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STEADY STATE ANALYSIS OF HYDRAZINE CATALYTIC
THRUSTERS FOR DIFFERENT TYPES OF CATALYSTS

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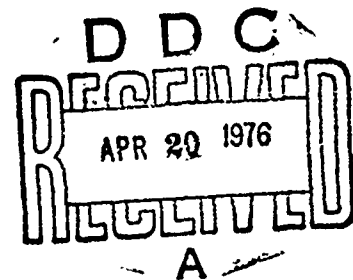
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A B S T R A C T

A theoretical analysis has been made to determine the characteristics of the steady state behaviour of a hydrazine decomposition engine. The model makes use of the simplification that consists in considering that the ammonia decomposition is only important at high temperatures. The results are valid for different types of catalysts with different stoichiometries and reaction rates for hydrazine and ammonia decomposition. Conditions at the chamber exit are obtained analytically as a function of chamber length and other characteristics, in particular expressions are given for fractional ammonia decomposition at the exit and the specific impulse obtained if the outgoing mixture would be expanded to the vacuum. Fluid and particle temperature distributions along the chamber are also calculated keeping, however, fixed a non-dimensional parameter expressing the ratio of the mass and heat transfer from the interstitial fluid to the particle.

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INTRODUCTION

The purpose of this work is to carry out a theoretical analysis that will provide a set of relations which can be used as a basis in a first step of the design of hydrazine engines with catalytic decomposition. In this work we shall only consider the steady state behaviour of the engines. Previous work on the theoretical aspects of this problem has been made by Kesten¹ and our research group of INTA². Works of a more practical nature, including experimental results, have been made by the Rocket Research Corporation³ and Adler et al⁴.

In all these works and in the present one, it is assumed that Hydrazine decomposes yielding Ammonia, Hydrogen and Nitrogen, and in turn Ammonia decomposes also giving Hydrogen and Nitrogen. Whereas in the previous works it was assumed that the stoichiometry of Hydrazine decomposition was given, in the present work we consider it as a parameter making the results more general.

Kesten¹ obtained extensive numerical results and provided a computer program for further calculations (Smith, Smith and Kesten⁵). Although he also gave some correlations of his numerical results for some range of variation of the parameters, it is always difficult to extend these correlations to other parameters of interest and to obtain the physical insight of the problem that an analytical solution provides. For example his correlation is only valid for the Shell 405 Catalyst and some particular conditions that were assumed regarding stoichiometry, velocity of reaction diffusion coefficients etc.. The previous work made at INTA² made some simplifications on the model of Kesten that did not affect essentially the physics of the problem, obtaining in turn a solution with all the characteristics of the reactor, that depended only on three non-dimensional parameters

of which only one appeared to be of importance. This model still retained some parameters fixed such as the kinetics of hydrazine decomposition and the reaction rate of ammonia decomposition. The present model is a further simplification of the previous work² that essentially consist in considering that ammonia decomposition is only important at temperatures close to the maximum one that is attained in the reactor. This simplification, in turn allows to retain as variable parameters of the problem all the characteristics determining the kinetics inside the particle and the transport coefficients, making the results applicable to other types of catalysts.

In the first part of the work the problem is stated and a review is made of the formulation of the problem as was presented in ref. 2, extending it in order to include more general characteristics of the catalyst. In the second part the problem is solved considering that there is no ammonia decomposition, providing a first approximation to some aspects of the solution of the problem. In the third part the ammonia decomposition effect is retained.

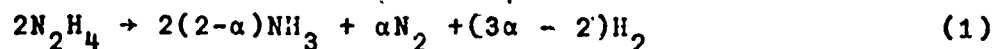
The temperature and concentration distributions along the chamber are calculated as functions of the particle surface temperature distribution, and in order to calculate it, it is necessary to solve a single ordinary differential equation of first order in which appear two parameters; one related to the ammonia decomposition rate and the other being the ratio of the heat and mass transfer coefficients from the interstitial fluid to the particle. A solution of this equation, appropriate for chamber exit conditions, is found that gives temperature, concentration of species, and specific impulse. This solution reproduces most of the aspects of Kesten's¹ correlation. However, in order to find other parameters such as maximum temperature and its

location in the chamber it is necessary to solve that differential equation. The nature of the solution of this differential equation depends critically on whether the parameter representing the ratio of the mass and heat transfer coefficients to the particle is smaller or larger than one; in ref. 2 it was assumed to be equal to one, whereas in ref. 1 it was taken to be less than one. The most fundamental difference in the results seems to be that for this number smaller than one a maximum is reached in the particle surface temperature, whereas if it is equal or larger than one, the particle surface temperature always decreases. Since the most important results for the designer are those corresponding to the chamber exit, and these are obtained analytically, we do not dwell much in the present work on the significance of this parameter, and reserve its discussion for future research. Numerical solutions are presented for this parameter equal to 0.6 that is approximately the value given in ref. 1.

1. FORMULATION OF THE PROBLEM

The problem to be studied consists on the steady state analysis of the catalytic chamber of a hydrazine microthruster for space propulsion.

In the chamber hydrazine flows between the interstices of the particles and diffuses to their interior, where it decomposes generating heat and products that are transferred to the interstitial fluid. In the kinetic model assumed here hydrazine decomposes according to the reaction



and in turn ammonia decomposes also



The hydrazine decomposition process is exothermic with a heat of reaction per unit mass of reactant H_h

$$H_h = 1930 + 300(1-\alpha)\text{BTU/lb} \quad (3)$$

and the ammonia decomposition is endothermic with

$$H_a = 1400 \text{ BTU/lb} \quad (4)$$

for temperatures around 1100°K appropriate for this study.

In refs. 1 and 2 it was assumed that $\alpha = 1$, and in ref 3 $\alpha=2/3$ for the Shell 405. In ref. 4 α was taken 2/3 for other type of catalyst.

The model used to study this problem in ref. 1 is

presented schematically in figure 1. Three regions of the microthruster are considered. In a first vapor region hydrazine enters the chamber in liquid form and decomposes slowly keeping its feeding temperature until it suddenly increases to the boiling temperature at the operating pressure. Then, follows a region where liquid hydrazine and its vapors and those of the decomposition products coexist. In this region hydrazine is being vaporized and the temperature of the mixture does not change. When all the hydrazine is vaporized in the third vapor region, the heat released in the hydrazine decomposition goes into heating the mixture increasing its temperature. This temperature increase continues until the endothermic ammonia decomposition process becomes more important than the exothermic hydrazine decomposition process, after that, when there is almost no hydrazine left, the temperature of the flow decreases.

In the previous work carried out at INTA² the following equations, in the vapor region, expressing that the variation rate of the different species along the chamber is equal to the rate at which they diffuse from the catalyst particles to the interstitial fluid, were considered

$$\left(\frac{G}{\rho_i}\right) \frac{dY_{ji}}{dz} = K_c^j (Y_{js} - Y_{ji}) A_p \quad (5)$$

$$j = h, a, H_2, N_2$$

Where G is the mass flow rate per unit cross section of the chamber, ρ is the density, Y the mass fraction of the different species, z the axial coordinate along the chamber, K_c a mass transfer coefficient that depends on the interstitial fluid properties, and A_p the external surface of catalyst particle per unit volume of bed. Subscripts and superscripts h and a

refer to hydrazine and ammonia respectively, and i and s to interstitial fluid and particle surface conditions.

A similar equation is given for the energy conservation

$$c_p G \frac{dT_i}{dz} = h_c (T_s - T_i) A_p \quad (6)$$

where c_p is an average specific heat at constant pressure of the gas mixture, T the temperature, and h_c a heat transfer coefficient. This is a system of five equations to determine the temperature and mass fractions of the four species in the interstitial fluid along the chamber. In order to complete the system, an equation of state is required, that we take to be of the form

$$P = \rho TR \left[\frac{Y_{H_2}}{2} + \frac{Y_{N_2}}{28} + \frac{Y_a}{17} + \frac{Y_h}{32} \right] \quad (7)$$

where the pressure P is assumed to be constant, that means that we neglect the pressure variations compared to the absolute value of the pressure (*); and also are required relations between the particle surface properties and the interstitial fluid properties that we are going now to present.

In ref. 2 an analytical solution was found of the problem of calculating the temperature and mass fractions distribution inside the particle by solving the heat and mass conservation equations of the four species, subject to the conditions expressing regularity at the particle center, and that the heat and species fluxes at the surface are those given by the right

(*) The validity of this assumption was checked in ref. 2.

hand side of equations (5) and (6). From this solution the required relations between the interstitial fluid properties and the particle/surface properties are found. The details of this resolution can be found in ref. 2, here we shall only reproduce the final results. In ref. 1 this particle problem was solved numerically, simultaneously with equations (5) and (6) along the chamber. In figure 2 is presented an schematic of the temperature and hydrazine and ammonia mass fractions distributions inside the catalyst as they were calculated in ref. 2. One of the main results, also appearing in ref. 1, is that the hydrazine mass fraction at the particle surface is very small compared to the corresponding interstitial value; the decomposition rate is controlled by external diffusion to the particle because the velocity of reaction of hydrazine is very large. Consequently, a detailed knowledge of the kinetic mechanism of hydrazine decomposition is not required for this study. Whereas hydrazine decomposes in a very thin region close to the particle surface, ammonia decomposes in a much thicker region that may or may not be small compared to the particle radius. The following equations expressing an species and energy balance at the particle surface give the required relations between the particle surface properties and the interstitial fluid properties

$$\rho_i K_c^j (Y_{ji} - Y_{js}) = - v_{aj} \frac{\rho_s D_{as} Y_{as}}{a} A - v_{hj} \rho_i K_c^h Y_{hi} \quad (8)$$

$$h_c (T_i - T_s) = H_a \frac{\rho_s D_{as} Y_{as}}{a} A - H_h K_c^h \rho_i Y_{hi} \quad (9)$$

$$Y_{hs} = 0 \quad (10)$$

where

$$j = a, N_2, H_2$$

and v_{ij} are the stoichiometric coefficients of the decomposition reactions, v_{aa} equals -1. The left hand sides of (8) and (9) represent the external fluxes to the particle (see equations (5) and (6)); the first and second terms of the right hand sides represent the ammonia and hydrazine decomposition contributions to the internal fluxes, respectively. a is the particle radius, A is a non-dimensional number proportional to the slope of the ammonia mass fraction at the inner edge of the hydrazine decomposition region, and D_a is the ammonia diffusion coefficient in the particle. For an ammonia decomposition rate of the form

$$k_a = b \exp(-T_a/T) \quad (11)$$

where b is constant^(*), A is found to be a function of the three non-dimensional parameters

$$\gamma = T_a/T_s \quad (12)$$

$$\phi^2 = k_{as} a^2 / D_{as} \quad (13)$$

$$\beta = \rho_s D_{as} H_a Y_{as} / (K_s T_s) \quad (14)$$

where K is the heat conduction coefficient inside the particle. The parameter β expresses the ratio of the mass and heat diffusion velocities inside the particle and is very small because, whereas the diffusion is controlled by the gas inside the particle, the heat transfer is controlled by the solid part, that usually

(*) according to ref. 1, b depends on the hydrogen mass fraction like $Y_{H_2}^{-1.6}$. However we may assume that hydrogen diffuses so fast in the particle that its mass fraction is constant equal to its surface value.

is made of aluminum or other good heatconducting metals. For $\gamma\beta = 0$, A takes the form

$$A = \phi \operatorname{cth} \phi - 1 \quad (15)$$

For large values of ϕ and moderately small values of $\gamma\beta$

$$A = \phi(1 + \gamma\beta/6) - (1 + \gamma\beta/9) + O(1/\phi_s) + O(\gamma^2\beta^2) + O(\beta) \quad (16)$$

As we shall see later, the contribution of ammonia decomposition is only important for large values of T_s , when ϕ is large, and we shall only retain the first term of the right hand side of (16).

We now need boundary conditions for equations (5) and (6) that give the temperature and mass fractions at the beginning of the vapor region. The initial temperature is the boiling temperature of hydrazine at the operating pressure

$$T = T_b(P) \quad (17)$$

The initial value of the hydrazine mass fraction, Y_{hb} , is determined by the fact that enough hydrazine should decompose to liberate the heat necessary for the hydrazine to vaporize and reach its boiling temperature. In ref. 2 this initial value is given in the form

$$T_f = T_b + \frac{H_h}{c_p} Y_{hb} \quad (18)$$

where T_f can be interpreted, as we shall see later, as the maximum temperature that would be reached by the mixture if there would not be ammonia decomposition. Ref. 2 gives for $\alpha = 1$

$$T_f = 1450^\circ\text{K} \quad (19)$$

Ref. 1 gives, also for $\alpha = 1$, the expression

$$T_f = 1420(1 + 0.035(P/70))^{\circ}K \quad (20)$$

, with p in atmospheres, that gives a slight variation of T_f with pressure from $1421^{\circ}K$ for $p = 1$ atm to $1463^{\circ}K$ for $p = 70$ atm. Refs. 3 and 4 give a value of $1650^{\circ}K$ for $\alpha = 2/3$. Using the same procedure of ref. 2, and taking for H_h the expression given in (3) the following correlation seems to be appropriate for T_f

$$T_f = 1450 + 600(1-\alpha) \quad (21)$$

that reproduces the results of refs. 1 to 4.

The initial values of the mass fractions of the other species are obtained as functions of the initial hydrazine mass fraction by considering that no-ammonia has been decomposed in the initial vaporization region. From the stoichiometry of equation (1) it is obtained:

$$Y_{ab} = \frac{17}{32} (2 - \alpha)(1 - Y_{hb}) \quad (22)$$

$$Y_{N_{2b}} = \frac{7}{16} \alpha (1 - Y_{hb}) \quad (23)$$

$$Y_{H_{2b}} = \frac{1}{32} (3\alpha - 2)(1 - Y_{hb}) \quad (24)$$

Before proceeding further, it should be noticed that three first integrals of the system of five differential equations (5) and (6) can be obtained by linear combination of these equations. On using the previous initial conditions (17) (18) and (22) to (24) it is obtained for these integrals

$$\frac{14}{17} Y_{ai} + Y_{N_{2i}} + \frac{7}{8} Y_{hi} = \frac{7}{8} \quad (25)$$

$$\frac{3}{17} Y_{ai} + Y_{H_{2i}} + \frac{1}{8} Y_{hi} = \frac{1}{8} \quad (26)$$

$$(Y_{ai} + (2-\alpha)\frac{17}{32} Y_{hi}) - \frac{c_p}{H_a} (T_i + \frac{H_h}{c_p} Y_{hi}) = (2-\alpha)\frac{17}{32} - \frac{c_p T_f}{H_a} \quad (27)$$

that allow to express three of the five independent variables as functions of the two others.

2. SOLUTION WITH NO-AMMONIA DECOMPOSITION

As it was stated in the introduction, the main simplification of this problem consists in considering that the ammonia decomposition is only important at high particle surface temperatures, close the maximum one reached with no ammonia decomposition. Consequently the solution obtained in this part will reproduce many of the aspects of the general solution obtained in the next part, and will also give the optimum value of the specific impulse.

To neglect ammonia decomposition corresponds to make the first terms on the right hand side of equations (8) and (9) equal to zero. With this approximation, and considering equation (10), equation (5) (for hydrazine) and equation (6) can be written in the form

$$\frac{G}{\rho_i} \frac{dY_{hi}}{dz} = - K_c^h A_p Y_{hi} \quad (28)$$

$$c_p G \frac{dT_i}{dz} = - H_h K_c^h \rho_i Y_{hi} \quad (29)$$

By dividing these two equations it is obtained

$$c_p \frac{dT_i}{dY_{hi}} = - H_h \quad (30)$$

and on integration with the initial condition (12)

$$T_i = T_f - \frac{H_h}{c_p} Y_{hi} \quad (31)$$

When all hydrazine is decomposed T_i reaches T_f . From (28) Y_{hi} goes to zero exponentially like

$$Y_{hi} = Y_{hb} e^{-\xi} \quad (32)$$

where

$$\xi = \frac{A}{G} \int_0^z K_c^h \rho_i dz \quad (33)$$

$K_c^h \rho_i$ is an slowly varying function of T_i that could in principle be considered constant with a certain average value of T_i between T_b and T_f .

By taking (31) into equations (25) to (27), we obtain the concentration of the species as functions of Y_{hi}

$$Y_{ai} = \frac{17}{32} (2-\alpha)(1-Y_{hi}) \quad (34)$$

$$Y_{N_{2i}} = \frac{7}{16} \alpha (1-Y_{hi}) \quad (35)$$

$$Y_{H_{2i}} = \frac{3\alpha-2}{32} (1-Y_{hi}) \quad (36)$$

For Y_{hi} equal to zero we obtain the conditions when all hydrazine is decomposed

$$T_f, \quad Y_{af} = \frac{17}{32} (2-\alpha), \quad Y_{N_{2f}} = \frac{7}{16} \alpha, \quad Y_{H_{2f}} = \frac{3\alpha-2}{32} \quad (37)$$

The corresponding specific impulse when this gas is expanded to the vacuum is

$$I_f^2 = \frac{2h_f}{g^2} = \frac{2}{g^2} \sum h_{jf} Y_{jf} \quad (38)$$

Using appropriate interpolation formulas for the enthalpies of the different species taken from ref. 6

$$I_f^2 = (56.49 + 2.44\alpha)T_f - (20117 - 8125\alpha) \quad (39)$$

and taking for T_f the expression (21)

$$I_f / (270 \text{ sec.}) = (1 + 0.326(1-\alpha) - 0.02(1-\alpha)^2)^{1/2} \approx 1 + 0.163(1 - \alpha) \quad (40)$$

The values of the particle surface properties are obtained from equations (8) and (9), with the ammonia production terms equal to zero, and the values of the interstitial properties calculated before in (31) and (34) to (36)

$$Y_{as} = 17/32 (2-\alpha) \left[1 + \left(\frac{K_c^h}{K_c^a} - 1 \right) Y_{hi} \right] \quad (41)$$

$$Y_{N_{2s}} = 7/16 \alpha \left[1 + \left(\frac{K_c^h}{K_{N_2}^h} - 1 \right) Y_{hi} \right] \quad (42)$$

$$Y_{H_{2s}} = \frac{3\alpha-2}{32} \left[1 + \left(\frac{K_c^h}{K_{H_2}^h} - 1 \right) Y_{hi} \right] \quad (43)$$

$$T_s = T_f \left[1 + \frac{H_h}{c_p T_f} \left(\frac{\rho_i K_c^h c_p}{h_c} - 1 \right) Y_{hi} \right] \quad (44)$$

For all K_c equal and

$$L_e = \frac{\rho_i K_c^h c_p}{h_c} = 1 \quad (45)$$

, as it was assumed in ref. 2, the surface properties do not change. If $L_e > 1$ T_s starts with a value higher than T_f . However, in general L_e is smaller than one, around 0.6.

When all the hydrazine is decomposed the particle surface properties and the interstitial properties become equal.

3. SOLUTION WITH AMMONIA DECOMPOSITION.

The ammonia decomposition effect is represented by the first term on the right hand side of equations (8) and (9). If we make equation (9) non-dimensional by dividing it by $h_c T_f$, the relative importance of the ammonia decomposition is measured by the term

$$\left[\frac{H_a \rho_s D_{as} Y_{as}}{h_c a T_f} \right] A$$

The first factor in brackets is small, of the order of 10^{-2} , so if this effect is going to be significant, A has to be large and the parameter ϕ must be large too. According to this, we take for A the expression given in equation (16) keeping only the first term of the right hand side. On using equations (11) and (13), we can rewrite the ammonia decomposition term as

$$J \exp \left(\frac{T_a}{2T_f} \frac{T_s - T_f}{T_s} \right)$$

where, for typical values of catalytic reactors,

$$J = H_a \rho_s D_{as}^{1/2} Y_{as}^{1/2} b_s \exp \left(- \frac{T_a}{2T_f} \right) / h_c T_f \quad (46)$$

is of order one. On the other hand the factor $\exp \left(\frac{T_a}{2T_f} \frac{T_s - T_f}{T_s} \right)$ experiences changes of order one for relatively small changes of T_s such that

$$\frac{T_f - T_s}{T_s} \sim \frac{T_f}{T_a} \ll 1$$

since T_a is much larger than T_f . According to Kesten¹ T_a has a

value of 27778°K. On the other hand, if T_s is smaller than T_f by an order of magnitude of one, the ammonia production term becomes very small. In the previous part it was seen that with no-ammonia decomposition the particle surface temperature was given by

$$T_s = T_f \left(1 - \frac{H_h}{c_p T_f} (1 - L_e) Y_{hi} \right)$$

If $L_e < 1$, T_s will be smaller than T_f in a quantity of order one (if Y_{hi} is also of order one) and it will be consistent with the analysis to neglect the ammonia decomposition effect. However, as Y_{hi} becomes smaller, T_s gets closer to T_f , and the ammonia decomposition effects should be taken into account. Consequently we can assume that the ammonia production term is only important for values of T_s close to T_f , and if J does not exhibit such a strong dependence on T_s as the exponential term, we can make the approximation of taking for J , given in equation (46), the value J_f , corresponding to take for the variables determining J the values given in equation (37). Equation (9) is then written in the form

$$\frac{T_i - T_s}{T_f} = J_f \exp\left(\frac{T_a}{2T_f} \frac{T_s - T_f}{T_f}\right) - \frac{H_h}{c_p T_f} L_e Y_{hi} \quad (47)$$

This equation has to be solved together with those determining the evolution of the hydrazine mass fraction, and the interstitial temperature

$$\frac{dY_{hi}}{d\xi} = -Y_{hi} \quad (48)$$

$$L_e \frac{dT_i}{d\xi} = T_s - T_i \quad (49)$$

where the non-dimensional distance ξ is defined in equation (33). By eliminating T_i between equations (47) and (49) and taking Y_{hi} as independent variable, a single differential equation is obtained for T_s

$$\begin{aligned} \frac{Y_{hi}}{T_f} \left[1 + J_f \frac{T_a}{2T_f} \exp\left(\frac{T_a}{2T_f} \frac{T_s - T_f}{T_f}\right) \right] \frac{dT_s}{dY_{hi}} = \\ = \frac{J_f}{L_e} \exp\left[\frac{T_a}{2T_f} \frac{T_s - T_f}{T_f}\right] - \frac{H_h}{c_p T_f} (1 - L_e) Y_{hi} \end{aligned} \quad (50)$$

to be solved with the initial condition

$$Y_{hi} = Y_{hib} ,$$

$$T_s = T_f (1 - (1 - L_e) \frac{H_h}{c_p T_f} Y_{hb}) \quad (51)$$

This initial condition is obtained from equation (47) by substituting in it the initial values of Y_{hi} and T_i (equs. (17) and (18)) and neglecting the effect of ammonia decomposition, that according to this theory is exponentially small initially.

In order to reduce the number of parameters we rewrite equation (50) and boundary condition (51) in the non-dimensional variables

$$\phi = \frac{T_a}{2T_f} \frac{T_s - T_f}{T_f} \quad (52)$$

$$y = Y_{hi} (1 - L_e) \frac{H_h}{c_p T_f} / (2T_f / T_a) \quad (53)$$

obtaining the equation

$$(1 + \lambda \exp \phi) \frac{d\phi}{dy} = \frac{\lambda}{L_e} \frac{\exp \phi}{y} - 1 \quad (54)$$

and boundary condition

$$y = 1/\delta, \quad \phi = -1/\delta \quad (55)$$

where

$$\lambda = J_f / (2T_f / T_a) \quad (56)$$

and

$$\delta = \frac{2T_f}{T_a} / (1 - L_e) \frac{H_h}{c_p T_f} Y_{hb} \quad (57)$$

δ is an small number, and at the entrance of the reactor ϕ is a large negative number that makes the term $\exp \phi$ in (54) exponentially small. The solution of (54) and (55) is initially of the form

$$\phi = -y$$

that coincides with the solution given by equation (44) for ammonia decomposition. Consequently, boundary condition (55) will be substituted by

$$\phi + y = 0, \quad y \rightarrow \infty \quad (55 a)$$

eliminating the parameter δ from this problem. In figure 3 is presented the solution of (54) and (55 a) for different values of λ and $L_e = 0.6$, giving the nondimensional particle temperature

ϕ , and the equivalent interstitial temperature, ϕ_i

$$\phi_i = \frac{T_a}{2T_f} \frac{T_i - T_f}{T_f} = \lambda \exp \phi - \frac{L_e}{1-L_e} y + \phi \quad (58)$$

as functions of $\xi_1 = -\ln y$

Since from (48) Y_{hi} is related to ξ through

$$Y_{hi} = Y_{hb} \exp(-\xi), \quad (59)$$

on using (53) and (57) it can be seen that ξ and ξ_1 are related by

$$\xi_1 = \xi + \ln \delta = \xi - |\ln \delta| \quad (60)$$

In figure 3 it can be seen that the particle surface temperature is initially smaller than the interstitial temperature and both increase, reaching a maximum first the particle temperature, and at a later stage the interstitial temperature. When the maximum of the interstitial temperature is reached, both the interstitial and particle surface temperature are equal. In ref. 2 it was assumed that $L_e = 1$, and the particle temperature always exhibited a decreasing behaviour, the same is expected for $L_e > 1$. However, it should be remarked that for $L_e > 1$ the particle surface temperature starts with values close to T_f and the ammonia decomposition is important from the beginning.

In figure 3 is also presented the asymptotic solution of (54) for large values of ξ . The corresponding asymptotic form of (54) is

$$\frac{d\phi}{d\xi_1} = -\frac{\lambda}{L_e} \exp \phi \quad (61)$$

that corresponds to neglect the effect of hydrazine decomposition compared to that of ammonia decomposition that is also small

$$y \ll \exp \phi \ll 1 \quad (62)$$

The solution of (60) is

$$\phi = - \ln \left[\frac{\lambda}{L_e} (\xi_1 + c) \right] \quad (63)$$

where c is a constant of integration that has been calculated from the numerical solutions and varies from -1.7 for $\lambda = 1$ to 2.8 for $\lambda = 100$. The solution presented in figure 3 corresponds to $c = 0$ that is consistent with considering ξ_1 large. In figure 3 it can be seen that if the chamber length is moderately larger than the distance corresponding to the attainment of the maximum interstitial temperature, the asymptotic solution gives a good approximation for both the interstitial and particle temperatures at the chamber exit. From equation (58) and condition (62) it can be seen that both temperatures are practically equal at the chamber exit.

An important parameter that is usually found in the literature is the fractional ammonia decomposition, $(1-F)$

$$F = Y_{ai}/Y_{af} = \frac{32}{17} Y_{ai}/(2-\alpha) \quad (64)$$

Expressing Y_{ai} as a function of $T_i \approx T_s$ with equation (27), assuming that $Y_{hi} \approx 0$, and then expressing T_s as function of ξ , through equations (52) and (63) we arrive at the asymptotic form of F

$$F = 1 - \frac{c_p T_f}{H_a} \frac{2T_f}{T_a} \frac{1}{Y_{af}} \ln \left[\frac{\lambda}{L_e} \xi_1 \right] \quad (65)$$

If we observe that the quantity

$$\epsilon = \frac{c_p T_f}{H_a} \frac{2T_f}{T_a} \frac{1}{Y_{af}}$$

is small, expression (65) can be approximately rewritten as

$$F \approx \left[\frac{\lambda}{L_e} \xi_1 \right]^{-\epsilon} \quad (66)$$

For $\alpha = 1$ and Shell 405, ϵ has a value of 0.26. A correlation for F has been obtained in ref. 1 that has many of the characteristics of equation (66), in particular, if we take for $\epsilon = 0.28$ the dependence of F on chamber length and mass flow rate turns out to be the same.

To complete this work we shall now give an expression relating the specific impulse that would be obtained expanding to the vacuum as a function of F .

$$(I_g)^2 = 2 \sum Y_j h_j \quad j = a, N_2, H_2 \quad (67)$$

where it has been assumed that the hydrazine concentration is zero. With this assumption the mass fractions of the hydrogen and nitrogen and the interstitial temperature can be expressed as functions of Y_{ai} by equations (25) to (27), and on using appropriate interpolation formulas for the enthalpies of the species, taken from ref. 6, the following expression, is obtained for the specific impulse as a function of F (equation (64)) and α

$$(I/I_f)^2 = 1 - \frac{0.3(2-\alpha)}{1+0.326(1-\alpha)} (1-F) [1-0.07(1-\alpha)+0.07(2-\alpha)(1-F)]$$

where I_f is the maximum specific impulse with no ammonia decomposition obtained in the previous section

$$I_f = 270(1 + 0.163(1-\alpha)) \text{ sec.}$$

and F is given in formulas (65) and (66).

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$$d \ll L$$

T_b = BOILING TEMPERATURE

T_0 = INJECTION TEMPERATURE

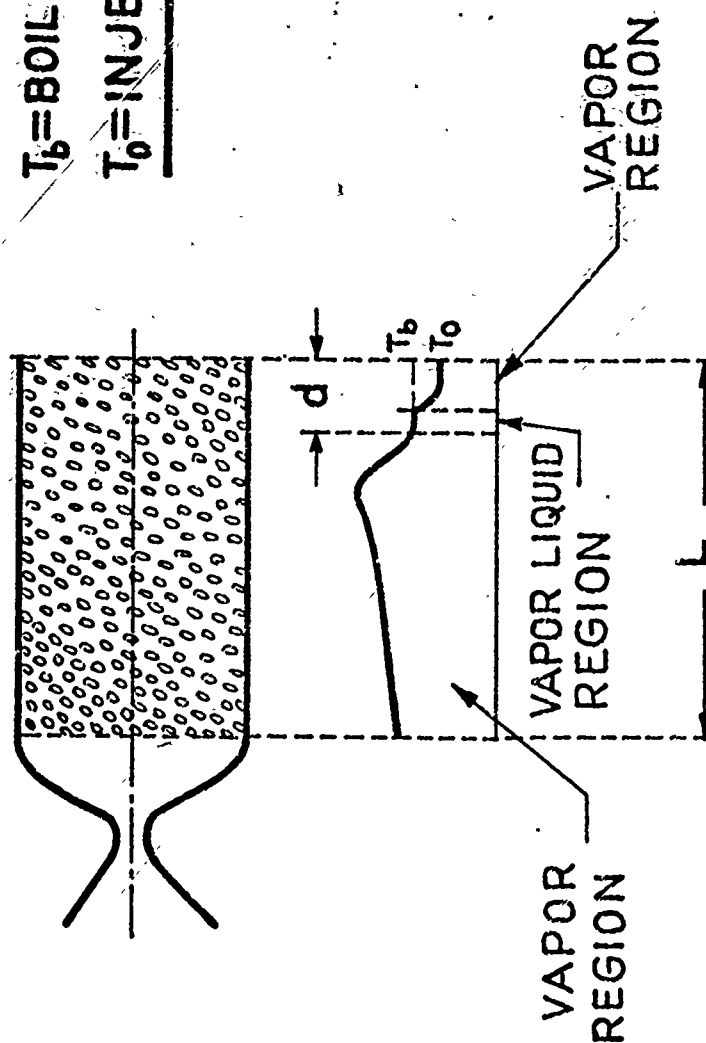
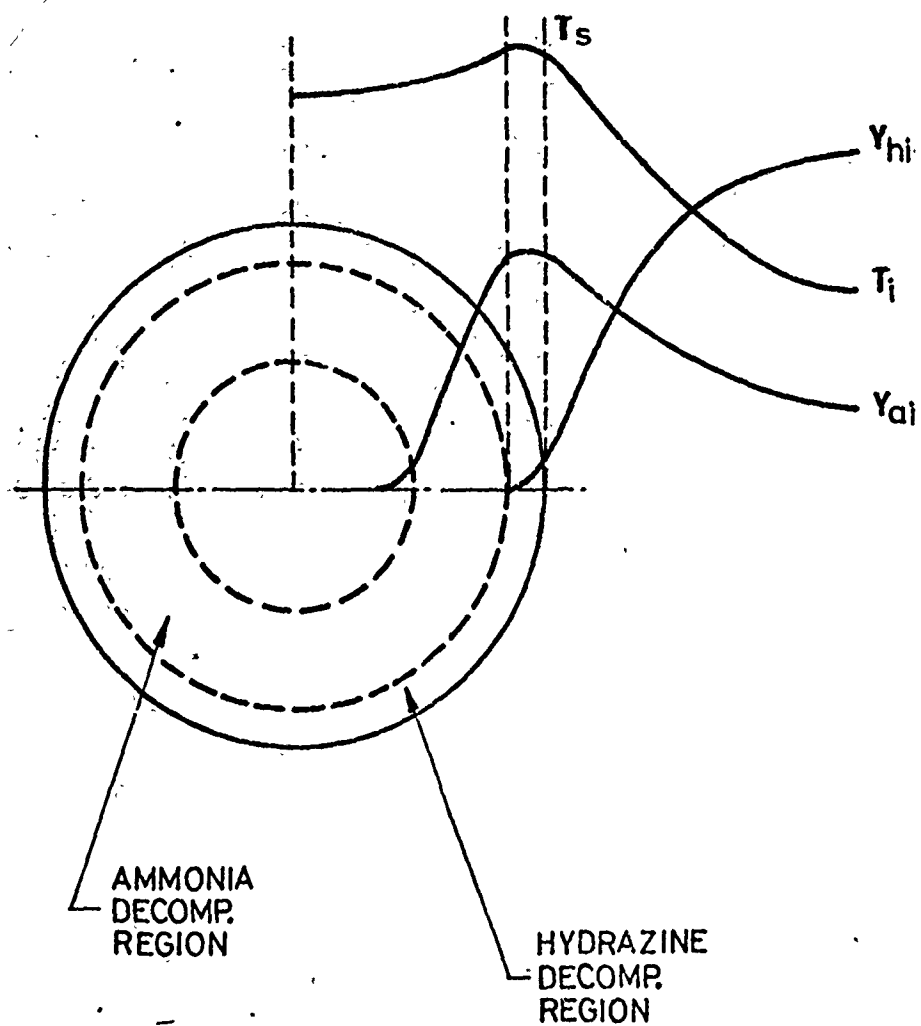


FIG. N°1

FIG. N°2



TEMPERATURE DISTRIBUTION ALONG THE CHAMBER

$\lambda = 1$

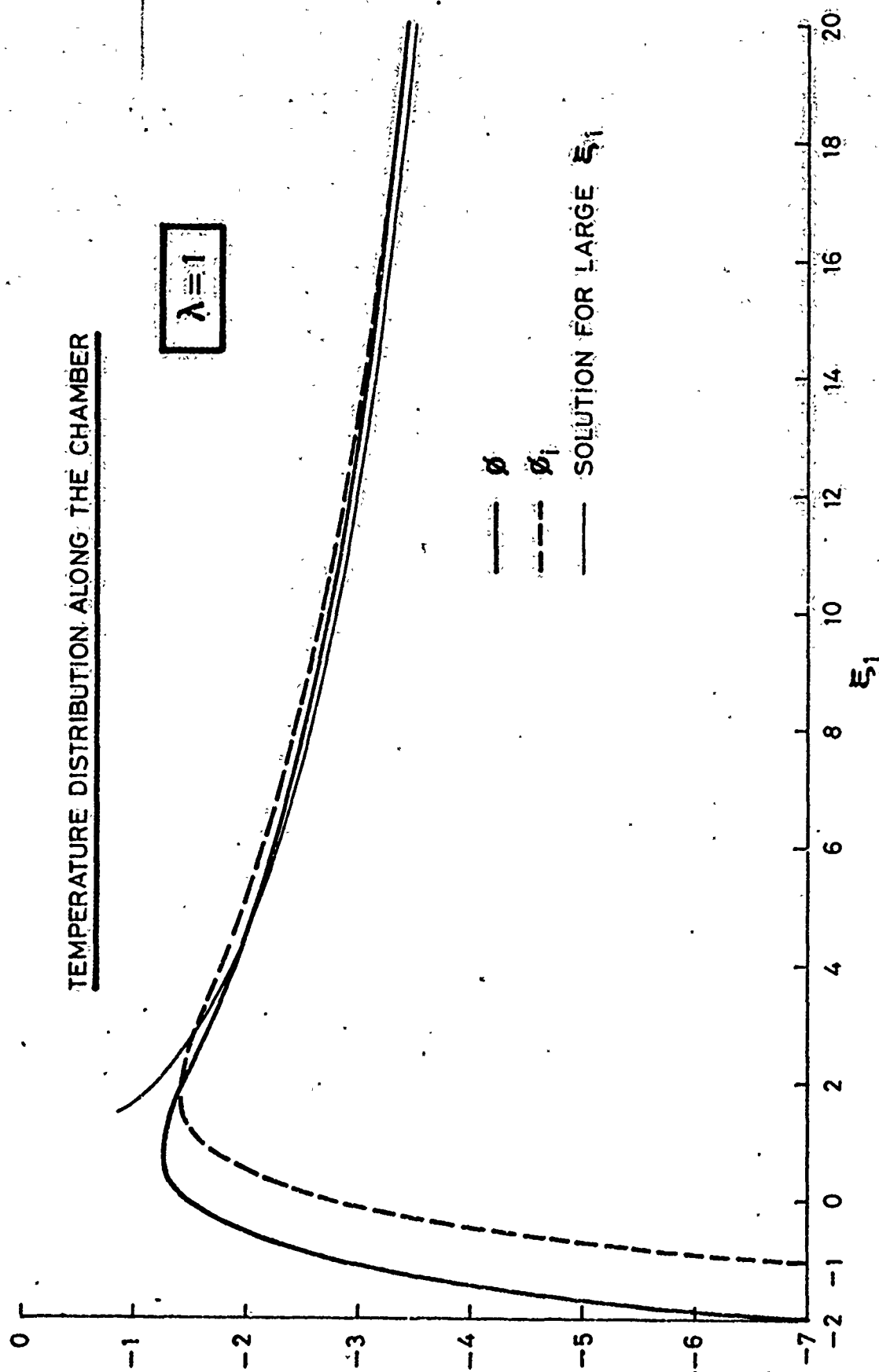


FIG. No 3a

TEMPERATURE DISTRIBUTION ALONG THE CHAMBER

$\lambda=10$

— ϕ
- - ϕ_1
— SOLUTION FOR LARGE ξ_1

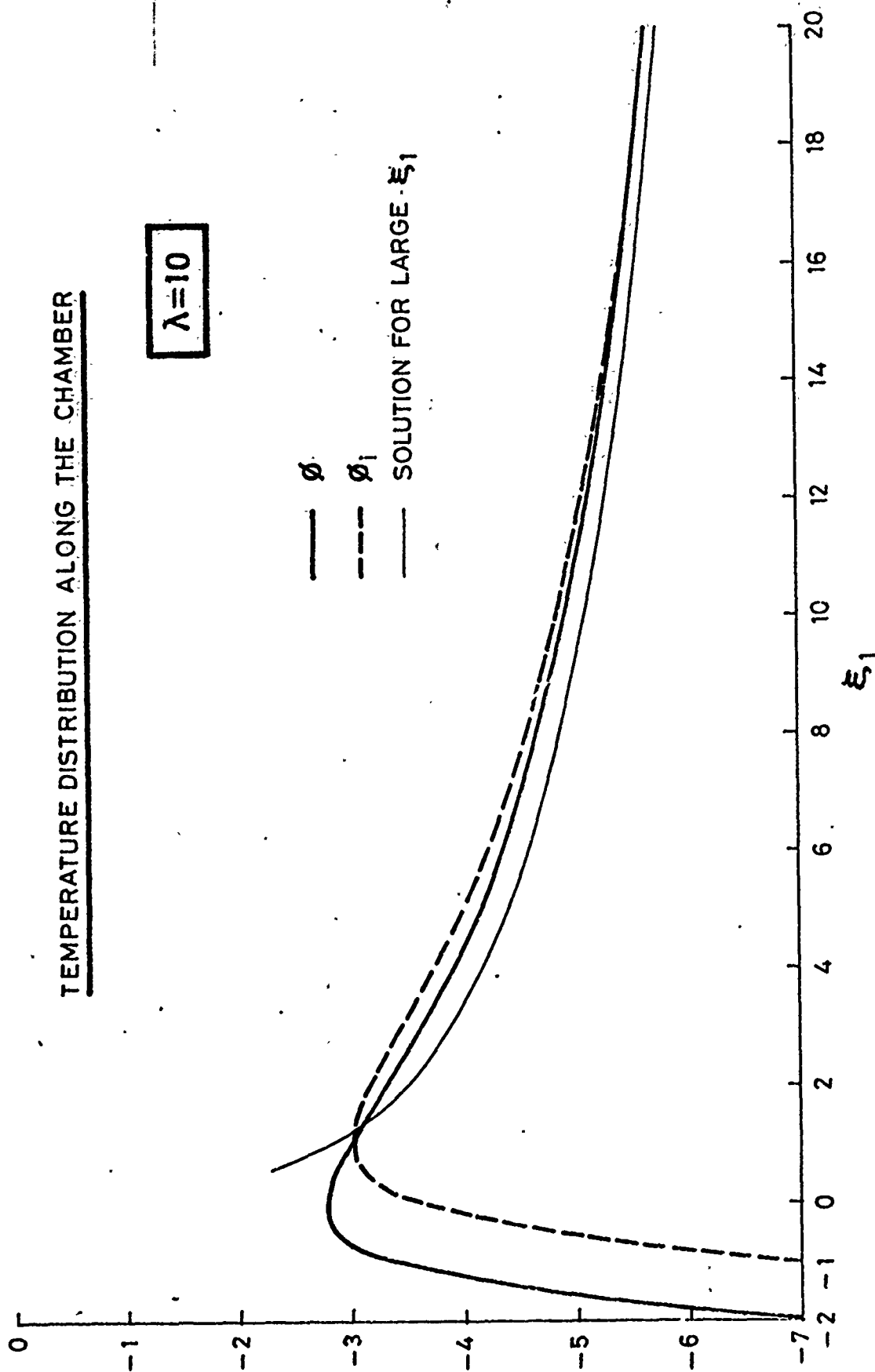


FIG. No 3b

TEMPERATURE DISTRIBUTION ALONG THE CHAMBER

$\lambda = 50$

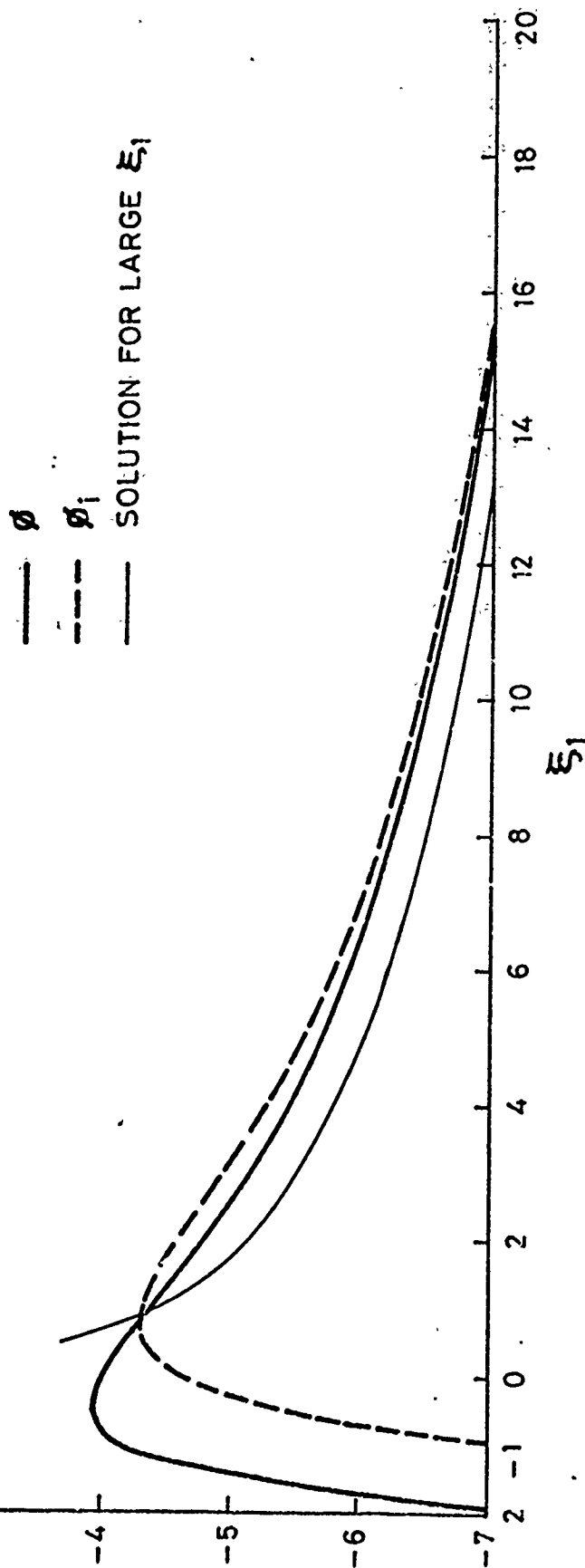


FIG. N^o 3_c