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Technical Report No. 51✓

STATISTICAL PERT: AN IMPROVED SUBNETWORK
ANALYSIS PROCEDURE

by

R. L. Sielken Jr., H. O. Hartley, R. K. Spoeri

Texas A&M Research Foundation
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ATTACHMENT I

STATISTICAL PERT: AN IMPROVED SUBNETWORK
ANALYSIS PROCEDURE

by

R. L. Sielken Jr., H. O. Hartley, R. K. Spoeri

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ATTACHMENT II

TABLE OF CONTENTS

Section		Page
0	Introduction	1
1	Analysis of a Subnetwork	2
1.1	Formation of Clusters	3
1.2	Approximate Subnetwork Completion Time Moments and Distribution	7
1.2.1	A Lower Bound on T_r and an Upper Bound on F	7
1.2.2	An Upper Bound on T_r and a Lower Bound on F	12
1.2.3	Summary	13
2	Comparison with Original Subnetwork Analysis Procedure .	14
3	Conclusion	24
	References	25
	Appendix A: Proof of Theorem 1	26
	Appendix B: New Subnetwork Analysis Program	31
	Appendix C: Original Subnetwork Analysis Program . . .	65
	Appendix D: Monte Carlo PERT Simulation Program	99

ABSTRACT

R. L. Sielken Jr., H. O. Hartley, R. K. Spoeri

 Statistical PERT is a new procedure for obtaining information about the distribution of a project's completion time when the project is comprised of a large number of activities and the time required to complete an individual activity once it can be begun is a random variable. The project is represented as an acyclic network whose arcs correspond to the project activities. This network is simplified by replacing various activity configurations by single equivalent activities and then decomposed into several subnetworks. The distribution and moments of each subnetwork's completion time are bounded and approximated on the basis of two points from each activity's completion time distribution by using some mathematical programming techniques and a new result in the theory of networks. The project's completion time distribution is then approximated by combining the approximate subnetwork distributions.

This report describes several refinements in the subnetwork analysis procedure. One major refinement greatly reduces the computational effort  in obtaining bounds on the project completion time moments and distribution. A second major refinement allows the two-point approximation of an activity's completion time distribution to better represent skewed distributions. The computer programs required to implement the new subnetwork analysis procedure are listed and documented.

Statistical PERT: An Improved Subnetwork Analysis Procedure

R. L. Sielken Jr., H. O. Hartley, R. K. Spoeri

The well-known Program Evaluation and Review Technique (PERT) is concerned with a 'project' comprised of a large number of 'activities' which are arranged as the arcs in a complex acyclic network (see e.g. Figure 1). The activities at any network node 'commence' as soon as all activities 'terminating' at that node are completed. The time required to complete an activity once it can be begun is a random variable, and hence the time needed to complete the entire project is also a random variable.

In Technical Report No. 48 "Statistical Critical Path Analysis in Acyclic Networks: Statistical PERT" a comprehensive new procedure for obtaining information on the project completion time and its distribution was described and illustrated. That procedure involved the following five general steps:

Step 1: Identification

Represent the project and its component activities in terms of an acyclic network with one source and one sink. Identify each activity's completion time distribution or at least two points on each activity's completion time distribution.

Step 2: Simplification

Replace various activity configurations and their associated completion time distributions by a single equivalent activity and completion time distribution.

Step 3: Decomposition

Decompose the simplified network into several subnetworks by separating parallel subnetworks and then separating the resulting subnetworks at each cut vertex. A cut vertex is any node such that every path from the source to the sink passes through it.

Step 4: Analysis

Each subnetwork arising from Step 3 is analyzed on the basis of two points from each component activity's completion time distribution. The result of this analysis is an approximation of each subnetwork's completion time distribution and the moments of this distribution.

Step 5: Synthesis

Combine the approximate subnetwork completion time distributions determined in Step 4. The result is an approximate completion time distribution for the entire project.

The purpose of this report is to document several refinements in the subnetwork analysis step, Step 4.

1. Analysis of a Subnetwork

The analytical procedure described in this section yields the following information on each subnetwork when each component activity's completion time distribution is replaced by a discrete two-point distribution:

- (a) Upper and lower bounds on the mean subnetwork completion time as well as the other moments of the subnetwork completion time.
- (b) Upper and lower bounds on the distribution function of the subnetwork completion time.

- (c) An approximate distribution function of the subnetwork completion time.

Each subnetwork is assumed to be an acyclic network with one source, one sink, and no cut vertices.

The analysis of each subnetwork involves essentially two parts:

1. The formation of "clusters" of activities whose effect on the subnetwork completion time seems to be interrelated.
2. The approximation of the subnetwork completion time moments and distribution on the basis of the clusters.

1.1 Formation of Clusters

The actual completion time distribution of each individual activity, A, in the subnetwork is replaced by a discrete distribution with probability P at the lower point, l_A , and $Q = 1-P$ at the upper point, u_A .

Let n be the number of activities in the subnetwork. Then for each of the 2^n combinations of the l_A 's and u_A 's there will be a subnetwork completion time (a critical path time). The r-th moment of these 2^n times will be denoted by T_r , and the distribution function of these times will be denoted by F. The approximation of the T_r 's (especially T_1 , the mean) and F is the goal of the subnetwork analysis. Since n will usually be fairly large, the complete enumeration of the 2^n critical path times will usually be unreasonable. Hence the activities which are most likely to be on the critical path through the subnetwork are identified and their joint behavior investigated.

The mean of the completion time distribution for activity A is defined to be $m_A = Pl_A + Qu_A$. The standard deviation of the completion time distribution for activity A is defined to be $s_A = \sqrt{Pl_A^2 + Qu_A^2 - m_A^2}$.

and is assumed to be positive. (The assumption that $s_A > 0$ is not really a practical restriction since the difference between a fixed activity completion time and one with a very small dispersion is negligible from a practical viewpoint.) The subnetwork's critical path when each activity's completion time is set equal to its mean will be referred to as the "original" critical path. The activities on this critical path will be referred to as "critical activities" with K equalling the number of such activities. Some non-critical activities might become critical if some of the completion times for the original critical activities were decreased. These activities are identified as follows. The completion time for one critical activity, say A , is set equal to $\max\{m_A - \lambda s_A, 0\}$ where λ is a non-negative algorithm parameter which the user specifies. All other completion times are set equal to their means. Then the longest path through the resulting network is determined. Any activities on this path which were not on the original critical path are now referred to as the "associates" of A since the effect of these "associates" on the networks' completion time is related to A 's completion time. This procedure is repeated for each original critical activity.

Each critical activity and its associates make up one "cluster". These K initial clusters are now "pooled" by combining any two clusters with at least one activity in common. In general there will still be more than one cluster, and many of the $n - K$ non-critical activities will not occur in any cluster.

The associates correspond to the activities which become critical when the completion times of the original critical activities are lowered. However, some of the originally non-critical activities may also become critical if their completion times exceed their means

and the completion times of the original critical activities are at their means. These activities are identified next. Each originally non-critical activity is investigated separately. If activity A is being investigated, then the completion time for A is set equal to $m_A + \theta s_A$ where θ is a non-negative algorithm parameter which the user specifies. The completion times for all other activities are set equal to their means, and the corresponding critical path determined. This critical path will either be the original critical path or a new path which includes A. In the latter case, the activities on the original critical path which are not on a new critical path containing A are called the "eliminants" of A. Thus, the effect of A's eliminants on the networks completion time is related to the completion time for A. Hence, A is added to any cluster which contains at least one of A's eliminants. After this procedure has been repeated for each originally non-critical activity, the resultant clusters are "pooled" again by combining any two clusters with at least one activity in common.

Although the number of clusters is reduced when the pooling on the basis of the associates occurs and then further reduced when the pooling on the basis of the eliminants occurs, there will generally remain more than one cluster and several activities not in any cluster.

In general the larger the values of λ and θ the greater the number of activities in the clusters and the smaller the number of clusters. In particular the procedure for forming the clusters has the following properties:

Property 1: If $\lambda_2 > \lambda_1$, then any activity which would be an associate of a critical activity A when $\lambda = \lambda_1$ would also be an associate of A when $\lambda = \lambda_2$.

- Property 2: If $\theta_2 > \theta_1$, then any critical activity which would be an eliminant of a non-critical activity A when $\theta = \theta_1$ would also be an eliminant of A when $\theta = \theta_2$.
- Property 3: For any originally non-critical activity A there exists θ_A such that A will have some eliminants for any $\theta \geq \theta_A$.
- Property 4: For any fixed value of λ , the set of activities in the union of the clusters is monotonically non-decreasing as $\theta \rightarrow \infty$.
- Property 5: There exists a finite value θ^* such that if $\theta \geq \theta^*$, then every activity would be in some cluster.
- Property 6: The number of clusters, originally K, is non-increasing as $\theta \rightarrow \infty$.
- Property 7: There exists a finite value θ^* such that if $\theta \geq \theta^*$, then there would only be one cluster.

Most of the properties of the cluster formation procedure are fairly straightforward; however, Property 7 requires some special justification. This justification is based on the following definition and theorem which is proven in Appendix A.

Definition: In any acyclic network a bridge over any two consecutive arcs A_1 and A_2 is any arc A_3 such that all paths from the source to the sink passing through A_3 do not pass through either A_1 or A_2 .

Theorem 1: In any acyclic network with no cut vertices there is at least one bridge for any pair of consecutive arcs.

Property 3 implies that all activities will belong to some cluster if $\theta \geq \theta^*$ and

$$\theta^* = \max\{\theta_A : A \text{ originally non-critical}\}.$$

Now consider any two consecutive activities A_1 and A_2 on the original critical path. Theorem 1 implies that there is a bridge over A_1 and A_2 , say A_3 . Since the original critical path passes through A_1 and A_2 , A_3 cannot be on the original critical path. Therefore, if $\theta \geq \theta^* \geq \theta_{A_3}$, A_1 and A_2 will be eliminants of A_3 and hence will be in the same cluster as A_3 . Thus, since each cluster contains at least one original critical activity and any two consecutive original critical path activities belong to the same cluster when $\theta \geq \theta^*$, there is only one cluster when $\theta \geq \theta^*$ and Property 7 is established.

1.2 Approximate Subnetwork Completion Time Moments and Distribution

1.2.1 A Lower Bound on T_r and an Upper Bound on F

For each cluster C let n_c denote the number of activities in C , and let $v = 1, \dots, 2^{n_c}$ index the 2^{n_c} configurations of activity completion times when

- (a) the completion time for each activity A not in C is equal to its lower point, l_A , and
- (b) the completion times for the activities in C are at each of the 2^{n_c} possible combinations of their upper and lower points.

Let

t_v = critical path time for the v -th configuration

and

p_v = probability of the v -th configuration

$$= \prod_{i=1}^{n_c} [P_i(1 - \delta_{v,i}) + Q_i \delta_{v,i}]$$

where

$\delta_{v,i} = 1$ if the time for the i -th activity in C is u_i in the v -th configuration
 $= 0$ if the time for the i -th activity in C is l_i in the v -th configuration.

Then

$$\hat{T}_r^-(C) \equiv \sum_{v=1}^{2^{n_c}} p_v t_v^r$$

and

$$\hat{F}^+(t; C) \equiv \sum_{v=1}^{2^{n_c}} p_v I_t(t_v)$$

where

$$\begin{aligned}
 I_t(t_v) &= 1 && \text{if } t_v \leq t \\
 &= 0 && \text{if } t_v > t
 \end{aligned}$$

are the r -th moment of the 2^{n_c} critical path times and their distribution function respectively. Let

$$T_r^-(\theta, \lambda) = \max_C \hat{T}_r^-(C)$$

and

$$F^+(t; \theta, \lambda) = \min_C \hat{F}^+(t; C)$$

which depend on θ and λ since the composition and number of clusters depend on θ and λ .

The first step in showing that $T_r^-(\theta, \lambda)$ is a lower bound for T_r is proving the following theorem:

Theorem 2: For any cluster C , any positive integer r , and any activity A not in C ,

$$\hat{T}_r^-(C \cup \{A\}) \geq \hat{T}_r^-(C).$$

Proof: Consider any particular critical path for a particular one of the 2^n combinations of upper and lower points involved in $\hat{T}_r^-(C)$. Consider the following two cases:

- (i) activity A with its completion time equal to l_A is on the critical path, and
- (ii) activity A with its completion time equal to l_A is not on the critical path.

Let $\delta_A = (u_A - l_A)$ and the particular critical path time be t . Then in case (i)

- (a) if the completion time for A is set equal to u_A , this will increase the r -th moment of the critical path time by $Q[(t + \delta_A)^r - t^r]$; and
- (b) if the completion time for A is set equal to l_A , this will not alter the r -th moment of the critical path time.

In case (ii)

- (a) if the completion time for A is set equal to u_A , this may increase the r -th moment of the critical path time by $Q[(t + \delta_A)^r - t^r]$ or less, and
- (b) if the completion time for A is set equal to l_A , this will not alter the r -th moment of the critical path time.

Therefore in either case the contribution to $\hat{T}_r^-(C \cup \{A\}) - \hat{T}_r^-(C)$ for this particular critical path will be between 0 and $Q[(t + \delta_A)^r - t^r]$. Since the contribution is non-negative for each particular combination,

$$\hat{T}_r^-(C \cup \{A\}) \geq \hat{T}_r^-(C).$$

QED

A straightforward application of Theorem 2 yields the following theorem:

Theorem 3: For any two clusters C_1 and C_2 and any positive integer r ,

$$\hat{T}_r^-(C_1 \cup C_2) \geq \max\{\hat{T}_r^-(C_1), \hat{T}_r^-(C_2)\}.$$

Property 2 of the cluster formation procedure implies that if θ is increased the clusters expand or are pooled. Thus, Theorems 2 and 3 imply that, for fixed λ , $T_r^-(\theta, \lambda)$ is non-decreasing as θ increases. Furthermore, Properties 5 and 7 together imply that for θ sufficiently large there is only one cluster and all of subnetwork activities are in that cluster. Hence, for θ sufficiently large $T_r^-(\theta, \lambda) = T_r$, and the following theorem is true:

Theorem 4:

- (a) $T_r^-(\theta, \lambda)$ is a non-decreasing function of θ for any fixed values of λ and r ;
- (b) there exists a finite value θ^* such that $\theta \geq \theta^*$ implies

$$T_r^-(\theta, \lambda) = T_r$$

for any λ , and r ; and

- (c) for any θ , λ , and r

$$T_r^-(\theta, \lambda) \leq T_r.$$

Similarly, the first step in showing that $F^+(t; \theta, \lambda)$ is an upper bound on $F(t)$ is the following theorem:

Theorem 5: For any cluster C , any value of t , and any activity A not in C ,

$$\hat{F}^+(t; C \cup \{A\}) \leq \hat{F}^+(t; C).$$

Proof: Consider any particular configuration of activity times used to determine $F^+(t; C)$ before C is augmented by A . When A is added to C , this particular configuration will appear once with A at its upper percentile and once with A at its lower percentile. When A is at its lower percentile, the configuration's critical path time is unchanged. However, when A is at its upper percentile, the configurations' critical path time is either unchanged or possibly increased if A were on the configuration's critical path. Thus, the addition of A leaves the cumulative probability associated with critical path times less than or equal to t either unchanged or decreased. QED

A straightforward extension of Theorem 5 is the following theorem:

Theorem 6: For any two clusters C_1 and C_2 and any t ,

$$\hat{F}^+(t; C_1 \cup C_2) \leq \min\{\hat{F}^+(t; C_1), \hat{F}^+(t; C_2)\}.$$

Since Property 2 of the cluster formation procedure implies that the clusters expand or are pooled if θ is increased, Theorem 5 and Theorem 6 together imply that, for all t and any fixed λ , $F^+(t; \theta, \lambda)$ is non-increasing function of θ . Furthermore, Properties 5 and 7 together imply that for θ sufficiently large there is only one cluster and all of the subnetwork activities are in that cluster. Hence, for θ sufficiently large $F^+(t; \theta, \lambda) = F(t)$ and the following theorem is true:

Theorem 7:

- (a) $F^+(t; \theta, \lambda)$ is a non-increasing function of θ for every t and any λ ;

- (b) there exists a finite value θ^* such that $\theta \geq \theta^*$ implies

$$F^+(t; \theta, \lambda) = F(t)$$

for every t and λ ; and

- (c) for any θ , λ , and t

$$F^+(t; \theta, \lambda) \geq F(t).$$

1.2.2 An Upper Bound on T_r and a Lower Bound on F

For each cluster C let n_c denote the number of activities in C , and let $v = 1, \dots, 2^{n_c}$ index the 2^{n_c} configurations of activity completion times when

- (a) the completion time for each activity not in the cluster is equal to its upper point, and
- (b) the completion times for the activities in the cluster are at each of the 2^{n_c} possible combinations of their upper and lower percentiles.

Let t_v , p_v , and $I_t(t_v)$ be as before and define

$$\hat{T}_r^+(C) = \sum_{v=1}^{2^{n_c}} p_v t_v^r,$$

$$T_r^+(\theta, \lambda) = \min_C \hat{T}_r^+(C)$$

$$\hat{F}^-(t; C) = \sum_{v=1}^{2^{n_c}} p_v I_t(t_v),$$

and

$$F^-(t; \theta, \lambda) = \max_C \hat{F}^-(t; C).$$

Then an argument completely analogous to that used to prove Theorem 4 and Theorem 7 leads to the following theorems:

Theorem 8:

- (a) $T_r^+(\theta, \lambda)$ is a non-increasing function of θ for any fixed values of λ and r ;
- (b) there exists a finite value θ^* such that $\theta \geq \theta^*$ implies

$$T_r^+(\theta, \lambda) = T_r$$

for any λ and r ; and

- (c) for any θ , λ , and r

$$T_r \leq T_r^+(\theta, \lambda).$$

Theorem 9:

- (a) There exists a finite value θ^* such that $\theta \geq \theta^*$ implies

$$F^-(t; \theta, \lambda) = F(t)$$

for every t and any λ ; and

- (b) for any θ , λ , and t

$$F^-(t; \theta, \lambda) \leq F(t).$$

1.2.3 Summary

Theorems 2-9 together imply that for any value of θ and λ chosen by the algorithm user:

- (a) $T_r^-(\theta, \lambda) \leq T_r \leq T_r^+(\theta, \lambda)$, for any positive integer r ; and
- (b) $F^-(t; \theta, \lambda) \leq F(t) \leq F^+(t; \theta, \lambda)$ for any t .

They also imply that, for any r , λ , and t ,

$$T_r^+(\theta, \lambda) - T_r^-(\theta, \lambda)$$

and

$$F^+(t; \theta, \lambda) - F^-(t; \theta, \lambda)$$

would decrease monotonically to zero if θ were increased. In fact, Theorems 2-9 imply that there exists a value θ^* which doesn't depend on r , λ , or t such that $\theta \geq \theta^*$ implies that

$$T_r^-(\theta, \lambda) = T_r = T_r^+(\theta, \lambda)$$

and

$$F^-(t; \theta, \lambda) = F(t) = F^+(t; \theta, \lambda).$$

A reasonable approximation for $F(t)$ is

$$\hat{F}(t; \theta, \lambda) = \frac{1}{2}[F^-(t; \theta, \lambda) + F^+(t; \theta, \lambda)].$$

2. Comparison with the Original Subnetwork Analysis Procedure

The original subnetwork analysis procedure documented in Technical Report No. 48 formed the clusters in essentially the same way as the new subnetwork analysis procedure described in this report except that in the original procedure P always equalled $\frac{1}{2}$ and θ and λ multiplied the point difference, $u_A - l_A$, instead of the standard deviation, $\sqrt{Pl_A^2 + Qu_A^2 - m_A^2}$.

In the original procedure $n_U = \sum_c n_c$ denotes the number of activities in the union of the clusters, and $T_r^+(\theta, \lambda)$ is defined to be the average of the r -th power of the 2^{n_U} critical path times when

- (a) the completion time for each activity not in the union of the clusters is equal to its upper point, and

- (b) the completion times for the activities in the union of the clusters are at each of the 2^{n_U} possible combinations of their upper and lower points.

Correspondingly, $F^-(t; \theta, \lambda)$ was defined to be the proportion of these 2^{n_U} critical path times that were less than or equal to t . Analogously, $F^+(t; \theta, \lambda)$ was the proportion of the 2^{n_U} critical path times less than or equal to t when

- (a) the completion time for each activity not in the union of the clusters is equal to its lower point, and
- (b) the completion times for the activities in the union of the clusters are at each of the 2^{n_U} possible combinations of their upper and lower points.

The only problem with this procedure is that 2^{n_U} may be quite large even for relatively small values of (θ, λ) . For example, if the original critical path contains 10 activities and each critical activity has one associate, then $2^{n_U} = 2^{20} = 1,048,576$, and the determination of $T_r^+(\theta, \lambda)$, $F^-(t; \theta, \lambda)$, and $F^+(t; \theta, \lambda)$ requires the evaluation of over 2 million critical path times. On the other hand, in the new procedure the determination of $T_r^-(\theta, \lambda)$, $T_r^+(\theta, \lambda)$, $F^-(t; \theta, \lambda)$, $F^+(t; \theta, \lambda)$ requires the evaluation of only $2 \sum_c 2^{n_c}$ critical path times, 80 in the example. Thus the new procedure greatly reduces the computational effort required to bound the project completion time moments and distribution.

A practical alternative to evaluating all 2^{n_U} critical path times called for in the original procedure is to randomly sample the 2^{n_U} critical path times when n_U is large and base the bounds on the sample critical path times. A computer program for the original procedure with a sampling option is documented in Appendix C. (This program supercedes

the subnetwork analysis program given in Technical Report No. 48.) It should be noted that the original subnetwork analysis procedure with the sampling option in effect is still superior to a simple Monte Carlo simulation of the subnetwork since the subnetwork analysis procedure

- (i) provides the information in terms of associates and eliminants about which activities play an important role in determining the subnetwork's completion time and the interactions among activities, and
- (ii) samples only those activities which most influence the subnetwork completion time,

Furthermore, the loss in accuracy due to sampling in the subnetwork analysis procedure seems to be quite minimal even for relatively small sample sizes - see for example Table 2 and Table 3.

All specific computational results documented in this report are for the project network given in Figure 1. (This network is the simplified network from a large naval PERT problem.) A listing of each activity's two-point approximation except for its P and Q is given in Table 1.

Sampling can also be used in the new subnetwork analysis procedure in the somewhat unlikely event that θ and λ are chosen so large that 2^{n_c} for some cluster C is too large. In this case the probability that the v -th configuration of activity completion times $v = 1, \dots, 2^{n_c}$ is selected on a sampling trial is

$$p_v = \prod_{i=1}^{n_c} [P_i(1 - \delta_{v,i}) + Q_i\delta_{v,i}]$$

where

Figure 1. A Project Network

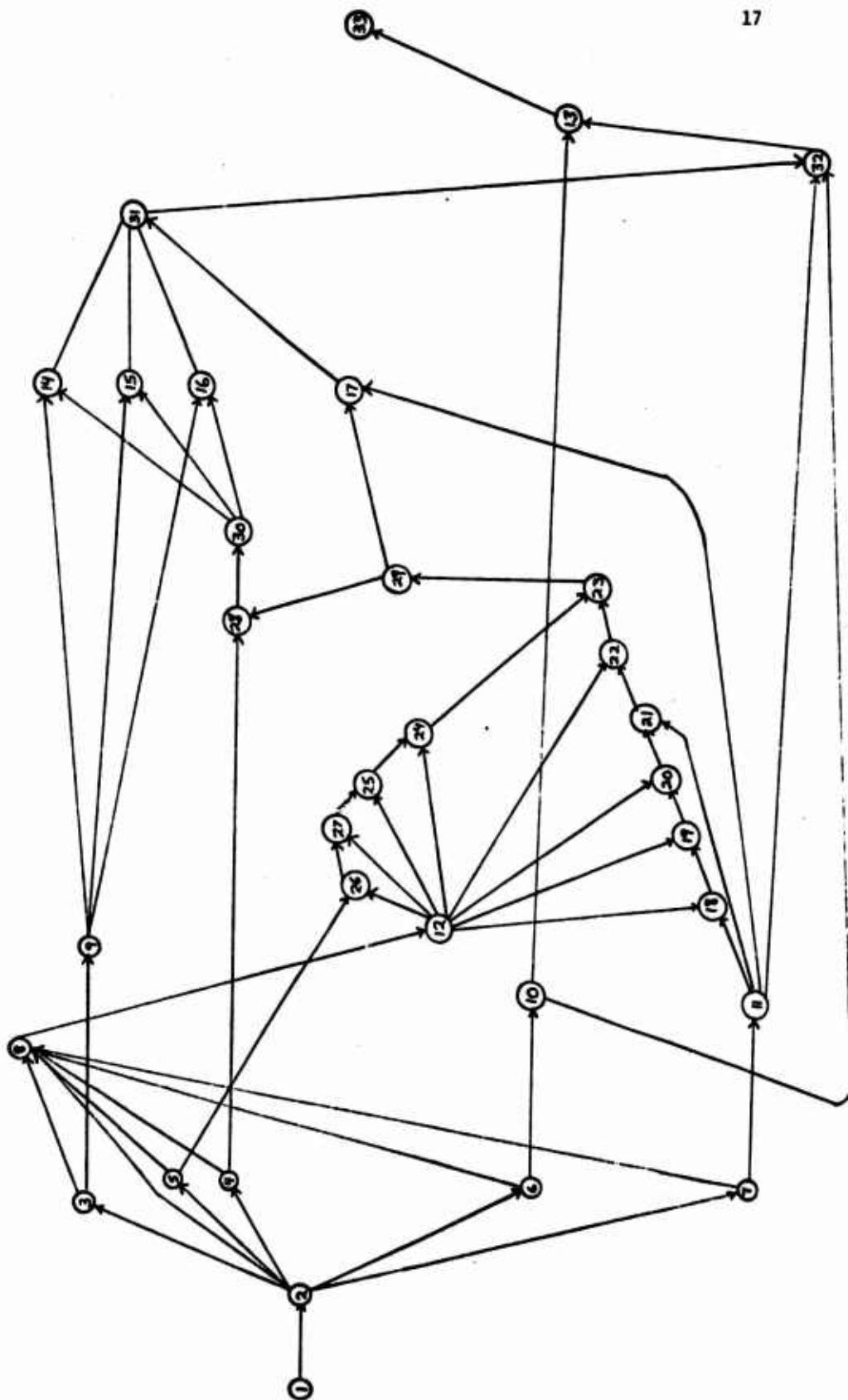


Table 1. The Upper and Lower Points in the Two-Point Approximations to the Activity Completion Time Distributions for the Project Network in Figure 1

Activity Number	Origin Node	Terminal Node	Lower Point	Upper Point
1	1	2	0.0	0.0
2	8	12	336.27	429.47
3	2	3	57.47	89.96
4	2	4	57.47	89.96
5	2	5	57.47	89.96
6	2	6	57.47	89.96
7	2	7	57.47	89.96
8	3	8	68.96	107.95
9	4	8	68.96	107.95
10	5	8	68.96	107.95
11	6	8	68.96	107.95
12	7	8	68.96	107.95
13	2	8	150.36	193.49
14	6	10	333.96	403.85
15	3	9	333.96	403.85
16	7	11	355.10	409.85
17	11	18	141.75	221.90
18	10	13	672.36	783.11
19	9	14	560.89	660.00
20	9	15	560.89	660.00
21	9	16	560.89	660.00
22	11	17	542.80	638.71
23	12	18	111.10	173.92
24	18	19	256.03	346.98
25	12	19	302.80	400.67
26	12	20	311.95	410.71
27	11	21	423.58	530.74
28	12	22	315.35	415.71
29	19	20	7.66	11.99
30	20	21	16.77	22.55
31	21	22	11.49	17.99
32	22	23	39.54	48.87
33	12	26	301.91	400.86
34	5	26	767.09	892.74
35	12	27	350.31	460.06
36	12	24	382.32	464.98
37	12	25	385.28	461.54
38	25	24	11.49	17.99
39	24	23	16.28	23.00
40	26	27	7.66	11.99
41	27	25	20.86	28.29
42	4	28	810.17	976.10
43	23	29	15.32	23.99

Activity Number	Origin Node	Terminal Node	Lower Point	Upper Point
44	28	30	15.32	23.99
45	14	31	57.47	89.96
46	16	31	49.81	77.96
47	15	31	53.64	83.96
48	17	31	88.12	137.94
49	31	32	3.83	6.00
50	32	13	49.81	77.96
51	13	33	109.01	152.63
52	11	32	745.50	811.32
53	10	32	714.74	799.83
54	30	14	0.0	0.0
55	30	16	0.0	0.0
56	30	15	0.0	0.0
57	29	17	0.0	0.0
58	29	28	0.0	0.0

Table 2. The Effect of Sampling in the Original Subnetwork Analysis
 Procedure: Percentiles of the Project Completion Time Distribution

Percentiles	Sample Sizes							
	$(\theta, \lambda) = (0,0)$			$(\theta, \lambda) = (.25,0)$				
	$2^{n_U} = 2^6$	2^5	2^4	$2^{n_U} = 2^{11}$	1000	500	200	100
.05	1360	1363	1377	1381	1381	1381	1384	1384
.10	1383	1390	1413	1409	1406	1411	1406	1414
.15	1403	1413	1423	1424	1421	1429	1424	1423
.20	1416	1423	1436	1441	1436	1443	1441	1446
.25	1430	1430	1446	1451	1448	1451	1451	1453
.30	1446	1453	1466	1458	1456	1461	1458	1463
.35	1459	1479	1472	1468	1466	1471	1468	1471
.40	1476	1486	1476	1478	1473	1478	1476	1478
.45	1499	1499	1512	1483	1483	1483	1483	1483
.50	1542	1542	1516	1488	1486	1491	1491	1488
.55	1578	1578	1575	1496	1496	1498	1498	1498
.60	1578	1578	1575	1501	1501	1505	1501	1503
.65	1602	1602	1579	1513	1510	1513	1513	1513
.70	1605	1605	1602	1515	1515	1515	1523	1515
.75	1605	1618	1605	1525	1525	1525	1528	1525
.80	1621	1618	1605	1533	1533	1530	1538	1530
.85	1622	1622	1618	1543	1543	1543	1545	1543
.90	1645	1622	1622	1558	1558	1558	1570	1577
.95	1648	1645	1648	1585	1585	1582	1587	1597
.975	1648	1648	1648	1607	1607	1605	1605	1622
.99	1648	1648	1648	1625	1625	1625	1622	1647
1.00	1648	1648	1648	1649	1649	1649	1649	1649

Table 3. The Effect of Skewed Activity Completion Time Distributions on the Subnetwork Analysis Procedures.

Estimated Percentiles of the Project Completion Time Distribution					
Percentile	Monte Carlo	Original Subnetwork Analysis Procedure (Sample Size = 1000/cluster)		New Subnetwork Analysis Procedure (Sample Size = 1000/cluster)	
		1st Run	2nd Run	1st Run	2nd Run
.05	1405	1402	1396	1370	1374
.10	1423	1416	1416	1390	1398
.15	1435	1430	1433	1402	1410
.20	1447	1438	1443	1425	1437
.25	1457	1448	1449	1437	1449
.30	1467	1453	1456	1450	1453
.35	1477	1462	1466	1461	1461
.40	1486	1469	1473	1484	1500
.45	1496	1476	1479	1504	1524
.50	1507	1481	1483	1528	1528
.55	1516	1487	1489	1528	1528
.60	1527	1495	1496	1528	1528
.65	1538	1500	1499	1563	1560
.70	1549	1510	1512	1567	1567
.75	1560	1517	1516	1579	1579
.80	1580	1526	1529	1587	1587
.85	1603	1541	1542	1598	1602
.90	1627	1550	1552	1610	1614
.95	1675	1575	1582	1622	1673
1.00	2019	1646	1648	1720	1720

* The "1st run" and "2nd run" correspond to two different samples with different initializations of the random number generator.

$\delta_{v,i} = 1$ if the time for the i -th activity in C is u_i in the v -th configuration

$= 0$ if the time for the i -th activity in C is l_i in the v -th configuration.

Then with $I_t(t_v)$ as before the estimated bounds from C are

$$\hat{T}_r^-(C) = \frac{2^{n_c}}{N} \sum_{v=1}^{2^{n_c}} w_v p_v t_v^r$$

and

$$\hat{F}^+(t; C) = \frac{2^{n_c}}{N} \sum_{v=1}^{2^{n_c}} w_v p_v I_t(t_v)$$

where w_v is the number of times the v -th configuration appears in the sample, N is the sample size, and t_v is the critical path time corresponding to the v -th configuration of activity completion times with the completion time for each activity A not in C equal to l_A . Similar modifications are made for $T_r^+(C)$ and $F^-(t; C)$. The corresponding estimators $T_r^-(\theta, \lambda)$, $T_r^+(\theta, \lambda)$, $F^-(t; \theta, \lambda)$, and $F^+(t; \theta, \lambda)$ are all unbiased.

A computer implementation of the new subnetwork analysis procedure including the sampling option is documented in Appendix B. In addition to reducing the computational effort, the new subnetwork analysis procedure allows the user to specify any probabilities (P, Q) for (l, u) instead of requiring $(1/2, 1/2)$. This would not be of great significance if all activity completion time distributions were symmetric. However, since many completion time distributions are skewed, the ability to specify (P, Q) can be a real advantage. To exemplify this advantage, the completion time for each activity in Figure 1 was taken to be a linearly

transformed chi-square random variable, $(\chi_3^2 - c_1)/c_2$, where c_1 and c_2 were determined so that the points in Table 1 corresponded to the 15-th and 85-th percentiles respectively. This made the activity's completion time distribution highly skewed. Then the corresponding project completion time distribution was approximated using

- (i) a Monte Carlo simulation of size 1000,
- (ii) the new subnetwork analysis procedure with $(P, Q) = (1/2, 1/2)$ for all activities, and
- (iii) the new procedure with each activity's (P, Q) chosen so that the mode and first two moments of its two-point approximation equalled the mode and first two moments of its transformed chi-square distribution.

The results are given in Table 3. If the Monte Carlo approximation is used as a basis for comparison, the value of being able to specify (P, Q) is obvious.

The Monte Carlo PERT simulation program used in the above experiment is documented in Appendix D and supercedes the Monte Carlo PERT program given in Technical Report No. 48.

In the special case where every activity's (P, Q) is $(1/2, 1/2)$, the $\hat{T}_r^-(C)$ and $\hat{F}^+(t; C)$ can be computed on the basis of the 2^n critical path times corresponding to

- (a) the completion time for each activity A not in C being equal to the mean, m_A , and
- (b) the completion times for the activities in C being at each of the 2^n possible combinations of their upper and lower points.

This is a change from the usual computation in that the activities not in C are at their means here instead of their lower points. This change

will tend to improve the estimators $T_r^-(\theta, \lambda)$ and $F^+(t; \theta, \lambda)$. However, this change is only guaranteed not to invalidate Theorems 4 and 7 when all (P, Q) are $(1/2, 1/2)$. The computer implementation of the new subnetwork analysis procedure includes the option to make this change.

3. Conclusion

The new subnetwork analysis procedure

- (i) forms associates, eliminants, and clusters in essentially the same way as the original procedure,
- (ii) bounds the project's completion time moments and distribution primarily on the basis of the individual clusters instead of on a pooled cluster, and
- (iii) approximates an activity's completion time distribution by a two-point distribution with possibly unequal probabilities for the two points instead of always equal probabilities.

The advantage of (ii) is that much much fewer critical path times need to be evaluated in determining the bounds on the project completion time. The advantage to (iii) is the ability to better approximate skewed activity completion time distributions.

Computer implementations of the new subnetwork analysis procedure, the original subnetwork analysis procedure, and a Monte Carlo simulation algorithm are documented in Appendices B, C, and D respectively. Both subnetwork analysis procedures include options to use sampling for large clusters.

References

Sielken, R. L. Jr., L. J. Ringer, H. O. Hartley, and E. Arseven,
"Statistical Critical Path Analysis in Acyclic Stochastic
Networks: Statistical PERT," Institute of Statistics, Texas
A&M University Project Themis Technical Report No. 48,
November 1974.

APPENDIX A

Proof of Theorem 1

The principle objective of this appendix is to prove the following theorem:

Theorem 1: In any acyclic network with no cut vertices there is at least one bridge for any pair of consecutive arcs.

The networks considered in this appendix are assumed to be acyclic, have no cut vertices, and have one source and one sink. Also the two arcs A_1 and A_2 are any two adjacent (consecutive) arcs with A_1 preceding A_2 .

Definition 1: A bridge over A_1 and A_2 is any arc, say A_3 , such that all paths from the source to the sink passing through A_3 do not pass through either A_1 or A_2 .

Definition 2: An origin violator of A_1 and A_2 is an arc, say A_3 , such that there exists a path from the terminal node of A_1 to the sink which passes through A_3 .

Definition 3. A terminal violator of A_1 and A_2 is an arc, say A_3 , such that there exists a path from the source to the terminal node of A_1 which passes through A_3 .

An intuitive feeling for these definitions can be obtained by considering any path P^* from the source to the sink which passes through A_1 and A_2 . If an arc A_3 is an origin violator, then there is a path from the source to the sink which follows along P^* through the terminal

Lemma 1: Every branch in the network is either a bridge over A_1 and A_2 , an origin violator of A_1 and A_2 , or a terminal violator of A_1 and A_2 .

Proof of Lemma 1: Suppose that A_3 is not a bridge. Then there exists a path P from the source to the sink which contains A_3 and either A_1 or A_2 .

Suppose that P contains A_1 . If $A_3 = A_1$ or A_3 precedes A_1 on P , then P contains a path from the source to the terminal node of A_1 which passes through A_3 , and A_3 would be a terminal violator. On the other hand, if A_3 follows A_1 on P , then P contains a path from the terminal node of A_1 to the sink which passes through A_3 , and A_3 would be an origin violator.

Suppose that P contains A_2 . If $A_3 = A_2$ or A_3 comes after A_2 on P , then P contains a path from the terminal node of A_1 (the origin node of A_2) to the sink which passes through A_3 , and A_3 would be an origin violator. If A_3 comes before A_2 on P , then P contains a path from the source to the terminal node of A_1 (the origin node of A_2) which passes through A_3 , and A_3 would be a terminal violator. This completes the proof of Lemma 1.

Lemma 2: A_1 is a terminal violator of A_1 and A_2 , and A_2 is an origin violator of A_1 and A_2 .

Lemma 3: Any arc A_3 cannot be both an origin violator of A_1 and A_2 and a terminal violator of A_1 and A_2 .

Proof of Lemma 3: Suppose that an arc A_3 is both an origin violator and a terminal violator. Since A_3 is an origin violator, there exists a path from the terminal node of A_1 to the origin node of A_3 . Since A_3 is a terminal violator, there exists a path from the terminal node of A_3 to the terminal node of A_1 . The existence of these two paths, however, implies the existence of a circuit which contradicts the given acyclic structure of the network. This completes the proof of Lemma 3.

Proof of Theorem 1: Since the terminal node of A_1 cannot be a cut vertex, there exists a path P from the source to the sink which does not pass through the terminal node of A_1 . Denote the arcs on P by $C_1, C_2, \dots, C_p$ with C_{i-1} preceding C_i on P .

Suppose that none of $C_1, C_2, \dots, C_p$ are bridges over A_1 and A_2 . Then Lemma 1 and Lemma 3 together imply that each of $C_1, C_2, \dots, C_p$ is either an origin violator of A_1 and A_2 or a terminal violator of A_1 and A_2 but not both. Since the origin node of C_1 is the source, C_1 cannot be an origin violator and must be a terminal violator.

Similarly, since the terminal node of C_p is the sink, C_p cannot be a terminal violator and must be an origin violator. Hence, there exists $j \geq 1$ such that $C_1, C_2, \dots, C_j$ are all terminal violators and C_{j+1} is an origin violator.

Since C_j is a terminal violator, there exists a path from the terminal node of C_j to the terminal node of A_1 . Furthermore, since C_{j+1} is an origin violator, there is a path from the terminal node of A_1 to the origin node of C_{j+1} (the terminal node of C_j). These two paths imply the existence of a circuit from the terminal node of

C_j to the terminal node of A_1 and then back to the terminal node of C_j . (The definition of P implies that the terminal node of C_j is not the terminal node of A_1 .) This contradicts the given acyclic structure of the network and completes the proof of Theorem 1.

APPENDIX B

New Subnetwork Analysis Program

The New Subnetwork Analysis Program is an implementation of the analytical procedure described in Section 1 of this report. The basic required input is

- (a) an acyclic network with one source and one sink,
- (b) two points from each component activity's completion time distribution
- (c) probability P to be associated with the lower point, and
- (d) specified values for the algorithm parameters θ and λ .

The output is mainly

- (a) upper and lower bounds on the moments of the network completion time, $T_r^+(\theta, \lambda)$ and $T_r^-(\theta, \lambda)$ $r = 1, 2, \dots, 10$;
- (b) upper and lower bounds on the distribution function of the network completion time, $F^+(\cdot; \theta, \lambda)$ and $F^-(\cdot; \theta, \lambda)$; and
- (c) an approximate network completion time distribution,

$$F(\cdot; \theta, \lambda) = \frac{1}{2} [F^+(\cdot; \theta, \lambda) + F^-(\cdot; \theta, \lambda)].$$

The basic computational technique for determining critical path times is the Simplex Algorithm. This algorithm is applied to the dual problem. The Simplex Algorithm is used instead of the standard network analysis techniques because the Simplex Algorithm is ideally suited for the type of parametric programming required to evaluate several critical path times when only the activity times vary from one problem to the next.

A listing of the Subnetwork Analysis Program and a program flowchart are given at the end of this appendix.

Specific Input Instructions:

Card 1. Col. 1-3: The number of activities in the network, Format (I3).

Col. 4-6: The number of nodes in the network, Format (I3).

For each activity one card with:

Col. 11-15: The origin node of the activity, Format (I5).

Col. 21-25: The terminal node of the activity, Format (I5).

Col. 31-40: The lower point on the activity's completion time distribution, Format (F10.0).

Col. 41-50: The upper point on the activity's completion time distribution, Format (F10.0).

Col. 51-60: The probability P to be associated with the lower point (1-P will be associated with the upper point), Format (F10.5).

Next Card. Col. 1: OPTON1. OPTON1=1 implies that the program will terminate after the clusters have been formed on the basis of associates and eliminants. OPTON#1 implies that the program will follow the normal procedure.

Col. 2: OPTON2. OPTON2=1 implies that the lower bounds on the moments of the project completion time and the upper bound on its distribution will be determined by using all activity times outside the cluster at their means instead of their lower points. This is only guaranteed to be a valid procedure when all $(P, Q) = (1/2, 1/2)$. OPTON2#1 implies that the program will follow the normal procedure.

Next Card. Col. 1-3: IEDF. The program computes an absolute upper and lower bound for the network completion time. This range is subdivided into IEDF equal parts and the approximate distribution function (F^+ , F^- , $\hat{F}$) values are printed at each of these dividing points. IEDF would usually be between 10 and 100. IEDF, Format (I3).

Next Card. Col. 1-5: θ , Format (F5.2).

Col.6-10: λ , Format (F5.2).

Next Card. Col.1-10: SAMSIZ. The number of activity time configurations to be randomly selected for explicit consideration in each cluster analysis. If $SAMSIZ < 0$ or $SAMSIZ > 2^{\frac{n}{c}}$, all activity time configurations will be explicitly considered - no random sampling will be done. Format (I10).

The nodes should be numbered 1, 2, ..., n with the source being number 1, the sink being number n, and the other node numbers being arbitrary. The activities should be numbered 1, 2, ... in any order desired.

Current Dimension Restrictions:

Currently the program is dimensioned for a maximum of

60 Activities

40 Nodes

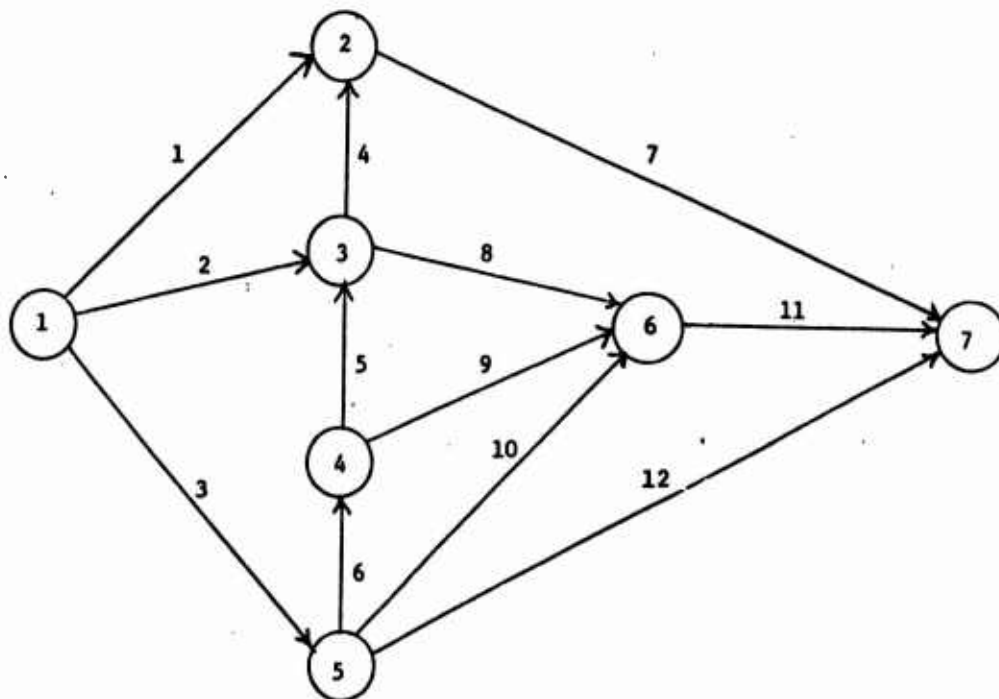
25 Clusters

25 Activities/Cluster and $IEDF \leq 100$.

Example:

The Program's input and output are illustrated in terms of the network in Figure B-1.

Figure B-1: New Subnetwork Analysis Program Example Network



SAMPLE INPUT

35

012C07

1	2	17.26	19.44	.5
1	3	19.26	21.44	.9
1	5	12.76	15.91	.3
3	2	3.51	4.01	.8
4	3	3.01	5.43	.75
5	4	3.52	4.25	.5
2	7	13.75	14.48	.65
3	6	5.05	8.43	.1
4	6	5.36	6.51	.7
5	6	8.78	11.44	.55
6	7	15.76	17.21	.5
5	7	14.32	18.35	.9

00

020

1. 1.

-1

SAMPLE OUTPUT

INITIAL INPUT

ACTIVITY	ORIGIN	TERMINAL	LOWER POINT	UPPER POINT	MEAN	STANDARD DEVIATION	PROB. LOWER ST.
1	1	2	17.2600	19.4400	18.3500	1.0900	0.5000
2	1	3	19.2600	21.4400	19.4750	0.6540	0.9000
3	1	5	12.7600	15.0100	14.3650	1.4435	0.3000
4	3	2	3.5100	4.2100	3.6100	0.2000	0.8000
5	4	3	3.0100	5.4300	3.6150	1.0479	0.7500
6	5	4	3.5200	4.2500	3.8850	0.3650	0.5000
7	2	7	13.7500	14.4900	14.0055	0.3482	0.6500
8	3	6	5.7500	9.4300	9.0920	1.0140	0.1000
9	4	6	5.3600	6.5100	5.7050	0.5270	0.7000
10	5	6	8.7800	11.4400	9.0770	1.3233	0.5500
11	6	7	15.7600	17.2100	16.0500	0.5800	0.8000
12	5	7	14.3200	18.3500	14.7230	1.2090	0.9000

THE CRITICAL PATH TIME WHEN EACH ACTIVITY'S COMPLETION TIME IS SET EQUAL TO ITS MEAN IS = 0.466070 02

THE 4 NODES ON THE CRITICAL PATH ARE AS FOLLOWS BEGINNING WITH THE TERMINAL NODE:

7. 6. 3. 4. 5. 1.

THE 5 CRITICAL ACTIVITIES ARE AS FOLLOWS BEGINNING WITH THE TERMINAL ACTIVITY:

11. 8. 5. 6. 3.

THETA = 0.100000 01 LAMBDA = 0.100000 01

A LOWER BOUND ON THE EXPECTED CRITICAL PATH TIME IS = 0.401000 02

THE 6 NODES ON THE CRITICAL PATH ARE AS FOLLOWS BEGINNING WITH THE TERMINAL NODE:

7. 6. 3. 4. 5. 1.

THE 5 CRITICAL ACTIVITIES ARE AS FOLLOWS BEGINNING WITH THE TERMINAL ACTIVITY:

11. 8. 5. 6. 3.

A UPPER BOUND ON THE EXPECTED CRITICAL PATH TIME IS = 0.512300 02

THE 6 NODES ON THE CRITICAL PATH ARE AS FOLLOWS BEGINNING WITH THE TERMINAL NODE:

7. 6. 3. 4. 5. 1.

THE 5 CRITICAL ACTIVITIES ARE AS FOLLOWS BEGINNING WITH THE TERMINAL ACTIVITY:

11. 8. 5. 6. 3.

THE ASSOCIATES ARE NOW IDENTIFIED:

THE NUMBER OF ASSOCIATES ASSOCIATED WITH THE	1-TH CRITICAL PATH ACTIVITY, I.E. ACTIVITY	11. IS = 0
THE NUMBER OF ASSOCIATES ASSOCIATED WITH THE	2-TH CRITICAL PATH ACTIVITY, I.E. ACTIVITY	8. IS = 0
THE NUMBER OF ASSOCIATES ASSOCIATED WITH THE	3-TH CRITICAL PATH ACTIVITY, I.E. ACTIVITY	5. IS = 0
THE NUMBER OF ASSOCIATES ASSOCIATED WITH THE	4-TH CRITICAL PATH ACTIVITY, I.E. ACTIVITY	6. IS = 0
THE NUMBER OF ASSOCIATES ASSOCIATED WITH THE	5-TH CRITICAL PATH ACTIVITY, I.E. ACTIVITY	3. IS = 0

THERE ARE 5 NONEMPTY CLUSTERS AFTER POOLING ON THE BASIS OF ASSOCIATES ONLY.

THE ACTIVITIES IN THE 1-TH CLUSTER ARE AS FOLLOWS:

11.

THE ACTIVITIES IN THE 2-TH CLUSTER ARE AS FOLLOWS:

9.

THE ACTIVITIES IN THE 3-TH CLUSTER ARE AS FOLLOWS:

5.

THE ACTIVITIES IN THE 4-TH CLUSTER ARE AS FOLLOWS:

6.

THE ACTIVITIES IN THE 5-TH CLUSTER ARE AS FOLLOWS:

3.

THE CLUSTER TO WHICH EACH ACTIVITY BELONGS:

(ZERO IMPLIES THAT THE ACTIVITY IS NOT IN ANY CLUSTER)

THE 1-TH ACTIVITY IS IN THE 0-TH CLUSTER
 THE 2-TH ACTIVITY IS IN THE 0-TH CLUSTER
 THE 3-TH ACTIVITY IS IN THE 5-TH CLUSTER
 THE 4-TH ACTIVITY IS IN THE 0-TH CLUSTER
 THE 5-TH ACTIVITY IS IN THE 3-TH CLUSTER
 THE 6-TH ACTIVITY IS IN THE 4-TH CLUSTER
 THE 7-TH ACTIVITY IS IN THE 2-TH CLUSTER
 THE 8-TH ACTIVITY IS IN THE 2-TH CLUSTER
 THE 9-TH ACTIVITY IS IN THE 0-TH CLUSTER
 THE 10-TH ACTIVITY IS IN THE 0-TH CLUSTER
 THE 11-TH ACTIVITY IS IN THE 1-TH CLUSTER
 THE 12-TH ACTIVITY IS IN THE 0-TH CLUSTER

THERE ARE 7 ACTIVITIES NOT IN ANY CLUSTER YET.

THE ELIMINANTS OF EACH NON-CRITICAL-PATH ACTIVITY ARE NOW DETERMINED:

THERE ARE 7 ACTIVITIES NOT ON THE CRITICAL PATH. THEY ARE AS FOLLOWS:

- 1.
- 2.
- 3.
- 4.
- 5.
- 6.
- 7.

THE COMPLETION TIME FOR THE 1-TH ACTIVITY HAS BEEN CHANGED TO C-19440D 02
THERE ARE 0 ELIMINANTS CORRESPONDING TO ACTIVITY 1

THE COMPLETION TIME FOR THE 2-TH ACTIVITY HAS BEEN CHANGED TO 0-20142D 02
THERE ARE 0 ELIMINANTS CORRESPONDING TO ACTIVITY 2

THE COMPLETION TIME FOR THE 4-TH ACTIVITY HAS BEEN CHANGED TO 0-39100D 01
THERE ARE 0 ELIMINANTS CORRESPONDING TO ACTIVITY 4

THE COMPLETION TIME FOR THE 7-TH ACTIVITY HAS BEEN CHANGED TO 0-14354D 02
THERE ARE 0 ELIMINANTS CORRESPONDING TO ACTIVITY 7

THE COMPLETION TIME FOR THE 9-TH ACTIVITY HAS BEEN CHANGED TO C-62320D 01
THERE ARE 0 ELIMINANTS CORRESPONDING TO ACTIVITY 9

THE COMPLETION TIME FOR THE 10-TH ACTIVITY HAS BEEN CHANGED TO 0-11300D 02
THERE ARE 0 ELIMINANTS CORRESPONDING TO ACTIVITY 10

THE COMPLETION TIME FOR THE 12-TH ACTIVITY HAS BEEN CHANGED TO C-15932D 02
THERE ARE 0 ELIMINANTS CORRESPONDING TO ACTIVITY 12

THERE ARE 5 CLUSTERS.

THERE ARE 1 ACTIVITIES IN THE 1-TH CLUSTER. THEY ARE AS FOLLOWS:

11.

1 CLUSTERS HAVE BEEN POOLED TO MAKE THIS CLUSTER. THEY WERE AS FOLLOWS:

1.

THERE ARE 1 ACTIVITIES IN THE 2-TH CLUSTER. THEY ARE AS FOLLOWS:

8.

1 CLUSTERS HAVE BEEN POOLED TO MAKE THIS CLUSTER. THEY WERE AS FOLLOWS:

2.

THERE ARE 1 ACTIVITIES IN THE 3-TH CLUSTER. THEY ARE AS FOLLOWS:

5.

1 CLUSTERS HAVE BEEN POOLED TO MAKE THIS CLUSTER. THEY WERE AS FOLLOWS:

1.

THERE ARE 1 ACTIVITIES IN THE 4-TH CLUSTER. THEY ARE AS FOLLOWS:

6.

1 CLUSTERS HAVE BEEN POOLED TO MAKE THIS CLUSTER. THEY WERE AS FOLLOWS:

4.

THERE ARE 1 ACTIVITIES IN THE 5-TH CLUSTER. THEY ARE AS FOLLOWS:

1.

1 CLUSTERS HAVE BEEN POOLED TO MAKE THIS CLUSTER. THEY WERE AS FOLLOWS:

5.

THE FOLLOWING TABLES WERE DETERMINED CONSIDERING ALL ACTIVITY CONFIGURATIONS.

THE INITIALIZATION PARAMETER FOR ANY SAMPLING IS $1Y =$

77

A LOWER BOUND. T-(1:THETA:LAMBDA). ON THE 1-TH MOMENT OF THE NETWORK COMPLETION TIME = 0.43142D 02
 A LOWER BOUND. T-(2:THETA:LAMBDA). ON THE 2-TH MOMENT OF THE NETWORK COMPLETION TIME = 0.18623D 04
 A LOWER BOUND. T-(3:THETA:LAMBDA). ON THE 3-TH MOMENT OF THE NETWORK COMPLETION TIME = 0.80428D 05
 A LOWER BOUND. T-(4:THETA:LAMBDA). ON THE 4-TH MOMENT OF THE NETWORK COMPLETION TIME = 0.34752D 07
 A LOWER BOUND. T-(5:THETA:LAMBDA). ON THE 5-TH MOMENT OF THE NETWORK COMPLETION TIME = 0.15023D 09
 A LOWER BOUND. T-(6:THETA:LAMBDA). ON THE 6-TH MOMENT OF THE NETWORK COMPLETION TIME = 0.84568D 10
 A LOWER BOUND. T-(7:THETA:LAMBDA). ON THE 7-TH MOMENT OF THE NETWORK COMPLETION TIME = 0.28108D 12
 A LOWER BOUND. T-(8:THETA:LAMBDA). ON THE 8-TH MOMENT OF THE NETWORK COMPLETION TIME = 0.12165D 14
 A LOWER BOUND. T-(9:THETA:LAMBDA). ON THE 9-TH MOMENT OF THE NETWORK COMPLETION TIME = 0.52667D 15
 A LOWER BOUND. T-(10:THETA:LAMBDA). ON THE 10-TH MOMENT OF THE NETWORK COMPLETION TIME = 0.22809D 17

AN UPPER BOUND ON THE NETWORK COMPLETION TIME DISTRIBUTION: F+(.:THETA:LAMBDA)

F+(0.40657D 02: 0.10000D 01: 0.10000D 01) = 0.10000D 00
 F+(0.41217D 02: 0.10000D 01: 0.10000D 01) = 0.10000D 00
 F+(0.41770D 02: 0.10000D 01: 0.10000D 01) = 0.10000D 00
 F+(0.42326D 02: 0.10000D 01: 0.10000D 01) = 0.10000D 00
 F+(0.42881D 02: 0.10000D 01: 0.10000D 01) = 0.10000D 00
 F+(0.43439D 02: 0.10000D 01: 0.10000D 01) = 0.10000D 00
 F+(0.43996D 02: 0.10000D 01: 0.10000D 01) = 0.10000D 01
 F+(0.44552D 02: 0.10000D 01: 0.10000D 01) = 0.10000D 01
 F+(0.45109D 02: 0.10000D 01: 0.10000D 01) = 0.10000D 01
 F+(0.45665D 02: 0.10000D 01: 0.10000D 01) = 0.10000D 01
 F+(0.46221D 02: 0.10000D 01: 0.10000D 01) = 0.10000D 01
 F+(0.46778D 02: 0.10000D 01: 0.10000D 01) = 0.10000D 01
 F+(0.47334D 02: 0.10000D 01: 0.10000D 01) = 0.10000D 01
 F+(0.47891D 02: 0.10000D 01: 0.10000D 01) = 0.10000D 01
 F+(0.48447D 02: 0.10000D 01: 0.10000D 01) = 0.10000D 01
 F+(0.49004D 02: 0.10000D 01: 0.10000D 01) = 0.10000D 01
 F+(0.49560D 02: 0.10000D 01: 0.10000D 01) = 0.10000D 01
 F+(0.50117D 02: 0.10000D 01: 0.10000D 01) = 0.10000D 01
 F+(0.50673D 02: 0.10000D 01: 0.10000D 01) = 0.10000D 01
 F+(0.51230D 02: 0.10000D 01: 0.10000D 01) = 0.10000D 01

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 PERMIT FULLY LEGIBLE PRODUCTION

AN UPPER ROUND. T+(1:THETA.LAMBDA). ON THE	1-TH MOMENT OF THE NETWORK COMPLETION TIME =	0.49415D 02
AN UPPER ROUND. T+(2:THETA.LAMBDA). ON THE	2-TH MOMENT OF THE NETWORK COMPLETION TIME =	0.24429D 04
AN UPPER ROUND. T+(3:THETA.LAMBDA). ON THE	3-TH MOMENT OF THE NETWORK COMPLETION TIME =	0.12083D 06
AN UPPER ROUND. T+(4:THETA.LAMBDA). ON THE	4-TH MOMENT OF THE NETWORK COMPLETION TIME =	0.59789D 07
AN UPPER ROUND. T+(5:THETA.LAMBDA). ON THE	5-TH MOMENT OF THE NETWORK COMPLETION TIME =	0.296C0D 09
AN UPPER ROUND. T+(6:THETA.LAMBDA). ON THE	6-TH MOMENT OF THE NETWORK COMPLETION TIME =	0.14661D 11
AN UPPER ROUND. T+(7:THETA.LAMBDA). ON THE	7-TH MOMENT OF THE NETWORK COMPLETION TIME =	0.72655D 12
AN UPPER ROUND. T+(8:THETA.LAMBDA). ON THE	8-TH MOMENT OF THE NETWORK COMPLETION TIME =	0.36023D 14
AN UPPER ROUND. T+(9:THETA.LAMBDA). ON THE	9-TH MOMENT OF THE NETWORK COMPLETION TIME =	0.17870D 16
AN UPPER ROUND. T+(10:THETA.LAMBDA). ON THE	10-TH MOMENT OF THE NETWORK COMPLETION TIME =	0.88694D 17

A LOWER ROUND ON THE NETWORK COMPLETION TIME DISTRIBUTION: F-(.;THETA;LAMUDA)

F-(	0.406570 02:	0.100000 01:	0.100000 01) =	0.000000 00
F-(	0.412130 02:	0.100000 01:	0.100000 01) =	0.000000 00
F-(	0.417700 02:	0.100000 01:	0.100000 01) =	0.000000 00
F-(	0.423260 02:	0.100000 01:	0.100000 01) =	0.000000 00
F-(	0.428820 02:	0.100000 01:	0.100000 01) =	0.000000 00
F-(	0.434380 02:	0.100000 01:	0.100000 01) =	0.000000 00
F-(	0.439940 02:	0.100000 01:	0.100000 01) =	0.000000 00
F-(	0.445500 02:	0.100000 01:	0.100000 01) =	0.000000 00
F-(	0.451060 02:	0.100000 01:	0.100000 01) =	0.000000 00
F-(	0.456620 02:	0.100000 01:	0.100000 01) =	0.000000 00
F-(	0.462180 02:	0.100000 01:	0.100000 01) =	0.000000 00
F-(	0.467740 02:	0.100000 01:	0.100000 01) =	0.000000 00
F-(	0.473300 02:	0.100000 01:	0.100000 01) =	0.000000 00
F-(	0.478860 02:	0.100000 01:	0.100000 01) =	0.000000 00
F-(	0.484420 02:	0.100000 01:	0.100000 01) =	0.000000 00
F-(	0.490000 02:	0.100000 01:	0.100000 01) =	0.000000 00
F-(	0.495560 02:	0.100000 01:	0.100000 01) =	0.000000 00
F-(	0.501120 02:	0.100000 01:	0.100000 01) =	0.000000 00
F-(	0.506680 02:	0.100000 01:	0.100000 01) =	0.000000 00
F-(	0.512240 02:	0.100000 01:	0.100000 01) =	0.000000 01

NOT AVAILABLE TO THE PUBLIC
 SECURITY INFORMATION
 EXCLUDED FROM AUTOMATIC
 DOWNGRADING AND
 DECLASSIFICATION

AN APPROXIMATE NETWORK COMPLETION TIME DISTRIBUTION:

$$F(.;:THETA_AMRDA) = .5 * (F(.;:THETA_LAMRDA) + F(.;:THETA_LAMHCA))$$

F(	0.406570	02:	0.100000	01:	0.100000	01:	=	C.500000-01
F(	0.412130	02:	0.100000	01:	0.100000	01:	=	C.500000-01
F(	0.417700	02:	0.100000	01:	0.100000	01:	=	C.500000-01
F(	0.423260	02:	0.100000	01:	0.100000	01:	=	C.500000-01
F(	0.428830	02:	0.100000	01:	0.100000	01:	=	C.500000-01
F(	0.434390	02:	0.100000	01:	0.100000	01:	=	C.500000-01
F(	0.439960	02:	0.100000	01:	0.100000	01:	=	C.500000 00
F(	0.445520	02:	0.100000	01:	0.100000	01:	=	C.500000 00
F(	0.451090	02:	0.100000	01:	0.100000	01:	=	C.500000 00
F(	0.456650	02:	0.100000	01:	0.100000	01:	=	C.500000 00
F(	0.462210	02:	0.100000	01:	0.100000	01:	=	C.500000 00
F(	0.467780	02:	0.100000	01:	0.100000	01:	=	C.500000 00
F(	0.473340	02:	0.100000	01:	0.100000	01:	=	C.500000 00
F(	0.478910	02:	0.100000	01:	0.100000	01:	=	C.500000 00
F(	0.484470	02:	0.100000	01:	0.100000	01:	=	C.500000 00
F(	0.490040	02:	0.100000	01:	0.100000	01:	=	C.500000 00
F(	0.495610	02:	0.100000	01:	0.100000	01:	=	C.500000 00
F(	0.501170	02:	0.100000	01:	0.100000	01:	=	C.500000 00
F(	0.506730	02:	0.100000	01:	0.100000	01:	=	C.500000 00
F(	0.512300	02:	0.100000	01:	0.100000	01:	=	C.100000 01

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C
C      NEW SUBNETWORK ANALYSIS PROGRAM
C
C      IMPLICIT REAL*8 (A-H,O-Z)
C      FOR THE SAKE OF IDENTIFYING THE APPROPRIATE DIMENSIONS, LET
C          M = THE NUMBER OF ACTIVITIES IN THE NETWORK
C          NMM = NUMBER OF NODES IN THE NETWORK
C          NMMP1 = NMM + 1
C          N = M + NMM
C          L = THE LENGTH OF THE CRITICAL PATH
C          C = THE MAXIMUM NUMBER OF BRANCHES IN A CLUSTER
C          IEDF = THE NUMBER OF DIVISIONS IN THE EMPIRICAL
C              DISTRIBUTION FUNCTION
C
C
C      INTEGER TAIL( M),HEAD( M),ASSGRP( L,L ),CLINCL(L,L),EGRP(L)
C      DIMENSION NINCL( C),INCLUS( L, C),NCLINC( L)
C      DIMENSION FD( IEDF),NLEFD(L, IEDF),NSAVE( IEDF),NIB(L)
C      DIMENSION      AVG( L ),THAT(L)
C      DIMENSION LEFT( M ),LEFTO( M ),NONCP(M)
C      DIMENSION INBASE(NMM),XNODE(NMM)
C      DIMENSION XB1(NMMP1),Y1(NMMP1),REDCOS(N),ISTAT(N)
C      DIMENSION ICRITP(L),NINAG(M),ICRITN(L+1),CTIME(N),COT(N)
C      DIMENSION KB(L),IBH(L),FLO(M),FHI(M),SIGMA(M),BIINV(NMMP1,NMMP1)
C      REAL      MOMENT(L,10)
C      DIMENSION PP(M),PQ(M)
C
C      OF COURSE THESE DIMENSIONS ARE MERELY UPPER BOUNDS
C
C      COMMON BIINV,REDCOS,CTIME,XB1,INBASE,HEAD,TAIL,NMMP1,NMM,N,ISTAT
C      COMMON M,MPI
C      INTEGER TAIL(60),HEAD(60),ASSGRP(25,25),CLINCL(25,25),EGRP(25)
C      INTEGER SAMSIZ,PANSAM
C      DIMENSION NINCL(25),INCLUS(25,25),ACLINC(25)
C      DIMENSION FD(100),NIB(25)
C      REAL*8 NLEFD(25,100),NSAVE(100)
C      DIMENSION AVG(25),THAT(25)
C      DIMENSION INBASE(40)
C      DIMENSION XNODE(40)
C      DIMENSION XB1(41),Y1(41),REDCOS(100),ISTAT(100)
C      DIMENSION BIINV(41,41),KB(25),IBH(25),FLO(60),FHI(60),SIGMA(60)
C      DIMENSION ICRITP(25),NINAG(50),ICRITN(26),COT(100),CTIME(100)
C      DIMENSION LEFT(60),LEFTO(60),NONCP(60)
C      REAL*8 LAMBDA,MOMENT(25,10)
C      DIMENSION PP(60),PQ(60)
C      INTEGER OPTON1,OPTON2
C
C      M = THE NUMBER OF ACTIVITIES IN THE NETWORK
C      NMM = THE NUMBER OF NODES IN THE PERT NETWORK
C      READ(5,100) M,NMM
100  FORMAT(2I3)
C      N=NMM+M
C      MP1=M+1
C      NMMP1=NMM+1
C
C      THE ACTIVITIES ARE DESCRIBED IN TERMS OF THEIR NODES
C      II=THE TAIL NODE, THE ORIGIN NODE
C      JJ=THE HEAD NODE, THE TERMINAL NODE
C      FLO = THE LOWER POINT

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C      FHI = THE UPPER POINT
C      SIGMA = (FHI - FLO)*DSORT( PP*(1-PP)) = STD. DEVIATION
C      PP = THE PROBABILITY OF THE LOWER POINT
C
DO 610 I=1,M
  READ(5,2501) II,JJ,FLO(I),FHI(I),PP(I)
  PQ(I)=1.00-PP(I)
  SIGMA(I)=(FHI(I)-FLO(I))*DSORT( PP(I)*PQ(I))
2501  FORMAT(10X,I5,5X,I5,5X,F10.0,F10.0,F10.5)
  COT(I)=PP(I)*FLO(I)+PQ(I)*FHI(I)
  CTIME(I) = COT(I)
  TAIL(I) = II
610  HEAD(I)=JJ
C      COT = THE ORIGINAL RIGHT-HAND SIDES, I.E. THE MEANS
C      OF FLO AND FHI
C      CTIME = THE CURRENT RIGHT-HAND SIDES
DO 55610 I=MP1,N
55610 CTIME(I) = 0.
C
C      OPTON1 =1 IMPLIES THAT THE PROGRAM WILL TERMINATE AFTER
C      THE CLUSTERS HAVE BEEN FORMED. NO BOUNDS ON
C      THE PROJECT COMPLETION TIME MOMENTS OR
C      DISTRIBUTION WILL BE DETERMINED.
C      OPTON1 NOT= 1 IMPLIES THAT THE NORMAL PROCEDURE WILL BE
C      FOLLOWED.
C      OPTON2 =1 IMPLIES THAT THE LOWER BOUNDS ON THE MOMENTS
C      AND THE UPPER BOUND ON THE DISTRIBUTION WILL
C      BE DETERMINED USING ALL ACTIVITY TIMES OUTSIDE
C      THE CLUSTER AT THEIR MEAN. THIS PROCEDURE IS
C      ONLY GUARANTEED TO BE VALID WHEN ALL
C      (P,0) = (.5,.5) .
C      OPTON2 NOT= 1 IMPLIES THAT THE NORMAL PROCEDURE WILL BE
C      FOLLOWED.
C
  READ(5,77551) OPTON1,OPTON2
77551 FORMAT(10I1)
  I=OPTON1+OPTON2
  IF(I.GE.1) WRITE(6,77553)
77553 FORMAT(1H1)
  IF(OPTON1.EQ.1) WRITE(6,77552)
77552 FORMAT(1H0,10X,'OPTION1=1 AND THE PROGRAM WILL TERMINATE AFTER THE
* CLUSTERS HAVE BEEN FORMED.',/,11X,'NO BOUNDS ON THE PROJECT COMPL
*ETION TIME MOMENTS OR DISTRIBUTION WILL BE DETERMINED.')
```

2702 FORMAT(1H ,13X,13.5X,13.7X,13.5X,F10.4,3X,F10.4,3X,F10.4,4X,F10.4,
*14X,F6.4)

THE FOLLOWING INDICATORS ARE USED:

IPARM = 1 IMPLIES THE CRITICAL PATH TIME WHEN ALL ACTIVITY
COMPLETION TIMES ARE SET EQUAL TO THEIR MEANS IS
BEING DETERMINED
IPARM = 2 IMPLIES THAT THE LOWER BOUND ON THE COMPLETION TIME
FOR THE SUBNETWORK IS BEING DETERMINED
IPARM = 3 IMPLIES THAT THE UPPER BOUND ON THE COMPLETION TIME
FOR THE SUBNETWORK IS BEING DETERMINED
IPARM > 3 WHEN INDEXL=0 IMPLIES THAT THE ASSOCIATES ARE
BEING DETERMINED

INDEXL=0 IMPLIES THAT INITIAL CLUSTERS ARE STILL BEING FORMED
INDEXL=1 IMPLIES THAT THE LEFTOVERS, THEIR ELIMINANTS, AND
POOLED CLUSTERS ARE BEING DETERMINED
INDEXL=2 IMPLIES THAT THE 2**NINCL() RUNS FOR EACH CLUSTER
ARE BEING MADE

ICBCP = 0 IMPLIES THAT THE PROCEDURE FOR DETERMINING UPPER
BOUNDS ON THE MOMENTS OF THE NETWORK COMPLETION TIME
AND LOWER BOUNDS ON THE DISTRIBUTION OF COMPLETION
TIMES HAS NOT BEEN BEGUN
ICBCP = 1 IMPLIES THAT THE PROCEDURE FOR DETERMINING UPPER
BOUNDS ON THE MOMENTS OF THE NETWORK COMPLETION TIME
AND LOWER BOUNDS ON THE DISTRIBUTION OF COMPLETION
TIMES IS BEING INITIALIZED

CONTINUE

ICBCP = 2 IMPLIES THAT THE PROCEDURE FOR DETERMINING UPPER
BOUNDS ON THE MOMENTS OF THE NETWORK COMPLETION TIME
AND LOWER BOUNDS ON THE DISTRIBUTION OF COMPLETION
TIMES IS BEING CARRIED OUT

IPARM=1
INDEXL=0
ICBCP=0

6010 CONTINUE

DO 104 I=1,NMM

104 INPASF(I)=M+I

DO 2001 J=1,M

2001 ISTAT(J)=0.

DO 2002 J=MP1,N

2002 ISTAT(J)=1

DO 10 II=1,NMMP1

DO 12 L=1,NMMP1

12 B1INV(L,II) = 0.

10 B1INV(II,II) = 1.

DO 30 I=1,NMM

30 XB1(I) = 0.

XB1(NMMP1) = 1.

TOLR1=1.0D-10

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C	START THE SIMPLEX ALGORITHM	180
C	SOLVE THE DUAL PROBLEM	181
C	THE NUMBER OF VARIABLES IS M REAL + NMM SLACKS	182
C	FOR A TOTAL OF N VARIABLES	183
C		184
350	CONTINUE	185
2800	DO 23 J=1,N	186
	RATS = 0.	187
	IF (ISTAT(J).EQ.1) GO TO 52800	188
	IF (J.GT.M) GO TO 22	189
	RATS = -B1INV(1,HEAD(J)+1)+B1INV(1,TAIL(J)+1) + CTIME(J)	190
	GO TO 52800	191
22	RATS = -B1INV(1,J-M+1)	192
52800	REDCOS(J)= RATS	193
23	CONTINUE	194
22800	CONTINUE	195
	IRMAX=1	196
	RMAX=REDCOS(1)	197
	DO 24 J=2,N	198
	IF(REDCOS(J) .LE. RMAX) GO TO 24	199
	RMAX=REDCOS(J)	200
	IRMAX=J	201
24	CONTINUE	202
	IF(RMAX .LE. TOLR1) GO TO 401	203
22824	CONTINUE	204
	DO 26 L=1,NMMP1	205
	IF (IRMAX.GT.M) GO TO 50026	206
	Y1(L) = -B1INV(L,TAIL(IRMAX)+1)+B1INV(L,HEAD(IRMAX)+1)	207
	GO TO 26	208
50026	Y1(L) = B1INV(L,IRMAX-M+1)	209
26	CONTINUE	210
	Y1(1) = Y1(1) - CTIME(IRMAX)	211
	NUMBER=1	212
	DO 27 L=2,NMMP1	213
27	IF(Y1(L) .LE. TOLR1) NUMBFR=NUMBER+1	214
	IF(NUMBER .EQ. NMM) GO TO 403	215
	RMIN=.99D 20	216
	IRMIN=0.	217
	DO 32 II=2,NMMP1	218
	IF(Y1(II).LE. TOLR1) GO TO 32	219
	RATS =XB1(II)/Y1(II)	220
	RR=RATS-RMIN	221
	IF(RR .GE. 0.D0) GO TO 32	222
	RMIN=RATS	223
	IRMIN=II	224
32	CONTINUE	225
	DO 33 J=2,NMMP1	226
	WW=B1INV(IRMIN,J)/Y1(IRMIN)	227
	DO 37 L=1,NMMP1	228
37	B1INV(L,J)=B1INV(L,J)-WW*Y1(L)	229
33	B1INV(IRMIN,J)=WW	230
C		231
C	UPDATE THE BASIC VARIABLES: INBASE AND XB1	232
C		233
	ISTAT(INBASE(IRMIN-1))=0	234
	ISTAT(IRMAX)=1	235
	INBASE(IRMIN-1)=IRMAX	236
	W=XB1(IRMIN)/Y1(IRMIN)	237
	DO 38 I=1,NMMP1	238
38	XB1(I)=XB1(I)-Y1(I)*W	239

	XBI(IRMIN)=W	240
	GO TO 350	241
403	WRITE(6,530)	242
530	FORMAT(1H0.5X,'NO FEASIBLE SOLUTION EXISTS. CHECK YOUR INPUT DATA	243
	*,')	244
	WRITE(6,850)	245
850	FORMAT(1H1)	246
	GO TO 999	247
C		248
C	END OF THE SIMPLEX ALGORITHM	249
C		250
401	CONTINUE	251
	IF(ICBCP.EQ.1) GO TO 6008	252
	IF(INDEXL.EQ.2) GO TO 3204	253
C		254
C	KKK= THE NUMBER OF NODES ON THE CRITICAL PATH	255
C	KB(L)= THE L-TH NODE IN THE CRITICAL PATH, COUNTING BACKWARDS	256
C	FROM THE TERMINAL NODE	257
C	KKB= THE NUMBER OF ACTIVITIES ON THE CRITICAL PATH	258
C	IBB(L)= THE L-TH ACTIVITY ON THE CRITICAL PATH, COUNTING	259
C	BACKWARDS FROM THE TERMINAL NODE	260
C		261
	CONTINUE	262
C		263
C	INBASE IS A SET OF M INTEGER VARIABLES WHICH INDICATE THE	264
C	COMPOSITION OF THE CURRENT BASIS. FOR EXAMPLE,	265
C	INBASE(K) = 7 IMPLIES THAT THE K-TH COLUMN IN THE BASIS B	266
C	CORRESPONDS TO THE 7-TH VARIABLE	267
C		268
C		269
C	ISTAT INDICATES THE BASIC STATUS OF EACH VARIABLE	270
C	ISTAT(K) = 1 IMPLIES THAT THE K-TH VARIABLE IS IN THE	271
C	DUAL BASIS	272
C	ISTAT(K) = 0 IMPLIES THAT THE K-TH VARIABLE IS NOT IN THE	273
C	DUAL BASIS	274
C		275
C		276
C	THE FOLLOWING STATEMENTS DETERMINE THE NODES AND ACTIVITIES ON	277
C	THE CRITICAL PATH	278
C		279
C		280
C	THE DUAL SOLUTION IMPLIES THE FOLLOWING OPTIMAL SOLUTION TO THE	281
C	PRIMAL PERT PROBLEM. HOWEVER SOME OF THE NODE TIMES(OTHER THAN	282
C	THE LAST ONE) MAY BE HIGHER THAN NECESSARY. THUS IN	283
C	DETERMINING THE CRITICAL PATH AN ALTERNATIVE OPTIMAL SOLUTION	284
C	MAY HAVE TO BE IDENTIFIED.	285
C	RIINV IS NOT CHANGED.	286
C		287
	DO 83002 I=1,NMM	288
83002	XNCOE(I)=BIINV(1,I+1)	289
	KKK=1	290
	KB(1)=NMM	291
83001	IK=KB(KKK)	292
C		293
C	DETERMINE WHETHER THE TIME TO REACH NODE IK IS NECESSARILY	294
C	AS LARGE AS INDICATED FROM THE DUAL SOLUTION	295
C		296
	SMIN=999999.	297
	ISMIN=0	298
	DO 83000 I=1,M	299

IF(HEAD(I).NF.IK) GO TO 83000	300
SLACK=XNODE(HEAD(I))-XNODE(TAIL(I))-CTIME(I)	301
IF(SLACK.GE.SMIN) GO TO 83000	302
SMIN=SLACK	303
ISMIN=I	304
83000 CONTINUE	305
IF(SMIN.LT.0.0001) GO TO 83003	306
C	307
C THE TIME FOR NODE IK WAS UNNECESSARILY LARGE	308
C	309
XNODE(IK)=XNODE(IK)-SMIN	310
KKK=KKK-1	311
GO TO 83001	312
83003 IRR(KKK)=ISMIN	313
KKK=KKK+1	314
KR(KKK)=TAIL(ISMIN)	315
IF(TAIL(ISMIN).GT.1) GO TO 83001	316
KKB=KKK-1	317
IF(INDEXL.EQ.1) GO TO 3121	318
IPARM=IPARM+1	319
IF(IPARM.GT.4) GO TO 2910	320
IF(IPARM.EQ.3) GO TO 6400	321
IF(IPARM.EQ.4) GO TO 6401	322
C	323
C ICRITP(L)= THE L-TH ACTIVITY ON THE ORIGINAL CRITICAL PATH	324
C KCPB= THE NUMBER OF ACTIVITIES ON THE ORIGINAL CRITICAL PATH	325
C	326
TOTAL = RIINV(1,NMMP1)	327
KCPB=KK9	328
ICRITN(1) = NMM	329
DO 2802 I=1,KCPB	330
ICRITN(I+1) = KR(I+1)	331
2802 ICRITP(I)=IRR(I)	332
X=TOTAL	333
WRITE(6,851) X	334
851 FORMAT(1H0,5X,'THE CRITICAL PATH TIME WHEN EACH ACTIVITY'S COMPLE	335
*TION TIME IS SET EQUAL TO ITS MEAN IS = ',D15.5)	336
WRITE(6,7606) KKK	337
7606 FORMAT(1H0,10X,'THE ',I3,' NODES ON THE CRITICAL PATH ARE AS FOLLO	338
*WS BEGINNING WITH THE TERMINAL NODE:')	339
WRITE(6,7707) (KR(I),I=1,KKK)	340
7707 FORMAT(15X,20(I3,' '))	341
WRITE(6,7711) KKB	342
7710 FORMAT(1H0,10X,'THE ',I3,' CRITICAL ACTIVITIES ARE AS FOLLOWS REGI	343
*NNING WITH THE TERMINAL ACTIVITY:')	344
WRITE(6,7707) (IRR(I),I=1,KKB)	345
READ(5,2920) THETA,LAMBDA	346
2920 FORMAT(2F5.2)	347
WRITE(6,3071) THETA,LAMBDA	348
3071 FORMAT(1H0,10X,'THETA = ',E15.5,' LAMBDA = ',E15.5)	349
C	350
C SAMSIZ = THE NUMBER OF ACTIVITY TIME CONFIGURATIONS TO BE	351
C RANDOMLY SELECTED FOR CONSIDERATION IN EACH CLUSTER	352
C	353
C NOTE: SINCE THIS IS A RANDOM SAMPLE , SOME PERCENTILE	354
C COMBINATIONS MAY BE CONSIDERED MORE THAN ONCE.	355
C	356
READ (5,3209) SAMSIZ	357
3209 FORMAT (I10)	358
C	359

```

C      THE COMPLETION TIME FOR ALL ACTIVITIES IS SET TO THEIR LOWER      360
C      PERCENTILE. THE RESULTING CRITICAL PATH TIME IS A LOWER          361
C      BOUND ON THE EXPECTED CRITICAL PATH TIME.                        362
C                                                                           363
      DO 6402 I=1,M                                                       364
6402  CTIME(I) = FLO(I)                                                  365
      CALL BINVA(52800)                                                  366
6400  CPLB= BINV(1,NMMP1)                                               367
      WRITE(6,6405) CPLB                                               368
6405  FORMAT(1H0,5X,'A LOWER BOUND ON THE EXPECTED CRITICAL PATH TIME IS 369
      * = ',F15.5)                                                    370
      WRITE(6,7606) KKK                                                371
      WRITE(6,7707) (KB(I),I=1,KKK)                                    372
      WRITE(6,7710) KKB                                               373
      WRITE(6,7707) (IBB(I),I=1,KKB)                                    374
C                                                                           375
C      THE COMPLETION TIME FOR ALL ACTIVITIES IS SET TO THEIR UPPER     376
C      PERCENTILE. THE RESULTING CRITICAL PATH TIME IS A UPPER         377
C      BOUND ON THE EXPECTED CRITICAL PATH TIME.                        378
C                                                                           379
      DO 6406 I=1,M                                                       380
6406  CTIME(I) = FHI(I)                                                  381
      CALL BINVA(52800)                                                  382
6401  CPUB= BINV(1,NMMP1)                                               383
      WRITE(6,6409) CPUB                                               384
6409  FORMAT(1H0,5X,'A UPPER BOUND ON THE EXPECTED CRITICAL PATH TIME IS 385
      * = ',E15.5)                                                    386
      WRITE(6,7606) KKK                                                387
      WRITE(6,7707) (KB(I),I=1,KKK)                                    388
      WRITE(6,7710) KKB                                               389
      WRITE(6,7707) (IBB(I),I=1,KKB)                                    390
C                                                                           391
C      FD(I) = THE LOWER BOUND ON THE EXPECTED CRITICAL PATH TIME      392
C      PLUS 1/IEDF OF THE DISTANCE TO THE UPPER BOUND                  393
C      NLEFD(IR,I) = THE SUM OF (THE CRITICAL PATH TIME FOR A        394
C      CONFIGURATION * THE PROBABILITY OF THE                         395
C      CONFIGURATION --- WHEN THE CRITICAL PATH TIME IS              396
C      <= FD(I) ) * (THE NUMBER OF POSSIBLE                          397
C      CONFIGURATIONS) / (THE SAMPLE SIZE)                            398
C      FOR THE IR-TH CLUSTER                                           399
C                                                                           400
C      FD AND NLEFD ARE USED TO BUILD AN 'EMPIRICAL' DISTRIBUTION OF   401
C      THE CRITICAL PATH TIMES                                         402
C                                                                           403
      C=(CPUB-CPLB)/IEDF                                                404
      DO 6412 K=1,KCPB                                                  405
      DO 6412 I=1,IEDF                                                  406
      FD(I)=CPLB+I*C                                                    407
6412  NLEFD( K,I)=0.DO                                                 408
C                                                                           409
C      THE ASSOCIATE GROUPS ARE NOW FORMED                             410
C                                                                           411
      WRITE(6,3165)                                                     412
3165  FORMAT(1H1,5X,'THE ASSOCIATES ARE NOW IDENTIFIED:')           413
      I1111=1                                                           414
      DO 2825 I=1,M                                                     415
2825  CTIME(I)=COT(I)                                                  416
      IWWWQ=ICRITP(I)                                                  417
      CHANG= LAMBDA*SIGMA(IWWWQ)                                       418
      TFX=COT(IWWWQ)-CHANG                                             419

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IF (TEX.LT.0.0) CHANG=COT(IWWWQ)
CTIME(IWWWQ)=COT(IWWWQ)-CHANG
CALL BINVA(62800)
2801 CONTINUE
IF (ISTAT(IWWWQ).EQ.1) CALL BINV1(622825,COT(IWWWQ),CTIME(IWWWQ),
*IWWWQ)
REDCOS(IWWWQ) = REDCOS(IWWWQ)+COT(IWWWQ)-CTIME(IWWWQ)
22825 CTIME(IWWWQ)=COT(IWWWQ)
IWWWQ=ICRITP(IIIII)
CHANG= LAMBDA*SIGMA(IWWWQ)
TEX=COT(IWWWQ)-CHANG
IF (TEX.LT.0.0) CHANG=COT(IWWWQ)
CTIME(IWWWQ)=COT(IWWWQ)-CHANG
IF (ISTAT(IWWWQ).EQ.1) CALL BINV1(622900,CTIME(IWWWQ),COT(IWWWQ),
*IWWWQ)
REDCOS(IWWWQ)=REDCOS(IWWWQ)-COT(IWWWQ)+CTIME(IWWWQ)
GO TO 22800

C
C DETERMINE ASSOCIATE GROUP
C
2910 NINAG(IIIII)=0
DO 2911 K=1,KKR
KK=1
2913 IF (IBR(K).EQ.ICRITP(KK)) GO TO 2911
IF (KK.GE.KCP3) GO TO 2912
KK=KK+1
GO TO 2913
2912 NINAG(IIIII)=NINAG(IIIII)+1
ASSGRP(IIIII,NINAG(IIIII))=IBR(K)
2911 CONTINUE
WRITE(6,2915) IIIII,ICRITP(IIIII),NINAG(I'IIII)
2915 FORMAT(1H0,10X,'THE NUMBER OF ASSOCIATES ASSOCIATED WITH THE ',I3,
*I-TH CRITICAL PATH ACTIVITY, I.E. ACTIVITY ',I3,', IS = ',I3)
IDUCK=NINAG(IIIII)
IF (IDUCK.EQ.0) GO TO 2810
WRITE(6,2916) (ASSGRP(IIIII,I),I=1,IDUCK)
2916 FORMAT(1H0,15X,'THE ACTIVITIES IN THE ASSOCIATE GROUP ARE AS FOLLO
*WS',/,15X,50(I3,.,.))
2810 IIIII=IIIII+1
IF (IIIII.LE.KCP3) GO TO 2801

C
C DETERMINE THE CLUSTERS
C
C THE CLUSTERS ARE POOLED TOWARD THE TERMINAL NODE
C NCLUS = THE NUMBER OF NON-EMPTY CLUSTERS
C NINCL(I) = THE NUMBER OF ACTIVITIES IN THE I-TH CLUSTER
C INCLUS(I,J) = THE J-TH ACTIVITY IN THE I-TH CLUSTER
C NCLINC(I) = THE NUMBER OF CLUSTERS COMPRISING THE I-TH
C CLUSTER AFTER POOLING
C CLINCL(I,J) = THE J-TH CLUSTER WHICH HAS BEEN POOLED INTO
C THE I-TH CLUSTER
C
C NCLINC AND CLINCL HELP KEEP TRACK OF WHICH CLUSTER THE
C CRITICAL PATH ACTIVITIES ARE IN
C
C
C BELOW FORMS CLUSTERS BY PUTTING EACH CRITICAL PATH ACTIVITY IN
C SEPARATE CLUSTER AND THEN ADDING EACH CRITICAL PATH ACTIVITY'S
C ASSOCIATES TO ITS CLUSTER
C

```

	NCLUS=KCPB	480
	DO 3020 I=1,KCPB	481
	NCLINC(I)=1	482
	CLINCL(I,1)=I	483
	NINCL(I) =NINAG(I)+1	484
	INCLUS(I,1)=ICRITP(I)	485
	IF(NINAG(I).EQ.0) GO TO 3020	486
	IDUCK=NINCL(I)	487
	DO 3021 J=2,IDUCK	488
	JJ=J-1	489
3021	INCLUS(I,J)=ASSGRP(I,JJ)	490
3020	CONTINUE	491
C		492
C	BELOW POOLS CLUSTERS FORMED FROM ASSOCIATES	493
C		494
	IA=0	495
3031	IA=IA+1	496
	IF(IA.GE.KCPB) GO TO 3030	497
	IF(NCLUS.EQ.1) GO TO 3030	498
	IDIA=NINCL(IA)	499
	IF(IDIA.EQ.0) GO TO 3031	500
	IAA=IA+1	501
	DO 3023 II=IAA,KCPB	502
	IDII=NINCL(II)	503
	IF(IDII.EQ.0) GO TO 3023	504
	DO 3025 I=1,IDIA	505
	DO 3025 J=1,IDII	506
	IF(INCLUS(II,J).EQ.INCLUS(IA,I)) GO TO 3027	507
3025	CONTINUE	508
	GO TO 3023	509
3027	NCLUS=NCLUS-1	510
	DO 3028 J=1,IDII	511
	DO 3029 I=1,IDIA	512
	IF(INCLUS(II,J).EQ.INCLUS(IA,I)) GO TO 3028	513
3029	CONTINUE	514
	NINCL(IA)=NINCL(IA)+1	515
	INCLUS(IA,NINCL(IA))=INCLUS(II,J)	516
3028	CONTINUE	517
	NINCL(II)=0	518
	NCLINC(IA)=NCLINC(IA)+1	519
	CLINCL(IA,NCLINC(IA)) = II	520
	NCLINC(II)=0	521
3023	CONTINUE	522
	GO TO 3031	523
3030	CONTINUE	524
C		525
C	BELOW DESCRIBES CLUSTERS AFTER POOLING BASED ON THE ASSOCIATES	526
C		527
	WRITE(6,3033) NCLUS	528
3033	FORMAT(1H1,10X,'THERE ARE ',I3,' NONEMPTY CLUSTERS AFTER POOLING O	529
	*N THE BASIS OF ASSOCIATES ONLY.')	530
	II=0	531
	DO 3034 I=1,KCPB	532
	IF(NINCL(I).EQ.0) GO TO 3034	533
	II=II+1	534
	IDUCJ=NINCL(I)	535
	WRITE(6,3035) I,(INCLUS(I,J),J=1,IDUCJ)	536
3035	FORMAT(1H0,10X,'THE ACTIVITIES IN THE ',I3,'-TH CLUSTER ARE AS FOL	537
	*LWS: ',/,15X,50(I3,'. '))	538
3034	CONTINUE	539

C		540
C	DESCRIBES WHERE EACH ACTIVITY IS BEFORE ELIMINANTS ARE	541
C	CONSIDERED	542
C		543
C		544
C	EXAMINE EACH ACTIVITY AND DETERMINE WHICH CLUSTER, IF ANY, IT IS	545
C	IN.	546
C	LEFT(I) = 0 IMPLIES THAT THE I-TH ACTIVITY IS NOT IN ANY	547
C	CLUSTER	548
C	LEFT(I) = J IMPLIES THE I-TH ACTIVITY IS IN THE J-TH	549
C	CLUSTER	550
C		551
	WRITE(6,3104)	552
3104	FORMAT(1H0,10X,'THE CLUSTER TO WHICH EACH ACTIVITY BELONGS:',/,15X	553
	*,'(ZERO IMPLIES THAT THE ACTIVITY IS NOT IN ANY CLUSTER)')	554
	DO 3101 I=1,M	555
	LEFT(I)=0	556
	DO 3102 J=1,KCPB	557
	IF(NINCL(J).EQ.0) GO TO 3102	558
	IDUCK=NINCL(J)	559
	DO 3110 K=1,IDUCK	560
	IF(I.EQ.INCLUS(J,K)) GO TO 3107	561
3110	CONTINUE	562
3102	CONTINUE	563
	GO TO 3101	564
3107	LEFT(I)=J	565
3101	WRITE(6,3103) I,LEFT(I)	566
3103	FORMAT(1H ,15X,'THE ',I3,'-TH ACTIVITY IS IN THE ',I3,'-TH CLUSTER	567
	*)	568
	INDEXL=1	569
C		570
C	LEFTOVERS ARE ACTIVITIES NOT IN CLUSTERS AFTER ASSOCIATES HAVE	571
C	BEEN CONSIDERED BUT BEFORE ELIMINANTS HAVE BEEN CONSIDERED	572
C		573
C		574
C	DETERMINE THE NUMBER OF LEFTOVERS. NLEFT	575
C	LEFTO(L) = J IMPLIES THAT THE L-TH LEFTOVER IS THE J-TH	576
C	ACTIVITY	577
C		578
	NLEFT=0	579
	DO 3122 J=1,M	580
	IF(LEFT(J).NE.0) GO TO 3122	581
	NLEFT=NLEFT+1	582
	LEFTO(NLEFT)=J	583
3122	CONTINUE	584
	WRITE(6,3123) NLEFT	585
3123	FORMAT(1H0,10X,'THERE ARE ',I3,' ACTIVITIES NOT IN ANY CLUSTER YET	586
	*T.')	587
	WRITE(6,3323)	588
3323	FORMAT(1H1,5X,'THE ELIMINANTS OF EACH NON-CRITICAL-PATH ACTIVITY ARE	589
	*RE NOW DETERMINED:')	590
C		591
C	ELIMINANTS FOR EACH NON-CRITICAL-PATH ACTIVITY ARE NOW	592
C	DETERMINED	593
C	NNCP = THE NUMBER OF ACTIVITIES NOT ON THE CRITICAL PATH	594
C	NONCP(LE) = THE LE-TH ACTIVITY NOT ON THE CRITICAL PATH	595
C		596
	NNCP=M-KCPB	597
	LF=0	598
	DO 5000 I=1,M	599

	J=1	600
5001	IF(I.EQ.ICRITP(J)) GO TO 5000	601
	J=J+1	602
	IF(J.LE.KCPB) GO TO 5001	603
5002	LE=LE+1	604
	NNNCP(LE)=1	605
5006	CONTINUE	606
	WRITE(6,5005) NNNCP	607
5005	FORMAT(1H0,15X,'THERE ARE ',I3,' ACTIVITIES NOT ON THE CRITICAL PA	608
	*TH. THEY ARE AS FOLLOWS:')	609
	IF(NNNCP.EQ.0) GO TO 3124	610
	DO 5006 I=1,LE	611
5006	WRITE(6,5007) I,NNNCP(I)	612
5007	FORMAT(1H,15X,I3,'. ',I3)	613
	IF(NNNCP.EQ.0) GO TO 3124	614
	LE=0	615
3126	LE=LE+1	616
	IF (ISTAT(IWWWQ).EQ.1) CALL BINVI(623127,COT(IWWWQ),CTIME(IWWWQ),	617
	*IWWWQ)	618
	REFCOS (IWWWQ) = RFDCOS(IWWWQ)-CTIME(IWWWQ)+COT(IWWWQ)	619
23127	CONTINUE	620
	CTIME(IWWWQ) = COT(IWWWQ)	621
	CTIME(NNCP(LE)) = COT(NNCP(LE)) + THETA*SIGMA(NNCP(LE))	622
	IF (ISTAT(NNCP(LE)).EQ.1) CALL BINVI(67756,CTIME(NNCP(LE)),	623
	* COT(NNCP(LE)),NNCP(LE))	624
	REFCOS(NNCP(LE))=RFDCOS(NNCP(LE))-COT(NNCP(LE))+CTIME(NNCP(LE)	625
	*)	626
7756	IWWWQ = NNCP(LE)	627
	WRITE(6,3152) NNCP(LE),CTIME(NNCP(LE))	628
3152	FORMAT(1H0,15X,'THE COMPLETION TIME FOR THE ',I3,'-TH ACTIVITY	629
	* HAS BEEN CHANGED TO ',E15.5)	630
	GO TO 22800	631
3121	CONTINUE	632
C		633
C	DETERMINE THE ELIMINANTS OF THE LE-TH ACTIVITY NOT ON THE	634
C	CRITICAL PATH	635
C	NE = THE NUMBER OF ELIMINANTS FOR THE LE-TH	636
C	ACTIVITY NOT ON THE CRITICAL PATH	637
C	EGRP(J) = THE J-TH ELIMINANT FOR THE LE-TH ACTIVITY	638
C	NOT ON THE CRITICAL PATH	639
C		640
	NE=0	641
	DO 3130 K=1,KCPB	642
	DO 3131 I=1,KKB	643
	IF(IRR(I).EQ.ICRITP(K)) GO TO 3130	644
3131	CONTINUE	645
	NE=NE+1	646
	EGRP(NE)=ICRITP(K)	647
3130	CONTINUE	648
	WRITE(6,3133) NE,NNCP(LE)	649
3133	FORMAT(1H0,10X,'THERE ARE ',I3,' FLIMINANTS CORRESPONDING TO ACTIV	650
	*ITY ',I3)	651
	IF(NE.EQ.0) GO TO 3171	652
	DO 3135 K=1,NE	653
3135	WRITE(6,3136) K,NNCP(LE),EGRP(K)	654
3136	FORMAT(1H,14X,'THE ',I3,'-TH ELIMINANT CORRESPONDING TO ACTIVITY	655
	*',I3,' IS ACTIVITY ',I3)	656
C		657
C	DETERMINE WHETHER NNCP(LE) IS AN ASSOCIATE	658
C	JA = 1 IF NNCP(LE) IS AN ASSOCIATE	659

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PERMIT FULLY LEGIBLE PRODUCTION**

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C          JA = 2 IF NONCP(LE) IS NOT AN ASSOCIATE 660
C 661
C          K=NONCP(LE) 662
C          JA=1 663
C          IF(LEFT(K).EQ.0) JA=2 664
C          IF(JA.EQ.2) GO TO 5010 665
C          IT=LEFT(K) 666
C 667
C          THE IT-TH CLUSTER IS EXPANDED TO INCLUDE ELIMINANTS 668
C 669
C          GO TO 5011 670
5010 CONTINUE 671
C 672
C          ITTT IS THE ACTIVITY NUMBER OF THE FIRST FLIMINANT 673
C          IT IS THE CLUSTER TO WHICH THE FIRST ELIMINANT CURRENTLY BELONG 674
C 675
C          ITTT=EGRP(1) 676
C          IT=LEFT(ITTT) 677
C          LEFT(NONCP(LE))=IT 678
C 679
C          THE IT-TH CLUSTER IS EXPANDED TO INCLUDE FLIMINANTS 680
C 681
C          NINCL(IT)=NINCL(IT)+1 682
C          INCLUS(IT,NINCL(IT))=NONCP(LE) 683
C          IF(NE.EQ.1) GO TO 3171 684
5011 DO 3172 J=JA,NE 685
C 686
C          IU IS THE ACTIVITY NUMBER OF THE NEXT ELIMINANT 687
C          IF IU IS IN CLUSTER K, THEN CLUSTER K IS POOLED INTO CLUSTER IT 688
C 689
C          IU=EGRP(J) 690
C          K=LEFT(IU) 691
3182 IF(IT.EQ.K) GO TO 3172 692
C          NCLUS=NCLUS+1 693
C          IW=NCLINC(K) 694
C          DO 3183 IA=1,IW 695
C          LEFT(ICRITP(CLINCL(K,IA)))=IT 696
C          NCLINC(IT)=NCLINC(IT)+1 697
3183 CLINCL(IT,NCLINC(IT))=CLINCL(K,IA) 698
C          NCLINC(K)=0 699
C          IW=NINCL(K) 700
C          NINCL(K)=0 701
C          DO 3184 IA=1,IW 702
C          LEFT(INCLUS(K,IA))=IT 703
C          NINCL(IT)=NINCL(IT)+1 704
3184 INCLUS(IT,NINCL(IT))=INCLUS(K,IA) 705
3172 CONTINUE 706
3171 CONTINUE 707
C          IF(LE.LT.NNCP) GO TO 3126 708
C 709
C          END OF POOLING BASED ON ELIMINANTS EXCEPT FOR THE FOLLOWING 710
C          DESCRIPTION 711
C 712
C          WRITE(6,3173) NCLUS 713
3173 FORMAT(1H1,05X,'THERE ARE ',I3,' CLUSTERS.')
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DO 3176 I=1,KCP9
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IF(NINCL(I).EQ.0) GO TO 3176
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IDD=NINCL(I)
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WRITE(6,3174) NINCL(I),I,(INCLUS(I,J),J=1,IDD)
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3174 FORMAT(1H0,10X,'THERE ARE ',I3,' ACTIVITIES IN THE ',I3,'-TH CLUST
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*ER. THEY ARE AS FOLLOWS:'./.20X.50(I3.','.')
IDUCK=NCLINC(I)
WRITE(6,3175) NCLINC(I).(CLINCL(I,J),J=1,IDUCK)
3175 FORMAT(1H0,15X,I3,' CLUSTERS HAVE BEEN POOLED TO MAKE THIS CLUSTER
*. THEY WERE AS FOLLOWS:'./.20X.50(I3.','.')
3176 CONTINUE
IF (SAMSIZ.LE.0) WRITE(6,6045)
IF (SAMSIZ.GT.0) WRITE(6,6056) SAMSIZ,SAMSIZ
6056 FORMAT( 5X,' THE FOLLOWING TABLES WERE DETERMINED CONSIDERING AT M
*OST '.I8,' ACTIVITY CONFIGURATIONS PER CLUSTER.'./.11X,' IF THERE A
*RE NO MORE THAN '.I8,' ACTIVITY CONFIGURATIONS IN A CLUSTER.'./.11
*X,' THEN ALL ACTIVITY CONFIGURATIONS ARE EXPLICITLY CONSIDERED, AND
* NO SAMPLING IS DONE.')
6045 FORMAT (5X,' THE FOLLOWING TABLES WERE DETERMINED CONSIDERING ALL
* ACTIVITY CONFIGURATIONS.')
C
C WARNING: THE SYSTEM SUBROUTINE CLOCK MAY NOT BE A PART OF
C ALL SYSTEMS. XTRAN NEEDS TO BE A RANDOM SEED.
C
CALL CLOCK(XTRAN)
IYUTS = XTRAN
WRITE (6,3238) IYUTS
3238 FORMAT (1H0,5X,'THE INITIALIZATION PARAMETER FOR ANY SAMPLING IS I
*Y = '.I10)
C
C STATEMENT NUMBER 3124 MARKS THE END OF POOLING CLUSTERS BASED
C ON LEFTOVERS AND ELIMINANTS
C
3124 CONTINUE
IF(OPTON1.EQ.1) GO TO 399
C
C THE FINAL CLUSTERS HAVE NOW BEEN DETERMINED
C THE 2**NINCL(I) RUNS ARE NOW MADE FOR ALL I WITH NINCL(I)>0.
C
C
6008 IF(ICRCP.EQ.1) ICRCP=2
INDEXL=2
IF(OPTON2.EQ.1) GO TO 87701
IF(ICRCP.EQ.2) GO TO 87701
DO 17701 I=1,M
17701 CNT(I)=FLD(I)
87701 CONTINUE
IR=0
3200 IR=IR+1
IF(IP.GT.KCPR) GO TO 3208
IF(NINCL(IR).EQ.0) GO TO 3200
DO 6031 I=1,10
6031 MOMENT(IR,I) = 0.00
DO 9101 I=1,IEND
9101 NLFFD(IR,I)=0.00
3310 IP = 0
NIR(IR)=0
C
C NIR = NUMBER OF ACTIVITY CONFIGURATIONS IN THE SAMPLE
C NSAVE = VECTOR CONTAINING THE UPPER BOUNDS ON THE NETWORK
C COMPLETION TIME DISTRIBUTION TO BE AVERAGED WITH THE
C LOWER BOUNDS ON THE NETWORK TO YIELD THE AVERAGE NETWORK
C COMPLETION TIME DISTRIBUTION.

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C		780
	IC=NINCL(IR)	781
	IR=2**NINCL(IR)	782
C	IDOALL = 1 MEANS ALL ACTIVITY CONFIGURATIONS ARE EXPLICITLY	783
C	CONSIDERED.	784
C	IDCALL = 0 MEANS TO SAMPLE.	785
C		786
C		787
	IDOALL = 0	788
	IF (SAMSIZ.LE.C.OP.SAMSIZ.GE.IB) IDOALL=1	789
	DO 3222 I=1,M	790
3222	CTIME(I) = COT(I)	791
	GO TO 20000	792
C		793
C	STATEMENT 3204 IS THE RE-ENTRY POINT FROM THE SIMPLEX	794
C	ALGORITHM WHEN CLUSTER BASED BOUNDS ARE BEING COMPUTED	795
C		796
3204	CONTINUE	797
	DO 6032 I=1,IC	798
6032	MOMENT(IR,I) = MOMENT(IR,I) + (B1INV(1,NMMP1)**I)*SPROB	799
9900	X= B1INV(1,NMMP1)	800
	X=X-1.0D-10	801
	I=0	802
6420	I=I+1	803
	IF(X.GT.FD(I)) GO TO 6420	804
	NLEFD(IR,I)=NLEFD(IR,I)+SPROB	805
10001	CONTINUE	806
20000	IP=IP+1	807
	NIR(IP)=NIR(IR)+1	808
C		809
C	GENERATE NEXT ACTIVITY CONFIGURATION TO BE EXPLICITLY	810
C	CONSIDERED.	811
C		812
	IF (IDOALL.EQ.0) GO TO 20500	813
	IF (IP.GT.IB) GO TO 3207	814
	RANSAM = IP	815
	GO TO 20501	816
20500	IF(NIR(IP).GT.SAMSIZ) GO TO 3207	817
	IYUTS = IYUTS*65539	818
	IF (IYUTS) 6210,6211,6211	819
6210	IYUTS = IYUTS+2147483647+1	820
6211	XRAN = IYUTS	821
	XRAN = XRAN*.4656613E-9	822
	RANSAM = XRAN*DFLOAT(IB-1) + 1	823
20501	CONTINUE	824
C		825
C	CONVERT THE RANDOM NUMBER, RANSAM, TO A BINARY NUMBER TO DEFINE AN	826
C	ACTIVITY CONFIGURATION.	827
C		828
	KRAN = RANSAM	829
	SPROB=1.00	830
	DO 8505 I=1,IC	831
	IHALF = KRAN/2	832
	IZ = KRAN - IHALF*2	833
	L = INCLUS(IR,I)	834
	CTIME(L) = IZ*FHI(L)-IZ*FLO(L) + FLO(L)	835
	SPROB=SPROB*(IZ*PQ(L)+(1-IZ)*PP(L))*2.00	836
C		837
C	SPROB= 2**NINCL(IR) * THE PROBABILITY OF THIS CONFIGURATION	838
C		839

8505	KRAN = IHALF	840
	CALL RINVA(62800)	841
3207	NIR(IR)=NIR(IR)-1	842
	DO 6030 I=1,10	843
6030	MOMENT(IR,I) = MOMENT(IR,I)/NIR(IR)	844
	GO TO 3200	845
3208	WRITE (6,6362)	846
6362	FORMAT (1H1)	847
	IF (ICRCP.EQ.2) GO TO 6011	848
	DO 10000 J=1,10	849
	ITMAX=1	850
	TMAX=0.	851
	DO 5021 I=1,KCPB	852
	IF(NINCL(I).EQ.0) GO TO 5021	853
	IF (MOMENT(I,J).LT.TMAX) GO TO 5021	854
	TMAX = MOMENT(I,J)	855
	ITMAX=I	856
5021	CONTINUE	857
	WRITE (6,5023) J,J,MOMENT(ITMAX,J)	858
5023	FORMAT (1H0,5X,'A LOWER BOUND, T-(',I2,';THETA,LAMBDA), ON THE '	859
*	I2,'-TH MOMENT OF THE NETWORK COMPLETION TIME = ',E15.5)	860
10000	CONTINUE	861
C		862
C		863
C	BEGIN THE PROCEDURE FOR DETERMINING UPPER BOUNDS ON THE	864
C	NETWORK COMPLETION TIME DISTRIBUTION	865
C		866
9011	CONTINUE	867
	DO 9007 IR=1,KCPB	868
	IF(NINCL(IR).EQ.0) GO TO 9007	869
	DO 9990 I=2,IEDF	870
	II=I-1	871
9990	NLEFD(IR,I)=NLEFD(IR,I)+NLEFD(IR,II)	872
	WNNIR=DFLCAT(NIR(IR))	873
	DO 3990 I=1,IEDF	874
3990	NLEFD(IR,I)=NLEFD(IR,I)/WNNIR	875
9007	CONTINUE	876
	DO 10111 IR=1,KCPB	877
	IF(NINCL(IR).GT.0) GO TO 10112	878
10111	CONTINUE	879
10112	IRR=IR	880
C		881
C	IRR = NON-EMPTY CLUSTER WITH THE SMALLEST INDEX	882
C		883
	DO 10119 IR=1,KCPB	884
	IF(NINCL(IR).EQ.0) GO TO 10119	885
	DO 10117 I=1,IEDF	886
	IF(NLEFD(IR,I).LT.NLEFD(IRR,I)) NLEFD(IRR,I)=NLEFD(IR,I)	887
10110	CONTINUE	888
10119	CONTINUE	889
	WRITE (6,6264)	890
	WRITE(6,9423)	891
9423	FORMAT(1H0,5X,'AN UPPER BOUND ON THE NETWORK COMPLETION TIME DISTR	892
*	IBUTION: F+(',I2,';THETA;LAMBDA)')	893
	DO 9421 I=1,IEDF	894
	NSAVE(I)=NLEFD(IRR,I)	895
	X=NLEFD(IRR,I)	896
9421	WRITE(6,9422) F(I),THETA,LAMBDA,X	897
9422	FORMAT(17X,'F+(',F15.5,';',E15.5,';',E15.5,'') = ',E15.5)	898
C		899

C	BEGIN THE PROCEDURE FOR DETERMINING UPPER BOUNDS ON THE MOMENTS	900
C	OF THE NETWORK COMPLETION TIME AND LOWER BOUNDS ON THE	901
C	DISTRIBUTION OF THE COMPLETION TIMES.	902
C		903
C		904
C	THE UPPER BOUNDS, $T+(R, \text{THETA}, \text{LAMBDA})$, ARE NOW DETERMINED.	905
C		906
C		907
C		908
C		909
C	FOR THE SAKE OF NUMERICAL ACCURACY THE PERT PROBLEM	910
C	WITH NEW ACTIVITY TIMES IS INITIALLY SOLVED FROM SCRATCH	911
C	INSTEAD OF UPDATING AN OLD SOLUTION. AFTER THIS	912
C	REINITIALIZATION, THE REMAINING CRITICAL PATH TIMES ARE	913
C	DETERMINED BY UPDATING THIS SOLUTION.	914
C		915
C		916
	ICRCP=1	917
	DO 6001 I=1,M	918
	CTIME(I)=FHI(I)	919
6001	COT(I)=FHI(I)	920
	GO TO 6010	921
6011	WRITE (6,6264)	922
6264	FORMAT(///)	923
	DO 10009 J=1,10	924
	ITMIN=IRR	925
	TMIN=MCMENT(IRR,J)	926
	DO 7101 I=1,KCPB	927
	IF(NINCL(I).EQ.0) GO TO 7101	928
	IF(MOMENT(I,J).GT.TMIN) GO TO 7101	929
	ITMIN=I	930
	TMIN=MCMENT(I,J)	931
7101	CONTINUE	932
10009	WRITE (6,6012) J,J,MOMENT(ITMIN,J)	933
6012	FORMAT (1H0,5X,'AN UPPER BOUND, $T+(I, I_2, \text{THETA}, \text{LAMBDA})$, ON THE '	934
	' I_2 ,'-TH MOMENT OF THE NETWORK COMPLETION TIME = ',E15.5)	935
	DO 66900 IR=1,KCPB	936
	IF(NINCL(IR).EQ.0) GO TO 66900	937
	DO 6900 I=1,IEDF	938
	II=I-1	939
6900	NLEFD(IR,I)=NLEFD(IR,I)+NLEFD(IR,II)	940
	WNNIR=DFLOAT(NIB(IR))	941
	DO 49990 I=1,IEDF	942
49990	NLEFD(IR,I)=NLEFD(IR,I)/WNNIR	943
66900	CONTINUE	944
	DO 20115 IR=1,KCPB	945
	IF(NINCL(IR).EQ.0) GO TO 20115	946
	DO 20110 I=1,IEDF	947
	IF(NLEFD(IR,I).GT.NLEFD(IRR,I)) NLEFD(IRR,I)=NLEFD(IR,I)	948
20110	CONTINUE	949
20115	CONTINUE	950
	WRITE (6,6362)	951
	WRITE(6,6423)	952
6423	FORMAT(1H0,5X,'A LOWER BOUND ON THE NETWORK COMPLETION TIME DISTR	953
	'IBUTION: F-(,;THETA;LAMBDA)')	954
	DO 6421 I=1,IEDF	955
	X=NLEFD(IRR,I)	956
6421	WRITE(6,6422) F(I),THETA,LAMBDA,X	957
6422	FORMAT(17X,'F-(,F15.5,;',E15.5,;',E15.5,') = ',E15.5)	958
C		959

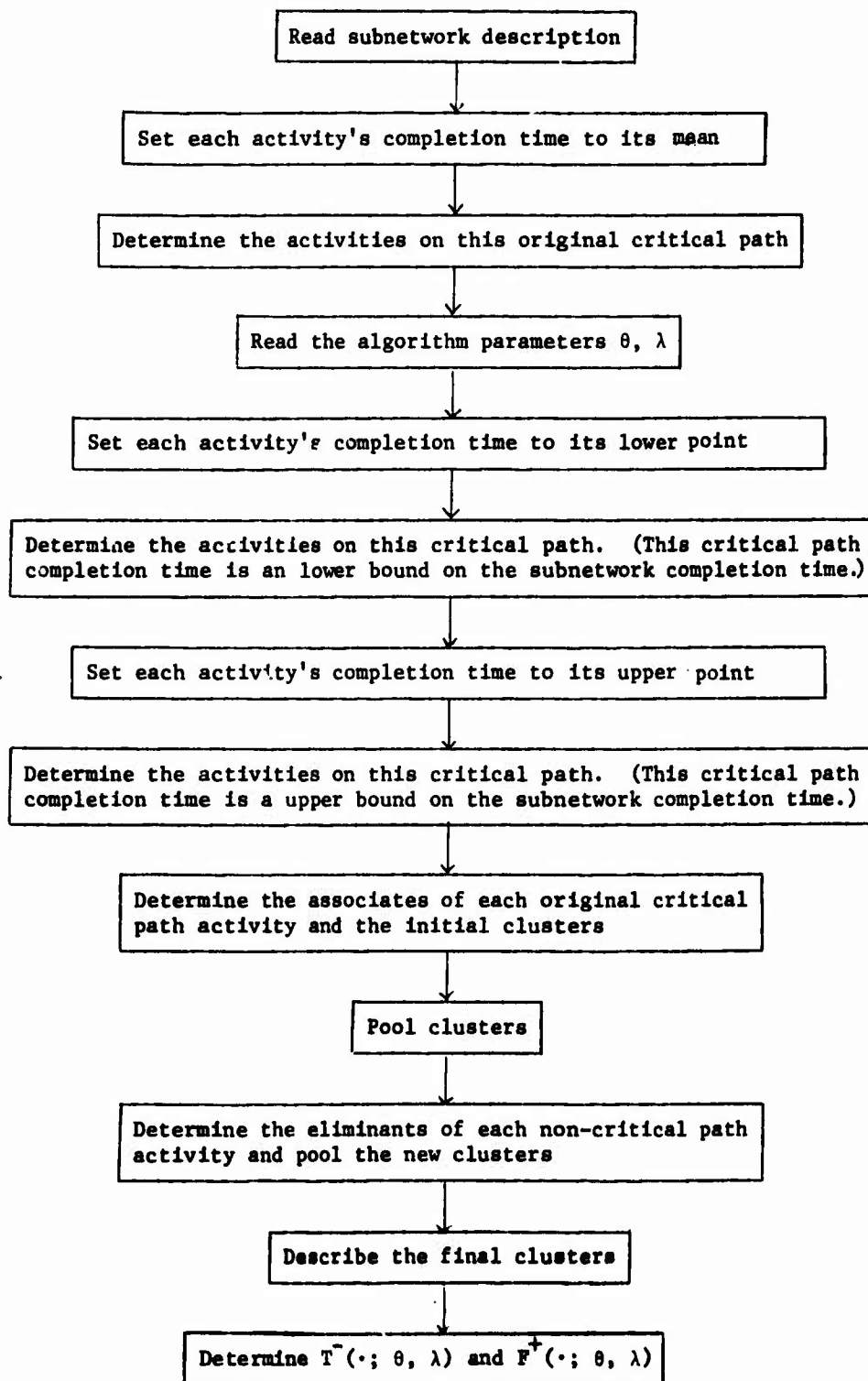
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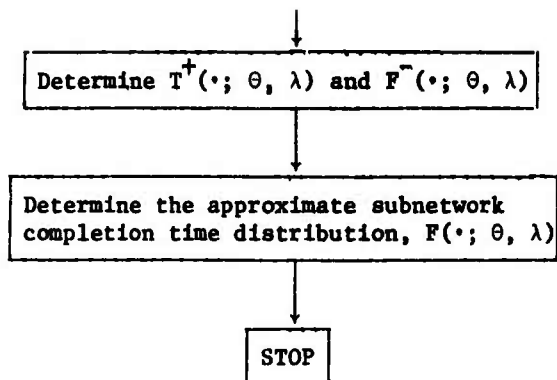
C      THE APPROXIMATE NETWORK COMPLETION TIME DISTRIBUTION
C
      WRITE (6,6362)
      WRITE(6,9472)
9472  FORMAT(1H0.5X,'AN APPROXIMATE NETWORK COMPLETION TIME DISTRIBUTION
*:',//,15X,'F(,;THETA,LAMBDA) = .5 * ( F(,;THETA,LAMBDA) + F(,;TH
*ETA,LAMBDA) )',//)
      DO 9471 I=1,IEDF
      X=.5D0*NSAVE(I)      +.5D0*NLEFD(IRR,I)
9471  WRITE(6,9473) FD(I),THETA,LAMBDA,X
9473  FORMAT(17X,' F(,E15.5,',',E15.5,',',E15.5,') = ',F15.5)
999  WRITE(6,850)
      STOP
      END
      SUBROUTINE BINVA(*)
      IMPLICIT REAL*8 (A-H,O-Z)
      COMMON B1INV,REDCOS,CTIME,XB1,INBASE,IHEAD,ITAIL,NMMP1,NMM,N,ISTAT
      COMMON M,MP1
      DIMENSION ISTAT(100),IHEAD(60),ITAIL(60),XB1(41)
      DIMENSION B1INV(41,41),INBASE(40),CTIME(100),REDCOS(100)
C
C      UPDATE THE FIRST ROW OF B1INV AFTER CHANGING CTIME
C
      DO 1 I=2,NMMP1
      B1INV(1,I) = 0.0
      DO 1 J=2,NMMP1
1  B1INV(1,I) = B1INV(1,I) + B1INV(J,I)*CTIME(INBASE(J-1))
C
C      UPDATE VALUE OF THE OBJECTIVE FUNCTION
C
      XB1(1) = B1INV(1,NMMP1)
      RETURN
      END
      SUBROUTINE BINV1 (*,TMNEW,TMOLD,ID)
      IMPLICIT REAL*8 (A-H,O-Z)
      COMMON B1INV,REDCOS,CTIME,XB1,INBASE,IHEAD,ITAIL,NMMP1,NMM,N,ISTAT
      COMMON M,MP1
      DIMENSION ISTAT(100),IHEAD(60),ITAIL(60),XB1(41)
      DIMENSION B1INV(41,41),INBASE(40),CTIME(100),REDCOS(100)
C
C      COMPUTE THE REDCOS CORRESPONDING TO ONE CHANGE IN CTIME
C
      DO 2 I=1,NMM
2  IF(INBASE(I).EQ.ID) II=I+1
      DIFF = TMNEW-TMOLD
      TMNEW IS THE NEW TIME AND TMOLD IS THE OLD TIME CORRESPONDING
      TO THE SINGLE CHANGE IN CTIME
C
      DO 1 K=1,M
      IF(ISTAT(K).EQ.1) GO TO 1
      REDCOS(K) = REDCOS(K)-DIFF*(B1INV(II,IHEAD(K)+1) -
* B1INV(II,ITAIL(K)+1))
1  CONTINUE
      DO 3 K=MP1,N
      IF (ISTAT(K).EQ.1) GO TO 3
      REDCOS(K) = REDCOS(K) - DIFF*B1INV(II,K-M+1)
3  CONTINUE
C
C      UPDATE THE FIRST ROW OF B1INV AFTER CHANGING CTIME
C

```

DO 10 I=2,NMMP1	1020
B1INV(1,I)=B1INV(1,I)+DIFF*B1INV(11,I)	1021
10 CONTINUE	1022
C	1023
C UPDATE VALUE OF THE OBJECTIVE FUNCTION	1024
C	1025
XB1(1) = B1INV(1,NMMP1)	1026
RETURN	1027
END	1028

New Subnetwork Analysis Program: Flowchart





APPENDIX C

Original Subnetwork Analysis Program

The Original Subnetwork Analysis Program is an implementation and extension of the analytical procedure described in Section 3 of Technical Report No. 48. The basic required input is

- (a) an acyclic network with one source and one sink,
- (b) two points from each component activity's completion time distribution, and
- (c) specified values for the algorithm parameters θ and λ .

The output is mainly

- (a) upper and lower bounds on the moments of the network completion time, $T_r^+(\theta, \lambda)$ and $T_r^-(\theta, \lambda)$ $r = 1, 2, \dots, 10$;
- (b) upper and lower bounds on the distribution function of the network completion time, $F^+(\cdot; \theta, \lambda)$ and $F^-(\cdot; \theta, \lambda)$; and
- (c) an approximate network completion time distribution,

$$F(\cdot; \theta, \lambda) = 1/2[F^+(\cdot; \theta, \lambda) + F^-(\cdot; \theta, \lambda)].$$

The main extension of this program is the inclusion of an option to consider only a random sample of the 2^{n_c} activity time configurations for a cluster C instead of explicitly evaluating the critical path time for all of the 2^{n_c} activity time configurations.

The basic computational technique for determining critical path times is the Simplex Algorithm. This algorithm is applied to the dual problem. The Simplex Algorithm is used instead of the standard network analysis techniques because the Simplex Algorithm is ideally suited for the type of parametric programming required to evaluate several critical path times when only the activity times vary from one problem to the next.

A listing of the Original Subnetwork Analysis Program and a program flowchart are given at the end of this appendix.

Specific Input Instructions:

Card 1. Col. 1-3 : The number of activities in the network, Format (I3).

Col. 4-6 : The number of nodes in the network, Format (I3).

For each activity one card with:

Col. 11-15: The origin node of the activity, Format (I5).

Col. 21-25: The terminal node of the activity, Format (I5).

Col. 31-40: The lower point on the activity's completion time distribution, Format (F10.0)

Col. 41-50: The upper point on the activity's completion time distribution, Format (F10.0)

Next Card. Col. 1: OPTON1. OPTON1=1 implies that the program will terminate after the clusters have been formed on the basis of associates and eliminants. OPTON $\neq$ 1 implies that the program will follow the normal procedure.

Next Card. Col. 1-3: IEDF. The program computes an absolute upper and lower bound for the network completion time. This range is subdivided into IEDF equal parts and the approximate distribution function (F^+ , F^- , $\hat{F}$) values are printed at each of these dividing points. IEDF would usually be between 10 and 100. IEDF, Format (I3).

Next Card. Col. 1-5 : θ , Format (F5.2).

Col. 6-10: λ , Format (F5.2)

Next Card. Col. 1-10: SAMSIZ. The number of activity time configurations to be randomly selected for explicit consideration in each cluster analysis. If $SAMSIZ < 0$ or $SAMSIZ > 2^n$, all activity time configurations will be explicitly considered - no random sampling will be done. Format (I10).

The nodes should be numbered 1, 2, ..., n with the source being number 1, the sink being number n, and the other node numbers being arbitrary. The activities should be numbered 1, 2, ... in any order desired.

Current Dimension Restrictions:

Currently the program is dimensioned for a maximum of

60 Activities

40 Nodes

25 Clusters

25 Activities/Cluster and $IEDF \leq 500$.

Example:

The program's input and output are illustrated in terms of the network in Figure C-1.

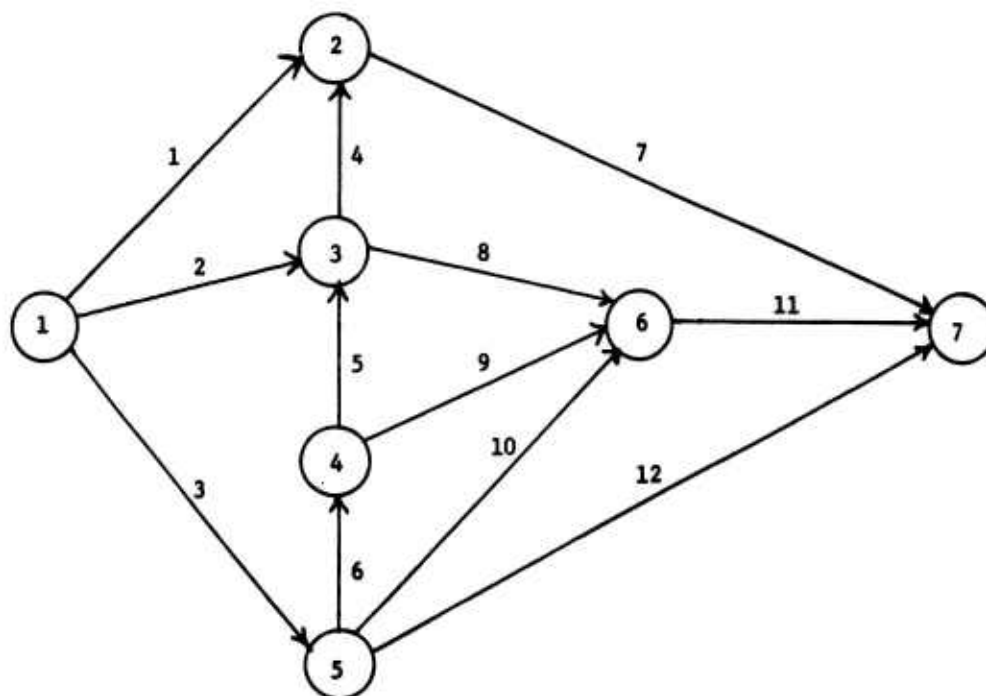


Figure C-1. Original Subnetwork Analysis Program Example Network

SAMPLE INPUT

69

012007

1	2	17.26	19.44
1	3	19.26	21.44
1	5	12.76	15.91
3	2	3.51	4.01
4	3	3.01	5.43
5	4	3.52	4.25
2	7	13.75	14.48
3	6	5.05	8.43
4	6	5.36	6.51
5	6	8.78	11.44
6	7	15.76	17.21
5	7	14.32	18.35

0

020

1. 1.

-1

SAMPLE OUTPUT

INITIAL INPUT

ACTIVITY	ORIGIN	TERMINAL	LOWER PERC.	UPPER PERC.	AVERAGE	PERCENTILT DIFFERENCE
1	1	2	17.2600	19.4400	18.3500	2.1800
2	1	3	19.2600	21.4400	20.3500	2.1800
3	1	5	12.7600	15.9100	14.3350	3.1500
4	3	2	3.5100	4.7100	3.7600	0.5000
5	4	3	3.6100	5.4300	4.2200	2.4200
6	5	4	3.5200	4.2500	3.9850	0.7300
7	2	7	13.7500	14.4800	14.1150	0.7300
8	3	6	5.0500	8.4300	6.7400	3.3800
9	4	6	5.3600	6.5100	5.9350	1.1500
10	5	6	8.7800	11.4400	10.1100	2.6600
11	6	7	15.7600	17.2100	16.4850	1.4500
12	5	7	14.3200	18.3500	16.3350	4.0300

THE CRITICAL PATH TIME WHEN EACH ACTIVITY'S COMPLETION TIME IS SET EQUAL TO ITS AVERAGE IS = 0.450650 C2

THE 6 NODES ON THE CRITICAL PATH ARE AS FOLLOWS BEGINNING WITH THE TERMINAL NODE:

7. 6. 3. 4. 5. 1.

THE 5 CRITICAL ACTIVITIES ARE AS FOLLOWS BEGINNING WITH THE TERMINAL ACTIVITY:

11. 9. 5. 6. 3.

THETA = 0.100000 C1 LAMBDA = 0.100000 01

A LOWER BOUND ON THE EXPECTED CRITICAL PATH TIME IS = 0.401000 C2

THE 4 NODES ON THE CRITICAL PATH ARE AS FOLLOWS BEGINNING WITH THE TERMINAL NODE:

7. 6. 3. 4. 5. 1.

THE 5 CRITICAL ACTIVITIES ARE AS FOLLOWS BEGINNING WITH THE TERMINAL ACTIVITY:

11. 8. 5. 6. 3.

A UPPER BOUND ON THE EXPECTED CRITICAL PATH TIME IS = 0.512300 C2

THE 6 NODES ON THE CRITICAL PATH ARE AS FOLLOWS BEGINNING WITH THE TERMINAL NODE:

7. 6. 3. 4. 5. 1.

THE 5 CRITICAL ACTIVITIES ARE AS FOLLOWS BEGINNING WITH THE TERMINAL ACTIVITY:

11. 8. 5. 6. 3.

THE ASSOCIATES ARE NOW IDENTIFIED:

THE NUMBER OF ASSOCIATES ASSOCIATED WITH THE	1-TH CRITICAL PATH ACTIVITY, I.E. ACTIVITY	11. IS = 0
THE NUMBER OF ASSOCIATES ASSOCIATED WITH THE	2-TH CRITICAL PATH ACTIVITY, I.E. ACTIVITY	8. IS = 0
THE NUMBER OF ASSOCIATES ASSOCIATED WITH THE	3-TH CRITICAL PATH ACTIVITY, I.E. ACTIVITY	5. IS = 1

THE ACTIVITIES IN THE ASSOCIATE GROUP ARE AS FOLLOWS

2.

THE NUMBER OF ASSOCIATES ASSOCIATED WITH THE	4-TH CRITICAL PATH ACTIVITY, I.E. ACTIVITY	6. IS = 0
THE NUMBER OF ASSOCIATES ASSOCIATED WITH THE	5-TH CRITICAL PATH ACTIVITY, I.E. ACTIVITY	3. IS = 1

THE ACTIVITIES IN THE ASSOCIATE GROUP ARE AS FOLLOWS

2.

THERE ARE 4 NAMEONLY CLUSTERS AFTER POOLING ON THE BASIS OF ASSOCIATES ONLY.

THE ACTIVITIES IN THE 1-TH CLUSTER ARE AS FOLLOWS:

11.

THE ACTIVITIES IN THE 2-TH CLUSTER ARE AS FOLLOWS:

9.

THE ACTIVITIES IN THE 3-TH CLUSTER ARE AS FOLLOWS:

5. 2. 3.

THE ACTIVITIES IN THE 4-TH CLUSTER ARE AS FOLLOWS:

6.

THE CLUSTER TO WHICH EACH ACTIVITY BELONGS:

(ZERO IMPLIES THAT THE ACTIVITY IS NOT IN ANY CLUSTER)

THE 1-TH ACTIVITY IS IN THE 0-TH CLUSTER
 THE 2-TH ACTIVITY IS IN THE 3-TH CLUSTER
 THE 3-TH ACTIVITY IS IN THE 3-TH CLUSTER
 THE 4-TH ACTIVITY IS IN THE 3-TH CLUSTER
 THE 5-TH ACTIVITY IS IN THE 3-TH CLUSTER
 THE 6-TH ACTIVITY IS IN THE 4-TH CLUSTER
 THE 7-TH ACTIVITY IS IN THE 0-TH CLUSTER
 THE 8-TH ACTIVITY IS IN THE 2-TH CLUSTER
 THE 9-TH ACTIVITY IS IN THE 3-TH CLUSTER
 THE 10-TH ACTIVITY IS IN THE 0-TH CLUSTER
 THE 11-TH ACTIVITY IS IN THE 1-TH CLUSTER
 THE 12-TH ACTIVITY IS IN THE 0-TH CLUSTER

THERE ARE 6 ACTIVITIES NOT IN ANY CLUSTER YET.

THE ELIMINANTS OF EACH NON-CRITICAL-PATH ACTIVITY ARE NOW DETERMINED:

THERE ARE 7 ACTIVITIES NOT ON THE CRITICAL PATH. THEY ARE AS FOLLOWS:

- 1.
- 2.
- 3.
- 4.
- 5.
- 6.
- 7.
- 8.
- 9.
- 10.
- 11.
- 12.

THE COMPLETION TIME FOR THE 1-TH ACTIVITY HAS BEEN CHANGED TO 0.205300 02

THERE ARE 0 ELIMINANTS CORRESPONDING TO ACTIVITY 1

THE COMPLETION TIME FOR THE 2-TH ACTIVITY HAS BEEN CHANGED TO 0.225300 02

THERE ARE 3 ELIMINANTS CORRESPONDING TO ACTIVITY 2
 THE 1-TH ELIMINANT CORRESPONDING TO ACTIVITY 2 IS ACTIVITY 5.
 THE 2-TH ELIMINANT CORRESPONDING TO ACTIVITY 2 IS ACTIVITY 6.
 THE 3-TH ELIMINANT CORRESPONDING TO ACTIVITY 2 IS ACTIVITY 3

THE COMPLETION TIME FOR THE 4-TH ACTIVITY HAS BEEN CHANGED TO 0.425000 01

THERE ARE 0 ELIMINANTS CORRESPONDING TO ACTIVITY 4

THE COMPLETION TIME FOR THE 7-TH ACTIVITY HAS BEEN CHANGED TO 0.149450 02

THERE ARE 0 ELIMINANTS CORRESPONDING TO ACTIVITY 7

THE COMPLETION TIME FOR THE 9-TH ACTIVITY HAS BEEN CHANGED TO 0.709500 01

THERE ARE 0 ELIMINANTS CORRESPONDING TO ACTIVITY 9

THE COMPLETION TIME FOR THE 10-TH ACTIVITY HAS BEEN CHANGED TO 0.127700 02

THERE ARE 0 ELIMINANTS CORRESPONDING TO ACTIVITY 10

THE COMPLETION TIME FOR THE 12-TH ACTIVITY HAS BEEN CHANGED TO 0.203050 02

THERE ARE 6 FLIMINANTS CORRESPONDING TO ACTIVITY 12

THERE ARE 3 CLUSTERS.

THERE ARE 1 ACTIVITIES IN THE 1-TH CLUSTER. THEY ARE AS FOLLOWS:

11.

1 CLUSTERS HAVE BEEN POOLED TO MAKE THIS CLUSTER. THEY WERE AS FOLLOWS:

1.

THERE ARE 1 ACTIVITIES IN THE 2-TH CLUSTER. THEY ARE AS FOLLOWS:

8.

1 CLUSTERS HAVE BEEN POOLED TO MAKE THIS CLUSTER. THEY WERE AS FOLLOWS:

2.

THERE ARE 4 ACTIVITIES IN THE 3-TH CLUSTER. THEY ARE AS FOLLOWS:

5. 2. 3. 6.

1 CLUSTERS HAVE BEEN POOLED TO MAKE THIS CLUSTER. THEY WERE AS FOLLOWS:

3. 5. 4.

THE NUMBER OF PERCENTILE COMBINATIONS EXPLICITLY CONSIDERED IN DETERMINING THE UPPER BOUNDS AND LOWER BOUNDS ON THE NETWORK COMPLETION TIME DISTRIBUTION AND THE UPPER BOUNDS ON ITS MOMENTS IS EQUAL TO -1

THE INITIALIZATION PARAMETER FOR THE SAMPLING IS $1Y =$

77

**COPY AVAILABLE TO DDC DOES NOT
PERMIT FULLY LEGIBLE PRODUCTION**

THE FOLLOWING TABLE WAS COMPUTED CONSIDERING ALL PERCENTILE COMBINATIONS.

A LOWER BOUND. $T-(1:\Theta\eta\lambda)$. ON THE 1-TH MOMENT OF THE NETWORK COMPLETION TIME =	0.45888D 02
A LOWER BOUND. $T-(2:\Theta\eta\lambda)$. ON THE 2-TH MOMENT OF THE NETWORK COMPLETION TIME =	0.21099D 04
A LOWER BOUND. $T-(3:\Theta\eta\lambda)$. ON THE 3-TH MOMENT OF THE NETWORK COMPLETION TIME =	0.97066D 05
A LOWER BOUND. $T-(4:\Theta\eta\lambda)$. ON THE 4-TH MOMENT OF THE NETWORK COMPLETION TIME =	0.44742D 07
A LOWER BOUND. $T-(5:\Theta\eta\lambda)$. ON THE 5-TH MOMENT OF THE NETWORK COMPLETION TIME =	0.20655D 09
A LOWER BOUND. $T-(6:\Theta\eta\lambda)$. ON THE 6-TH MOMENT OF THE NETWORK COMPLETION TIME =	0.95494D 10
A LOWER BOUND. $T-(7:\Theta\eta\lambda)$. ON THE 7-TH MOMENT OF THE NETWORK COMPLETION TIME =	0.44215D 12
A LOWER BOUND. $T-(8:\Theta\eta\lambda)$. ON THE 8-TH MOMENT OF THE NETWORK COMPLETION TIME =	0.20533D 14
A LOWER BOUND. $T-(9:\Theta\eta\lambda)$. ON THE 9-TH MOMENT OF THE NETWORK COMPLETION TIME =	0.95211D 15
A LOWER BOUND. $T-(10:\Theta\eta\lambda)$. ON THE 10-TH MOMENT OF THE NETWORK COMPLETION TIME =	0.44279D 17

THE FOLLOWING TABLE WAS COMPUTED CONSIDERING ALL PERCENTILE COMBINATIONS.

AN UPPER BOUND. $T+(1:\Theta\eta\lambda)$. ON THE 1-TH MOMENT OF THE NETWORK COMPLETION TIME =	0.45888D 02
AN UPPER BOUND. $T+(2:\Theta\eta\lambda)$. ON THE 2-TH MOMENT OF THE NETWORK COMPLETION TIME =	0.21123D 04
AN UPPER BOUND. $T+(3:\Theta\eta\lambda)$. ON THE 3-TH MOMENT OF THE NETWORK COMPLETION TIME =	0.97530D 05
AN UPPER BOUND. $T+(4:\Theta\eta\lambda)$. ON THE 4-TH MOMENT OF THE NETWORK COMPLETION TIME =	0.45170D 07
AN UPPER BOUND. $T+(5:\Theta\eta\lambda)$. ON THE 5-TH MOMENT OF THE NETWORK COMPLETION TIME =	0.20983D 09
AN UPPER BOUND. $T+(6:\Theta\eta\lambda)$. ON THE 6-TH MOMENT OF THE NETWORK COMPLETION TIME =	0.97769D 10
AN UPPER BOUND. $T+(7:\Theta\eta\lambda)$. ON THE 7-TH MOMENT OF THE NETWORK COMPLETION TIME =	0.45688D 12
AN UPPER BOUND. $T+(8:\Theta\eta\lambda)$. ON THE 8-TH MOMENT OF THE NETWORK COMPLETION TIME =	0.21412D 14
AN UPPER BOUND. $T+(9:\Theta\eta\lambda)$. ON THE 9-TH MOMENT OF THE NETWORK COMPLETION TIME =	0.10066D 16
AN UPPER BOUND. $T+(10:\Theta\eta\lambda)$. ON THE 10-TH MOMENT OF THE NETWORK COMPLETION TIME =	0.47434D 17

THE FOLLOWING TABLE WAS COMPUTED CONSIDERING ALL PERCENTILE COMBINATIONS.

A LOWER BOUND ON THE NETWORK COMPLETION TIME DISTRIBUTION: $F(-; \theta: \text{THETA}; \text{LAMRDA})$

F(-)	0.405570 C2:	0.100000 C1:	0.100000 C1:	0.156250-01
F(-)	0.412130 C2:	0.100000 C1:	0.100000 C1:	0.312500-01
F(-)	0.417700 C2:	0.100000 C1:	0.100000 C1:	0.468750-01
F(-)	0.423260 C2:	0.100000 C1:	0.100000 C1:	0.637500-01
F(-)	0.428830 C2:	0.100000 C1:	0.100000 C1:	0.812500 00
F(-)	0.434390 C2:	0.100000 C1:	0.100000 C1:	0.100000 00
F(-)	0.439960 C2:	0.100000 C1:	0.100000 C1:	0.206880 00
F(-)	0.445520 C2:	0.100000 C1:	0.100000 C1:	0.312500 00
F(-)	0.451080 C2:	0.100000 C1:	0.100000 C1:	0.392630 00
F(-)	0.456650 C2:	0.100000 C1:	0.100000 C1:	0.468750 00
F(-)	0.462210 C2:	0.100000 C1:	0.100000 C1:	0.531250 00
F(-)	0.467780 C2:	0.100000 C1:	0.100000 C1:	0.625000 00
F(-)	0.473340 C2:	0.100000 C1:	0.100000 C1:	0.697500 00
F(-)	0.478910 C2:	0.100000 C1:	0.100000 C1:	0.781250 00
F(-)	0.484470 C2:	0.100000 C1:	0.100000 C1:	0.843750 00
F(-)	0.490040 C2:	0.100000 C1:	0.100000 C1:	0.875000 00
F(-)	0.495600 C2:	0.100000 C1:	0.100000 C1:	0.906250 00
F(-)	0.501170 C2:	0.100000 C1:	0.100000 C1:	0.937500 00
F(-)	0.506730 C2:	0.100000 C1:	0.100000 C1:	0.968750 00
F(-)	0.512300 C2:	0.100000 C1:	0.100000 C1:	0.100000 01

THE FOLLOWING TABLE WAS COMPUTED CONSIDERING ALL PERCENTILE COMBINATIONS.

AN UPPER BOUND ON THE NETWORK COMPLETION TIME DISTRIBUTION: $F(+; \theta: \text{THETA}; \text{LAMRDA})$

F(+)	0.405570 C2:	0.100000 C1:	0.100000 C1:	0.156250-01
F(+)	0.412130 C2:	0.100000 C1:	0.100000 C1:	0.312500-01
F(+)	0.417700 C2:	0.100000 C1:	0.100000 C1:	0.468750-01
F(+)	0.423260 C2:	0.100000 C1:	0.100000 C1:	0.637500-01
F(+)	0.428830 C2:	0.100000 C1:	0.100000 C1:	0.812500 00
F(+)	0.434390 C2:	0.100000 C1:	0.100000 C1:	0.100000 00
F(+)	0.439960 C2:	0.100000 C1:	0.100000 C1:	0.206880 00
F(+)	0.445520 C2:	0.100000 C1:	0.100000 C1:	0.312500 00
F(+)	0.451080 C2:	0.100000 C1:	0.100000 C1:	0.392630 00
F(+)	0.456650 C2:	0.100000 C1:	0.100000 C1:	0.468750 00
F(+)	0.462210 C2:	0.100000 C1:	0.100000 C1:	0.531250 00
F(+)	0.467780 C2:	0.100000 C1:	0.100000 C1:	0.625000 00
F(+)	0.473340 C2:	0.100000 C1:	0.100000 C1:	0.697500 00
F(+)	0.478910 C2:	0.100000 C1:	0.100000 C1:	0.781250 00
F(+)	0.484470 C2:	0.100000 C1:	0.100000 C1:	0.843750 00
F(+)	0.490040 C2:	0.100000 C1:	0.100000 C1:	0.875000 00
F(+)	0.495600 C2:	0.100000 C1:	0.100000 C1:	0.906250 00
F(+)	0.501170 C2:	0.100000 C1:	0.100000 C1:	0.937500 00
F(+)	0.506730 C2:	0.100000 C1:	0.100000 C1:	0.968750 00
F(+)	0.512300 C2:	0.100000 C1:	0.100000 C1:	0.100000 01

AN APPROXIMATE NETWORK COMPLETION TIME DISTRIBUTION:

```

F(:,THETA.LAMBDA) = .5 * ( F+(:,THETA.LAMBDA) + F-(:,THETA.LAMBDA) )

F( 0.406570 02: 0.100000 01: 0.100000 01) = 0.156250-01
F( 0.412130 02: 0.100000 01: 0.100000 01) = 0.312500-01
F( 0.417700 02: 0.100000 01: 0.100000 01) = 0.468750-01
F( 0.423260 02: 0.100000 01: 0.100000 01) = 0.937500-01
F( 0.428830 02: 0.100000 01: 0.100000 01) = 0.125000 00
F( 0.434390 02: 0.100000 01: 0.100000 01) = 0.187500 00
F( 0.439960 02: 0.100000 01: 0.100000 01) = 0.296880 00
F( 0.445520 02: 0.100000 01: 0.100000 01) = 0.312500 00
F( 0.451090 02: 0.100000 01: 0.100000 01) = 0.330630 00
F( 0.456650 02: 0.100000 01: 0.100000 01) = 0.468750 00
F( 0.462210 02: 0.100000 01: 0.100000 01) = 0.531250 00
F( 0.467780 02: 0.100000 01: 0.100000 01) = 0.625000 00
F( 0.473340 02: 0.100000 01: 0.100000 01) = 0.687500 00
F( 0.478910 02: 0.100000 01: 0.100000 01) = 0.781250 00
F( 0.484470 02: 0.100000 01: 0.100000 01) = 0.843750 00
F( 0.490040 02: 0.100000 01: 0.100000 01) = 0.975000 00
F( 0.495600 02: 0.100000 01: 0.100000 01) = 0.936250 00
F( 0.501170 02: 0.100000 01: 0.100000 01) = 0.937500 00
F( 0.506730 02: 0.100000 01: 0.100000 01) = 0.968750 00
F( 0.512300 02: 0.100000 01: 0.100000 01) = 0.100000 01

```

ORIGINAL SUBNETWORK ANALYSIS PROGRAM

```

C      IMPLICIT REAL*8 (A-H,O-Z)

```

```

C      FOR THE SAKE OF IDENTIFYING THE APPROPRIATE DIMENSIONS, LET

```

```

C      M = THE NUMBER OF ACTIVITIES IN THE NETWORK

```

```

C      NMM = NUMBER OF NODES IN THE NETWORK

```

```

C      NMMP1 = NMM + 1

```

```

C      N = M + NMM

```

```

C      L = THE LENGTH OF THE CRITICAL PATH

```

```

C      C = THE MAXIMUM NUMBER OF BRANCHES IN A CLUSTER

```

```

C      IEDF = THE NUMBER OF DIVISIONS IN THE EMPIRICAL
C      DISTRIBUTION FUNCTION

```

```

C      INTEGER TAIL( M),HEAD( M),ASSGRP( L,L ),CLINCL(L,L),EGRP(L)
C      DIMENSION NINCL( C),INCLUS( L, C),NCLINC( L)
C      DIMENSION NLEFD( IEDF),FD( IEDF),NSAVE( IEDF)
C      DIMENSION IZZ( C),AVG( L),THAT(L)
C      DIMENSION INBASE(NMM),XNODE(NMM)
C      DIMENSION LEFT( M ),LEFTO( M ),NONCP( M)
C      DIMENSION XB1( NMMP1),Y1( NMMP1),REDCOS( N ),ISTAT( N)
C      DIMENSION ICRTIP( L ),NINAG( M ),ICRTIN( L+1 ),CTIME( N ),COT( N)
C      DIMENSION KB( L ),IBB( L ),F25( M ),F75( M ),SIGMA( M ),B1INV( NMMP1, NMMP1 )
C      REAL      MOMENT( L,10)

```

```

C      OF COURSE THESE DIMENSIONS ARE MERELY UPPER BOUNDS

```

```

C      COMMON B1INV,REDCOS,CTIME,XB1,INBASE,HEAD,TAIL,NMMP1,NMM,N,ISTAT
C      COMMON M,MP1
C      INTEGER TAIL(60),HEAD(60),ASSGRP(25,25),CLINCL( 25,25),EGRP(25)
C      INTEGER SAMSIZ,RANSAM
C      DIMENSION NINCL(25),INCLUS(25,25),NCLINC(25)
C      DIMENSION FD(500),NLEFD(500),NSAVE(500)
C      DIMENSION AVG(25),THAT(25)
C      DIMENSION INBASE(40)
C      DIMENSION XNODE(40)
C      DIMENSION XB1(41),Y1(41),REDCOS(100),ISTAT(100)
C      DIMENSION B1INV(41,41),KB(25),IBB(25),FLO(60),FHI(60),SIGMA(60)
C      DIMENSION ICRTIP(25),NINAG(50),ICRTIN(26),COT(100),CTIME(100)
C      DIMENSION LEFT(60),LEFTO(60),NONCP(60)
C      REAL*8 LAMBDA,MOMENT(25,10)

```

```

C      M = THE NUMBER OF ACTIVITIES IN THE NETWORK

```

```

C      NMM = THE NUMBER OF NODES IN THE PERT NETWORK

```

```

100 READ(5,100)  M,NMM

```

```

FORMAT(2I3)

```

```

N=NMM+M

```

```

MP1=M+1

```

```

NMMP1=NMM+1

```

```

C      THE ACTIVITIES ARE DESCRIBED IN TERMS OF THEIR NODES

```

```

C      II=THE TAIL NODE, THE ORIGIN NODE

```

```

C      JJ=THE HEAD NODE, THE TERMINAL NODE

```

```

C      FLO = THE LOWER PERCENTILE

```

```

C      FHI = THE UPPER PERCENTILE

```

```

C      SIGMA = FLO - FHI

```

```

DO 610 I=1,M

```


C	ICBCP = 1 IMPLIES THAT THE PROCEDURE FOR DETERMINING UPPER	120
C	BOUNDS ON THE MOMENTS OF THE NETWORK COMPLETION TIME	121
C	AND LOWER BOUNDS ON THE DISTRIBUTION OF COMPLETION	122
C	TIMES IS BEING INITIALIZED	123
C	ICBCP = 2 IMPLIES THAT THE PROCEDURE FOR DETERMINING UPPER	124
C	BOUNDS ON THE MOMENTS OF THE NETWORK COMPLETION TIME	125
C	AND LOWER BOUNDS ON THE DISTRIBUTION OF COMPLETION	126
C	TIMES IS BEING CARRIED OUT	127
C		128
C		129
C	IDLB = 0 IMPLIES THAT THE PROCEDURE FOR DETERMINING A UPPER	130
C	BOUND ON THE NETWORK COMPLETION TIME DISTRIBUTION HAS NOT	131
C	BEGUN	132
C	IDLB = 1 IMPLIES THAT THE UPPER BOUND ON THE NETWORK	133
C	COMPLETION TIME DISTRIBUTION IS BEING DETERMINED	134
C		135
	IPARM=1	136
	INDEXL=0	137
	ICBCP=0	138
	IDLB = 0	139
6000	CONTINUE	140
	DO 104 I=1,NMM	141
104	INBASE(I)=M+1	142
	DO 2001 J=1,M	143
2001	ISTAT(J)=0.	144
	DO 2002 J=MP1,N	145
2002	ISTAT(J)=1	146
	DO 10 II=1,NMMP1	147
	DO 12 L=1,NMMP1	148
12	BIINV(L,II) = 0.	149
10	BIINV(II,II) = 1.	150
	DO 30 I=1,NMM	151
30	XB1(I) = 0.	152
	XB1(NMMP1) = 1.	153
	TOLR1=1.0D-10	154
C		155
C	START THE SIMPLEX ALGORITHM	156
C	SOLVE THE DUAL PROBLEM	157
C	THE NUMBER OF VARIABLES IS M REAL + NMM SLACKS	158
C	FOR A TOTAL OF N VARIABLES	159
C		160
350	CONTINUE	161
2800	DO 23 J=1,N	162
	RATS = 0.	163
	IF (ISTAT(J).EQ.1) GO TO 52800	164
	IF (J.GT.M) GO TO 22	165
	RATS = -BIINV(1,HEAD(J)+1)+BIINV(1,TAIL(J)+1) + CTIME(J)	166
	GO TO 52800	167
22	RATS = -BIINV(1,J-M+1)	168
52800	REDCOS(J)= RATS	169
23	CONTINUE	170
22800	CONTINUE	171
	IRMAX=1	172
	RMAX=REDCOS(1)	173
	DO 24 J=2,N	174
	IF(REDCOS(J) .LE. RMAX) GO TO 24	175
	RMAX=REDCOS(J)	176
	IRMAX=J	177
24	CONTINUE	178
	IF(RMAX .LE. TOLR1) GO TO 401	179

22824	CONTINUE	180
	DO 26 L=1,NMMP1	181
	IF (IRMAX.GT.M) GO TO 50026	182
	Y1(L) =-B1INV(L,TAIL(IRMAX)+1)+B1INV(L,HEAD(IRMAX)+1)	183
	GO TO 26	184
50026	Y1(L) = B1INV(L,IRMAX-M+1)	185
26	CONTINUE	186
	Y1(1) = Y1(1) - CTIME(IRMAX)	187
	NUMBER=0	188
	DO 27 L=2,NMMP1	189
27	IF(Y1(L).LE. TOLR1) NUMBER=NUMBER+1	190
	IF(NUMBER.EQ. NMM) GO TO 403	191
	RMIN=.99D 20	192
	IRMIN=0.	193
	DO 32 II=2,NMMP1	194
	IF(Y1(II).LE. TOLR1) GO TO 32	195
	RATS =XB1(II)/Y1(II)	196
	RR=RATS-RMIN	197
	IF(RR.GE. 0.00) GO TO 32	198
	RMIN=RATS	199
	IRMIN=II	200
32	CONTINUE	201
	DO 33 J=2,NMMP1	202
	WW=B1INV(IRMIN,J)/Y1(IRMIN)	203
	DO 37 L=1,NMMP1	204
37	B1INV(L,J)=B1INV(L,J)-WW*Y1(L)	205
33	B1INV(IRMIN,J)=WW	206
C		207
C	UPDATE THE BASIC VARIABLES: INBASE AND XB1	208
C		209
	I STAT(INBASE(IRMIN-1))=0	210
	I STAT(IRMAX)=1	211
	INBASE(IRMIN-1)=IRMAX	212
	W=XB1(IRMIN)/Y1(IRMIN)	213
	DO 38 I=1,NMMP1	214
38	XB1(I)=XB1(I)-Y1(I)*W	215
	XB1(IRMIN)=W	216
	GO TO 350	217
403	WRITE(6,530)	218
530	FORMAT(1H0,5X,'NO FEASIBLE SOLUTION EXISTS. CHECK YOUR INPUT DATA	219
	*,')	220
	WRITE(6,850)	221
850	FORMAT(1H1)	222
	GO TO 999	223
C		224
C	END OF THE SIMPLEX ALGORITHM	225
C		226
401	CONTINUE	227
C		228
C	KKK= THE NUMBER OF NODES ON THE CRITICAL PATH	229
C	KR(L)= THE L-TH NODE IN THE CRITICAL PATH, COUNTING BACKWARDS	230
C	FROM THE TERMINAL NODE	231
C	KKB= THE NUMBER OF ACTIVITIES ON THE CRITICAL PATH	232
C	IBR(L)= THE L-TH ACTIVITY ON THE CRITICAL PATH, COUNTING	233
C	BACKWARDS FROM THE TERMINAL NODE	234
C		235
	IF(ICBCP.EQ.1) GO TO 6008	236
	IF(INDFXL.EQ.2) GO TO 3204	237
C		238
C	INBASE IS A SET OF M INTEGER VARIABLES WHICH INDICATE THE	239

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C	COMPOSITION OF THE CURRENT BASIS. FOR EXAMPLE,	240
C	INBASE(K) = 7 IMPLIES THAT THE K-TH COLUMN IN THE BASIS B	241
C	CORRESPONDS TO THE 7-TH VARIABLE	242
C		243
C		244
C	ISTAT INDICATES THE BASIC STATUS OF EACH VARIABLE	245
C	ISTAT(K) = 1 IMPLIES THAT THE K-TH VARIABLE IS IN THE	246
C	DUAL BASIS	247
C	ISTAT(K) = 0 IMPLIES THAT THE K-TH VARIABLE IS NOT IN THE	248
C	DUAL BASIS	249
C		250
C		251
C	THE FOLLOWING STATEMENTS DETERMINE THE NODES AND ACTIVITIES ON	252
C	THE CRITICAL PATH	253
C		254
C		255
C	THE DUAL SOLUTION IMPLIES THE FOLLOWING OPTIMAL SOLUTION TO THE	256
C	PRIMAL PERT PROBLEM. HOWEVER SOME OF THE NODE TIMES(OTHER THAN	257
C	THE LAST ONE) MAY BE HIGHER THAN NECESSARY. THUS IN	258
C	DETERMINING THE CRITICAL PATH AN ALTERNATIVE OPTIMAL SOLUTION	259
C	MAY HAVE TO BE IDENTIFIED.	260
C	BIINV IS NOT CHANGED.	261
C		262
	DO 83002 I=1,NMM	263
83002	XNODE(I)=BI[NV(1,I+1)	264
	KKK=1	265
	KR(1)=NMM	266
83001	IK=KB(KKK)	267
C		268
C	DETERMINE WHETHER THE TIME TO REACH NODE IK IS NECESSARILY	269
C	AS LARGE AS INDICATED FROM THE DUAL SOLUTION	270
C		271
	SMIN=999999.	272
	ISMIN=0	273
	DO 83000 I=1,M	274
	IF(HEAD(I).NE.IK) GO TO 83000	275
	SLACK=XNODE(HEAD(I))-XNODE(TAIL(I))-CTIME(I)	276
	IF(SLACK.GE.SMIN) GO TO 83000	277
	SMIN=SLACK	278
	ISMIN=I	279
83000	CONTINUE	280
	IF(SMIN.LT.0.0001) GO TO 83003	281
C		282
C	THE TIME FOR NODE IK WAS UNNECESSARILY LARGE	283
C		284
	XNODE(IK)=XNODE(IK)-SMIN	285
	KKK=KKK-1	286
	GO TO 83001	287
83003	IBB(KKK)=ISMIN	288
	KKK=KKK+1	289
	KB(KKK)=TAIL(ISMIN)	290
	IF(TAIL(ISMIN).GT.1) GO TO 83001	291
	KKR=KKK-1	292
	IF(INDEXL.EQ.1) GO TO 3121	293
	IPARM=IPARM+1	294
	IF(IPARM.GT.4) GO TO 2910	295
	IF(IPARM.EQ.3) GO TO 6400	296
	IF(IPARM.EQ.4) GO TO 6401	297
C		298
C	ICRITP(L)= THE L-TH ACTIVITY ON THE ORIGINAL CRITICAL PATH	299

C	KCPB= THE NUMBER OF ACTIVITIES ON THE ORIGINAL CRITICAL PATH	300
C		301
	TOTAL = RIINV(1,NMMP1)	302
	KCPB=KKB	303
	ICFITN(1) = NMM	304
	DO 2802 I=1,KCPB	305
	ICRITN(I+1) = KB(I+1)	306
2802	ICRITP(I)=IBB(I)	307
	X=TOTAL	308
	WRITE(6,851) X	309
851	FORMAT(1H0,5X,'THE CRITICAL PATH TIME WHEN EACH ACTIVITY'S COMPLE	310
	*TION TIME IS SET EQUAL TO ITS AVERAGE IS = ',E15.5)	311
	WRITE(6,7606) KKK	312
7606	FORMAT(1H0,10X,'THE ',13,' NODES ON THE CRITICAL PATH ARE AS FOLLO	313
	*WS BEGINNING WITH THE TERMINAL NODE:')	314
	WRITE(6,7707) (KB(I),I=1,KKK)	315
7707	FORMAT(15X,20(13,' '))	316
	WRITE(6,7710) KKB	317
7710	FORMAT(1H0,10X,'THE ',13,' CRITICAL ACTIVITIES ARE AS FOLLOWS BEGI	318
	*NNING WITH THE TERMINAL ACTIVITY:')	319
	WRITE(6,7707) (IBB(I),I=1,KKB)	320
	READ(5,2920) THETA,LAMBDA	321
2920	FORMAT(2F5.2)	322
	WRITE(6,3071) THETA,LAMBDA	323
3071	FORMAT(1H0,10X,'THETA = ',E15.5,' LAMBDA = ',E15.5)	324
C		325
C	SAMSI2 = THE NUMBER OF ACTIVITY TIME CONFIGURATIONS TO BE	326
C	RANDOMLY SELECTED FOR CONSIDERATION IN EACH CLUSTER	327
C		328
C	NOTE: SINCE THIS IS A RANDOM SAMPLE , SOME PERCENTILE	329
C	COMBINATIONS MAY BE CONSIDERED MORE THAN ONCE.	330
C		331
	READ (5,3209) SAMSI2	332
3209	FORMAT (I10)	333
C		334
C	THE COMPLETION TIME FOR ALL ACTIVITIES IS SET TO THEIR LOWER	335
C	PERCENTILE. THE RESULTING CRITICAL PATH TIME IS A LOWER	336
C	BOUND ON THE EXPECTED CRITICAL PATH TIME.	337
C		338
	DO 6402 I=1,M	339
6402	CTIME(I) = FLO(I)	340
	CALL BINVA(62800)	341
6400	CPLB= RIINV(1,NMMP1)	342
	WRITE(6,6405) CPLB	343
6405	FORMAT(1H0,5X,'A LOWER BOUND ON THE EXPECTED CRITICAL PATH TIME IS	344
	* = ',F15.5)	345
	WRITE(6,7606) KKK	346
	WRITE(6,7707) (KB(I),I=1,KKK)	347
	WRITE(6,7710) KKB	348
	WRITE(6,7707) (IBB(I),I=1,KKB)	349
C		350
C	THE COMPLETION TIME FOR ALL ACTIVITIES IS SET TO THEIR UPPER	351
C	PERCENTILE. THE RESULTING CRITICAL PATH TIME IS A UPPER	352
C	BOUND ON THE EXPECTED CRITICAL PATH TIME.	353
C		354
	DO 6406 I=1,M	355
6406	CTIME(I) = FHI(I)	356
	CALL BINVA(62800)	357
6401	CPUR= RIINV(1,NMMP1)	358
	WRITE(6,6409) CPUR	359

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6409  FORMAT(1H0,5X,'A UPPER BOUND ON THE EXPECTED CRITICAL PATH TIME IS 360
* = ',F15.5) 361
WRITE(6,7606) KKK 362
WRITE(6,7707) (KB(I),I=1,KKK) 363
WRITE(6,7710) KKR 364
WRITE(6,7707) (IRB(I),I=1,KKR) 365
C 366
C FD(I) = THE LOWER BOUND ON THE EXPECTED CRITICAL PATH TIME 367
C PLUS 1/IEDF OF THE DISTANCE TO THE UPPER BOUND 368
C NLEFD(I) = THE OBSERVED NUMBER OF CRITICAL PATH TIMES 369
C THAT ARE < OR= FD(I) 370
C FD AND NLEFD ARE USED TO BUILD AN 'EMPIRICAL' DISTRIBUTION OF 371
C THE CRITICAL PATH TIMES 372
C 373
C C=(CPUB-CPLB)/IEDF 374
DO 6412 K=1,KCPB 375
DO 6412 I=1,IEDF 376
FD(I)=CPLB+I*C 377
6412 NLEFD(I) = 0 378
C 379
C THE ASSOCIATE GROUPS ARE NOW FORMED 380
C 381
WRITE(6,3165) 382
3165 FORMAT(1H1,5X,'THE ASSOCIATES ARE NOW IDENTIFIED:') 383
IIII=1 384
DO 2825 I=1,M 385
2825 CTIME(I)=COT(I) 386
IWWWQ=ICRITP(I) 387
CHANG= LAMBDA*SIGMA(IWWWQ) 388
TFX=COT(IWWWQ)-CHANG 389
IF(TEX.LT.0.0) CHANG=COT(IWWWQ) 390
CTIME(IWWWQ)=COT(IWWWQ)-CHANG 391
CALL RINVA(62800) 392
2801 CONTINUE 393
IF(ISTAT(IWWWQ).EQ.1) CALL BINV1(622825,COT(IWWWQ),CTIME(IWWWQ), 394
*IWWWQ) 395
REDCOS(IWWWQ) = REDCOS(IWWWQ)+COT(IWWWQ)-CTIME(IWWWQ) 396
22825 CTIME(IWWWQ)=COT(IWWWQ) 397
IWWWQ=ICRITP(IIII) 398
CHANG= LAMBDA*SIGMA(IWWWQ) 399
TEX=COT(IWWWQ)-CHANG 400
IF(TEX.LT.0.0) CHANG=COT(IWWWQ) 401
CTIME(IWWWQ)=COT(IWWWQ)-CHANG 402
IF(ISTAT(IWWWQ).EQ.1) CALL BINV1(622800,CTIME(IWWWQ),COT(IWWWQ), 403
*IWWWQ) 404
REDCOS(IWWWQ)=REDCOS(IWWWQ)-COT(IWWWQ)+CTIME(IWWWQ) 405
GO TO 22800 406
C 407
C DETERMINE ASSOCIATE GROUP 408
C 409
2910 NINAG(IIII)=0 410
DO 2911 K=1,KKR 411
KK=1 412
-2913 IF(IRB(K).EQ.ICRITP(KK)) GO TO 2911 413
IF(KK.GE.KCPB) GO TO 2912 414
KK=KK+1 415
GO TO 2913 416
2912 NINAG(IIII)=NINAG(IIII)+1 417
ASSGRP(IIII,NINAG(IIII))=IRB(K) 418
2911 CONTINUE 419

```

```

WRITE(6,2915) IIIII,ICRITP(IIIII),NINAG(IIIII)
2915 FORMAT(1H0,10X,'THE NUMBER OF ASSOCIATES ASSOCIATED WITH THE ',I3,
*' -TH CRITICAL PATH ACTIVITY, I.E. ACTIVITY ',I3,', IS = ',I3)
IDUCK=NINAG(IIIII)
IF(IDUCK.EQ.0) GO TO 2810
WRITE(6,2916) (ASSGRP(IIIII,I),I=1,IDUCK)
2916 FORMAT(1H0,15X,'THE ACTIVITIES IN THE ASSOCIATE GROUP ARE AS FOLLO
*WS',/,15X,50(I3,', '))
2810 IIIII=IIIII+1
IF(IIIII.LE.KCPR) GO TO 2801

C
C      DETERMINE THE CLUSTERS
C
C      THE CLUSTERS ARE POOLED TOWARD THE TERMINAL NODE
C      NCLUS = THE NUMBER OF NON-EMPTY CLUSTERS
C      NINCL(I) = THE NUMBER OF ACTIVITIES IN THE I-TH CLUSTER
C      INCLUS(I,J) = THE J-TH ACTIVITY IN THE I-TH CLUSTER
C      NCLINC(I) = THE NUMBER OF CLUSTERS COMPRISING THE I-TH
C      CLUSTER AFTER POOLING
C      CLINCL(I,J) = THE J-TH CLUSTER WHICH HAS BEEN POOLED INTO
C      THE I-TH CLUSTER
C
C      NCLINC AND CLINCL HELP KEEP TRACK OF WHICH CLUSTER THE
C      CRITICAL PATH ACTIVITIES ARE IN
C
C      BELOW FORMS CLUSTERS BY PUTTING EACH CRITICAL PATH ACTIVITY IN
C      SEPARATE CLUSTER AND THEN ADDING EACH CRITICAL PATH ACTIVITY'S
C      ASSOCIATES TO ITS CLUSTER
C
NCLUS=KCPR
DO 3020 I=1,KCPR
  NCLINC(I)=1
  CLINCL(I,1)=1
  NINCL(I)=NINAG(I)+1
  INCLUS(I,1)=ICRITP(I)
  IF(NINAG(I).EQ.0) GO TO 3020
  IDUCK=NINCL(I)
  DO 3021 J=2,IDUCK
    JJ=J-1
3021  INCLUS(I,J)=ASSGRP(I,JJ)
3020  CONTINUE
C
C      BELOW POOLS CLUSTERS FORMED FROM ASSOCIATES
C
IA=0
3031  IA=IA+1
  IF(IA.GE.KCPR) GO TO 3030
  IF(NCLUS.EQ.1) GO TO 3030
  IDIA=NINCL(IA)
  IF(IDIA.EQ.0) GO TO 3031
  IAA=IA+1
  DO 3023 II=IAA,KCPR
    IDII=NINCL(II)
    IF(IDII.EQ.0) GO TO 3023
    DO 3025 I=1,IDIA
      DO 3025 J=1,IDII
        IF(INCLUS(II,J).EQ.INCLUS(IA,I)) GO TO 3027
3025  CONTINUE
      GO TO 3023

```

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3027	NCLUS=NCLUS-1	480
	DO 3028 J=1,IDI1	481
	DO 3029 I=1,IDI1	482
	IF(INCLUS(I,J).EQ.INCLUS(IA,I)) GO TO 3028	483
3029	CONTINUE	484
	NINCL(IA)=NINCL(IA)+1	485
	INCLUS(IA,NINCL(IA))=INCLUS(I,J)	486
3028	CONTINUE	487
	NINCL(I)=0	488
	NCLINC(IA)=NCLINC(IA)+1	489
	CLINCL(IA,NCLINC(IA))=I	490
	NCLINC(I)=0	491
3023	CONTINUE	492
	GO TO 3031	493
3030	CONTINUE	494
C		495
C	BELOW DESCRIBES CLUSTERS AFTER POOLING BASED ON THE ASSOCIATES	496
C		497
	WRITE(6,3033) NCLUS	498
3033	FORMAT(1H1,10X,'THERE ARE ',I3,' NONEMPTY CLUSTERS AFTER POOLING O	499
	*N THE BASIS OF ASSOCIATES ONLY.')	500
	II=0	501
	DO 3034 I=1,KCPB	502
	IF(NINCL(I).EQ.0) GO TO 3034	503
	II=II+1	504
	IDUCJ=NINCL(I)	505
	WRITE(6,3035) I,(INCLUS(I,J),J=1,IDUCJ)	506
3035	FORMAT(1H0,10X,'THE ACTIVITIES IN THE ',I3,'-TH CLUSTER ARE AS FOL	507
	*LWS: ',/,15X,50(I3,'.')	508
3034	CONTINUE	509
C		510
C	DESCRIBES WHERE EACH ACTIVITY IS BEFORE ELIMINANTS ARE	511
C	CONSIDERED	512
C		513
C		514
C	EXAMINE EACH ACTIVITY AND DETERMINE WHICH CLUSTER, IF ANY, IT IS	515
C	IN.	516
C	LEFT(I) = 0 IMPLIES THAT THE I-TH ACTIVITY IS NOT IN ANY	517
C	CLUSTER	518
C	LEFT(I) = J IMPLIES THE I-TH ACTIVITY IS IN THE J-TH	519
C	CLUSTER	520
C		521
	WRITE(6,3104)	522
3104	FORMAT(1H0,10X,'THE CLUSTER TO WHICH EACH ACTIVITY BELONGS: ',/,15X	523
	* (ZERO IMPLIES THAT THE ACTIVITY IS NOT IN ANY CLUSTER)')	524
	DO 3101 I=1,M	525
	LEFT(I)=0	526
	DO 3102 J=1,KCPB	527
	IF(NINCL(J).EQ.0) GO TO 3102	528
	IDUCK=NINCL(J)	529
	DO 3110 K=1,IDUCK	530
	IF(I.EQ.INCLUS(J,K)) GO TO 3107	531
3110	CONTINUE	532
3102	CONTINUE	533
	GO TO 3101	534
3107	LEFT(I)=J	535
3101	WRITE(6,3103) I,LEFT(I)	536
3103	FORMAT(1H,15X,'THE ',I3,'-TH ACTIVITY IS IN THE ',I3,'-TH CLUSTER	537
	*')	538
	INDEXL=1	539

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C
C      LEFTOVERS ARE ACTIVITIES NOT IN CLUSTERS AFTER ASSOCIATES HAVE
C      BEEN CONSIDERED BUT BEFORE ELIMINANTS HAVE BEEN CONSIDERED
C
C      DETERMINE THE NUMBER OF LEFTOVERS. NLEFT
C      LEFTO(L) = J  IMPLIES THAT THE L-TH LEFTOVER IS THE J-TH
C      ACTIVITY
C
      NLEFT=0
      DO 3122 J=1,M
      IF(LEFT(J).NE.0) GO TO 3122
      NLEFT=NLEFT+1
      LEFTO(NLEFT)=J
3122  CONTINUE
      WRITE(6,3123) NLEFT
3123  FORMAT (1H0,10X,'THERE ARE ',I3,' ACTIVITIES NOT IN ANY CLUSTER YE
      *T. ')
      WRITE(6,3323)
3323  FORMAT(1H1,5X,'THE ELIMINANTS OF EACH NON-CRITICAL-PATH ACTIVITY A
      *RE NOW DETERMINED:')
C
C      ELIMINANTS FOR EACH NON-CRITICAL-PATH ACTIVITY ARE NOW
C      DETERMINED
C      NNNCP = THE NUMBER OF ACTIVITIES NOT ON THE CRITICAL PATH
C      NONCP(LE) = THE LE-TH ACTIVITY NOT ON THE CRITICAL PATH
C
      NNNCP=M-KCP3
      LE=0
      DO 5000 I=1,M
      J=1
5001  IF(I.EQ.ICRITP(J)) GO TO 5000
      J=J+1
      IF(J.LE.KCP3) GO TO 5001
5002  LE=LE+1
      NONCP(LE)=I
5003  CONTINUE
      WRITE(6,5005) NNNCP
5005  FORMAT(1H0,05X,'THERE ARE ',I3,' ACTIVITIES NOT ON THE CRITICAL PA
      *TH. THEY ARE AS FOLLOWS:')
      IF(NNNCP.EQ.0) GO TO 3124
      DO 5006 I=1,LE
5006  WRITE(6,5007) I,NONCP(I)
5007  FORMAT(1H,15X,I3,' ',I3)
      IF(NNNCP.EQ.0) GO TO 3124
      LE=0
3126  LE=LE+1
      IF (ISTAT(IWWWQ).EQ.1) CALL RINVI(623127,COT(IWWWQ),CTIME(IWWWQ),
      *IWWWQ)
      PEDCOS (IWWWQ) = PEDCOS(IWWWQ)-CTIME(IWWWQ)+COT(IWWWQ)
23127 CONTINUE
      CTIME(IWWWQ) = COT(IWWWQ)
      CTIME(NONCP(LE)) = COT(NONCP(LE)) + THETA*SIGMA(NONCP(LE))
      IF (ISTAT(NONCP(LE)).EQ.1) CALL RINVI(67756,CTIME(NONCP(LE)),
      *COT(NONCP(LE)),NONCP(LE))
      PEDCOS(NONCP(LE))=PEDCOS(NONCP(LE))-COT(NONCP(LE))+CTIME(NONCP(LE)
      *)
7756 IWWWQ = NONCP(LE)
      WRITE(6,3152) NONCP(LE),CTIME(NONCP(LE))
3152  FORMAT(1H0,///,5X,'THE COMPLETION TIME FOR THE ',I3,'-TH ACTIVITY

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* HAS BEEN CHANGED TO *.F15.5)
GO TO 22800
3121 CONTINUE
C
C      DETERMINE THE ELIMINANTS OF THE LE-TH ACTIVITY NOT ON THE
C      CRITICAL PATH
C      NE      = THE NUMBER OF ELIMINANTS FOR THE LE-TH
C              ACTIVITY NOT ON THE CRITICAL PATH
C      EGRP(J)  = THE J-TH ELIMINANT FOR THE LE-TH ACTIVITY
C              NOT ON THE CRITICAL PATH
C
C      NE=0
C      DO 3130 K=1,KCPB
C      DO 3131 I=1,KKB
C      IF(1BB(I).EQ.ICRITP(K)) GO TO 3130
3131 CONTINUE
C      NE=NE+1
C      EGRP(NE)=ICRITP(K)
3130 CONTINUE
C      WRITE(6,3133) NE,NONCP(LE)
3133 FORMAT(1H0,10X,'THERE ARE ',I3,' ELIMINANTS CORRESPONDING TO ACTIV
*ITY ',I3)
C      IF(NE.EQ.0) GO TO 3171
C      DO 3135 K=1,NE
3135 WRITE(6,3136) K,NONCP(LE),EGRP(K)
3136 FORMAT(1H ,14X,'THE ',I3,'-TH ELIMINANT CORRESPONDING TO ACTIVITY
* ',I3,' IS ACTIVITY ',I3)
C
C      DETERMINE WHETHER NONCP(LE) IS AN ASSOCIATE
C      JA = 1 IF NONCP(LE) IS AN ASSOCIATE
C      JA = 2 IF NONCP(LE) IS NOT AN ASSOCIATE
C
C      K=NONCP(LE)
C      JA=1
C      IF(LEFT(K).EQ.0) JA=2
C      IF(JA.EQ.2) GO TO 5010
C      IT=LEFT(K)
C
C      THE IT-TH CLUSTER IS EXPANDED TO INCLUDE ELIMINANTS
C
C      GO TO 5011
5010 CONTINUE
C
C      ITTT IS THE ACTIVITY NUMBER OF THE FIRST ELIMINANT
C      IT IS THE CLUSTER TO WHICH THE FIRST ELIMINANT CURRENTLY BELONG
C
C      ITTT=EGRP(1)
C      IT=LEFT(ITTT)
C      LEFT(NONCP(LE))=IT
C
C      THE IT-TH CLUSTER IS EXPANDED TO INCLUDE ELIMINANTS
C
C      NINCL(IT)=NINCL(IT)+1
C      INCLUS(IT,NINCL(IT))=NONCP(LE)
C      IF(NE.EQ.1) GO TO 3171
5011 DO 3172 J=JA,NE
C
C      IU IS THE ACTIVITY NUMBER OF THE NEXT ELIMINANT
C      IF IU IS IN CLUSTER K, THEN CLUSTER K IS POOLED INTO CLUSTER IT
C

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IU=EGRP(J)
K=LEFT(IU)
3182 IF(IT.EQ.K) GO TO 3172
NCLUS=NCLUS-1
IW=NCLINC(K)
DO 3183 IA=1,IW
LEFT(ICRITP(CLINCL(K,IA)))=IT
NCLINC(IT)=NCLINC(IT)+1
3183 CLINCL(IT,NCLINC(IT))=CLINCL(K,IA)
NCLINC(K)=0
IW=NINCL(K)
NINCL(K)=0
DO 3184 IA=1,IW
LEFT(INCLUS(K,IA))=IT
NINCL(IT)=NINCL(IT)+1
3184 INCLUS(IT,NINCL(IT))=INCLUS(K,IA)
3172 CONTINUE
3171 CONTINUE
IF(LE.LT.NNNCP) GO TO 3126

C
C      END OF POOLING BASED ON FLIMINANTS EXCEPT FOR THE FOLLOWING
C      DESCRIPTION
C
WRITE(6,3173) NCLUS
3173 FORMAT(1H1,25X,'THERE APE ',I3,' CLUSTERS.')
DO 3175 I=1,KCPB
IF(NINCL(I).EQ.0) GO TO 3176
IDD=NINCL(I)
WRITE(6,3174) NINCL(I),I,(INCLUS(I,J),J=1,IDD)
3174 FORMAT(1H0,10X,'THERE ARE ',I3,' ACTIVITIES IN THE ',I3,'-TH CLUST
*ER. THEY ARE AS FOLLOWS:',/,20X,50(I3,' '))
IDUCK=NCLINC(I)
WRITE(6,3175) NCLINC(I),(CLINCL(I,J),J=1,IDUCK)
3175 FORMAT(1H0,15X,I3,' CLUSTERS HAVE BEEN POOLED TO MAKE THIS CLUSTER
*. THEY WERE AS FOLLIWS:',/,20X,50(I3,' '))
3176 CONTINUE
WRITE(6,2776) SAMSIZ
2776 FORMAT(//5X, ' THE NUMBER OF PERCENTILE COMBINATIONS EXP
*LICITLY CONSIDERED IN DETERMINING THE UPPER BOUNDS AND LOWER BOUND
*S',/6X,'ON THE NETWORK COMPLETION TIME DISTRIBUTION AND THE UPPER
*BOUNDS ON ITS MOMENTS IS EQUAL TO ',I5)
CALL CLOCK(XRAN)
IYUTS = XRAN
WRITE(6,3238) IYUTS
3238 FORMAT(1H0,5X,'THE INITIALIZATION PARAMETER FOR THE SAMPLING IS I
*Y = ',I10)

C
C      STATEMENT NUMBER 3124 MARKS THE END OF POOLING CLUSTERS BASED
C      ON LEFTOVERS AND ELIMINANTS
C
3124 CONTINUE

C
C      THE FINAL CLUSTERS HAVE NOW BEEN DETERMINED
C      THE 2**NINCL(I) RUNS ARE NOW AVERAGED FOR ALL I WITH NINCL(I)>0
C      THE IZZ( ) ARE USED TO REPRESENT ALL OF THE 2**NINCL
C      POSSIBILITIES.
C      THIS IS WHERE THE BINARY REPRESENTATION IS CONSTRUCTED.
C
6008 IF(ICBCP.EQ.1) ICBCP=2

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INDEXL=2	720
IR=0	721
3200 IR=IR+1	722
DO 6031 I=1,10	723
6031 MOMENT(IR,I) = 0.0	724
IF (IR.GT.KCPB) GO TO 3208	725
IF (NINCL(IR).EQ.0) GO TO 3200	726
IDUCK=NCLINC(IR)	727
DO 3410 I=1,IDUCK	728
K=CLINCL(IR,I)	729
3410 CONTINUE	730
L=ICRITP(K)	731
3310 IP = 0	732
NIR= 0	733
C	734
C NIB = NUMBER OF PERCENTILE COMBINATIONS IN THE SAMPLE	735
C PNIB = PERCENTAGE OF THE TOTAL NUMBER OF PERCENTILE COMBINATIONS	736
C EXPLICITLY CONSIDERED	737
C NSAVE = VECTOR CONTAINING THE LOWER ROUNDS ON THE NETWORK	738
C COMPLETION TIME DISTRIBUTION TO BE AVERAGED WITH THE	739
C UPPER ROUNDS ON THE NETWORK TO YIELD THE AVERAGE NETWORK	740
C COMPLETION TIME DISTRIBUTION.	741
C KNIB = NIB ASSOCIATED WITH NSAVE.	742
C	743
C	744
IC=NINCL(IR)	745
IB=2**NINCL(IR)	746
C IDOALL = 1 MEANS ALL PERCENTILE COMBINATIONS ARE EXPLICITLY	747
C CONSIDERED.	748
C IDOALL = 0 MEANS TO SAMPLE.	749
C	750
C	751
IDOALL = 0	752
IF (SAMSIZ.LE.0.OP.SAMSIZ.GE.IB) IDOALL=1	753
DO 3222 I=1,M	754
3222 CTIME(I) = COT(I)	755
GO TO 20000	756
C	757
C STATEMENT 3204 IS THE RE-ENTRY POINT FROM THE DUAL SIMPLEX	758
C ALGORITHM WHEN A CLUSTER AVERAGE IS BEING COMPUTED	759
C	760
3204 CONTINUE	761
IF (IDLR.EQ.1) GO TO 9900	762
DO 6032 I=1,10	763
6032 MOMENT(IR,I) = MOMENT(IR,I) + B1INV(1,NMMP1)**I	764
IF (ICBCP.NE.2) GO TO 10001	765
9900 X= B1INV(1,NMMP1)	766
X=X-1.00-10	767
I=0	768
6420 I=I+1	769
IF (X.GT.FD(I)) GO TO 6420	770
NLEFD(I) = NLEFD(I) + 1	771
10001 CONTINUE	772
20000 IP=IP+1	773
NIB = NIB+1	774
C	775
C GENERATE NEXT PERCENTILE COMBINATIONS TO BE EXPLICITLY	776
C CONSIDERED.	777
C	778
IF (IDOALL.EQ.0) GO TO 20500	779

IF (IP.GT.IB) GO TO 3207	780
RANSAM = IP	781
GO TO 20501	782
20500 IF (NIB.GT.SAMSIZE) GO TO 3207	783
IYUTS = IYUTS*65539	784
IF (IYUTS) 6210,6211,6211	785
6210 IYUTS = IYUTS+2147483647+1	786
6211 XCRAN = IYUTS	787
XCRAN = XCRAN*.4656613F-9	788
RANSAM = XCRAN*DFLOAT(IB-1) + 1	789
20501 CONTINUE	790
C	791
C CONVERT THE RANDOM NUMBER, RANSAM, TO A BINARY NUMBER TO DEFINE A	792
C PERCENTILE COMBINATION.	793
C	794
KCRAN = RANSAM	795
DO 8505 I=1,IC	796
IHALF = KCRAN/2	797
IZ = KCRAN - IHALF*2	798
L = INCLUS(IP,I)	799
CTIME(L) = IZ*FHT(L)-IZ*FLO(L) + FLO(L)	800
8505 KCRAN = IHALF	801
CALL BINVA(62800)	802
3207 NIB = NIB-1	803
IF (IDLB.EQ.1) GO TO 9011	804
DO 6030 I=1,10	805
6030 MOMENT(IR,I) = MOMENT(IR,I)/NIB	806
IF (ICBCP.EQ.2) GO TO 6011	807
GO TO 3200	808
3208 WRITE (6,6362)	809
6362 FORMAT (1H1)	810
PNIB = DFLOAT(NIB*100)/DFLOAT(IB)	811
IF (SAMSIZE.LE.0) WRITE(6,6045)	812
IF (SAMSIZE.GT.0) WRITE(6,6055) SAMSIZE	813
6056 FORMAT(5X,' THE FOLLOWING TABLE WAS COMPUTED CONSIDERING AT MOST	814
*,IB,' PERCENTILE COMBINATIONS .')	815
6046 FORMAT(5X,' THE FOLLOWING TABLE WAS COMPUTED CONSIDERING ONLY ',	816
* IB,' PERCENTILE COMBINATIONS OR ',F6.2,' PERCENT OF ALL COMBINA	817
* TIONS.')	818
6045 FORMAT (5X,' THE FOLLOWING TABLE WAS COMPUTED CONSIDERING ALL PERC	819
* ENTILE COMBINATIONS.')	820
DO 10000 J=1,10	821
ITMAX=0	822
TMAX=0.	823
DO 5021 I=1,KCPB	824
IF(NINCL(I).EQ.0) GO TO 5021	825
IF (MOMENT(I,J).LE.TMAX) GO TO 5021	826
TMAX = MOMENT(I,J)	827
ITMAX=I	828
5021 CONTINUE	829
WRITE (6,5023) J,J,MOMENT(ITMAX,J)	830
5023 FORMAT (1H0,5X,'A LOWER BOUND. T=(',I2,';THETA,LAMRDA), ON THE '	831
*, I2 ,'-TH MOMENT OF THE NETWORK COMPLETION TIME = ',F15.5)	832
10000 CONTINUE	833
C	834
C BEGIN THE PROCEDURE FOR DETERMINING UPPER BOUNDS ON THE MOMENTS	835
C OF THE NETWORK COMPLETION TIME AND LOWER BOUNDS ON THE	836
C DISTRIBUTION OF THE COMPLETION TIMES.	837
C	838
C	839

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C      POOL ALL OF THE PREVIOUS CLUSTERS INTO ONE CLUSTER
C      NCL = THE INDEX OF THE RESULTANT POOLED CLUSTER
C      NNCL = THE NUMBER OF ACTIVITIES IN THIS POOLED CLUSTER
C
      IF(NCLUS.GT.1) GO TO 6000
      I=0
      6001 I=I+1
      IF(NINCL(I).EQ.0) GO TO 6001
      NCL=I
      GO TO 6002
      6000 NMAX = 0
      DO 6003 I=1,KCPB
      IF(NINCL(I).LE.NMAX) GO TO 6003
      NCL = I
      NMAX=NINCL(I)
      6003 CONTINUE
      DO 6004 I=1,KCPB
      IF(NINCL(I).EQ.0) GO TO 6004
      IF(I.EQ.NCL) GO TO 6004
      K=NINCL(NCL)
      JJ=NINCL(I)
      NINCL(I)=0
      DO 6005 J=1,JJ
      K=K+1
      6005 INCLUS(NCL,K)=INCLUS(I,J)
      NINCL(NCL)=NINCL(NCL)+JJ
      6004 CONTINUE
      6002 NNCL=NINCL(NCL)
C
C
C      THE UPPER BOUNDS, T+(R,THETA,LAMBDA), ARE NOW DETERMINED.
C
C
C      FOR THE SAKE OF NUMERICAL ACCURACY THE PERT PROBLEM
C      WITH NEW ACTIVITY TIMES IS INITIALLY SOLVED FROM SCRATCH
C      INSTEAD OF UPDATING AN OLD SOLUTION. AFTER THIS
C      REINITIALIZATION, THE REMAINING CRITICAL PATH TIMES ARE
C      DETERMINED BY UPDATING THIS SOLUTION.
C
      DO 6006 I=1,M
      CTIME(I)=FHI(I)
      6006 COT(I)=FHI(I)
      DO 6007 J=1,NNCL
      I =INCLUS(NCL,J)
      6007 CTIME(I)=.5*(FLO(I)+FHI(I))
      DO 7101 I=1,IEDF
      7101 NLEFD(I) = 0
      ICRCP=1
      GO TO 6010
      6011 WRITE (6,6264)
      6264 FORMAT(///)
      PNIB = DFLDAT(NIB*100)/DFLOAT(IB)
      IF (IDOALL.EQ.0) WRITE(6,6046) NIB,PNIB
      IF (IDOALL.EQ.1) WRITE(6,6045)
      DO 10009 J=1,10
      10009 WRITE (6,6012) J,J,MOMENT(NCL,J)
      6012 FORMAT (1H0,5X,'AN UPPER BOUND, T+(',I2,';THETA,LAMBDA), ON THE '
      *,I2,'-TH MOMENT OF THE NETWORK COMPLETION TIME = ',E15.5)
      DO 6900 I=2,IEDF

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        II=I-1
6900  NLEFD(I) = NLEFD(I) + NLEFD(II)
        WRITE (6,6362)
        PNIB = DFL0AT(NIB*100)/DFL0AT(IB)
        IF (ID0ALL.EQ.0) WRITE(6,6046) NIB,PNIB
        IF (ID0ALL.EQ.1) WRITE(6,6045)
        WRITE(6,6423)
6423  FORMAT(1H0,5X, 'A LOWER BOUND ON THE NETWORK COMPLETION TIME DISTR
*IBUTION: F-(.;THETA;LAMBDA)')
        KNIB = NIB
        DO 6421 I=1,IEDF
        NSAVE(I)=NLEFD(I)
        X=NLEFD(I)
        X=X/NIIB
6421  WRITE(6,6422) FD(I),THETA,LAMBDA,X
6422  FORMAT(17X,'F-(',E15.5,',',E15.5,',',E15.5,') = ',E15.5)
C
C
C      BEGIN THE PROCEDURE FOR DETERMINING UPPER BOUNDS ON THE
C      NETWORK COMPLETION TIME DISTRIBUTION
C
C      FOR THE SAKE OF NUMERICAL ACCURACY THE PERT PROBLEM
C      WITH NEW ACTIVITY TIMES IS INITIALLY SOLVED FROM SCRATCH
C      INSTEAD OF UPDATING AN OLD SOLUTION. AFTER THIS
C      REINITIALIZATION, THE REMAINING CRITICAL PATH TIMES ARE
C      DETERMINED BY UPDATING THIS SOLUTION.
        DO 9006 I=1,M
        CTIME(I)=FLO(I)
9006  COT(I)=FLO(I)
        DO 9007 J=1,NNCL
        I =INCLUS(NCL,J)
9007  CTIME(I)=.5*(FLO(I )+FH I(I ))
        DO 9101 I=1,IFDF
9101  NLEFD(I) = 0
        ICBCP = 1
        IDLB = 1
        GO TO 6010
9011  DO 9990 I=2,IEDF
        II=I-1
9990  NLEFD(I) = NLEFD(I) + NLEFD(II)
        WRITE (6,6254)
        PNIB = DFL0AT(NIB*100)/DFL0AT(IB)
        IF (ID0ALL.EQ.0) WRITE(6,6046) NIB,PNIB
        IF (ID0ALL.EQ.1) WRITE(6,6045)
        WRITE(6,9423)
9423  FORMAT(1H0,5X, 'AN UPPER BOUND ON THE NETWORK COMPLETION TIME DISTR
*IBUTION: F+(.;THETA;LAMBDA)')
        DO 9421 I=1,IEDF
        X=NLEFD(I)
        X=X/NIIB
9421  WRITE(6,9422) FD(I),THETA,LAMBDA,X
9422  FORMAT(17X,'F+(',E15.5,',',E15.5,',',E15.5,') = ',E15.5)
C
C      THE APPROXIMATE NETWORK COMPLETION TIME DISTRIBUTION
        WRITE (6,6362)
C
        WRITE(6,9472)
9472  FORMAT(1H0,5X, 'AN APPROXIMATE NETWORK COMPLETION TIME DISTRIBUTION
*:',//,15X,'F(.;THETA,LAMBDA) = .5 * ( F+(.;THETA,LAMBDA) + F-(.;TH
*ETA,LAMBDA) )',//)

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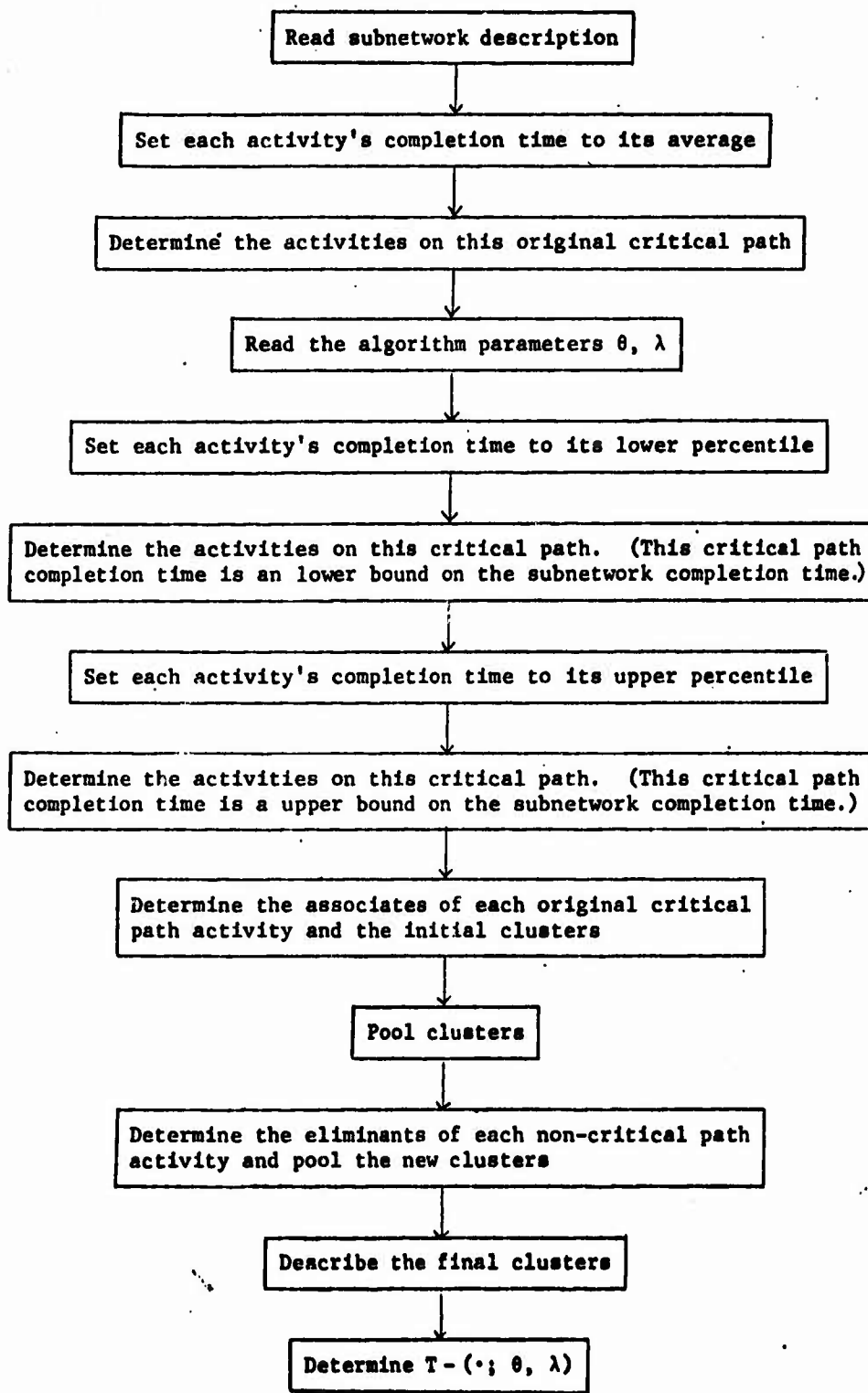
DO 9471 I=1,IEDF
X = .5*DFLOAT(NSAVE(I))/KNIR+.5*DFLOAT(NLEFD(I))/NIR
9471 WRITE(6,9473) FD(I),THETA,LAMRDA,X
9473 FORMAT(17X,' F(',E15.5,',',E15.5,',',E15.5,') = ',E15.5)
999 WRITE(6,850)
STOP
END
SUBROUTINE BINVA(*)
IMPLICIT REAL*8 (A-H,O-Z)
COMMON BIINV,REDCOS,CTIME,XB1,INBASE,IHEAD,ITAIL,NMMP1,NMM,N,ISTAT
COMMON M,MPI
DIMENSION ISTAT(100),IHEAD(60),ITAIL(60),XB1(41)
DIMENSION BIINV(41,41),INBASE(40),CTIME(100),REDCOS(100)
C
C      UPDATE THE FIRST ROW OF BIINV AFTER CHANGING CTIME
C
DO 1 I=2,NMMP1
BIINV(1,I) = 0.0
DO 1 J=2,NMMP1
1 BIINV(1,I) = BIINV(1,I) + BIINV(J,I)*CTIME(INBASE(J-1))
C
C      UPDATE VALUE OF THE OBJECTIVE FUNCTION
C
XB1(1) = BIINV(1,NMMP1)
RETURN
END
SUBROUTINE BINVI (*,TMNEW,TMOLD,ID)
IMPLICIT REAL*8 (A-H,O-Z)
COMMON BIINV,REDCOS,CTIME,XB1,INBASE,IHEAD,ITAIL,NMMP1,NMM,N,ISTAT
COMMON M,MPI
DIMENSION ISTAT(100),IHEAD(60),ITAIL(60),XB1(41)
DIMENSION BIINV(41,41),INBASE(40),CTIME(100),REDCOS(100)
C
C      COMPUTE THE REDCOS CORRESPONDING TO ONE CHANGE IN CTIME
C
DO 2 I=1,NMM
2 IF(INBASE(I).EQ.ID) II=I+1
DIFF = TMNEW-TMOLD
C
C      TMNEW IS THE NEW TIME AND TMOLD IS THE OLD TIME CORRESPONDING
C      TO THE SINGLE CHANGE IN CTIME
C
DO 1 K=1,M
IF(ISTAT(K).EQ.1) GO TO 1
REDCOS(K) = REDCOS(K)-DIFF*(BIINV(II,IHEAD(K)+1) -
* BIINV(II,ITAIL(K)+1))
1 CONTINUE
DO 3 K=MPI,N
IF (ISTAT(K).EQ.1) GO TO 3
REDCOS(K) = REDCOS(K) - DIFF*BIINV(II,K-M+1)
3 CONTINUE
C
C      UPDATE THE FIRST ROW OF BIINV AFTER CHANGING CTIME
C
DO 10 I=2,NMMP1
BIINV(1,I)=BIINV(1,I)+DIFF*BIINV(II,I)
10 CONTINUE
C
C      UPDATE VALUE OF THE OBJECTIVE FUNCTION
C
XB1(1) = BIINV(1,NMMP1)

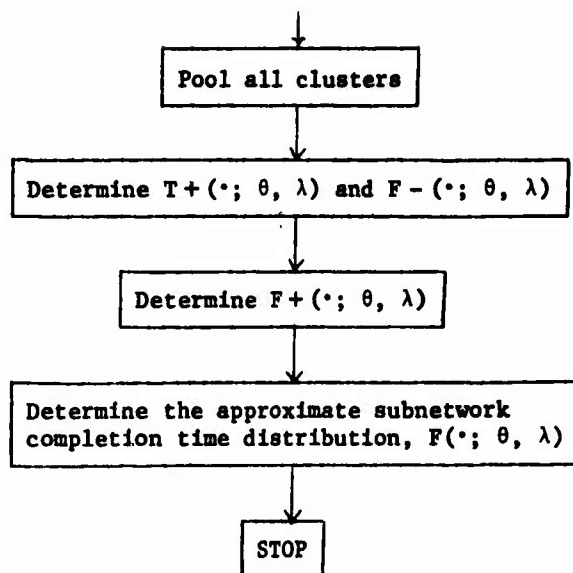
```

RETURN
END

1020
1021

Original Subnetwork Analysis Program: Flowchart





APPENDIX D

Monte Carlo PERT Simulation Program

The Monte Carlo PERT Simulation Program will generate a random sample of network completion times. The required input is

- (a) an acyclic network with one sink,
- (b) parameters describing each activity's completion time distribution, and
- (c) the size of the random sample to be generated.

Currently, the program generates each activity's random sample of completion times from a chi-square distribution with 3 degrees of freedom that has been linearly transformed to have specified 15-th and 85-th percentiles. The activity time distribution can be easily changed. The basic output is the ordered sample of random network completion times and the corresponding empirical distribution function. The critical paths associated with the sample of network completion times are not determined.

The generation of the random sample of network completion times involves only one network but varying values of the individual activity completion times. It is computationally faster to find the network completion time for a new set of activity completion times by "updating" the network completion time for a previous set of activity completion times than it is to start all over each time. Since the Simplex Algorithm applied to the dual of the PERT problem is ideally suited for this type of "updating", the basic computational technique for determining the network completion times is the Simplex Algorithm (see e.g., G. Hadley, Linear Programming).

A listing of the Monte Carlo Simulation Program is given at the end of this appendix.

Specific Input Instructions:

- Card 1. Col. 1-3 : The number of activities in the network, Format (I3).
Col. 4-6 : The number of nodes in the network, Format (I3).
- Card 2. Col. 11-15: The number of random network completion times to be generated, Format (I5).
Col. 21-25: The number of parameters needed for generating the random activity completion times, Format (I5).

For each activity one card with:

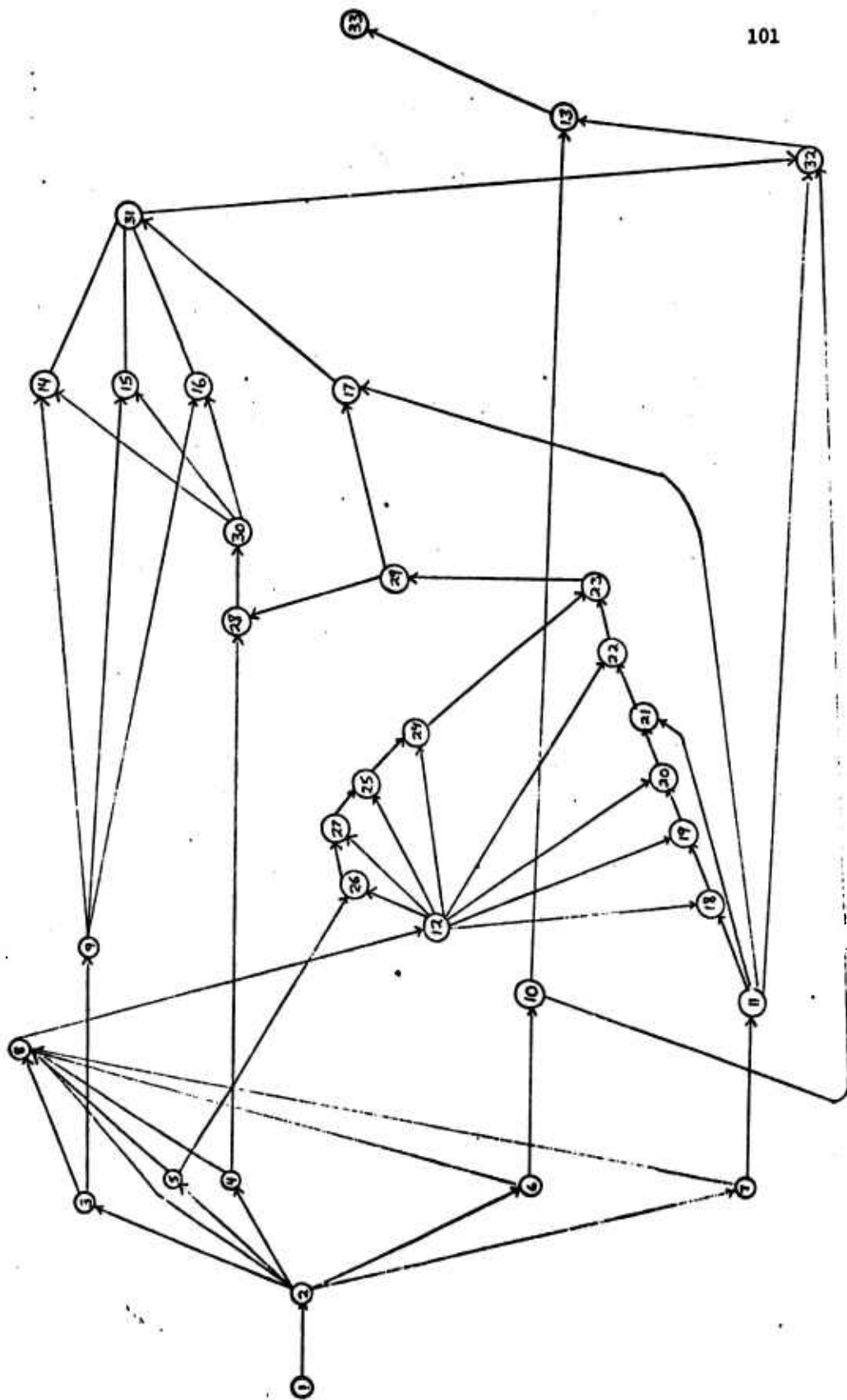
- Col. 11-15: The origin node of the activity, Format (I5)
Col. 21-25: The terminal node of the activity, Format (I5)
Col. 31-40: Parameter 1. The 15-th percentile of the activity's completion time distribution, Format (F10.4).
Col. 41-50: Parameter 2. The 85-th percentile of the activity's completion time distribution, Format (F10.4)

The nodes should be numbered 1, 2, ..., n with the sink being number n.

Example:

The program's input and output are illustrated in terms of the network in Figure D-1.

Figure D-1. Monte Carlo PERT Simulation Program Example Network



SAMPLE INPUT

102

58 33				
	5	2		
01	1	2		
02	8	12	336.27	429.47
03	2	3	57.47	89.96
04	2	4	57.47	89.96
05	2	5	57.47	89.96
06	2	6	57.47	89.96
07	2	7	57.47	89.96
08	3	8	68.96	107.95
09	4	8	68.96	107.95
10	5	8	68.96	107.95
11	6	8	68.96	107.95
12	7	8	68.96	107.95
13	2	8	150.36	193.49
14	6	10	333.96	403.85
15	3	9	333.96	403.85
16	7	11	355.10	409.85
17	11	18	141.75	221.90
18	10	13	672.36	783.11
19	9	14	560.89	660.00
20	9	15	560.89	660.00
21	9	16	560.89	660.00
22	11	17	542.80	638.71
23	12	18	111.10	173.92
24	18	19	256.03	346.98
25	12	19	302.80	400.67
26	12	20	311.95	410.71
27	11	21	423.58	530.74
28	12	22	315.35	415.71
29	19	20	7.66	11.99
30	20	21	16.77	22.55
31	21	22	11.49	17.99
32	22	23	39.54	48.87
33	12	26	301.91	400.86
34	5	26	767.09	892.74
35	12	27	350.31	460.06
36	12	24	382.32	464.98
37	12	25	385.28	461.54
38	25	24	11.49	17.99
39	24	23	16.28	23.00
40	26	27	7.66	11.99
41	27	25	20.86	28.29
42	4	28	810.17	976.10
43	23	29	15.32	23.99
44	28	30	15.32	23.99
45	14	31	57.47	89.96
46	16	31	49.81	77.96
47	15	31	53.64	83.96
48	17	31	88.12	137.94
49	31	32	3.83	6.00
50	32	13	49.81	77.96
51	13	33	109.01	152.63
52	11	32	745.50	811.32
53	10	32	714.74	799.83
54	30	14		
55	30	16		
56	30	15		
57	29	17		

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103

SAMPLE OUTPUT

INITIAL INPUT		TERMINAL	PARAMETER 1	PARAMETER 2	PARAMETER 3 (NONE SPECIFIED)	PARAMETER 4 (NONE SPECIFIED)	PARAMETER 5 (NONE SPECIFIED)
ACTIVITY	ORIGIN						
1	1	2	0.0	0.0			
2	8	12	336.2700	429.4700			
3	3	3	57.4700	99.9600			
4	2	4	57.4700	89.9600			
5	2	5	57.4700	89.9600			
6	2	6	57.4700	89.9600			
7	2	7	57.4700	89.9600			
8	3	8	68.9600	107.9500			
9	4	8	68.9600	107.9500			
10	5	8	68.9600	107.9500			
11	6	8	68.9600	107.9500			
12	7	8	68.9600	107.9500			
13	2	8	150.3600	193.4900			
14	6	10	333.9600	403.8500			
15	3	9	333.9600	403.8500			
16	7	11	333.9600	403.8500			
17	11	11	333.9600	403.8500			
18	10	13	141.7500	221.9600			
19	9	14	672.3600	783.1100			
20	9	15	560.8900	660.0000			
21	9	16	560.8900	660.0000			
22	11	17	542.8000	638.7100			
23	12	18	111.1000	173.9200			
24	18	19	256.0300	346.9800			
25	12	19	302.8000	400.6700			
26	12	20	311.9500	410.7100			
27	11	21	423.5800	530.7400			
28	12	22	315.3500	415.7100			
29	19	20	7.6600	11.9900			
30	20	21	16.7700	22.5500			
31	21	22	11.4900	17.9900			
32	22	23	39.5400	48.8700			
33	12	24	301.9100	400.8600			
34	5	26	767.0900	892.7400			
35	12	27	350.3100	460.0600			
36	12	24	382.3200	464.9800			
37	12	25	385.2800	461.5400			
38	25	24	11.4900	17.9900			
39	24	23	16.2800	23.0000			
40	26	27	7.6600	11.9900			
41	27	25	20.8600	28.2900			
42	4	28	810.1700	976.1000			
43	23	29	15.3200	23.9900			
44	28	30	15.3200	23.9900			
45	14	31	57.4700	89.9600			
46	14	31	49.8100	77.9600			
47	15	31	53.6400	83.9600			
48	17	31	88.1200	137.9400			
49	31	32	3.8300	6.0000			
50	32	13	49.8100	77.9600			
51	13	33	109.0100	152.6300			
52	11	32	745.5000	811.3200			
53	30	14	714.7400	799.8300			
54	30	14	0.0	0.0			
55	30	15	0.0	0.0			
56	30	15	0.0	0.0			
57	29	17	0.0	0.0			

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58 29 28 0.0 0.3

THE CRITICAL PATH TIMES FOR 5 TRIALS ARE AS FOLLOWS: (TIME/OBSERVED PERCENTILE)

0-148370+04	0-152120+04	0-153220+04	0-153270+04	0-155870+04
0-203000+00	0-400000+00	0-600000+00	0-800000+00	0-100000+01

```

C
C      MONTE CARLO PERT
C
C      IMPLICIT REAL*8 (A-H,O-Z)
C      FOR THE SAKE OF IDENTIFYING THE APPROPRIATE DIMENSIONS, LET
C          M = THE NUMBER OF ACTIVITIES IN THE NETWORK
C          NMM = NUMBER OF NODES IN THE NETWORK
C          NMMP1 = NMM + 1
C          N = M + NMM
C          NTRIAL = NUMBER OF SIMULATED SETS OF ACTIVITY
C          COMPLETION TIMES
C          NPARM = NUMBER OF PARAMETERS NEEDED FOR GENERATING RANDOM
C          ACTIVITY COMPLETION TIMES FROM A PARTICULAR DISTRIBUTION
C
C      INTEGER TAIL( M),HEAD( M),PARM(M,NPARM),TIMES(NTRIAL)
C      DIMENSION BIINV(NMMP1,NMMP1),CTIME(N),XB1(NMMP1),Y1(NMMP1)
C      DIMENSION REDCOS(N),ISTAT(N),INBASE(NMM),CDF(5),NSPEC(20)
C
C      CURRENTLY THE DIMENSIONS ARE SET FOR
C          M=60
C          NMM=40
C          NTRIAL=2000
C
C      COMMON BIINV,REDCOS,CTIME,XB1,INBASE,HEAD,TAIL,NMMP1,NMM,N,ISTAT
C      COMMON M,MPI
C      INTEGER TAIL(60),HEAD(60),TRIAL,NSPEC(20)
C      DIMENSION INBASE(40),PARM(60,5),CDF(5),TIMES(2000)
C      DIMENSION BIINV(41,41),CTIME(100)
C      DIMENSION XB1(41),Y1(41),REDCOS(100),ISTAT(100)
C      DATA NSPEC(1),NSPEC(5),NSPEC(9),NSPEC(13),NSPEC(17)/5*(NON*/
C      DATA NSPEC(2),NSPEC(6),NSPEC(10),NSPEC(14),NSPEC(18)/5*(F SP*/
C      DATA NSPEC(3),NSPEC(7),NSPEC(11),NSPEC(15),NSPEC(19)/5*(ECIF*/
C      DATA NSPEC(4),NSPEC(8),NSPEC(12),NSPEC(16),NSPEC(20)/5*(ED)*/
C      DATA BLANKS/' '
C      M = THE NUMBER OF ACTIVITIES IN THE NETWORK
C      NMM = THE NUMBER OF NODES IN THE PERT NETWORK
C      READ(5,100) M,NMM
100  FORMAT(2I3)
C      N=NMM+M
C      MPI=M+1
C      NMMP1=NMM+1
C      TRIAL = 0
C
C      SIMULATION VARIABLES
C
C          NTRIAL = NUMBER OF SIMULATED SETS OF ACTIVITY
C          COMPLETION TIMES
C          NPARM = NUMBER OF PARAMETERS NEEDED FOR GENERATING RANDOM
C          ACTIVITY COMPLETION TIMES FROM A PARTICULAR DISTRIBUTION
C          TIMES = VECTOR CONTAINING THE OPTIMUM VALUE FOR EACH TRIAL
C          PARM(I,1) = THE LOWER PERCENTILE POINT FOR THE I-TH ACTIVITY
C          PARM(I,2) = THE UPPER PERCENTILE POINT FOR THE I-TH ACTIVITY
C
C      READ (5,2501) NTRIAL,NPARM
C
C      THE ACTIVITIES ARE DESCRIBED IN TERMS OF THEIR NODES

```

C	II=THE TAIL NODE, THE ORIGIN NODE	60
C	JJ=THE HEAD NODE, THE TERMINAL NODE	61
C		62
	DO 610 I=1,M	63
	READ(5,2501) II,JJ,(PARM(I,J),J=1,NPARM)	64
2501	FORMAT(10X,15,5X,15, 5X,5F10.4)	65
	TAIL(I) = II	66
610	HEAD(I)=JJ	67
	NNPARM = 4*NPARM	68
	DO 222 I=1,NNPARM	69
222	NSPEC(I) = BLANKS	70
	WRITE(6,2700)	71
2700	FORMAT(1H1,15X,'INITIAL INPUT')	72
	WRITE(6,2701)	73
2701	FORMAT(1H0,10X,'ACTIVITY ORIGIN TERMINAL PARAMETER 1 P	74
	*APAMETER 2 PARAMETER 3 PARAMETER 4 PARAMETER 5')	75
	WRITE (6,2509) (NSPEC(I),I=1,20)	76
2509	FORMAT (41X,4A4,4(1X,4A4))	77
	DO 2704 I=1,M	78
2704	WRITE(6,2702) I,TAIL(I),HEAD(I),(PARM(I,J),J=1,NPARM)	79
2702	FORMAT(1H ,13X,13,5X,13,7X,13,1X,5(7X,F10.4))	80
C		81
	DO 104 I=1,NMM	82
104	INBASE(I)=M+I	83
	DO 2001 J=1,M	84
2001	ISTAT(J)=0.	85
	DO 2002 J=MP1,N	86
2002	ISTAT(J)=1	87
	DO 10 II=1,NMMP1	88
	DO 12 L=1,NMMP1	89
12	B1INV(L,II) = 0.	90
10	B1INV(II,II) = 1.	91
	DO 30 I=1,NMM	92
30	XB1(I) = 0.	93
	XB1(NMMP1) = 1.	94
	TOLR1=1.0D-10	95
C		96
	DO 55610 I=MP1,N	97
55610	CTIME(I) = 0.	98
350	CONTINUE	99
C		100
C	GENERATE A SET OF ACTIVITY COMPLETION TIMES	101
C		102
	TRIAL = TRIAL + 1	103
	CALL PANTIM (CTIME,PARM,M)	104
	CALL BINVA (52800)	105
C	START THE SIMPLEX ALGORITHM	106
C	SOLVE THE DUAL PROBLEM	107
C	THE NUMBER OF VARIABLES IS M REAL + NMM SLACKS	108
C	FOR A TOTAL OF N VARIABLES	109
C		110
2800	DO 23 J=1,N	111
	RATS = 0.	112
	IF (ISTAT(J).EQ.1) GO TO 52800	113
	IF (J.GT.M) GO TO 22	114
	RATS = -B1INV(1,HEAD(J)+1)+B1INV(1,TAIL(J)+1) + CTIME(J)	115
	GO TO 52800	116
22	RATS = -B1INV(1,J-M+1)	117
52800	REDCOS(J)= RATS	118
23	CONTINUE	119

22800	CONTINUE	120
	IRMAX=1	121
	RMAX=PEDCOS(1)	122
	DO 24 J=2,N	123
	IF(PEDCOS(J) .LE. RMAX) GO TO 24	124
	RMAX=PEDCOS(J)	125
	IRMAX=J	126
24	CONTINUE	127
	IF(RMAX .LE. TOLR1) GO TO 401	128
22824	CONTINUE	129
	DO 26 L=1,NMMP1	130
	IF (IRMAX.GT.M) GO TO 50026	131
	Y1(L) =-B1INV(L,TAIL(IRMAX)+1)+B1INV(L,HEAD(IRMAX)+1)	132
	GO TO 26	133
50026	Y1(L) = B1INV(L,IRMAX-M+1)	134
26	CONTINUE	135
	Y1(1) = Y1(1) - CTIME(IRMAX)	136
	NUMBER=0	137
	DO 27 L=2,NMMP1	138
27	IF(Y1(L) .LE. TOLR1) NUMBER=NUMBER+1	139
	IF(NUMBER .EQ. NMM) GO TO 403	140
	RMIN=.99D 20	141
	IRMIN=0.	142
	DO 32 II=2,NMMP1	143
	IF(Y1(II).LE. TOLR1) GO TO 32	144
	RATS =XB1(II)/Y1(II)	145
	RR=RATS-RMIN	146
	IF(RR .GE. 0.D0) GO TO 32	147
	RMIN=RATS	148
	IRMIN=II	149
32	CONTINUE	150
	DO 33 J=2,NMMP1	151
	WW=B1INV(IRMIN ,J)/Y1(IRMIN)	152
	DO 37 L=1,NMMP1	153
37	B1INV(L,J)=B1INV(L,J)-WW*Y1(L)	154
33	B1INV(IRMIN ,J)=WW	155
C		156
C	UPDATE THE BASIC VARIABLES: INBASE AND XB1	157
C		158
	ISTAT(INBASE(IRMIN-1))=C	159
	ISTAT(IRMAX)=1	160
	INBASE(IRMIN-1)=IRMAX	161
	W=XB1(IRMIN)/Y1(IRMIN)	162
	DO 38 I=1,NMMP1	163
38	XB1(I)=XB1(I)-Y1(I)*W	164
	XB1(IRMIN)=W	165
	GO TO 2800	166
403	WRITE(6,530)	167
530	FORMAT(1H0.5X,'NO FEASIBLE SOLUTION EXISTS. CHECK YOUR INPUT DATA	168
	,)	169
	WRITE(6,850)	170
850	FORMAT(1H1)	171
	GO TO 999	172
C		173
C	END OF THE SIMPLEX ALGORITHM	174
C		175
401	TIMES(TRIAL) = B1INV(1,NMMP1)	176
	IF (TRIAL.LT.NTRIAL) GO TO 350	177
C		178
C	INBASE IS A SET OF NMM INTEGER VARIABLES WHICH INDICATE THE	179

```

C      COMPOSITION OF THE CURRENT BASIS.  FOR EXAMPLE,
C      INBASE(K) = 7  IMPLIES THAT THE K-TH COLUMN IN THE BASIS B
C      CORRESPONDS TO THE 7-TH VARIABLE
C
C      ISTAT INDICATES THE BASIC STATUS OF EACH VARIABLE
C      ISTAT(K) = 1  IMPLIES THAT THE K-TH VARIABLE IS IN THE
C      DUAL BASIS
C      ISTAT(K) = 0  IMPLIES THAT THE K-TH VARIABLE IS NOT IN THE
C      DUAL BASIS
C
C      ORDER THE RANDOMLY GENERATED NETWORK COMPLETION TIMES
C
3000  LIMIT = NTRIAL
      IPASS = NTRIAL-1
      ICHNG = 1
      DO 4001 J=1,IPASS
      LIMIT = LIMIT-1
      IF (ICHNG.EQ.0) GO TO 3060
      ICHNG = 0
      DO 4002 I=1,LIMIT
      IF (TIMES(I).LT.TIMES(I+1)) GO TO 4002
      TEMP = TIMES(I+1)
      TIMES(I+1) = TIMES(I)
      TIMES(I) = TEMP
      ICHNG = 1
4002  CONTINUE
4001  CONTINUE
C
C      DESCRIBE THE ORDERED NETWORK COMPLETION TIMES
C
3060  WRITE(6,9662)
9662  FORMAT(1H1)
      WRITE(6,3012) NTRIAL
3012  FORMAT(1H0,10X,'THE CRITICAL PATH TIMES FOR ',15,' TRIALS ARE AS
*  FOLLOWS: (TIME/OBSERVED PERCENTILE)')
      LINE = (NTRIAL+4)/5
      DO 3050 J=1,LINE
      I2 = (J-1)*5+1
      I3 = J*5
      ICNT = 0
      IF (NTRIAL-I2+1.LT.5) I3= NTRIAL
      DO 3051 K=I2,I3
      ICNT = ICNT+1
3051  CDF(ICNT) = DFLOAT(K)/DFLOAT(NTRIAL)
      WRITE (6,3011) (TIMES(I),I= I2,I3)
3011  FORMAT(1H0,5X,5(6X,F15.5))
      WRITE (6,3013) (CDF(K),K=1,ICNT)
3013  FORMAT (6X,5(6X,F15.5))
3050  CONTINUE
999   STOP
      END
      SUBROUTINE BINVA(*)
      IMPLICIT REAL*8 (A-H,O-Z)
      COMMON BINV,REDCOS,CTIME,XBI,INBASE,IHEAD,ITAIL,NMMP1,NMM,N,ISTAT
      COMMON M,MPI
      DIMENSION ISTAT(100),IHEAD(60),ITAIL(60),XBI(41)

```

```

DIMENSION BIINV(41,41),INBASE(40),CTIME(100),REDCOS(100)
240
C
241
C      UPDATE THE FIRST ROW OF BIINV AFTER CHANGING CTIME
242
C
243
DO 1 I=2,NMMP1
244
BIINV(1,I) = 0.0
245
DO 1 J=2,NMMP1
246
1 BIINV(1,I) = BIINV(1,I) + BIINV(J,I)*CTIME(INBASE(J-1))
247
C
248
C      UPDATE VALUE OF THE OBJECTIVE FUNCTION
249
C
250
XB1(1) = BIINV(1,NMMP1)
251
RETURN
252
END
253
SUBROUTINE RANTIM (CTIME,PARM,M)
254
C
255
C      SUBROUTINE RANTIM GENERATES M RANDOM TIMES FROM A SPECIFIED
256
C      DISTRIBUTION WITH PARAMETERS CONTAINED IN PARM(60,5) AND RETURNS
257
C      WITH THE RESULTS IN CTIME(99).
258
C
259
IMPLICIT REAL*8 (A-H,O-Z)
260
DATA J/0/,IY/19447/,TPI/6.2831853/
261
DIMENSION PARM(60,5),CTIME(99),SAVTIM(99)
262
IF (J.NE.0) GO TO 30
263
C
264
C      THE FOLLOWING GENERATES A CHI SQUARE RANDOM DEViate WITH 3 D.F.'S
265
C      TRANSFORMED TO MAKE THE LOWER POINT CORRESPOND TO THE 15-TH
266
C      PERCENTILE AND THE UPPER POINT CORRESPOND TO THE 85-TH PERC.
267
C
268
C      PARM(3,I) = THE PERCENTILE DIFFERENCE
269
C
270
DO 5 I=1,M
271
PARM(I,3) = PARM(I,2)-PARM(I,1)
272
5 IF (PARM(I,3).EQ.0) CTIME(I) = PARM(I,1)
273
30 J = J+1
274
IF (MOD(J,2).EQ.1) GO TO 20
275
C
276
DO 15 I=1,M
277
IF (PARM(I,3).EQ.0 ) GO TO 15
278
CTIME(I) = SAVTIM(I)
279
15 CONTINUE
280
RETURN
281
C
282
C
283
C      GENERATE A PAIR OF COMPLETION TIMES FOR EACH ACTIVITY.
284
C      THE BOX-MULLER METHOD IS USED TO GENERATE A PAIR OF
285
C      NORMAL DEVIATES
286
C      U1 AND U2 = UNIFORM RANDOM NUMBERS
287
C      W*DSIN(AN) = STANDARD NORMAL RANDOM VARIABLE
288
C      W*DCOS(AN) = STANDARD NORMAL RANDOM VARIABLE
289
C      CHI3 = A CHI-SQUARE RANDOM VARIABLE -- GENERATED USING THE
290
C      METHOD OF WILSON AND HILFERTY - PROC. NAT. ACADEMY OF
291
C      SCIENCE, 1931
292
C      C1 AND C2 TRANSFORM THE CHI-SQUARE WITH 3 D.F.
293
C
294
20 DO 3002 I=1,M
295
IF (PARM(I,3).EQ.0) GO TO 3002
296
IY = IY*65539
297
IF (IY) 3015,3016,3016
298
3015 IY = IY + 2147483647 + 1
299

```

3016	YFL = IY	300
	U1 = YFL*.465613E-9	301
	IY = IY*65539	302
	IF (IY) 3025,3026,3026	303
3025	IY = IY + 2147483647 + 1	304
3026	YFL = IY	305
	U2 = YFL*.465613E-9	306
	W = -2.*DLOG(U2)	307
	W = DSQRT(W)	308
	AN = TPI*U1	309
	C2 = 4.51927/PARM(I,3)	310
	C1 = .79777-PARM(I,1)*C2	311
	CTIME(I) = ((.392530614*W*DSIN(AN)+1.335416269)**3-C1)/C2	312
	SAVTIM(I) = ((.392530614*W*DCOS(AN)+1.335416269)**3-C1)/C2	313
3002	CONTINUE	314
	RETURN	315
	END	316

unclassified

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See page following Table of Contents

9 THEMIS optimization research program,

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