENT OF MECHANICAL & AEROSPACE ENGINEERING, NEWARK, DELAWARE 19711	10. PROGRAM ELEMENT, PROJECT, TASK AREA & WORK UNIT NUMBERS: 681307, 9782-04, 61102F
11. CONTROLLING OFFICE NAME AND ADDRESS: AIR FORCE OFFICE OF SCIENTIFIC RESEARCH/NA, 1400 WILSON BOULEVARD, ARLINGTON, VIRGINIA 22209	12. REPORT DATE: [redacted] 1975
	13. NUMBER OF PAGES: 92
14. MONITORING AGENCY NAME & ADDRESS (if different from Controlling Office)	15. SECURITY CLASS. (of this report): UNCLASSIFIED
	15a. DECLASSIFICATION/DOWNGRADING SCHEDULE

16. DISTRIBUTION STATEMENT (of this Report)

Approved for public release; distribution unlimited.

17. DISTRIBUTION STATEMENT (of the abstract entered in Block 20, if different from Report)

18. SUPPLEMENTARY NOTES

19. KEY WORDS (Continue on reverse side if necessary and identify by block number)

ELASTIC STABILITY
PLATES
COMPOSITE MATERIALS
TRANSVERSE SHEAR DEFORMATION

20. ABSTRACT (Continue on reverse side if necessary and identify by block number)

Methods of analysis are developed for determining the critical inplane buckling loads for plates of composite materials with a wide variety of boundary conditions. Although the expressions developed are restricted to specially orthotropic construction of one lamina, their use is more general, and can be used for laminated specially orthotropic plates that are symmetric with respect to the plate midsurface (i.e. no bending-stretching coupling). It is seen that the effects of transverse shear deformation can be significant, not only in altering the magnitude of the buckling loads but even in altering the wave number.

DD FORM 1 JAN 73 1473 EDITION OF 1 NOV 65 IS OBSOLETE

UNCLASSIFIED

SECURITY CLASSIFICATION OF THIS PAGE (When Data Entered)