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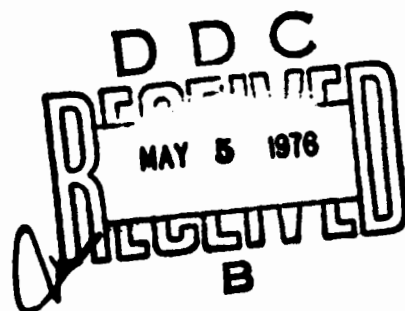
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Flux-Corrected Transport Modules for Solving Generalized Continuity Equations

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March 1976



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FLUX-CORRECTED TRANSPORT MODULES FOR SOLVING GENERALIZED CONTINUITY EQUATIONS

1. Introduction.

This report documents two new Fortran subroutines for solving generalized continuity equations of the form

$$\begin{aligned} \frac{\partial \rho}{\partial t} = & - \frac{1}{r^{\alpha-1}} \frac{\partial}{\partial r} (r^{\alpha-1} \rho V) \\ & + \frac{1}{r^{\alpha-1}} \frac{\partial}{\partial r} (r^{\alpha-1} D_1) + C_2 \frac{\partial D_2}{\partial r} + D_3. \end{aligned} \quad (1)$$

These routines, ETBFCT and PRBFCT, use the latest flux-corrected transport techniques.^{1,2,3} The underlying algorithm gives fourth-order accurate phases and minimum residual diffusion. The loops all vectorize to take full advantage of the pipeline architecture of the NRL Texas Instruments Advanced Scientific Computer. Use of internal scratch and vector temporary memory has been minimized. A rather general facility for handling the source terms in Eq. (1) above has been provided so that coupled, multidimensional non-linear sets of equations, e.g. the equations of ideal compressible hydrodynamics, can be solved using one or two simple Fortran modules.

The routine ETBFCT can treat Cartesian, cylindrical, or spherical one-dimensional systems and PRBFCT treats periodic Cartesian systems. Using these routines, multidimensional problems are solved by timestep splitting techniques. The finite-difference grid at the end

of the timestep can differ from the grid at the beginning of the timestep. Thus Lagrangian and Sliding Rezone calculations in any of the three coordinate systems are as easy and efficient to perform as Eulerian calculations. When the fluid velocity is zero but the grid is moved, the routines deliver a positive, conservative interpolation.

The concept of phoenical antidiffusion³ has been generalized to Lagrangian numerical grids. A modification is included to incorporate the excellent nonlinear properties of explicit antidiffusion without sacrificing the performance of phoenical antidiffusion for passive convection. In terms of the error criteria of FCT/I, II, and III for our canonical convection problem, ETBFCT and PRBFCT have an average absolute error of 0.038, the smallest error yet attained by a simple general purpose algorithm.

The two routines are coded in Fortran and listings are shown in the appendices. Appendix A contains a listing of ETBFCT while Appendix B presents PRBFCT. Appendix C contains a progressing shock test program using ETBFCT with results of a test run. Appendix D contains a bursting diaphragm test program and results of a test run also using ETBFCT. Appendices E and F contain test programs using PRBFCT with test runs, the former solving a squarewave test problem and the latter solving a nonlinear diaphragm problem. Section II describes the algorithm used in the two routines.

II. The Algorithm.

The algorithm used in ETEFCT is described below. The differences between this algorithm and that in PRBFCT are minimal and are given at the end of the section. This section also attempts to explain the reasoning behind the design decisions taken in constructing these routines.

Figure 1 shows a one-dimensional tube of fluid where the velocities are constrained to move along the tube. The variable r measures length along the tube and U^f is the fluid flow velocity along r . The points in the center of the indicated cells are the finite-difference grid points. There are N of them and their positions at the beginning of a numerical timestep are denoted by $\{r_i^0\} (i = 1, \dots, N)$. At the end of a timestep of duration δt the grid points are at $\{r_i^n\}$, where

$$r_i^n = r_i^0 + U_i^g \delta t \quad \text{for } i = 1, \dots, N \quad (2)$$

Here $U_i^g \delta t$, the grid displacement, is zero for an Eulerian representation and equals $U_i^f \delta t$, the fluid displacement, in a Lagrangian mesh.

Figure 1 also shows the basic cell volumes $\{\Lambda_i\}$ and the cell interface areas $\{A_{i+\frac{1}{2}}\}$. The interface areas are assumed to be perpendicular to the tube and hence to $\{U_i^f\}$. The change in the total amount of a convected quantity in a cell is just the flux of that quantity into and out of the cell through the interfaces. Both $\{\Lambda_i\} (i = 1, \dots, N)$ and $\{A_{i+\frac{1}{2}}\} (i = 0, \dots, N)$ have to be calculated using both new and old grid positions and they have to be properly

related for the representation to be fully consistent.

Figure 1 has N grid points and $N+1$ interfaces including the two at the ends of the tube. The positions of the interfaces are denoted by $\{r_{i+\frac{1}{2}}\}(i = 0, \dots, N)$ with superscript o or n for "old" or "new" mesh respectively. In the following we consider a single timestep. These interface positions are calculated as

$$r_{i+\frac{1}{2}}^{o,n} = \frac{1}{2} [r_i^{o,n} + r_{i+1}^{o,n}] \quad \text{for } i = 1, \dots, N-1. \quad (3)$$

The end point positions, $r_{\frac{1}{2}}^{o,n}$ and $r_{N+\frac{1}{2}}^{o,n}$, have to be specified by the user and might be the location of bounding walls, for example.

Equation (3) could have been more general since there is no hard requirement that the interfaces be halfway between the grid points. The above choice is simple, accurate, and very efficient.

We also need the flux of fluid across the interface which moves from $r_{i+\frac{1}{2}}^o$ to $r_{i+\frac{1}{2}}^n$ during the cycle. The velocities of the fluid and grid are known at the grid points and therefore the velocity of the fluid relative to the grid is

$$\delta U_i = U_i^f - U_i^g \quad \text{for } i = 1, \dots, N. \quad (4)$$

Since the fluxes out of one cell into the next are needed on the cell interfaces, we naturally form

$$\delta U_{i+\frac{1}{2}} = \frac{1}{2} [\delta U_i + \delta U_{i+1}] \quad \text{for } i = 1, \dots, N-1. \quad (5)$$

The end points $\delta U_{\frac{1}{2}}$ and $\delta U_{N+\frac{1}{2}}$ are calculated using the specified end points, $r_{\frac{1}{2}}^{o,n}$ and $r_{N+\frac{1}{2}}^{o,n}$, and two end-point velocities $U_{\frac{1}{2}}^f$ and $U_{N+\frac{1}{2}}^f$.

Thus

$$\begin{aligned} \delta U_{\frac{1}{2}} &= U_{\frac{1}{2}}^f - \frac{(r_{\frac{1}{2}}^n - r_{\frac{1}{2}}^o)}{\delta t}, \text{ and} \\ \delta U_{N+\frac{1}{2}} &= U_{N+\frac{1}{2}}^f - \frac{(r_{N+\frac{1}{2}}^n - r_{N+\frac{1}{2}}^o)}{\delta t}. \end{aligned} \quad (6)$$

To complete determination of the flux we need the density at the cell interfaces. This is taken to be

$$\rho_{i+\frac{1}{2}}^o = \frac{1}{2} [\rho_{i+1}^o + \rho_i^o] \quad \text{for } i = 1, \dots, N-1. \quad (7)$$

Again we need the values on the interfaces $i = \frac{1}{2}$ and $i = N+\frac{1}{2}$. Since they are needed several times using updated ρ values, the formulae

$$\rho_0 = L_B \rho_1 \quad \text{and} \quad (8)$$

$$\rho_{N+1} = R_B \rho_N$$

are used to calculate densities one cell beyond the last cell. Then $(\rho_1 - \rho_0)$ and $(\rho_{N+1} - \rho_N)$ are always defined at the first and last

interfaces and Eq. (7) above gives us

$$\begin{aligned}\rho_{\frac{1}{2}}^0 &= \left(\frac{1}{2} + \frac{L_B}{2}\right) \rho_1^0, \text{ and} \\ \rho_{N+\frac{1}{2}}^0 &= \left(\frac{1}{2} + \frac{R_B}{2}\right) \rho_N^0.\end{aligned}\tag{9}$$

Using the above definitions the convective transport part of the continuity equation can be solved.

$$\begin{aligned}\Lambda_1^0 \rho_1^* &= \Lambda_1^0 \rho_1^0 - \delta t \rho_{1+\frac{1}{2}}^0 A_{1+\frac{1}{2}} \delta U_{1+\frac{1}{2}} \\ &\quad + \delta t \rho_{1-\frac{1}{2}}^0 A_{1-\frac{1}{2}} \delta U_{1-\frac{1}{2}}\end{aligned}\tag{10}$$

for $i = 1, \dots, N$. The lefthand side $\Lambda_1^0 \rho_1^*$ has not yet undergone the compression or expansion implied by the difference between the Λ_1^0 and Λ_1^n . The source terms have not yet been used and the diffusion and antidiffusion portions of flux correction remain to be included.

The source terms in Eq. (1), if nonzero, are added into Eq. (10) as follows:

$$\begin{aligned}\Lambda_1^0 \rho_1^T &= \Lambda_1^0 \rho_1^* + \frac{\delta t}{2} A_{1+\frac{1}{2}} (D_{1,1+1} + D_{1,1}) - \frac{\delta t}{2} A_{1-\frac{1}{2}} (D_{1,1} + D_{1,1-1}) \\ &\quad + \frac{\delta t}{4} C_{2,1} (A_{1+\frac{1}{2}} + A_{1-\frac{1}{2}}) (D_{2,1+1} - D_{2,1-1}) + \delta t \Lambda_1^0 D_{3,1}\end{aligned}\tag{11}$$

for $i = 2, \dots, N-1$. The end values, $i = 1$ and $i = N$ are treated

using D_R and D_L , the user specified right and left boundary values of D , in place of the interface average at the boundaries. The routines ETBFCT and PRBFCT are designed so that other source terms can be added easily to the formalism. The three source terms given provide a simple set adequate to describe many important fluid dynamic problems.

The diffusion stage of this FCT algorithm also includes the compression according to the following equation

$$\begin{aligned} \Lambda_i^n \tilde{\rho}_i = & \Lambda_i^O \rho_i^T + v_{i+\frac{1}{2}} \Lambda_{i+\frac{1}{2}} (\rho_{i+1}^O - \rho_i^O) \\ & - v_{i-\frac{1}{2}} \Lambda_{i-\frac{1}{2}} (\rho_i^O - \rho_{i-1}^O) \end{aligned} \quad (12)$$

for $i = 1, \dots, N$. The diffusion coefficients $\{v_{i+\frac{1}{2}}\}$ will be defined shortly and are chosen to reduce phase errors from second to fourth order. The interface volumes $\{\Lambda_{i+\frac{1}{2}}\}$ appear in Eq. (12) in order that $\{v_{i+\frac{1}{2}}\}$ can be dimensionless. They are defined to be

$$\Lambda_{i+\frac{1}{2}} = \frac{1}{2}(\Lambda_{i+1}^n + \Lambda_i^n) \quad \text{for } i = 1, \dots, N-1. \quad (13)$$

with the boundary interface values taken to be

$$\Lambda_{\frac{1}{2}} = \Lambda_1^n \quad \text{and} \quad \Lambda_{N+\frac{1}{2}} = \Lambda_N^n. \quad (14)$$

Remember that the diffusion and antidiffusion stages in FCT are

basically nonphysical and are included only to reduce or eliminate certain types of numerical error. Therefore, I have made most choices in the averaging on the basis of efficiency.

The convective transport, sources, compression, and diffusion have been broken into the two stages of Eqs. (10-11) and (12) because we need to compute antidiffusive fluxes using $\{\rho_i^T\}$ to obtain our generalized phoenical algorithm. If we compute the antidiffusive flux using $\{\tilde{\rho}_i\}$, the algorithm will exhibit residual diffusion when the grid is Lagrangian and even in the special case when the grid and the fluid are stationary. The uncorrected antidiffusive fluxes are defined to be

$$F_{i+\frac{1}{2}} = \kappa_{i+\frac{1}{2}} \Lambda_{i+\frac{1}{2}} \left[\rho_{i+1}^T - \rho_i^T \right] \quad (15)$$

for $i = 0, \dots, N$. Simple linear antidiffusion of the transported-diffused solution $\{\tilde{\rho}_i\}$ would give

$$\Lambda_1^n \rho_1^n = \Lambda_1^n \tilde{\rho}_1 - F_{i+\frac{1}{2}} + F_{i-\frac{1}{2}} \quad (16)$$

for $i = 1, \dots, N$. The generalized phoenical antidiffusion is designed so that

$$\Lambda_1^n \rho_1^n = \Lambda_1^o \rho_1^o \quad (17)$$

when the grid is Lagrangian and hence $\{\delta U_{i+\frac{1}{2}}\}$ vanishes in Eq. (10).

Substituting Eq. (10) and Eq. (11) into Eq. (12) yields

$$\begin{aligned}\Lambda_1^n \rho_1 &= \Lambda_1^O \rho_1^O + v_{1+\frac{1}{2}} \Lambda_{1+\frac{1}{2}} (\rho_{1+1}^O - \rho_1^O) \\ &\quad - v_{1-\frac{1}{2}} \Lambda_{1-\frac{1}{2}} (\rho_1^O - \rho_{1-1}^O)\end{aligned}\tag{18}$$

since $\rho_i^T = \rho_i^O$ in the Lagrangian case without sources. The anti-diffusion of Eqs. (15) and (16) used on Eq. (18) gives

$$\begin{aligned}\Lambda_1^n \rho_1^n &= \Lambda_1^O \rho_1^O + (v_{1+\frac{1}{2}} - \mu_{1+\frac{1}{2}}) \Lambda_{1+\frac{1}{2}} (\rho_{1+1}^O - \rho_1^O) \\ &\quad - (v_{1-\frac{1}{2}} - \mu_{1-\frac{1}{2}}) \Lambda_{1-\frac{1}{2}} (\rho_1^O - \rho_{1-1}^O).\end{aligned}\tag{19}$$

The desired generalized phenical result can be achieved in Eq. (19)

as long as $v_{1+\frac{1}{2}} = \mu_{1+\frac{1}{2}}$ whenever the grid is Lagrangian.

As explained in an earlier paper,¹ the choices

$$v_{1+\frac{1}{2}} = \frac{1}{6} + \frac{\epsilon_{1+\frac{1}{2}}^2}{3},\tag{20}$$

$$\mu_{1+\frac{1}{2}} = \frac{1}{6} - \frac{\epsilon_{1+\frac{1}{2}}^2}{6}$$

reduce the relative phase errors in a locally uniform grid region to fourth order. If we define

$$\epsilon_{i+\frac{1}{2}} \equiv A_{i+\frac{1}{2}} \delta U_{i+\frac{1}{2}} \frac{\delta t}{2} \left[\frac{1}{\Lambda_i^n} + \frac{1}{\Lambda_{i+1}^n} \right], \quad (21)$$

for $i = 0, \dots, N$, the diffusion and antidiffusion coefficients are equal in the Lagrangian case as required. Furthermore, $\{\epsilon_{i+\frac{1}{2}}\}$ is computed directly from existing quantities without an extra divide because $(\Lambda_i^0)^{-1}$ and $(\Lambda_i^n)^{-1}$ are computed for use elsewhere anyway.

With $\{F_{i+\frac{1}{2}}\}$ from Eq. (15) as the raw antidiffusive flux and with $\{\tilde{\rho}_i\}$ known from Eq. (12), the quantities $\{S_{i+\frac{1}{2}}\}$ can be defined having the sign of $[\tilde{\rho}_{i+1} - \tilde{\rho}_i]$ and the magnitude unity. The corrected antidiffusive flux is then calculated from the formula

$$F_{i+\frac{1}{2}}^C = S_{i+\frac{1}{2}} \max \left\{ 0, \min \left[|F_{i+\frac{1}{2}}|, S_{i+\frac{1}{2}} \Lambda_{i+1}^n (\tilde{\rho}_{i+2} - \tilde{\rho}_{i+1}), \right. \right. \\ \left. \left. S_{i+\frac{1}{2}} \Lambda_i^n (\tilde{\rho}_i - \tilde{\rho}_{i-1}) \right] \right\} \quad (22)$$

for $i = 1, \dots, N-1$. For the corrected fluxes $F_{\frac{1}{2}}^C$ and $F_{N+\frac{1}{2}}^C$, the $\min[\dots, \dots, \dots]$ term in Eq. (22) above contains only two terms; the term coming from differences which are undefined beyond the boundary is effectively dropped.

The result is then computed as in Eq. (16) where the corrected flux $\{F_{i+\frac{1}{2}}^C\}$ replaces $\{F_{i+\frac{1}{2}}\}$. The density returned to the user is

$$\rho_i^n = \tilde{\rho}_i - \frac{1}{\Lambda_i^n} [F_{i+\frac{1}{2}}^C - F_{i-\frac{1}{2}}^C]. \quad (23)$$

A few of the geometric variables used above have yet to be defined. The obvious choice of volume elements in Cartesian, cylindrical, and spherical geometries is used in ETRFCT

$$\begin{aligned} \Lambda_1^{o,n} &= [r_{1+\frac{1}{2}}^{o,n} - r_{1-\frac{1}{2}}^{o,n}] && \langle \text{Cartesian} \rangle, \\ \Lambda_1^{o,n} &= \pi [(r_{1+\frac{1}{2}}^{o,n})^2 - (r_{1-\frac{1}{2}}^{o,n})^2] && \langle \text{cylindrical} \rangle, \quad (24) \\ \Lambda_1^{o,n} &= \frac{4\pi}{3} [(r_{1+\frac{1}{2}}^{o,n})^3 - (r_{1-\frac{1}{2}}^{o,n})^3] && \langle \text{spherical} \rangle. \end{aligned}$$

The corresponding interface areas are taken to be:

$$\begin{aligned} A_{1+\frac{1}{2}} &= 1 && \langle \text{Cartesian} \rangle, \\ A_{1+\frac{1}{2}} &= \pi [r_{1+\frac{1}{2}}^o + r_{1+\frac{1}{2}}^n] && \langle \text{cylindrical} \rangle, \quad (25) \\ A_{1+\frac{1}{2}} &= \frac{4\pi}{3} [(r_{1+\frac{1}{2}}^o)^2 + r_{1+\frac{1}{2}}^o r_{1+\frac{1}{2}}^n + (r_{1+\frac{1}{2}}^n)^2] && \langle \text{spherical} \rangle. \end{aligned}$$

These interface areas are time and space centered, but other centered choices are also possible. These particular definitions are forced if a constant density profile is to remain constant and unchanged when $\{U_1^f\} = 0$ and the grid is rezoned arbitrarily. If L_B and R_B are unity, the rezone can even move fluid into and out of the system and the density will still be constant.

The ETBFCT algorithm has been shown to generalize phoenical antidiffusion to Lagrangian systems by requiring zero residual linear diffusion whenever $U_i^f = U_i^g$. ETBFCT has also been designed to deliver back the same constant no matter how the system is rezoned, provided that new interfaces don't cross old grid point locations. Finally, in an Eulerian representation the similarity solution with constant density and linearly increasing velocity is reproduced to second order. That is, when $\dot{r}_i = C(t)\bar{r}_i$, the continuity equation for $\rho(r,t)$ has the solution

$$\rho(r,t) = \rho(t) = \rho(0) \exp\left[-\alpha \int_0^t C(t)dt\right]. \quad (26)$$

Assuming C is constant (at least during one timestep) and starting with $\{\rho_i^0\} = \rho^0$ (a constant), Eq. (10) becomes

$$\Lambda_1^0 \rho_1^T = \Lambda_1^0 \rho^0 - \rho^0 C \delta t \left[A_{1+\frac{1}{2}} \bar{r}_{1+\frac{1}{2}} - A_{1-\frac{1}{2}} \bar{r}_{1-\frac{1}{2}} \right]. \quad (27)$$

Since the grid isn't moving, $\left[A_{1+\frac{1}{2}} \bar{r}_{1+\frac{1}{2}} - A_{1-\frac{1}{2}} \bar{r}_{1-\frac{1}{2}} \right] = \alpha \Lambda_1^0 + \text{higher order terms}$. Dividing Eq. (27) through by Λ_1^0 gives

$$\rho_1^T = \rho^0 [1 - \alpha C \delta t + \text{higher order terms}]. \quad (28)$$

In other words the spatial constancy of ρ is preserved and ρ compresses or expands according to whether C is greater than or less than zero. Again, $\alpha = 1, 2, 3$ for Cartesian, cylindrical, or

spherical coordinates. The higher order terms can be chosen to give second-order accurate approximations to Eq. (26). This would be done by letting $\{U_1^f\}$ be modified to be the velocity that a fluid element at the grid point i would have halfway through the time-step. A slight acceleration is involved since the velocity field changes in a frame of reference moving with the fluid.

The algorithm used in PRBFCT differs from the above only in rather obvious ways. The code is restricted to the Cartesian geometry and thus the interface areas are all taken to be unity. The PRBFCT routine requires all input arrays to be specified at $N+1$ points where the $N+1$ st value must equal the first value. The only exception is the array of radii input to NGRIDP. There $RADN(N+1) = RADN(1) + \text{SYSTEM LENGTH}$. Obviously no special treatment is required at either of the boundaries. For example, Eqs. (8) are replaced by

$$\rho_0 = \rho_N \quad \text{and} \quad (8')$$

$$\rho_{N+1} = \rho_1.$$

Similarly, Eqs. (14) are replaced by

$$\Lambda_{\frac{1}{2}} = \frac{1}{2}(\Lambda_N^n + \Lambda_1^n) \quad \text{and} \quad (14')$$

$$\Lambda_{N+\frac{1}{2}} = \Lambda_{\frac{1}{2}}.$$

Table 1 shows the correspondence between the mnemonic Fortran notation used in ETBFCT and in PRBFCT and the mathematical notation used above. This table and the comments appearing in the source listings of the routines themselves should make the actual Fortran implementation of the algorithm quite easy to follow.

III. How to Use ETBFCT.

Use of ETBFCT is described below. Examples of its use are given with test results in the appendices. Using PRBFCT is virtually the same--differing only in simplifications of the calling sequences which are allowed by the periodic geometry. ETBFCT has several entries to perform the five distinct tasks required to advance the general continuity equation one timestep:

1. OGRIDE(N) - which sets up the old grid, i.e. at the beginning of the timestep(based on the most recent new grid).
2. NGRIDE(RADN, N, RADR, RADL, ALPHA) - which sets up the new grid, i.e. the mesh on which the new densities will be specified at the end of the timestep.
3. VELOCE(U, N, UR, UL, DT) - which calculates all velocity dependent terms such as $\{\delta U_{1+\frac{1}{2}}\}$ and the diffusion and anti-diffusion coefficients $\{v_{1+\frac{1}{2}}\}$ and $\{\mu_{1+\frac{1}{2}}\}$.
4. SOURCE(N, DT, MODES, C, D, DR, DL) - which can be called once or repeatedly to build up composite source terms of several types.
5. ETRFCT(RHOO, RHON, N, RBC, LBC) - which takes the old densities RHOO on the old mesh and calculates the new densities RHON on the new mesh using flux-corrected transport.

A sixth entry, CONSRE(RHO, N, CSUM), calculates the conservation sum

$$CSUM = \sum_{i=1}^N \Lambda_i^n \rho_i \quad (28)$$

where the quantity summed can be any grid quantity but where the cell volumes $\{\Lambda_i^n\}$ are for the grid set up by the most recent call to NGRIDE. The listings of the two subroutines in the appendices contain prompts for all of the arguments in the form of comment cards with each entry.

The five tasks are programmed separately to eliminate all unnecessary recalculation of quantities which appear in several different continuity equations having the same flow field (VELOCE) or which must be recalculated only when the grid is changed (OGRIDE and NGRIDE). The source terms (SOURCE) are treated separately because they are not used at all in many continuity equations, because the calling sequences would be unwieldy if all possible source terms were allowed at once, and because it is sometimes necessary to accumulate sources from several terms of the same form. Each of these five tasks is examined in some detail below.

OGRIDE is called whenever the user wishes to copy geometric variables determined during the most recent call to NGRIDE into the arrays set aside for old grid values. This leaves the original new-grid arrays free to be reinitialized with a new grid. Thus, when the grid is changed every timestep, as in a Lagrangian calculation, a call to NGRIDE usually must be preceded by a call to OGRIDE to store off all the grid variables from the previous cycle (which are

now the old grid variables).

Separating the OGRIDE and the NGRIDE functions also allows HGRIDE to be called twice, as for a half step and then a whole step from the same starting point, without having to reestablish the old grid after the half step.⁶ This separation of function also means that NGRIDE has to be called at the very beginning of the calculation to establish the first grid variables before they can be copied over into the old grid arrays by OGRIDE.*

The only argument to OGRIDE is N, the number of points in the mesh. NGRIDE has five arguments. The user stores the N values of the new grid positions in RADN(1), ..., RADN(N). RADR is the new value of $r_{N+\frac{1}{2}}^n$, the right side system boundary position, and RADL is the new value of $r_{\frac{1}{2}}^n$, the left side system boundary position. If the grid is being used unchanged for many timesteps, NGRIDE and OGRIDE only have to be called once. The final argument, ALPHA, takes the values 1, 2, and 3 for Cartesian, cylindrical and spherical systems, respectively. The algorithm can be applied to other 1D problems, of course, by generating a block of code to calculate the cell volumes and interface areas appropriate to any given metric. The basic subroutine includes only three most often-used coordinate systems.

The next step is to call VELOCE (also having five arguments).

* Notes of warning: 1. When a multidimensional problem is being solved, both NGRIDE and OGRIDE have to be called every cycle. This is true even in the Eulerian case since the x and y grids (for example) will overwrite any information from the other left in ETBFCT. 2. A call to NGRIDE must follow a call to OGRIDE before all variables are initialized. Therefore the sequence NGRIDE(--), OGRIDE(--), NGRIDE(--), is actually required to completely initialize the routine.

VELOCE is called when the flow velocity is changed. The fluid flow velocity at the N grid points is stored in $U(1), \dots, U(N)$ before calling VELOCE. These velocities are assumed to apply as averages over the timestep and hence U_i is really the flow velocity at the position $\bar{r}_i = \frac{1}{2}[r_i^0 + r_i^n]$. In the calling sequence UR and UL are the fluid velocities at $\bar{r}_{N+\frac{1}{2}}$ and $\bar{r}_{\frac{1}{2}}$ respectively, the right and left boundaries of the system. DT is the current timestep. During the half step of hydrodynamic calculations use $0.5 * DT$.

After VELOCE has been called for $\{U_i\}$, each of the generalized continuity equations for which $\{U_i\}$ is the appropriate flow velocity can be solved by calling ETBFCT. If a particular continuity equation requires source terms, SOURCE, with seven arguments, is called once for each source term just before calling ETBFCT. Again N is the number of grid points and DT the timestep. MODES is an integer currently limited to the values 1, 2, or 3. When $MODES = 1$, the source term $(\nabla \cdot \underline{D}_1)$ in Eq. (1) is computed conservatively. When $MODES = 2$, the source term $(\underline{C}_2 \cdot \nabla \underline{D}_2)$ in Eq. (1) is computed. When $MODES = 3$, the source term $\underline{D}_3$ in Eq. (1) is formed. The entry SOURCE is designed so new special-purpose source calculations can be added and referenced by $MODES = 4, 5, 6$, etc. When $MODES = 1$ and 3, the array C is not used so any variable name can be used in the calling sequence.

Since the sources $C(1), \dots, C(N)$ and $D(1), \dots, D(N)$ are defined at the grid points, DR and DL are required, the values of D at the right and left boundaries of the system ($\bar{r}_{N+\frac{1}{2}}$ and $\bar{r}_{\frac{1}{2}}$). A

logical variable LSOURC in ETBFCT is set .TRUE. whenever SOURCE is called to accumulate source terms. Whenever ETBFCT is called, the calculations involving source terms are only performed when LSOURC has been set .TRUE. by a previous call to SOURCE. On exiting, ETBFCT always sets LSOURC = .FALSE. Therefore source terms should be calculated immediately before they are to be used. For example, if source terms for an energy continuity equation are calculated before the momentum continuity equation is advanced, they will be lost when the call to ETBFCT is made to solve the momentum equation. Then momentum and energy equations will be solved incorrectly.

When the grid, the flow field, and the source terms have been completely described, the desired quantity ρ is convected numerically by calling ETBFCT. The values RHOO(1), ..., RHOO(N) are the N grid point density values to be advanced, RHON(1), ..., RHON(N) are the variables into which the new values $\{\rho_i^n\}$ are to be stored. RHOO and RHON may be the same storage locations if one can afford to destroy the old values. RBC and LBC are real variables into which quantities describing the right and left boundary conditions on $\{\rho_i\}$ are stored. ETBFCT assumes that

$$\rho_{N+1} = RBC \cdot \rho_N, \quad (29)$$

$$\rho_0 = LBC \cdot \rho_1$$

so that average densities at the boundaries and fluxes through the

boundaries can be determined. Choosing $RBC = 1.0$ or $LBC = 1.0$ means that the value beyond the boundary equals the value just inside the boundary. The value on the boundary, being the average of these, is therefore the same as the value just inside the boundary. Choosing $RBC = -1.0$ or $LBC = -1.0$ means that the value on the boundary is supposed to be zero, the average of the inside value and its negative. Other values of RBC or LBC might be considered for special applications.

Boundary conditions at a solid impermeable wall on the right side, for example, are established by setting $RBC = 1.0$ for density and energy variables but $RBC = -1.0$ for the momentum variable. Setting $UR = 0.0$ in the corresponding call to VELOCE also forces the convective flux of ρ , V , and E through the wall to zero (see diaphragm test problem - ETBFCT).

As another example, if we wanted to set up an outflow condition at the left hand boundary of the system, $LBC = 1.0$ would again be required so the value of ρ just beyond the system boundary would equal the value just within. The velocity $UL = U(1)$ is set in the corresponding call to VELOCE to extrapolate the velocity of outflow or inflow (see shock test problem - ETBFCT).

At the axis of a cylindrical or spherical coordinate system we have to set $RADL = 0.0$ and $UL = 0.0$ in the appropriate calls to NGRID and VELOCE. We would also use $LBC = 1.0$ to prevent a diffusive source of ρ at the origin $r = 0$. $RBC = -1.0$ or $LBC = -1.0$ would be used in a true mirror symmetry situation such as solving the continuity equation for the perpendicular velocity or momentum

at a solid wall or axis.

With the boundary conditions, as with the use of ETBFCT and its various entries, study of the concrete test program examples in the appendices gives techniques for a spectrum of uses. Tables 2,3, and 4 present three templates for using ETBFCT or PRBFCT for passive convection, Eulerian hydrodynamics, and Lagrangian hydrodynamics. In the latter two examples the following three coupled equations for compressible inviscid flow are solved:

$$\begin{aligned}\frac{\partial \rho}{\partial t} &= - \frac{1}{r^{\alpha-1}} \frac{\partial}{\partial r} (r^{\alpha-1} \rho U), \\ \frac{\partial(\rho U)}{\partial t} &= - \frac{1}{r^{\alpha-1}} \frac{\partial}{\partial r} (r^{\alpha-1} \rho U U) - \frac{\partial}{\partial r}(P), \\ \frac{\partial E}{\partial t} &= - \frac{1}{r^{\alpha-1}} \frac{\partial}{\partial r} (r^{\alpha-1} E U) - \frac{1}{r^{\alpha-1}} \frac{\partial}{\partial r} (\alpha^{-1} P U)\end{aligned}\tag{30}$$

where the pressure P is related to the internal energy E by $E = P/(\gamma-1) + \frac{1}{2}\rho U^2$. These equations (and templates) also apply to the examples used in the appendices.

The use of PRBFCT is identical except, of course, the geometry and boundary conditions are automatically periodic. Thus entry OGRIDE(N) becomes merely OGRIDP(NP) but the new grid entry simplifies to NGRIDP(RADN,NP). Neither RADR, RADL, nor ALPHA are required. Notice, however, that $NP = N+1$ is the number of grid points which are solved for and which must be initialized. Of course $U(NP) = U(1)$, $C(NP) = C(1)$, $D(NP) = D(1)$, and $RHOO(NP) = RHOO(1)$. The routines calculate $RHON(NP) = RHON(1)$. The burden is placed on the user to

fill the N+1st point because $RADN(NP) \neq RADN(1)$. In fact the periodic system length, $SL = RADN(NP) - RADN(1)$, is usually constant even in a Lagrangian calculation.

The call to the sources entry has the name and arguments `SOURCP(NP, DT, MODES, C, D)`. The basic FCT convection routine is called `PRBFCT(RHOO, RHON, NP)`. The conservation sum is performed by calling `CONSRP(RHO, NP, CSUM)`. Notice again that arguments `N` for `ETBFCT` are replaced by `NP = N+1` in using `PRBFCT`.

IV. Summary.

The report has described two new Fortran subroutines for solving generalized continuity equations using flux-corrected transport algorithms optimized for speed and accuracy. These routines, `ETBFCT` and `PRBFCT`, treat a wide range of geometries, grid configurations, boundary conditions, and source terms. "Hooks" have been left in the codes to add additional geometries and source terms. This report describes the low-phase error FCT algorithm used and the programs reproduced in the appendices are substantially self-documenting. Section III above describes how to use `ETBFCT` and `PRBFCT` in some detail and Tables 2-4 give templates for three of the most common types of problem to which these FCT routines could be applied. Here I would like to reiterate the philosophy behind the structure adopted for these routines.

To solve Eq. (1) numerically for even one timestep requires a lot of input data and many independent calculations have to be performed. The logically separate components of this calculation have been made into distinct subroutine entries requiring

separate calls. This separation of function has two advantages. First, many different types of calculation can be performed using the same code by mixing and matching the various functions (entries) in different orders and with different arguments. Second, functions and calculations which are common to several different continuity equations or which are unchanged over several timesteps do not need to be performed more than once. Thus optimization is achieved by eliminating repeat calculations even though a slight cost is obviously exacted to construct the extra calling sequences. The resulting subroutines are general and flexible and yet are actually faster than the original SHASTA³ algorithms.

On the CDC 7600 using the Livermore CHATR compiler the two major entries ETBFCT and PRBFCT require 150 μ sec overhead plus about 25 μ sec per grid point per call. The Huntsville ASC executes the code at about the same speed. The NRL ASC will be about 10 times faster. The ASC execution time on a one pipe configuration is given by the linear relation

$$t_{1X \text{ ASC}} = 160 + 2.52 \times n \text{ } \mu\text{sec} \quad (31)$$

and the 2 pipe timing (the NRL configuration) is

$$t_{2X \text{ ASC}} = 195 + 1.27 \times n \text{ } \mu\text{sec.} \quad (32)$$

This means that a 100 x 100 two-dimensional hydrodynamics program (four variables) which is timestep split would require about 0.5 sec

per timestep. This number is roughly a factor of six faster than the 0.5 sec per step reported for the SHAS2D code³ for a 40 x 40 grid.

The memory requirement for ETRFCT (or PRBFCT) varies depending on the value assigned to the parameter NPT which designates the length of internal storage and scratch arrays. The size of the routines (in decimal) is roughly

$$\#(\text{words}) = 1800 + 13 \times \text{NPT} \quad (23)$$

since there are 18 distinct storage arrays used. NPT must be at least 1 larger than the maximum size of a 1D system passed to ETRFCT (or PRBFCT) for solution.

As a final note of caution I wish to point out to the potential user of these routines that they have been checked carefully but by no means has every possible application been tested. Many combinations of calls and valid applications have not even been thought of yet. Thus there may yet be bugs which appear in esoteric applications which have not yet been scared out. This warning is particularly important because FCT algorithms are notoriously forgiving and uncomplaining. They seldom blow up (unless $U\delta t/\delta x > \frac{1}{2}$ say) but rather the solution degrades gracefully and unspectacularly. Therefore always check your numbers carefully, particularly near the boundaries. Don't just look at the exponents of the answers.

Any suggestions for improvement or error reports, either for the subroutines or for this write-up, will be gratefully received by the author and incorporated into future editions of this report and into updates to the routines themselves.

Acknowledgments

I would like to acknowledge the contributions of my colleagues, David Book, John Gardner, Klaus Hain, and Steve Zalesak, to the development of flux-corrected transport during the last five years.

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References

1. Boris, J. P. and Book, D. L. (1976). Flux-Corrected Transport III: Minimal Error FCT Algorithms [FCT/III]. J. Comp. Phys. (to be published).
2. Boris, J. P. and Book, D. L. (1976). Solution of Continuity Equations by the Method of Flux-Corrected Transport, Chap. 11, Vol. 16, Methods of Computational Physics (Academic Press, New York).
3. Boris, J. P. and Book, D. L. (1973). Flux-Corrected Transport I: SHASTA, A Fluid Transport Algorithm that Works [FCT/I]. J. Comp. Phys. 11, 38 (1973).
4. Book, D. L., Boris, J. P., and Hain, K. H. (1975). Flux-Corrected Transport II: Generalizations of the Method [FCT/II]. J. Comp. Phys. 18, 248 (1975).
5. Boris, J. P., Gardner, J. H., and Zalesak, S. (1975). Atmospheric Hydrocodes Using FCT Algorithms. NRL Memorandum Rept. No. 3081.
6. Boris, J. P. (1972), SHAS2D, A Fully Compressible Hydrodynamics Code in Two Dimensions. NRL Memorandum Rept. No. 2542.

Table 1. Mnemonic Correspondence of Physical Variables and Fortran Notation in ETBFCT and PRBFCT.

<u>Fortran Variable</u>	<u>Physical Variable</u>	<u>Comments</u>
FLXH(I)	multiple usage	used for convective and diffusive fluxes
DIFF(I)	multiple usage	used for various grid differences
RHOL	$\rho_{N+1}^O = R_B \rho_N^O$	extrapolated value of ρ past left boundary
RHOR	$\rho_O^O = L_B \rho_1^O$	extrapolated value of ρ past right boundary
LORHOT(I)	$\Lambda_1^O \rho_1^*$ and $\Lambda_1^O \rho_1^T$	transported mass elements
RHOT(I)	ρ_1^T	transported, "sourced" and compressed densities
LNRHOT(I)	$\Lambda_1^n \tilde{\rho}_1$ and $\Lambda_1^n \rho_1^n$	transported/diffused mass elements
FSGN(I)	$S_{i-\frac{1}{2}}$	sign of grid differences of $\tilde{\rho}_1$
FABS(I)	$ F_{i-\frac{1}{2}} $	absolute value of anti-diffusive flux
SOURCE(I)	multiple usage	summed source term (units of $\Lambda_1 \rho_1$)
RHOO(I)	ρ_1^O	input grid point densities
RHON(I)	ρ_1^n	calculated output grid point densities
ADUDTH(I)	$\delta t A_{i-\frac{1}{2}} \delta U_{i-\frac{1}{2}}$	convective mass flux over density
EPSH(I)	$\epsilon_{i-\frac{1}{2}}$	nondimensional interface velocities
NULH(I)	$\Lambda_{i-\frac{1}{2}} v_{i-\frac{1}{2}}$	diffusion coefficient x volume of cell
MULH(I)	$\Lambda_{i-\frac{1}{2}} u_{i-\frac{1}{2}}$	antidiffusion coefficient x volume of cell
RADN(I)	r_i^n	new grid positions
RNH(I)	$r_{i-\frac{1}{2}}^n$	new interface positions
LBC	L_B	boundary condition factors
RBC	R_B	boundary condition factors

Table 1. (Cont.)

<u>Fortran Variable</u>	<u>Physical Variable</u>	<u>Comments</u>
AH(I)	$A_{i-\frac{1}{2}}$	interface areas (time centered)
LN(I)	Λ_i^n	new cell volumes
LH(I)	$\Lambda_{i-\frac{1}{2}}^n$	new interface cell volumes
RLH(I)	$1/\Lambda_i^n$	reciprocal new cell volumes
ADUPTH(I)	$A_{i-\frac{1}{2}}(r_{i-\frac{1}{2}}^n - r_{i-\frac{1}{2}}^o)$	grid motion times interface area
RLH(I)	$\sim 1/\Lambda_{i-\frac{1}{2}}^n$	used in non-dimensional relative velocities
C(I), D(I)	C_i, D_i	source term variables (multiple usage)
DR, DL	D_R, D_L	right and left boundary values of D
LO(I)	Λ_i^o	old cell volumes
RLC(I)	$1/\Lambda_i^o$	reciprocal old cell volumes
ROH(I)	$r_{i-\frac{1}{2}}^o$	old interface positions
N(NP)	$N(N+1)$	number of interior grid points (plus 1)

Entry and Arguments

ETBFCT(RHOO, RHON, N, RBC, LPC)
 PRPFCT(RHOO, RHON, NP)
 VELOCE(U, N, UR, UL, DT)
 VELOCP(U, NP, DT)
 NGRIDE(RADN, N, RADR, RADL, ALPHA A)
 NGRIDP(RADN, NP)
 SOURCE(N, DT, MODES, C, D, DR, DL)
 SOURCP(NP, DT, MODES, C, D)
 OGRIDE(N)
 OGRIDP(NP)
 CONSRE(RHO, N, CSUM)
 CONSRP(RHO, NP, CSUM)

Purpose

convect RHOO into RHON using FCT
 calculated velocity-dependent coefficients
 calculate geometry and coefficients for a new grid
 calculate and accumulate source terms
 set up the old grid variables
 compute the conservation sum

Table 2.

Logical Sequence of FCT Calls for Passive
Convection (advance ρ , $U(x,t)$ given)

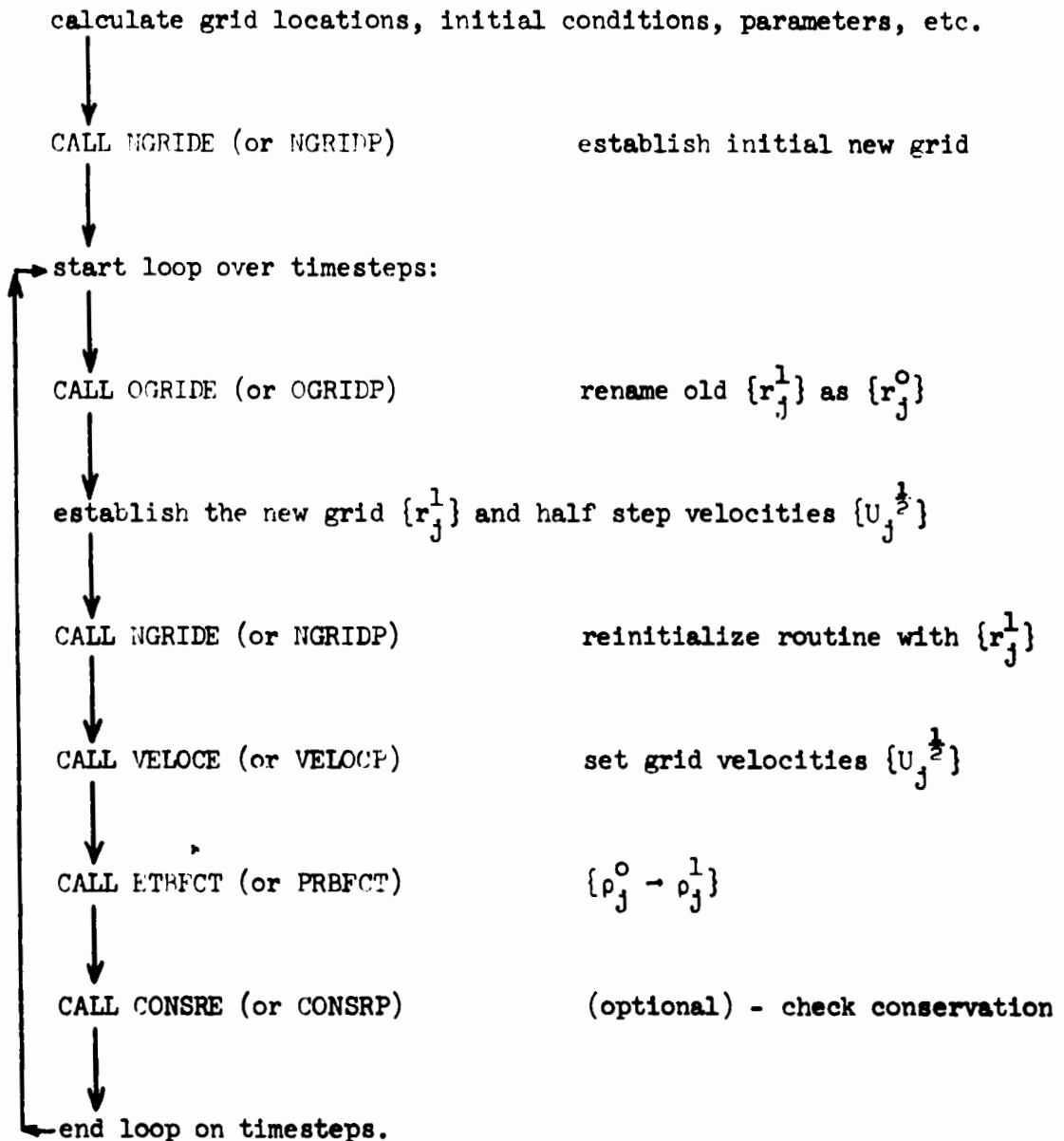


Table 3. Logical Sequence of FCT Calls for Eulerian

Hydrodynamics (advancing ρ , ρU , E)

calculate grid locations, initial conditions, parameters, etc.

CALL NGRIDE (or NGRIDP) establish initial new grid

CALL OGRIDE (or OGRIDP) rename also as old grid

CALL NGRIDE (or NGRIDP) set up the desired grid

→ start loop over timesteps:

begin half step calculations:

calculate $\{U_j^0\}$ and $\{P_j^0\}$ using old ρ , ρU , and E

CALL VELOCE (or VELOCP) set grid velocities $\{U_j^0\}$

CALL ETBFCT (or PRBFCT) $\{\rho_j^0 \rightarrow \rho_j^{\frac{1}{2}}\}$

CALL SOURCE (or SOURCP) set $-\nabla P^0$ for momentum "sources"

CALL ETBFCT (or PRBFCT) $\{\rho_j^0 U_j^0 \rightarrow \rho_j^{\frac{1}{2}} U_j^{\frac{1}{2}}\}$

CALL SOURCE (or SOURCP) set $-\nabla \cdot P^0 U^0$ for energy "sources"

CALL ETBFCT (or PRBFCT) $\{E_j^0 \rightarrow E_j^{\frac{1}{2}}\}$

begin whole step calculations:

calculate $\{U_j^{\frac{1}{2}}\}$ and $\{P_j^{\frac{1}{2}}\}$ using half step ρ , ρU , E

CALL VELOCE (or VELOCP) set grid velocities $\{U_j^{\frac{1}{2}}\}$

CALL ETBFCT (or PRBFCT) $\{\rho_j^0 \rightarrow \rho_j^1\}$

CALL SOURCE (or SOURCP) set $-\nabla P^{\frac{1}{2}}$ for momentum "sources"

CALL ETBFCT (or PRBFCT) $\{\rho_j^0 U_j^0 \rightarrow \rho_j^1 U_j^1\}$

CALL SOURCE (or SOURCP) set $-\nabla \cdot P^{\frac{1}{2}} U^{\frac{1}{2}}$ for energy "sources"

CALL ETBFCT (or PRBFCT) $\{E_j^0 \rightarrow E_j^1\}$

→ end loop on timesteps.

Table 4. Logical Sequence of PCT Calls for Lagrangian

Hydrodynamics (advancing ρ , ρU , E)

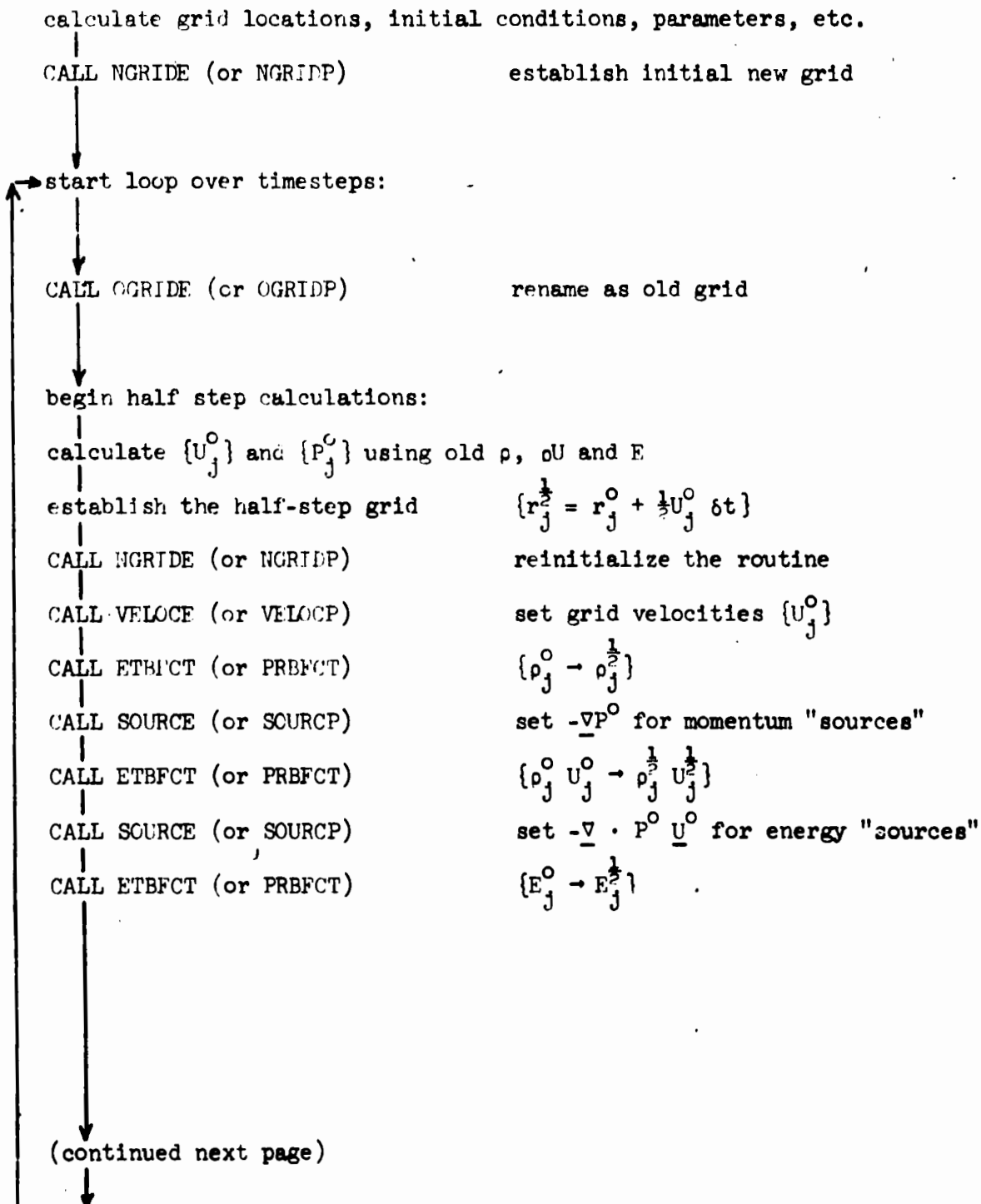
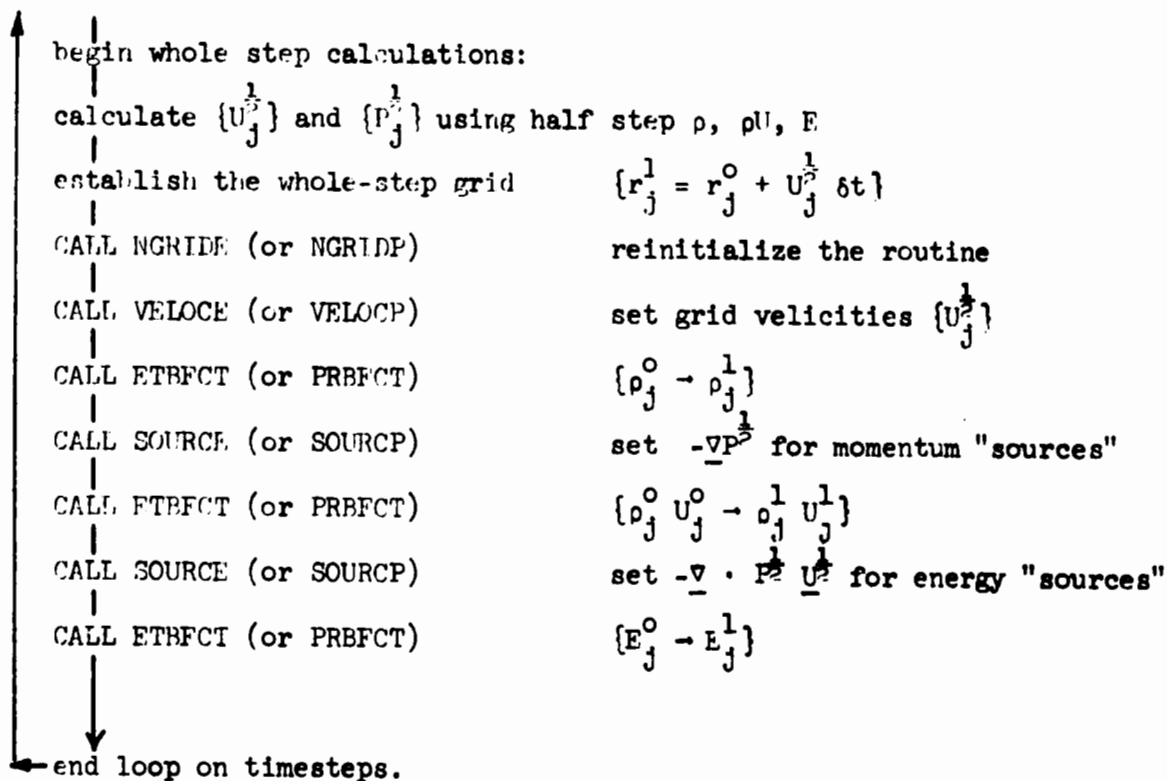


Table 4. (Continued)



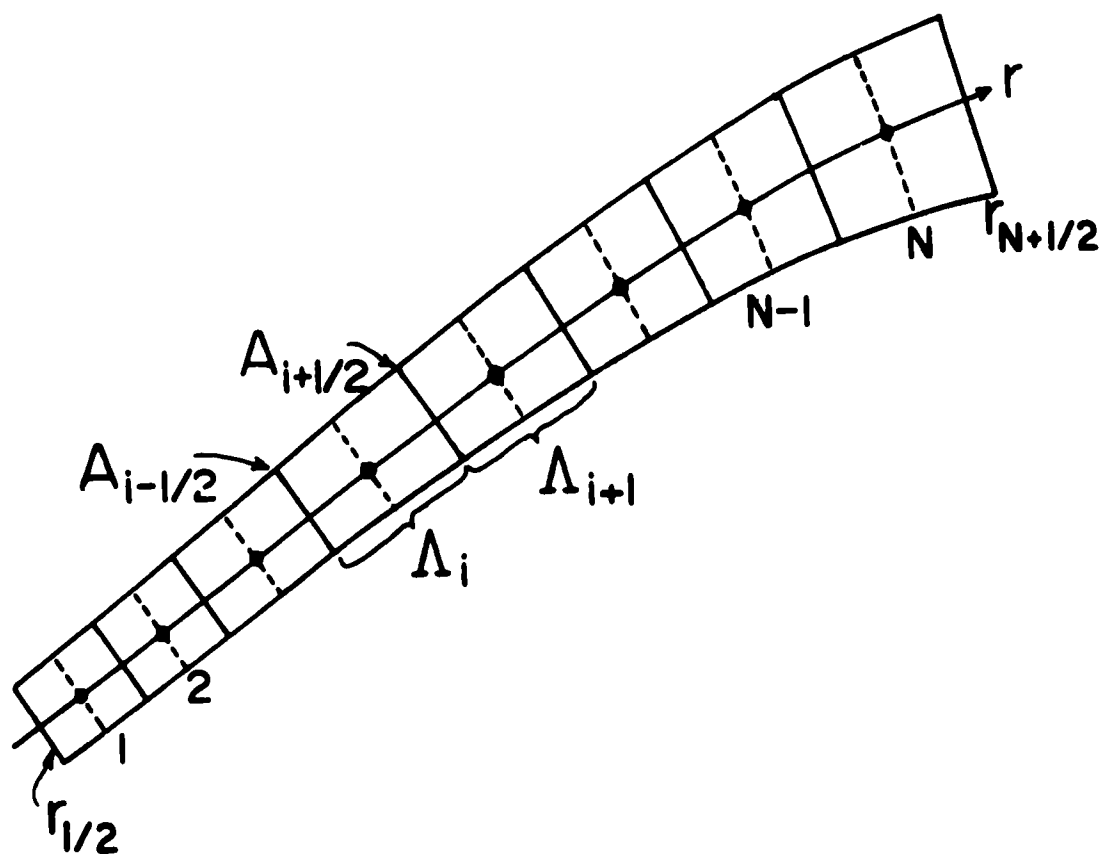


Fig. 1 — General geometry interpretation of the ETBFCT grid. The subroutine considers only Cartesian, cylindrical (radial) and spherical (radial) systems but the notation also allows other geometries with a more general metric.

Appendix A LISTING OF ETBFCT

```

SUBROUTINE ETBFCT (RHOD, RHON, N, KBC, LBC)
C
      REAL      RHOD(N), RHON(N)
CD
CD *****
CD
CD ETBFCT (RHOD, RHON, N, KBC, LBC)                                D.3
CD ORIGINATOR - J.P.BORIS                                         CODE 7705, NRL      OCT.1975
CD
CD DESCRIPTION: THIS ROUTINE SOLVES GENERALIZED CONTINUITY EQUATIONS
CD OF THE FORM  $\partial \text{RHOD} / \partial T = -\text{DIV}(\text{RHOD} * \text{V}) + \text{SOURCES}$  IN EITHER
CD CARTESIAN, CYLINDRICAL, OR SPHERICAL GEOMETRY. THE FINITE-DIFFER-
CD ENCE GRID CAN BE EULERIAN, SLIDING MESH, OR LAGRANGIAN AND CAN
CD BE ARBITRARILY SPACED. THE ALGORITHM USED IS A LOW-PHASE-ERROR FCT
CD ALGORITHM, VECTORIZED AND OPTIMIZED FOR SPEED.
CD
CD ARGUMENTS: IN THIS ROUTINE THE RIGHT BOUNDARY AT RADR IS HALF A
CD CELL BEYOND THE LAST GRID POINT N AT RADN(N) AND THE LEFT BOUNDARY
CD AT RADL IS HALF A CELL BEFORE THE FIRST GRID POINT AT RADN(1).
CD RHOD REAL ARRAY(N) GRID POINT DENSITIES AT START OF STEP 1
CD RHON REAL ARRAY(N) GRID POINT DENSITIES AT END OF STEP 0
CD N INTEGER NUMBER OF INTERIOR GRID POINTS 1
CD KBC REAL RIGHT BOUNDARY CONDITION FACTOR 1
CD LBC REAL LEFT BOUNDARY CONDITION FACTOR 1
CD
CD LANGUAGE AND LIMITATIONS: THE SUBROUTINE ETBFCT IS A MULTIPLE-
CD ENTRY FORTRAN ROUTINE IN SINGLE PRECISION (32 BITS ASC). THE ASC
CD PARAMETER STATEMENT IS USED TO SET SYMBOLICALLY THE INTERNAL ARRAY
CD DIMENSIONS. UNDERFLOWS ARE POSSIBLE WHEN THE FUNCTION BEING TRANS-
CD PORTED HAS MANY ZEROES. THE CALCULATIONS GENERALLY MISCONSERVE BY
CD ONE OR TWO BITS PER CYCLE. THE RELATIVE PHASE AND AMPLITUDE ERRORS
CD (FOR SMOOTH FUNCTIONS) ARE TYPICALLY SEVERAL PERCENT FOR CHARAC-
CD TERISTIC LENGTHS OF 1 - 2 CELLS (WAVELENGTHS OF ORDER 10 CELLS).
CD SHOCKS ARE GENERALLY ACCURATE TO BETTER THAN 1 PERCENT. THIS SUB-
CD ROUTINE MUST BE COMPILED WITH THE Y OPTION TO FORCE STORAGE AND
CD RETENTION OF INTERNAL VARIABLES. ALTERNATIVELY A COMMON BLOCK CAN
CD BE ADDED TO ACCOMPLISH THE SAME END.
CD
CD ENTRY POINTS: UGRIDE, NGRIDE, VELOC, SJURGE, CONSR, SEE #11
CD OF THE DETAILED DOCUMENTATION (SEE THE LISTING BELOW) FOR THE EX-
CD PLANATION AND USE OF THE ARGUMENTS TO THESE OTHER ENTRIES.
CD
CD NO AUXILIARY OR LIBRARY ROUTINES ARE CALLED BY ETBFCT.
CD *****

```

```

C
PARAMETER NPT = 202
LOGICAL LSOURC
REAL RBC, LBC, MASK1, MASK2, MASK3
REAL SOURCE(NPT), SCRH(NPT), RHOT(NPT), DIFF(NPT)
REAL ADUDTH(NPT), FLXH(NPT), MULH(NPT), MULH(NPT)
REAL LNRHOT(NPT), FSGN(NPT), FABS(NPT), EPSH(NPT)
REAL LORHOT(NPT), TERP(NPT), TERM(NPT), ADUDTH(NPT)
REAL LO(NPT), LN(NPT), LH(NPT), RLO(NPT), RLN(NPT)
REAL RNH(NPT), RHN(NPT), RLH(NPT), AH(NPT)
DATA MASK1, MASK2, MASK3 /280000000, 1.0, 27FFFFFFF/
DATA RDN /NPT*1.0/, P1, FTPI /3.1415927, 4.1887902/
DATA SOURCE/NPT*0.0/, LSOURC/F/
EQUIVALENCE (EPSH(1), SCRH(1)), (LNRHOT(1), LORHOT(1))
EQUIVALENCE (FLXH(1), SCRH(1)), (FSGN(1), RHOT(1))
EQUIVALENCE (FABS(1), SCRH(1)), (TERP(1), SOURCE(1))
EQUIVALENCE (TERM(1), SOURCE(1))

C
C CALCULATE THE DIFFUSIVE AND CONVECTIVE FLUXES.
NP = N + 1
DO 11 I = 2, N
FLXH(I) = 0.5*ADUDTH(I)*(RHOU(I) + RHOU(I-1))
11 DIFF(I) = MULH(I)*(RHOU(I) - RHOU(I-1))
RHOL = RHOU(1)*LBC
RHUR = RHOU(N)*RBC
DIFF(1) = MULH(1)*(RHOU(1) - RHOL)
DIFF(NP) = MULH(NP)*(RHOU - RHOU(N))
FLXH(1) = 0.5*ADUDTH(1)*(RHOU(1) + RHOL)
FLXH(NP) = 0.5*ADUDTH(NP)*(RHUR + RHOU(N))

C
C CALCULATE LAMBDAD*RHOT, THE TRANSPORTED MASS ELEMENTS.
DO 12 I = 1, N
12 LORHOT(I) = LO(I)*RHOU(I) - FLXH(I+1) + FLXH(I)

C
C ADD IN THE SOURCE TERMS AS APPROPRIATE.
IF (.NOT. LSOURC) GO TO 14
DO 13 I = 1, N
13 LORHOT(I) = LORHOT(I) + SOURCE(I)

C
C CALCULATE THE PHOENICAL ANTIDIFFUSIVE FLUXES HERE.
DO 14 I = 1, N
14 RHOT(I) = LORHOT(I)*RLO(I)
DO 17 I = 2, N
17 FLXH(I) = MULH(I)*(RHOT(I) - RHOT(I-1))
FLXH(1) = MULH(1)*(RHOT(1) - LBC*RHOT(1))
FLXH(NP) = MULH(NP)*(RBC*RHOT(N) - RHOT(N))

C
C DIFFUSE THE SOLUTION RHOT USING OLD FLUXES.
DO 18 I = 1, N
18 LNRHOT(I) = LORHOT(I) + DIFF(I+1) - DIFF(I)

```

```

C
C      CALCULATE THE TRANSPORTED/DIFFUSED DENSITY AND GRID DIFFERENCES.
      DO 19 I = 1, N
19      RHOT(I) = L*RHOT(I)*RLN(I)
      DO 20 I = 2, N
20      DIFF(I) = RHOT(I) - RHOT(I-1)
      DIFF(1) = RHOT(1) - LBC*RHOT(1)
      DIFF(NP) = RBC*RHOT(N) - RHOT(N)
C
C      CALCULATE THE SIGN AND MAGNITUDE OF THE ANTIDIFFUSIVE FLUX.
      DO 21 I = 1, NP
21      FSGN(I) = AND(MASK1, DIFF(I))
      DO 22 I = 1, NP
22      FABS(I) = AND(MASK3, FLXH(I))
      FSGN(I) = OR(MASK2, FSGN(I))
C
C      CALCULATE THE FLUX-LIMITING CHANGES IN THE RIGHT AND THE LEFT.
      DO 23 I = 1, N
23      TERP(I) = FSGN(I)*LN(I)*DIFF(I+1)
      TERP(NP) = 1.0E+75
      DO 24 I = 1, NP
24      FABS(I) = AMIN1(TERP(I), FABS(I))
      DO 25 I = 2, NP
25      TERM(I) = FSGN(I)*LN(I-1)*DIFF(I-1)
      TERM(1) = 1.0E+75
C
C      CORRECT THE FLUXES COMPLETELY NOW.
      DO 35 I = 1, NP
35      DIFF(I) = AMIN1(FABS(I), TERM(I))
      DO 36 I = 1, NP
36      FLXH(I) = AMAX1(0.0, DIFF(I))
      DO 37 I = 1, NP
37      FLXH(I) = FSGN(I)*FLXH(I)
C
C      CALCULATE THE NEW FLUX-CORRECTED DENSITIES.
      DO 40 I = 1, N
40      LNRHOT(I) = LNRHOT(I) - FLXH(I+1) + FLXH(I)
      DO 41 I = 1, N
41      SOURCE(I) = 0.0
      RHON(I) = LNRHOT(I)*RLN(I)
      LSOURC = .FALSE.
      RETURN

```

ENTRY VELOC (U, N, UR, UL, DT)

REAL U(N)

```

CD      * * * * *
CD
CD      VELOC (U, N, UR, UL, DT)
CD      DESCRIPTION: THIS ENTRY CALCULATES ALL VELOCITY-DEPENDANT COEFFS.
CD
CD      ARGUMENTS:
CD      U      REAL ARRAY(N)      FLOW VELOCITY AT THE GRID POINTS      I
CD      N      INTEGER            NUMBER OF INTERIOR GRID POINTS      I
CD      UR      REAL              VELOCITY OF FLOW AT RIGHT BOUNDARY    I
CD      UL      REAL              VELOCITY OF FLOW AT LEFT BOUNDARY      I
CD      DT      REAL              STEPSIZE FOR THE TIME INTEGRATION      I
CD
CD      * * * * *
C
C      CALCULATE THE INTERFACE AREA X VELOCITY DIFFERENTIAL X DT.
C      NP = N + 1
C      DTH = 0.5*DT
C      DO 101 I = 2, N
101      ADUDTH(I) = AH(I)*DTH*(U(I) + U(I-1)) - ADUGTH(I)
C      ADUDTH(1) = AH(1)*DT*UL - ADUGTH(1)
C      ADUDTH(NP) = AH(NP)*DT*UR - ADUGTH(NP)
C
C      CALCULATE THE HALF-CELL EPSILON (V*DT/DX)
C      DO 102 I = 1, NP
102      EPSH(I) = ADUDTH(I)*RLH(I)
C
C      NEXT CALCULATE THE DIFFUSION AND ANTIDIFFUSION COEFFICIENTS.
C      VARIATION WITH EPSILON MEANS FOURTH-ORDER ACCURATE PHASES.
C      DO 103 I = 1, NP
103      NULH(I) = 0.16666667 + 0.3333333*EPSH(I)*EPSH(I)
C      MULH(I) = 0.25 - 0.5*NULH(I)
C      DO 104 I = 1, NP
104      MULH(I) = LH(I)*NULH(I)
C      MULH(I) = LH(I)*MULH(I)
C      RETURN
C
C
C      -----
C
C      ENTRY NGRIDE (RADN, N, RADR, RADL, ALPHA)
C
C      INTEGER  ALPHA
C      REAL    RADN(N)
C
C      * * * * *
CD
CD
CD      NGRIDE (RADN, N, RADR, RADL, ALPHA)
CD      DESCRIPTION: THIS ENTRY SETS NEW GEJMETRY VARIABLES AND COEFFS.
CD
CD      ARGUMENTS:

```

```

CJ RADN REAL ARRAY(N) NEW GRID POINT POSITIONS I
CJ N INTEGER NUMBER OF INTERNAL GRID POINTS I
CJ RADR REAL POSITION OF THE RIGHT BOUNDARY I
CJ RAUL REAL POSITION OF THE LEFT BOUNDARY I
CJ ALPHA INTEGER = 1 FOR CARTESIAN GEOMETRY I
CJ = 2 FOR CYLINDRICAL GEOMETRY I
CJ = 3 FOR SPHERICAL GEOMETRY I
CJ .....
C
C CALCULATE THE NEW HALF-CELL POSITIONS AND GRID CHANGES.
      NP = N + 1
      DO 202 I = 2, N
202  RNH(I) = 0.5*(RADN(I) + RADN(I-1))
      RNH(1) = RAUL
      RWH(NP) = RADR
C
C CALCULATE THE THREE COORDINATE SYSTEMS.
      GO TO (203, 206, 209), ALPHA
C
C CARTESIAN COORDINATES.
203  DO 204 I = 1, NP
204  AH(I) = 1.0
      DO 205 I = 1, N
205  LH(I) = RNH(I+1) - RNH(I)
      GO TO 213
C
C CYLINDRICAL COORDINATES.
206  DO 207 I = 1, NP
      DIFF(I) = RNH(I)*RNH(I)
207  AH(I) = PI*(RCH(I) + RNH(I))
      DO 208 I = 1, N
208  LH(I) = PI*(DIFF(I+1) - DIFF(I))
      GO TO 213
C
C SPHERICAL COORDINATES.
209  DO 210 I = 1, NP
      DIFF(I) = RNH(I)*RNH(I)*RNH(I)
210  SCRH(I) = (RCH(I) + RNH(I))*RCH(I)
      DO 211 I = 1, NP
211  AH(I) = FTP*(SCRH(I) + RNH(I)*RNH(I))
      DO 212 I = 1, N
212  LH(I) = FTP*(DIFF(I+1) - DIFF(I))
C
C NOW THE GEOMETRIC VARIABLES WHICH ARE SYSTEM INDEPENDANT.
213  DO 214 I = 2, N
214  LH(I) = 0.5*(LH(I) + LH(I-1))
      LH(1) = LH(1)
      LH(NP) = LH(N)
      DO 215 I = 1, N

```

```

215     RLN(I) = 1.0/LV(I)
      DO 216 I = 1, NP
216     ADUGTH(I) = AH(I)*(KXNH(I) - ROM(I))
      DU 217 I = 2, N
217     RLH(I) = 0.5*(RLN(I) + RLN(I-1))
      RLH(1) = RLN(1)
      RLH(NP) = RLN(N)
      RETURN

C
C
C -----
C
ENTRY SOURCE (N, DT, MODES, C, D, DR, DL)

C
      REAL      C(N), D(N)

C
CD *****
CD
CD SOURCE (N, DT, MODES, C, D, DR, DL)
CD DESCRIPTION: THIS ENTRY ACCUMULATES DIFFERENT SOURCE TERMS.
CD
CD ARGUMENTS:
CD N          INTEGER          NUMBER OF INTERIOR GRID POINTS      I
CD DT         REAL            STEPSIZE FOR THE TIME INTEGRATION    I
CD MODES      INTEGER          = 1  COMPUTES + DIV (D)              I
CD                                     = 2  COMPUTES + C*GRAD (D)      I
CD                                     = 3  ADDS + D TO THE SOURCES    I
CD C          REAL ARRAY(I)    ARRAY OF SOURCE VARIABLES AT GRID PTS I
CD D          REAL ARRAY(N)    ARRAY OF SOURCE VARIABLES AT GRID PTS I
CD DR         REAL            RIGHT BOUNDARY VALUE OF D             I
CD DL         REAL            LEFT BOUNDARY VALUE OF D              I
CD
CD *****
C
      NP = N + 1
      DTH = 0.5*DT
      DTQ = 0.25*DT
      GO TO (310, 320, 330), MODES

C
C + DIV(D) IS COMPUTED CONSERVATIVELY AND ADDED TO THE SOURCES.
310   DO 311 I = 2, N
311   SCRH(I) = DTH*AH(I)*(C(I) + D(I-1))
      SCRH(1) = DT*AH(1)*DL
      SCRH(NP) = DT*AH(NP)*DR
      DU 312 I = 1, N
312   SOURCE(I) = SOURCE(I) + SCRH(I+1) - SCRH(I)
      LSOURC = .TRUE.
      RETURN

C
C + C*GRAD(D) IS COMPUTED EFFICIENTLY AND ADDED TO THE SOURCES.

```

```

320      DO 321 I = 2, N
321      SCRH(I) = DTQ*(D(I) + D(I-1))
          SCRH(I) = DTH*DL
          SCRH(NP) = JTH*DL
          DJ 322 I = 1, N
322      DIFF(I) = SCRH(I+1) - SCRH(I)
          DU 323 I = 1, N
323      SOURCE(I) = SOURCE(I) + C(I)*(AH(I+1) + AH(I))*DIFF(I)
          LSOURC = .TRUE.
          RETURN

C
C      * J IS ADDED TO THE SOURCES IN AN EXPLICIT FORMULATION.
330      DO 331 I = 1, N
331      SOURCE(I) = SOURCE(I) + DT*LU(I)*J(I)
          LSOURC = .TRUE.
          RETURN

C
C
C      -----
C
C      ENTRY JGRIDE (N)
C
C      * * * * *
C
C      UGRIDE (N)
C      DESCRIPTION: THIS ENTRY COPIES OLD GRID AND GEOMETRY VARIABLES.
C
C      ARGUMENTS:
C      N          INTEGER          NUMBER OF INTERIOR GRID POINTS          I
C
C      * * * * *
C
C      COPY THE PREVIOUSLY NEW GRID VALUES TO BE USED AS THE OLD GRID.
          NP = N + 1
          DO 401 I = 1, N
              LU(I) = LN(I)
401      RLU(I) = RLN(I)
          DO 402 I = 1, NP
              RQH(I) = RQH(I)
402
          RETURN

C
C
C      -----
C
C      ENTRY CCNSRE (RHC, N, CSUM)
C
C      REAL      RHC(N)
C
C      * * * * *
C
C

```

```

CD   CCONSR (RHO, N, CSUM)
CD   DESCRIPTION: THIS ENTRY COMPUTES THE OSTENSIBLY CONSERVED SUM.
CD
CD   ARGUMENTS:
CD   RHO    REAL ARRAY(N)    GRID PT VALUES FOR CONSERVATION SUM    I
CD   N      INTEGER          NUMBER OF INTERIOR GRID POINTS        I
CD   CSUM   REAL             VALUE OF THE CONSERVATION SUM OF RHO    0
CD
CD   * * * * *
CD
C    CCAPUTE THE OSTENSIBLY CONSERVED TOTAL MASS (BEWARE YOUR B.C.)
      CSUM = 0.0
      DO 501 I = 1, N
501   CSUM = CSUM + LN(I)*RHO(I)
      RETURN
END

```

The above subroutine may be punched up directly from the listing. On computers other than the Texas Instruments ASC the occurrences of NPT throughout the declarations section should be replaced by a fixed point number, i.e., 202 in the present example, which is at least one larger than the number of points in the longest system to be solved. This routine compiles at K level optimization on the ASC and the Y and P options are also required to retain internal variables across entry boundaries and to suppress underflows.

Appendix B LISTING OF PRBFCT

```

SUBROUTINE PRBFCT (RHOD, RHODV, NP)
C
C     REAL    RHOD(NP), RHODV(NP)
C
C     * * * * *
C
C     PRBFCT (RHOD, RHODV, NP)                                D.3
C     ORIGINATOR - J.P. JORIS                                CODE 7706, NRL    OCT.1975
C
C     DESCRIPTION: THIS ROUTINE SOLVES GENERALIZED CONTINUITY EQUATIONS
C     OF THE FORM  $\partial \text{RHOD} / \partial T = -\text{DIV}(\text{RHOD} \cdot \text{V}) + \text{SOURCES}$  WITH PERIODIC
C     BOUNDARY CONDITIONS IN CARTESIAN GEOMETRY. THE FINITE-DIFFERENCE
C     GRID CAN BE EULERIAN, SLIDING REGION, OR LAGRANGIAN AS LONG AS THE
C     SYSTEM REMAINS PERIODIC WITH A FIXED SYSTEM SIZE. THE ALGORITHM IS
C     A LOW-PHASE-ERROR FCT ALGORITHM, VECTORIZED AND OPTIMIZED.
C
C     ARGUMENTS: IN THIS ROUTINE ALL INPUT VARIABLES MUST BE GIVEN AT
C     NP = N + 1 POINTS WHERE RHOD(NP) = RHOD(1) AND THE LENGTH OF THE
C     SYSTEM IS RADN(NP) - RADN(1).
C     RHOD REAL ARRAY(NP) GRID POINT DENSITIES AT START OF STEP 1
C     RHODV REAL ARRAY(NP) GRID POINT DENSITIES AT END OF STEP 0
C     NP INTEGER NUMBER OF GRID POINTS PLUS ONE 1
C
C     LANGUAGE AND LIMITATIONS: THE SUBROUTINE PRBFCT IS A MULTIPLE-
C     ENTRY FORTRAN ROUTINE IN SINGLE PRECISION (32 BITS ASC). THE ASC
C     PARAMETER STATEMENT IS USED TO SET SYMBOLICALLY THE INTERNAL ARRAY
C     DIMENSIONS. UNDERFLOWS ARE POSSIBLE WHEN THE FUNCTION BEING TRANS-
C     PORTED HAS MANY ZEROS. THE CALCULATIONS GENERALLY MISCONSERVE BY
C     ONE OR TWO BITS PER CYCLE. THE RELATIVE PHASE AND AMPLITUDE ERRORS
C     (FOR SMOOTH FUNCTIONS) ARE TYPICALLY SEVERAL PERCENT FOR CHARAC-
C     TERISTIC LENGTHS OF 1 - 2 CELLS (WAVELENGTHS OF ORDER 10 CELLS).
C     SHOCKS ARE GENERALLY ACCURATE TO BETTER THAN 1 PERCENT. THIS SUB-
C     ROUTINE MUST BE COMPILED WITH THE V OPTION TO FORCE STORAGE AND
C     RETENTION OF INTERNAL VARIABLES. ALTERNATIVELY A COMMON BLOCK CAN
C     BE ADDED TO ACCOMPLISH THE SAME END.
C
C     ENTRY POINTS: JGRIDP, NGRIDP, VELJCP, SJURCP, CONSRP, SEE #11
C     OF THE DETAILED DOCUMENTATION (OR THE LISTING BELOW) FOR THE EX-
C     PLANATION AND USE OF THE ARGUMENTS TO THESE OTHER ENTRIES.
C
C     NO AUXILIARY OR LIBRARY ROUTINES ARE CALLED BY PRBFCT.
C
C     * * * * *
C
C     PARAMETER NPT = 202

```

```

LOGICAL LSOURC
REAL MASK1, MASK2, MASK3
REAL SOURCE(NPT), SCRH(NPT), RHOT(NPT), DIFF(NPT)
REAL ADUGTH(NPT), FLXH(NPT), MULH(NPT), MULH(NPT)
REAL LNRHOT(NPT), FSGN(NPT), FABS(NPT), EPSH(NPT)
REAL LORHOT(NPT), TERP(NPT), TERM(NPT), ADUDTH(NPT)
REAL LQ(NPT), LN(NPT), LH(NPT), RLO(NPT), RLN(NPT)
REAL RNH(NPT), RUH(NPT), RLH(NPT)
DATA MASK1, MASK2, MASK3 /23000000, 1.0, Z7FFFFFFF/
DATA ROM /NPT*1.0/, SOURCE/NPT*0.0/, LSOURC/F/
EQUIVALENCE (EPSH(1), SCRH(1)), (LNRHOT(1), LORHOT(1))
EQUIVALENCE (FLXH(1), SCRH(1)), (FSGN(1), RHOT(1))
EQUIVALENCE (FABS(1), SCRH(1)), (TERP(1), SOURCE(1))
EQUIVALENCE (TERM(1), SOURCE(1))

C
C CALCULATE THE DIFFUSIVE AND CONVECTIVE FLUXES.
N = NP - 1
DO 11 I = 2, NP
FLXH(I) = 0.5*ADUDTH(I)*(RHOT(I) + RHOT(I-1))
11 DIFF(I) = MULH(I)*(RHOT(I) - RHOT(I-1))
FLXH(1) = FLXH(NP)
DIFF(1) = DIFF(NP)

C
C CALCULATE LAMBDA0*RHOT, THE TRANSPORTED MASS ELEMENTS.
DO 12 I = 1, N
12 LORHOT(I) = LQ(I)*RHOT(I) - FLXH(I+1) + FLXH(I)

C
C ADD IN THE SOURCE TERMS AS APPROPRIATE.
IF (.NOT. LSOURC) GO TO 14
DO 13 I = 1, N
13 LORHOT(I) = LORHOT(I) + SOURCE(I)

C
C CALCULATE THE PHECNICAL ANTIDIFFUSIVE FLUXES HERE.
14 DO 16 I = 1, N
16 RHOT(I) = LORHOT(I)*RLO(I)
RHOT(NP) = RHOT(1)
DO 17 I = 2, NP
17 FLXH(I) = MULH(I)*(RHOT(I) - RHOT(I-1))
FLXH(1) = FLXH(NP)

C
C DIFFUSE THE SOLUTION RHOT USING OLD FLUXES.
DO 18 I = 1, N
18 LNRHOT(I) = LORHOT(I) + DIFF(I+1) - DIFF(I)

C
C CALCULATE THE TRANSPORTED/DIFFUSED DENSITY AND GRID DIFFERENCES.
DO 19 I = 1, N
19 RHOT(I) = LNRHOT(I)*RLN(I)
RHOT(NP) = RHOT(1)
DO 20 I = 2, NP
20 DIFF(I) = RHOT(I) - RHOT(I-1)

```

```

      DIFF(1) = DIFF(NP)
C
C      CALCULATE THE SIGN AND MAGNITUDE OF THE ANTIDIFFUSIVE FLUX.
      DO 21 I = 1, NP
21      FSGN(I) = AND(MASK1, DIFF(I))
      DO 22 I = 1, NP
      FABS(I) = AND(MASK3, FLXH(I))
22      FSGN(I) = OR(MASK2, FSGN(I))
C
C      CALCULATE THE FLUX-LIMITING CHANGES ON THE RIGHT AND THE LEFT.
      DO 23 I = 1, N
23      TERP(I) = FSGN(I)*LN(I)*DIFF(I+1)
      TERP(NP) = TERP(1)
      DO 24 I = 1, NP
24      FABS(I) = AMIN1(TERP(I), FABS(I))
      DO 25 I = 2, NP
25      TERM(I) = FSGN(I)*LN(I-1)*DIFF(I-1)
      TERM(1) = TERM(NP)
C
C      CORRECT THE FLUXES COMPLETELY NOW.
      DO 35 I = 1, NP
35      DIFF(I) = AMIN1(FABS(I), TERM(I))
      DO 36 I = 1, NP
36      FLXH(I) = AMAX1(0.0, DIFF(I))
      DO 37 I = 1, NP
37      FLXH(I) = FSGN(I)*FLXH(I)
C
C      CALCULATE THE NEW FLUX-CORRECTED DENSITIES.
      DO 40 I = 1, N
40      LNRHQT(I) = LNRHQT(I) - FLXH(I+1) + FLXH(I)
      LNRHQT(NP) = LNRHQT(1)
      DO 41 I = 1, NP
41      SOURCE(I) = 0.0
      RHON(I) = LNRHQT(I)*RLN(I)
      LSOURC = .FALSE.
      RETURN
C
C
C      -----
C      ENTRY VELOCP (U, NP, DT)
C
C      REAL      U(NP)
C
C      * * * * *
C      VELOCP (U, NP, DT)
C      DESCRIPTION: THIS ENTRY CALCULATES ALL VELOCITY-DEPENDANT COEFFS.
C      ARGUMENTS:

```

```

CD      U      REAL ARRAY(NP)      FLOW VELOCITY AT THE GRID POINTS      I
CD      NP      INTEGER      NUMBER OF GRID POINTS PLUS ONE      I
CD      DT      REAL      STEPSIZE FOR THE TIME INTEGRATION      I
CD
CD      * * * * *
C
C      CALCULATE THE INTERFACE AREA X VELOCITY DIFFERENTIAL X DT.
C      N = NP - 1
C      DTH = 0.5*DI
C      DO 101 I = 1, NP
101      ADUOTH(I) = DTH*(U(I) + U(I-1)) - ADUGTH(I)
C      ADUOTH(1) = ADUOTH(NP)
C
C      CALCULATE THE HALF-CELL EPSILON (V*DT/OX).
C      DO 102 I = 1, NP
102      EPSH(I) = ADUOTH(I)*RLH(I)
C
C      NEXT CALCULATE THE DIFFUSION AND ANTIDIFFUSION COEFFICIENTS.
C      VARIATION WITH EPSILON MEANS FOURTH-ORDER ACCURATE PHASES.
C      DO 103 I = 1, NP
103      MULH(I) = 0.16666667 + 0.3333333*EPSH(I)*EPSH(I)
C      DO 104 I = 1, NP
104      MULH(I) = LH(I)*MULH(I)
C      MULH(1) = LH(1)*MULH(1)
C      RETURN
C
C
C      -----
C
C      ENTRY NGRIDP (RADN, NP)
C
C      REAL      RADN(NP)
C
C      * * * * *
CD
CD      NGRIDP (RADN, NP)
CD      DESCRIPTION: THIS ENTRY SETS NEW GEOMETRY VARIABLES AND COEFFS.
CD
CD      ARGUMENTS:
CD      RADN      REAL ARRAY(NP)      NEW GRID POINT POSITIONS      I
CD      NP      INTEGER      NUMBER OF GRID POINTS PLUS ONE      I
CD
CD      * * * * *
C
C      CALCULATE THE NEW HALF-CELL POSITIONS AND GRID CHANGES.
C      N = NP - 1
C      DO 202 I = 1, NP
202      RNH(I) = 0.5*(RADN(I) + RADN(I-1))
C      RNH(1) = RADN(1) - 0.5*(RADN(NP) - RADN(N))

```

```

C
C      CALCULATE THE CELL VOLUMES AND OTHER GEOMETRIC VARIABLES FOR
C      RETENTION AND GLOBAL USE.
      DO 205 I = 1, N
205      LN(I) = RNH(I+1) - RNH(I)
          LN(NP) = LN(1)
          DO 213 I = 2, NP
213      LH(I) = 0.5*(LN(I) + LN(I-1))
          LH(1) = LH(NP)
          DO 214 I = 1, NP
214      RLN(I) = 1.0/LN(I)
          DO 215 I = 1, NP
215      ADUGTH(I) = (RNH(I) - ROM(I))
          DO 216 I = 2, NP
216      RLH(I) = 0.5*(RLN(I) + RLN(I-1))
          RLH(1) = RLH(NP)
          RETURN

C
C
C      -----
C
C      ENTRY SOURCP (NP, DT, MODES, C, D)
C
C      REAL      C(NP), D(NP)
C
C      * * * * *
C
C      SOURCP (NP, DT, MODES, C, D)
C      DESCRIPTION: THIS ENTRY ACCUMULATES DIFFERENT SOURCE TERMS.
C
C      ARGUMENTS:
C      NP      INTEGER      NUMBER OF GRID POINTS PLUS ONE      I
C      DT      REAL         STEPSIZE FOR THE TIME INTEGRATION    I
C      MODES   INTEGER      = 1  COMPUTES + DIV (D)              I
C                                     = 2  COMPUTES + C*GRAD (D)   I
C                                     = 3  ADDS + D TO THE SOURCES  I
C      C      REAL ARRAY(NP)  ARRAY OF SOURCE VARIABLES AT GRID PTS I
C      D      REAL ARRAY(NP)  ARRAY OF SOURCE VARIABLES AT GRID PTS I
C
C      * * * * *
C
C      N = NP - 1
C      DTH = 0.5*DT
C      GO TO (310, 320, 330), MODES
C
C      + DIV(D) IS COMPUTED CONSERVATIVELY AND ADDED TO THE SOURCES.
310      DO 311 I = 2, NP
311      SCRH(I) = DTH*(U(I) + D(I-1))
          SCRH(1) = SCRH(NP)
          DO 312 I = 1, N

```

```

312     SOURCE(I) = SOURCE(I) + SCRH(I+1) - SCRH(I)
        SOURCE(NP) = SOURCE(1)
        LSOURC = .TRUE.
        RETURN
C
C   + C*GRAD(D) IS COMPUTED EFFICIENTLY AND ADDED TO THE SOURCES.
320     DO 321 I = 2, NP
321     SCRH(I) = DTH*(D(I) + D(I-1))
        SCRH(1) = SCRH(NP)
        DO 322 I = 1, N
322     SOURCE(I) = SOURCE(I) + C(I)*(SCRH(I+1) - SCRH(I))
        SOURCE(NP) = SOURCE(1)
        LSOURC = .TRUE.
        RETURN
C
C   + U IS ADDED TO THE SOURCES IN AN EXPLICIT FORMULATION.
330     DO 331 I = 1, NP
331     SOURCE(I) = SOURCE(I) + DT*LO(I)*D(I)
        LSOURC = .TRUE.
        RETURN
C
C
C   -----
C
C   ENTRY OGRIDP (NP)
C
C   * * * * *
C
C   UGRIDP (NP)
C   DESCRIPTION: THIS ENTRY COPIES OLD GRID AND GEOMETRY VARIABLES.
C
C   ARGUMENTS:
C   NP      INTEGER      NUMBER OF GRID POINTS PLUS ONE      I
C
C   * * * * *
C
C   COPY THE PREVIOUSLY NEW GRID VALUES TO BE USED AS THE OLD GRID.
        N = NP - 1
        DO 401 I = 1, NP
            ROH(I) = RNH(I)
            LO(I) = LN(I)
            RLO(I) = RLN(I)
401     RETURN
C
C
C   -----
C
C   ENTRY CGNSRP (RHU, NP, CSUM)
C
C       REAL      RHU(NP)

```

```

C
CD *****
CD
CD CONSKP (RHO, NP, CSUM)
CD DESCRIPTION: THIS ENTRY COMPUTES THE OSTENSIBLY CONSERVED SUM.
CD
CD ARGUMENTS:
CD RHO    REAL ARRAY(NP)    GRID PT VALUES FOR CONSERVATION SUM    I
CD NP     INTEGER          NUMBER OF GRID POINTS PLUS ONE          I
CD CSUM   REAL              VALUE OF THE CONSERVATION SUM OF RHO    O
CD *****
CD
C    COMPUTE THE OSTENSIBLY CONSERVED TOTAL MASS (BEWARE YOUR B.C.)
      N = NP - 1
      CSUM = 0.0
      DO 501 I = 1, N
501   CSUM = CSUM + LN(I)*RHO(I)
      RETURN
END

```

The above subroutine may be punched up directly from the listing. On computers other than the Texas Instruments ASC the occurrences of NP throughout the declarations section should be replaced by a fixed point number, i.e., 202 in the present example, which is at least one larger than the number of points in the longest system to be solved. This routine compiles at K level optimization on the ASC and the Y and P options are also required to retain internal variables across entry boundaries and to suppress underflows.

Appendix C A PROGRESSING SHOCK TEST PROGRAM USING ETBFCT

This test program initializes a one-dimensional progressing Mach 5 shock. The right and left hand boundary conditions are set to the Rankine-Hugoniot jump conditions for a shock travelling to the right. The results of the calculation are reproduced below after 0, 100, and 200 cycles. Note that the column containing the grid positions has been removed in printing this appendix since the cells are equally spaced. This test problem is used in FCT I with the original SHASTA algorithm.

```

C      ETBFCT PROGRESSING SHOCK PROBLEM      DEC. 1975      SPL VERSION
C      THIS PROGRAM RUNS A VERY SIMPLE SHOCK PROBLEM USING ETBFCT
C      FOR BOTH THE HALF AND THE WHOLE STEP. THE FLUID IS IDEAL AND
C      INVISCID WITH GAMMA = 5/3. A VARIABLE GRID AND THREE GEOMETRIES
C      ARE ALLOWED AND LAGRANGIAN AS WELL AS EULERIAN MOTIONS ARE PER-
C      Mitted BY SETTING FLAG = 1.0.
C
C      INTEGER  ALPHA
C      REAL     MACH
C      REAL     ROLD(200), RNEW(200), VOLD(200), VNEW(200)
C      REAL     POLD(200), PNEW(200), EJLD(200), ENEW(200)
C      REAL     TOLD(200), PV(200), ZERO(200), UNIT(200)
C      REAL     SCR1(200), SCR2(200), RAO(200), RACN(200)
C      REAL     RVOLD(200), RVNEW(200)
C
C      1000  FORMAT ('1 AFTER STEP NO. ',15,' DX = ', F7.3, ' AND DT = ',
C      1      F7.5, '/')
C      1001  FORMAT (2X, 13, 1P5E12.4, 12X, E12.4)
C      1002  FORMAT (' 1 DENSITY TEMPERATURE PRESSURE ',
C      1      ' VELOCITY ENERGY POSITION ', '/')
C      1003  FORMAT ('0 CONSERVATION SUMS ', //, 2X, 13, 1P5E12.4, 12X,
C      1      3E12.4)
C
C      * * * * *
C      THE SHOCK PARAMETERS ARE SET UP. CHANGE FOR DIFFERENT CASES.
C      * * * * *
C      GAMMA0 = 1.66667
C      GAMMAH = GAMMA0 - 1.0
C      MACH = 5.0
C      V0 = 3.0
C      TRIGHT = 1.0
C      LRIGHT = 1.0

```

```

C
C THE RANKINE-HUGONIOT CONDITIONS ARE SET FOR BOUNDARIES.
  PRIGHT = TRIGHT*RRIGHT
  SRIGHT = SQRT(GAMMAO*PRIGHT/RRIGHT)
  VRIGHT = -MACH*SRIGHT
  VLEFT = VRIGHT*(GAMMAO - 1.0 + 2.0/MACH**2)/(GAMMAO + 1.0)
  RLEFT = RRIGHT*VRIGHT/VLEFT
  PLEFT = PRIGHT - RLEFT*VLEFT**2 + RRIGHT*VRIGHT**2
  VRIGHT = VRIGHT + VO
  VLEFT = VLEFT + VO
  RVRIGH = VRIGHT*RRIGHT
  RVLEFT = VLEFT*RLEFT
  ERIGHT = PRIGHT/(GAMMAO - 1.0) + 0.5*RRIGHT*VRIGHT**2
  ELEFT = PLEFT/(GAMMAO - 1.0) + 0.5*RLEFT*VLEFT**2

C
C *****
C CONTROL PARAMETERS ARE INITIALIZED. CHANGE FOR DIFFERENT CASES.
C *****
  ALPHA = 1
  MAXSTP = 201
  IPHINT = 50
  FLAG = 0.0
  DELTAX = 1.0
  DELTAL = DELTAX
  DELTAR = DELTAX*(RLEFT/RRIGHT)*FLAG + (1.0 - FLAG)*DELTAX
  DELTAT = .05
  NX = 50
  LPRINT = NX
  MX = 10

C
C *****
C THE PHYSICAL VARIABLES ARE INITIALIZED.
C *****
  DO 11 I = 1, NX
11  RADN(I) = (FLOAT(I-MX) - 0.5)*DELTAR
  DO 12 I = 1, MX
12  RADN(I) = (FLOAT(I-MX) - 0.5)*DELTAL
  RADR = RADN(NX) + 0.5*DELTAR
  RADL = RADN(1) - 0.5*DELTAL
  CALL NGRIDE (RADN, NX, RADR, RADL, ALPHA)
  DO 13 I = 1, NX
  RNEW(I) = RRIGHT
  VNEW(I) = VRIGHT
  PNEW(I) = PRIGHT
  RVNEW(I) = RVRIGH
  ZENJ(I) = 0.0
  UNIT(I) = 1.0
13  ENEM(I) = ERIGHT
  DO 14 I = 1, MX
  RNEW(I) = RLEFT

```

```

VNEW(I) = VLEFT
PNEW(I) = PLEFT
RVNEW(I) = RVLEFT
14 ENEW(I) = ELEFT
C
C *****
C THE LGOP OVER TIMESTEPS.
C *****
C DJ 9999 ISTEP = 1, MAXSTP
C
C INITIALIZE THE QUANTITIES FOR THE CYCLE.
C DJ 20 I = 1, NX
RADO(I) = RADN(I)
ROLD(I) = RVEW(I)
RVOLD(I) = RVNEW(I)
EOLD(I) = ENEW(I)
VOLD(I) = RVOLD(I)/ROLD(I)
POLD(I) = GAMMA*(EOLD(I) - 0.5*RVOLD(I)*VOLD(I))
TOLD(I) = POLD(I)/ROLD(I)
20 PV(I) = POLD(I)*VOLD(I)
C
C THE RESULTS ARE PRINTED.
C IF (MOD(ISTEP-1,IPRINT) .NE. 0) GO TO 22
JSTEP = ISTEP - 1
WRITE (6, 1003) JSTEP, DELTAX, DELTAT
WRITE (6, 1002)
WRITE (6, 1001) (I, ROLD(I), TOLD(I), POLD(I), VOLD(I),
1 EOLD(I), RADO(I), I=1, LPRINT)
CALL CONSRE (ROLD, NX, RHOSUM)
CALL CONSRE (POLD, NX, PRESUM)
CALL CONSRE (EOLD, NX, ERGSUM)
CALL CONSRE (RVOLD, NX, RHVSUM)
PRESUM = PRESUM/GAMMA
WRITE (6, 1003) NX, RHOSUM, PRESUM, RHVSUM, ERGSUM
22 CONTINUE
C
C RESET GRID AND VELOCITIES FOR THE HALF STEP.
C CALL OGRID (NX)
DTGRID = 0.5*DELTAT*FLAG
DO 30 I = 1, NX
30 RADN(I) = RAD(I) + DTGRID*VOLD(I)
RADR = RADR + DTGRID*VRIGHT
RADL = RADL + DTGRID*VLEFT
CALL NGRID (RADN, NX, RADR, RADL, ALPHA)
CALL VELOC (VOLD, NX, VRIGHT, VLEFT, 0.5*DELTAT)
C
C ALL QUANTITIES ARE INTEGRATED FORWARD HALF A STEP USING THE OLD
C VALUES ON THE RIGHT HAND SIDE FOR FORMING DERIVATIVES.
C DO 31 I = 1, NX
SCR1(I) = -POLD(I)

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```

31  SCR2(I) = -PV(I)
    CALL ETBFCT (RJLD, RNEW, NX, 1.0, 1.0)
    RNEW(1) = RLEFT
    RNEW(NX) = RRIGHT
    CALL SOURCE (NX, 0.5*DELTAT, 2, UNIT, SCR1, -PRIGHT, -PLEFT)
    CALL ETBFCT (RVULD, RVNEW, NX, 1.0, 1.0)
    RVNEW(1) = RVLEFT
    RVNEW(NX) = RVRIGHT
    CALL SOURCE (NX, 0.5*DELTAT, 1, ZERO, SCR2, -PRIGHT*VRIGHT,
1      -PLEFT*VLEFT)
    CALL ETBFCT (EJLD, ENEW, NX, 1.0, 1.0)
    ENEW(1) = ELEFT
    ENEW(NX) = ERIGHT
C
C  THE INTERMEDIATE QUANTITIES ARE CALCULATED.
    DO 32 I = 1, NX
      VNEW(I) = RVNEW(I)/RNEW(I)
      PNEW(I) = GAMMA*(ENEW(I) - 0.5*RVNEW(I)*VNEW(I))
32  PV(I) = PNEW(I)*VNEW(I)
C
C  RESET GRID AND VELOCITIES FOR THE WHOLE STEP.
    DTGRID = DELTAT*FLAG
    DO 33 I = 1, NX
33  RADN(I) = RADG(I) + DTGRID*VNEW(I)
      RADR = RADR + 0.5*DTGRID*VRIGHT
      RADL = RADL + 0.5*DTGRID*VLEFT
      CALL NGRIDE (RADN, NX, RADR, RADL, ALPHA)
      CALL VELOCE (VNEW, NX, VRIGHT, VLEFT, DELTAT)
C
C  THE INTEGRATION STEP IS PERFORMED WITH THE COMPLETE TIMESTEP.
    DO 34 I = 1, NX
      SCR1(I) = -PNEW(I)
34  SCR2(I) = -PV(I)
      CALL ETBFCT (RJLD, RNEW, NX, 1.0, 1.0)
      RNEW(1) = RLEFT
      RNEW(NX) = RRIGHT
      CALL SOURCE (NX, DELTAT, 2, UNIT, SCR1, -PRIGHT, -PLEFT)
      CALL ETBFCT (RVULD, RVNEW, NX, 1.0, 1.0)
      RVNEW(1) = RVLEFT
      RVNEW(NX) = RVRIGHT
      CALL SOURCE (NX, DELTAT, 1, ZERO, SCR2, -PRIGHT*VRIGHT,
1      -PLEFT*VLEFT)
      CALL ETBFCT (EJLD, ENEW, NX, 1.0, 1.0)
      ENEW(1) = ELEFT
      ENEW(NX) = ERIGHT
C
9999  CONTINUE
C
      STOP
    END

```

AFTER STEP NO. 0 DX = 1.000 AND DT = 0.05000

I	DENSITY	TEMPERATURE	PRESSURE	VELOCITY	ENERGY
1	3.5714E 00	8.6800E 00	3.1000E 01	1.1926E 00	4.9040E 01
2	3.5714E 00	8.6800E 00	3.1000E 01	1.1926E 00	4.9040E 01
3	3.5714E 00	8.6800E 00	3.1000E 01	1.1926E 00	4.9040E 01
4	3.5714E 00	8.6800E 00	3.1000E 01	1.1926E 00	4.9040E 01
5	3.5714E 00	8.6800E 00	3.1000E 01	1.1926E 00	4.9040E 01
6	3.5714E 00	8.6800E 00	3.1000E 01	1.1926E 00	4.9040E 01
7	3.5714E 00	8.6800E 00	3.1000E 01	1.1926E 00	4.9040E 01
8	3.5714E 00	8.6800E 00	3.1000E 01	1.1926E 00	4.9040E 01
9	3.5714E 00	8.6800E 00	3.1000E 01	1.1926E 00	4.9040E 01
10	3.5714E 00	8.6800E 00	3.1000E 01	1.1926E 00	4.9040E 01
11	1.0000E 00	1.0000E 00	1.0000E 00	-3.4550E 00	7.4684E 00
12	1.0000E 00	1.0000E 00	1.0000E 00	-3.4550E 00	7.4684E 00
13	1.0000E 00	1.0000E 00	1.0000E 00	-3.4550E 00	7.4684E 00
14	1.0000E 00	1.0000E 00	1.0000E 00	-3.4550E 00	7.4684E 00
15	1.0000E 00	1.0000E 00	1.0000E 00	-3.4550E 00	7.4684E 00
16	1.0000E 00	1.0000E 00	1.0000E 00	-3.4550E 00	7.4684E 00
17	1.0000E 00	1.0000E 00	1.0000E 00	-3.4550E 00	7.4684E 00
18	1.0000E 00	1.0000E 00	1.0000E 00	-3.4550E 00	7.4684E 00
19	1.0000E 00	1.0000E 00	1.0000E 00	-3.4550E 00	7.4684E 00
20	1.0000E 00	1.0000E 00	1.0000E 00	-3.4550E 00	7.4684E 00
21	1.0000E 00	1.0000E 00	1.0000E 00	-3.4550E 00	7.4684E 00
22	1.0000E 00	1.0000E 00	1.0000E 00	-3.4550E 00	7.4684E 00
23	1.0000E 00	1.0000E 00	1.0000E 00	-3.4550E 00	7.4684E 00
24	1.0000E 00	1.0000E 00	1.0000E 00	-3.4550E 00	7.4684E 00
25	1.0000E 00	1.0000E 00	1.0000E 00	-3.4550E 00	7.4684E 00
26	1.0000E 00	1.0000E 00	1.0000E 00	-3.4550E 00	7.4684E 00
27	1.0000E 00	1.0000E 00	1.0000E 00	-3.4550E 00	7.4684E 00
28	1.0000E 00	1.0000E 00	1.0000E 00	-3.4550E 00	7.4684E 00
29	1.0000E 00	1.0000E 00	1.0000E 00	-3.4550E 00	7.4684E 00
30	1.0000E 00	1.0000E 00	1.0000E 00	-3.4550E 00	7.4684E 00
31	1.0000E 00	1.0000E 00	1.0000E 00	-3.4550E 00	7.4684E 00
32	1.0000E 00	1.0000E 00	1.0000E 00	-3.4550E 00	7.4684E 00
33	1.0000E 00	1.0000E 00	1.0000E 00	-3.4550E 00	7.4684E 00
34	1.0000E 00	1.0000E 00	1.0000E 00	-3.4550E 00	7.4684E 00
35	1.0000E 00	1.0000E 00	1.0000E 00	-3.4550E 00	7.4684E 00
36	1.0000E 00	1.0000E 00	1.0000E 00	-3.4550E 00	7.4684E 00
37	1.0000E 00	1.0000E 00	1.0000E 00	-3.4550E 00	7.4684E 00
38	1.0000E 00	1.0000E 00	1.0000E 00	-3.4550E 00	7.4684E 00
39	1.0000E 00	1.0000E 00	1.0000E 00	-3.4550E 00	7.4684E 00
40	1.0000E 00	1.0000E 00	1.0000E 00	-3.4550E 00	7.4684E 00
41	1.0000E 00	1.0000E 00	1.0000E 00	-3.4550E 00	7.4684E 00
42	1.0000E 00	1.0000E 00	1.0000E 00	-3.4550E 00	7.4684E 00
43	1.0000E 00	1.0000E 00	1.0000E 00	-3.4550E 00	7.4684E 00
44	1.0000E 00	1.0000E 00	1.0000E 00	-3.4550E 00	7.4684E 00
45	1.0000E 00	1.0000E 00	1.0000E 00	-3.4550E 00	7.4684E 00
46	1.0000E 00	1.0000E 00	1.0000E 00	-3.4550E 00	7.4684E 00
47	1.0000E 00	1.0000E 00	1.0000E 00	-3.4550E 00	7.4684E 00
48	1.0000E 00	1.0000E 00	1.0000E 00	-3.4550E 00	7.4684E 00
49	1.0000E 00	1.0000E 00	1.0000E 00	-3.4550E 00	7.4684E 00
50	1.0000E 00	1.0000E 00	1.0000E 00	-3.4550E 00	7.4684E 00

CONSERVATION SUMS

50 7.5714E 01 5.2500E 02 -9.5606E 01 7.8913E 02

AFTER STEP NO. 100 DX = 1.000 AND DT = 0.05000

I	DENSITY	TEMPERATURE	PRESSURE	VELOCITY	ENERGY
1	3.5714E 00	8.6800E 00	3.1000E 01	1.1926E 00	4.9040E 01
2	3.5693E 00	8.6766E 00	3.0970E 01	1.1951E 00	4.9003E 01
3	3.5672E 00	8.6734E 00	3.0940E 01	1.1969E 00	4.8965E 01
4	3.5672E 00	8.6733E 00	3.0940E 01	1.1969E 00	4.8965E 01
5	3.5672E 00	8.6733E 00	3.0940E 01	1.1970E 00	4.8965E 01
6	3.5672E 00	8.6732E 00	3.0939E 01	1.1970E 00	4.8964E 01
7	3.5671E 00	8.6733E 00	3.0939E 01	1.1970E 00	4.8963E 01
8	3.5671E 00	8.6733E 00	3.0939E 01	1.1970E 00	4.8963E 01
9	3.5672E 00	8.6733E 00	3.0939E 01	1.1970E 00	4.8964E 01
10	3.5672E 00	8.6735E 00	3.0940E 01	1.1968E 00	4.8965E 01
11	3.5674E 00	8.6736E 00	3.0942E 01	1.1968E 00	4.8968E 01
12	3.5675E 00	8.6736E 00	3.0943E 01	1.1965E 00	4.8968E 01
13	3.5680E 00	8.6744E 00	3.0950E 01	1.1964E 00	4.8979E 01
14	3.5686E 00	8.6741E 00	3.0954E 01	1.1956E 00	4.8982E 01
15	3.5699E 00	8.6754E 00	3.0971E 01	1.1946E 00	4.9003E 01
16	3.5705E 00	8.6762E 00	3.0978E 01	1.1942E 00	4.9013E 01
17	3.5705E 00	8.6773E 00	3.0982E 01	1.1939E 00	4.9018E 01
18	3.5704E 00	8.6779E 00	3.0983E 01	1.1935E 00	4.9017E 01
19	3.5700E 00	8.6807E 00	3.0990E 01	1.1935E 00	4.9027E 01
20	3.5710E 00	8.6851E 00	3.1005E 01	1.1922E 00	4.9045E 01
21	3.5642E 00	8.6896E 00	3.1015E 01	1.1908E 00	4.9052E 01
22	3.5680E 00	8.6958E 00	3.1027E 01	1.1911E 00	4.9071E 01
23	3.5682E 00	8.6957E 00	3.1028E 01	1.1912E 00	4.9073E 01
24	3.5426E 00	8.7084E 00	3.0851E 01	1.1796E 00	4.8741E 01
25	3.5096E 00	8.7749E 00	3.0797E 01	1.1535E 00	4.8530E 01
26	1.1728E 00	2.7403E 00	3.2138E 00	-2.8035E 00	9.4296E 00
27	9.9771E-01	9.1574E-01	9.1364E-01	-3.4952E 00	7.4648E 00
28	9.9771E-01	9.8940E-01	9.8713E-01	-3.4635E 00	7.4648E 00
29	9.9893E-01	9.9807E-01	9.9700E-01	-3.4573E 00	7.4656E 00
30	9.9996E-01	1.0012E 00	1.0012E 00	-3.4545E 00	7.4682E 00
31	9.9996E-01	1.0003E 00	1.0003E 00	-3.4549E 00	7.4682E 00
32	9.9994E-01	1.0002E 00	1.0002E 00	-3.4550E 00	7.4682E 00
33	9.9993E-01	1.0002E 00	1.0001E 00	-3.4550E 00	7.4682E 00
34	9.9993E-01	1.0002E 00	1.0001E 00	-3.4550E 00	7.4682E 00
35	9.9993E-01	1.0002E 00	1.0001E 00	-3.4550E 00	7.4681E 00
36	9.9993E-01	1.0002E 00	1.0001E 00	-3.4550E 00	7.4681E 00
37	9.9993E-01	1.0002E 00	1.0001E 00	-3.4550E 00	7.4681E 00
38	9.9993E-01	1.0002E 00	1.0001E 00	-3.4550E 00	7.4681E 00
39	9.9993E-01	1.0002E 00	1.0001E 00	-3.4550E 00	7.4681E 00
40	9.9995E-01	1.0002E 00	1.0002E 00	-3.4549E 00	7.4681E 00
41	1.0000E 00	1.0002E 00	1.0002E 00	-3.4549E 00	7.4683E 00
42	1.0000E 00	1.0001E 00	1.0001E 00	-3.4549E 00	7.4684E 00
43	1.0000E 00	1.0002E 00	1.0001E 00	-3.4549E 00	7.4684E 00
44	1.0000E 00	1.0001E 00	1.0001E 00	-3.4549E 00	7.4684E 00
45	1.0000E 00	1.0001E 00	1.0001E 00	-3.4549E 00	7.4684E 00
46	1.0000E 00	1.0001E 00	1.0001E 00	-3.4549E 00	7.4684E 00
47	1.0000E 00	1.0000E 00	1.0000E 00	-3.4550E 00	7.4684E 00
48	1.0000E 00	1.0000E 00	1.0000E 00	-3.4550E 00	7.4684E 00
49	1.0000E 00	1.0000E 00	1.0000E 00	-3.4550E 00	7.4684E 00
50	1.0000E 00	1.0000E 00	1.0000E 00	-3.4550E 00	7.4684E 00

CONSERVATION SUMS

50 1.1429E 02 1.2015E 03 2.0068E 01 1.4129E 03

AFTER STEP NO. 200 DX = 1.000 AND DT = 0.05000

I	DENSITY	TEMPERATURE	PRESSURE	VELOCITY	ENERGY
1	3.5714E 00	8.6800E 00	3.1000E 01	1.1924E 00	4.9040E 01
2	3.5701E 00	8.6779E 00	3.0981E 01	1.1940E 00	4.9016E 01
3	3.5696E 00	8.6771E 00	3.0974E 01	1.1945E 00	4.9008E 01
4	3.5696E 00	8.6771E 00	3.0974E 01	1.1945E 00	4.9008E 01
5	3.5704E 00	8.6783E 00	3.0985E 01	1.1937E 00	4.9021E 01
6	3.5713E 00	8.6802E 00	3.1000E 01	1.1926E 00	4.9039E 01
7	3.5713E 00	8.6801E 00	3.1000E 01	1.1926E 00	4.9039E 01
8	3.5722E 00	8.6815E 00	3.1012E 01	1.1916E 00	4.9054E 01
9	3.5726E 00	8.6821E 00	3.1018E 01	1.1913E 00	4.9062E 01
10	3.5730E 00	8.6829E 00	3.1024E 01	1.1908E 00	4.9069E 01
11	3.5730E 00	8.6829E 00	3.1024E 01	1.1907E 00	4.9069E 01
12	3.5733E 00	8.6833E 00	3.1029E 01	1.1904E 00	4.9073E 01
13	3.5734E 00	8.6836E 00	3.1030E 01	1.1903E 00	4.9077E 01
14	3.5736E 00	8.6840E 00	3.1033E 01	1.1900E 00	4.9080E 01
15	3.5736E 00	8.6840E 00	3.1033E 01	1.1900E 00	4.9080E 01
16	3.5736E 00	8.6841E 00	3.1034E 01	1.1899E 00	4.9080E 01
17	3.5736E 00	8.6841E 00	3.1034E 01	1.1899E 00	4.9080E 01
18	3.5736E 00	8.6839E 00	3.1033E 01	1.1899E 00	4.9079E 01
19	3.5738E 00	8.6830E 00	3.1031E 01	1.1901E 00	4.9077E 01
20	3.5738E 00	8.6826E 00	3.1030E 01	1.1901E 00	4.9075E 01
21	3.5738E 00	8.6823E 00	3.1029E 01	1.1903E 00	4.9074E 01
22	3.5738E 00	8.6822E 00	3.1029E 01	1.1903E 00	4.9074E 01
23	3.5734E 00	8.6819E 00	3.1024E 01	1.1905E 00	4.9068E 01
24	3.5727E 00	8.6832E 00	3.1023E 01	1.1907E 00	4.9066E 01
25	3.5718E 00	8.6840E 00	3.1017E 01	1.1910E 00	4.9059E 01
26	3.5709E 00	8.6860E 00	3.1017E 01	1.1912E 00	4.9058E 01
27	3.5686E 00	8.6903E 00	3.1013E 01	1.1913E 00	4.9051E 01
28	3.5672E 00	8.6930E 00	3.1009E 01	1.1915E 00	4.9046E 01
29	3.5666E 00	8.6944E 00	3.1009E 01	1.1917E 00	4.9046E 01
30	3.5659E 00	8.6944E 00	3.1004E 01	1.1919E 00	4.9039E 01
31	3.5656E 00	8.6950E 00	3.1003E 01	1.1923E 00	4.9038E 01
32	3.5651E 00	8.6949E 00	3.0999E 01	1.1925E 00	4.9031E 01
33	3.5649E 00	8.6947E 00	3.0996E 01	1.1925E 00	4.9029E 01
34	3.5648E 00	8.6946E 00	3.0995E 01	1.1926E 00	4.9027E 01
35	3.5646E 00	8.6943E 00	3.0992E 01	1.1927E 00	4.9022E 01
36	3.5645E 00	8.6941E 00	3.0990E 01	1.1931E 00	4.9022E 01
37	3.5642E 00	8.6935E 00	3.0985E 01	1.1933E 00	4.9015E 01
38	3.5642E 00	8.6934E 00	3.0985E 01	1.1934E 00	4.9015E 01
39	3.5337E 00	8.7022E 00	3.0751E 01	1.1832E 00	4.8600E 01
40	3.5053E 00	8.7171E 00	3.0556E 01	1.1726E 00	4.8243E 01
41	1.2046E 00	2.7879E 00	3.3584E 00	-2.6437E 00	9.2473E 00
42	9.9734E-01	9.9754E-01	8.9515E-01	-3.5037E 00	7.4644E 00
43	9.9734E-01	9.9650E-01	9.8388E-01	-3.4654E 00	7.4644E 00
44	9.9879E-01	9.9736E-01	9.9416E-01	-3.4578E 00	7.4652E 00
45	1.0000E 00	1.0013E 00	1.0014E 00	-3.4543E 00	7.4684E 00
46	1.0000E 00	1.0002E 00	1.0002E 00	-3.4549E 00	7.4685E 00
47	1.0000E 00	1.0001E 00	1.0001E 00	-3.4550E 00	7.4684E 00
48	1.0000E 00	1.0000E 00	1.0000E 00	-3.4550E 00	7.4684E 00
49	1.0000E 00	1.0000E 00	1.0000E 00	-3.4550E 00	7.4684E 00
50	1.0000E 00	1.0000E 00	1.0000E 00	-3.4550E 00	7.4684E 00

CONSERVATION SUMS

50 1.5291E 02 1.8779E 03 1.3564E 02 2.0372E 03

Appendix D AN EXPLODING DIAPHRAGM TEST PROBLEM USING ETBFCT

This test program performs a one-dimensional exploding diaphragm calculation with hardwall (symmetry) conditions at both ends of the system. The program uses ETBFCT to solve the ideal fluid equations on a system 50 cells long with equally spaced cells. The check provided by comparing these results (cells 1-50) with results of Appendix E (cells 32-81) is a very stringent test in that any coding inconsistency would surface. The results after cycle 0, 150, and 300 are printed below with grid positions removed.

```

C      ETBFCT SIMPLE DIAPHRAGM PROBLEM      DEC. 1975      SPL VERSION
C      THIS PROGRAM RUNS A SIMPLE DIAPHRAGM PROBLEM USING ETBFCT
C      FOR BOTH THE HALF AND THE WHOLE STEP. THE FLUID IS IDEAL AND
C      INVISCID WITH GAMMA = 5/3. A VARIABLE GRID AND THREE GEOMETRIES
C      ARE ALLOWED AND LAGRANGIAN AS WELL AS EULERIAN MOTIONS ARE PER-
C      Mitted BY SETTING FLAG = 1.0.
C
C      INTEGER  ALPHA
C      REAL     ROLD(200), RNEW(200), VOL0(200), VNEW(200)
C      REAL     POLD(200), PNEW(200), EULD(200), ENEW(200)
C      REAL     TULD(200), PV(200), ZERO(200), UNIT(200)
C      REAL     SCR1(200), SCR2(200), XADD(200), RADN(200)
C      REAL     RVOLD(200), RVNEW(200)
C
C      1000  FORMAT ('1 AFTER STEP NO. ', I5, ' DX = ', F7.3, ' AND DT = ',
C      1      F7.5, '/')
C      1001  FORMAT (2X, I3, 1PSE12.4, 12X, E12.4)
C      1002  FORMAT ('1 I DENSITY TEMPERATURE PRESSURE ',
C      1      ' VELOCITY ENERGY POSITION ', '/')
C      1003  FORMAT ('0 CONSERVATION SUMS ', //, 2X, I3, 1PE12.4, 12X,
C      1      3E12.4)
C
C      * * * * *
C      THE SHOCK PARAMETERS ARE SET UP. CHANGE FOR DIFFERENT CASES.
C      * * * * *
C      GAMMA0 = 1.66667
C      GAMMAH = GAMMA0 - 1.0
C      VRIGHT = 0.0
C      VLEFT = 0.0
C      TRIGHT = 1.0
C      TLEFT = 10.0
C      RRIGHT = 1.0

```

```

RLEFT = 1.0
RVRIGH = VRIGHT*RRIGHT
RVLEFT = VLEFT*RLEFT
PRIGHT = TRIGHT*RRIGHT
PLEFT = TLEFT*RLEFT
ERIGHT = PRIGHT/(GAMMAO - 1.0) + 0.5*RRIGHT*VRIGHT**2
ELEFT = PLEFT/(GAMMAO - 1.0) + 0.5*RLEFT*VLEFT**2
C
C *****
C CONTROL PARAMETERS ARE INITIALIZED. CHANGE FOR DIFFERENT CASES.
C *****
    ALPHA = 1
    MAXSTP = 301
    IPRINT = 50
    FLAG = 0.0
    DELTAX = 1.0
    DELTAL = DELTAX
    DELTAR = DELTAX*(RLEFT/RRIGHT)*FLAG + (1.0 - FLAG)*DELTAX
    DELTAT = .05
    NX = 50
    LPRINT = NX
    MX = 30
C
C *****
C THE PHYSICAL VARIABLES ARE INITIALIZED.
C *****
    DO 11 I = 1, NX
11      RADN(I) = (FLOAT(I-NX) - 0.5)*DELTAR
    DO 12 I = 1, NX
12      RADN(I) = (FLOAT(I-NX) - 0.5)*DELTAL
        RAOR = RADN(NX) + 0.5*DELTAR
        RAOL = RADN(1) - 0.5*DELTAL
        CALL NGRIDE (RADN, NX, RAOR, RAOL, ALPHA)
    DO 13 I = 1, NX
        RNEW(I) = RRIGHT
        VNEW(I) = VRIGHT
        PNEW(I) = PRIGHT
        RVNEW(I) = RVRIGH
        ZERO(I) = 0.0
        UNIT(I) = 1.0
13      ENEN(I) = ERIGHT
    DO 14 I = 1, NX
        RNEW(I) = RLEFT
        VNEW(I) = VLEFT
        PNEW(I) = PLEFT
        RVNEW(I) = RVLEFT
14      ENEN(I) = ELEFT
C
C *****
C THE LOOP OVER TIMESTEPS.

```

```

C      * * * * *
C      DJ 9999 ISTEP = 1, MAXSTP
C
C      INITIALIZE THE QUANTITIES FOR THE CYCLE.
      DO 20 I = 1, NX
        RADU(I) = RADN(I)
        ROLD(I) = RNEW(I)
        RVOLD(I) = RVNEW(I)
        EOLD(I) = ENEW(I)
        VOLD(I) = RVOLD(I)/ROLD(I)
20      POLD(I) = GAMMA*(EOLD(I) - 0.5*RVOLD(I)*VOLD(I))
      DO 21 I = 1, NX
        POLD(I) = AMAX1 (POLD(I), 0.1*PRIGHT)
        TOLD(I) = POLD(I)/ROLD(I)
21      PV(I) = POLD(I)*VOLD(I)
C
C      THE RESULTS ARE PRINTED.
      IF (MOD(ISTEP-1,IPRINT) .NE. 0) GO TO 22
      JSTEP = ISTEP - 1
      WRITE (6, 1000) JSTEP, DELTAX, DELTAT
      WRITE (6, 1002)
      WRITE (6, 1001) (I, ROLD(I), TOLD(I), POLD(I), VOLD(I),
1      EOLD(I), RADU(I), I=1,LPRINT)
      CALL CONSRE (ROLD, NX, RHOSUM)
      CALL CONSRE (POLD, NX, PRESUM)
      CALL CONSRE (EOLD, NX, ERGSUM)
      CALL CONSRE (RVOLD, NX, RHVSUM)
      PRESUM = PRESUM/GAMMA
      WRITE (6, 1003) NX, RHOSUM, PRESUM, RHVSUM, ERGSUM
22      CONTINUE
C
C      RESET GRID AND VELOCITIES FOR THE HALF STEP.
      CALL OGRID (NX)
      DTGRID = 0.5*DELTAT*FLAG
      DO 30 I = 1, NX
30      RADN(I) = RADU(I) + DTGRID*VOLD(I)
      CALL NGRID (RADN, NX, RADR, RADL, ALPHA)
      CALL VELOC (VOLD, NX, 0.0, 0.0, 0.5*DELTAT)
C
C      ALL QUANTITIES ARE INTEGRATED FORWARD HALF A STEP USING THE OLD
C      VALUES ON THE RIGHT HAND SIDE FOR FORMING DERIVATIVES.
      DO 31 I = 1, NX
        SCR1(I) = -POLD(I)
        SCR2(I) = -PV(I)
31      CALL ETBFCT (ROLD, RNEW, NX, 1.0, 1.0)
      CALL SOURCE (NX, 0.5*DELTAT, 2, UYIT, SCR1, -POLD(NX),
1      -POLD(1))
      CALL ETBFCT (RVOLD, RVNEW, NX, -1.0, -1.0)
      CALL SOURCE (NX, 0.5*DELTAT, 1, ZERO, SCR2, 0.0, 0.0)
      CALL ETBFCT (EOLD, ENEW, NX, 1.0, 1.0)

```

```

C
C   THE INTERMEDIATE QUANTITIES ARE CALCULATED.
      DO 32 I = 1, NX
        VNEW(I) = RVNEW(I)/RNEW(I)
32    PNEW(I) = GAMMA*(ENEW(I) - 0.5*RVNEW(I)*VNEW(I))
      DO 33 I = 1, NX
        PNEW(I) = AMAX1 (PNEW(I), 0.1*PRIGHT)
33    PV(I) = PNEW(I)*VNEW(I)
C
C   RESET GRID AND VELOCITIES FOR THE WHOLE STEP.
      DTGRID = DELTAT*FLAG
      DO 34 I = 1, NX
34    RADN(I) = RADJ(I) + DTGRID*VNEW(I)
        CALL NGRID (RADN, NX, RADR, RADL, ALPHA)
        CALL VELOC (VNEW, NX, 0.0, 0.0, DELTAT)
C
C   THE INTEGRATION STEP IS PERFORMED WITH THE COMPLETE TIMESTEP.
      DO 35 I = 1, NX
        SCR1(I) = -PNEW(I)
35    SCR2(I) = -PV(I)
        CALL ETBFCT (ROLD, RNEW, NX, 1.0, 1.0)
        CALL SOURCE (NX, DELTAT, 2, UNIT, SCR1, -PNEW(NX), -PNEW(1))
        CALL ETBFCT (RVOLD, RVNEW, NX, -1.0, -1.0)
        CALL SOURCE (NX, DELTAT, 1, ZERO, SCR2, 0.0, 0.0)
        CALL ETBFCT (EOLD, ENEW, NX, 1.0, 1.0)
C
9999  CONTINUE
C
      STOP
      END

```

0 9x 3

1.000 AND DT = 0.05000

I	DENSITY	TEMPERATURE	PRESSURE	VELOCITY	ENERGY
---	---------	-------------	----------	----------	--------

CONSERVATION SUMS

57 5,0000E 01 4,0000E 02 0,0000E 00 4,0000E 02

AFTER STEP NO. 150 DX = 1.000 AND DT = 0.05000

I	DENSITY	TEMPERATURE	PRESSURE	VELOCITY	ENERGY
1	9.2838E-01	9.5233E 00	8.8413E 00	3.0518E-02	1.3262E 01
2	9.2730E-01	9.4741E 00	8.7380E 00	1.7350E-01	1.3121E 01
3	9.1867E-01	9.3872E 00	8.5487E 00	3.1171E-01	1.2867E 01
4	8.9935E-01	9.3259E 00	8.3873E 00	4.0537E-01	1.2655E 01
5	8.7507E-01	9.1421E 00	8.0000E 00	5.1264E-01	1.2115E 01
6	8.6057E-01	9.0531E 00	7.7909E 00	5.8365E-01	1.1833E 01
7	8.4270E-01	8.9193E 00	7.5163E 00	7.1894E-01	1.1492E 01
8	8.1929E-01	8.7583E 00	7.1756E 00	7.5942E-01	1.1000E 01
9	7.9929E-01	8.6130E 00	6.8850E 00	8.6309E-01	1.0625E 01
10	7.8988E-01	8.5450E 00	6.7495E 00	9.7740E-01	1.0501E 01
11	7.6356E-01	8.3588E 00	6.3825E 00	1.0305E 00	9.9791E 00
12	7.4510E-01	8.2148E 00	6.1208E 00	1.1015E 00	9.6331E 00
13	7.3709E-01	8.1777E 00	6.0277E 00	1.2375E 00	9.6059E 00
14	7.1493E-01	7.9769E 00	5.7030E 00	1.3008E 00	9.1593E 00
15	6.9311E-01	7.8587E 00	5.4469E 00	1.3499E 00	8.8019E 00
16	6.8732E-01	7.7819E 00	5.3486E 00	1.4675E 00	8.7630E 00
17	6.7572E-01	7.7078E 00	5.2083E 00	1.5154E 00	8.5882E 00
18	6.6651E-01	7.6279E 00	5.0841E 00	1.5348E 00	8.4111E 00
19	6.6417E-01	7.6208E 00	5.0615E 00	1.5543E 00	8.3945E 00
20	6.6354E-01	7.6143E 00	5.0524E 00	1.5698E 00	8.3961E 00
21	6.6277E-01	7.6008E 00	5.0376E 00	1.5716E 00	8.3749E 00
22	6.6277E-01	7.6029E 00	5.0390E 00	1.5696E 00	8.3749E 00
23	6.6277E-01	7.6121E 00	5.0451E 00	1.5608E 00	8.3749E 00
24	6.6409E-01	7.6371E 00	5.0870E 00	1.5534E 00	8.4341E 00
25	6.6822E-01	7.6432E 00	5.1074E 00	1.5350E 00	8.4883E 00
26	6.6802E-01	7.6792E 00	5.1299E 00	1.5329E 00	8.4795E 00
27	6.7028E-01	7.6595E 00	5.1340E 00	1.5276E 00	8.4831E 00
28	6.7036E-01	7.6527E 00	5.1300E 00	1.5334E 00	8.4831E 00
29	6.7018E-01	7.6333E 00	5.1157E 00	1.5336E 00	8.4617E 00
30	6.6939E-01	7.6298E 00	5.1073E 00	1.5463E 00	8.4612E 00
31	6.6944E-01	7.6005E 00	5.0881E 00	1.5470E 00	8.4332E 00
32	6.6827E-01	7.6091E 00	5.0850E 00	1.5490E 00	8.4291E 00
33	6.6443E-01	7.6633E 00	5.0918E 00	1.5411E 00	8.4266E 00
34	6.5889E-01	7.7781E 00	5.0938E 00	1.5437E 00	8.4210E 00
35	6.5250E-01	7.8118E 00	5.0972E 00	1.5445E 00	8.4240E 00
36	6.5245E-01	7.8171E 00	5.1003E 00	1.5430E 00	8.4270E 00
37	6.5220E-01	7.8208E 00	5.1007E 00	1.5425E 00	8.4270E 00
38	6.5217E-01	7.8206E 00	5.1004E 00	1.5425E 00	8.4264E 00
39	6.5217E-01	7.8206E 00	5.1004E 00	1.5425E 00	8.4264E 00
40	6.5588E-01	7.4281E 00	5.0948E 00	1.5726E 00	8.4903E 00
41	1.0207E 00	4.9818E 00	5.0850E 00	1.5308E 00	8.8229E 00
42	1.4454E 00	3.4907E 00	5.0456E 00	1.5335E 00	9.2679E 00
43	1.8083E 00	2.8531E 00	5.1592E 00	1.5593E 00	9.9370E 00
44	2.0224E 00	2.5358E 00	5.1283E 00	1.5421E 00	1.0097E 01
45	2.1594E 00	2.3527E 00	5.0803E 00	1.5296E 00	1.0147E 01
46	2.2478E 00	2.2209E 00	5.0387E 00	1.5484E 00	1.0270E 01
47	2.2828E 00	2.1220E 00	4.8442E 00	1.6269E 00	1.0287E 01
48	2.2921E 00	2.1147E 00	4.8171E 00	1.6544E 00	1.0407E 01
49	2.7292E 00	1.9862E 00	5.4206E 00	1.3895E 00	1.0765E 01
50	3.0356E 00	2.3527E 00	7.1417E 00	9.4510E-01	1.2068E 01

CONSERVATION SUMS

50 4.9997E 01 4.3311E 02 6.5222E 01 4.7998E 02

AFTER STEP NO. 300 DX = 1.000 AND DT = 0.05000

I	DENSITY	TEMPERATURE	PRESSURE	VELOCITY	ENERGY
1	4.1694E-01	5.5884E 00	2.3300E 00	3.3472E-02	3.4953E 00
2	4.1869E-01	5.6037E 00	2.3462E 00	3.8535E-02	3.5196E 00
3	4.2171E-01	5.6278E 00	2.3733E 00	5.4317E-02	3.5606E 00
4	4.2603E-01	5.6549E 00	2.4091E 00	9.2037E-02	3.6155E 00
5	4.3203E-01	5.7151E 00	2.4691E 00	1.2770E-01	3.7071E 00
6	4.3591E-01	5.7476E 00	2.5054E 00	1.5719E-01	3.7635E 00
7	4.4227E-01	5.8041E 00	2.5669E 00	1.7723E-01	3.8573E 00
8	4.4935E-01	5.8806E 00	2.6425E 00	2.2218E-01	3.9788E 00
9	4.5735E-01	5.9319E 00	2.7130E 00	2.5814E-01	4.0847E 00
10	4.6247E-01	5.9669E 00	2.7595E 00	2.8967E-01	4.1587E 00
11	4.6773E-01	6.0256E 00	2.8184E 00	3.3136E-01	4.2532E 00
12	4.7514E-01	6.0823E 00	2.8900E 00	3.7932E-01	4.3692E 00
13	4.8153E-01	6.1213E 00	2.9476E 00	4.2188E-01	4.4602E 00
14	4.8562E-01	6.1448E 00	2.9936E 00	4.6329E-01	4.5426E 00
15	4.9092E-01	6.2262E 00	3.0566E 00	5.0018E-01	4.6462E 00
16	4.9762E-01	6.2945E 00	3.1323E 00	5.4873E-01	4.7733E 00
17	5.0446E-01	6.3527E 00	3.2047E 00	6.0066E-01	4.8980E 00
18	5.1034E-01	6.4015E 00	3.2669E 00	6.3353E-01	5.0028E 00
19	5.1596E-01	6.4463E 00	3.3260E 00	6.6802E-01	5.1081E 00
20	5.2237E-01	6.4910E 00	3.3911E 00	7.0564E-01	5.2167E 00
21	5.3109E-01	6.5574E 00	3.4826E 00	7.5794E-01	5.3768E 00
22	5.3959E-01	6.6276E 00	3.5761E 00	8.0832E-01	5.5405E 00
23	5.4523E-01	6.6751E 00	3.6395E 00	8.4345E-01	5.6531E 00
24	5.4765E-01	6.6988E 00	3.6686E 00	8.9285E-01	5.7212E 00
25	5.5181E-01	6.7024E 00	3.6985E 00	9.8976E-01	5.8179E 00
26	5.6114E-01	6.8223E 00	3.8282E 00	9.9949E-01	6.0226E 00
27	5.8716E-01	6.9524E 00	4.0822E 00	9.1790E-01	6.3706E 00
28	7.9629E-01	8.3905E 00	6.6813E 00	-2.6742E-01	1.0050E 01
29	1.0217E 00	1.0028E 01	1.0245E 01	-9.3299E-01	1.5812E 01
30	1.0217E 00	1.0481E 01	1.0708E 01	-9.3299E-01	1.6507E 01
31	1.0212E 00	1.0529E 01	1.0752E 01	-8.6211E-01	1.6507E 01
32	1.0095E 00	1.0616E 01	1.0717E 01	-7.6102E-01	1.6368E 01
33	1.0058E 00	1.0697E 01	1.0755E 01	-6.5286E-01	1.6386E 01
34	1.0068E 00	1.0793E 01	1.0866E 01	-5.8076E-01	1.6469E 01
35	1.0091E 00	1.0956E 01	1.1056E 01	-5.6049E-01	1.6743E 01
36	1.0104E 00	1.1149E 01	1.1265E 01	-4.8289E-01	1.7015E 01
37	1.0143E 00	1.1155E 01	1.1318E 01	-4.2461E-01	1.7063E 01
38	1.0144E 00	1.1151E 01	1.1311E 01	-3.9178E-01	1.7045E 01
39	1.0151E 00	1.1157E 01	1.1325E 01	-3.1800E-01	1.7039E 01
40	1.0150E 00	1.0893E 01	1.1057E 01	-2.9061E-01	1.6628E 01
41	1.0150E 00	1.0761E 01	1.0922E 01	-2.2441E-01	1.6408E 01
42	1.1313E 00	9.5511E 00	1.0805E 01	-1.8579E-01	1.6227E 01
43	1.4226E 00	7.3500E 00	1.0456E 01	-1.4774E-01	1.5700E 01
44	1.8910E 00	5.4623E 00	1.0329E 01	-1.1147E-01	1.5505E 01
45	2.6492E 00	3.7545E 00	9.9467E 00	-7.8695E-02	1.4928E 01
46	3.1849E 00	3.0358E 00	9.6687E 00	-6.5131E-02	1.4510E 01
47	3.1880E 00	2.9633E 00	9.4469E 00	3.2425E-02	1.4172E 01
48	3.1880E 00	2.7558E 00	8.7854E 00	3.8414E-02	1.3180E 01
49	3.1886E 00	2.5988E 00	8.0742E 00	2.3847E-02	1.2115E 01
50	3.0748E 00	2.6160E 00	8.0435E 00	-9.2880E-04	1.2065E 01

CONSERVATION SUMS

50 4.9992E 01 4.7497E 02 -1.7846E 00 4.7991E 02

Appendix E CONVECTION TEST PROBLEMS USING PRBFCT

This is a convection test program with constant velocity using PRBFCT. The 100 cell periodic system used in the three articles FCT I, II, and III, is initialized. Three problems are run here--the standard square wave test, a sinusoid with 5 wavelengths in the system, and a linear peak function. The results after 0, 400, and 800 cycles are reproduced below. Comparisons with other methods are readily available from FCT I, II, and III. The square wave is 20 cells across and the flow velocity is 0.2 cells per timestep.

```

C      PRBFCT SIMPLE CONVECTION PROBLEM      DEC. 1975      SPL VERSION
C
C      THIS PROGRAM RUNS A PERIODIC CONVECTION PROBLEM USING PRBFCT.
C      FULLY VARIABLE CARTESIAN SPATIAL GRIDS ARE ALLOWED. THREE DIFFER-
C      ENT QUANTITIES ARE CONVECTED USING CONSTANT VELOCITY. THEY ARE THE
C      USUAL SQUARE WAVE, A SINUSOID, AND A PEAK FUNCTION.
C
C      REAL      ROLD(200), RNEW(200), RADD(200), RADN(200)
C      REAL      VNEW(200), RVNEW(200), EOLD(200), ENEW(200)
C      REAL      RVOLD(200)
C
C      1000      FORMAT ('1 AFTER STEP NO. ', I5, ' DX = ', F7.3, ' AND DT = ',
C      1          F7.5, '/')
C      1001      FORMAT (2X, I3, 1P4E12.4)
C      1002      FORMAT ('1 I SQ WAVE SINUSOID PEAK FUN. ',
C      1          ' POSITION ', /)
C      1003      FORMAT ('0 CONSERVATION SUMS ', //, 2X, I3, 1P3E12.4)
C      1004      FORMAT ('1 ', 5X)
C
C      * * * * *
C      THE INITIAL PARAMETERS ARE SET UP. CHANGE FOR DIFFERENT CASES.
C      * * * * *
C      RRIGHT = 0.5
C      RLEFT = 1.5
C      RVHIGH = 0.0
C      RVLEFT = 1.0
C      ERIGHT = 0.0
C      ELEFT = 1.0
C      VKRIGHT = 1.0
C
C      * * * * *
C      CONTROL PARAMETERS ARE INITIALIZED. CHANGE FOR DIFFERENT CASES.

```

```

C      * * * * *
      FLAG = 0.0
      IPRINT = 100
      MAXSTP = 1001
      DELTAX = 1.0
      DELTAT = .20
      NX = 100
      NXP = NX + 1
      LPRINT = NXP
      MX = 21

C      * * * * *
C      THE PHYSICAL VARIABLES ARE INITIALIZED.
C      * * * * *
      DO 11 I = 1, NXP
11      RADN(I) = FLOAT(I-1)*DELTAX
          CALL NGRIDP (RADN, NXP)
          DO 13 I = 1, NXP
              RNEW(I) = RRIGHT
              VNEW(I) = VRIGHT
              RVNEW(I) = RVRIGH + SIN(6.283185*FLOAT(I-1)/FLOAT(MX-1))
13      ENEN(I) = ERIGHT
          DO 14 I = 2, NX
              RNEW(I) = RNEW(I) + RLEFT
              ARG = (FLOAT(I-1) - 0.5*FLOAT(MX))/(0.5*FLOAT(MX))
14      ENEN(I) = ENEN(I) + ELEFT*(1.0 - ABS(ARG))

C      * * * * *
C      THE LOOP OVER TIMESTEPS.
C      * * * * *
      DO 9999 ISTEP = 1, MAXSTP

C      INITIALIZE THE QUANTITIES FOR THE CYCLE.
      DO 20 I = 1, NXP
          RADO(I) = RADN(I)
          ROLD(I) = RNEW(I)
          RVOLD(I) = RVNEW(I)
20      EOLD(I) = ENEN(I)

C      THE RESULTS ARE PRINTED.
      IF (MOD(ISTEP-1,IPRINT) .NE. 0) GO TO 22
      JSTEP = ISTEP - 1
      WRITE (6, 1003) JSTEP, DELTAX, DELTAT
      WRITE (6, 1002)
      WRITE (6, 1001) (I, ROLD(I), RVOLD(I), EOLD(I), RADO(I),
1          I = 1, 50)
      WRITE (6, 1004)
      WRITE (6, 1001) (I, ROLD(I), RVOLD(I), EOLD(I), RADO(I),
1          I = 51, LPRINT)
      CALL CONSRP (ROLD, NXP, RHOSUM)

```

```

      CALL CONSRP (RVOLD, NXP, RHVSUM)
      CALL CJNSRP (EOLD, NXP, ERGSUM)
      WRITE (6, 1003) NX, RHOSUM, RHVSJM, ERGSUM
22    CONTINUE
C
C    RESET GRID AND VELOCITIES FOR THE NEXT STEP.
      CALL OGRIDP (NXP)
      DTGRID = DELTAT*FLAG
      DO 34 I = 1, NXP
34    RADN(I) = RADG(I) + DTGRID*VNEW(I)
      CALL NGRIDP (RADN, NXP)
      CALL VELOCP (VNEW, NXP, DELTAT)
C
C    THE INTEGRATION STEP IS PERFORMED.
      CALL PRBFCT (ROLD, RNEW, NXP)
      CALL PRBFCT (RVOLD, RVNEW, NXP)
      CALL PRBFCT (EOLD, ENEW, NXP)
C
9999  CONTINUE
C
      STOP
      END

```

AFTER STEP NO. $\Delta X = 1.000$ AND $\Delta T = 0.20000$

I	SQ WAVE	SINUSOID	PEAK FREQ.	POSITION
1	5.0000E-01	0.0000E 00	0.0000E 00	0.0000E 00
2	2.0000E 00	3.0902E-01	9.5238E-02	1.0000E 00
3	2.0000E 00	5.8779E-01	1.9048E-01	2.0000E 00
4	2.0000E 00	8.0902E-01	2.8571E-01	3.0000E 00
5	2.0000E 00	9.5106E-01	3.8095E-01	4.0000E 00
6	2.0000E 00	1.0000E 00	4.7619E-01	5.0000E 00
7	2.0000E 00	9.5106E-01	5.7143E-01	6.0000E 00
8	2.0000E 00	8.0902E-01	6.6667E-01	7.0000E 00
9	2.0000E 00	5.8779E-01	7.6190E-01	8.0000E 00
10	2.0000E 00	3.0902E-01	8.5714E-01	9.0000E 00
11	2.0000E 00	1.5815E-06	9.5238E-01	1.0000E 01
12	2.0000E 00	-3.0902E-01	9.5238E-01	1.1000E 01
13	2.0000E 00	-5.8779E-01	8.5714E-01	1.2000E 01
14	2.0000E 00	-8.0902E-01	7.6190E-01	1.3000E 01
15	2.0000E 00	-9.5106E-01	6.6667E-01	1.4000E 01
16	2.0000E 00	-1.0000E 00	5.7143E-01	1.5000E 01
17	2.0000E 00	-9.5106E-01	4.7619E-01	1.6000E 01
18	2.0000E 00	-8.0902E-01	3.8095E-01	1.7000E 01
19	2.0000E 00	-5.8779E-01	2.8571E-01	1.8000E 01
20	2.0000E 00	-3.0902E-01	1.9048E-01	1.9000E 01
21	2.0000E 00	-2.2093E-06	9.5238E-02	2.0000E 01
22	5.0000E-01	3.0901E-01	0.0000E 00	2.1000E 01
23	5.0000E-01	5.8779E-01	0.0000E 00	2.2000E 01
24	5.0000E-01	8.0902E-01	0.0000E 00	2.3000E 01
25	5.0000E-01	9.5106E-01	0.0000E 00	2.4000E 01
26	5.0000E-01	1.0000E 00	0.0000E 00	2.5000E 01
27	5.0000E-01	9.5106E-01	0.0000E 00	2.6000E 01
28	5.0000E-01	8.0902E-01	0.0000E 00	2.7000E 01
29	5.0000E-01	5.8779E-01	0.0000E 00	2.8000E 01
30	5.0000E-01	3.0902E-01	0.0000E 00	2.9000E 01
31	5.0000E-01	3.7908E-06	0.0000E 00	3.0000E 01
32	5.0000E-01	-3.0901E-01	0.0000E 00	3.1000E 01
33	5.0000E-01	-5.8779E-01	0.0000E 00	3.2000E 01
34	5.0000E-01	-8.0901E-01	0.0000E 00	3.3000E 01
35	5.0000E-01	-9.5106E-01	0.0000E 00	3.4000E 01
36	5.0000E-01	-1.0000E 00	0.0000E 00	3.5000E 01
37	5.0000E-01	-9.5106E-01	0.0000E 00	3.6000E 01
38	5.0000E-01	-8.0902E-01	0.0000E 00	3.7000E 01
39	5.0000E-01	-5.8779E-01	0.0000E 00	3.8000E 01
40	5.0000E-01	-3.0902E-01	0.0000E 00	3.9000E 01
41	5.0000E-01	-4.4187E-06	0.0000E 00	4.0000E 01
42	5.0000E-01	3.0901E-01	0.0000E 00	4.1000E 01
43	5.0000E-01	5.8779E-01	0.0000E 00	4.2000E 01
44	5.0000E-01	8.0901E-01	0.0000E 00	4.3000E 01
45	5.0000E-01	9.5106E-01	0.0000E 00	4.4000E 01
46	5.0000E-01	1.0000E 00	0.0000E 00	4.5000E 01
47	5.0000E-01	9.5106E-01	0.0000E 00	4.6000E 01
48	5.0000E-01	8.0902E-01	0.0000E 00	4.7000E 01
49	5.0000E-01	5.8779E-01	0.0000E 00	4.8000E 01
50	5.0000E-01	3.0902E-01	0.0000E 00	4.9000E 01

51	5.0000E-01	6.0002E-04	0.0000E 00	5.0000E 01
52	5.0000E-01	-3.0901E-01	0.0000E 00	5.1000E 01
53	5.0000E-01	-5.8777E-01	0.0000E 00	5.2000E 01
54	5.0000E-01	-8.0901E-01	0.0000E 00	5.3000E 01
55	5.0000E-01	-9.5105E-01	0.0000E 00	5.4000E 01
56	5.0000E-01	-1.0000E 00	0.0000E 00	5.5000E 01
57	5.0000E-01	-9.5104E-01	0.0000E 00	5.6000E 01
58	5.0000E-01	-8.0903E-01	0.0000E 00	5.7000E 01
59	5.0000E-01	-5.8780E-01	0.0000E 00	5.8000E 01
60	5.0000E-01	-3.0903E-01	0.0000E 00	5.9000E 01
61	5.0000E-01	-7.5817E-04	0.0000E 00	6.0000E 01
62	5.0000E-01	3.0900E-01	0.0000E 00	6.1000E 01
63	5.0000E-01	5.8777E-01	0.0000E 00	6.2000E 01
64	5.0000E-01	8.0901E-01	0.0000E 00	6.3000E 01
65	5.0000E-01	9.5105E-01	0.0000E 00	6.4000E 01
66	5.0000E-01	1.0000E 00	0.0000E 00	6.5000E 01
67	5.0000E-01	9.5104E-01	0.0000E 00	6.6000E 01
68	5.0000E-01	8.0902E-01	0.0000E 00	6.7000E 01
69	5.0000E-01	5.8780E-01	0.0000E 00	6.8000E 01
70	5.0000E-01	3.0903E-01	0.0000E 00	6.9000E 01
71	5.0000E-01	1.3932E-05	0.0000E 00	7.0000E 01
72	5.0000E-01	-3.0901E-01	0.0000E 00	7.1000E 01
73	5.0000E-01	-5.8777E-01	0.0000E 00	7.2000E 01
74	5.0000E-01	-8.0901E-01	0.0000E 00	7.3000E 01
75	5.0000E-01	-9.5105E-01	0.0000E 00	7.4000E 01
76	5.0000E-01	-1.0000E 00	0.0000E 00	7.5000E 01
77	5.0000E-01	-9.5104E-01	0.0000E 00	7.6000E 01
78	5.0000E-01	-8.0903E-01	0.0000E 00	7.7000E 01
79	5.0000E-01	-5.8780E-01	0.0000E 00	7.8000E 01
80	5.0000E-01	-3.0903E-01	0.0000E 00	7.9000E 01
81	5.0000E-01	-2.0281E-05	0.0000E 00	8.0000E 01
82	5.0000E-01	3.0900E-01	0.0000E 00	8.1000E 01
83	5.0000E-01	5.8777E-01	0.0000E 00	8.2000E 01
84	5.0000E-01	8.0900E-01	0.0000E 00	8.3000E 01
85	5.0000E-01	9.5105E-01	0.0000E 00	8.4000E 01
86	5.0000E-01	1.0000E 00	0.0000E 00	8.5000E 01
87	5.0000E-01	9.5104E-01	0.0000E 00	8.6000E 01
88	5.0000E-01	8.0903E-01	0.0000E 00	8.7000E 01
89	5.0000E-01	5.8780E-01	0.0000E 00	8.8000E 01
90	5.0000E-01	3.0903E-01	0.0000E 00	8.9000E 01
91	5.0000E-01	1.1373E-05	0.0000E 00	9.0000E 01
92	5.0000E-01	-3.0900E-01	0.0000E 00	9.1000E 01
93	5.0000E-01	-5.8777E-01	0.0000E 00	9.2000E 01
94	5.0000E-01	-8.0901E-01	0.0000E 00	9.3000E 01
95	5.0000E-01	-9.5105E-01	0.0000E 00	9.4000E 01
96	5.0000E-01	-1.0000E 00	0.0000E 00	9.5000E 01
97	5.0000E-01	-9.5104E-01	0.0000E 00	9.6000E 01
98	5.0000E-01	-8.0903E-01	0.0000E 00	9.7000E 01
99	5.0000E-01	-5.8781E-01	0.0000E 00	9.8000E 01
100	5.0000E-01	-3.0904E-01	0.0000E 00	9.9000E 01
101	5.0000E-01	-1.7722E-05	0.0000E 00	1.0000E 02

CONSERVATION SUMS

100	5.0000E 01	9.1976E-04	1.0476E 01
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AFTER STEP NR. 400 DX = 1.000 AND DT = 0.20000

I	SG WAVE	SINUSOID	PEAK FLIN.	POSITION
1	1.3910E 00	5.6443E-02	8.0748E-02	0.0000E 00
2	1.0086E 00	4.2293E-01	3.3563E-02	1.0000E 00
3	7.3124E-01	6.7436E-01	9.6859E-03	2.0000E 00
4	5.7549E-01	8.0489E-01	1.1734E-03	3.0000E 00
5	5.1262E-01	8.4564E-01	0.0000E 00	4.0000E 00
6	5.0009E-01	8.4779E-01	2.5915E-64	5.0000E 00
7	5.0000E-01	8.4779E-01	2.5915E-64	6.0000E 00
8	5.0000E-01	8.4766E-01	5.1871E-64	7.0000E 00
9	5.0000E-01	7.3164E-01	6.9467E-61	8.0000E 00
10	5.0000E-01	3.6763E-01	7.3316E-60	9.0000E 00
11	5.0000E-01	-5.6442E-02	9.9781E-60	1.0000E 01
12	5.0000E-01	-4.2293E-01	9.6610E-60	1.1000E 01
13	5.0000E-01	-6.7436E-01	1.1276E-59	1.2000E 01
14	5.0000E-01	-8.0489E-01	2.1165E-53	1.3000E 01
15	5.0000E-01	-8.4564E-01	4.9207E-53	1.4000E 01
16	5.0000E-01	-8.4779E-01	4.7094E-53	1.5000E 01
17	5.0000E-01	-8.4779E-01	7.8988E-53	1.6000E 01
18	5.0000E-01	-8.4766E-01	2.1004E-51	1.7000E 01
19	5.0000E-01	-7.3164E-01	1.9029E-51	1.8000E 01
20	5.0000E-01	-3.6763E-01	1.7677E-49	1.9000E 01
21	5.0000E-01	5.6440E-02	5.5571E-48	2.0000E 01
22	5.0000E-01	4.2293E-01	5.4361E-48	2.1000E 01
23	5.0000E-01	6.7435E-01	9.2953E-47	2.2000E 01
24	5.0000E-01	8.0489E-01	2.4689E-44	2.3000E 01
25	5.0000E-01	8.4564E-01	3.0350E-44	2.4000E 01
26	5.0000E-01	8.4779E-01	7.4855E-44	2.5000E 01
27	5.0000E-01	8.4779E-01	8.4076E-43	2.6000E 01
28	5.0000E-01	8.4766E-01	3.0560E-39	2.7000E 01
29	5.0000E-01	7.3164E-01	2.8071E-39	2.8000E 01
30	5.0000E-01	3.6763E-01	4.9601E-39	2.9000E 01
31	5.0000E-01	-5.6434E-02	1.4463E-37	3.0000E 01
32	5.0000E-01	-4.2292E-01	1.3276E-37	3.1000E 01
33	5.0000E-01	-6.7435E-01	2.2802E-37	3.2000E 01
34	5.0000E-01	-8.0489E-01	2.4228E-34	3.3000E 01
35	5.0000E-01	-8.4564E-01	4.1889E-34	3.4000E 01
36	5.0000E-01	-8.4779E-01	6.9145E-33	3.5000E 01
37	5.0000E-01	-8.4779E-01	9.3019E-33	3.6000E 01
38	5.0000E-01	-8.4766E-01	1.3021E-31	3.7000E 01
39	5.0000E-01	-7.3165E-01	1.7204E-31	3.8000E 01
40	5.0000E-01	-3.6764E-01	3.9598E-31	3.9000E 01
41	5.0000E-01	5.6428E-02	7.3733E-31	4.0000E 01
42	5.0000E-01	4.2292E-01	2.1844E-30	4.1000E 01
43	5.0000E-01	6.7435E-01	1.4091E-28	4.2000E 01
44	5.0000E-01	8.0489E-01	1.2590E-27	4.3000E 01
45	5.0000E-01	8.4564E-01	9.0004E-26	4.4000E 01
46	5.0000E-01	8.4779E-01	1.5284E-25	4.5000E 01
47	5.0000E-01	8.4779E-01	1.4626E-25	4.6000E 01
48	5.0000E-01	8.4766E-01	1.6964E-25	4.7000E 01
49	5.0000E-01	7.3165E-01	7.4962E-24	4.8000E 01
50	5.0000E-01	3.6765E-01	2.5550E-23	4.9000E 01

51	5,0000E-01	-5,6426E-02	2,3087E-23	5,0000E 01
52	5,0000E-01	-4,2292E-01	3,0271E-23	5,1000E 01
53	5,0000E-01	-6,7435E-01	1,3736E-21	5,2000E 01
54	5,0000E-01	-8,0489E-01	1,3802E-21	5,3000E 01
55	5,0000E-01	-8,4564E-01	2,2206E-21	5,4000E 01
56	5,0000E-01	-8,4779E-01	3,2233E-20	5,5000E 01
57	5,0000E-01	-8,4779E-01	2,9355E-20	5,6000E 01
58	5,0000E-01	-8,4766E-01	2,8609E-19	5,7000E 01
59	5,0000E-01	-7,3165E-01	3,9048E-19	5,8000E 01
60	5,0000E-01	-3,6765E-01	3,5708E-17	5,9000E 01
61	5,0000E-01	5,6425E-02	3,1433E-17	6,0000E 01
62	5,0000E-01	4,2292E-01	4,8248E-17	6,1000E 01
63	5,0000E-01	6,7435E-01	1,2917E-15	6,2000E 01
64	5,0000E-01	8,0489E-01	1,8404E-15	6,3000E 01
65	5,0000E-01	8,4564E-01	2,5979E-14	6,4000E 01
66	5,0000E-01	8,4779E-01	4,9111E-14	6,5000E 01
67	5,0000E-01	8,4779E-01	4,6223E-14	6,6000E 01
68	5,0000E-01	8,4766E-01	6,3013E-12	6,7000E 01
69	5,0000E-01	7,3165E-01	1,8472E-10	6,8000E 01
70	5,0000E-01	3,6765E-01	1,8461E-09	6,9000E 01
71	5,0000E-01	-5,6422E-02	2,4775E-09	7,0000E 01
72	5,0000E-01	-4,2291E-01	4,0637E-08	7,1000E 01
73	5,0000E-01	-6,7435E-01	5,4165E-06	7,2000E 01
74	5,0000E-01	-8,0489E-01	1,9602E-05	7,3000E 01
75	5,0001E-01	-8,4565E-01	2,5292E-05	7,4000E 01
76	5,0002E-01	-8,4779E-01	2,9222E-05	7,5000E 01
77	5,0005E-01	-8,4779E-01	4,4679E-05	7,6000E 01
78	5,0128E-01	-8,4766E-01	4,4081E-05	7,7000E 01
79	5,0350E-01	-7,3165E-01	7,6551E-05	7,8000E 01
80	7,1586E-01	-3,6764E-01	1,2116E-02	7,9000E 01
81	1,1073E 00	5,6438E-02	5,7537E-02	8,0000E 01
82	1,4899E 00	4,2293E-01	1,0610E-01	8,1000E 01
83	1,7675E 00	6,7436E-01	1,5442E-01	8,2000E 01
84	1,9236E 00	8,0489E-01	2,2332E-01	8,3000E 01
85	1,9867E 00	8,4564E-01	3,3219E-01	8,4000E 01
86	1,9993E 00	8,4779E-01	4,7651E-01	8,5000E 01
87	1,9994E 00	8,4779E-01	6,2666E-01	8,6000E 01
88	1,9994E 00	8,4766E-01	7,4670E-01	8,7000E 01
89	1,9994E 00	7,3164E-01	8,1625E-01	8,8000E 01
90	1,9994E 00	3,6763E-01	8,3975E-01	8,9000E 01
91	1,9994E 00	-5,6444E-02	8,4119E-01	9,0000E 01
92	1,9994E 00	-4,2293E-01	8,4115E-01	9,1000E 01
93	1,9994E 00	-6,7436E-01	8,4078E-01	9,2000E 01
94	1,9994E 00	-8,0489E-01	8,0960E-01	9,3000E 01
95	1,9994E 00	-8,4564E-01	7,2100E-01	9,4000E 01
96	1,9994E 00	-8,4779E-01	6,1923E-01	9,5000E 01
97	1,9993E 00	-8,4779E-01	5,0234E-01	9,6000E 01
98	1,9980E 00	-8,4766E-01	3,7799E-01	9,7000E 01
99	1,9959E 00	-7,3164E-01	2,5805E-01	9,8000E 01
100	1,7824E 00	-3,6763E-01	1,5607E-01	9,9000E 01
101	1,3910E 00	5,6443E-02	8,0748E-02	1,0000E 02

CONSERVATION SUMS

100 7,9984E 01 7,8753E-06 1,0475E 01

AFTER STEP NO. 800 DX = 1,000 AND DT = 0.20000

I	SO WAVE	SINUSOID	PEAK FUN.	POSITION
1	5.0000E-01	6.0168E-02	5.6765E-36	0.0000E 00
2	5.0000E-01	4.1335E-01	6.0710E-36	1.0000E 00
3	5.0000E-01	6.5258E-01	9.1109E-36	2.0000E 00
4	5.0000E-01	7.7537E-01	1.7490E-35	3.0000E 00
5	5.0000E-01	8.1308E-01	7.4875E-34	4.0000E 00
6	5.0000E-01	8.1493E-01	7.8591E-34	5.0000E 00
7	5.0000E-01	8.1493E-01	1.0858E-33	6.0000E 00
8	5.0000E-01	8.1474E-01	2.4775E-33	7.0000E 00
9	5.0000E-01	7.1150E-01	8.6281E-32	8.0000E 00
10	5.0000E-01	3.5450E-01	7.9481E-32	9.0000E 00
11	5.0000E-01	-6.0164E-02	1.2613E-31	1.0000E 01
12	5.0000E-01	-4.1335E-01	2.8552E-30	1.1000E 01
13	5.0000E-01	-6.5258E-01	9.1036E-30	1.2000E 01
14	5.0000E-01	-7.7537E-01	1.0113E-29	1.3000E 01
15	5.0000E-01	-8.1307E-01	8.1600E-29	1.4000E 01
16	5.0000E-01	-8.1493E-01	7.4378E-29	1.5000E 01
17	5.0000E-01	-8.1493E-01	1.2149E-28	1.6000E 01
18	5.0000E-01	-8.1474E-01	1.8615E-27	1.7000E 01
19	5.0000E-01	-7.1151E-01	1.2977E-26	1.8000E 01
20	5.0000E-01	-3.5451E-01	1.3819E-26	1.9000E 01
21	5.0000E-01	6.0157E-02	1.5641E-26	2.0000E 01
22	5.0000E-01	4.1335E-01	3.3337E-26	2.1000E 01
23	5.0000E-01	6.5257E-01	9.9104E-25	2.2000E 01
24	5.0000E-01	7.7537E-01	1.3060E-24	2.3000E 01
25	5.0000E-01	8.1307E-01	1.2861E-24	2.4000E 01
26	5.0000E-01	8.1493E-01	1.8782E-24	2.5000E 01
27	5.0000E-01	8.1493E-01	7.5962E-23	2.6000E 01
28	5.0000E-01	8.1474E-01	1.3254E-22	2.7000E 01
29	5.0000E-01	7.1151E-01	1.2586E-22	2.8000E 01
30	5.0000E-01	3.5452E-01	1.4625E-22	2.9000E 01
31	5.0000E-01	-6.0153E-02	4.5049E-21	3.0000E 01
32	5.0000E-01	-4.1334E-01	8.1900E-21	3.1000E 01
33	5.0000E-01	-6.5257E-01	9.2358E-21	3.2000E 01
34	5.0000E-01	-7.7537E-01	3.0317E-20	3.3000E 01
35	5.0000E-01	-8.1307E-01	1.8982E-19	3.4000E 01
36	5.0000E-01	-8.1493E-01	1.7903E-19	3.5000E 01
37	5.0000E-01	-8.1493E-01	2.8229E-19	3.6000E 01
38	5.0000E-01	-8.1474E-01	4.0682E-18	3.7000E 01
39	5.0000E-01	-7.1151E-01	8.8223E-18	3.8000E 01
40	5.0000E-01	-3.5452E-01	8.1726E-18	3.9000E 01
41	5.0000E-01	6.0154E-02	9.9531E-18	4.0000E 01
42	5.0000E-01	4.1334E-01	4.5993E-16	4.1000E 01
43	5.0000E-01	6.5257E-01	4.2804E-16	4.2000E 01
44	5.0000E-01	7.7537E-01	8.0792E-16	4.3000E 01
45	5.0000E-01	8.1307E-01	2.2600E-16	4.4000E 01
46	5.0000E-01	8.1493E-01	8.9121E-11	4.5000E 01
47	5.0000E-01	8.1493E-01	1.0346E-10	4.6000E 01
48	5.0000E-01	8.1474E-01	3.3974E-10	4.7000E 01
49	5.0000E-01	7.1151E-01	2.1590E-07	4.8000E 01
50	5.0000E-01	3.5452E-01	1.5470E-06	4.9000E 01

51	5.0000E-01	-4.7149E-02	2.6919E-06	5.0000E 01
52	5.0000E-01	-4.1334E-01	3.0099E-06	5.1000E 01
53	5.0000E-01	-4.5257E-01	2.9784E-06	5.2000E 01
54	5.0000E-01	-7.7537E-01	3.1664E-06	5.3000E 01
55	5.0000E-01	-8.1307E-01	3.8373E-06	5.4000E 01
56	5.0011E-01	-8.1493E-01	4.2831E-05	5.5000E 01
57	5.0010E-01	-8.1493E-01	1.5421E-04	5.6000E 01
58	5.0020E-01	-8.1474E-01	1.0229E-02	5.7000E 01
59	5.3634E-01	-7.1150E-01	2.3976E-02	5.8000E 01
60	7.5433E-01	-3.5451E-01	3.0028E-02	5.9000E 01
61	1.1351E 00	6.0167E-02	3.4362E-02	6.0000E 01
62	1.4682E 00	4.1335E-01	5.6465E-02	6.1000E 01
63	1.7224E 00	6.5258E-01	1.1750E-01	6.2000E 01
64	1.8831E 00	7.7538E-01	2.2445E-01	6.3000E 01
65	1.9648E 00	8.1308E-01	3.6422E-01	6.4000E 01
66	1.9944E 00	8.1494E-01	5.1752E-01	6.5000E 01
67	1.9987E 00	8.1494E-01	6.5092E-01	6.6000E 01
68	1.9987E 00	8.1474E-01	7.4767E-01	6.7000E 01
69	1.9987E 00	7.1149E-01	8.0267E-01	6.8000E 01
70	1.9987E 00	3.5449E-01	8.2353E-01	6.9000E 01
71	1.9987E 00	-6.0173E-02	8.2605E-01	7.0000E 01
72	1.9987E 00	-4.1336E-01	8.2605E-01	7.1000E 01
73	1.9987E 00	-6.5258E-01	8.2605E-01	7.2000E 01
74	1.9987E 00	-7.7538E-01	8.1352E-01	7.3000E 01
75	1.9987E 00	-8.1308E-01	7.4013E-01	7.4000E 01
76	1.9985E 00	-8.1494E-01	6.2753E-01	7.5000E 01
77	1.9985E 00	-8.1494E-01	4.9788E-01	7.6000E 01
78	1.9984E 00	-8.1474E-01	3.6759E-01	7.7000E 01
79	1.9616E 00	-7.1149E-01	2.4994E-01	7.8000E 01
80	1.7126E 00	-3.5450E-01	1.5406E-01	7.9000E 01
81	1.3616E 00	6.0170E-02	8.3978E-02	8.0000E 01
82	1.0287E 00	4.1335E-01	3.8729E-02	8.1000E 01
83	7.7502E-01	6.5258E-01	1.3784E-02	8.2000E 01
84	6.1484E-01	7.7538E-01	2.9691E-03	8.3000E 01
85	5.3349E-01	8.1308E-01	1.2730E-04	8.4000E 01
86	5.0422E-01	8.1494E-01	0.0000E 00	8.5000E 01
87	5.0000E-01	8.1494E-01	9.1799E-45	8.6000E 01
88	5.0000E-01	8.1474E-01	4.9408E-44	8.7000E 01
89	5.0000E-01	7.1149E-01	4.3059E-43	8.8000E 01
90	5.0000E-01	3.5450E-01	3.7811E-43	8.9000E 01
91	5.0000E-01	-6.0169E-02	5.4220E-43	9.0000E 01
92	5.0000E-01	-4.1335E-01	5.6249E-41	9.1000E 01
93	5.0000E-01	-6.5258E-01	1.8653E-40	9.2000E 01
94	5.0000E-01	-7.7537E-01	1.6854E-40	9.3000E 01
95	5.0000E-01	-8.1308E-01	2.2343E-40	9.4000E 01
96	5.0000E-01	-8.1493E-01	6.4535E-39	9.5000E 01
97	5.0000E-01	-8.1493E-01	2.3373E-38	9.6000E 01
98	5.0000E-01	-8.1474E-01	2.1060E-38	9.7000E 01
99	5.0000E-01	-7.1149E-01	4.1873E-38	9.8000E 01
100	5.0000E-01	-3.5450E-01	3.2323E-37	9.9000E 01
101	5.0000E-01	6.0168E-02	5.6765E-36	1.0000E 02

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100	7.9965E 01	1.2379E-04	1.0474E 01
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Appendix F AN EXPLODING DIAPHRAGM TEST PROBLEM WITH PERIODIC SYMMETRY USING PRBFCT

This test program performs a one-dimensional exploding diaphragm calculation in periodic geometry using PRBFCT. The system is 100 cells long and a symmetric initial condition is chosen which is equally well represented by a system 50 cells long with symmetry boundary conditions (Appendix D). The check provided by comparing these results (cells 32-81) with the results of Appendix D (cells 1-50) is a very stringent test in that any coding inconsistency would surface. The results after cycle 0, 150, and 300 are reproduced below. Note that the equally-spaced grid positions have been removed from the print-outs below.

```

C      PRBFCT PERIODIC DIAPHRAGM PROBLEM   DEC. 1975       SPL VERSION
C
C      THIS PROGRAM RUNS A PERIODIC DIAPHRAGM PROBLEM USING PRBFCT
C      FOR BOTH THE HALF AND THE WHOLE STEP. THE FLUID IS IDEAL AND
C      INVISCID WITH GAMMA = 5/3. FULLY VARIABLE CARTESIAN SPATIAL GRIDS
C      ARE ALLOWED AND LAGRANGIAN AS WELL AS EULERIAN MOTIONS ARE PER-
C      MITTED BY SETTING FLAG = 1.0.
C
C      REAL      ROLD(200), RNEW(200), WOLD(200), VNEW(200)
C      REAL      POLD(200), PNEW(200), EOLD(200), ENEW(200)
C      REAL      TOLD(200), PV(200),      ZERO(200), UNIT(200)
C      REAL      SCR1(200), SCR2(200), MADO(200), RADN(200)
C      REAL      RVOLD(200), RVNEW(200)
C
C      1000      FORMAT ('1 AFTER STEP NO. ', I5, ' DX = ', F7.3, ' AND DT = ',
C      1          F7.5, '/')
C      1001      FORMAT (2X, I3, 1P5E12.4, 12X, E12.4)
C      1002      FORMAT (' I DENSITY TEMPERATURE PRESSURE ',
C      1          ' VELOCITY ENERGY POSITION ', '/')
C      1003      FORMAT ('0 CONSERVATION SUMS ', //, 2X, I3, 1P5E12.4, 12X,
C      1          3E12.4)
C
C      * * * * *
C      THE INITIAL PARAMETERS ARE SET UP. CHANGE FOR DIFFERENT CASES.
C      * * * * *
C      GAMMA0 = 1.66667
C      GAMMAH = GAMMA0 - 1.0
C      TRIGHT = 1.0
C      RRIGHT = 1.0
C      TLEFT = 10.0
C      RLEFT = 1.0
C      PRIGHT = TRIGHT*RRIGHT

```

```

PLEFT = TLEFT*RLEFT
VRIGHT = 0.0
VLEFT = 0.0
RVRIGH = VRIGHT*RRIGHT
RVLEFT = VLEFT*RLEFT
ERIGHT = PRIGHT/(GAMMA0 - 1.0) + 0.5*RRIGHT*VRIGHT**2
ELEFT = PLEFT/(GAMMA0 - 1.0) + 0.5*RLEFT*VLEFT**2
C
C *****
C CONTROL PARAMETERS ARE INITIALIZED. CHANGE FOR DIFFERENT CASES.
C *****
MAXSTP = 301
IPRINT = 50
FLAG = 0.0
DELTAX = 1.0
DELTAT = .05
NX = 100
NXP = NX + 1
LPRINT = NXP
MX = 61
C
C *****
C THE PHYSICAL VARIABLES ARE INITIALIZED.
C *****
11 DO 11 I = 1, NXP
RADN(I) = FLOAT(I-1)*DELTAX
CALL NGRIDP (RADN, NXP)
DJ 13 I = 1, NXP
RNEW(I) = RRIGHT
VNEW(I) = VRIGHT
PNEW(I) = PRIGHT
RVNEW(I) = RVRIGH
ZERO(I) = 0.0
UNIT(I) = 1.0
13 ENEN(I) = ERIGHT
DO 14 I = 2, MX
KNEW(I) = RLEFT
VNEW(I) = VLEFT
PNEW(I) = PLEFT
RVNEW(I) = RVLEFT
14 ENEN(I) = ELEFT
C
C *****
C THE LOOP OVER TIMESTEPS.
C *****
DO 9999 ISTEP = 1, MAXSTP
C
C INITIALIZE THE QUANTITIES FOR THE CYCLE.
DO 20 I = 1, NXP
RAD0(I) = RADN(I)

```

```

      RJLD(I) = RNEW(I)
      RVOLD(I) = RVNEW(I)
      EOLD(I) = ENEW(I)
      VJLD(I) = RVOLD(I)/ROLD(I)
20    POLD(I) = GAMMA*(EJLD(I) - 0.5*RVOLD(I)*VOLD(I))
      DO 21 I = 1, NXP
      POLD(I) = AMAX1(POLD(I), 0.1*PRIGHT)
      TJLD(I) = POLD(I)/ROLD(I)
21    PV(I) = POLD(I)*VOLD(I)

C
C    THE RESULTS ARE PRINTED.
      IF (MOD(ISTEP-1,IPRINT) .NE. 0) GO TO 22
      JSTEP = ISTEP - 1
      WRITE (6, 1000) JSTEP, DELTAX, DELTAT
      WRITE (6, 1002)
      WRITE (6, 1001) (I, ROLD(I), TJLD(I), POLD(I), VOLD(I),
1      EOLD(I), RADU(I), I=1,LPRINT)
      CALL CONSRP (ROLD, NXP, RHOSUM)
      CALL CONSRP (RVOLD, NXP, RHVSUM)
      CALL CONSRP (POLD, NXP, PRESUM)
      CALL CONSRP (EOLD, NXP, ERGSUM)
      PRESUM = PRESUM/GAMMA
      WRITE (6, 1003) NX, RHOSUM, PRESUM, RHVSUM, ERGSUM
22    CONTINUE

C
C    RESET GRID AND VELOCITIES FOR THE HALF STEP.
      CALL OGRIDP (NXP)
      DTGRID = 0.5*DELTAT*FLAG
      DO 30 I = 1, NXP
30    RADN(I) = RADJ(I) + DTGRID*VOLD(I)
      CALL NGRIDP (RADN, NXP)
      CALL VELOC (VOLD, NXP, 0.5*DELTAT)

C
C    ALL QUANTITIES ARE INTEGRATED FORWARD HALF A STEP USING THE OLD
C    VALUES ON THE RIGHT HAND SIDE FOR FORMING DERIVATIVES.
      DO 31 I = 1, NXP
31    SCR1(I) = -POLD(I)
      SCR2(I) = -PV(I)
      CALL PR3FCT (ROLD, RNEW, NXP)
      CALL SOURCP (NXP, 0.5*DELTAT, 2, UNIT, SCR1)
      CALL PR3FCT (RVOLD, RVNEW, NXP)
      CALL SOURCP (NXP, 0.5*DELTAT, 1, ZERO, SCR2)
      CALL PR3FCT (EJLD, ENEW, NXP)

C
C    THE INTERMEDIATE QUANTITIES ARE CALCULATED.
      DO 32 I = 1, NXP
32    VNEW(I) = RVNEW(I)/RNEW(I)
      PNEW(I) = GAMMA*(ENEW(I) - 0.5*RVNEW(I)*VNEW(I))
      DO 33 I = 1, NXP
      PNEW(I) = AMAX1(PNEW(I), 0.1*PRIGHT)

```

```

33      PV(I) = PNEW(I)*VNEW(I)
C
C      RESET GRID AND VELOCITIES FOR THE WHOLE STEP.
      DTGRID = DELTAT*FLAG
      DO 34 I = 1, NXP
34      RADN(I) = RADJ(I) + DTGRID*VNEW(I)
      CALL NGRIDP (RADN, NXP)
      CALL VELOC (VNEW, NXP, DELTAT)
C
C      THE INTEGRATION STEP IS PERFORMED WITH THE COMPLETE TIMESTEP.
      DO 35 I = 1, NXP
35      SCR1(I) = -PNEW(I)
      SCR2(I) = -PV(I)
      CALL PRBFCT (ROLD, RNEW, NXP)
      CALL SOURCP (VXP, DELTAT, 2, UNIT, SCR1)
      CALL PRBFCT (RVOLD, RVNEW, NXP)
      CALL SOURCP (NXP, DELTAT, 1, ZERO, SCR2)
      CALL PRBFCT (EOLD, ENEW, NXP)
C
9999    CONTINUE
C
      STOP
      END

```

0 0X 3

1.000 AND DT = 0.05000

[illegible]

CONSERVATION SUMS

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AFTER STEP NO. 150 UX = 1.000 AND DT = 0.05000

I	DENSITY	TEMPERATURE	PRESSURE	VELOCITY	ENERGY
1	6.6944E-01	7.6006E 00	5.0882E 00	-1.5470E 00	8.4332E 00
2	6.6939E-01	7.6299E 00	5.1074E 00	-1.5462E 00	8.4612E 00
3	6.7019E-01	7.6333E 00	5.1154E 00	-1.5336E 00	8.4617E 00
4	6.7036E-01	7.6527E 00	5.1301E 00	-1.5333E 00	8.4831E 00
5	6.7028E-01	7.6596E 00	5.1341E 00	-1.5276E 00	8.4831E 00
6	6.6803E-01	7.6793E 00	5.1300E 00	-1.5327E 00	8.4796E 00
7	6.6823E-01	7.6434E 00	5.1075E 00	-1.5350E 00	8.4484E 00
8	6.6610E-01	7.6372E 00	5.0871E 00	-1.5533E 00	8.4342E 00
9	6.6278E-01	7.6122E 00	5.0452E 00	-1.5607E 00	8.3750E 00
10	6.6278E-01	7.6030E 00	5.0391E 00	-1.5695E 00	8.3750E 00
11	6.6278E-01	7.6009E 00	5.0377E 00	-1.5715E 00	8.3750E 00
12	6.6355E-01	7.6143E 00	5.0525E 00	-1.5697E 00	8.3962E 00
13	6.6418E-01	7.6219E 00	5.0616E 00	-1.5543E 00	8.3946E 00
14	6.6652E-01	7.6280E 00	5.0842E 00	-1.5347E 00	8.4113E 00
15	6.7573E-01	7.7078E 00	5.2084E 00	-1.5153E 00	8.5884E 00
16	6.8732E-01	7.7819E 00	5.3487E 00	-1.4674E 00	8.7631E 00
17	6.9311E-01	7.8587E 00	5.4470E 00	-1.3499E 00	8.8020E 00
18	7.1493E-01	7.9769E 00	5.7030E 00	-1.3008E 00	9.1593E 00
19	7.3709E-01	8.1777E 00	6.0277E 00	-1.2375E 00	9.6059E 00
20	7.4510E-01	8.2148E 00	6.1208E 00	-1.1015E 00	9.6331E 00
21	7.6356E-01	8.3588E 00	6.3825E 00	-1.0305E 00	9.9791E 00
22	7.8987E-01	8.5450E 00	6.7495E 00	-9.7741E-01	1.0501E 01
23	7.9929E-01	8.6139E 00	6.8850E 00	-8.6309E-01	1.0625E 01
24	8.1929E-01	8.7583E 00	7.1755E 00	-7.5942E-01	1.1000E 01
25	8.4270E-01	8.9192E 00	7.5163E 00	-7.1894E-01	1.1492E 01
26	8.6057E-01	9.0531E 00	7.7908E 00	-5.8365E-01	1.1833E 01
27	8.7507E-01	9.1421E 00	8.0000E 00	-5.1264E-01	1.2115E 01
28	8.9935E-01	9.3260E 00	8.3873E 00	-4.0537E-01	1.2655E 01
29	9.1067E-01	9.3872E 00	8.5487E 00	-3.1171E-01	1.2867E 01
30	9.2230E-01	9.4741E 00	8.7380E 00	-1.7351E-01	1.3121E 01
31	9.2838E-01	9.5234E 00	8.8413E 00	-3.0519E-02	1.3262E 01
32	9.2838E-01	9.5234E 00	8.8413E 00	3.0516E-02	1.3262E 01
33	9.2230E-01	9.4741E 00	8.7380E 00	1.7351E-01	1.3121E 01
34	9.1067E-01	9.3872E 00	8.5487E 00	3.1172E-01	1.2867E 01
35	8.9935E-01	9.3259E 00	8.3873E 00	4.0537E-01	1.2655E 01
36	8.7507E-01	9.1421E 00	8.0000E 00	5.1264E-01	1.2115E 01
37	8.6057E-01	9.0531E 00	7.7908E 00	5.8365E-01	1.1833E 01
38	8.4270E-01	8.9193E 00	7.5163E 00	7.1894E-01	1.1492E 01
39	8.1929E-01	8.7583E 00	7.1756E 00	7.5942E-01	1.1000E 01
40	7.9929E-01	8.6140E 00	6.8850E 00	8.6308E-01	1.0625E 01
41	7.8987E-01	8.5450E 00	6.7495E 00	9.7740E-01	1.0501E 01
42	7.6356E-01	8.3588E 00	6.3825E 00	1.0305E 00	9.9791E 00
43	7.4510E-01	8.2148E 00	6.1208E 00	1.1015E 00	9.6331E 00
44	7.3709E-01	8.1777E 00	6.0277E 00	1.2375E 00	9.6059E 00
45	7.1493E-01	7.9769E 00	5.7030E 00	1.3008E 00	9.1593E 00
46	6.9311E-01	7.8587E 00	5.4469E 00	1.3499E 00	8.8019E 00
47	6.8732E-01	7.7819E 00	5.3486E 00	1.4675E 00	8.7630E 00
48	6.7572E-01	7.7078E 00	5.2083E 00	1.5154E 00	8.5882E 00
49	6.6652E-01	7.6270E 00	5.0841E 00	1.5348E 00	8.4111E 00
50	6.6417E-01	7.6208E 00	5.0615E 00	1.5543E 00	8.3945E 00

51	6.6354E-01	7.6142E	00	5.0524E	00	1.5698E	00	8.3961E	00
52	6.6278E-01	7.6008E	00	5.0377E	00	1.5716E	00	8.3749E	00
53	6.6278E-01	7.6029E	00	5.0390E	00	1.5696E	00	8.3749E	00
54	6.6278E-01	7.6121E	00	5.0451E	00	1.5608E	00	8.3749E	00
55	6.6609E-01	7.6371E	00	5.0870E	00	1.5534E	00	8.4341E	00
56	6.6822E-01	7.6433E	00	5.1074E	00	1.5350E	00	8.4483E	00
57	6.6802E-01	7.6792E	00	5.1299E	00	1.5328E	00	8.4795E	00
58	6.7028E-01	7.6595E	00	5.1340E	00	1.5276E	00	8.4831E	00
59	6.7036E-01	7.6527E	00	5.1300E	00	1.5334E	00	8.4831E	00
60	6.7018E-01	7.6333E	00	5.1157E	00	1.5336E	00	8.4617E	00
61	6.6939E-01	7.6298E	00	5.1073E	00	1.5462E	00	8.4612E	00
62	6.6944E-01	7.6005E	00	5.0881E	00	1.5470E	00	8.4332E	00
63	6.6927E-01	7.6091E	00	5.0850E	00	1.5490E	00	8.4291E	00
64	6.6444E-01	7.6633E	00	5.0918E	00	1.5411E	00	8.4266E	00
65	6.5489E-01	7.7781E	00	5.0938E	00	1.5437E	00	8.4210E	00
66	6.5250E-01	7.8118E	00	5.0972E	00	1.5445E	00	8.4240E	00
67	6.5245E-01	7.8171E	00	5.1003E	00	1.5430E	00	8.4270E	00
68	6.5228E-01	7.8208E	00	5.1007E	00	1.5425E	00	8.4270E	00
69	6.5217E-01	7.8206E	00	5.1004E	00	1.5425E	00	8.4264E	00
70	6.5217E-01	7.8206E	00	5.1004E	00	1.5425E	00	8.4264E	00
71	6.8508E-01	7.4281E	00	5.0948E	00	1.5726E	00	8.4903E	00
72	1.0207E	4.9818E	00	5.0850E	00	1.5306E	00	8.8230E	00
73	1.4454E	3.4907E	00	5.0455E	00	1.5335E	00	9.2678E	00
74	1.8083E	2.8531E	00	5.1592E	00	1.5593E	00	9.9370E	00
75	2.0224E	2.5358E	00	5.1284E	00	1.5421E	00	1.0097E	01
76	2.1594E	2.3527E	00	5.0903E	00	1.5296E	00	1.0147E	01
77	2.2670E	2.2209E	00	5.0347E	00	1.5484E	00	1.0270E	01
78	2.2828E	2.1220E	00	4.8442E	00	1.6268E	00	1.0287E	01
79	2.2921E	2.1147E	00	4.8472E	00	1.6544E	00	1.0407E	01
80	2.7292E	1.9862E	00	5.4206E	00	1.3895E	00	1.0765E	01
81	3.0356E	2.3527E	00	7.1417E	00	9.4510E-01	-01	1.2068E	01
82	3.0356E	2.3527E	00	7.1418E	00	-9.4508E-01	-01	1.2068E	01
83	2.7291E	1.9862E	00	5.4206E	00	-1.3894E	00	1.0765E	01
84	2.2921E	2.1147E	00	4.8472E	00	-1.6543E	00	1.0407E	01
85	2.2828E	2.1220E	00	4.8441E	00	-1.6268E	00	1.0287E	01
86	2.2670E	2.2209E	00	5.0347E	00	-1.5484E	00	1.0270E	01
87	2.1594E	2.3527E	00	5.0903E	00	-1.5296E	00	1.0147E	01
88	2.0224E	2.5358E	00	5.1283E	00	-1.5421E	00	1.0097E	01
89	1.8083E	2.8531E	00	5.1592E	00	-1.5593E	00	9.9369E	00
90	1.4454E	3.4907E	00	5.0455E	00	-1.5335E	00	9.2678E	00
91	1.0207E	4.9817E	00	5.0849E	00	-1.5306E	00	8.8228E	00
92	6.8507E-01	7.4280E	00	5.0947E	00	-1.5726E	00	8.4902E	00
93	6.5217E-01	7.8206E	00	5.1004E	00	-1.5424E	00	8.4263E	00
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95	6.5220E-01	7.8208E	00	5.1007E	00	-1.5425E	00	8.4269E	00
96	6.5244E-01	7.8172E	00	5.1002E	00	-1.5430E	00	8.4270E	00
97	6.5250E-01	7.8118E	00	5.0972E	00	-1.5445E	00	8.4240E	00
98	6.5489E-01	7.7780E	00	5.0938E	00	-1.5437E	00	8.4210E	00
99	6.6443E-01	7.6634E	00	5.0918E	00	-1.5411E	00	8.4266E	00
100	6.6824E-01	7.6093E	00	5.0850E	00	-1.5490E	00	8.4291E	00
101	6.6944E-01	7.6006E	00	5.0882E	00	-1.5470E	00	8.4332E	00

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100 9.9993E 01

8.6622E 02 7.7350E-04 9.5996E 02

AFTER STEP NO. 300 DX = 1.000 AND DT = 0.05000

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2	1.0217E 00	1.0481E 01	1.0700E 01	9.3296E-01	1.6507E 01
3	1.0217E 00	1.0020E 01	1.0245E 01	9.3296E-01	1.5812E 01
4	7.9633E-01	8.3906E 00	6.6817E 00	2.6766E-01	1.0051E 01
5	5.8710E-01	6.9527E 00	4.0825E 00	-9.1783E-01	6.3710E 00
6	5.6114E-01	6.8223E 00	3.8283E 00	-9.9985E-01	6.0227E 00
7	5.5183E-01	6.7024E 00	3.6986E 00	-9.8973E-01	5.8181E 00
8	5.4765E-01	6.6989E 00	3.6687E 00	-8.9280E-01	5.7213E 00
9	5.4523E-01	6.6752E 00	3.6395E 00	-8.4338E-01	5.6531E 00
10	5.3959E-01	6.6277E 00	3.5763E 00	-8.0831E-01	5.5807E 00
11	5.3110E-01	6.5575E 00	3.4827E 00	-7.5790E-01	5.3765E 00
12	5.2237E-01	6.4919E 00	3.3912E 00	-7.0557E-01	5.2160E 00
13	5.1597E-01	6.4464E 00	3.3261E 00	-6.6799E-01	5.1083E 00
14	5.1035E-01	6.4016E 00	3.2671E 00	-6.3389E-01	5.0030E 00
15	5.0447E-01	6.3527E 00	3.2048E 00	-6.0060E-01	4.8981E 00
16	4.9764E-01	6.2986E 00	3.1324E 00	-5.6870E-01	4.7735E 00
17	4.9092E-01	6.2263E 00	3.0566E 00	-5.0011E-01	4.6463E 00
18	4.8563E-01	6.1687E 00	2.9937E 00	-4.6322E-01	4.5427E 00
19	4.8154E-01	6.1214E 00	2.9477E 00	-4.2184E-01	4.4644E 00
20	4.7517E-01	6.0823E 00	2.8901E 00	-3.7926E-01	4.3694E 00
21	4.6774E-01	6.0257E 00	2.8185E 00	-3.3125E-01	4.2533E 00
22	4.6249E-01	5.9671E 00	2.7597E 00	-2.8963E-01	4.1589E 00
23	4.5737E-01	5.9320E 00	2.7131E 00	-2.5806E-01	4.0848E 00
24	4.4936E-01	5.8807E 00	2.6425E 00	-2.2209E-01	3.9749E 00
25	4.4227E-01	5.8042E 00	2.5670E 00	-1.7717E-01	3.8575E 00
26	4.3592E-01	5.7477E 00	2.5056E 00	-1.5710E-01	3.7637E 00
27	4.3204E-01	5.7152E 00	2.4692E 00	-1.2765E-01	3.7073E 00
28	4.2603E-01	5.6550E 00	2.4092E 00	-9.1901E-02	3.6158E 00
29	4.2173E-01	5.6279E 00	2.3735E 00	-5.4264E-02	3.5608E 00
30	4.1869E-01	5.6038E 00	2.3463E 00	-3.8432E-02	3.5197E 00
31	4.1695E-01	5.5885E 00	2.3301E 00	-3.3371E-02	3.4994E 00
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38	4.4227E-01	5.8041E 00	2.5670E 00	1.7724E-01	3.8574E 00
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44	4.8153E-01	6.1213E 00	2.9476E 00	4.2186E-01	4.4642E 00
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