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SUMMARY

The compounding technique of obtaining approximate solutions for stress intensity factors is applied to periodically stiffened sheets containing cracks. The basic compounding method requires a simple modification if the stiffener actually crosses the crack. Probable errors in compounded results are shown to be small (a few per cent) by comparing some compounded results with known results. The stress intensity factor is calculated for a crack located asymmetrically between stiffeners.

Departmental Reference: Mat 250

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* Lecturer, University of Southampton.

** Vacation student, University of Leeds.

CONTENTS

1 INTRODUCTION

Many structural components of aircraft contain cracks which may have existed at manufacture or may have been initiated subsequently. Under fluctuating service loads, these cracks can grow by fatigue. It thus becomes necessary to know whether or not a cracked structure can support the service loads, i.e. is it safe to operate, and, if it is safe, by how much does the crack need to grow before it becomes unsafe. Both the residual strength of a cracked structure and the rate of crack-growth are controlled by the stress intensity factor K - a parameter which characterizes the stress-field at the tip of a crack. It is important therefore to be able to calculate this parameter for cracks in real structures. But these often are complex geometrical configurations for which it may be, and usually is, difficult and costly to obtain stress intensity factors of sufficient accuracy; it is only for relatively simple geometries that solutions for K are generally available.

Recently a new method has been developed[1,2] for determining approximate stress intensity factors for complex configurations with many boundaries by compounding the known results of several simpler configurations. This Report describes how the compounding method can be extended to obtain stress intensity factors for cracks in plane sheets having localized line stiffeners (see Fig.1a) - a geometrical configuration which models the locally stiffened metal-sheet construction widely used in many aircraft structural components (see Fig.1b). There are two cases to consider; if the component is made by integral machining or by bonding of the stiffeners to the sheet then the stiffeners are attached continuously along the sheet; or, if localized fasteners, e.g. rivets (see Figs.1a and 1b) are used then the attachment is at discrete points. In both cases, known solutions for cracks in sheets with a simple stiffener are compounded to give approximate stress intensity factors for cracks in sheets with many stiffeners.

If the crack runs beyond a stiffener then, whether the stiffener is broken or not, the presence of the stiffener changes the shape of the crack and it is necessary, see section 2, to modify the basic compounding method[1,2]. The accuracy of the method is assessed by comparing the solutions for some configurations having known results, see section 3.1, and the errors are shown to be small (a few per cent). The method is used to obtain a new solution for the stress intensity factor for a crack located asymmetrically between two stiffeners in a periodic set (see section 3.2).

The potential of the compounding method for simplifying the analysis of the effects of many practical features of stiffener design and engineering is discussed in section 5.

2 MODIFIED THEORY OF COMPOUNDING

In order to extend compounding methods to obtain stress intensity factors for cracks in sheets with multiple stiffeners, it is necessary to modify the basic method[1,2] since stiffeners, regarded as boundaries, are different from the boundaries previously considered[1,2]. In developing the compounding method[1,2] each additional boundary introduced was subjected to forces (usually of zero magnitude) which were independent of the crack shape or size. However the forces acting on stiffeners exist because of the mismatch in displacements between the sheet and the stiffener; this mismatch is due entirely to the presence of the crack in the sheet and is a function of the shape and size of the crack. Thus the effect, on the crack, of introducing an additional stiffener is modified by the stiffeners already present with the result that boundary interactions are not necessarily negligible when the boundaries are stiffeners. This interaction is small if the crack is between stiffeners and may be ignored, but it must be taken into account if a stiffener crosses the crack since the shape of the crack is then much altered.

The modification is illustrated (see Fig.2) by considering a sheet with a periodic array of stiffeners spaced a distance b apart with a crack of length $2a$ $(a < b)$ which is perpendicular to and centred about one of the stiffeners. A uniaxial tensile stress σ is applied to the sheet remote from and perpendicular to the crack. In order to maintain strain compatibility remote from the crack a stress of $(E_2/E_1)\sigma$ is applied to each stiffener; E_1 and E_2 are the Young's moduli of the sheet and stiffener respectively. It is convenient to label the stiffeners S_n with n positive integers to the right of the crack and negative integers to the left. If, with a single stiffener S_0 which crosses the crack, the stress intensity factor is K_0 then, the displacement of the crack near the tip is proportional to K_0. The shape of the tip of this crack is thus representative of any crack with a stress intensity factor K_0. A crack of length $2a'$ in an unstiffened sheet has a stress intensity factor K_0 providing a' is given by

$$\sigma\sqrt{\pi a'} = K_0 \quad . \tag{1}$$

K_0 may be written in terms of the normalized stress intensity factor as

$$K_0 = Q_0\bar{K} \qquad (2)$$

where $\bar{K}$ is the stress intensity factor in the absence of all boundaries (stiffeners), i.e.

$$\bar{K} = \sigma\sqrt{\pi a} \quad . \qquad (3)$$

Equations (1), (2) and (3) combine to give

$$a' = Q_0^2 a \quad . \qquad (4)$$

The effect of adding other stiffeners to the sheet containing the crack 2a is assumed to be the same as the effect of adding stiffeners to an infinite sheet containing a crack of length 2a'. The distance from the near tip of the crack to the additional stiffener is kept the same, i.e.

$$b' - a' = b - a \quad , \qquad (5)$$

where b' is the distance from the centre of the crack of length 2a' to the stiffener.

The basic compounding equation for calculating resultant stress intensity factors K_r, neglecting boundary interactions, is given by[1,2]

$$K_r = \bar{K} + \sum_{\text{all } n} (K_n - \bar{K}) \quad , \qquad (6)$$

where K_n is the stress intensity factor for a crack in the presence of the boundary B_n. In the present case the crack of length 2a which is crossed by a central stiffener, is represented by another crack of length 2a' with a stress intensity factor K_0 in the absence of all other stiffeners. The compounding equation thus becomes

$$K_r = K_0 + \sum_{n=-\infty}^{\infty} (K_n' - K_0) \quad , \qquad n \neq 0, \quad (7)$$

where K'_n is the effect of the stiffener S_n only, acting on the crack of length $2a'$. In terms of normalized stress intensity factors Q'_n, K'_n becomes

$$K'_n = Q'_n K_0 = Q'_n Q_0 \bar{K} \quad . \tag{8}$$

From equations (7) and (8) the following equation for Q_r $(= K_r/\bar{K})$ is obtained

$$Q_r = Q_0 \left[1 + \sum_{n=-\infty}^{n=\infty} (Q'_n - 1) \right] \quad , \qquad n \neq 0. \tag{9}$$

In equation (9) Q_r and Q_0 relate to the original crack of length $2a$, whereas Q'_n relates to the replacement crack of length $2a'$. The procedure to obtain Q_r is therefore first to obtain Q_0 and hence a', from equation (4), and then Q'_n for a crack of length $2a'$ in the presence of each one of the additional stiffeners S_n.

3 APPLICATION TO STIFFENED SHEETS

Poe[3,4] has obtained the stress intensity factors for cracks located in sheets with periodic arrays of riveted stiffeners; the rivets are spaced a distance h apart along the stiffeners. The crackline is always perpendicular to the stiffeners and passes through a rivet site at each stiffener. The cracks are located either symmetrically between two stiffeners (unbroken[3]) or centred about a single stiffener (unbroken[3] or broken[4]); the configurations are shown in Figs.2a and 2b where the stiffeners are labelled with positive integers to the right and negative integers to the left of the crack. The ancillary configurations required are shown in Figs.3a and 3b; the stress intensity factors for these configurations have been obtained by Bloom and Sanders[5]. For the crack located between two stiffeners the solution to ancillary configuration 3a only is required. A sample calculation is shown in detail in the Appendix.

3.1 Test solutions

The first test configuration considered is that of a crack of length $2a$ located symmetrically between two of the stiffeners a distance b apart. A tensile stress σ is applied to the sheet, remote from and perpendicular to the crack, and a stress of $(E_2/E_1)\sigma$ is applied to the stiffeners in order to maintain strain compatibility. Because of symmetry only one tip of the crack (e.g. the right hand tip in Fig.2a) need be considered. If the normalized stress intensity factor due to the nth stiffener is Q_n then the unmodified compounding formula (equation (6)) becomes

$$Q_r = 1 + \sum_{n=-\infty}^{n=\infty} (Q_n - 1) \quad , \qquad n \neq 0, \quad (10)$$

where Q_r ($= K_r/\bar{K}$) is the normalized resultant stress intensity factor for the complete configuration. The normalizing factor $\bar{K}$ is the stress intensity factor in the absence of all boundaries, and is given by equation (3).

The major contribution to Q_r comes from the nearest pair of stiffeners ($n = \pm 1$); the contribution from the next pair of stiffeners ($n = \pm 2$) is negligible over most of the range of a/b, the maximum being <1% at $a/b = 0.45$. Contributions from all other stiffeners ($|n| > 2$) have been ignored. The compounded results for this configuration are obtained from the results of ancillary configurations given by Bloom and Sanders[5]. The results of Poe[3], for this configuration, can be obtained as functions of a/b for various h/b values and various values of a stiffness parameter μ. The stiffness parameter s, used in this Report, is the ratio of the stiffnesses of the stiffener and the sheet, i.e.

$$s = \frac{AE_2}{btE_1} \qquad (11)$$

where A is the cross sectional area of the stiffener and t the thickness of the sheet. The parameters μ and s are related by

$$\mu = \frac{1}{1 + s} \quad . \qquad (12)$$

The results of Bloom and Sanders for the ancillary configurations can be obtained as functions of a/b for various values of h/a and the parameter λ. The parameter λ is defined as

$$\lambda = \frac{2E_1at}{AE_2} \qquad (13)$$

which can be written in terms of s as

$$\lambda = \frac{2}{s}\left(\frac{a}{b}\right) . \tag{14a}$$

In the modified version a' replaces a and λ' replaces λ for the contribution from the stiffeners which do not cross the cracks; λ' is given by

$$\lambda' = \frac{2}{s}\left(\frac{a'}{b}\right) = Q_0^2\lambda . \tag{14b}$$

Bloom and Sanders[5] show that, for the values of λ and a/h over the range of a/b considered here, the results for $Q_n (n = \pm 1, \pm 2)$ are indistinguishable from those for continuously attached stiffeners ($a/h = \infty$) obtained by Greif and Sanders[6]. Since more data are available on continuously attached stiffeners, curves of Q (near tip) and Q (far tip) were obtained from the work of Greif and Sanders[6]; these are shown as functions of a/d for various values of λ in Figs.4 and 5. d is the distance from the centre of the crack to the stiffener.

The compounded results for the opening mode stress intensity factor ($Q_r = K_I/(\sigma\sqrt{\pi a})$) are compared, in Table 1, with those of Poe[3] for the same configuration for $0.0 \leqslant a/b \leqslant 0.45$ for $s = 1.0$ and $h/b = 1/12$. The differences are very small ($\leqslant 1\%$) and are no greater than the possible inaccuracies in reading the graphical results. The differences will be even less for smaller values of s and larger values of h/b since the effect of the stiffener decreases in both cases. The value of $s = 1.0$ was chosen since it is about the maximum value of s in aircraft applications.

Table 1

Comparison of values of $K_I/(\sigma\sqrt{\pi a})$ for a crack located symmetrically between two stiffeners in a periodically stiffened sheet ($s = 1.0$, $h/b = 1/12$)

a/b	Compounded results	Ref.14
0.00	1.00	1.00
0.10	0.98	0.99
0.20	0.96	0.96
0.30	0.92	0.92
0.40	0.82	0.81
0.45	0.72	0.72
0.50	stiffener site	

The next test configuration considered is that of a crack of length $2a$ located symmetrically about one of the stiffeners in a periodic array spaced a distance b apart (see Fig.2b). The stiffener across the crack can be either unbroken[3] or broken[4]. The resultant stress intensity factor normalized with respect to $\bar{K}$ is obtained from the modified compounding formula (equation (9)), which is

$$Q_r = Q_0\left[1 + \sum_{n=-\infty}^{n=\infty} (Q'_n - 1)\right] , \qquad n \neq 0. \tag{15}$$

The summation term contains the effects of stiffeners not across the crack which are spaced a distance b apart, except for the two nearest the crack which are a distance $2b'$ apart. The term in square brackets has the same form as equation (10) and it is convenient to write equation (15) as

$$Q_r = Q_0 Q'_r \tag{16}$$

Q_0 for the unbroken stiffener, obtained from the work of Bloom and Sanders[5], is shown plotted as a function of h/a for various λ in Fig.6. Q'_r is obtained from Figs.4 and 5 in the same way as was Q_r in equation (10). Q_0 for a broken stiffener also obtained from the work of Bloom and Sanders[5], is shown plotted as a function of $1/\lambda$ for various values of h/a in Fig.7. Again Q'_r is obtained from Figs.4 and 5. The results for the opening mode stress intensity factor for $s = 1.0$ for both the unbroken central stiffener ($h/b = 1/12$) and the broken central stiffener ($h/b = 1/6$) are shown in Table 2. The maximum difference between the compounded solutions and the numerical solutions due to Poe[2,3] is ~5% for $0.25 \leq a/b \leq 0.90$.

Table 2

Comparison of values of $K_I/(\sigma\sqrt{\pi a})$ for a crack located symmetrically about the central stiffener in a periodically stiffened sheet ($s = 1.0$)

a/b	Central stiffener unbroken (h/b = 1/12)		Central stiffener broken (h/b = 1/6)	
	Compounded results	Ref.14	Compounded results	Ref.15
0.25	0.67	0.68	1.72	1.78
0.50	0.66	0.67	1.32	1.36
0.75	0.64	0.65	1.11	1.12
0.90	0.56	0.59	0.91	0.91
1.00	next stiffener			

The above configurations have also been studied for the case when the stiffeners are continuously attached to the sheet. The results are indistinguishable within the approximations used from those for riveted stiffeners except when the crack is very close to a stiffener, i.e. the distance between the crack tip and the stiffener is $\leqslant h$. Too few results are available for the ancillary configurations when the crack tip is close to the stiffener to enable this region to be investigated fully.

3.2 New solution

In this section the solution is derived for a crack which is located asymmetrically between two continuously attached stiffeners in a periodically stiffened sheet. A stress σ is applied to the sheet remote from and perpendicular to the crack; in order to maintain strain compatibility a stress $(E_2/E_1)\sigma$ is applied to the stiffener remote from the crack. The stiffeners to the right of the crack are labelled with positive integers (+n) and those to the left with negative integers (−n); the distance from the centre of the crack to the nth stiffener to the right is b_{+n} and the distance to the nth stiffener to the left is b_{-n}. This configuration is shown in Fig.8a; the required ancillary configuration is shown in Fig.8b. The distance d from the centre of the crack to the stiffener in the ancillary configuration is b_{+n} for stiffeners on the right and b_{-n} for stiffeners on the left of the crack. If the stiffeners are a distance b apart then

$$b_{+n} + b_{-n} = (2n - 1)b \quad ; \quad n = 1,2, \ldots, \infty \qquad (17)$$

The resultant normalized stress intensity factor is given by equation (10). Because of the asymmetry the two tips will have different stress intensity factors. The contributions from each stiffener (Q_{+n} or Q_{-n}) for the tip under consideration can be obtained from the curves in Figs.4 and 5. The results, for $s = 1.0$, for both tips are shown in Fig.9; $K_I/(\sigma\sqrt{\pi a})$ is plotted as a function of a/b for various values of b_{+1}/b.

4 DISCUSSION

The compounding method has been applied to crack problems in periodically stiffened sheets; a modification to the basic method is necessary if a stiffener crosses the crack. The errors in the approximate stress intensity factors increase as the crack-length increases and as the relative stiffness of the stiffener to that of the sheet increases. The maximum error for a wide range of crack-lengths and stiffness ratios is a few per cent (Tables 1 and 2) which is within normal engineering tolerances. The compounding method can be extended readily to other stiffened configurations, but direct application to some other stiffener configurations may be limited by the lack of data for the required ancillary configurations. For example data are required on small cracks near stiffeners (the difference between continuously attached and riveted stiffeners would be important) and on cracks which are located asymmetrically behind a single stiffener.

An important consequence of the use of compounding is that it is now necessary to have data for simple ancillary configurations only; curves of $K_I/\bar{K}$ for many such configurations are contained in the Compendium of Stress Intensity Factors[7]. The importance of design parameters such as distribution of stiffeners, relative stiffnesses, type of attachment, flexibility of rivets and sheet curvature can be studied using a simple structure with a single stiffener. Results for a structure with multiple stiffeners can then be compounded from those for the simple structure. The method of compounding can be applied to problems with both plane boundaries and stiffeners, e.g. a crack in the vicinity of a cut-out in a multiple-stiffened sheet.

5 CONCLUSIONS

(1) The compounding method developed for plane problems can be modified to obtain approximate stress intensity factors for cracks in stiffened sheets.

(2) The errors are within engineering tolerances for many applications.

(3) The effect of many practical features of stiffness design and engineering, such as fastener flexibility, can now be studied in simple configurations and the results extended to complex structures by using the compounding method.

Appendix

SAMPLE CALCULATION
(see section 3.1)

This Appendix contains the detailed calculation of the stress intensity factor for the crack in Fig.2b.

The basic variables are $a/b = 0.9$, $s = 1$ and $h/b = 1/12$ from which $h/a = h/b \times b/a = 0.093$.

Using equation (14a) gives $\lambda = (2/s)(a/b) = 1.8$ and Q_0 obtained from Fig.6 for $h/a = 0.093$ and $\lambda = 1.8$ is given by

$$Q_0 = 0.686 \quad . \tag{A-1}$$

If, for the nth stiffener b' is written b'_n then equation (5) becomes

$$b'_n - a' = b_n - a$$

which with equation (4) gives

$$b'_n = b_n - a + Q_0^2 a$$

and hence

$$\frac{a'}{b'_n} = \frac{Q_0^2\left(\frac{a}{b}\right)}{|n| + \left(Q_0^2 - 1\right)\frac{a}{b}} \quad . \tag{A-2}$$

For the right-hand crack tip in Fig.2b the effect of the first pair of stiffeners ($n = \pm 1$) is obtained as follows. Substitute a'/b'_1 , determined from equation (A-2) with $n = 1$, for a/d and λ' for λ in Fig.4 to obtain the value of Q'_{+1} which is found to be

$$Q'_{+1} = 0.856 \quad . \tag{A-3}$$

Substitute a'/b'_{-1} , determined from equation (A-2) with $n = -1$, for a/d and λ' for λ in Fig.5 to obtain the value of Q'_{-1} which is found to be

$$Q'_{-1} = 0.962 \quad . \tag{A-4}$$

Repeating this procedure for the next pair of stiffeners $n = \pm 2$ gives

$$Q'_{+2} = 0.995$$

and (A-5)

$$Q'_{-2} = 0.997 \quad .$$

Contributions from further pairs of stiffeners $|n| > 2$ are negligible. Hence the normalized stress intensity factor, given by equation (15) becomes

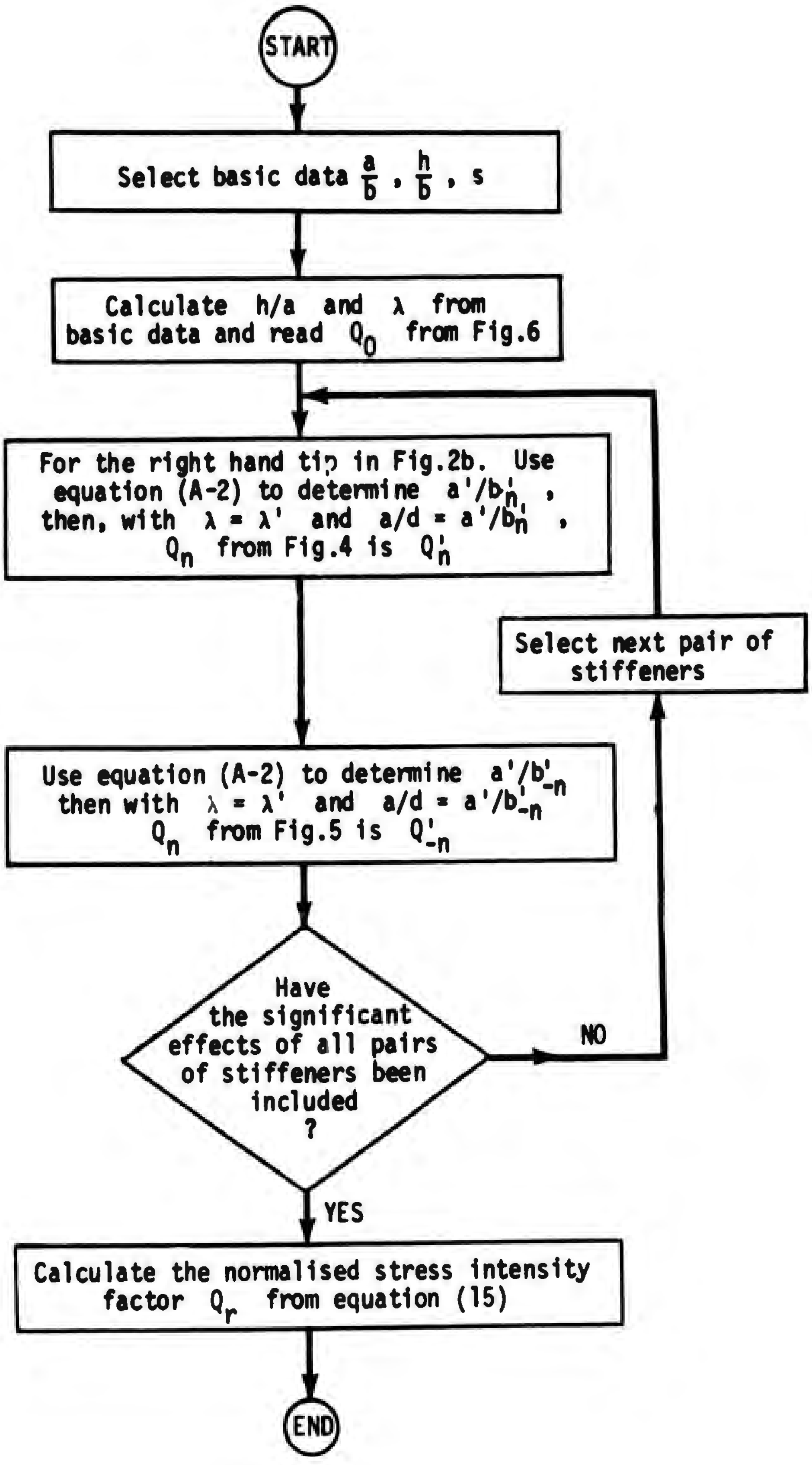
$$Q_r = Q_0 \left[Q'_{+1} + Q'_{-1} + Q'_{+2} + Q'_{-2} - 3 \right]$$

which from equations (A-1) to (A-5) gives

$$Q_r = 0.56 \tag{A-6}$$

as given in Table 2.

A flow chart of the above sample calculation is shown.



Procedure of a sample calculation using the compounding method

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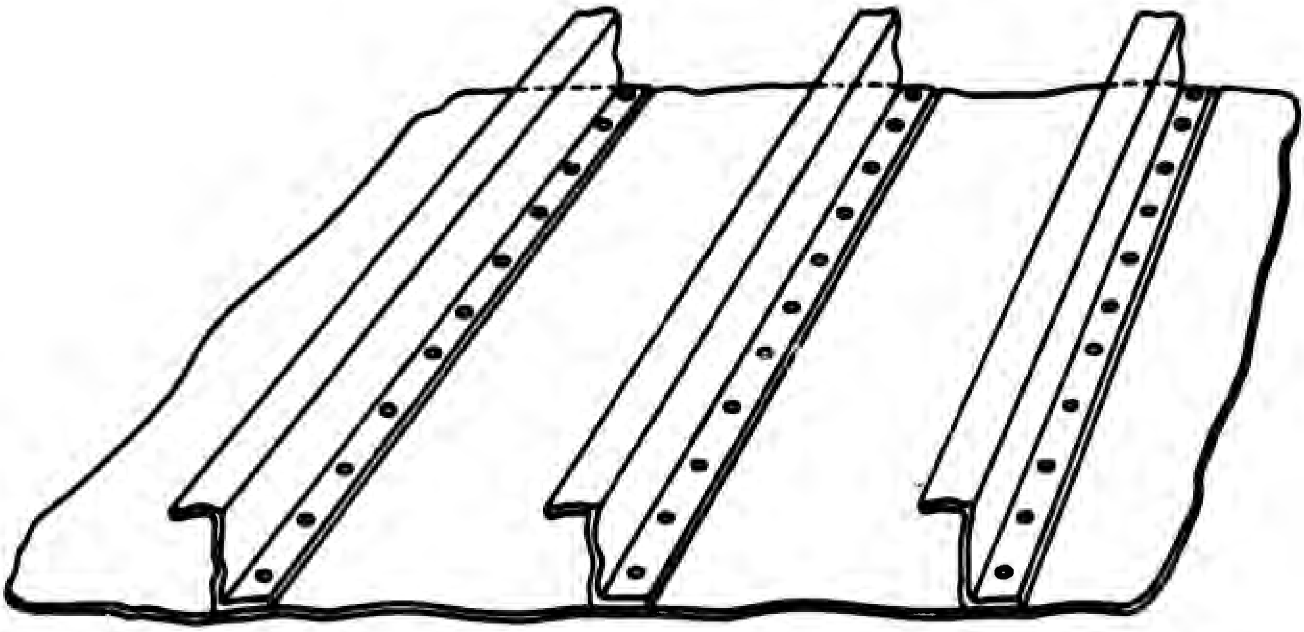

Fig.1a A typical stiffened aircraft structure

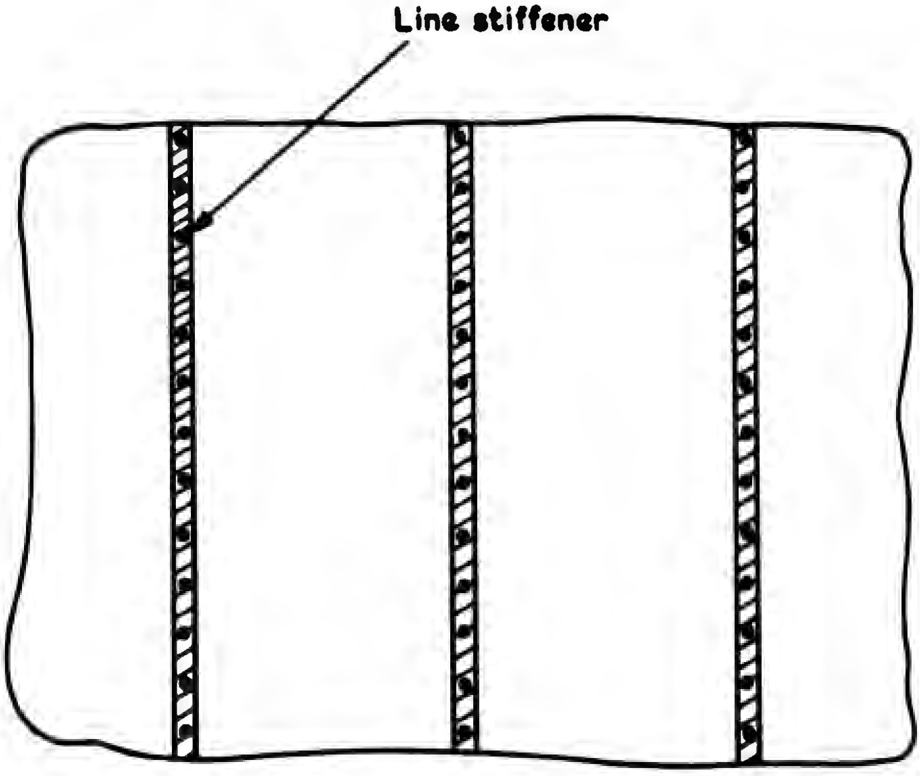

Fig.1b A model stiffened sheet

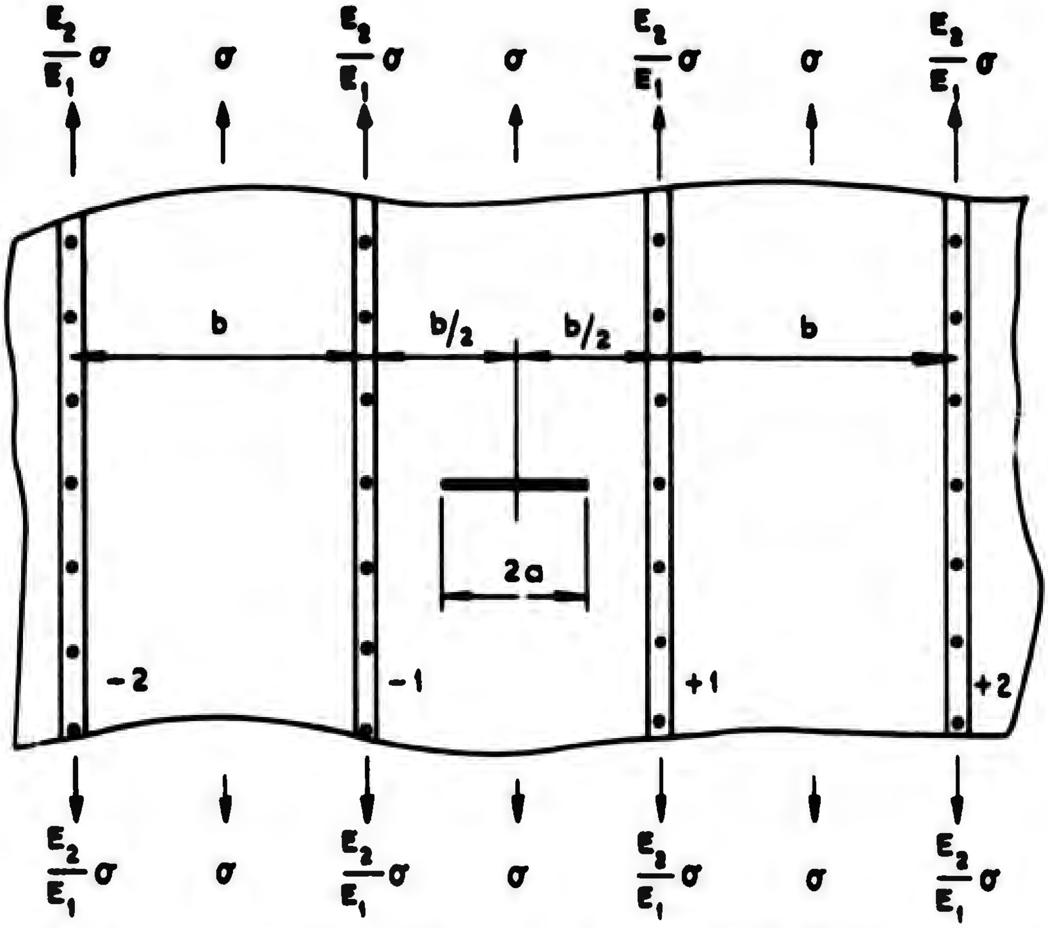

Fig. 2a Crack located symmetrically between two of the riveted stiffeners in a periodically stiffened sheet

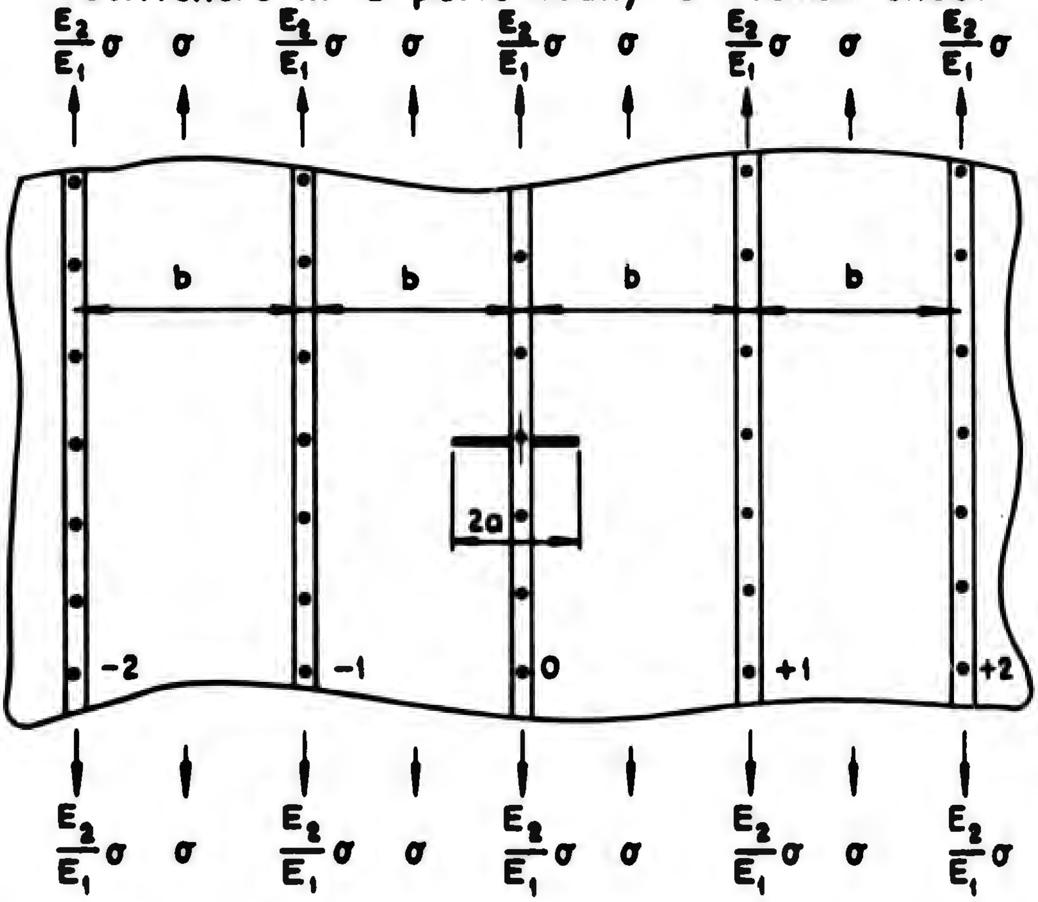

Fig. 2b Crack located symmetrically about one of the riveted stiffeners in a periodically stiffened sheet

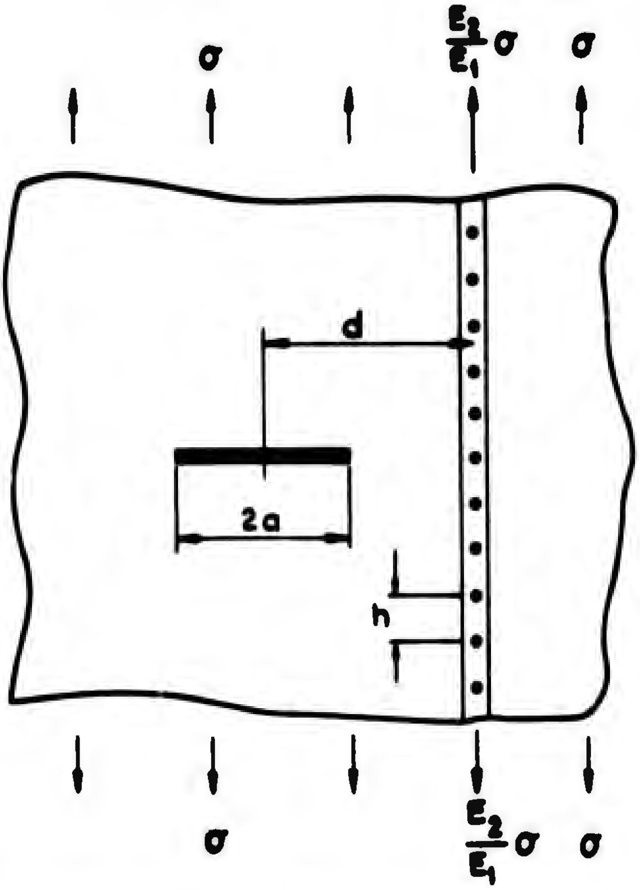

Fig. 3a Crack near a single riveted stiffener (ancillary configuration for 2a and 2b)

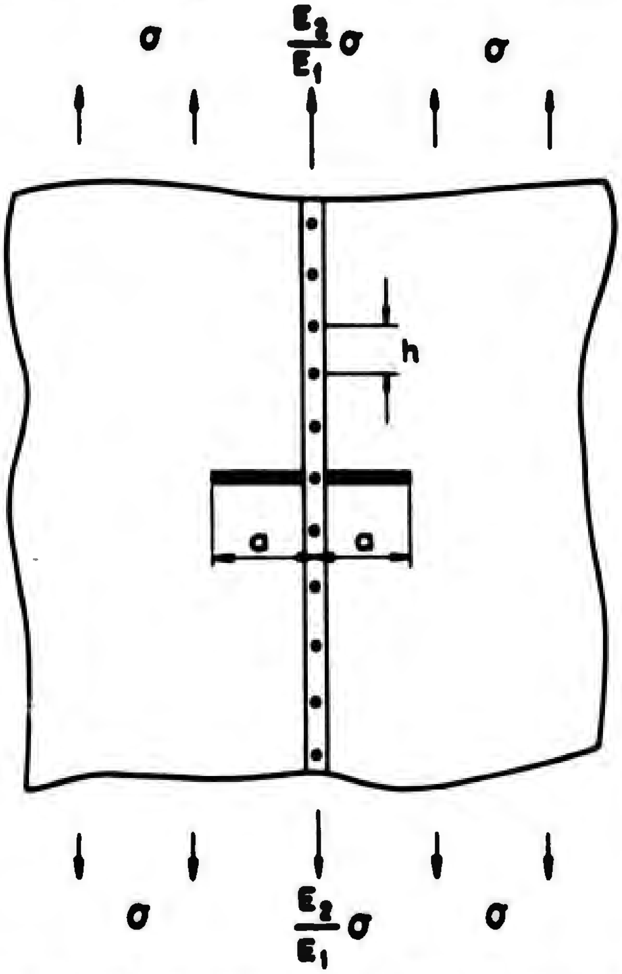

Fig. 3b Crack located symmetrically about a single riveted stiffener (ancillary configuration for 2 b)

Fig.4

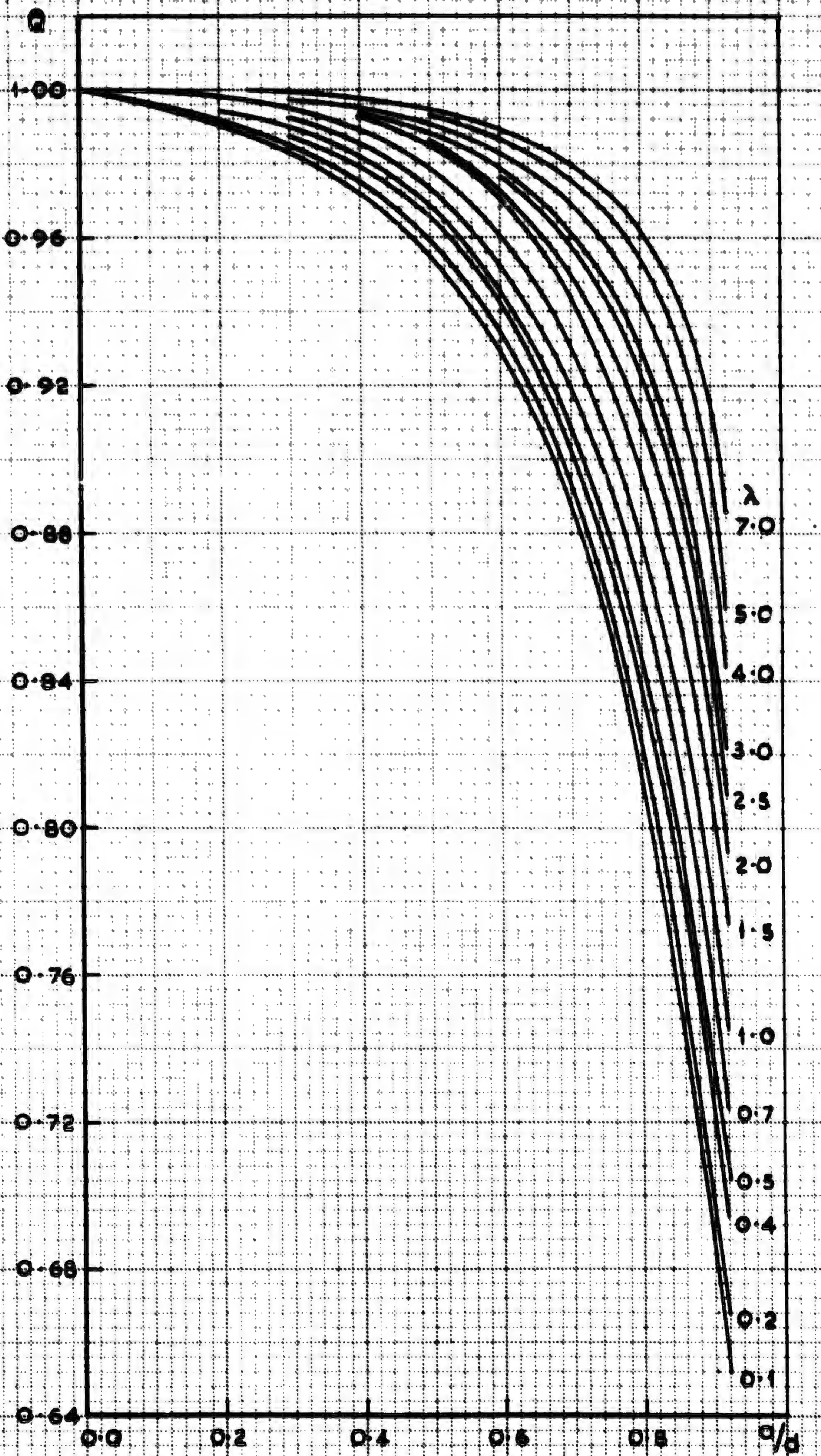

Fig.4 Normalised stress intensity factor for a crack near a continuously attached stiffener (near tip)

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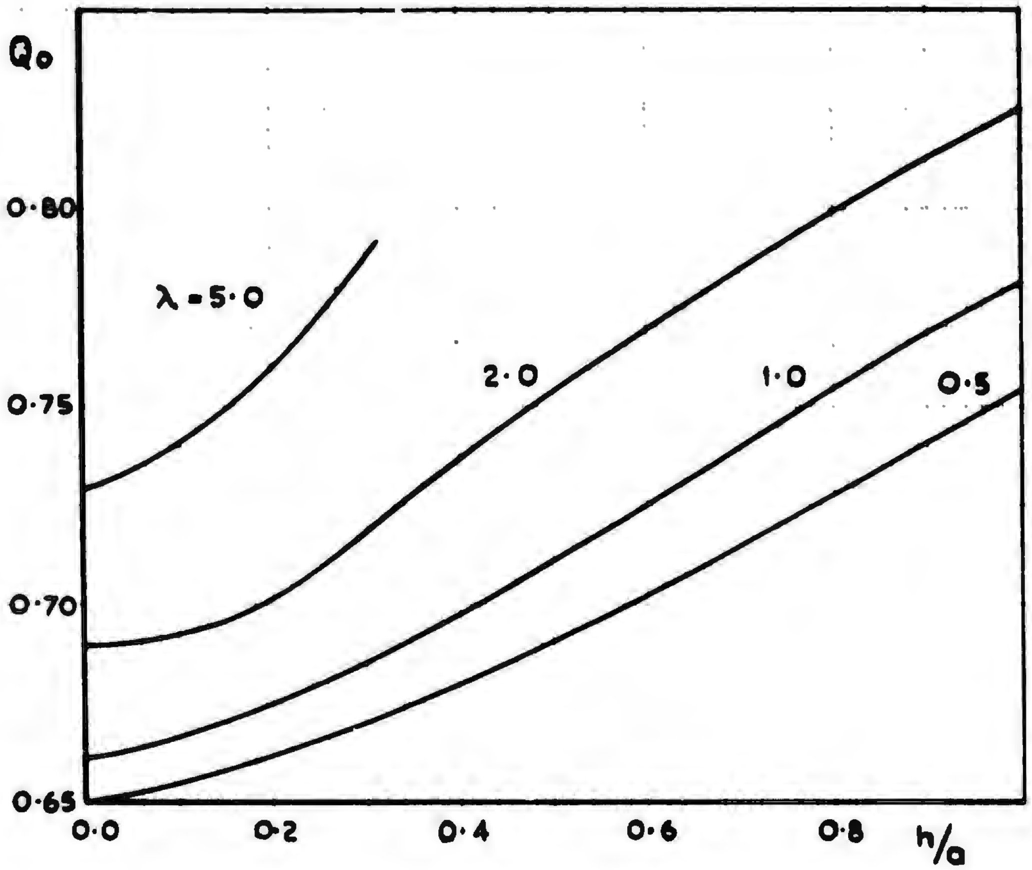

Fig.6 Normalised stress intensity factor for a crack symmetrically located about an unbroken riveted stiffener

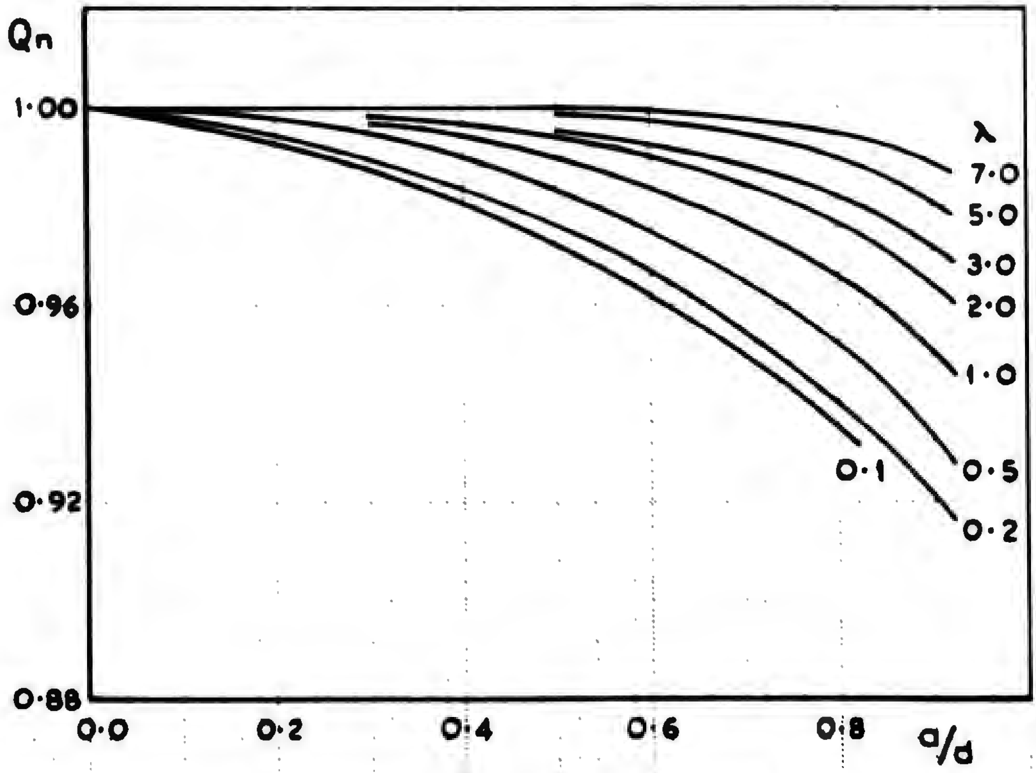

Fig.5 Normalised stress intensity factor for a crack near a continuously attached stiffener (far tip)

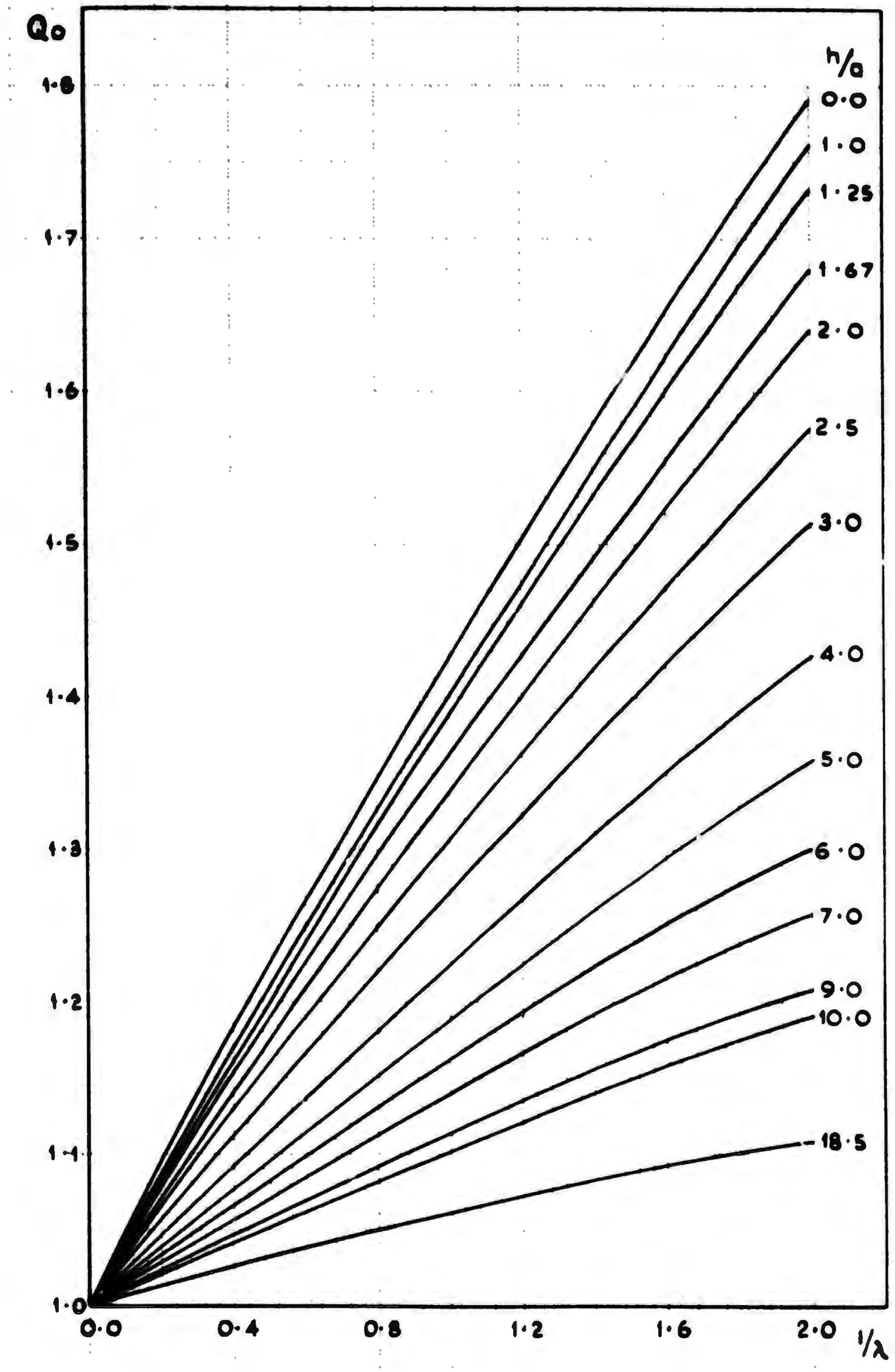

Fig.7 Normalised stress intensity factor for a crack symmetrically located about a broken riveted stiffener

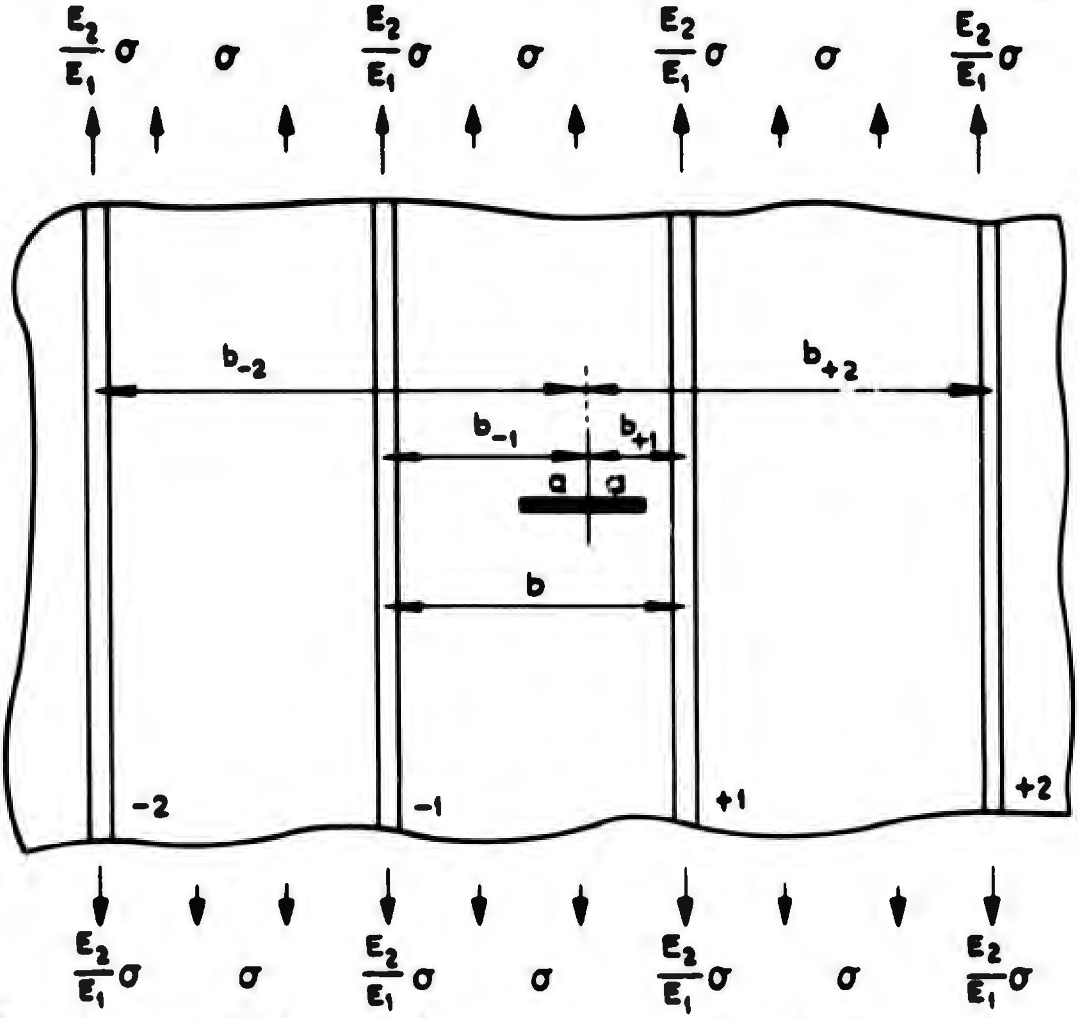

Fig. 8a A crack located asymmetrically between two of the stiffeners in a periodically stiffened sheet

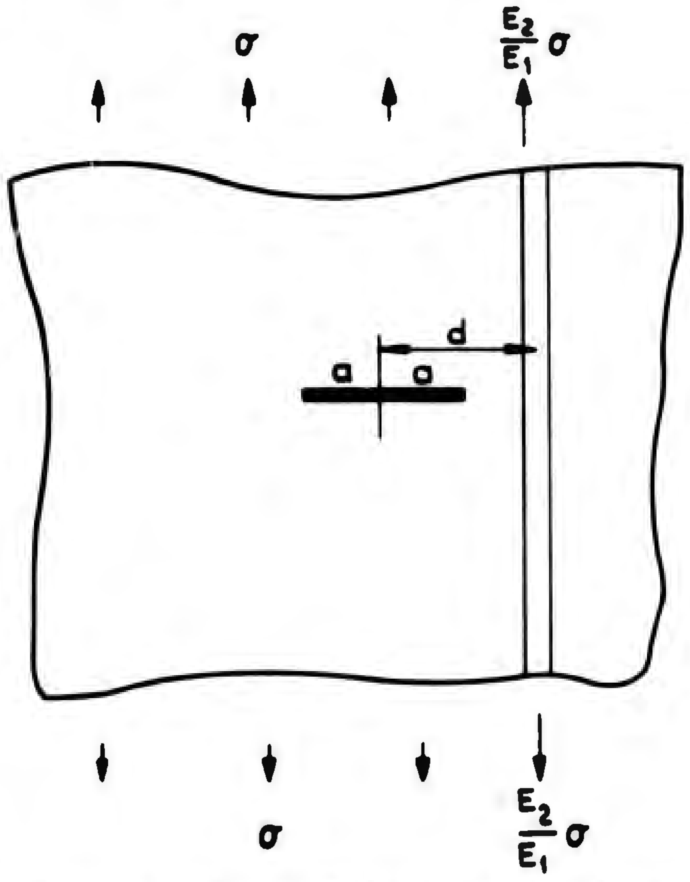

Fig. 8b The ancillary configuration for a crack near a stiffener

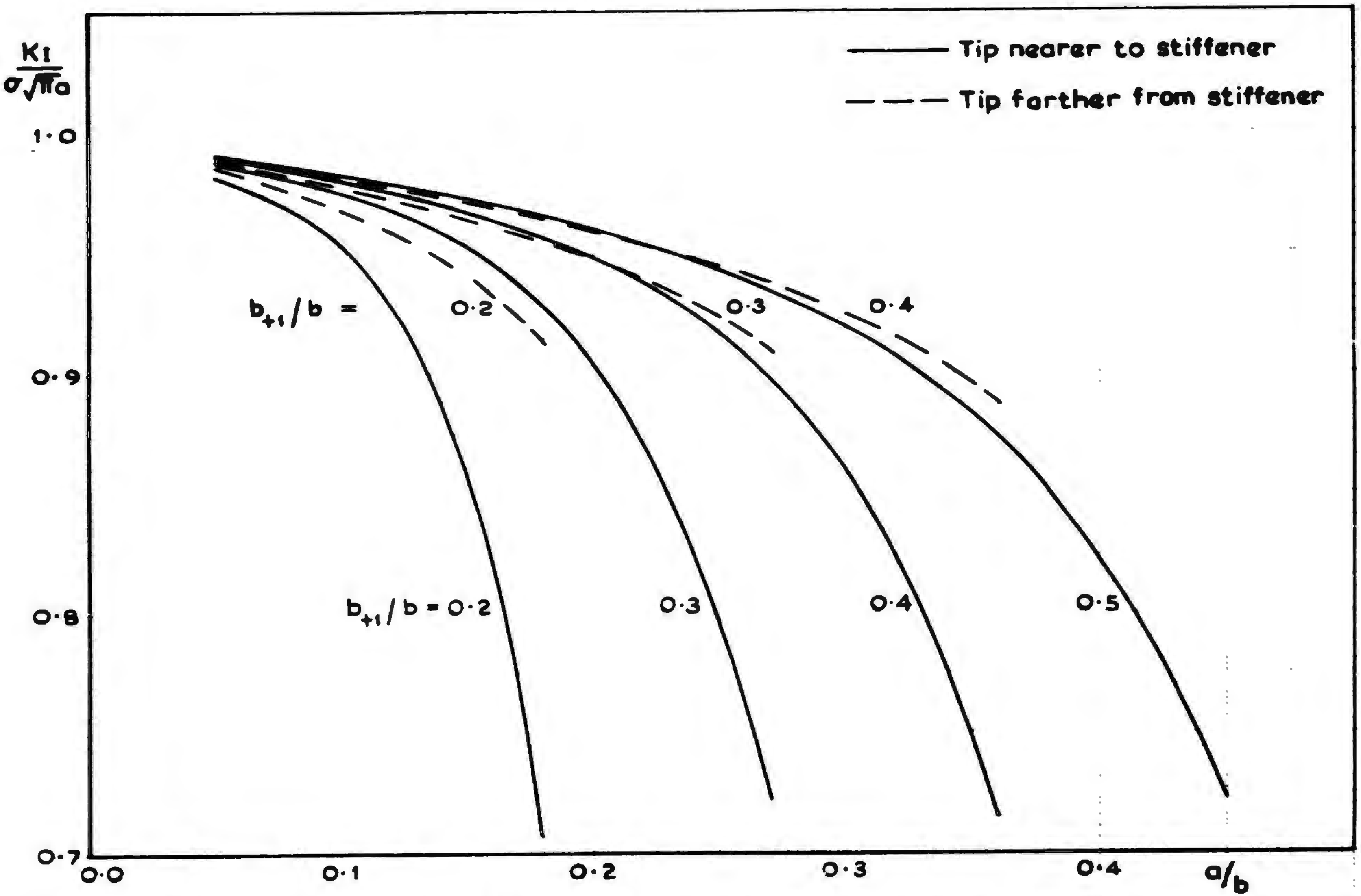

Fig.9 Normalized stress intensity factors for a crack located asymmetrically between two of the stiffeners in a periodically stiffened sheet

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Cracks*. Stress intensity factor. Stiffeners.

17. Abstract

The compounding technique of obtaining approximate solutions for stress intensity factors is applied to periodically stiffened sheets containing cracks. The basic compounding method requires a simple modification if the stiffener actually crosses the crack. Probable errors in compounded results are shown to be small (a few per cent) by comparing some compounded results with known results. The stress intensity factor is calculated for a crack located asymmetrically between stiffeners.